Gamma discounters are short-termist

Conference on

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Motivation

- Should one use a smaller rate to discount more distant benefits?
- Simple answer by Weitzman (1998, 2001): Yes, because future interest rates are uncertain.
- 1400 cites on Google Scholar…
- Norway, UK, France have used the argument for public policy evaluation.
- In this paper, I examine the economic foundations of this argument, which has been developed in a theoretical vacuum.

Gamma Discounting: Weitzman (AER 2001)

- Consider a simple safe project with
  - an initial cost *C*,
  - a future benefit *F* occurring at date *t*.
- If the interest rate is *r*, one can transfer the future benefit to today by a loan of *Fexp(-rt)*, yielding a net benefit

$$NPV = -C + F\exp(-rt)$$

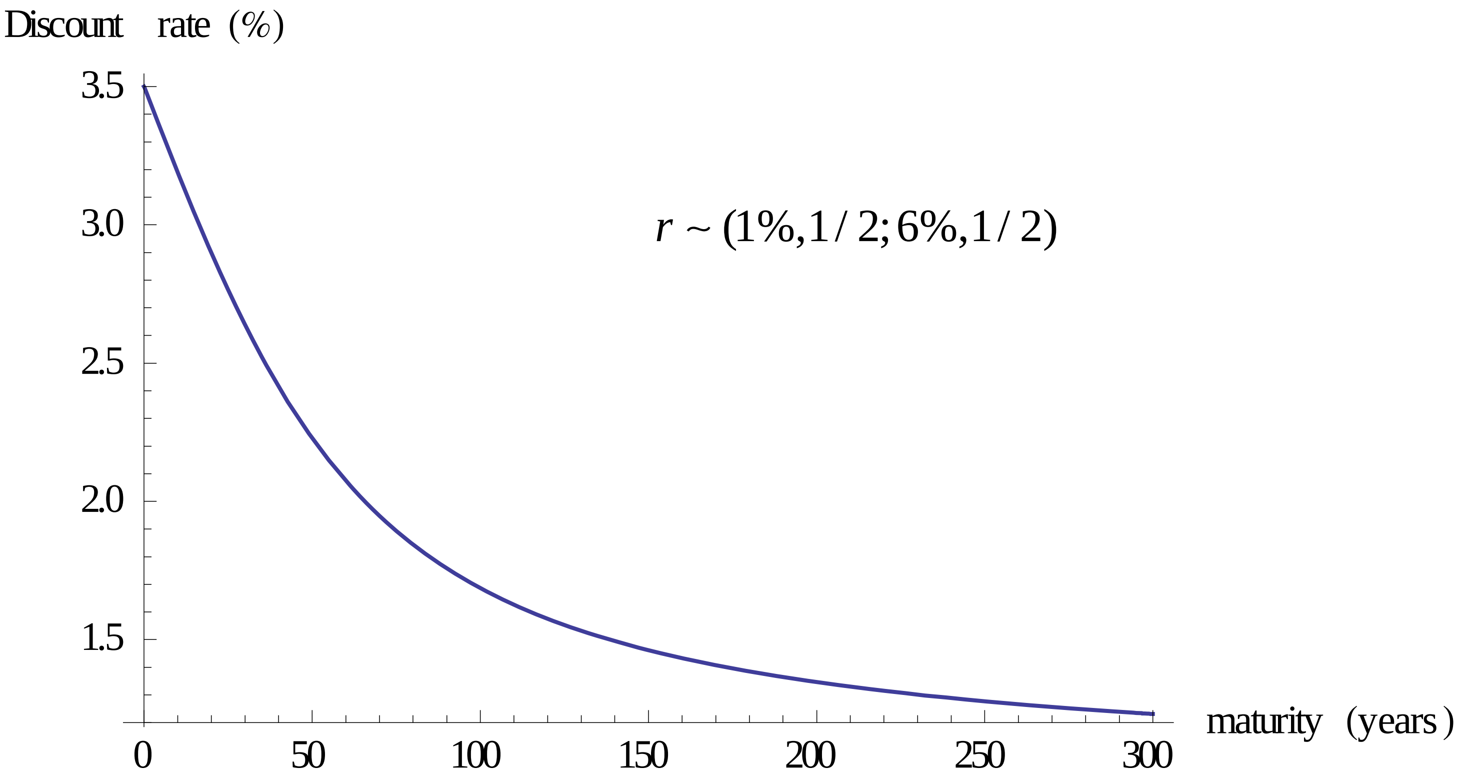
- Invest if NPV is positive.
- Suppose now that *r* is uncertain. A risk-neutral DM should invest if the expected NPV is positive, i.e., if the NPV is positive using a certainty equivalent DR $r^W_{0\to t}$ defined by

$$\exp(-r^W_{0\to t}t) = E\exp(-rt)$$

Gamma Discounting: Example

$$\exp(-r^W_{0\to t} t) = E\exp(-rt)$$

- $r^W_{0\to t}$ is decreasing and tends to the smallest possible r.
- Weitzman (2001) proposes a gamma distribution for r, hence the terminology.



But what if…

- If the interest rate is r, one can transfer the current cost to the terminal date t by a loan of C, yielding a net benefit

$$NFV = -C\exp(rt) + F$$

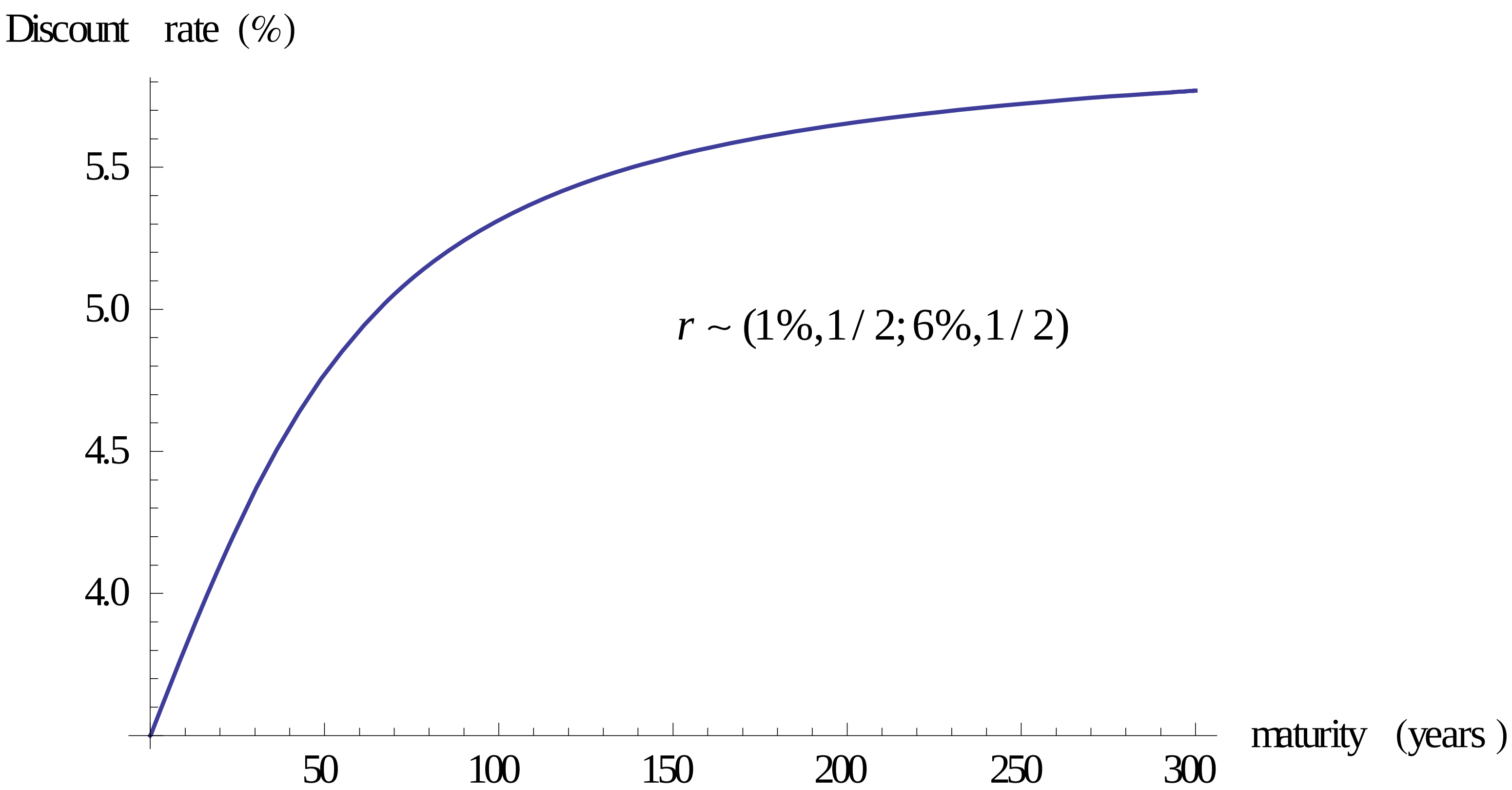
- Invest if NFV is positive.
- Gollier (2004): Suppose now that r is uncertain. A risk-neutral DM should invest if the expected NFV is positive, i.e., if the NPV is positive using a certainty equivalent DR $r^G_{0\to t}$ defined by

$$\exp(r^G_{0\to t}t) = E\exp(rt)$$

Expected NFV: Example

$$\exp(r^G_{0\to t} t) = E \exp(rt)$$

- $r^G_{0\to t}$ is increasing and tends to the largest possible r.



Weitzman-Gollier puzzle

- Lack of an economic foundation for gamma discounting, and for DDR.
- Under risk neutrality, interest rates are constant (and equal to the rate of impatience). We must add risk aversion into the picture.
- Unsolved controversy: Pazner and Razin (1975), Newell and Pizer (2003), Hepburn and Groom (2007), Groom, Koundouri, Panopoulou and Pantelidis (2007), Gollier, Koundouri and Pantelidis (2008), Buchholz and Schumacher (2008), Freeman (2010), Freeman and Groom (2010), Weitzman and Gollier (2010), Weitzman (2010), Traeger (2013), Arrow et alii (2013 a,b), Szekeres (2013), Heal and Millner (2013).

Lucas tree economy

$$W_\tau = \sum_{t=\tau} e^{-\delta(t-\tau)} E_\tau u(c_t)$$

- WLOG, we assume $\delta = 0$.
- Consistent with uncertainties on growth and on interest rates.
- Social discount rate and equilibrium interest rates:

« Price equals MRS »

$$r_{\tau\to t} = \frac{-1}{t-\tau} \ln \frac{E_\tau u'(c_t)}{u'(c_\tau)}$$

Why discounting?

$$r_{\tau \to t} = \frac{-1}{t-\tau} \ln \frac{E_\tau u'(c_t)}{u'(c_\tau)}$$

- If $u'(c)=c^{-\gamma}$ and c_t is a geometric brownian process, then

$$r_{\tau \to t} = \gamma\mu - 0.5\gamma^2\sigma^2$$

In a growing economy, investing has the adverse effect to raise intetemporal inequalities

Precautionary effect

The gamma discount problem with two periods

$t=0 \qquad\qquad\qquad\qquad t=1 \qquad\qquad\qquad\qquad t=2$

$c_0 \qquad\qquad\qquad\qquad\quad c_1 \qquad\qquad\qquad\qquad\quad c_2$

- Long rate today:

$$e^{-2r_{0\to 2}} = \frac{Eu'(c_2)}{u'(c_0)}$$

- Spot short rates:

$$e^{-r_{0\to 1}} = \frac{Eu'(c_1)}{u'(c_0)} \qquad e^{-r_{1\to 2}} = \frac{E_1 u'(c_2)}{u'(c_1)}$$

- Can one recover the efficient long rate from the knowledge of the distribution of the spot rates alone?
- Gamma discounting:

$$\exp(-2r^W_{0\to 2}) = E\exp(-r_{0\to 1} - r_{1\to 2})$$

$$r^W_{0\to 2} = r_{0\to 2}\,?$$

Equivalence results

- **Proposition 1**: *If the representative agent is a discounted expected utility maximizer, there are three equivalent ways to define the efficient long discount rate:*

$$
\begin{aligned}
e^{-2r_{0\to 2}} &= \frac{Eu'(c_2)}{u'(c_0)} \\
&= \left(E\left[\frac{u'(c_2)}{Eu'(c_2)} e^{r_{0\to 1}+r_{1\to 2}} \right] \right)^{-1} \\
&= E\left[\frac{u'(c_1)}{Eu'(c_1)} e^{-r_{0\to 1}-r_{1\to 2}} \right].
\end{aligned}
$$

- These characterizations fail to attain the objective envisioned by Weitzman to fully characterize the price of long-dated safe assets from the distribution of future spot interest rates alone.

Possible bridge between the efficient rates and the gamma rates

$$e^{-2r_{0\to 2}} = E\left[\frac{u'(c_1)}{Eu'(c_1)} e^{-r_{0\to 1} - r_{1\to 2}}\right].$$

$$e^{-2r^W_{0\to 2}} = Ee^{-r_{0\to 1} - r_{1\to 2}}$$

- Risk neutrality: $r^W_{0\to 2} = r_{0\to 2} = r_{0\to 1} = r_{1\to 2} = 0.$
- Early resolution of uncertainty (initial version)

Gamma discounters are short termist

- **Proposition 2:** *Suppose that the future consumption level* c_1 *and the future spot rate* $r_{1\to 2}$ *are positively correlated (PQD). Then, the gamma discount rate* $r^W_{0\to 2}$ *defined is <u>larger</u> than the efficient discount rate* $r_{0\to 2}$.

- Intuition: The NPV borrowing strategy has a negative beta.
  - Weitzman fails to recognize this hedging benefit of the ENPV valuation strategy.

Decreasing discount rates

- **Proposition 3:** *Suppose that growth rates are positively correlated (PQD) and that relative prudence is uniformly larger than unity. Then the efficient long discount rates have a decreasing term structure.*

- Intuition: Magnification of LT risk => Increase the intensity of the precautionary motive to invest for the LT.

Example

$$u'(c) = c^{-\gamma}$$

$$x_\tau = \ln c_\tau / c_{\tau-1}$$

$$x_1, x_2, \ldots \; i.i.d. \; N(\theta, \sigma^2)$$

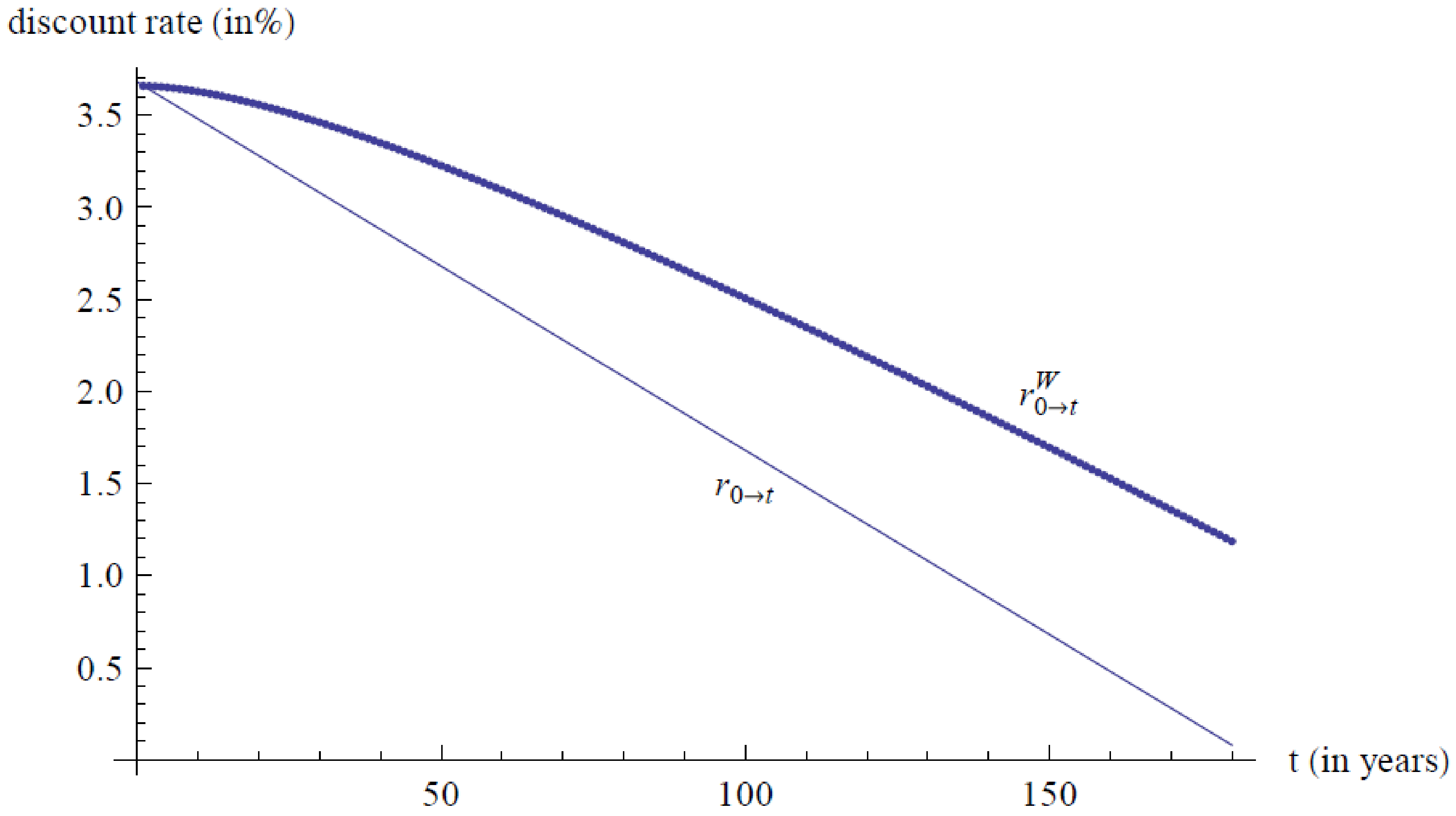
$$\theta \sim N(\mu, \sigma_0^2)$$

$$\gamma = 2$$

$$\sigma = 4\%$$

$$\mu = 2\%$$

$$\sigma_0 = 1\%$$



Conclusion

- The gamma discounting rule used in France, Norway and the UK to impose DDR has no economic foundation.

- Weitzman is "*mostly right for the wrong reasons*".