SELLING DREAMS: ENTREPRENEUR OPTIMISM AND COLLATERAL USE IN FINANCIAL CONTRACTING

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Abstract. Entrepreneurs seeking credit are prone to understating the risks they face and adopt too optimistic a view of their future prospects in order to reduce the anxiety involved with high-stakes, long-term ventures. This tendency exposes them to predatory lending practices that are detrimental in material terms, potentially justifying borrower protection policies on paternalistic grounds. We model borrowers overoptimism in credit markets where strategic profit-maximizing lenders design contracts that shape borrowers' incentives to appraise their projects. Borrowers are wishful thinkers and optimally bias their perceived probability of having a successful project in light of the anticipatory utility benefits of being overoptimistic and the cost of overoptimism that arises from impaired decision-making. In a monopolistic setting, we predict that loan terms are predatory and harm deluded borrowers in material terms when the borrower's risk is observable. When the risk is not observed, incorporating wishful thinking modifies the predictions of classical screening models: separating risks requires giving up a larger rent to high-risk borrowers and, consistent with empirical evidence, lenders may relinquish attempts to separate types and offer pooling contracts that feature positive collateral requirements and overoptimism. The case for paternalistic protection does not hold: the behavioral trait of borrowers increases their material payoff rather and does not expose them to predation. In competitive markets, the tendency towards optimism benefits high-risk borrowers and exacerbates cross-subsidization from low- to high-risks, and can also lead to pooling equilibria. In line with US lending market conditions before the financial crisis, competitive markets are more likely to give rise to overoptimism and collateralized loans to high-risk borrowers when credit is cheap and entrepreneurial profits are high. **Keywords**: optimal expectations, optimism, wishful thinking, financial contracting, asymmetric information, common values.

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1. Introduction

People like being optimistic because they savor the anticipation of future riches and because it helps them to be less anxious, especially about long-term, high-stakes ventures in which the outcome is uncertain but the sleepless nights spent worrying are not. However, when it distorts an individual's decisions and exposes her to being taken advantage of in the marketplace, optimism is associated with real costs. Projecting confidence is harmless, but genuinely underestimating the risk of adverse contingencies leads to undervaluing insurance and to taking too positive a view of apparent market opportunities that are truly detrimental.

Positively biased beliefs and the potential for disadvantageous transactions are particularly acute in credit markets. Survey evidence suggests that US households systematically overestimate their own credit rating and downplay the risk of bankruptcy.[1] At the same time, predatory lenders issue loans that seek to exploit consumer optimism and naiveté.[2] In particular, an optimistic borrower tends to be overly willing to pledge collateral and can thus be sold a profitable loan with a high likelihood of default and equity extraction. United States government agencies, such as the Federal Trade Commission, the Federal Deposit Insurance Corporation and General Accountability Office,[3] recognize that some borrowers accept loans that are detrimental to their material welfare, and these agencies thus increasingly state a paternalistic case for consumer protection.

Our objective in this paper is to formalize and evaluate the case for consumer protection in lending markets. We do so by considering a model where borrowers have malleable beliefs about how likely they are to be able to repay loans. We assume that borrowers derive anticipatory utility from the expectation of future payoffs and, following Brunnermeier and Parker (2005), we assume that borrowers choose beliefs that optimally trade off these affective benefits from optimism with the material costs that arise from agreeing to contractual terms that appear advantageous, but are detrimental in light of the true risk. We let borrowers interact with lenders in a standard model of project financing under asymmetric information about the risk of borrowers' projects and derive the qualitative and quantitative impact of malleable beliefs on equilibrium contract design. We thus provide a theory of when and why overoptimism arises in lending markets[4] and derive conditions under which this overoptimism results in worse material outcomes for borrowers and inefficient lending arrangements.

Our main result is that in both monopolistic and competitive contexts, lenders constrained by adverse selection design contracts that induce borrowers to appraise their projects realistically as long anticipatory concerns are limited. Borrowers are not overoptimistic in equilibrium and, far

[1]See evidence from the Freddie Mac Consumer Credit Survey cited in Perry (2008)

[2]See Morgan (2007), Bond et al. (2009) and the large legal literature referenced therein.

[3]See FTC (2000); GAO (2004); FDIC (2005)

[4]While overoptimistic beliefs about risks and future prospects are a commonly observed phenomenon that has been documented among experimental subjects (e.g. Babcock and Loewenstein, 1997; Camerer and Lovallo, 1999 and Mayraz, 2011) entrepreneurs (e.g. Arabsheibani et al., 2000 and Landier and Thesmar, 2009), CEOs (Malmendier and Tate, 2005) and students (Kaniel et al., 2010), they are by no means ubiquitous. It is this important to identify the contexts in which overoptimism arises and predation is a potential concern.

from exposing them to predation, their focus on *anticipatory* payoffs actually contributes to raising their equilibrium *material* payoff. If borrower attach more importance to anticipatory concerns, lenders give up on inducing realism and issue single contract offers that induce overoptimism on behalf of high-risk borrowers. Such pooling offers feature positive amounts of collateral and correspond to inefficient speculative trade, through which lenders exploit the ex-post differences in beliefs. Even in this case, despite their inaccurate valuation of collateral, high-risk borrowers still earn larger rents than they would, absent the behavioral bias. Therefore, as long as asymmetric information about the risk of borrowers' projects prevails, the informational contraints faced by lenders are sufficient protection for borrowers and paternalistic intervention is not justified. Malleable beliefs actually improve allocative efficiency, even though monopoly lenders receive less profit and low-risk borrowers face a higher cross-subsidy uner competition.

Our model involves entrepreneurs who seek a loan to finance an investment project, such as a risky business venture. Borrowers are heterogeneous with respect to the risk of their project and are endowed with partially liquid assets that may be pledged as collateral. Lenders have access to liquid funds and offer contracts that specify a repayment if the borrower's project succeeds and a transfer of collateral in case of failure. Collateral is a strictly less efficient way of transferring resources but is a valid screening instrument. The behavioral friction comes in the form of anticipatory utility from the expectation of future payoffs and from the borrowers' ability to bias their belief about the probability of success in an effort to inflate these anticipatory benefits. Borrowers trade off the cost arising from impaired decision making with the affective benefits from overoptimism, and choose the belief that maximizes a weighted sum of anticipatory and material payoffs, i.e. they form *optimal expectations*. The weight placed on anticipatory utility concerns therefore parameterizes the extent of the behavioral bias and can be set to zero, yielding standard preferences as a natural benchmark.

Lenders are rational profit-maximizers and are aware of the borrowers' bias. They consider the costs and benefit of inducing realism or delusion on behalf of borrowers and select the profit-maximizing option, subject to the constraints given by their information and their market power. Whether lenders wish to induce realism or not results from two conflicting considerations. On one hand, overoptimism both generates a psychological benefit for borrowers and distorts the rate at which exchanging collateral for repayment is deemed acceptable, so the malleability of borrowers' beliefs is potentially a profit opportunity for lenders. On the other hand, overoptimism also creates an *entitlement effect*, whereby the borrower reassesses her outside option upwards and effectively commits to refuse offers that a realistic borrower would deem acceptable, thus depressing lenders' profits. The magnitude of this entitlement effect depends crucially on the difference between the outside options of each borrower type, which are exogenously given to a monopoly lender and are endogenous in competitive settings, as borrowers' outside options are generated by the contract offers of competitors.

If the entitlement effect is dominant, lenders wish to induce realistic appraisals on behalf of borrowers. If the borrower's type is symmetrically known, this is achieved at no cost through the use of latent contracts (not selected along the equilibrium path by a realistic borrower) with large amounts of collateral and low repayment rates that act as a deterrent. However, asymmetric information prevents the use of such latent contracts: the contract that high-risk borrowers would take up if they deluded themselves must be the same contract that low-risk borrowers accept in equilibrium. Therefore, a lender cannot treat the problem of inducing realism for high-risk borrowers separately from his dealings with low-risk borrowers. We start by establishing conditions under which our model exhibits a version of predatory lending. This occurs when a monopoly lender either knows the borrower's type (symmetric information) or faces advantageous selection because low-risk and high-risk borrowers have similar outside options. The lender is then able to manage the high-risk borrower's cognitive incentives without distorting the contract intended for low-risk borrowers and as a result, the optimal offer induces delusion and leaves high-risk borrowers worse off in material terms. Collateral requirements, which are not used when borrowers have standard preferences, are then used solely as an extractive device, as overoptimistic borrowers underestimate the cost of pledging collateral. High-risk borrowers effectively buy dreams, adopt inaccurate beliefs, and accept contracts which deliver worse outcomes than their outside option. Under advantageous selection, if anticipatory concerns are large enough, this extraction motive leads the lender to demand collateral from high-risk and low-risk borrowers alike. This causes inefficiencies but does not harm low-risk borrowers, who never earn rents in this context.

The mechanisms of such predatory lending practices rely crucially on the lenders' ability to identify the type of borrower they are facing. However, a more realistic assumption is that borrowers possess superior information about the risk associated to their project. Lenders may well possess superior information regarding aggregate investment conditions and aggregate risk, but they stand at a disadvantage regarding the idiosyncratic component of risk, which is the crucial element that enables discrimination between lenders. We thus move on to the more realistic setting of asymmetric information, and therefore lenders cannot condition their offer on the type of borrower faced. We show that predatory lending behavior does not follow from the malleability of borrowers' beliefs and that our findings overturn the received wisdom of paternalistic protection.

Our central result pertains to a monopoly lender facing adverse selection. We first recall the classical results of Bester (1985) and Besanko and Thakor (1987): under standard preferences, borrower types are separated in the optimal offer and collateral requirements perform a screening function. However, since the borrower may delude herself, a lender seeking to separate borrowers needs to ensure not only incentive compatibility, but also realism on behalf of high-risk types. We show (section 4.2.1) that inducing realism is a strictly more demanding requirement and that incentive compatibility is redundant. As the weight placed on anticipatory utility concerns increases, the efficiency of collateral as a device for separating borrowers decreases and the rent necessary to induce separation increases, with negative consequences for the lender's profit.

Building on this result, we then characterize optimal contract design. The first part of proposition 4 states that when the weight placed on anticipatory utility is low, the lender induces high-risk borrowers to appraise their projects realistically. Borrowers are separated and exhibit no overoptimism in equilibrium; and there is no qualitative difference with respect to the screening menu that obtains under standard preferences: low-risk borrowers pledge collateral in return for a smaller repayment and high-risk borrowers receive a rent. Quantitatively, with respect to the standard preferences benchmark, the lender optimally chooses lower-powered separation incentives (low-risk borrowers pledge less collateral) and gives up a larger rent to high-risk borrowers. The lender's profit decreases with the weight of anticipatory utility even though the borrowers' propensity towards optimism is not activated in equilibrium.

As the weight of anticipatory concerns become large, the rent that needs to be surrendered to high-risk borrowers to keep them realistic eventually erodes the lender's profit to the point that the profit-maximizing contract design involves inducing high-risk borrowers to be overoptimistic. This equilibrium (second part of proposition 4) differs qualitatively from the equilibrium that obtains under standard preferences and features a single pooling offer with positive collateral. In the pooling equilibrium, collateral stops functioning as a screening device and instead becomes the support for speculative bets on the probability of failure, through which the lender exploits the difference in beliefs. The lender's gains from speculative trade still fall short of compensating the loss incurred by giving up on separating borrower types and the lender obtains strictly less profit than if he was facing borrowers with non-malleable beliefs. Finally, the equilibrium outcome and the extent of the behavioral friction have no impact on the equilibrium payoff of low-risk borrowers.

Section 5 analyzes competitive screening. In a competitive market for loans, outside options are derived from the equilibrium offers of competitors, ensuring that adverse selection holds. Lenders face the same challenge to inducing realism as in the monopoly case, and must similarly allow for an extra informational rent if high-risk borrowers are to be separated. However, dissipation of total profits ensures that this rent must come at the expense of low-risk borrowers rather than from a reduction in profit. We show (Proposition 7) that when the weight borrowers place on anticipatory utility is low, lenders induce realism on behalf of high-risk borrowers in equilibrium. The additional rent required for separation with respect to the benchmark comes at the expense of low-risk borrowers, who fund a cross-subsidy which increases with the weight of anticipatory utility concerns. When the weight of anticipatory utility concerns becomes large enough, the competitive equilibrium features a pooling contract with positive collateral. High-risk borrowers benefit from their bias both in material and anticipatory terms, in the sense of securing a larger cross-subsidy than with standard preferences. However, their equilibrium loans feature a positive amount of collateral which is justified only by the ex-post difference in beliefs and is therefore wasteful.

The main implication of our results is that informational constraints faced by lenders are enough to protect borrowers from the negative consequences of their malleable beliefs. Even though our model features no direct cost of delusion and further assumes that the borrower's cognitive bias is commonly known and amenable to exploitation by lenders, the latter do not benefit from the borrowers' bias as long as adverse selection prevails. We show that competition in the credit market is sufficient but not necessary for borrowers to be protected: it is because competitive equilibria feature adverse selection that the behavioral bias does not translate into negative material payoffs.

Both in monopolistic and competitive environments, the joint hypothesis of malleable beliefs and asymmetric information plays a crucial role: in a symmetric information context, contract menus can be tailored to the borrower's type and latent "threat contracts", which are never picked up on the equilibrium path, can induce realism at no cost to the lender. Under asymmetric information, lenders cannot distinguish between optimistic high-risk borrowers and realistic low-risk borrowers, and must therefore offer the same menu to both. The contract taken up by a deluded high-risk borrower is the low-risk borrowers' equilibrium contract and therefore distorting it induces a real cost whenever adverse selection prevails.

The pooling of risk types with positive collateral requirements that arises in both monopolistic and competitive markets with motivated beliefs is a novel prediction specific to this setting, as under standard preferences, borrower types are either screened or are pooled on contracts that specify no collateral transfer. Our finding aligns with empirical evidence which indicates that collateral use is prevalent across risk classes and not necessarily correlated with ex-ante measures of risk (Berger and Udell, 1990), even though the use of collateral is responsive to changes in ex-ante information asymmetry (Berger et al., 2011).

Because competition between lenders channels rents to the borrower, the risk-free rate and entrepreneurial profits affect equilibrium outcomes, particularly whether borrowing classes are pooled are separated. We find (proposition 8) that, fixing the weight placed on anticipatory utility concerns, equilibria with collateralization and overoptimism on behalf of high-risk borrowers are more likely to emerge in competitive markets when the risk-free rate or the opportunity cost of funds is low and the returns to entrepreneurial projects are high. The negative correlation between the risk-free rate and the likelihood of collateral use in loan contracts that is implied by our model is supported by findings in Jiménez et al. (2006).

Related Literature. Our paper contributes to a large and growing literature on consumers with behavioral biases that face strategic firms.[5] As in the model of Gabaix and Laibson (2006) and many of the models developed in Spiegler (2011), competition between firms in our model does little to discipline the behavioral bias of consumers. Instead, when competitive forces allow

[5]See Ellison (2006) for a review of some of the early papers in this literature.

borrowers to obtain larger rents, the dream of owning a low-risk project becomes even more desirable so deterring delusion becomes more costly.

Since our model treats beliefs as an endogenous outcome and not a primitive, whether or not a borrower ultimately deludes herself is determined in the equilibrium. Our approach is therefore distinct from previous work that studies optimal contract design with optimists and assumes that the level of overoptimism or overconfidence is fixed and exogenous to the model (Sandroni and Squintani, 2007; Landier and Thesmar, 2009; de la Rosa, 2011; Eliaz and Spiegler, 2008 and Spinnewijn, 2013). In Sandroni and Squintani (2007), for example, exogenous overoptimism leads to an insurance provider having no choice but to offer the same contract to overly optimistic high-risk agents and low-risk agents. In our case, whether or not the pooling of risk-types occurs in equilibrium is an endogenous outcome and depends on parameters. Our approach is also distinct from works that focus on the selection of agents in the market. In Simsek (2013), a belief disagreement is postulated and the key mechanism is one of market disciplining through endogenous borrowing constraints, while Manove and Padilla (1999) analyze the provision of credit at the intensive margin, with a focus on loan size and exclusion.

The idea that individuals may bias their beliefs in the service of psychological needs is not new in the economics literature. Brunnermeier and Parker (2005), Brunnermeier et al. (2007) and Bénabou (2013), like us, emphasize an anticipatory utility motive for biased beliefs, while Akerlof and Dickens (1982) stress a related motive of cognitive dissonance reduction. Carrillo and Mariotti (2000) and Bénabou and Tirole (2002) point to the motivational benefits of strategic ignorance and optimism. In Caplin and Leahy (2001), Caplin and Eliaz (2003), Kőszegi (2003) and Kőszegi (2006), agents' utility also directly depends on their beliefs, but beliefs are not motivated and actively chosen as in our model.

The role of contract design in shaping cognitive incentives is studied by Menichini et al. (2010) and Immordino et al. (2011), who explore the interplay between a rational principal and an agent with motivated beliefs in a moral hazard setting. Their focus on managerial incentives and moral hazard leads them to emphasize the selection of managers on the basis of cognitive traits and does not readily translate into insights relevant to lending markets.

Recent empirical work supports the notion that beliefs are motivated by affective benefits: Eil and Rao (2011), Mobius et al. (2011) and Mayraz (2011) uncover self-serving overoptimism and biased information processing in the economic decision making of experimental subjects. Kunda (1990) reviews a sizable psychology literature that provides evidence for motivated belief formation. Oster et al. (2013) study the health beliefs and economic behavior of people at risk of Huntington disease and find evidence for mental processes best described by the optimal expectations paradigm. In their data, individuals' propensity to delude themselves appears to depend on the costs associated with optimism, such as those stemming from inadequate life-cycle decisions, as well as its affective benefits.

We show that the entitlement effect is leveraged into higher material payoffs and therefore the behavioral bias of agents improves their material outcomes. Consistent with this point, Charness et al. (2011) find that subjects in an experiment become overconfident about their ability because their confidence is effective in compelling potential competitors to opt out of a tournament. In their setting, private information about one's true ability is critical to making overconfidence profitable. The entitlement effect also relates to the literature on bargaining with obstinate types (see Abreu and Gul, 2000; Compte and Jehiel, 2002) and echoes processes at play in Bénabou and Tirole (2009), with a reversal: in Bénabou and Tirole (2009), bargainers define themselves by their actions and refuse offers because this allows them to validate self-image concerns, whereas our entitlement effect is a case of fooling oneself in order to credibly demand a better treatment.

The next section introduces our basic model before sections 3 and 4 consider monopoly lending under symmetric and asymmetric information respectively. Section 5 studies the case of competition with privately informed borrowers. Section 6 is concerned with welfare and borrower protection and section 7 concludes. Proofs are gathered in appendices.

2. Setup

2.1. Technology, material payoffs and contracts

A risk-neutral borrower seeks a fixed-sized investment G for a project that may either succeed and yield a positive return y or fail and yield no return. A borrower's risk of failure θ may either be high (θ_H) or low (θ_L), with $1 > \theta_H > \theta_L > 0$. The proportion of high-risk types $\nu = \mathbb{P}[\theta = \theta_H] \in (0, 1)$ is common knowledge. The expected surplus generated by investment is $(1 - \theta)y - G$ and we assume that all projects are valuable:

$$(1 - \theta_H)y - G \geq 0 \tag{1}$$

Since $\theta_L < \theta_H$, the same property holds for the low-risk project. The parameter G compounds the cost of the investment and the opportunity cost of funds faced by the lender. Therefore, decreases in the cost of funds or in the cost of investment both translate into a decrease in G.

Success is observable and verifiable, so contracts specify a repayment R from the borrower to the lender if the project is successful, and a non-negative amount of collateral C that is transferred if the project fails. The expected *material* payoff of a borrower with type θ if she accepts contract (R, C) is given by

$$U_B(\theta, R, C) = (1 - \theta)(y - R) - \theta C \tag{2}$$

A risk-neutral lender's expected payoff from contracting with a borrower with a borrower with type θ is given by

$$U_I(\theta, R, C) = (1-\theta)R + \theta\delta(C)C - G \tag{3}$$

We do not impose any constraints on the borrower's ability to pledge collateral, but we rule out fully secured loans by assuming that collateral is not perfectly transferable, so that if the project fails, the borrower loses C, but the lender only obtains $\delta(C)C$ with $\delta(C) \leq 1$, so the extent of value destruction is given by $(1-\delta(C))C$.

While most models of costly collateral use a constant loss specification $\delta(C) = \bar{\bar{\delta}} < 1$, we assume that $\delta(.)$ is a strictly decreasing function: the borrower's assets are heterogeneous in their transferability and the borrower pledges the most transferable assets first. For example, her cash and treasury bills will be pledged before she considers pledging her car, for which the wedge between private and market value is presumably larger. We further assume that the first unit of collateral is perfectly transferable: $\delta(0) = 1$: substituting collateral for repayment thus involves no destruction of value at the margin when $C = 0$, so using the screening technology is free at the margin.[6]

For tractability reasons, we assume a linear functional form:

$$\delta(C) = \max(1 - \chi C, 0),\ \chi > 0 \tag{4}$$

Since the rate of recovery keeps decreasing, any amount of collateral over $(2\chi)^{-1}$ is worthless to the lender, but may still be required from borrowers for incentives reasons. On a formal level, using a strictly decreasing recovery rate makes the lender's preferences in the (R, C) space strictly convex and prevents systematic corner solutions in the monopolistic screening problem.

If they do not receive any contract, borrowers receive a type-dependent outside option payoff $\bar{U}_\theta$. To avoid pathological cases, we assume that high risks have a strictly positive outside option payoff, that low risks' outside options are at least as valuable as high risks' and that no borrower type needs more than the entire surplus generated by the project to be willing to trade with the lender:

$$0 < \bar{U}_H < \bar{U}_L \tag{5a}$$

$$\bar{U}_\theta \leq (1-\theta)\,y - G \tag{5b}$$

We define and use extensively the virtual "reservation contract" $\left(\bar{R}, \bar{C}\right)$ that yields either type their reservation utilities. This contract is unique and deemed "virtual" because it may not correspond to an actual feasible contract, as collateral $\bar{C}$ may be negative. Geometrically in the

[6] This feature enables us to draw analogies with insurance models based on expected utility, in which the risk premium associated with a small deviation around full insurance is also of second order and the screening technology (partial insurance) therefore comes at no cost at the margin around the efficient (full insurance) allocation.

(C, R) plane, this contract corresponds to the intersection of indifference lines associated with reservation utility levels.

The link between reservation utilities and reservation contracts is as follows:

$$\bar{U}_\theta = (1-\theta)\,(y-\bar{R}) - \theta\bar{C} \tag{6a}$$

$$\bar{C} = (\theta_H - \theta_L)^{-1}\left[(1-\theta_H)\,\bar{U}_L - (1-\theta_L)\,\bar{U}_H\right] \tag{6b}$$

$$\bar{R} = y - (\theta_H - \theta_L)^{-1}\left[\theta_H\bar{U}_L - \theta_L\bar{U}_H\right] \tag{6c}$$

2.2. **Optimal expectations: anticipatory utility and choice of beliefs**

A crucial building block of our model is anticipatory utility: the borrower does not only care about the material payoffs in (2), but also derives utility from anticipating future outcomes. A borrower who believes her type to be $\tilde{\theta}$ and accepts contract offer (R, C) expects to receive a future utility $U_B(\tilde{\theta}, R, C)$. This anticipatory utility term depends on the borrower's belief $\tilde{\theta}$ and not on her actual risk, and the difference is a motive for belief manipulation: when a high-risk borrower believes that she is of low risk, she inflates the anticipatory utility component. On the other hand, if distorted beliefs cause her to agree to detrimental contractual terms, this causes a reduction in the material payoffs component. Different contract options are appraised using the possibly distorted belief $\tilde{\theta}$ and the eventual choice may not be in the best interest of the borrower's material payoff $U_B\,(\theta, R, C)$ which depends on her true risk.

To provide modeling discipline on the borrower's belief selection, we use a criterion in the spirit of Brunnermeier and Parker (2005) and suppose that the borrower, after observing contract offers, chooses her beliefs to maximize the weighted sum of material and anticipatory payoffs

$$U_B\,(\theta, R, C) + sU_B\left(\tilde{\theta}, R, C\right) \tag{7}$$

This tradeoff is carried out with the understanding that the choice of contract (R, C) is constrained to be optimal given the adopted belief $\tilde{\theta}$. The parameter $s \geq 0$ measures the weight the borrower places on anticipatory feelings relative to material payoffs.

We share with Brunnermeier and Parker the postulate that there is no direct cognitive cost to belief distortions but that the entire cost stems from impaired decision-making. We also share the restriction that the borrower cannot commit to future choices that would not be optimal given adopted beliefs, and the parametrization of the tradeoff. Our formulation departs from that of Brunnermeier and Parker by restricting borrowers to formulate beliefs about their risk class, instead of selecting general probability distributions over the eventual state of the world.

Having borrowers formulate beliefs about their risk class has the intuitive interpretation of identifying with an existing population. Formally, this assumption preserves a lot of the benchmark model's structure: for example, a high-risk borrower who believes to be a low risk also believes her outside option to be that of a low risk, hence cannot inflate her subjective view of her outside option independently from her view of the inside options. Borrowers may believe they belong to a different risk class, but they retain full knowledge of the set of contracts and options available and cannot use "commitment through beliefs" (being blind to an option or assigning it imaginary punitive penalties is an indirect way of committing.

We further constrain chosen beliefs to be an element of the set of types $\{\theta_L, \theta_H\}$, so borrower does not have the option of believing that she belongs to a risk class that has zero probability. This implies that low-risk borrowers have nothing to gain from belief manipulation and we accordingly focus on the behavior and cognitive processes of high-risk borrowers. This assumption greatly improves tractability and spares us the need to define continuation payoffs inside and outside of the lender-borrower relationship for any possible belief $\tilde{\theta}$. More importantly, restricting the possibilities for delusion makes our model a minimal departure from the rational benchmark.

We investigate the robustness of our results in appendix B by allowing both types of borrowers to adopt any belief about their probability of failure. We focus on the case of a monopolist facing adverse selection and are able to show that the extension results in few qualitative differences with our baseline model. In particular, it is the low-risk borrower's contract which is used to discipline high-risk borrowers' cognition, and low-risk borrowers' optimism can be disciplined at no additional cost to the lender.

2.3. Timing and equilibrium concept

We focus on the monopoly case. Changes induced by competition are highlighted in section 5.

$t = 0$: The lender offers a menu of contracts $\mathcal{C} = \{(R_i, C_i)\}_i$

$t = 1$: The borrower observes both her type $\theta \in \{\theta_L, \theta_H\}$ and the menus of contracts available. She chooses her belief $\tilde{\theta} \in \{\theta_L, \theta_H\}$ so as to maximize $U_B(\theta, \tilde{R}, \tilde{C}) + sU_B(\tilde{\theta}, \tilde{R}, \tilde{C})$.

$t = 2$: The borrower chooses her favored contract $(\tilde{R}, \tilde{C}) \in \mathcal{C}$ or her outside option given her belief $\tilde{\theta}$.

$t = 3$: Material payoffs $U_B(\theta, \tilde{R}, \tilde{C})$ and $U_I(\theta, \tilde{R}, \tilde{C})$ are realized.

Note that at time $t = 2$ the notation $\tilde{R}$ and $\tilde{C}$ reflects the fact that contracts are chosen based on the borrower's beliefs $\tilde{\theta}$, which need not correspond to her actual type θ.

We study subgame-perfect equilibria and use the lender's preferences for tie-breaking procedures. We can then represent monopoly allocations as the solutions of optimization programs, taking the beliefs and contract choices of borrowers as control variables. In that sense we treat optimal expectations requirements as a form of cognitive moral hazard and characterize offers that induce realism and delusion separately before determining which contracts deliver the highest value to lenders and are therefore offered in equilibrium.

3. Monopoly lending under symmetric information

3.1. Type-independent reservation utilities

This section considers the choices of a monopoly lender faced with a high-risk borrower when the lender knows that risk. It is intended as an expository aid and a benchmark for the more relevant case of asymmetric information.

We consider a monopoly lender who faces a borrower of known risk θ_H and with the ability to delude herself into thinking that she is the low-risk type, i.e. $\tilde{\theta} = \theta_L$. We assume that the reservation utilities of both high-risk and low-risk borrowers are equal, and we further set their common value to zero, i.e. $\bar{U}_H = \bar{U}_L = 0$. [7] Faced with a borrower with non-malleable belief θ_H, the monopoly lender would offer the single contract $(R, C) = (y, 0)$, which exactly induces participation, appropriates the entire surplus, and is non-wasteful since no collateral requirements are stipulated. To account for the malleability of beliefs, write (R_H, C_H) for the contract that the borrower finds most attractive at time $t = 2$ if she is realistic and believes her risk to be θ_H, and (R_L, C_L) for the most-preferred contract of an overoptimistic high-risk borrower with belief $\tilde{\theta}_H = \theta_L$.[8]

If the lender induces realism on behalf of the borrower, he must incentivize participation so his maximum attainable expected profit is the total surplus from the project $(1 - \theta_H)y - G$. We show generally in appendix D that there exists a contract menu that induces realism and delivers this profit level. However, the monopolist can attain higher profits by taking advantage of the malleability of the borrower's beliefs.

In order to induce delusion, the lender's contract (R_L, C_L) must satisfy the following requirements:

$$U_B\left(\theta_H, R_L, C_L\right) + sU_B\left(\theta_L, R_L, C_L\right) \geq (1+s)U_B\left(\theta_H, R_L, C_L\right) \tag{8a}$$

$$U_B\left(\theta_H, R_L, C_L\right) + sU_B\left(\theta_L, R_L, C_L\right) \geq (1+s)\bar{U}_H = 0 \tag{8b}$$

$$U_B\left(\theta_L, R_L, C_L\right) \geq \bar{U}_L = 0 \tag{8c}$$

If a realistic borrower prefers contract (R_L, C_L) to her outside option, inequality (8a) stipulates that viewed from $t = 1$, the borrower's utility from deluding herself and picking the contract (R_L, C_L) exceeds the utility from remaining realistic and choosing the same contract (R_L, C_L). If a realistic borrower prefers the outside option, inequality (8b) states that at $t = 1$, the borrower prefers deluding herself rather than remaining realistic and obtaining her reservation utility. Inequality (8c) assures that, once a borrower has decided to delude herself at $t = 1$ and evaluates contract (R_L, C_L) at $t = 2$ in light of her belief $\tilde{\theta} = \theta_L$, she finds it preferable to her perceived outside option.

[7] This calibration violates the strict inequalities in assumptions (5b) and (5a) but results in simpler exposition.

[8] If the lender prefers that a borrower with belief $\tilde{\theta}$ select her outside option, contract $(R_{\tilde{\theta}}, C_{\tilde{\theta}})$ can be the reservation contract $(y, 0)$ which uniquely delivers the outside option payoff to either borrower type.

Without loss of generality, the lender offers a single contract (R_L, C_L) and arrives at his preferred allocation by maximizing profit subject to (8b). Even though the contract is intended for a high-risk borrower, we denote it by (R_L, C_L) to emphasize that it is targeted at a borrower who assesses her own probability of failure to be *low*.

$$\begin{aligned} \underset{\{R_L, C_L \geq 0\}}{Max} \quad & U_I(\theta_H, R_L, C_L) \\ s.t. \quad & \left\{ U_B\left(\theta_H, R_L, C_L\right) + sU_B\left(\theta_L, R_L, C_L\right) \geq 0 \right. \end{aligned} \tag{9}$$

The solution of (9) always satisfies constraints (8a) and (8c), so they can be omitted. For $s > 0$, the solution delivers a strictly higher profit than the best realism-inducing contract, which implies that inducing delusion is optimal. Proposition 1 describes the profit-maximizing contract offer and the equilibrium allocation.[9]

Proposition 1. (Symmetric information monopoly lending when $\bar{U}_H = \bar{U}_L = 0$)
In equilibrium, for any $s > 0$, the lender offers the contract (R_L, C_L) that solves program (9). It induces delusion and is taken up by the borrower. Collateral requirements and repayment are given by

$$C_L = \frac{1}{2}\,\frac{s\left(\theta_H - \theta_L\right)}{\chi\,\theta_H\left[(1-\theta_H) + s\left(1-\theta_L\right)\right]} \;\; and \;\; R_L = y - \frac{1}{2}\,\frac{s\left(\theta_H - \theta_L\right)\left(\theta_H + s\,\theta_L\right)}{\chi\,\theta_H\left[(1-\theta_H) + s\left(1-\theta_L\right)\right]^2}$$

When borrowers' weight on anticipatory payoffs converges towards $s = 0$, the incentive to engage in delusion is removed and the lender's does not gain from the malleability of beliefs. The equilibrium allocation converges to the standard preferences benchmark $(R = y, C = 0)$. The use of positive collateral in the case of symmetric information lending thus derives from the joint presence of anticipatory utility and the possibility to bias beliefs. There is no convergence of equilibrium beliefs as the cost incurred by holding incorrect beliefs vanishes faster than the benefit, so the distorted assessment has no material consequences.

A deluded borrower underestimates her risk of failure and is willing to accept contracts with positive collateral provided that she is reimbursed for each unit of collateral with a decrease in repayment of at least $\theta_L/(1-\theta_L)$ units. From the lender's objective perspective, these odds are better than actuarially fair odds, which correspond to a $\theta_H/(1-\theta_H)$ decrease in repayment for a unit increase in collateral requirements. Hence, there exist gains from trade from contracting with a deluded borrower and the monopolist benefits from inducing delusion. Nonetheless, the lender refrains from fully appropriating the gains from trade, otherwise the borrower's anticipatory benefits from delusion would be driven down to zero and the prospect of material losses would lead the borrower to remain realistic.

[9] The lender's offer may contain an arbitrary number of offers not taken up along the equilibrium path. Only equilibrium *allocations* are uniquely determined.

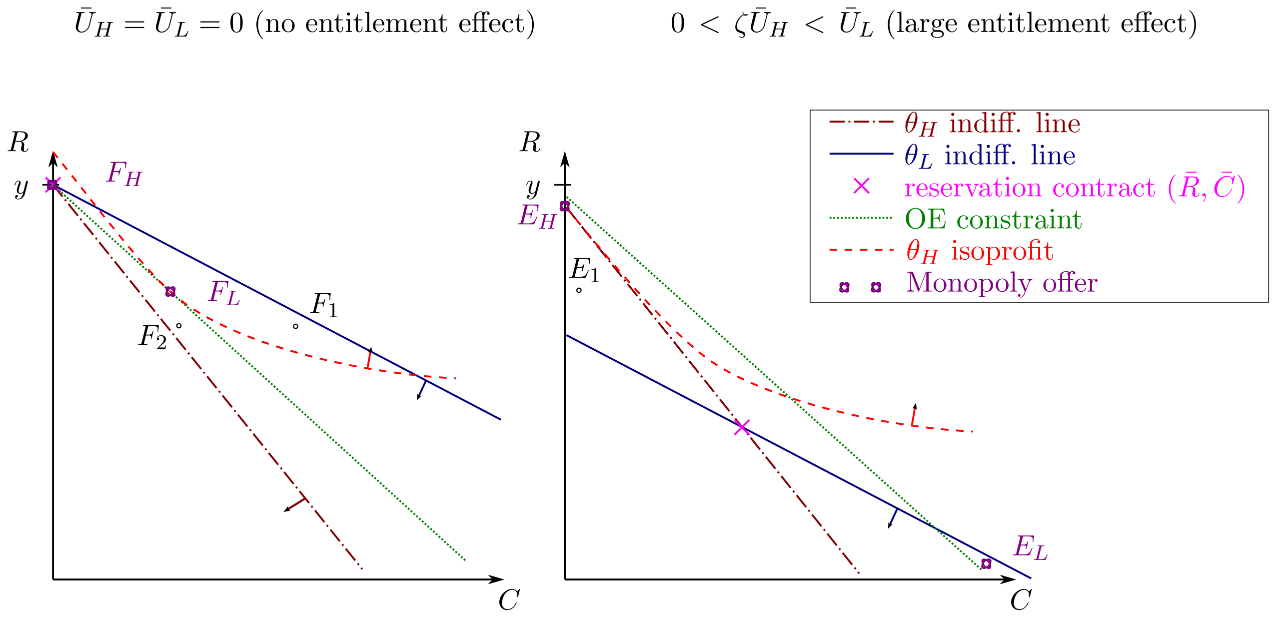

FIGURE 1. Realism-inducing and delusion-inducing contract offers.

The profit-maximizing offer reflects the optimal expectation constraint (8b). At the optimal offer, the following first-order condition holds

$$\theta_H \left(1 - \chi C - \chi C\right) = \frac{\left(1 - \theta_H\right)\left(\theta_H + s\,\theta_L\right)}{\left(\left(1 - \theta_H\right) + s\left(1 - \theta_L\right)\right)} \tag{10}$$

where the left-hand side is the marginal benefit of increasing collateral: with probability θ_H, the borrower transfers an increased amount, which is valued at $(1 - \chi C)$ and decreases the value of inframarginal collateral by χC. The right-hand side represents the marginal cost of increasing collateral: with probability $(1 - \theta_H)$, the lender pockets a repayment that has to be reduced by a factor of $(\theta_H + s\theta_L) / ((1 - \theta_H) + s\,(1 - \theta_L))$ to assure that delusion remains the borrower's optimal expectation.

To gain intuition for the equilibrium allocation, consider the left panel of figure 1. The straight indifference curves that reflect the reservation utilities of both types of borrowers pass through the "reservation contract" point $(\bar{R}, \bar{C}) = (y, 0)$ and are oriented: borrowers always prefer less repayment and collateral. The low-risk type's indifference curve is flatter than that of the high-risk type because she is at every point more willing to accept collateral in exchange for a given lowering of repayment. The set of contracts that are acceptable to the borrower at $t = 2$ is thus strictly larger if the borrower deludes herself into thinking that her project has a low risk, and contracts such as F_1 or F_2 are acceptable at time $t = 2$ if the borrower adopts belief $\tilde{\theta} = \theta_L$. However, the lender needs to refrain from capturing all of the $t = 2$ gains from trade, otherwise delusion would no longer be optimal for the borrower: whereas a contract such as F_1 balances a small loss in material terms against a large anticipatory benefit, a contract such as F_2 occasions a large material loss and offers only a relatively small anticipatory benefit.[10] The requirement that the contract picked by self-assessed low risks induce delusion at time $t = 1$ is that the corresponding point lie below the OE line, which represents constraint (8b) and lies between the high and low-risk types' indifference curves: in the figure, while contract F_2 induces delusion, contract F_1 does not. The contract pair (F_H, F_L) satisfies tangency condition (10) and represents the monopolist's optimal delusion-inducing offer. Since delusion occurs on the equilibrium path, only the active offer F_L is identified. Offering contract F_H as in the picture, or letting the borrower pick her outside option if she were to remain realistic — by offering only the contract F_L— would achieve the same result.

The equilibrium contract offer sets a weighted sum of the borrower's material and psychological payoffs equal to zero, her outside option. Since the lender only cares about his material gain, this is done by providing the borrower with a psychological rent, while imposing negative material payoffs on her. Negative material payoffs are a more general feature of monopoly lending under symmetric information and can also occur when the reservation utilities of the borrower's types differ.

3.2. **Type-dependent reservation utilities**

The assumption that the borrowers' outside options are equal is the natural one to make if the lender is absolutely essential to the project's financing in the sense that borrowers would not pursue their investment project absent funding by the lender but would instead be employed in unrelated activities for which they would use skills uncorrelated with the entrepreneurial talent reflected in their project's riskiness. However, low-risk borrowers are endowed with an unambiguously better project in the sense of first-order stochastic dominance. If there exist alternative, inefficient ways of financing the projects such as self-financing of a scaled-down version, we expect the low-risk type to have a more attractive outside option than the high-risk type. This is also true if a borrower's entrepreneurial talent is positively correlated with her employment prospects outside the lending relationship. Finally, a type-independent pooling contract with positive collateral contract offered by a competitive fringe of lenders would also generate type-dependent reservation utilities that favor low-risk borrowers. Under that interpretation, we denote such a reservation contract by $(\bar{R}, \bar{C})$ and outside options are given by $\bar{U}_\theta = U_B(\theta, \bar{R}, \bar{C})$. One can then check that for any value $\bar{C} > 0$, we have $\bar{U}_L > \zeta \bar{U}_H$, with $\zeta := (1 - \theta_H)^{-1} (1 - \theta_L) > 1$, so strongly unbalanced outside options emerge naturally in this context. The right panel of figure 1 depicts the case of $\bar{C} > 0$ and hence $\bar{U}_L > \zeta \bar{U}_H$.

When low-risk borrowers' prospects are better than those of high risks, delusion has an independent *entitlement effect*: being overoptimistic essentially commits the borrower to reject any

[10] The material loss is $1-\theta_H$ times the vertical distance between the contract point F_i and the high-risk indifference curve that contains the reservation contract, while the anticipatory benefit is $1 - \theta_L$ times the vertical distance between the contract point F_i and the low-risk indifference curve.

offer deemed less favorable than her inflated outside option. Therefore, when choosing between delusion-inducing and realism-inducing contracts, the lender must consider that delusion raises the borrower's self-assessed outside option and therefore the rent that must be surrendered. The right panel of figure 1 depicts the case of a large entitlement effect. We see that adopting the belief $\tilde{\theta} = \theta_L$ implies that the borrower at $t = 2$ prefers her outside option over a contract such as E_1, although the material payoff delivered by E_1 is larger than the material payoff delivered by the outside option evaluated in the light of her actual risk θ_H. Therefore, delusion on behalf of the borrower *reduces* the set of acceptable contracts and the lender finds it preferable to induce the borrower to remain realistic. Such cognitive incentives are provided at no cost by offering a menu of two contracts, one destined for realistic borrowers, which is taken up in equilibrium, and one "threat contract" such as E_L that a deluded high-risk borrower would prefer to her equilibrium contract and that contains high collateral requirements, thus making it costly in material terms. At time 1, the borrower chooses to remain realistic because if she were to become overoptimistic, she could not refrain from agreeing to take on contract E_L. The threat contract is never taken up in equilibrium, so it comes at no cost to the lender.

In appendix D, we consider a general distribution of outside options. We then characterize the optimal lending contracts and show that the results from the previous section extend as long as borrowers' outside options are similar. However, if the entitlement effect is large, the monopolist chooses to induce realism at no cost through the use of an offer such as (E_H, E_L) in figure 1. The general feature of the symmetric monopoly setup is that borrowers are harmed in material terms. However, the contracts offered by the symmetrically informed monopoly lender are not robust to informational frictions.

4. Monopoly lending under asymmetric information

Under asymmetric information, the same menu must be offered to both borrower types. Deluded high-risk borrowers and realistic low-risk borrowers must be offered, and agree to, the same contract, and therefore the cognition of high-risk borrowers cannot be managed independently from the lender's contract with low risks. On top of ensuring incentive compatibility, the lender must manage the borrowers' cognition and do so through the use of a single, type-independent offer, although that offer may contain multiple contracts. As is the case in symmetric information settings, the outside options of both borrower types play a key role is determining whether the lender finds it profitable to induce realism on behalf of high-risk borrowers.

4.1. Contract design without anticipatory utility concerns

We begin by setting up the monopoly's problem under asymmetric information absent behavioral frictions. We focus on deterministic offers whereby the monopolist may choose to exclude ($\rho_\theta = 0$) or include ($\rho_\theta = 1$) borrower types in her offer but may not use probabilistic credit rationing. If a borrower type is excluded, there are no restrictions on negative collateral values but this representation is simply a device that allows the lender to assign either type their outside option. Collateral requirements satisfy a standard sorting condition ($(U_B)''_{\theta C} = -1 < 0$) and are thus a suitable instrument to deter mimicking by high-risk borrowers, as low-risk borrowers find it cheaper to pledge collateral. However, they are not a valid instrument to deter mimicking by low-risk borrowers.

$$\underset{\{\rho_H C_H \geq 0, \rho_L C_L \geq 0, R_H, R_L, \rho_H, \rho_L\}}{Max} \quad \nu\rho_H U_I(\theta_H, R_H, C_H) + (1-\nu)\rho_L U_I(\theta_L, R_L, C_L)$$
$$s.t. \quad \begin{cases} U_B(\theta_H, R_H, C_H) - \bar{U}_H \geq 0 & \langle IR_H \rangle \\ U_B(\theta_L, R_L, C_L) - \bar{U}_L \geq 0 & \langle IR_L \rangle \\ U_B(\theta_H, R_H, C_H) - U_B(\theta_H, R_L, C_L) \geq 0 & \langle IC_H \rangle \\ U_B(\theta_L, R_L, C_L) - U_B(\theta_L, R_H, C_H) \geq 0 & \langle IC_L \rangle \end{cases} \tag{11}$$

As is standard in asymmetric information contexts, a nonempty subset of constraints are redundant, but the specific subset depends on parameter values. Type-by-type materially efficient allocations do not generically induce self-selection but the direction of mimicking incentives allows us to separate the cases of advantageous selection and adverse selection.

Definition 1. (Advantageous and Adverse selection)
We say that the monopoly lender faces adverse selection iff:

$$(1-\theta_L)\bar{U}_H \leq (1-\theta_H)\bar{U}_L \tag{12}$$

. Otherwise, the monopolist faces advantageous selection:

$$(1-\theta_L)\bar{U}_H > (1-\theta_H)\bar{U}_L \tag{13}$$

By denoting $\zeta := (1-\theta_H)^{-1}(1-\theta_L) > 1$, we can equivalently say that adverse selection holds iff $\bar{U}_L \geq \zeta\bar{U}_H$.

This standard characterization has to do with the zero-collateral contracts that exactly induce allocation type-by-type and solve the benchmark planner's problem under symmetric information absent anticipatory utility concerns. If the high-risk type is charged with a higher repayment than the low-risk type's, then she prefers the low-risk type's allocation and vice-versa. In an asymmetric information setting, offering both contracts to both borrower types leads to pooling on the lowest-repayment allocation. Raising the repayment demanded in the pooling offer leads one borrower type to drop out and select her outside option. If it is the high-risk borrower that drops out, then raising repayment retains only the least profitable borrower, a case of adverse selection.

4.1.1. *Advantageous selection*

If there is advantageous selection, a monopolist that wants to include both borrower types is constrained to offer the low repayment rate that induces high risks to participate. Alternatively, the lender chooses to exclude high-risk borrowers if their proportion is small enough.

Proposition 2. (Monopoly contract design without anticipatory utility concerns, advantageous selection)

Under advantageous selection (13), there exists a threshold proportion of high-risks $\bar{\nu} < 1$ such that for $\nu < \bar{\nu}$, high-risk types are excluded and low risks receive no rent.

$$\rho_H = 0, \rho_L = 1, C_L = 0, U_B(\theta_L, R_L, 0) = \bar{U}_L \tag{14}$$

For $\nu \geq \bar{\nu}$, high-risk types are included and the offer pools both types: $R_H = R_L, C_H = C_L$. Low risks receive a rent.

$$\rho_H = 1, \rho_L = 1, C_L = 0, U_B(\theta_H, R_H, 0) = \bar{U}_H, U_B(\theta_L, R_L, 0) - \bar{U}_L = \zeta\bar{U}_H - \bar{U}_L \tag{15}$$

The threshold parameter $\bar{\nu}$ is given as follows (note that $\bar{C} < 0$)

$$-\frac{\bar{C}(\theta_H - \theta_L)}{-\bar{C}(\theta_H - \theta_L) + (1 - \theta_H)\left((1 - \theta_H)y - G - \bar{U}_H\right)} \tag{16}$$

The lender does not use collateral requirements, as increasing the collateral intended for high-risk borrowers worsens the low risk type's incentive constraint. Excluding high-risk borrowers enables the monopolist to leave no rents to low-risk borrowers at the cost of foregoing efficient loans.

4.1.2. *Adverse selection*

If there is adverse selection, starting from a zero-collateral loan offer that attracts both borrower types and exactly induces participation of low risks, increasing the repayment rate causes low risks to drop out first, leaving only high risks. The monopolist does not exclude borrowers but instead uses collateral requirements as a screening device.

Proposition 3. (Monopoly contract design without anticipatory utility concerns, adverse selection)

Under adverse selection (12), no borrower is excluded: $\rho_H = \rho_L = 1$. There exists a threshold proportion of high-risks $\hat{\nu} \in (0, 1)$ such that for $\nu < \hat{\nu}$, the monopolist's problem has an interior solution, while the monopolist chooses a corner solution if $\nu \geq \hat{\nu}$. For $\nu < \hat{\nu}$, the monopolist offers the pair of contracts that solve program (11) with constraints IC_H and IR_L binding and IC_L and IR_H slack. The profit-maximizing collateral requirements for high and low-risk borrowers *respectively are given by*

$$C_H = 0 \quad \textit{and} \quad C_L = \frac{1}{2} \frac{(\theta_H - \theta_L)\ \nu}{(1-\theta_L)\ (1-\nu)\ \theta_L \chi} \tag{17}$$

Repayment values solve the saturated constraints (IC_H, IR_L).

For $\nu \geq \hat{\nu}$*, constraints* IC_H*,* IR_L *and* IR_H *are binding at the solution of program* (11)*. Neither borrower type earns any rent and collateral requirements are given by*

$$C_H = 0 \quad \textit{and} \quad C_L = \bar{C} = (\theta_H - \theta_L)^{-1} \left[(1-\theta_H)\, \bar{U}_L - (1-\theta_L)\ \bar{U}_H\right] \tag{18}$$

The threshold $\hat{\nu}$ *satisfies:*

$$\frac{\bar{U}_L}{1-\theta_L} - \frac{\bar{U}_H}{1-\theta_H} = \frac{\hat{\nu}(\theta_H - \theta_L)^2}{2\,(1-\hat{\nu})\,\chi\theta_L\,(1-\theta_H)\,(1-\theta_L)^2} > 0 \tag{19}$$

Starting from the incentive compatible and efficient pooling contracts with $C_H = C_L = 0$, $R_H = R_L$ and $U_B(\theta_L, R_L, C_L) = \bar{U}_L$, the lender finds it profitable to increase the collateral of the mimicked low-risk type and to accept a second-order efficiency loss in return for a first-order gain from relaxing the high-risk borrower's incentive compatibility constraint. Assuming interiority, this process continues until the marginal costs and benefits of the distortion are equal, as given by first-order condition (20):

$$(1-\nu)\,\theta_L\,(1 -\ \chi\, C_L -\ \chi\, C_L) + \lambda_H \theta_H = \mu_L \theta_L \tag{20}$$

where λ_H and μ_L are the Lagrange multipliers associated with the high-risk type's incentive compatibility and the low-risk type's participation constraint respectively. The left-hand side of (20) is the marginal benefit of increasing collateral to the lender: with probability θ_L, a low-risk borrower transfers an increased amount, which is valued at $(1 - \chi\, C_L)$ and decreases the value of inframarginal collateral by χC_L. Furthermore, raising collateral requirements on low-risk borrowers relaxes the incentive constraint on high-risk borrowers. The right-hand side embodies the marginal cost of increasing collateral and corresponds to the decrease in repayment necessary to maintain the participation of the low-risk type.

The assumption of perfect transferability of the first infinitesimal unit of collateral (4) guarantees that pooling borrowers, while obviously incentive-compatible, is always dominated. Under adverse selection, the monopolist always separates borrower types:

Corollary 1. *Under adverse selection, for* $s = 0$*, high-risk and low-risk types take distinct contracts, and low-risk contracts always contain positive collateral requirements:* $C_H = 0 < C_L$*.*

4.2. **Contract design with anticipatory utility concerns**

We now revert to the assumption that borrowers have nonzero anticipatory utility concerns and may thus choose to deny their actual risk for the sake of a rosy view of the future. We proceed by deriving a monopolist's preferred realism-inducing set of contracts, in which borrower types are separated, and then compare these to the most profitable contracts that yield delusion on behalf of high-risk borrowers. We can then characterize the equilibrium, in which the lender simply chooses the contract offers that yield the highest profits.

4.2.1. *Properties of realism-inducing offers*

A lender who tries to induce realism on behalf of the borrower, i.e. $\tilde{\theta}_H = \theta_H$ and $\tilde{\theta}_L = \theta_L$, maximizes her profits by solving the following general program: [11]

$$\underset{\{C_H,C_L,R_H,R_L\}}{Max} \quad \nu U_I(\theta_H,R_H,C_H) + (1-\nu)U_I(\theta_L,R_L,C_L)$$
$$s.t. \begin{cases} U_B(\theta_H,R_H,C_H) - \bar{U}_H \geq 0 & \langle IR_H \rangle \\ U_B(\theta_L,R_L,C_L) - \bar{U}_L \geq 0 & \langle IR_L \rangle \\ U_B(\theta_H,R_H,C_H) - U_B(\theta_H,R_L,C_L) \geq 0 & \langle IC_H \rangle \\ U_B(\theta_L,R_L,C_L) - U_B(\theta_L,R_H,C_H) \geq 0 & \langle IC_L \rangle \\ (1+s)U_B(\theta_H,R_H,C_H) - U_B(\theta_H,R_L,C_L) - sU_B(\theta_L,R_L,C_L) \geq 0 & \langle OE_{H,H} \rangle \end{cases} \tag{21}$$

In addition to assuring incentive compatibility, the lender needs to provide the cognitive incentives that induce realism on behalf of the borrower. These cognitive incentives are embodied in the optimal expectation constraint $OE_{H,H}$[12]. Program (21) nests program (11) as a special case where $s = 0$, as constraint $OE_{H,H}$ collapses into the incentive constraint IC_H.

Our first observation is that in solving program (21), the lender can ignore the high-risk type's incentive compatibility constraint IC_H, because the optimal expectations constraint $OE_{H,H}$ is uniformly tighter, a point made precise in the following lemma.

Lemma 1. (Redundancy of the incentive constraint IC_H)
Any offer $\{C_H \geq 0, C_L \geq 0, R_H, R_L\}$ that satisfies $OE_{H,H}$ and IR_H also satisfies IC_H, strictly so for $s > 0$ and either $C_L > 0$ or $\bar{U}_H > 0$.

Lemma 1 implies that the standard screening menu derived from program (11) would induce delusion on behalf of the high-risk type with $s > 0$. Recall that the incentive compatibility constraint for the high-risk type is saturated at the optimal menu when $s = 0$. The high-risk borrower therefore receives identical material payoffs regardless of whether she remains realistic and chooses the contract destined for her or deludes herself and chooses the low risk-type's contract.

[11] We treat the possibility of exclusion separately.

[12] We show below that the optimal expectation constraint is tighter than the incentive constraint for high-risk borrowers and the opposite is true for low-risk borrowers: if material payoffs leave the low-risk borrower indifferent between two contracts, anticipatory utility considerations push her towards the non-pessimistic beliefs. We therefore omit constraints pertaining to the low-risk borrowers' optimal expectations.

Since the material payoff is the same either way, there is no downside to enjoying the higher emotional payoff that optimism provides when $s > 0$. A lender who is trying to get the borrower to adopt realistic beliefs must therefore provide additional incentives beyond those of standard screening, either with additional collateral for the low-risk type or with a lower repayment for the high-risk type.

These realism incentives are most efficiently delivered, not by increasing the punishment on a delusion path (an increase in C_L), but by increasing the rewards for realists in the form of a lower repayment, while decreasing the power of the incentive scheme (the difference $C_H - C_L$). This last point follows because collateral requirements are less efficient at deterring overoptimism than they are at deterring mimicking. Formally, increasing low risks' collateral requirements decreases the high risk's informational rent at a lower rate, the higher the weight on anticipatory utility s. The informational rent of high-risk borrowers is defined as $U_B(\theta_H, R_H, 0) - \bar{U}_H$, and can be seen as a function of C_L while adjusting the low-risk borrower's contract along her indifference curve.

Lemma 2. (Information rent of realistic high-risk borrowers and collateral use)
Write $Re_H(C_L)$ for $U_B(\theta_H, R_H, 0) - \bar{U}_H$, where the value of R_H saturates constraint $OE_{H,H}$ and the value of R_L is adjusted so that $U_B(\theta_L, R_L, C_L)$ is kept constant. We have

$$Re'_H(C_L) = -\frac{(\theta_H - \theta_L)}{(1+s)(1-\theta_L)} \tag{22}$$

A given increase in collateral C_L reduces the informational rent of realistic high-risk borrowers by a lower amount, the higher the value of s.

Lemma 1 and lemma 2 have a straightforward geometrical illustration in figure 2. For any contract (R_H, C_H) on the iso-utility line IC_H, the pair of contracts (R_H, C_H) and (R_L, C_L) satisfies incentive compatibility whenever (R_L, C_L) lies above IC_H. To induce realism, however, contract (R_L, C_L) has to lie above the OE line, which is a more stringent requirement. This property leads the informational rent given up in realism-inducing offers to rise with s. The marginal impact of collateral on the information rent is related to the slope of the OE line, which equals $(1 - \theta_H + s(1-\theta_L))^{-1}(-(\theta_H + s\theta_L))$ and lies between the slopes of each type's indifference line. On a delusion path, the high-risk borrower only evaluates the expected cost of increases in collateral using the biased probability assessment θ_L. The deterrent effect of increased collateral is thus lower when applied to a (potentially) deluded borrower than when applied to a realist tempted by mimicking.

4.2.2. *Properties of delusion-inducing offers*

Consider a lender who wants to induce delusion as the $t = 1$ optimal expectation for high-risk borrowers. Since deluded high-risk borrowers and realistic low-risk borrowers hold the same beliefs at $t = 2$, they must pick the same contract (R_L, C_L). If the high-risk borrower were to remain realistic she would pick contract (R_H, C_H), so the monopolist must structure contracts so that this

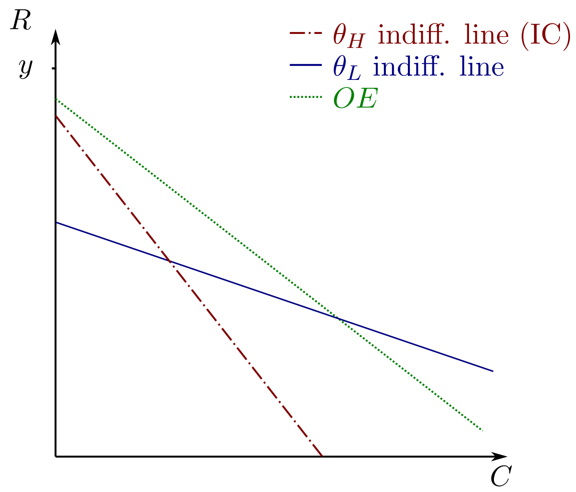

FIGURE 2. Geometrical illustration of lemma 1 and lemma 2. (R_L, C_L) contracts that make any contract on the indifference line IC_H realism-inducing must lie above the OE line.

does not happen along the equilibrium path. The monopolist wishing to induce delusion solves the following program:

$$\underset{\{C_L\geq 0, R_L, C_H, R_H, \rho_H\}}{Max} \quad \nu\rho_H U_I(\theta_H, R_L, C_L) + (1-\nu)U_I(\theta_L, R_L, C_L)$$

$$s.t. \quad \begin{cases} U_B(\theta_H, R_H, C_H) - \bar{U}_H \geq 0 & \langle IR_H \rangle \\ U_B(\theta_L, R_L, C_L) - \bar{U}_L \geq 0 & \langle IR_L \rangle \\ U_B(\theta_H, R_H, C_H) - U_B(\theta_H, R_L, C_L) \geq 0 & \langle IC_H \rangle \\ U_B(\theta_H, R_L, C_L) + sU_B(\theta_L, R_L, C_L) - (1+s)\, U_B(\theta_H, R_H, C_H) \geq 0 & \langle OE_{H,L} \rangle \\ \text{either } C_H \geq 0 \text{ or } C_H = \bar{C} \end{cases} \tag{23}$$

The same line OE as in figure 2 still represents the optimal expectation requirement, except that the low-risk borrower's contract must lie *below* the OE line to induce delusion. If borrowers face a single offer that is individually-rational for both types, borrowers face no material cost to optimism, as they pick the same contract regardless of beliefs. The high-risk borrower is thus induced to adopt belief θ_L. [13]

It follows that there is no loss of generality in having the monopoly offer a single contract, while a realistic high-risk borrower selects either contract (R_L, C_L) or her outside option.

Lemma 3. (Delusion-inducing menus contain only one contract)
There is no loss of generality in offering a single contract in program (23). If the solution contract $(R_{L,\langle 23\rangle}, C_{L,\langle 23\rangle})$ *satisfies high risks' individual rationality, then* $(R_H, C_H) = (R_{L,\langle 23\rangle}, C_{L,\langle 23\rangle})$ *is a suitable high-risk off-equilibrium contract. If* $(R_{L,\langle 23\rangle}, C_{L,\langle 23\rangle})$ *does not satisfy the high risks'*

[13] When faced with a single pooling offer (R, C) that is individually rational for both types, both borrower types adopt belief θ_L. This result obtains provided that $s\,(\theta_H - \theta_L)\,(y - R + C) \geq 0$, which is implied by assumption (5a).

individual rationality, then $(R_H, C_H) = (\bar{R}, \bar{C})$ is a suitable high-risk off-equilibrium contract and realistic high-risk borrowers are excluded.

We now study specific realism- and delusion-inducing offers under both adverse and advantageous selection and establish the optimal contract design.

4.2.3. *Adverse selection: realism-inducing offers*

We assume that the proportion of high-risk types is small enough that the solution to the benchmark screening problem is interior and given by (17) and treat the other case in appendix C. The profit maximizing realism-inducing menu of contracts solves program (21) in which several constraints are relaxed. We have seen that IC_H must be slack, and under an innocuous restriction, constraints IR_H and IC_L may also be omitted.

Lemma 4. (Monopoly realism-inducing separating contracts, adverse selection)
Assuming $\nu < \hat{\nu}$[14], there exists a threshold $\bar{s} > 0$ such that for $0 < s \leq \bar{s}$ the solution to program (68), which relaxes all constraints except $OE_{H,H}$ and IR_L, also solves the more general program (21) and constitutes the lender-preferred offer in the class of offers that separate borrower types and induce realism. [15] *The profit-maximizing collateral requirements are given by*

$$C_{H,\langle 68 \rangle} = 0 \quad and \quad C_{L,\langle 68 \rangle} = \frac{1}{2} \frac{\nu\,(\theta_H - \theta_L)}{(1-\nu)\,\chi\,\theta_L\,(1-\theta_L)\,(1+s)} \tag{24}$$

Saturating constraints $OE_{H,H}$ and IR_L gives the corresponding repayment values. The threshold $\bar{s}$ is defined by

$$\bar{s} = \frac{1}{2}\left(\sqrt{1 + 2\frac{\nu\,(\theta_H - \theta_L)}{\chi\,\theta_L\,\bar{U}_L\,(1-\theta_L)\,(1-\nu)}} - 1\right) \tag{25}$$

Starting from the benchmark interior solution, as the weight of anticipatory utility concerns increases, the optimal collateral requirement $C_{L,\langle 68 \rangle}$ is decreasing in s. The strength of separation incentives is reduced and both types' contracts converge towards the pooling offer that attracts both types and leaves no rent to low risks. This result follows from lemma 2 and the comparison between the delusion incentives and mimicking incentives of high-risk borrowers. Accordingly, a monopoly lender decreases the ineffective collateral requirements $C_{L,\langle 68 \rangle}$ and decreases the repayment asked of the high-risk type, $R_{H,\langle 68 \rangle}$, to preserve the borrowers' separation incentives.

[14] The threshold $\hat{\nu}$ is defined in proposition 3.

[15] In the proof of lemma 4 we show that the solution to the relaxed program (68) solves program (21) as long as $s \leq \bar{s}$. For $s > \bar{s}$, it is the incentive compatibility constraint of low-risk borrowers that first binds among the omitted constraints. Importantly, we will establish that inducing *delusion* rather than *realism* is optimal when $s > \bar{s}$. The relaxed program therefore characterizes all *relevant* realism-inducing offers.

Since $C_{H,\langle 68\rangle}$ is always zero and $R_{H,\langle 68\rangle}$ is decreasing in s, the payoff of high-risk agents $U_B(\theta_H, R_H, C_H)$ is increasing in s. The high-risk borrower effectively uses her anticipatory utility concerns as a threat to embrace the dream of owning a low-risk project. Deterring delusion requires allowing realists to make a lower repayment than in a standard screening setting.

4.2.4. *Adverse selection: delusion-inducing offers*

As established in section (4.2.2), without loss of generality, delusion-inducing offers contain only one contract.

Lemma 5. (Monopoly delusion-inducing pooling contract)
When $s > 0$ and $\nu < \hat{\nu}$, the profit maximizing offer that induces delusion consists of a single pooling contract which solves program (23). Repayment and collateral requirements are given by

$$C_{L,\langle 23\rangle} = \frac{\nu\left(\theta_H - \theta_L\right)}{2\left(1-\theta_L\right)\chi\mathbb{E}\left[\theta\right]} \quad \textit{and} \quad R_{L,\langle 23\rangle} = y - \frac{\theta_L \nu\left(\theta_H - \theta_L\right)}{2(1-\theta_L)^2\chi\mathbb{E}\left[\theta\right]} - \frac{\bar{U}_L}{(1-\theta_L)}$$

Notably, despite the pooling of risk types, the lender-preferred amount of collateral $C_{P,\langle 23\rangle}$ is positive. The logic behind this can be gleaned from the first-order condition of the lender's maximization problem.

$$\mathbb{E}\left[\theta\right]\left[(1-\chi C_L) - \chi C_L\right] = \frac{\theta_L}{(1-\theta_L)}\left(1-\mathbb{E}\left[\theta\right]\right)$$

At the optimum, the marginal revenue from increasing collateral, i.e. the $(1-2\chi C_L)$ units that are obtained with probability $\mathbb{E}\left[\theta\right]$, has to be equal to the marginal cost, which arises from the fact that in order to maintain participation, the repayment must be lowered by $\theta_L/\left(1-\theta_L\right)$, weighted by the average probability of repayment $(1-\mathbb{E}\left[\theta\right])$. At $C_L = 0$, a one unit increase in collateral that is accompanied by a $(1-\mathbb{E}\left[\theta\right])^{-1}\mathbb{E}\left[\theta\right]$ unit decrease in repayment is profit-neutral, but since all agents are self-assessed low-risk borrowers, they only require a $(1-\theta_L)^{-1}\theta_L$ unit decrease in repayment to be kept indifferent. It is therefore profitable to raise collateral requirements and exploit the speculative benefits generated by the difference in beliefs.

4.2.5. *Adverse selection: equilibrium*

We now bring together the two previous sections and establish that a monopolist constrained by adverse selection uses separating, realism-inducing offers when s is small and switches to a pooling, delusion-inducing offer when s is large.

Proposition 4. (Monopolistic lending under adverse selection with anticipatory utility concerns)
There exists a unique threshold $s^* > 0$ such that

- *for $0 \leq s < s^*$, the lender-preferred contract offers induce realism and are described by lemma 4,*

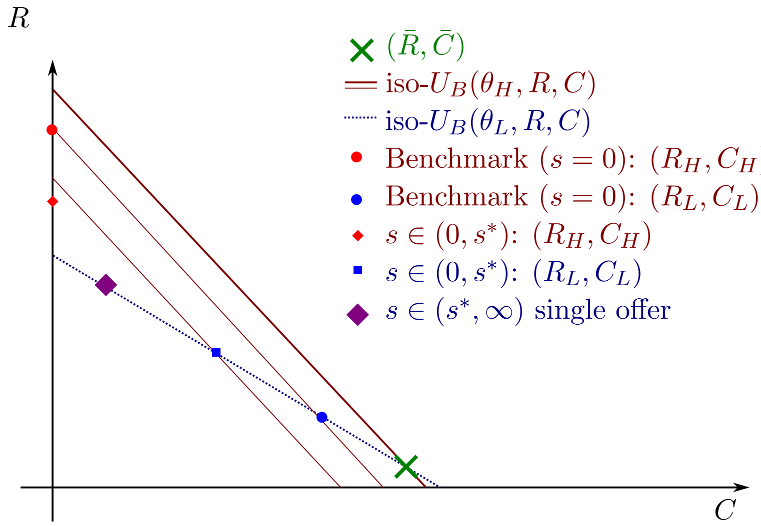

FIGURE 3. Changes in equilibrium contract design as s increases. For $s \leq s^*$, borrower types are separated and only low-risk borrowers pledge collateral. High-risk borrowers earn a rent that increases with s. For $s > s^*$, the monopolist issues a pooling offer

- *for $s > s^*$, the profit-maximizing contract offer induces delusion on behalf of the high-risk borrower and is described by lemma 5.*

As we have seen, when s is small, a high-risk borrower's utility from mimicking and her utility from actually believing that she is a low-risk type are almost equal, and separating types is costly for the lender but still preferable to issuing a pooling offer. As s increases, however, screening becomes more and more costly and the profit of a lender who attempts to separate borrowers decreases, because a larger and larger information rent needs to be given up to the high-risk borrower. Eventually, offering a pooling contract and allowing the high-risk borrower to delude herself becomes more profitable. Figure 3 illustrates the effect of increasing anticipatory utility concerns on equilibrium contract design.

If the weight of anticipatory utility concerns $s > s^*$, then the equilibrium of our contracting game features pooling of risk types and positive collateral requirements for both types of borrower. The pooling equilibrium is consistent with the empirical finding that ex-ante credit risk and collateral use are not significantly correlated although collateral use is ubiquitous (Berger and Udell, 1990). In the standard case of $s = 0$, on the other hand, corollary 1 tells us that the high-risk borrower never pledges any collateral and risk-types never select identical contracts in equilibrium. The standard model with ex-ante private information therefore always predicts a negative risk-collateral correlation. Of course, we do not claim to provide the only explanation for a lack of risk-collateral correlation. Most notably, introducing moral hazard into a model of collateralized loans also breaks the link between risk and collateral that a model with only ex-ante private information predicts. The pooling equilibrium is also consistent with the empirical evidence on small business financing in the US that suggests that optimists are neither charged higher interest rates nor required to pledge collateral more often than realistic borrowers (Dai and Ivanov, 2010). However, we make this assertion with some caution, since agents that are realistic in our model are also of a lower risk and hence, adding a second dimension of heterogeneity.

4.2.6. *Advantageous selection: realism-inducing offers*

Under advantageous selection, delusion increases the set of lending terms that the borrower finds acceptable and high-risk borrowers earn no rent in the benchmark. The rationale for realism-inducing contracts is the exclusion of high-risk borrowers rather than reducing their informational rent.

Exclusive contract offers solve

$$\underset{\{R_L, C_L\}}{Max} \quad (1-\nu)\, U_I\left(\theta_L, R_L, C_L\right)$$
$$s.t. \quad \begin{cases} U_B\left(\theta_L, R_L, C_L\right) - U_B\left(\theta_L, \bar{R}, \bar{C}\right) \geq 0 & (IR_L) \\ U_B\left(\theta_H, \bar{R}, \bar{C}\right) - U_B\left(\theta_H, R_L, C_L\right) \geq 0 & (IC_H) \\ (1+s)U_B\left(\theta_H, \bar{R}, \bar{C}\right) - U_B\left(\theta_H, R_L, C_L\right) - sU_B\left(\theta_L, R_L, C_L\right) \geq 0 & (OE_{H,H}) \end{cases} \tag{26}$$

We know from section 4.2.1 that constraint (IC_H) is redundant. The following lemma characterizes the optimal offer.

Lemma 6. (Monopoly realism-inducing exclusion offers)
Profit maximizing, realism-inducing offers exclude high-risk borrowers. There exists a threshold $\check{s} > 0$ *such that for* $s \leq \check{s}$*, excluding high-risk borrowers does not require collateral and the optimal offer assigns to low-risk borrowers the zero-collateral contract that exactly induces their participation:*

$$R_L = \frac{\bar{C}\theta_L}{(1-\theta_L)} + \frac{(1-\theta_L)\,\bar{R}}{(1-\theta_L)}, \quad C_L = 0 \tag{27}$$

The lender extracts the entire surplus and leaves no rent to low-risk borrowers.

For $s > \check{s}$*, excluding high-risk borrowers is best achieved by requiring that low-risk borrowers pledge collateral and the optimal offer is given by:*

$$R_L = \bar{R} + \frac{\bar{C}\theta_L}{(1-\theta_L)} - \theta_L\left(s-\check{s}\right)\left(y-\bar{R}+\bar{C}\right) \quad C_L = \left(s-\check{s}\right)\left(1-\theta_L\right)\left(y-\bar{R}+\bar{C}\right) \tag{28}$$

The lender still delivers low risks their outside option (no rent) but fails to extract the entire surplus.

The threshold $\check{s}$ *is given by*

$$\check{s} = -\frac{\bar{C}}{\left(y-\bar{R}+\bar{C}\right)\left(1-\theta_L\right)} \tag{29}$$

For low values of s, the higher repayment rate levied on low-risk borrowers is sufficient to induce high-risk borrowers to appraise their risk realistically and stay out of the market. As s rises, the psychological rent generated by optimism starts to make up for the higher repayment and starting from $s = \bar{s}$, the monopolist can no longer raise repayments without excluding low-risk borrowers. Collateral requirements enable the lender to discipline high-risk borrowers' cognition and exclude them while maintaining participation of low-risk borrowers.

4.2.7. *Advantageous selection: delusion-inducing offers*

Delusion-inducing offers depend on the proportion of high risks and the weight of anticipatory concerns. We describe the solution to program (23), where realistic high-risks receive their outside option.

Lemma 7. (Monopoly delusion-inducing pooling contract)
There are three possible types of solution to program (23), parameterized by boundary functions $0 < s_D^-(\nu) < s_D^+(\nu)$. For $s \leq s_D^-(\nu)$, the optimal delusion-inducing offer involves no collateral and a repayment which increases with s: [16]

$$C_L = 0, \quad R_L = \frac{(1-\theta_H)\,y - \bar{U}_H + \big((1-\theta_L)\,y - \bar{U}_H\big)\,s}{(1-\theta_H) + (1-\theta_L)\,s} \tag{30a}$$

For $s_D^-(\nu) \leq s < s_D^+(\nu)$, the optimal delusion-inducing offer involves a positive amount of collateral that increases with s

$$C_L = \frac{\big(\bar{U}_L - \bar{U}_H\big)\,(s-\check{s})\,(1-\theta_L)}{(\theta_H - \theta_L)} \tag{30b}$$

Repayment obtains by saturating constraint (IR_L) in program (23).

For $s \geq s_D^+(\nu)$, the offer does not depend on s. Collateral is chosen to optimally exploit the ex-post difference in beliefs:

$$C_L = \frac{\nu\,(\theta_H - \theta_L)}{2\,(1-\theta_L)\,\chi\mathbb{E}\,[\theta]} \quad \text{and} \quad R_L = y - \frac{\theta_L\nu\,(\theta_H - \theta_L)}{2(1-\theta_L)^2\chi\mathbb{E}\,[\theta]} - \frac{\bar{U}_L}{(1-\theta_L)} \tag{30c}$$

The lower boundary $s_D^-(\nu)$ equals $\check{s}$ for $\nu \leq (1+\check{s})^{-1}$ and $(1-\nu)\nu^{-1}$ otherwise. The upper boundary obtains as follows:

$$s_D^+(\nu) = \frac{1}{2}\,\frac{2\chi\,(-1+\theta_L)\,(\nu\theta_H + \theta_L - \nu\theta_L)\,\bar{C} - \nu\theta_L + \nu\theta_H}{(-1+\theta_L)^2\,\chi\,(\nu\theta_H + \theta_L - \nu\theta_L)\,\big(y - \bar{R} + \bar{C}\big)} \tag{30d}$$

Under advantageous selection, the lender unambiguously benefits from the borrowers' bias and can always simply replicate the benchmark offer. Generally, while the precise form of the optimal delusion-inducing offer depends on parameter configurations, the general pattern is that the lender uses moderate collateral requirements as long as inducing delusion is a concern at the margin, which happens for $s \leq s_D^+(\nu)$. For these parameter values, there exist subjectively-assessed gains from trade at $t = 2$, in the sense that all borrowers are convinced that they belong to a low risk class and would be willing to further exchange collateral for repayment at a $\theta_L(1-\theta_L)^{-1}$ rate.

[16] This last point follows from the properties of the optimal expectation constraint and can be checked by simple differentiation of the repayment with respect to s, yielding $\left(((1-\theta_H) + (1-\theta_L)\,s)^2\right)^{-1}\left(\bar{U}_H\,(\theta_H - \theta_L)\right)$.

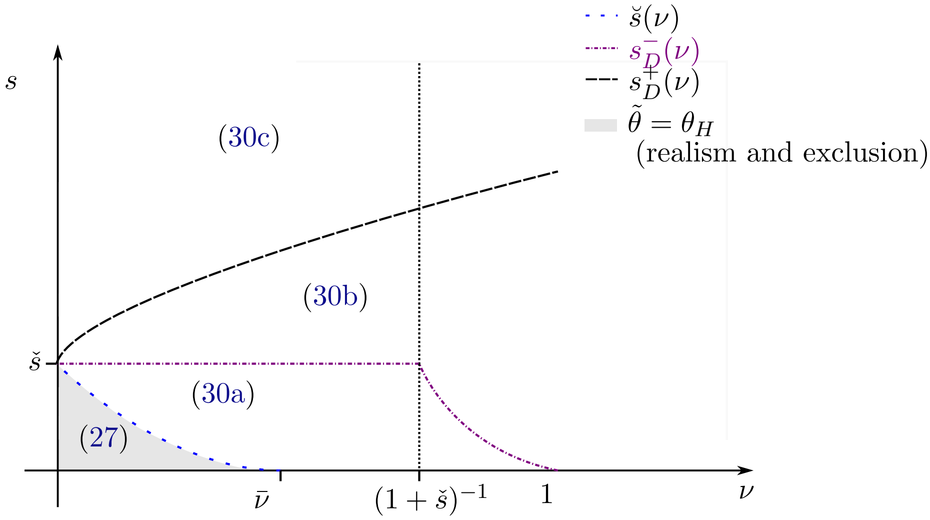

FIGURE 4. Optimal lending under advantageous selection

The monopolist effectively commits not to exploit such gains from trade as doing so would deter optimism at $t = 1$. For $s > s_D^+(\nu)$, there is no such concern and the lender can fully exploit his ex-post infomrmational advantage and extract value in the form of collateral.

4.2.8. *Advantageous selection: equilibrium*

We now bring together the two previous sections and establish that a monopolist constrained by advantageous selection rations loans for a strictly smaller set of parameters than in the benchmark and never excludes borrowers as long as the weight of anticipatory utility concerns exceeds $\check{s}$.

Proposition 5. (Monopolistic lending under advantageous selection with anticipatory utility concerns)

There exists a unique boundary $\check{s}(\nu)$ such that

- *for $s \leq \check{s}(\nu)$, high-risk borrowers are realistic and excluded using offer* (30a),
- *for $s \geq \check{s}(\nu)$, high-risk borrowers are overoptimistic and loans are as described in* (30)

The boundary $\check{s}(\nu)$ is decreasing and satisfies $\check{s}(\hat{\nu}) = 0$ and $\check{s}(0) = \check{s}$. For $\nu \geq \hat{\nu}$, $\check{s}(\nu) = 0$

Figure 4 summarizes proposition 5 graphically.

Under advantageous selection, the malleability of beliefs leads the lender not to exclude high-risk borrowers as often. The entitlement effect is negative and therefore the lender is not concerned by adverse effects of inducing delusion, and can effectively sells dreams. For low values of s still consistent with inclusion of high-risk borrowers ($s \leq \check{s}$ and $\nu \geq \check{\nu}(s)$, the equilibrium repayment increases with s and the lender does not require collateral. In effect, this is a pure dream-selling transaction whereby the borrower accepts detrimental terms and is compensated in anticipatory terms.

If the weight parameter s is large enough (larger than $s_D^-(\nu)$), the lender induces delusion and uses collateral requirements in a speculative manner. High-risk borrowers receive negative material payoffs for any $s > 0$.

5. Competitive lending market under asymmetric information

The case of a competitive credit market is empirically relevant. Moreover, anticipatory utility concerns and the potential for delusion are likely to be especially important in competitive settings because competitive forces channel rents to borrowers and make the dream of owning a low-risk project even more desirable.

Modeling competition between lenders is associated with some challenges, even in competitive screening settings that are devoid of the added complications our agent's behavioral biases introduce. In the seminal work by Rothschild and Stiglitz (1976), who study competitive insurance markets with adverse selection, firms post menus of contract offers and the borrower chooses her favored contract from the set of available offers. Pure-strategy equilibria of such games may fail to exist, because cross-subsidization is impossible in equilibrium, yet separating menus of contracts are less profitable than cross-subsidizing offers if the proportion of high-risk types is low enough. We are interested in the comparative statics of our credit market's competitive equilibrium with respect to an economy's fundamentals such as the risk-free rate and the profitability of entrepreneurial projects. Equilibrium non-existence poses a serious problem to us because it either makes the comparative statics impossible to derive or, at the very least, difficult to interpret when an exogenous parameter shifts both the equilibrium allocation and the existence region.

We follow the contributions of Wilson (1977), Miyazaki (1977) and Spence (1978) (hereafter WMS), who posit that cream-skimming deviations that attract only low-risk types rely on a rather implausible form of cooperation on behalf of competitors: when facing a firm that employs cross-subsidization from low-risk to high-risk borrowers, poaching low-risk borrowers is profitable for a competing firm, but only if the original firm does indeed carry on servicing high-risk borrowers at a loss. To get around this, these authors make use of an equilibrium concept that, in our setting, translates into the following definition.

Definition 2. *A menu of contracts is a WMS equilibrium if no lender can offer a different menu that earns positive profits right away and continues to be profitable after competitors have dropped all unprofitable contracts in response to the original lender's move.*

Recent papers by Netzer and Scheuer (2012) and Mimra and Wambach (2011) have devised more elaborate structures and rules for the contracting game between firms that, in combination with conventional equilibrium notions like subgame perfection, can give rise to unique equilibrium allocations that correspond to those of the WMS equilibrium.[17] These authors therefore provide a game theoretic foundation for the WMS equilibrium concept.

5.1. Competition for borrowers without anticipatory utility concerns

This section describes the benchmark case of $s = 0$. The following program is the informationally constrained second-best problem of maximizing the low-risk type's utility subject to lenders breaking even, the absence of cross-subsidization from high to low-risk borrowers, and incentive compatibility.

$$\begin{aligned} &\underset{\{C_H \geq 0, C_L \geq 0, R_H, R_L\}}{Max} \quad U_B(\theta_L, R_L, C_L) \\ &s.t. \quad \begin{cases} \nu\, U_I(\theta_H, R_H, C_H) + (1-\nu) U_I(\theta_L, R_L, C_L) \geq 0 & \langle P \rangle \\ U_I(\theta_H, R_H, C_H) \leq 0 & \langle P_H \rangle \\ U_B(\theta_H, R_H, C_H) - U_B(\theta_H, R_L, C_L) \geq 0 & \langle IC_H \rangle \end{cases} \end{aligned} \tag{31}$$

We omit the participation constraints that features in the monopoly setting because we assume that competitive pressures channel enough of the surplus to both types of borrowers to yield payoffs that are larger than their respective reservation utilities. We denote the unique solution of the above program by $((R_{L,\langle 31\rangle}, C_{L,\langle 31\rangle}), (R_{H,\langle 31\rangle}, C_{H,\langle 31\rangle}))$, where the subscripts denote the type a contract is aimed at. Proposition 6 characterizes this allocation and asserts that for $s = 0$, it is the unique WMS equilibrium allocation.

Proposition 6. (Competitive screening without anticipatory utility concerns)
i) When $s = 0$, the contracts $(R_{L,\langle 31\rangle}, C_{L,\langle 31\rangle})$ and $(R_{H,\langle 31\rangle}, C_{H,\langle 31\rangle})$ are the unique WMS equilibrium allocation.
ii) In equilibrium, lenders make zero profits, borrowers are separated and there exists a threshold $\bar{\nu} \in (0, 1)$ such that

- *for $\nu < \bar{\nu}$ low-risk borrowers cross-subsidize high-risk borrowers and collateral requirements are given by $C_{L,\langle 31\rangle} = (2\theta_L \chi (1-\theta_L)(1-\nu))^{-1}((\theta_H - \theta_L)\nu)$ and $C_{H,\langle 31\rangle} = 0$,*
- *for $\nu \geq \bar{\nu}$ contracts make zero profit type by type and collateral requirements are uniquely defined by $C_{H,\langle 31\rangle} = 0$ and the joint saturation of constraints P, P_H and IC_H.*

To establish that $((R_{L,\langle 31\rangle}, C_{L,\langle 31\rangle}), (R_{H,\langle 31\rangle}, C_{H,\langle 31\rangle}))$ constitutes a WMS equilibrium we check that there exist no incentive-compatible and profitable deviating contract offers that attract both borrower types, and no offers that attract one type only, taking into account the removal of unprofitable offers. In the following argument we make use of the fact that the solution to 31 has the properties that the lender offering $((R_{L,\langle 31\rangle}, C_{L,\langle 31\rangle}), (R_{H,\langle 31\rangle}, C_{H,\langle 31\rangle}))$ makes zero profits and that the lending terms offered to high-risk borrowers are efficient, i.e. that $C_{H,\langle 31\rangle} = 0$.

[17] The construction of Netzer and Scheuer (2012) features a vanishing cost of contract withdrawal and that firms can only withdraw their entire menu of contract offers but not individual contracts, while that of Mimra and Wambach (2011) introduces several withdrawal periods and the possibility of firm entry at each withdrawal stage.

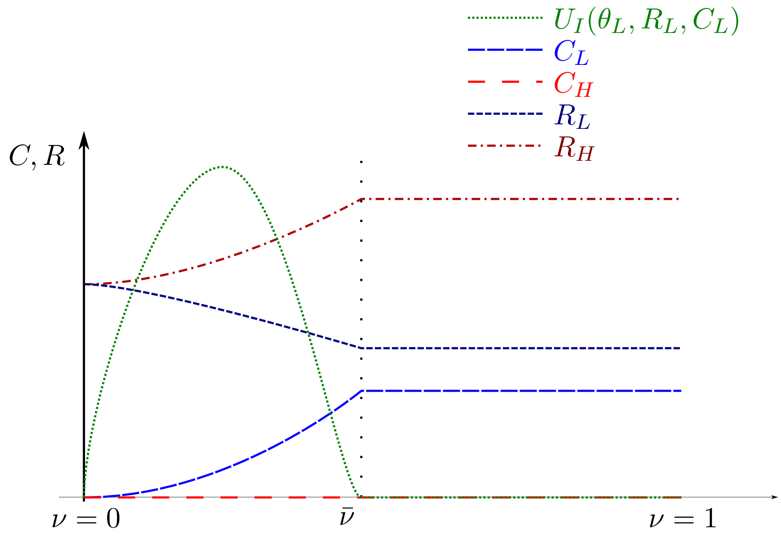

FIGURE 5. Competitive equilibrium offers and cross-subsidy as a function of the proportion of high-risk borrowers when $s = 0$.

If the cross-subsidization constraint P_H is not binding at the optimum, then the solution to 31 is an incentive-constrained optimal or Pareto optimal allocation by construction. It follows that there can be no other incentive-compatible offer that attracts both borrowers while delivering higher profit. If the cross-subsidization constraint is binding at the optimum, then any incentive-compatible offer that attracts both borrowers and yields positive total profit must make positive profits on high-risk types. But since $C_{H,\langle 31\rangle} = 0$ and profits on the high-risk borrowers is zero, there are neither efficiency improvements nor scope for more favorable lending terms to make a deviation that makes profits on the high risk borrower's possible.

Since the lender that offers the WMS allocation makes weakly negative profits on the high-risk type while lending at efficient terms, it is not possible for a deviation to attract only the high-risk type and be profitable. Attracting the low-risk type only may well be profitable, particularly when the cross-subsidization constraint is active. However, such a contract would cause the WMS offer to make losses and be withdrawn. The high-risk type would then also select the deviating contract offer, thereby rendering it unprofitable.

Figure 5 depicts the equilibrium contract design and cross-subsidy as a function of the proposition of high-risk borrowers ν. In the limit, as ν approaches zero, the equilibrium allocation approaches that of a pooling contract with zero collateral. If, contrary to what we assume, collateral was not free at the margin, then the equilibrium contract would also feature pooling and zero collateral for very high values of ν. As we shall see, all this changes when lenders contract with a borrower who has anticipatory utility concerns. Then pooling becomes a more likely occurrence and corresponding collateral requirements are positive.

5.2. **Competitive lending with anticipatory utility concerns**

Once anticipatory utility concerns are introduced, the separation of risk-types becomes more costly. At the same time, letting high-risk borrowers delude themselves makes it possible to exploit differences in beliefs in an effort to channel some of the high-risk borrower's rent to the low-risk borrower. In this section, we define the notions of borrower-preferred separating and pooling contracts, where the contracts we study are borrower-preferred in that they maximize the low-risk borrower's utility, given certain constraints. We then show when the separating and when the pooling contracts constitute the unique equilibrium allocation of our contracting game in the competitive setting.

5.2.1. *Borrower-preferred separating contracts*

In the presence of anticipatory utility concerns, separating allocations not only need to satisfy incentive compatibility, but also have to induce realism. Consider the following program, which maximizes the utility of the low-risk type subject to a break even constraint on the lender, the absence of cross-subsidization from high to low-risk borrowers and an optimal expectations constraint that assures realism on behalf of the high-risk borrower.[18]

$$\underset{\{C_H, C_L, R_H, R_L\}}{Max} \quad U_B(\theta_L, R_L, C_L)$$
$$s.t. \quad \begin{cases} \nu\, U_I(\theta_H, R_H, C_H) + (1-\nu)\, U_I(\theta_L, R_L, C_L) \geq 0 & \langle P \rangle \\ U_I(\theta_H, R_H, C_H) \geq 0 & \langle P_H \rangle \\ (1+s)\, U_B(\theta_H, R_H, C_H) - U_B(\theta_H, R_L, C_L) - sU_B(\theta_L, R_L, C_L) \geq 0 & \langle OE_{H,H} \rangle \end{cases} \tag{32}$$

We define the borrower-preferred separating contracts $((R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}), (R_{H,\langle 32\rangle}, C_{H,\langle 32\rangle}))$ as the solution to (32). Simply solving program (32) then yields the following lemma.

Lemma 8. (Borrower-preferred separating contracts)
Lenders make zero profits, the high-risk borrower remains realistic and there exists a threshold $\bar{\nu}(s) \in (0,1)$ *such that*

- *for* $\nu < \bar{\nu}(s)$ *the solution to* (32) *features cross-subsidization from low to high-risk borrowers and collateral requirements are given by* $C_{L,\langle 32\rangle} = \frac{1}{2(1+s)} \frac{(\theta_H-\theta_L)\,\nu}{\theta_L \chi (1-\theta_L)\,(1-\nu)}$ *and* $C_{H,\langle 32\rangle} = 0$,
- *for* $\nu \geq \bar{\nu}(s)$ *collateral requirements are uniquely defined by* $C_{H,\langle 32\rangle} = 0$ *and the joint saturation of constraints* P, P_H *and* $OE_{H,H}$.

Similar to the case of $s = 0$, we can thus define a cutoff $\bar{\nu}(s)$ in the proportion of high-risk borrowers such that there is cross-subsidization for $\nu \leq \bar{\nu}(s)$, while contracts break even type-by-type for $\nu > \bar{\nu}(s)$. Note that $\bar{\nu}$ increases with s, which implies that for a fixed proportion

[18] For high values of s the solution to this program may violate incentive compatibility of the *low-risk* type. We choose to omit the IC_L because it never binds when our equilibrium features the separating allocation.

of high-risk borrowers ν, the solution of (32) may feature no cross-subsidization for $s = 0$ and cross-subsidization for $s > 0$.

5.2.2. *Borrower-preferred pooling contract*

We define the borrower-preferred pooling contract $(R_{P,\langle 33\rangle}, C_{P,\langle 33\rangle})$ as the contract offer that maximizes the utility of the low-risk borrower subject to a zero-profit constraint, where profits are evaluated at average risk because high-risk types also select the contract.

$$
\begin{aligned}
&\underset{\{C\geq 0, R\geq 0\}}{Max} \quad U_B(\theta_L, R, C) \\
&\qquad s.t. \quad \left\{ U_I(\mathbb{E}[\theta], R, C) \geq 0 \quad \langle P \rangle \right.
\end{aligned}
\tag{33}
$$

If a single contract is offered, high-risk borrowers choose to delude themselves, since they would choose the same offer regardless of whether they are deluded or realistic and biased expectations therefore do not entail the cost of picking an unfavorable contract. Conversely, since a deluded high-risk borrower and a low-risk borrower always select the same contract, any contract that induces delusion may as well be offered in isolation, i.e. be the only contract offer on the table. The following lemma characterizes the borrower-preferred pooling contract and is obtained by solving program (33).

Lemma 9. (Borrower-preferred pooling contract)
Lenders make zero profits, the high-risk borrower deludes herself and the collateral requirement for both types is given by $C_{P,\langle 33\rangle} = \frac{1}{2}\frac{(\theta_H - \theta_L)\,\nu}{\chi\,(1-\theta_L)\mathbb{E}[\theta]}$.

When contract menus are restricted to contain a single contract, high-risk borrowers strictly prefer adopting overoptimistic expectations and there is no incentive for delusion at the margin. Competition then presents firms with the sole challenge of attracting self-assessed low-risk borrowers, while they know the population of potential applicants to be an unbiased sample of the entire population of borrowers. The amount of collateral reflects a trade-off between exploiting the difference in beliefs and limiting the dead-weight losses due to collateral. Any pooling offer that features a different amount of collateral can be improved upon, while any pooling offer with the same amount of collateral but more repayment can be undercut. The borrower-preferred pooling contract thus survives any deviation by a firm using a pooling offer.

5.2.3. *Competitive lending market equilibrium*

With our definitions of the two sets of borrower-preferred contracts in hand we can now characterize the equilibrium of our contracting game with anticipatory utility concerns.

Proposition 7. (Asymmetric information competitive lending market equilibrium)
There exists a unique threshold $s^{**} > 0$ *such that*

- *for* $s < s^{**}$ *the unique WMS equilibrium outcome features realism* ($\tilde{\theta}_H = \theta_H$) *and borrowers select the borrower-preferred separating contracts characterized in lemma 8,*

- *for $s > s^{**}$ the unique WMS equilibrium outcome features delusion ($\tilde{\theta}_H = \theta_L$) and borrowers select the borrower-preferred pooling contract characterized in lemma 9,*
- *s^{**} is the unique value of s that solves $U_B(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}) = U_B(\theta_L, R_{P,\langle 33\rangle}, C_{P,\langle 33\rangle})$.*

The equilibrium allocation is given by the borrower-preferred pooling contract when it yields a higher utility for the low-risk type than the borrower-preferred separating contracts; otherwise, it is given by the borrower-preferred separating contracts. The borrower-preferred separating contracts are offered for low values of s, when screening is relatively cheap. However, as s rises, an increasingly high rent needs to be given up to the high-risk type to keep her realistic, because she would receive greater and greater anticipatory utility benefits from believing she has a low risk of failure. Eventually, for a high enough s, lenders renounce screening and offer a single contract that is taken up by both types. The fact that the borrower-preferred pooling contract can make use of positive collateral requirements to transfer some rent from high to low-risk borrowers then makes the renouncement of screening even more desirable for the low-risk type.

Loosely speaking, the equilibrium allocation maximizes the utility of the low-risk type, subject to whether or not it is preferable for the low-risk type to have the high-risk type delude herself. In equilibrium, as much of the rent as is possible, subject to equilibrium constraints, is thus channeled to the low-risk type. If a profitable deviation that can hope to attract the low-risk borrower exists, this deviation would then necessarily cause the equilibrium allocation to make losses and hence, be withdrawn. This, in turn, renders any deviation aimed at the low-risk type unprofitable. More subtle reasoning also assures that there is no profitable way for a deviation to attract the high-risk borrower. In the case of $s < s^{**}$ this is evident, because a lender offering the borrower-preferred separating contract always makes weakly negative profits on the high-risk type.

5.3. Comparative statics: optimism driven by cheap credit and booms

We explore the comparative statics of the competitive equilibrium by asking how shifts in the parameters impact on the threshold s^{**}. A shift in parameters that decreases s^{**} makes it more likely that we observe the borrower-preferred pooling equilibrium allocation with delusion on behalf of high-risk borrowers, while the likelihood of observing the separating equilibrium allocation and realism on behalf of borrowers increases with parameter shifts that increase the s^{**} threshold.

Proposition 8. (Comparative statics in a competitive lending market)
The threshold s^{**} and therefore the likelihood that we observe realism and the separation of borrower types is

- *increasing in the opportunity cost of funds G*
- *decreasing in the return of the project y.*

We thus find that the incidence of delusion and the collateralization of loans destined for high-risk borrowers is inversely related to the cost of funds G. This entails that we would expect more delusion in an economy in which lenders or banks are able to borrow at a low risk-free rate. The intuition for this is as follows: competitive markets channel the rents created by a decrease in the cost of funds to the borrowers. The lower is G, the higher are the returns that accrue to the borrower when the project is a success because less needs to be repaid to the lender for the lender to break even. This provides a cognitive incentive for the high-risk borrower to delude herself into thinking that the state in which these larger returns are realized occurs relatively more often. As a result the optimal expectations constraint tightens and screening borrowers becomes more expensive and may thus be relinquished.

In our equilibrium, high-risk types only pledge collateral when $s > s^{**}$, which means that our model predicts that an economy with a competitive lending market should see an increase in the use of collateral if interest rates are low. This is precisely what Jiménez et al. (2006) find in their investigation of the likelihood of collateral use in a large sample of Spanish business loans. They also remark that they know of no theory or any previous evidence on the relation between the use of collateral and macroeconomic conditions.

Other things equal, an increase in y has a similar effect on the borrower's returns in the good state of the world as a decrease in G, and hence also increases the incentive to believe that the success probability of the project is high. The comparative statics of our model suggest that we expect overoptimism to prevail during economic booms and when interest rates are low, leading to disillusion later and excessive transfer of collateral following crashes.

6. Efficiency, distributional concerns and borrower protection

We now address the implications of our results on on the distribution of the surplus and on the efficiency of lending terms. Equilibria under both adverse-selection monopoly and competitive lending have the same structure, but the critical thresholds s^* and s^{**} are naturally different, as are the euilibrium utility leves of both agents. The following discussion uses monopoly terminology but results extend to competitive markets.

6.1. Adverse selection: adaptiveness of motivated beliefs

Brunnermeier and Parker (2005) address concerns that agents with optimal expectations might be driven to extinction by agents with rational beliefs by pointing out that optimal expectations respond to the costs of mistakes and that they are therefore harder to exploit than a fixed bias. Moreover, they highlight that some environments favor agents who take on more risk and that there is a biological link between happiness and better health.

We propose a more direct evolutionary benefit of optimal expectations. When agents have private information about the variable that is susceptible to belief manipulation, malleable beliefs make the agent better off in material terms. In our model, under adverse selection, the high-risk borrower strictly benefits from her bias in material terms, whether realism or delusion is featured in equilibrium.

Corollary 2. *In the presence of ex-ante asymmetric information and adverse selection, the high-risk borrower's material payoffs* $U_B(\theta_H, R_H, C_H)$ *are always higher when she has anticipatory utility concerns and may delude herself* ($s > 0$) *than when she does not have anticipatory utility concerns* ($s = 0$)*. Furthermore, her material payoffs increase with the weight of anticipatory concerns as long as* $s \leq s^*$*.*

When $s > s^*$, the boost in material payoffs comes from the deluded high-risk type being offered a contract that is designed according to borrowers' average risk and that has to assure the participation of the low-risk type with her better outside option. When $0 < s < s^*$, material payoffs are higher because of the threat of delusion and the entitlement effect, although delusion happens entirely off the equilibrium path and is precluded by the lender's contract offer.

In a world of asymmetric information it may therefore make sense for parents to encourage their children to envision a rosy future, dream big and believe in themselves. Similar advice can often be found in "How to make it in business" books and the popular press.[19] We provide one rationale for why such advice might have some substance to it.

Our model rhus combines affective and evolutionary benefits of self-deception. Trivers (2011), for example, contends that happiness is not an end in itself in evolutionary terms and that we instead delude ourselves to better delude others, where deluding others may carry evolutionary benefits. We propose a theory of how anticipatory benefits or a desire for rosy expectations may constitute a means to evolutionary ends, in contexts where privately informed agents interact with others in strategic settings.

6.2. Adverse selection: efficiency of loans

Under adverse selection, no borrower is excluded so analysing the efficiency of loans only requires checking the amount of collateral involved in loans. Lending terms become unambiguously more efficient as s increases, provided it remains below s^*. Then the total amount of collateral transferred increases as s crosses the s^* threshold.

Corollary 3. *The prevalence of collateral (unconditional probability of accepting a contract with positive collateral) jumps from* $1 - \nu$ *to* 1 *as* s *becomes larger than* s^**. The expected amount of collateral transferred equals* $(1+s)^{-1}\left((2(1-\theta_L)\chi)^{-1}(\nu(\theta_H - \theta_L))\right)$ *for* $s < s^*$ *and jumps to* $(2(1-\theta_L)\chi)^{-1}(\nu(\theta_H - \theta_L))$ *for* $s > s^*$*.*

in accordance with intuition, the presence of distorted beliefs in equilibrium leads to increased average levels and more frequent transfers of collateral. However, this does not imply that a

[19]See, for example, the op-ed piece entitled "Irrational optimism: An Essential Trait for Entrepreneurs" in Forbes. Available online at http://www.forbes.com/sites/shafqatislam/2012/09/25/irrational-optimism-an-essential-trait-for-entrepreneurs/

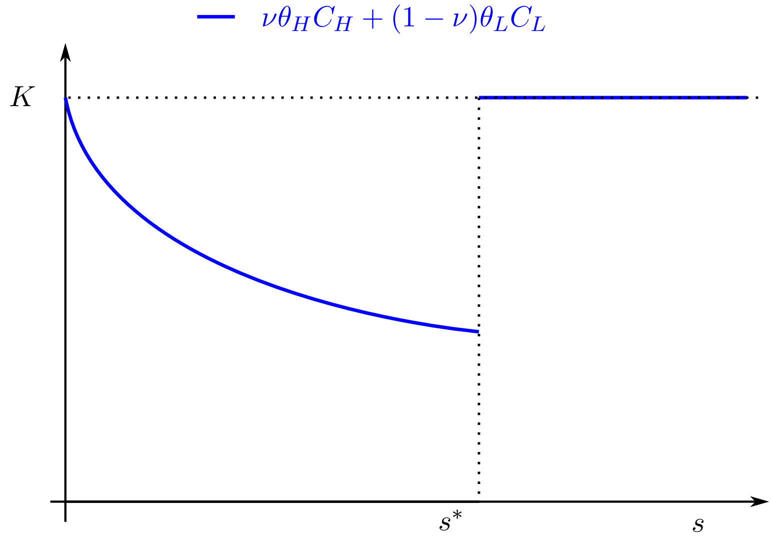

FIGURE 6. The expected amount of collateral that is transferred in equilibrium, where $K = \frac{\nu(\theta_H - \theta_L)}{2(1-\theta_L)\chi}$

transition from separating to pooling equilibrium occasions increases distortions and leads less efficient lending: the separating equilibrium features infrequent (with probability $(1-\nu)\theta_L$) transfer of a relatively large amount of collateral, while the pooling equilibrium features the frequent (with probability $(1-\nu)\theta_L + \nu\theta_H = \mathbb{E}[\theta]$) transfers of a relatively small amount of collateral. Jensen's inequality, applied to the mapping $C \mapsto \delta C^2$, implies that, for a given amount of expected collateral seizure, less spread-out distribution occasions fewer distortions. We therefore cannot rule out that the pooling equilibrium features less value destruction than bechmark screening equilibria.

Since we have seen that positive collateral requirements in contracts aimed at high-risk borrowers always go hand in hand with equilibrium delusion in our model, it is tempting to conclude that a social planner that is exclusively concerned with material efficiency would always like to curb borrowers' potential to delude themselves. However, the case of asymmetric monopoly lending illustrates why the social planner might not eliminate a high-risk borrower's potential for delusion, even if that was possible. As long as s is small enough in the sense of proposition 4, the potential for delusion actually leads to more efficient screening in equilibrium, as it reduces collateral.

6.3. Predatory lending

The flood of foreclosures in the US housing market has renewed interest amongst researchers and policy makers in predatory lending practices. The term has been taken to refer to any lending practice that inflicts harm on borrowers (General Accountability Office, 2004). Bond et al. (2009) make the point that it is to be expected that any loan may sometimes make borrowers worse off ex-post. Predatory lending therefore must imply that borrowers are put in a worse position than their outside option ex ante or in expectation. The concept of predatory lending naturally has us consider a setting in which the lender has some market power and the borrower's expected payoffs, unlike in the case of competition, are close to her outside option.

We have seen in corollary 4 that our psychologically enriched high-risk borrower is always left with negative expected payoffs when she faces a symmetrically informed monopoly lender. In light of our model, one may take the perspective that what looks like predatory lending may in fact be a contract that is tailored to a borrower's desire to believe in a rosy future and, once psychological payoffs are considered, a borrower is never worse off than her outside option. On the other hand, society may have reasons to discount borrowers' anticipatory utility, to dislike negative material payoffs on behalf of borrowers or to be averse to high expected transfers of collateral. For example, overcollateralization and negative material payoffs may exert negative externalities on dependents of borrowers or even the macroeconomy.

Previous papers have focused on cognitive and informational advantages on behalf of the lender to account for predatory lending practices. It is clear, for example, that there is scope for predatory lending if a borrower is naive, confused or easily taken advantage of by outright fraud. A recent economics literature shows that predatory lending may also arise in models in which lenders have some market power and are better informed about the borrower's risk than the borrower is (Morgan, 2007; Inderst, 2008; and Bond et al., 2009).

We argue that material harm to borrowers may arise even in situations in which borrowers and lenders are symmetrically informed ex-ante. In a sense, the informational "advantage" of the lender arises endogenously in our model. In particular, if the beliefs of ex-ante symmetrically informed lenders and borrowers could be elicited after they signed a loan contract, one would find an optimistic bias on behalf of the borrower and realism on behalf of the lender.

Theories of predatory lending that are based on an ex-ante informational advantage on behalf of the lender tend to imply that policy makers should try to erode a lender's informational advantage by educating borrowers. However, such a strategy may be a lot less effective if our theory captures the underlying reality, because a borrower who is willfully ignorant is likely to be less receptive to information that contradicts her desired belief than a borrower that has been mislead or was simply uninformed.

This section illustrates that, in the presence of optimal expectations, the welfare effect of any policy aimed at lending markets is likely to depend on unobserved psychological factors such as the weight the borrower places on anticipatory utility, as well as on how the potential for delusion interacts with factors like the information asymmetries between lenders and borrowers and borrowers' outside options. Rather than providing concrete policy recommendations, we therefore highlight the difficulty involved in designing policy when a model of collateralized loans features agents with our enriched psychology, even if a social planner places no weight on psychological payoffs. This difficulty calls for a better understanding of the psychological processes at play.

7. Conclusion

We study the interaction between borrowers, whose beliefs are malleable and motivated by anticipatory utility, and strategic lenders. Whether borrowers are induced to appraise their projects realistically depends on the contracts designed, and overoptimistic borrowers can be induced to pledge collateral without actuarially fair compensation, leading to worse material outcomes when they face a monopolistic lender under advantageous selection. However, we show that the borrowers' proclivity towards overoptimism does not harm them so long as adverse selection prevails: genuinely lower risks are present in the market and secure better equilibrium terms. Under competition, adverse selection must prevail so overoptimistic borrowers actually benefit from their bias, but this result does not follow from the change in market power.

We consider ex-ante asymmetric information as the only informational friction. While this is the natural departure point, one may wish to introduce moral hazard into a model of financial contracting with borrowers who form optimal expectations. Then, whether or not a lender will attempt to encourage delusion is likely to depend on the borrower's production function and specifically, on whether effort and self-assessed ability are strategic substitutes or complements. It would also be interesting to study the implications of optimal expectations in a model that, unlike our model, allows for credit rationing.

Our framework may fruitfully be applied to other contexts in which firms try to profitably influence their customer's beliefs. Consider, for example, the quality choice and pricing decisions of producers of consumer goods, especially in markets for products that impact on consumers' health, a domain that is associated with a high prevalence of wishful thinking. Moreover, insurance providers may, like the lenders in our model, be concerned about shaping cognitive incentives. Studying how beliefs about health risks are endogenously determined in insurance markets could yield important insights for an empirical literature that has recently taken to employing subjective beliefs as exogenous explanatory variables (see, for example, Finkelstein and McGarry, 2006 and Hendren, 2013).

Previous empirical work on unrealistic optimism in financial markets has generally focused on the effect of optimism on behavior. For example, Landier and Thesmar (2009) show that optimists take on more short-term debt than realists. In our model optimism is an outcome, which points to an interesting empirical endeavor that treats unrealistic optimism as a dependent variable. We may, for example, explore whether the presence of unrealistic optimism, as measured by the wedge between ex-ante subjective expectations and ex-post realizations of risk, is impacted upon by exogenous variation in the risk-free rate or entrepreneurial profits. Of course, our model also provides novel predictions that may be tested even if a suitable measure of optimism cannot be constructed. For example, we predict the opportunity cost of funds and entrepreneurial profits to impact on the correlation between a borrower's risk and her likelihood of pledging collateral in equilibrium.

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Appendix A. Discussion: Restrictions on the timing of delusion

A key element of our model is the interaction between two incarnations of the borrower, her self at $t = 1$ and her self at $t = 2$. The borrower at $t = 1$ has unbiased beliefs and chooses the beliefs of the borrower at $t = 2$, anticipating both the contract choice that different beliefs induce and the anticipatory utility benefits at $t = 2$. If a high-risk borrower at $t = 1$ deludes herself, her $t = 2$ incarnation will act as if to maximize the material payoffs of a low-risk borrower. Both the belief and the subjective utility function of a given borrower at $t = 1$ and $t = 2$ may thus differ.

An important simplifying assumption inherent in the timing of our game is that a lender is able to commit to the set of contracts she offers to the borrower. We thereby rule out scenarios in which a lender offers a contract that induces delusion only to switch it for a more profitable contract once she faces the deluded type at $t = 2$, which, if anticipated by the borrower, may discourage delusion in the first place.

In a similar vein, the timing restricts when a borrower is able to delude herself. We assume that a borrower can only delude herself *before* she picks a contract and, as a result, delusion comes at the cost of potentially choosing a contract that is detrimental to her material payoffs. We therefore rule out that a borrower remains realistic throughout the contract selection stage and only deludes herself once the ink on the contract has dried and there is no more cost of delusion (except perhaps from renegotiation). We think this assumption is realistic because delusion is probably most likely to occur when a borrower is first confronted with the task of evaluating her probability of failure, which happens at $t = 1$ in our model. Consistent with this, evidence from experimental psychology suggests that subjects only respond to incentives to make accurate judgments based on a piece of information that is given to them, if the incentives are provided before they encounter the information Tetlock (1985, 1983).

Another reason for thinking that an agent is unlikely to remain realistic only to delude herself after a contract has been signed, is that the contract she has signed will serve as hard evidence of the borrower's actual risk. Oster et al. (2013) document the denial of health risks by people at risk of Huntington disease. The power of hard evidence in counteracting delusion is reflected in their finding that those who are diagnosed by a genetic test as having Huntington disease behave and believe markedly differently from people who do not have the disease; while those who only receive a more noisy signal in the form of symptoms or the genetic predisposition, revealed by a parent's death from Huntington's disease, behave as if they have no risk of having the disease. Put simply, it appears as though a noisy signal can be readily forgotten, but hard evidence cannot be censored.

We try to ground our modeling assumptions in empirical realism, but some facets of our model are open to discussion. For example, one may wish to allow borrowers to deceive themselves at a time of their choosing rather than imposing that delusion happens before a contract is signed.

Delusion after the contracting stage may have interesting implications for whether or not agents wish to renegotiate contracts.

Appendix B. Robustness to intermediate beliefs

We carry out an analysis of the properties of implementable offers and show a result that plays a similar role to the revelation principle in more standard settings, whereby there is no loss of generality in limiting the lender to offer only three contracts (one for each borrower type and an additional "threat contract" disciplining borrower beliefs). We then investigate the properties of the solution and find few qualitative differences with our benchmark model: borrowers are separated when the weight on anticipatory emotions is small and up to a positive threshold value, then they are pooled and all accept a positive amount of collateral. Equilibrium material utility levels of high-risk borrowers are increasing in the weight of anticipation s up to the point where pooling prevails. On the lender side, we see that even though relaxing the restriction on admissible beliefs that can be induced, the equilibrium payoff of lenders is lower than in our benchmark case. This finding lends credence to the interpretation of motivated beliefs as a form of cognitive moral hazard: it is not too surprising that enlarging the set of possible borrower deviations reduces the payoff of the lender.

In our benchmark model, the borrowers' optimal expectations must belong to the set of actual types: $\tilde{\theta} \in \{\theta_L, \theta_H\}$. That assumption is made for analytical tractability and because it enables a straightforward interpretation of delusion as assuming one belongs to a different risk class. A natural consequence of that assumption is that a version of the revelation principle holds, whereby the number of contracts issued need not exceed the number of borrower types – two. However, the optimal expectations framework has been known since Spiegler (2008) to exhibit sensitiveness to seemingly irrelevant alternatives[20] when beliefs are unrestricted, so the restriction to two contracts may entail a loss of generality once such restrictions are removed.

In this section, we adopt an entirely agnostic viewpoint and allow borrowers to adopt as their belief any distribution over the unit interval, with no reference to the actual objective probabilities. This modeling is in keeping with the original Brunnermeier and Parker (2005) formulation, whereby only support restrictions pertaining to final states constrain the decision maker's beliefs. In addition, lenders are free to offer latent contracts, not destined to be taken up along the equilibrium path, in order to discipline borrowers' beliefs.

[20] Spiegler constructs an example with three possible actions, with a decision maker behaving according to the optimal expectations model and identified with choice rule $c(.)$ such that $c(\{a_s, a_r\}) = \{a_s, a_r\}$ and $c(\{a_s, a_r, a_{r'}\}) = \{a_s\}$. Action a_r is revealed weakly preferred over action a_s and yet drops out of the choice correspondence while action a_s remains upon inclusion of the seemingly irrelevant alternative $a_{r'}$.

B.1. **Intermediate beliefs: timing and notation**

B.1.1. *Timing*

The timing unfolds as in section 2.3, except at $t = 1$.

- $t = 0$: Lenders offer contracts $\mathcal{C} = \{(R_i, C_i)\}_{i \in I_{\mathcal{C}}}$
- $t = 1$: The borrower observes both her type $\theta \in \{\theta_L, \theta_H\}$ and the set of contracts available $\mathcal{C}$. She chooses her belief $\tilde{\theta} \in [0, 1]$ so as to maximize the undiscounted sum of $U_B(\theta, \tilde{R}, \tilde{C})$, her material payoffs at $t = 3$; and $sU_B(\tilde{\theta}, \tilde{R}, \tilde{C})$, her anticipatory payoff at $t = 2$.
- $t = 2$: The borrower chooses her favored contract $(\tilde{R}, \tilde{C}) \in \mathcal{C}$ or her outside option given her belief $\tilde{\theta}$. She receives anticipatory utility from her expectation of material payoffs $\tilde{\theta}$: $sU_B(\tilde{\theta}, \tilde{R}, \tilde{C})$.
- $t = 3$: Material payoffs $U_B(\theta, \tilde{R}, \tilde{C})$ and $U_I(\theta, \tilde{R}, \tilde{C})$ are realized.

The only relevant feature of belief distributions over θ is the implied binary distribution over outcomes, i.e. the probability of failure. A borrower that believes her probability of failure θ to be distributed according to a cdf $F_B(.)$over $[0, 1]$ and a borrower who believes her probability of failure to be a single point $\tilde{\theta}_A \in [0, 1]$ are observationally identical and behave in identical ways provided that $\tilde{\theta}_A$ equals the expectation of distribution F_B. For any given contract set, the set of utility-maximizing contracts is the same, hence the borrower's behavior at time $t = 2$ and her material,and anticipatory utilities are uniquely determined by $\mathbb{E}\left[\tilde{\theta}\right]$, so there is no loss of generality in considering that consumers adopt point beliefs. Distributions may however be a more accurate description of the mental processes at play and reflect significant uncertainty over the parameter θ on behalf of the borrowers.

B.1.2. *Notation*

We first introduce some useful notation and then show that both the set of contracts and the set of types can be naturally ordered in the sense that one contract is favored by the least optimistic types (θ close to 1), the next contract is favored by lower types, and so on up to the lowest (most optimistic) types. This observation enables a parsimonious treatment of incentive compatibility.

The set of contracts $\mathcal{C} := \{(R_i, C_i)\}, i \in I_{\mathcal{C}}$ is finite and all contracts are distinct. For an arbitrary belief θ, denote by $\sigma_\theta \subseteq \mathcal{C}$ the set of utility-maximizing contracts at time $t = 2$ and by V_θ the associated continuation value, satisfying

$$\forall (R_j, C_j) \in \sigma_\theta, U_B(\theta, R_j, C_j) = V_\theta \tag{34}$$

$$\forall (R_i, C_i) \in \mathcal{C}, U_B(\theta, R_i, C_i) \leq V_\theta \tag{35}$$

In the reverse direction, for each contract $(R_j, C_j) \in \mathcal{C}$, we denote by γ_j the set of borrower types that view contract (R_j, C_j) as utility-maximizing:

$$\gamma_j := \{\theta \in [0, 1], U_B(\theta, R_j, C_j) = V_\theta\} \tag{36}$$

Tautologically, we have the equivalency $\theta \in \gamma_j \Leftrightarrow (R_j, C_j) \in \sigma_\theta$. If a given set γ_j is empty, then (R_j, C_j) is a truly irrelevant alternative and shall be omitted from $\mathcal{C}$ without loss of generality.

Lemma 10. (Covering of the belief space induced by the set of contracts)
The sets γ_j are closed intervals and cover the belief space $[0, 1]$.

Proof. Since a finite set has a maximal element, every θ must belong to some set γ_j. Linearity of utility in θ implies that each set γ_j is convex, hence an interval: consider contract (R_j, C_j), two points $\theta, \theta' \in \gamma_j$, $\lambda \in (0, 1)$ and any other contract $(R_i, C_i) \neq (R_j, C_j)$ in $\mathcal{C}$. We show that the convex combination $\lambda\theta + (1 - \lambda)\theta'$ must belong in γ_j.

$$U_B(\theta, R_i, C_i) \leq U_B(\theta, R_j, C_j) \text{ and } U_B(\theta', R_i, C_i) \leq U_B(\theta', R_j, C_j) \quad (37a)$$

$$\lambda U_B(\theta, R_i, C_i) + (1 - \lambda) U_B(\theta', R_i, C_i) \leq \lambda U_B(\theta, R_j, C_j) + (1 - \lambda) U_B(\theta', R_j, C_j) \quad (37b)$$

$$U_B(\lambda\theta + (1 - \lambda)\theta', R_i, C_i) \leq U_B(\lambda\theta + (1 - \lambda)\theta', R_j, C_j) \quad (37c)$$

Closedness follows from the definition of γ_j and continutity of utility with respect to θ: taking a converging sequence (θ_n) with limit θ and any contract (R_i, C_i), we have $U_B(\theta_n, R_j, C_j) \geq U_B(\theta_n, R_i, C_i)$, hence the limit result $U_B(\theta, R_j, C_j) \geq U_B(\theta, R_i, C_i)$, implying $\theta \in \gamma_j$. □

Each set γ_j is a nonempty, convex and closed subset of $[0; 1]$ so its smallest element is well-defined and denoted by

$$t_j := \min \gamma_j \quad (38)$$

We can re-order contracts in $\mathcal{C}$ so that the t_j are decreasing: $t_0 \geq t_1 \ldots$. Then the set of contracts is explicitly ordered, with higher-numbered contracts being more attractive to more optimistic types. We now show two important properties (39) and (40) that pertain to the ordering of contracts. First, an indifference condition must hold at every belief t_j but the lowest one. Second, the ordering of contracts is reflected in an ordering of the net trade $R - C$, the unweighted difference between repayment and collateral.

Lemma 11. (Indifference at t_i and monotonicity of net trade)
A borrower with belief $t_i > 0$ must be indifferent between contract i and contract $i + 1$:

$$\forall (j, j + 1) \in I^2_{\mathcal{C}},\ U_B(t_j, R_j, C_j) = U_B(t_j, R_{j+1}, C_{j+1}) \quad (39)$$

$$\forall (j, j + 1) \in I^2_{\mathcal{C}},\ R_{j+1} - C_{j+1} < R_j - C_j \quad (40)$$

Proof. Property (39): For $\lambda \in (0, 1)$, consider $T_j(\lambda) := (1 - \lambda)t_j + \lambda t_{j+1} \leq t_j$. The function $h(\lambda) := U_B(T_j(\lambda), R_j, C_j) - U_B(T_j(\lambda), R_{j+1}, C_{j+1})$ is continuous in λ. Since $t_j \in \gamma_j$ and $t_{j+1} \in \gamma_j$, we have $h(0) \geq 0$ and $h(1) \leq 0$. If $h(0) = \epsilon > 0$, then there must exist $\bar{\lambda} \in (0,1)$ such that $h(\bar{\lambda}) = \epsilon/2$, but then $T_j(\bar{\lambda}) < t_j$ and $T_j(\bar{\lambda}) \in \gamma_j$, which contradicts (38).

Property (40): The utility function has strictly increasing differences in $(\theta, (R{-}C))$, so the standard monotonicity result of self-selection models obtains : for any $(R', C') \in \sigma_{\theta'}$ and $(R, C) \in \sigma_\theta$, incentive constraints imply

$$\left(R - C - (R' - C')\right)\left(\theta - \theta'\right) \geq 0 \tag{41}$$

a weak inequality version of (40). We argue further that the inequality is strict. Rewrite the indifference condition (39) as

$$\left(R_j - C_j - (R_{j+1} - C_{j+1})\right) t_j + (R_{j+1} - R_j) = 0 \tag{42}$$

If equality holds in (40), then the first term in (42) is zero and therefore $R_{j+1} = R_j$, but then by equality in (40), we have $C_{j+1} = C_j$ also, contradicting the assumption that contracts are all distinct. □

B.1.3. *Restrictions on reservation utilities*

To limit the degrees of freedom involved in the choice of reservation utilities, we assume they are all derived from a single reservation contract $(\bar{R}, \bar{C})$, possibly with $\bar{C} < 0$:

$$\forall \theta \in [0,1]\,, \bar{U}_\theta = U_B\left(\theta, \bar{R}, \bar{C}\right) = (1-\theta)\left(y - \bar{R}\right) - \theta\bar{C} \tag{43}$$

Defining reservation contracts is without loss of generality in the two-type case, as given a pair $(\bar{U}_L, \bar{U}_H)$ we can always define

$$\bar{C} = \frac{(1-\theta_H)\,\bar{U}_L}{\theta_H - \theta_L} - \frac{(1-\theta_L)\,\bar{U}_H}{\theta_H - \theta_L} \tag{44a}$$

$$\bar{R} = y - \frac{\theta_H \bar{U}_L}{\theta_H - \theta_L} + \frac{\bar{U}_H \theta_L}{\theta_H - \theta_L} \tag{44b}$$

Identifying reservation utilities to the payoff of a type-independent contract is an assumption that entails a loss of generality relative to specifying a general term $\bar{U}_\theta$ for all $\theta \in [0,1]$. It does, however, put structure on entitlement effects and allow for a simple interpretations, either litteraly through the representation of a competitive fringe, or in the more abstract sense of correlation between entrepreneurial talents and opportunities outside of the entrepreneur's project. A positive correlation generates a positive entitlement effect, while the absence of correlation is associated with a negative one.

We further impose that the reservation contract is weakly dominated by some contract in $\mathcal{C}$ for any possible belief:

$$\forall \theta \in [0,1]\,, \exists (R_j, C_j) \in \mathcal{C}, U_B(\theta, R_j, C_j) \geq U_B(\theta, \bar{R}, \bar{C}) \tag{45}$$

This assumption is innocuous as long as no borrower is excluded along the equilibrium path and $\bar{C} \geq 0$: any off-path strategy that involves a borrower picking the reservation contract can be replicated by offering the reservation contract as part of the offer $\mathcal{C}$. This assumption is not suitable when dealing with the possibility of exclusion.

B.2. **Intermediate beliefs: implementability**

B.2.1. *Implementation of (contract, belief) pairs*

We have seen that we can associate uniquely the vector $(t_i)_{i\in I_\mathcal{C}}$ of belief thresholds to the offer $\mathcal{C}$. These belief values play an important role as they are the beliefs selected by borrowers with anticipatory utility. With reference to the borrower types θ_H, θ_L, we say that an offer $\mathcal{C}$ implements the contract pair $((R_H, C_H), (R_L, C_L))$ with optimal expectations $(\tilde{\theta}_H, \tilde{\theta}_L)$ if and only if $\{(R_H, C_H), (R_L, C_L)\} \subseteq \mathcal{C}$ and

$$U_B(\tilde{\theta}_H, R_H, C_H) = V_{\tilde{\theta}_H} \text{ and } U_B(\tilde{\theta}_L, R_H, C_H) = V_{\tilde{\theta}_L} \tag{46a}$$

$$\forall\theta \in [0;1], \forall(R_j, C_j) \in \sigma_\theta, U_B(\theta_H, R_H, C_H) + sV_{\tilde{\theta}_H} \geq U_B(\theta_H, R_j, C_j) + sV_\theta \tag{46b}$$

$$\forall\theta \in [0;1], \forall(R_j, C_j) \in \sigma_\theta, U_B(\theta_L, R_L, C_L) + sV_{\tilde{\theta}_L} \geq U_B(\theta_L, R_j, C_j) + sV_\theta \tag{46c}$$

Note that in this definition, the contract subscript H refers to the actual type, not the optimal offer from the viewpoint of an agent with belief $\tilde{\theta} = \theta_H$. Requirement (46a) mandates the optimality of behavior at time 2: incentive compatibility using subjective beliefs. Requirements (46b),(46c) reformulate the optimal expectations criterion at time 1.

B.2.2. *Analysis*

Borrowers with anticipatory utility may bias their beliefs towards optimism so long as the material cost is not too large. As long as a borrower is not indifferent between two contracts at stage 2, a small amount of optimism does not affect the material payoff but improves the anticipatory payoff received at stage 1. If optimism does not cause the borrower to select another contract, there is no material cost and therefore no discipline on the motivated beliefs. Therefore optimal expectations $(\tilde{\theta}_H, \tilde{\theta}_L)$ have to coincide with the belief thresholds t_i of offer $\mathcal{C}$.

Lemma 12. (Indifference condition)
Assume that offer $\mathcal{C}$ implements the contract pair $((R_H, C_H), (R_L, C_L))$ with optimal expectations $(\tilde{\theta}_H, \tilde{\theta}_L)$. Assume that $s > 0$ and $\bar{U}_H > 0$. Beliefs $\tilde{\theta}_H$ and $\tilde{\theta}_L$ must be such that there exist (i, j) both in $I_\mathcal{C}$ such that $\tilde{\theta}_H = t_i$ and $\tilde{\theta}_L = t_j$.

Proof of lemma 12.

Proof. First, note that necessarily, $y - R_H + C_H$ must be positive. If $y - R_H + C_H$ is non-positive, then since $C_H \geq 0$, we have $y - R_H = C_H = 0$ and for any θ, $U_B(\theta, R_H, C_H) = 0$ and $V_{\tilde{\theta}_H} = 0$.

Incentive compatibility implies:

$$0 < U_B\left(\theta_H, \bar{R}, \bar{C}\right) \leq V_{\theta_H} \tag{47a}$$

$$0 < (1+s)\, V_{\theta_H} \tag{47b}$$

$$U_B\left(\theta_H, R_H, C_H\right) + sV_{\tilde{\theta}_H} < (1+s)\, V_{\theta_H} \tag{47c}$$

a violation of (46b). If $\tilde{\theta}_H$ is not minimal in any of the γ sets it belongs to, then in particular for contract (R_H, C_H), there exists another belief $T < \tilde{\theta}_H$ such that $(R_H, C_H) \in \sigma_T$. Adopting such a belief while still picking the same contract delivers a strict increment in anticipatory utility, at no material cost:

$$U_B\left(T, R_H, C_H\right) - U_B\left(\tilde{\theta}_H, R_H, C_H\right) = \left(y - R_H + C_H\right)\left(\tilde{\theta}_H - T\right) > 0 \tag{48a}$$

$$U_B\left(\theta_H, R_H, C_H\right) + sU_B\left(T, R_H, C_H\right) > U_B\left(\theta_H, R_H, C_H\right) + sU_B\left(\tilde{\theta}_H, R_H, C_H\right) \tag{48b}$$

□

While downward incentive compatibility constraints are binding for optimal contracts in standards screening problems, we see here that binding perceived incentive constraints are actually a feature of any implementable belief-contract pair. In particular, unless some borrower is induced to adopt optimal expectation $\tilde{\theta} = 0$, the most optimistic interior belief induced must be sustained by a "threat contract" that precludes a further optimistic distortion of beliefs. That contract is necessarily latent and only active off the equilibrium path.

Geometrical representation in the $\theta - U$ space. Examining the mapping from contracts to utility levels allows us to represent incentive compatibility in a flexible way. The utility level delivered by each contract (R, C) can be seen as a one-degree polynomial in θ, represented by a line with slope $-((y - R) + C)$ and intercept $y - R$. With the addition of the reservation contract $(\bar{R}, \bar{C})$, a menu of contracts defines a menu of downward-sloping lines. Lemma 12 imposes the borrower's indifference at the two belief levels t_H and t_L, which correspond to intersections of the utility schedules. Optimality of the $t = 2$ decision implies that we may identify a menu offer with the upper envelope of these mappings, the properties of which are summarized in the following lemma:

Lemma 13. (Upper-envelope properties)
Any incentive-compatible menu defines a piecewise-affine mapping from beliefs to indirect utilities $\theta \mapsto V_\theta$ with the following properties: Participation, in the sense that $\forall \theta \in [0;1], V_\theta \geq U_B(\theta, \bar{R}, \bar{C})$, continuity, monotonicity and convexity.

The mapping is piecewise-affine and continuous by construction. Monotonicity is inherited from the fact that each component is itself non-increasing as the slope of the portion associated with contract (R, C) is $-((y - R) + C)$. The only nontrivial result is the convexity, which follows from relation (40).

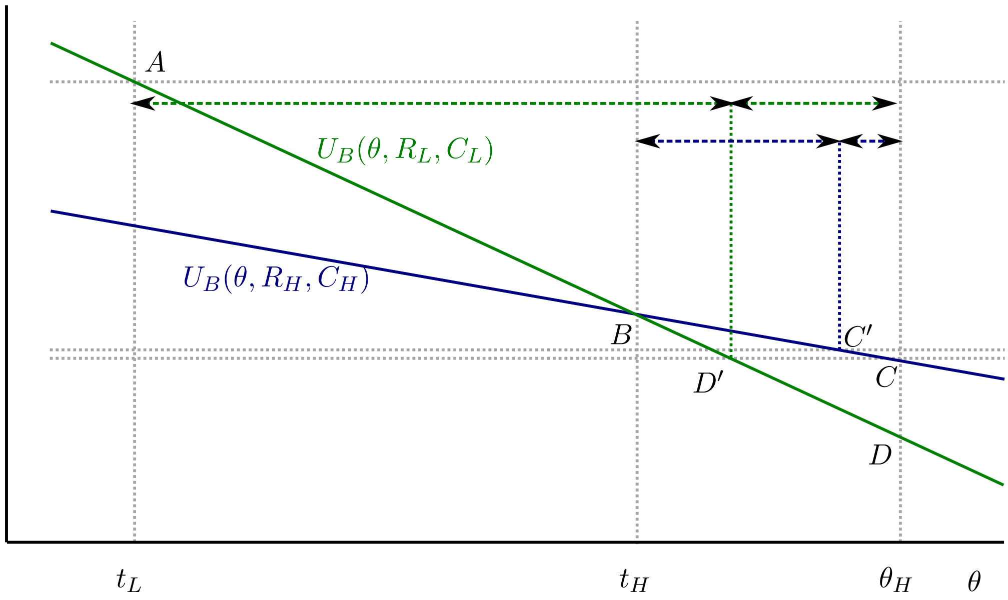

FIGURE 7. Geometrical representation of the optimal expectation requirement.

Just as incentive compatibility translates into an upper envelope property and a sequence of critical threshold beliefs, the optimal expectation requirement (46b) further has a geometrical representation, illustrated in figure 7, in which we assume that the two borrower types pick consecutive contracts as established by lemma 14. The left-hand side of (46b) is the weighted average of $U_B(\theta_H, R_H, C_H)$ and $U_B(t_H, R_H, C_H)$, so geometrically it corresponds to the height of a point C' such that $CC' = sBC'$. Similarly, the right-hand side corresponds to the altitude of point D' such that $DD' = sAD'$. For inequality (46b) to hold, the material loss associated with picking the alternative option (R_L, C_L), which corresponds to the vertical CD distance, must be large enough as to offset the psychological gain associated with the vertical distance between points A and B.

Using this characterization, we can argue geometrically that for any positive s, a minimum amount of optimism is unavoidable. Indeed, realism ($t_H = \theta_H$) corresponds to equating points B, C and D, but then the entire segment AD lies above point B, contradicting the optimal expectation requirement (46b). That the high-risk borrower must exhibit a positive amount of optimism is stated as lemma 16 in the next section, and proven analytically.

B.2.3. *The number of contracts in a monopolist's menu*

We focus on the monopolist's offer and show that a desired material allocation (contracts taken, with no references to the corresponding beliefs) can be brought about by using only 3 contracts at most. The lender only takes into account material outcomes and maximizes

$$\nu U_I(\theta_H, R_H, C_H) + (1 - \nu) U_I(\theta_L, R_L, C_L) \tag{49}$$

The offer $\mathcal{C}$ must implements the contract pair $((R_H, C_H), (R_L, C_L))$ with some optimal expectations $(\tilde{\theta}_H, \tilde{\theta}_L)$ that satisfy requirements (46a) and (46b),(46c) , and are determined as part of the offer: $\tilde{\theta}_H$ and $\tilde{\theta}_L$ are choice variables along with $((R_H, C_H), (R_L, C_L))$.

It is not necessarily obvious how many contracts a monopolist's offer should feature. Even if only two contracts are effectively chosen along the equilibrium path, additional latent contracts act as threats and can be used to discipline the borrower's cognition. On the other hand, additional options introduce additional constraints to the inducement of any given (t_H, t_L) pair of beliefs.

While we cannot appeal to any result with the generality of the revelation principle, we can use elementary arguments to establish that with two borrower types, the monopolist need only use one contract for each type along with "threat" contracts that discipline the beliefs of the low-risk borrower. We first establish that starting from a menu with inactive contracts (not taken up in equilibrium), removing inactive intermediate contracts that lie between active contracts still implements the initial allocation, although the belief implemented for the highest type is then more optimistic. With two borrower types, repeated application of this principle leaves us free to consider offers containing no contracts ranked below the one destined to low risks or above the one destined for high risks. We show further that the high risks' offer need not be ranked higher than 0.

Lemma 14. (Pruning intermediate contracts)
Consider an offer $\mathcal{C}$ with at least four contracts, associated belief levels $t_k, k \in I_{\mathcal{C}}$, such that contracts $((R_H, C_H), (R_L, C_L))$ and beliefs $\tilde{\theta}_H = t_i$ and $\tilde{\theta}_L = t_j$, with $j \geq i + 2$, are implemented, and $-\bar{R} + \bar{C} + R_j - C_j \geq 0$.

The pruned offer obtained by removing contracts $i + 1$ through $j - 1$ implements the contract pair $((R_H, C_H), (R_L, C_L))$ with optimal expectations $\tilde{\theta}_L = t_j, \tilde{\theta}_H = T_i \in [t_j, t_i]$.

The requirement $-\bar{R} + \bar{C} + R_j - C_j \geq 0$, which imposes that contract j be ranked lower than the reservation contract, is necessary for lemma 14 to be general but is innocuous for the case of a monopoly lender, as offers that violate the requirement use an excessive amount of collateral. Furthermore, the contract intended for high-risk borrowers can be assumed to have rank 0.

Lemma 15. (Pruning low-rank contracts)
Consider an implementable offer $\mathcal{C}$ with at least four contracts, associated belief levels $t_k, k \in I_{\mathcal{C}}$, such that contracts $((R_H, C_H), (R_L, C_L))$ and beliefs $\tilde{\theta}_H = t_i$ and $\tilde{\theta}_L = t_{i+1}$ are implemented. The pruned offer obtained by removing contracts 0 through $i - 1$ implements the contract pair $((R_H, C_H), (R_L, C_L))$ with optimal expectations $\tilde{\theta}_L = t_{i+1}, \tilde{\theta}_H = t_i$.

Proof. Removing any contract of rank $i - 1$ or less has no impact on the indifference condition at t_i and the removal of latent contracts cannot make requirement (46b) less easy to satisfy. $\square$

The previous lemmas imply that the monopolist may without loss of generality offer only three contracts: one per borrower type, as well as a "threat contract" disciplining low-risk borrowers.

Lemma 16. (Positive amount of optimism for the high-risk type)
Assume offer $\mathcal{C}$ pruned according to lemmas 15 and 14 implements the contract pair $((R_H, C_H), (R_L, C_L))$ with optimal expectations $(\tilde{\theta}_H, \tilde{\theta}_L)$. Borrower type θ_H displays a positive amount of optimism in the sense that $\tilde{\theta}_H < \theta_H$. Borrower type θ_L displays a non-negative amount of optimism: $\tilde{\theta}_L \leq \theta_L$.

Lemma 17. (A single threat contract is sufficient to discipline low-risk borrowers' beliefs)
Assume the pruned offer $\mathcal{C}$ with belief thresholds (t_i) implements the contract pair $((R_H, C_H), (R_L, C_L))$ with optimal expectations $(\tilde{\theta}_H, \tilde{\theta}_L)$. According to lemmas 15 and 14, assume belief thresholds are $\tilde{\theta}_H = t_0, \tilde{\theta}_L = t_1$. There exists a 3-contract offer that implements $((R_H, C_H), (R_L, C_L))$ with optimal expectations $(\tilde{\theta}_H, \tilde{\theta}_L)$.

B.3. Intermediate beliefs: optimal offer

We have shown that simple optimisation techniques enable us to identify the monopolist's optimal offer by choosing $(\tilde{\theta}_H = t_H = t_0, \tilde{\theta}_L = t_L = t_1)$, $(R_H, C_H) = (R_0, C_0 \geq 0)$ and $(R_L, C_L) = (R_1, C_1 \geq 0)$ so as to maximize

$$\nu U_I(\theta_H, R_H, C_H) + (1 - \nu) U_I(\theta_L, R_L, C_L) \tag{50}$$

subject to requirements (40), (39), (46c), (46b). A suitable (belief-contract) threat is given by $t_2 = 0$ in the solution to (94). Choosing $t_0 \neq t_1$ separates borrowers while choosing $t_0 = t_1$ pools them. The choice of belief levels t_0, t_1 makes the solution to this program unwieldy, but numerically straightforward.

B.3.1. *Qualitative results*

We provide an illustration based on the simulated example in figure 8, with the benchmark offer without intermediate beliefs described in proposition 4 and represented by narrow lines for comparison. The main result that we gather from the comparison is the qualitative similarity between the optimal offers in the benchmark and intermediate belief cases. Both offers approach the standard screening offer as s is close to zero, and the optimal offer exhibits a discontinuous jump around some positive value s^*, although the actual threshold is lower in the benchmark case. For $s < s^*$ the optimal offer is separating, while it is pooling for $s > s^*$, and the pooling offer is characterized by aggregate optimism and features a positive amount of collateral. We view this as a robustness property that reinforces the results of the simpler model with belief restrictions.

Several differences are also apparent on the figure. With intermediate beliefs, both borrowers exhibit optimism for $s > 0$: $t_H < \theta_H$ and $t_L < \theta_L$. This is directly imposed by the interplay between incentive compatibility and optimal expectation requirements, as shown in lemma 16.

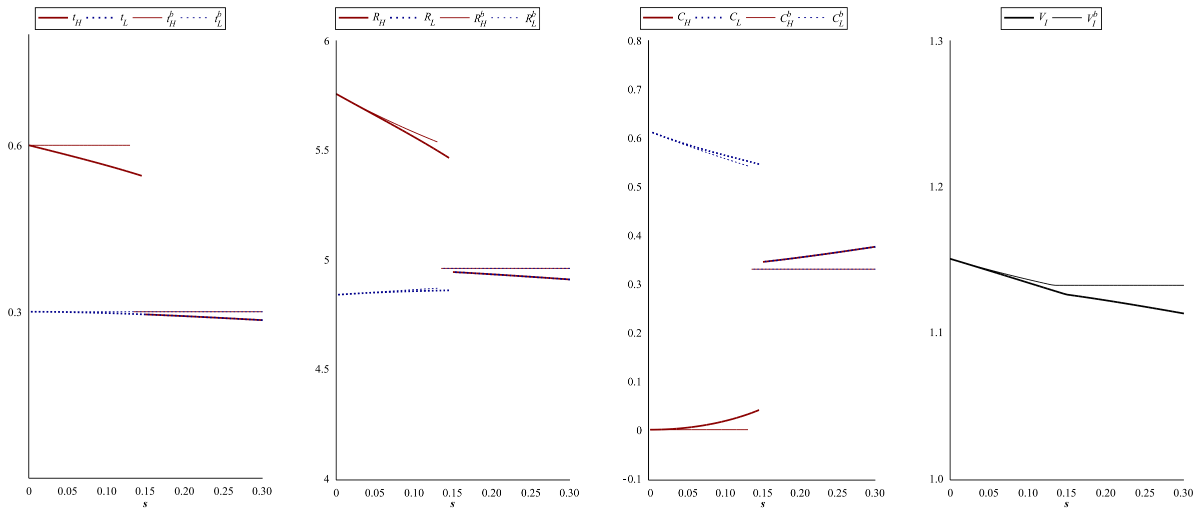

Figure 8. The optimal offer with and without (benchmark) allowing for intermediate beliefs, as a function of the weight parameter s. Parameters: $\theta_L = 3/10, \theta_H = 6/10, \bar{R} = 9/2, \bar{C} = 7/5, G = 2, \chi = 1/2, y = 6, \nu = 3/10$

Consistent with the difference in beliefs, a positive amount of collateral is offered to both types. The distance between t_L and θ_L is positive but remains relatively small as long as borrowers are separated. However, despite the ability to profit from the difference in beliefs, the increased flexibility imposes a net cost on the lender, as evidenced in the fourth panel: whatever the value of $s > 0$, profits are higher in the benchmark case. Enlarging the set of possible borrower deviations does not benefit the lender, justifying a characterization of motivated beliefs as cognitive moral hazard. Finally, note that the pooling offer is constant in the benchmark case and varies with s in the case of intermediate beliefs: the lender surrenders an increased material and psychological rent.

Appendix C. Monopoly screening under alternative assumptions about outside options

If the proportion of high risks is high enough that the monopolist uses the maximum level of collateral attainable until neither type earns rents in the benchmark, introducing small but positive anticipatory utility concerns only affects the low-risk borrowers' contract and separation incentives (the amount of collateral levied on low risks) are *increased.*

The high-risk contract remains fixed whatever the weight on anticipatory utility, while the low-risk contract features an increasing amount of collateral. At the optimal solution, both participation constraints are binding, along with the optimal expectations constraint.

$$\underset{\{C_H \geq 0, C_L, R_H, R_L\}}{Max} \quad \nu\, U_I\left(\theta_H, R_H, C_H\right) + (1-\nu)\, U_I\left(\theta_L, R_L, C_L\right)$$

$$s.t. \quad \begin{cases} U_B\left(\theta_L, R_L, C_L\right) - \bar{U}_L \geq 0 & \langle IR_L \rangle \\ U_B\left(\theta_H, R_H, C_H\right) - \bar{U}_H \geq 0 & \langle IR_H \rangle \\ (1+s)\, U_B\left(\theta_H, R_H, C_H\right) - U_B\left(\theta_H, R_L, C_L\right) - sU_B\left(\theta_L, R_L, C_L\right) \geq 0 & \langle OE_{H,H} \rangle \end{cases} \tag{51}$$

For $\nu \geq \hat{\nu}$ in the sense of proposition 3, there exists a threshold $\hat{s} > 0$ such that, for $s \leq \hat{s}$ the solution to characterized by a tight upper limit on collateral:

$$R_{H,\langle 51 \rangle} = \bar{R} + \frac{\theta_H}{1-\theta_H}\bar{C}, \qquad C_{H,\langle 51 \rangle} = 0 \tag{52a}$$

$$R_{L,\langle 51 \rangle} = \bar{R} - \theta_L\left(y - \bar{R} + \bar{C}\right)s, \qquad C_{L,\langle 51 \rangle} = \bar{C} + (1-\theta_L)\left(y - \bar{R} + \bar{C}\right)s \tag{52b}$$

When $s \geq \hat{s}$ the solution to program (21) is the solution to (68) as above provided that $\hat{s} \leq \bar{s}$. [21]

Appendix D. Monopoly lending under symmetric information with type-dependent reservation utilities

The optimal choice of a monopoly lender in our setup can be characterized according to the distance between the outside options of both types, weighted according to the probability of success. In a symmetric monopoly context, the positive entitlement effect is the counterpart of the adverse selection condition.

Definition 3. (Characterizing the amplitude of the entitlement effect)
We can partition the parameter space according to the difference between outside options, which determine the properties of optimal delusion-inducing offers.
Negative or mild entitlement effect:

$$\frac{\bar{U}_L}{1-\theta_L} - \frac{\bar{U}_H}{1-\theta_H} \leq 0 \text{ or equivalently } \bar{C} \leq 0 \tag{53a}$$

Large entitlement effect:

$$\frac{\bar{U}_L}{1-\theta_L} - \frac{\bar{U}_H}{1-\theta_H} \geq 0 \text{ or equivalently } \bar{C} \geq 0 \tag{53b}$$

By denoting $\zeta := (1-\theta_H)^{-1}(1-\theta_L) > 1$, we can equivalently say that (53b) holds if and only if $\bar{U}_L \geq \zeta \bar{U}_H$. While (53a) holds, we say that the entitlement effect is negative if $\bar{U}_L \leq \bar{U}_H$ and that it is mild if $\bar{U}_L > \bar{U}_H$.

Our assumption in section 3.1 implies that condition (53a) holds and the entitlement effect is null, and as we saw, the lender always finds it optimal to induce delusion on the part of the

[21] If not, the characterization is more complex but that realism-inducing contracts are dominated.

borrower: $\tilde{\theta}_H = \theta_L$. If condition (53b) holds, the monopolist faces a non-trivial choice between inducing delusion or realism. We now proceed to characterize the profit-maximizing contract offers in both the delusion-inducing and realism-inducing classes.

D.1. **Delusion-inducing offers**

A monopolist inducing delusion solves problem (54). Without loss of generality, offers can be limited to a single contract, with the borrower selecting the high-risk's outside option in the off-equilibrium path subgame following $\tilde{\theta} = \theta_H$.

$$\underset{\{C_L \geq 0, R_L\}}{Max} \quad \theta_H \left(1 - \chi C_L\right) C_L + \left(1 - \theta_H\right) R_L - G$$
$$s.t. \quad \begin{cases} U_B\left(\theta_L, R_L, C_L\right) \geq \bar{U}_L \quad (\mu_L) \\ U_B\left(\theta_H, R_L, C_L\right) + sU_B\left(\theta_L, R_L, C_L\right) \geq (1+s)\,\bar{U}_H \quad (\kappa_{H,L}) \end{cases} \tag{54}$$

When the optimal expectations constraint (associated with $\kappa_{H,L}$) is active, the monopolist is providing incentives for delusion at the margin. There is no rent at $t = 1$, in the sense that the borrower is indifferent between delusion and realism. When the constraint is slack, the borrower strictly prefers to be deluded and earns a rent at $t = 1$: the monopolist fails to capture some of the psychological rent that is created, even beyond the loss associated with the use of collateral. The participation constraint, on the other hand, is enforced at stage 2 and relates to the agents' preference for the offered contract over her outside option. A slack constraint is associated with a positive rent, as is the case in section 3.1. The following lemma characterizes possible solutions to program (54), and the associated timing of rents.

Lemma 18. (Delusion-inducing monopolist: candidate solutions)
There are three possible candidate solutions to program (54). Along with the slackness conditions (55), they can be used to recover the optimal offer (R_L, C_L).

$$\left[U_B\left(\theta_L, R_L, C_L\right) - \bar{U}_L\right] \mu_L = 0 \tag{55a}$$

$$\left[U_B\left(\theta_H, R_L, C_L\right) + sU_B\left(\theta_L, R_L, C_L\right) - (1+s)\,\bar{U}_H\right] \kappa_{H,L} = 0 \tag{55b}$$

Slack participation constraint ($t = 2$ rent):

$$C_{L,\langle 56a\rangle} = \frac{1}{2}\,\frac{s\left(\theta_H - \theta_L\right)}{\left(\left(1-\theta_H\right) + s\left(1-\theta_L\right)\right)\theta_H \chi}, \quad \mu_L = 0,\ \kappa_{H,L} = \frac{\left(1-\theta_H\right)}{\left(\left(1-\theta_H\right) + s\left(1-\theta_L\right)\right)} \tag{56a}$$

Active participation constraint, slack delusion-inducing constraint ($t = 1$ rent):

$$C_{L,\langle 56b\rangle} = \frac{1}{2}\,\frac{\left(\theta_H - \theta_L\right)}{\theta_H \chi\left(1-\theta_L\right)}, \qquad \mu_L = \frac{\left(1-\theta_H\right)}{\left(1-\theta_L\right)},\ \kappa_{H,L} = 0 \tag{56b}$$

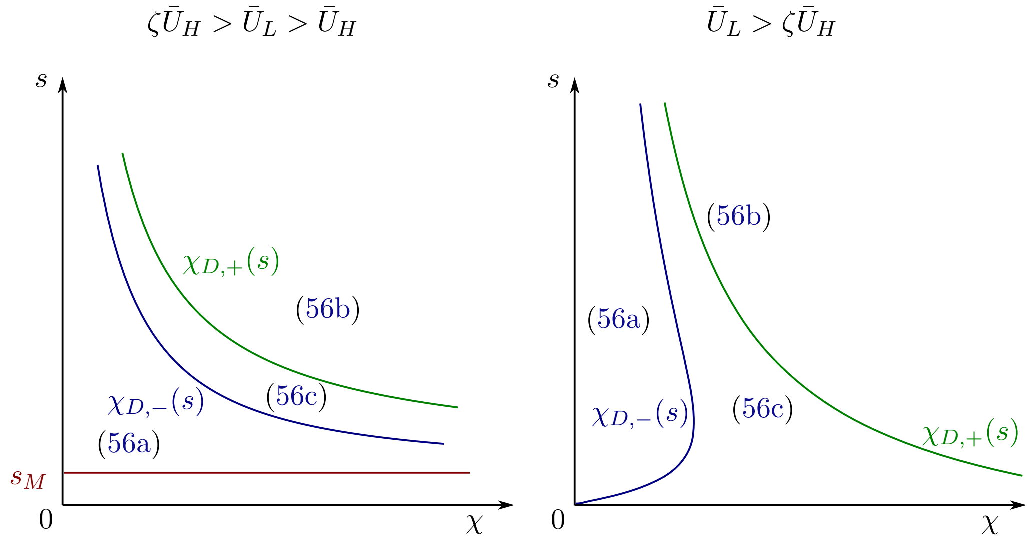

FIGURE 9. Optimal delusion-inducing offers for mild (L) and positive (R) entitlement effect

Both constraints active (no rents):

$$C_{L,\langle 56c\rangle} = (1 + (1-\theta_L)\, s) \left(\frac{(1-\theta_H)\, \bar{U}_L}{(\theta_H - \theta_L)} - \frac{(1-\theta_L)\, \bar{U}_H}{(\theta_H - \theta_L)} \right) + (1-\theta_L)\, s \left(\frac{\theta_H \bar{U}_L}{(\theta_H - \theta_L)} - \frac{\bar{U}_H \theta_L}{(\theta_H - \theta_L)} \right),\ 0 \le \mu_L,\ 0 \le \kappa_{H,L} \tag{56c}$$

Depending on the values of $\bar{U}_L, \bar{U}_H, \chi, s$, these solutions may or may not be valid in the sense of satisfying all the constraints of program (54) and featuring non-negative collateral.

If $\bar{U}_L \le \bar{U}_H$, which implies (53a), the only valid candidate is (56a).

If $\bar{U}_L > \zeta \bar{U}_H$ and for any value of s, there exist thresholds $\chi_{D,-}(s), \chi_{D,+}(s)$ such that $0 \le \chi_{D,-}(s) \le \chi_{D,+}(s)$ and the optimal solution is (56a),(56c),(56b) for χ in $(0, \chi_{D,-}(s))]$, $(\chi_{D,-}(s), \chi_{D,+}(s))$, $(\chi_{D,+}(s), \infty)$ respectively.

If $\zeta \bar{U}_H \ge \bar{U}_L > \bar{U}_H$, there exists a threshold value $s_M > 0$ such that if $s \le s_M$, the only valid candidate is (56a), while if $s > s_M$, there exist thresholds $\chi_{D,-}(s), \chi_{D,+}(s)$ such that $0 \le \chi_{D,-}(s) \le \chi_{D,+}(s)$ and the optimal solution is (56a),(56c),(56b) for χ in $(0, \chi_{D,-}(s))]$, $(\chi_{D,-}(s), \chi_{D,+}(s))$, $(\chi_{D,+}(s), \infty)$ respectively. The properties of these thresholds are summarized in figure 9.

Proof. In appendix E. □

Lemma 18 emphasizes that the positive $t = 2$ rent (and leftover gains from trade) exhibited in our introductory example (section 3.1) is not the only possible feature of delusion-inducing offers. In such a solution, the $t = 2$ rent is non-negative as a by-product of delusion incentives but the outside option of low-risk borrowers does not enter in their determination. In more general settings, participation considerations may come into conflict with the provision of delusion incentives at the margin.

If the entitlement effect is mild, then the participation constraint is slack, resulting in a positive $t = 2$ rent. The marginal tradeoff which determines the amount of collateral taken up is then identical to the tradeoff exhibited in equation (10):

$$\theta_H (1 - 2\chi\, C_L) = \frac{(1 - \theta_H)(\theta_H + s\,\theta_L)}{((1 - \theta_H) + s\,(1 - \theta_L))} \tag{57}$$

By contrast, if the lender can reap the entirety of the benefits of speculative trade and still not worry about incentivizing delusion at the margin, then the marginal trade-off determining collateral simply reflects the difference in ex post beliefs:

$$\theta_H (1 - 2\chi\, C_L) = \frac{(1 - \theta_H)\ \theta_L}{1 - \theta_L} \tag{58}$$

The marginal cost of additional collateral (right-hand side) is a reduction in repayment at a rate of $(1 - \theta_L)^{-1} (\theta_L)$ reflecting low-risk odds, which is lower than the marginal cost of providing delusion incentives, $((1 - \theta_H) + s\,(1 - \theta_L))^{-1} ((1 - \theta_H)(\theta_H + s\,\theta_L))$. The level of collateral satisfying condition (58) therefore exceeds the value satisfying (57).

The reason why participation constraints enter into consideration is related to the entitlement effect. As in the type-independent reservation utility case, the psychological benefit of overoptimism implies that the monopolist can extract a higher repayment than a realistic borrower would accept to pay, and can further make use of collateral as a speculative trade, leveraging the difference in beliefs. However, as the outside prospects of low-risk borrowers improve, the higher repayment may be unacceptable to overoptimistic borrowers at $t = 2$. In that case, the monopolist's program is solved by (56b), in which delusion is warranted without additional cost at the margin, while the monopolist exploits the entirety of gains from trade stemming from the difference in beliefs, or by (56c), in which the borrower is indifferent between realism and denial at $t = 1$, and earns no rent at $t = 2$ either.

When the entitlement effect is large, low-risk borrowers have a high outside option and their repayment has to be lowered in accordance. Therefore, inducing realism on the part of borrowers becomes attractive to the lender, even while foregoing the benefits of speculative trade entirely, because a higher repayment can be demanded from realistic borrowers.

D.2. **Realism-inducing offers**

When designing an incentive scheme which induces a borrower to be realistic, a lender has to compensate for the psychological rent associated with overoptimism, either by sweetening the terms offered to realists or by increasing the material cost of delusion. At face value, borrowers' predisposition towards overoptimism ought to make the enforcement of realism costly. However, since the material penalty for delusion only has to be dealt off the equilibrium path, enforcing realism comes at virtually no cost to the lender, as we prove in lemma 19.

The monopolist can induce realism on the part of the borrower by offering a menu of two offers, one destined for realistic borrowers, which is taken up in equilibrium, and one "threat contract" that a deluded high-risk borrower would prefer to her equilibrium contract. At $t = 1$, the borrower chooses to remain realistic because if she were to become overoptimistic, she could not refrain from agreeing to take on a large amount of collateral, effectively accepting a large side-bet at unfair odds against her success probability, and this would ultimately be detrimental to her welfare.

A suitable realism-inducing menu must satisfy:

$$(1+s)\,U_B\left(\theta_H, R_H, C_H\right) \geq U_B\left(\theta_H, R_L, C_L\right) + sU_B\left(\theta_L, R_L, C_L\right) \tag{59a}$$

$$U_B\left(\theta_L, R_L, C_L\right) \geq \text{Max}\left\{U_B\left(\theta_L, R_H, C_H\right), \bar{U}_L\right\} \tag{59b}$$

$$U_B\left(\theta_H, R_H, C_H\right) \geq \bar{U}_H \tag{59c}$$

Lemma 19. (Realism-inducing menus)
Any H-individually rational (R_H, C_H) contract can be supplemented with a "threat" offer (R_L, C_L) in a manner that induces realism. The offer (R_L, C_L) satisfies incentive compatibility

$$U_B(\theta_L, R_L, C_L) \geq U_B(\theta_L, R_H, C_H), \quad U_B(\theta_H, R_L, C_L) \leq U_B(\theta_H, R_H, C_H)$$

and induces realism:

$$(1+s)U_B(\theta_H, R_H, C_H) \geq U_B(\theta_H, R_L, C_L) + sU_B(\theta_L, R_L, C_L)$$

Among realism-inducing offers, the monopolist's most profitable offer is the zero-collateral offer that leaves no rent to the borrower:

$$C_H = 0, \quad y - R_H = \frac{\bar{U}_H}{(1-\theta_H)} \tag{60}$$

Proof. The last claim follows directly from assumption 4 on the cost of collateral. For the purpose of supplementing an arbitrary offer (R_H, C_H), one suitable threat contract is

$$R_L = R_H - \theta_L\left(y - R_H + C_H\right)s \tag{61a}$$

$$C_L = C_H + s\left(\left(1-\theta_L\right)\left(y - R_H\right) + \left(1-\theta_L\right)C_H\right) \tag{61b}$$

This offer actually leaves a deluded borrower indifferent between the two contracts, and leaves the borrower indifferent between realism and delusion at $t = 1$. For any $s > 0$, other threat contracts exist that do not rely on the borrower's indifference. □

D.3. **Optimal lending**

We now characterise the lender's optimal lending as a function of the weight of anticipatory utility concerns, the difference in reservation utilities and the transferability of collateral.

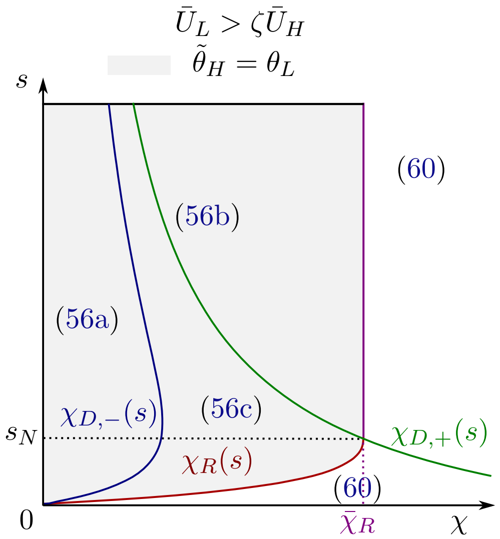

FIGURE 10. Optimal solutions when the entitlement effect is large

Proposition 9. (Optimal lending to high-risk borrowers with type-dependent utilities)
If the entitlement effect is mild or negative (condition (53a) holds), the monopolist induces delusion: $\tilde{\theta} = \theta_L$. Lemma 18 characterizes the optimal offer.
If the entitlement effect is large (condition (53b) holds), the monopolist induces realism if and only if either of two conditions are satisfied:

$$\chi > \bar{\chi}_R := \frac{(\theta_H - \theta_L)}{4\,(1-\theta_L)\,\theta_H \bar{C}} \tag{62a}$$

$$s \le s_N := \frac{\bar{C}}{(1-\theta_L)\left(y - \bar{R} + \bar{C}\right)} \quad \text{and } \bar{\chi}_R \ge \chi \ge \chi_R(s) \tag{62b}$$

with $\chi_R(s) := \left(\theta_H\,(1-\theta_L)\,C_{L,\langle 56c\rangle}{}^2\right)^{-1}\left((\theta_H-\theta_L)\left((1-\theta_L)\left(y-\bar{R}+\bar{C}\right)s\right)\right)$. *Geometrically, the solution is given in figure 10.*

If the entitlement effect is large, outside option considerations imply that the monopolist wants to induce realism. On the other hand, delusion generates a psychological rent which can partly be extracted through the use of collateral.

For $s \ge \bar{s}_N$, $\bar{\chi}_R$ is the threshold above which collateral is not sufficiently transferable (side bets are too costly) to justify the inducement of delusion: even though a higher surplus would be generated, it cannot be extracted cheaply enough, and the monopolist induces realism. If on the other hand the marginal cost of side-bets is relatively low ($\chi \le \bar{\chi}_R$), then inducing delusion is optimal. When $s \le \bar{s}_N$, psychological rents are insufficiently large and the monopolist must give incentives for delusion *at the margin.* This comes at a direct cost and diminishes the value of inducing delusion.

To summarize, when the monopolist chooses to induce realism, this is achieved costlessly through the use of latent "threat" contracts. However, the monopolist chooses to induces delusion when the entitlement effect is not too strong, and even when a strong entitlement effect seems to make the inducement of realism preferable, the monopolist may still induce delusion if there is a strong potential for psychological rent and if the cost of collateral is not too high.

As s converges to zero, the monopoly solution must converge to the rational benchmark solution. Indeed, for s close to 0, $\chi_R(s)$ is also close to $\chi_R(0) = 0$ and therefore $\chi \geq \chi_R(s)$, so one of the conditions (62) must hold: the monopolist enforces realism because the entitlement effect dominates any psychological rent considerations.

In terms of efficiency and equilibrium payoffs, we note that the lender can only profit from the possibility of delusion (equilibrium profit weakly increases with s) while the efficiency of loan contracts can only worsen with $s > 0$, as all equilibria featuring delusion have positive collateral and are inefficient. The following corollary to proposition 9 establishes that a borrower who faces an informed lender always obtains a non-positive material rent.

Corollary 4. *If the lender and the borrower are symmetrically informed, then for any $\bar{U}_H$ and $\bar{U}_L$, the high-risk borrower' equilibrium contract yields worse material payoffs than her outside option, i.e. $U_B(\theta_H, R, C) \leq \bar{U}_H$. Both lender profit and collateral use increase with the weight of anticipatory utility concerns s.*

In equilibrium, a borrower obtains exactly her reservation utility in expected material terms if she is realistic. A deluded borrower obtains strictly less than her reservation utility. Profit-maximizing contracts may therefore therefore harm the borrower in expected terms, an outcome that is in line with some definitions of predatory lending. Furthermore, part of the profit extraction involves the transfer of collateral, which occasions value destruction. Implications of our model for the debate surrounding predatory lending are discussed in more detail in section 6.3.

Appendix E. Proofs omitted in the main text

We make repeated use of the fact that the adverse selection (12) and large entitlement effect (53b) conditions translate into $\bar{C} \geq 0$, while the assumption that $\bar{U}_H < \bar{U}_L$ translates into

$$y - \bar{R} + \bar{C} > 0 \tag{63}$$

E.1. Proof of proposition 1

Proof. We first check that constraints (8a) and (8c) are satisfied. This is readily verified by computing material utilities:

$$U_B\left(\theta_H, R_{L,\langle 9\rangle}, C_{L,\langle 9\rangle}\right) = -\frac{1}{2}\frac{(\theta_H - \theta_L)^2 s^2}{((1-\theta_H) + s(1-\theta_L))^2 \theta_H \chi} < 0 \tag{64a}$$

$$U_B\left(\theta_L, R_{L,\langle 9\rangle}, C_{L,\langle 9\rangle}\right) = \frac{1}{2}\frac{(\theta_H - \theta_L)^2 s}{((1-\theta_H) + s(1-\theta_L))^2 \theta_H \chi} > 0 \tag{64b}$$

Furthermore, a general argument (see section 3.2) establishes that the lender-preferred realism-inducing offer has the borrower select $(R_H, C_H) = (y, 0)$. $(R_L, C_L) = (y, 0)$ is feasible in program

(9), and therefore yields weakly less profit than the optimal solution. This establishes that the monopolist prefers inducing delusion, strictly so if $s > 0$. ☐

E.2. Proof of corollary 4

Proof. This follows directly from a revealed preference argument: since the realism-inducing offer (60) is always feasible, the lender's payoff must exceed $U_I(\theta_H, y - (1-\theta_H)^{-1}\left(\bar{U}_H\right), 0)$. In (65b), we denote the lender's payoff in excess of that level by $\Delta \geq 0$. The sum of material payoffs must equal the total surplus, as specified in (65a), which enables us to bound the material payoff of high-risk borrowers from above in (65c).

$$U_I\left(\theta_H, R, C\right) + U_B\left(\theta_H, R, C\right) = -\theta_H \chi C^2 - G + y - \theta_H y \tag{65a}$$

$$U_I\left(\theta_H, R, C\right) = y\left(1-\theta_H\right) - \bar{U}_H - G + \Delta, \text{ with } \Delta \geq 0 \tag{65b}$$

$$U_B\left(\theta_H, R, C\right) = -\theta_H \chi C^2 - \Delta + \bar{U}_H \leq \bar{U}_H \tag{65c}$$

☐

E.3. Proof of proposition 3

Proposition 3 is a special case of proposition 4 for $s = 0$.

E.4. Proof of lemma 1

Proof. We provide a geometrical argument in line with figure 2 and an analytical argument that is less intuitive but highlights the rationale for the statement on slackness of the incentive compatibility constraint *for $s > 0$ and either $C_L > 0$ or $\bar{U}_H > 0$*. For a given individually rational contract (R_H, C_H), we characterize geometrically the set of (R_L, C_L) offers that complete (R_H, C_H) in a manner consistent with the constraint $OE_{H,H}$ as follows:

$$R_L - R_H \geq \underbrace{\frac{s\left(\theta_H - \theta_L\right)\left(\left(y - R_H\right) + C_H\right)}{\left(\left(1-\theta_H\right) + s\left(1-\theta_L\right)\right)}}_{\geq 0} - \underbrace{\frac{\left(\theta_H + s\,\theta_L\right)}{\left(\left(1-\theta_H\right) + s\left(1-\theta_L\right)\right)}}_{\in\left[(1-\theta_L)^{-1}\theta_L,(1-\theta_H)^{-1}\theta_H\right]}\left(C_L - C_H\right) \tag{66}$$

Collateral is non-negative and $U_B(\theta_H, R_H, C_H) \geq \bar{U}_H \geq 0$. Therefore, $(y - R_H) \geq 0$ and $(y - R_H) + C_H \geq 0$. In the (C, R) space, and in figure 2, this defines a line that is less steep than the iso-utility lines of high-risk borrowers and lies uniformly above that line. Furthermore this line lies above (C_H, R_H) as long as IR_H holds.

Analytically, we translate the assumption that $OE_{H,H}$ and IR_H are satisfied as follows: write $\Delta_1 \geq 0$ for the right-hand side of the IR_H constraint and similarly with $\Delta_2 \geq 0$ and the $OE_{H,H}$ constraint, so

$$U_B\left(\theta_H, R_H, C_H\right) - \bar{U}_H = \Delta_1 \tag{67a}$$

$$(1+s)U_B\left(\theta_H, R_H, C_H\right) - \left(U_B\left(\theta_H, R_L, C_L\right) + sU_B\left(\theta_L, R_L, C_L\right)\right) = \Delta_2 \tag{67b}$$

Suppose that the IC_H constraint is violated and write $-\Delta_3 < 0$ for the value of its right-hand side.

$$U_B\left(\theta_H, R_H, C_H\right) - U_B\left(\theta_H, R_L, C_L\right) = -\Delta_3 \tag{67c}$$

By substituting (67a) and (67b) into (67c), we obtain the following rewriting :

$$\left(\theta_H - \theta_L\right)\, s\left(\Delta_1 + C_L + \bar{U}_H\right) + \left(1-\theta_H\right)\Delta_2 = -\Delta_3\left[1 - \theta_H + s\,\left(1-\theta_L\right)\right] \tag{67d}$$

Since $C_L \geq 0$ and $\bar{U}_H \geq 0$, this contradicts our assumptions on the signs of $\Delta_1, \Delta_2, \Delta_3$.

□

E.5. **Proof of lemma 4**

We note for reference the relaxed program that enables us to describe the solution to the full program (21) without ambiguity.

$$\underset{\{C_H\geq 0, C_L, R_H, R_L\}}{Max} \quad \nu\, U_I\left(\theta_H, R_H, C_H\right) + \left(1-\nu\right) U_I\left(\theta_L, R_L, C_L\right)$$
$$s.t.\quad \begin{cases} U_B\left(\theta_L, R_L, C_L\right) - \bar{U}_L \geq 0 & \langle IR_L\rangle \\ \left(1+s\right)U_B\left(\theta_H, R_H, C_H\right) - U_B\left(\theta_H, R_L, C_L\right) - sU_B\left(\theta_L, R_L, C_L\right) \geq 0 & \langle OE_{H,H}\rangle \end{cases} \tag{68}$$

Proof. We examine the omitted constraints from program (21) and check that the solution to (68) satisfies them.

- $\langle IC_L, OE_{L,L}\rangle$: Evaluated at the optimum, constraints IC_L and $OE_{L,L}$ rewrite into, respectively:

$$0 \leq D_1\left(-2\chi s\theta_L \bar{U}_L\left(1-\theta_L\right)\left(1+s\right)\left(1-\nu\right) + \nu(\theta_H - \theta_L)\right) \tag{69a}$$
$$0 \leq D_2\left(-2\chi s\theta_L \bar{U}_L\left(1-\theta_L\right)\left(1+s\right)\left(1-\nu\right) + \nu\left(1-\theta_H\right)s + \nu\left(1-\theta_L\right)\right), \tag{69b}$$

  with D_1, D_2 positive. Since $\nu\left(1-\theta_H\right)s + \nu\left(1-\theta_L\right) - \nu\left(\theta_H - \theta_L\right) = \nu\left(1-\theta_H\right)\left(1+s\right) > 0$, the relevant bound on s is given by (69a). Isolating s, we obtain the condition given in lemma 4.

- $\langle IC_H\rangle$: We already established that optimal expectation constraint is tighter for the high-risk borrower. We confirm that at the solution, constraint $\langle IC_H\rangle$ rewrites into

$$0 \leq \left[2\theta_L\left(1+s\right)^2\left(1-\theta_L\right)^2\left(1-\nu\right)\chi\right]^{-1}\left(s(\theta_H-\theta_L)^2\nu + 2\chi s\theta_L\bar{U}_L\left(1-\theta_L\right)\left(1+s\right)\left(1-\nu\right)\left(\theta_H-\theta_L\right)\right)$$

  which is satisfied, as we assume (5a) holds, so $\bar{U}_L > \bar{U}_H \geq 0$ and all the other terms are also positive.

- $\langle IR_H\rangle$: we impose that the high-risk reservation utility is non-negative and that assumption $\nu \leq \hat{\nu}$ (as defined in (19)) holds and write $\Delta_1 \geq 0$ for $\hat{\nu} - \nu$. At the candidate solution, constraint IR_H reads as:

$$
\begin{aligned}
0 \leq K^{-1} \times \\
& [2\chi\theta_L (1-\theta_L)(1+s)((1-\theta_H) + s(1-\theta_L))(1-\nu)\Delta_1 \\
& + 2\chi s \theta_L (1-\theta_L)^2 (1+s)(1-\nu)\bar{U}_H \\
& + \nu (1-\theta_L)(\theta_H - \theta_L) s^2 + \nu ((1-\theta_L) + (1-\theta_H))(\theta_H - \theta_L) s]
\end{aligned} \tag{70}
$$

with $K = 2(1+s)^2 \theta_L \chi (1-\theta_L)^2 (1-\nu)(1-\theta_H)(\theta_H - \theta_L)^{-1} > 0$. This is satisfied and therefore constraint IR_H may be omitted. Note that as s converges to 0, the only term that does not vanish is proportional to Δ_1, so the value $\hat{\nu}$ is indeed the critical one. □

E.6. Proof of lemma 5

Proof. The solution identified in the lemma solves program (71).

$$
\begin{aligned}
\underset{\{C_L \geq 0, R_L\}}{Max} \quad & \nu U_I(\theta_H, R_L, C_L) + (1-\nu) U_I(\theta_L, R_L, C_L) \\
s.t. \quad & \left\{ U_B(\theta_L, R_L, C_L) - \bar{U}_L \geq 0 \quad \langle IR_L \rangle \right.
\end{aligned} \tag{71}
$$

The high-risk contract can be taken to be the reservation contract. Delusion is the optimal expectation if $U_B(\theta_H, R_{L,\langle 71 \rangle}, C_{L,\langle 71 \rangle}) + s\bar{U}_L \geq (1+s)\bar{U}_H$. First, note that $C_{L,\langle 71 \rangle} \leq C_{L,\langle 68 \rangle}(s = 0)$ as the ratio between the two equals $\frac{(1-\nu)\theta_L}{(1-\nu)\theta_L + \nu\theta_H}$. Furthermore, $(R_{L,\langle 68 \rangle}, C_{L,\langle 68 \rangle})$ is revealed preferred over the high-risk's outside option and therefore, any offer which delivers the same utility to low risks with less collateral is also incentive-compatible.

□

E.7. Proof of proposition 4

Proof. First, we note that the payoff associated with (68) decreases with s, while the payoff associated with (23) does not depend on s. Second, we can express the difference between the value of the two programs $(V_{I,\langle 68 \rangle} - V_{I,\langle 23 \rangle})$ as proportional to a polynomial of degree 2 in s, and show that there exists a unique a threshold s^* such that the polynomial is positive for small values of s up to a positive threshold s^* and negative for values of s above s^*.

$$V_{I,\langle 68 \rangle} - V_{I,\langle 23 \rangle} = K\left(\alpha_0 + \alpha_1 s + \alpha_2 s^2\right) \text{ with } K > 0, \alpha_0 > 0, \alpha_1 < 0, \alpha_2 < 0 \tag{72a}$$

$$K = \frac{1}{4} \frac{\nu\ (\theta_H - \theta_L)}{\theta_L (1-\theta_L)^2 (1-\nu)(1+s)^2 (\nu\ (\theta_H - \theta_L) + \theta_L)\chi} \tag{72b}$$

$$\alpha_0 = \nu^2 (\theta_H - \theta_L)\ \theta_H \tag{72c}$$

$$\alpha_1 = -2(1-\nu)\theta_L \left[2\chi\ (1-\theta_L)(\nu\ (\theta_H - \theta_L) + \theta_L)\bar{U}_L + \nu\ (\theta_H - \theta_L)\right] \tag{72d}$$

$$\alpha_2 = -(1-\nu)\ \theta_L \left[4\chi\ (1-\theta_L)(\nu\ (\theta_H - \theta_L) + \theta_L)\bar{U}_L + \nu\ (\theta_H - \theta_L)\right] \tag{72e}$$

s^* is therefore uniquely defined as the positive root of the polynomial in (72) and has an explicit analytical expression, if not a particularly simple one.

We furthermore have $s^* < \bar{s}$, which guarantees that relaxing the delusion-inducing program is done with no loss of generality. Indeed, if it were not the case, then for $s = \bar{s}$, constraint IC_L would be saturated in program (21). Then the proposed solution $\left(C_{H,\langle 68\rangle}, C_{L,\langle 68\rangle}, R_{H,\langle 68\rangle}, R_{L,\langle 68\rangle}\right)$ is dominated by a delusion-inducing pooling offer $(C_{H,\langle 68\rangle}, R_{H,\langle 68\rangle})$ which leaves the low-risk borrower indifferent (constraint IC_L is saturated by assumption) and improves the efficiency of trade. This contradicts the assumption that inducing realism is profitable in a neighborhood of $s = \bar{s}$. □

E.8. **Proof of corollary 3**

Proof. The amount of collateral transferred is $(1-\nu)\,\theta_L C_{L,\langle 68\rangle}$ for a realism-inducing offer, or $(\nu\,\theta_H + (1-\nu)\,\theta_L)\,C_{P,\langle 23\rangle}$ for a denial-inducing offer. Straightforward algebra enables us to arrive at the expressions in text. □

E.9. **Proof of corollary 2**

Proof. Taking the derivative of $R_{H,\langle 68\rangle}$ with respect to s yields

$$\frac{dR_{H,\langle 68\rangle}}{ds} = \frac{-\nu\,(\theta_H-\theta_L)^2}{\chi\,\theta_L\,(1-\theta_L)^2\,(1+s)^3\,(1-\nu)\,(1-\theta_H)} - \frac{\bar{U}_L\,(\theta_H-\theta_L)}{(1-\theta_L)\,(1+s)^2\,(1-\theta_H)} < 0 \quad (73)$$

$R_{H,\langle 68\rangle}$ therefore decreases with s, while $C_{H,\langle 68\rangle} = 0$. The material payoff of high-risk borrower is increasing in s so long as the profit-maximizing offer induces realism. The material payoff $U_B(\theta_H, R, C)$ may jump downward as s passes the threshold s^*, but it cannot be lower than in the $s = 0$ case. Note that IC_H is binding in the separating equilibrium of $s = 0$. Since the pooling contract lies on the reservation iso-utility line of the low-risk borrower and our observation on expected collateral transfers (corollary 3) establishes that the level of pooling collateral is lower than $C_{L,\langle 68\rangle}$, it then follows that the pooling allocation yields a higher material payoff to the high-risk borrower than her $s = 0$ allocation does. □

E.10. **Proof of lemma 3**

Proof. Contract (R_H, C_H) only enters program (23) as part of the off-equilibrium utility level $u_H = U_B\,(\theta_H, R_H, C_H)$. In geometrical terms, moving the contract along a high-risk's indifference curve is irrelevant. Furthermore, since the utility level $U_B\,(\theta_H, R_H, C_H)$ only appears in constraint $OE_{H,H}$, decreasing it down to the lowest possible level leads to the maximum lender's profit. □

E.11. **Proof of lemma 6**

Proof. First, any realism-inducing contract that does not exclude high-risk borrowers is dominated. To see this, consider the candidate offer's indirect utility levels $u_H, u_L \geq \bar{U}_L$ and construct the delusion-inducing, no-collateral offer $(R_P, 0)$ such that $R_P = min(R_{P,IR_L}, R_{P,OE_{H,L}})$ where R_{P,IR_L} and $R_{P,OE_{H,L}}$ saturate constraints IR_L and $OE_{H,L}$ (with utility level u_H). By construction, we have $R_{P,OE_{H,L}} \geq R_H$ by lemma 1. If $R_P = R_{P,IR_L}$, no other individually-rational contract makes higher profit with low risks. If $R_P = R_{P,OE_{H,L}}$, then by lemma 1, we must have $u_L > u_B(\theta_L, R_P, 0)$

so moving from contract (R_L, C_L) to (R_P, C_P) improves the lender's payoff. Either way the lender's payoff is higher type by type with offer $(R_P, 0)$.

We therefore restrict attention to realism-inducing contracts that exclude high-risk borrowers, solving either of the following program:

$$\begin{aligned} \underset{\{R_L, C_L\}}{Max} \quad & (1-\nu) U_I(\theta_L, R_L, C_L) \\ s.t. \quad & \left\{ U_B(\theta_L, R_L, C_L) - U_B\left(\theta_L, \bar{R}, \bar{C}\right) \geq 0 \quad (\mu_L) \right. \end{aligned} \tag{74}$$

$$\begin{aligned} \underset{\{R_L, C_L\}}{Max} \quad & (1-\nu) U_I(\theta_L, R_L, C_L) \\ s.t. \quad & \begin{cases} U_B(\theta_L, R_L, C_L) - U_B\left(\theta_L, \bar{R}, \bar{C}\right) \geq 0 \quad (\mu_L) \\ U_B\left(\theta_H, \bar{R}, \bar{C}\right) + sU_B\left(\theta_H, \bar{R}, \bar{C}\right) - U_B(\theta_H, R_L, C_L) - sU_B(\theta_L, R_L, C_L) \geq 0 \quad (\kappa_{H,H}) \end{cases} \end{aligned} \tag{75}$$

□

E.12. Proof of proposition 6

Proposition 6, while an important benchmark, can be treated as a special case of proposition 7. It is discussed in section E.15.2: for $s = 0$, the borrower-preferred pooling offer yields lower utility to the low-risk borrower than the separating offer, and is therefore always dominated. Part *ii* of the proposition is obtained by simply solving program (31).

E.13. Proof of proposition 7: separating offers

To prove proposition 7 we proceed as follows: we first show that there is no profitable separating deviation to the borrower-preferred separating contracts by an entrant using a *separating allocation*, or by an entrant attracting either low-risk types only or high-risk types only, while preserving realism as an optimal expectation. Then we show that the borrower-preferred pooling contract cannot be profitably deviated upon by another pooling offer. Finally, to show that the allocation in proposition 7 is a WMS equilibrium, we argue that $s < s^{**}$ implies that the borrower-preferred separating allocation does not allow for a profitable deviation that induces delusion and pools risk types, and that $s > s^{**}$ implies that the borrower-preferred pooling contract does not allow for a profitable separating deviation. We conclude the proof by showing that the allocation in proposition 7 constitutes the unique equilibrium outcome.

First, we establish that the borrower-preferred separating contract cannot be improved upon by an entrant using a separating allocation, or by an entrant attracting either low-risk types only or high-risk types only, while preserving realism as an optimal expectation.

We shall assume that the solution to program (32) satisfies incentive-compatibility for low risks, i.e. $U_B(\theta_L, R_L, C_L) \geq U_B(\theta_L, R_H, C_H)$. This translates into a restriction that s be lower than some threshold $\check{s} > 0$, which is trivially true for $s = 0$ because of the single-crossing condition.

This restriction does not affect optimal lending because $\check{s} > s^{**}$ and the WMS equilibrium features pooling for $s > s^{**}$.

Suppose a lender issues the borrower-preferred separating menu

$$\mathcal{M}_{\langle 32\rangle} := \left\{\left(R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}\right), \left(R_{H,\langle 32\rangle}, C_{H,\langle 32\rangle}\right)\right\} \tag{76}$$

Following entry, the delusion and contract choice stages must unfold and lead to either one type or both types joining the new offer (an entrant attracting no borrower makes zero profit by definition). We now show that no offer can deliver positive profit, taking into account the potential for withdrawal.

E.13.1. *Constraint P_H is not saturated in the solution to (32)*

We first consider the case where $\nu < \nu(s)$, so the optimal solution features cross-subsidization.

a. No entrant can profitably attract both types with a separating offer.

Suppose that another entrant offers a menu $\mathcal{M}_{dv2} := \{(R_{L,dv2}, C_{L,dv2}), (R_{H,dv2}, C_{H,dv2})\}$ that satisfies the following properties:

$$(1+s)\, U_B\left(\theta_H, R_{H,dv2}, C_{H,dv2}\right) \geq U_B\left(\theta_H, R_{L,dv2}, C_{L,dv2}\right) + sU_B\left(\theta_L, R_{L,dv2}, C_{L,dv2}\right) \tag{77a}$$

$$U_B\left(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}\right) \leq U_B\left(\theta_L, R_{L,dv2}, C_{L,dv2}\right) \tag{77b}$$

$$U_B\left(\theta_H, R_{H,\langle 32\rangle}, C_{H,\langle 32\rangle}\right) \leq U_B\left(\theta_H, R_{H,dv2}, C_{H,dv2}\right) \tag{77c}$$

$$0 < \nu U_I\left(\theta_H, R_{H,dv2}, C_{H,dv2}\right) + (1-\nu)\, U_I\left(\theta_L, R_{L,dv2}, C_{L,dv2}\right) \tag{77d}$$

That is, menu M_{dv2} offers to both types a better allocation than they receive under menu $\mathcal{M}_{\langle 32\rangle}$ while still inducing realism, and yields strictly positive profits. Consider now a small amount $\delta > 0$ and the menu

$$M_\delta := \left\{\left(R_{L,dv2} - \delta, C_{L,dv2}\right), \left(R_{H,dv2} - \delta\frac{(1-\theta_H) + s\,(1-\theta_L)}{(1-\theta_H) + s\,(1-\theta_H)}, C_{H,dv2}\right)\right\} \tag{78}$$

By construction, M_δ satisfies constraint $OE_{H,H}$ and satisfies constraint P for δ small enough. However, we know that the menu $\mathcal{M}_{\langle 32\rangle}$ solves (32) even if constraint P_H is omitted, yet

$$U_B\left(\theta_L, R_{L,dv2} - \delta, C_{L,dv2}\right) > U_B\left(\theta_L, R_{L,dv2}, C_{L,dv2}\right) \geq U_B(\left(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}\right) \tag{79}$$

This contradicts the optimality of $\mathcal{M}_{\langle 32\rangle}$.

b. No entrant can profitably attract high-risk borrowers only.

Suppose now that an entrant offers a contract designed to attract only high-risk borrowers, while low-risk borrowers still choose their contract in $\mathcal{M}_{\langle 32\rangle}$.

$$0 \leq (s+1)\, U_B\left(\theta_H, R_{H,dvH}, C_{H,dvH}\right) - U_B\left(\theta_H, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}\right) - sU_B\left(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}\right) \tag{80a}$$

$$U_B\left(\theta_L, R_{H,dvH}, C_{H,dvH}\right) \leq U_B\left(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}\right) \tag{80b}$$

$$U_B\left(\theta_H, R_{H,\langle 32\rangle}, C_{H,\langle 32\rangle}\right) \leq U_B\left(\theta_H, R_{H,dvH}, C_{H,dvH}\right) \tag{80c}$$

$$0 < U_I\left(\theta_H, R_{H,dvH}, C_{H,dvH}\right) \tag{80d}$$

Using quantities $\Delta_1, \Delta_2, \Delta_3$ all weakly positive, we can write:

$$U_I\left(\theta_H, R_{H,\langle 32\rangle}, C_{H,\langle 32\rangle}\right) = -\Delta_1 \tag{81a}$$

$$U_B\left(\theta_H, R_{H,dvH}, C_{H,dvH}\right) = U_B\left(\theta_H, R_{H,\langle 32\rangle}, C_{H,\langle 32\rangle}\right) + \Delta_2 \tag{81b}$$

$$\left(U_B + U_I\right)\left(\theta_H, R_{H,dvH}, C_{H,dvH}\right) = \left(U_B + U_I\right)\left(\theta_H, R_{H,\langle 32\rangle}, C_{H,\langle 32\rangle}\right) - \Delta_3 \tag{81c}$$

Equation (81c) is non-trivial and obtains because $C_{H,\langle 32\rangle} = 0$. So joint surplus is maximized in the high risks's offer. However, conditions (81) imply that the new offer cannot be profitable.

$$U_I\left(\theta_H, R_{H,dvH}, C_{H,dvH}\right) = -\Delta_1 - \Delta_3 - \Delta_2 \leq 0 \tag{82}$$

c. Attracting low-risk borrowers with a separating offer cannot yield positive profits.
Since constraint P_H is not saturated in the solution to (32), while constraint P must bind, it follows that the incumbent lender is making a strict loss on high-risk borrowers and must therefore withdraw offer $\mathcal{M}_{\langle 32\rangle}$ after low-risk borrowers switch offers. Since the entrant's offer leads to withdrawal of $\mathcal{M}_{\langle 32\rangle}$, the entrant's problem is one of attracting both types while maintaining optimal expectations, as in point a, and we established that the profit from such an offer is nonpositive.

E.13.2. *Constraint P_H is saturated in the solution to* (32)

We now consider the case of $\nu \geq \nu(s)$, when the solution to (32) features no cross-subsidization and profit is zero type by type. The incumbent lender's ability to withdraw offers is now irrelevant, as long as separation incentives are maintained.

d. No new offer can generate profit on high-risk types.
Irrespective of whether it attracts low-risk borrowers, a deviating offer aimed at attracting high-risk types must offer them an inducement:

$$U_B\left(\theta_H, R_H, C_H\right) \leq U_B\left(\theta_H, R_{H,dv}, C_{H,dv}\right) \tag{83}$$

However, the argument in point b applies: the incumbent's offer to high-risk borrowers already maximises joint surplus ($C_H = 0$), while funneling enough rents towards the high-risk type as to drive profits to zero. There can therefore be no profits on the high-risk offer.

e. No entrant can profitably attract both types while maintaining separation.

Any entrant attracting both types must make nonpositive profits on high risks. By the same argument as in point *a*, starting from a profitable separating offer, one can construct an improvement over the solution to program (32). The existence of a profitable deviation would therefore contradict optimality in (32).

f. No entrant can profitably attract high risks while maintaining separation.

Point *d* establishes that entrants' profits on high risks must be nonpositive.

g. No entrant can profitably attract low risks while maintaining separation.

Attracting low-risk borrowers and maintaining separation requires the entrant's offer $\mathcal{M}_{dvL} := \{(R_{L,dvL}, C_{L,dvL})\}$ to satisfy the following properties

$$U_B(\theta_L, R_L, C_L) \leq U_B(\theta_L, R_{L,dvL}, C_{L,dvL}) \tag{84a}$$

$$U_B(\theta_H, R_{L,dvL}, C_{L,dvL}) + sU_B(\theta_L, R_{L,dvL}, C_{L,dvL}) \leq (s+1)\, U_B(\theta_H, R_H, C_H) \tag{84b}$$

$$0 < U_I(\theta_L, R_{L,dvL}, C_{L,dvL}) \tag{84c}$$

Together, (84c) and (84a) imply that $\mathcal{M}_{dvL}$ must improve efficiency with respect to $\mathcal{M}_{32}$, that is, $C_{L,dvL} < C_{L,\langle 32\rangle}$. However, any offer which attracts low-risk borrowers with a lower amount of collateral fails to induce realism and maintain separation. Writing $C_{L,dvL} = C_{L,\langle 32\rangle} - \Delta_1$, we impose that offer $\mathcal{M}_{dvL}$ is attractive

$$R_{L,dvL} = -\frac{-\theta_L \Delta_1 + \theta_L R_L - R_L}{(1-\theta_L)} - \Delta_2$$

which establishes that constraint $OE_{H,H}$ is violated:

$$\begin{aligned}
&(1+s)\, U_B(\theta_H, R_H, C_H) - U_B(\theta_H, R_{L,dvL}, C_{L,dvL}) - sU_B(\theta_L, R_{L,dvL}, C_{L,dvL}) \\
&= -\frac{(\theta_H - \theta_L)\,\Delta_1}{(1-\theta_L)} - \Delta_2\left((1-\theta_H) + s(1-\theta_L)\right) < 0
\end{aligned}$$

and that the entrant's offer therefore fails to induce separation.

E.14. **Proof of proposition 7: pooling offers**

Now, we establish that the pooling offer described in lemma 9 is robust to any deviation aimed at attracting both borrowers on the same contract. The argument presented here is simpler because there is no analog to constraint (P_H) in program (85) below. We may use duality arguments directly. Any deviation that attracts both borrowers, induces delusion and solves program (85) with $\bar{u}_L = U_B\left(\theta_L, R_{P,\langle 33\rangle}, C_{P,\langle 33\rangle}\right)$:

$$\begin{aligned}
\underset{\{R_P, C_P \geq 0\}}{Max} \quad & \nu U_I(\theta_H, R_P, C_P) + (1-\nu)\, U_I(\theta_L, R_P, C_P) \\
s.t. \quad & \left\{ U_B(\theta_L, R_P, C_P) - \bar{u}_L \geq 0 \quad \langle \tilde{IR_L} \rangle \right.
\end{aligned} \tag{85}$$

Since this program is exactly dual to (33), its value is zero. There is thus no strictly profitable offer available to potential entrants.

By the same argument, any pooling offer different from that described in lemma 9 leaves open the possibility for an entrant offering the borrower-optimal pooling offer, with a small increase in repayment, i.e. offering $(R_{P,\langle 33\rangle} + \delta, C_{P,\langle 33\rangle})$, for δ small, still induces delusion, attracts all borrower types, and yields positive profits.

E.15. **Proof of proposition 7: general deviations**

We finally consider the possibility that an entrant may offer a separating allocation when the incumbent's offer pools borrowers, or vice-versa. We argue that the properties of the equilibrium obtain by comparing the values of programs (32)and (33).

E.15.1. *Existence and uniqueness of threshold $s^{**} > 0$*

We begin by arguing that there exists a unique threshold s^{**} such that for $s < s^{**}$ we have $U_B(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}) > U_B(\theta_L, R_{P,\langle 33\rangle}, C_{P,\langle 33\rangle})$ and for $s > s^{**}$ we have $U_B(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}) < U_B(\theta_L, R_{P,\langle 33\rangle}, C_{P,\langle 33\rangle})$. Note that that the value of program (32) strictly decreases with s, while that of program (33) does not depend on s. Indeed, s only appears in the optimal expectation constraint in program (32). Denoting by $\mathcal{L}$ the Lagrangian associated with this program and saturating the $OE_{H,H}$ constraint, we find that the constraint becomes tighter as s increases and, by the envelope theorem, the value of the program strictly decreases (multiplier $\kappa_{H,H}$ is always strictly positive at the solution).

$$\frac{d\mathcal{L}}{ds} = \kappa_{H,H}\left(-R_H - \theta_H y + R_H\theta_H - C_H\theta_H + R_L + \theta_L y - \theta_L R_L + C_L\theta_L\right)$$
$$\frac{d\mathcal{L}}{ds} = -\frac{\kappa_{H,H}\left(\theta_H - \theta_L\right)\left(y - R_L + C_L\right)}{1+s} < 0$$

For $s = 0$, assuming (P_H) is not binding in (32), the separating program provides an increment of value equal to

$$U_B(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}) - U_B(\theta_L, R_{P,\langle 33\rangle}, C_{P,\langle 33\rangle}) =$$
$$\frac{1}{4}\,\frac{\theta_H(\theta_H - \theta_L)^2\nu^3}{\chi\,(1-\theta_L)\,\mathbb{E}\left[\theta\right](1-\nu)\,(1-\mathbb{E}\left[\theta\right])\,\theta_L} > 0$$

For $s = 0$, assuming (P_H) is binding, the low-risk borrower's contract must lie on the $U_I(\theta_L, R, C) = 0$ locus, while the pooling contract lies on the $U_I(\mathbb{E}\left[\theta\right], R, C) = 0$ locus. These two curves never intersect in (C, R) space, as their slopes and height are ranked similarly: $(1-\theta_L)^{-1}G < (1-\mathbb{E}\left[\theta\right])^{-1}G$ and $(1-\theta_L)\theta_L - 2\chi < (1-\mathbb{E}\left[\theta\right])^{-1}\mathbb{E}\left[\theta\right] - 2\chi$. Therefore, the low-risk borrower must attain a higher utility in the separating allocation for $s = 0$. For $s = 0$, the WMS equilibrium is therefore always separating.

As s becomes large, on the other hand, the value of $U_B(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle}) - U_B(\theta_L, R_{P,\langle 33\rangle}, C_{P,\langle 33\rangle})$ becomes arbitrarily small. By continuity of the value functions (which follows directly from Berge's theorem), monotonicity and the conditions at the boundaries, there exists a unique threshold s^{**} that equalizes the values of both programs.

E.15.2. *Robustness to general deviations*

We construct an equilibrium in which an incumbent lender offers the equilibrium allocation described in proposition 7 and an additional inactive or *latent* offer $(R = G/(1-\theta_H), C = 0)$ when $s > s^{**}$.

For $s < s^{**}$, the incumbent lender offers the separating allocation. An entrant aiming to attract both borrowers while inducing delusion must attract low-risk borrowers and the entrant's offer cannot achieve more than the value of the dual program (85), with $\bar{u}_L = v_1 = U_B(\theta_L, R_{L,\langle 32\rangle}, C_{L,\langle 32\rangle})$. We know that the value of program (85) with $\bar{u}_L = v_2 = U_B(\theta_L, R_{L,\langle 33\rangle}, C_{L,\langle 33\rangle})$ equals 0, and since $s < s^{**}$, we have $v_1 > v_2$. Therefore, entry cannot be profitable. Furthermore, we established in section E.13 that no entrant can obtain positive profits by offering realism-inducing contracts.

For $s > s^{**}$, the incumbent lender offers the pooling allocation and the latent contract $(R = G/(1-\theta_H), C = 0)$. An entrant aiming to achieve positive profits must attract both borrowers and induce separation. Furthermore, the existence of the latent contract translates into a non-positivity constraint on profit, as we show below. Equation (86a) translates the requirement that the entrant's offer attract high risks, while (86b) follows from surplus accounting. Together, they imply (86c), or $U_I(\theta_H, R_H, C_H) \leq 0$, precisely constraint (P_H) in program (32).

$$U_B(\theta_H, R_H, C_H) = U_B\left(\theta_H, \frac{G}{-\theta_H + 1}, 0\right) + \Delta_1 \tag{86a}$$

$$U_B(\theta_H, R_H, C_H) + U_I(\theta_H, R_H, C_H) = (-\theta_H + 1)\, y - G - \Delta_2 \tag{86b}$$

$$U_I(\theta_H, R_H, C_H) = -\Delta_2 - \Delta_1 \tag{86c}$$

By definition of threshold s^{**}, and by the same duality argument as above, the profit from an entrant's offer cannot be strictly positive.

This section established that the allocation described in proposition 7 is indeed an equilibrium. Next, we argue that for any value $s < s^{**}$ or $s > s^{**}$, the equilibrium allocation is in fact unique.

E.15.3. *Equilibrium uniqueness*

Competition between lenders implies that candidate equilibrium offers must leave no aggregate profit to the incumbent lender. For $s < s^{**}$, consider any other zero-profit realism-inducing offer $M_{ac} := \{(R_{H,ac}, C_{H,ac}), (R_{L,ac}, C_{L,ac})\}$ as a candidate equilibrium. Lenders cannot make a strictly positive profit on high-risk borrowers. Indeed, since the incentive constraint (IC_L) is slack, a single contract $(R_{H,ac} - \epsilon, C_{H,ac})$ would otherwise be profitable for $\epsilon > 0$ small enough. It follows that any candidate equilibrium allocation must solve program (32) and therefore, must achieve a lower value than its solution. Thus, from the true equilibrium allocation $\mathcal{M}_{\langle 32\rangle}$, one can construct an improvement offer simply by using offer $\left\{(R_{L,\langle 32\rangle} + \delta, C_{L,\langle 32\rangle}), (R_{H,\langle 32\rangle}, C_{H,\langle 32\rangle})\right\}$, for δ small enough, which still induces separation and achieves strictly positive profit.

Likewise for $s > s^{**}$, the borrower-preferred pooling allocation generates the highest possible surplus among pooling allocations. Any other profitable offer generates less surplus, and therefore must leave room for entry.

E.16. **Proof of proposition 8**

Proof. Assume that the solution to (32) features cross-subsidization. Implicit differentiation is facilitated by the fact that only the value of program (32) is decreasing in s, while that of program (33) does not depend on s. We differentiate the values of (33) and (32) with respect to the relevant parameters and study their difference. If it is positive, the threshold s^{**} decreases with the relevant parameter, so delusion is more likely. For an increase in y, we obtain:

$$\frac{(1-\theta_L)\;\nu\, s\,(\theta_H-\theta_L)}{s\,(1-\theta_L)+(1-\mathbb{E}\,[\theta])}$$

For a decrease in G, we obtain:

$$\frac{(1-\theta_L)\;\nu\, s\,(\theta_H-\theta_L)}{(1-\mathbb{E}\,[\theta])\,(s\,(1-\theta_L)+(1-\mathbb{E}\,[\theta]))}$$

The same arguments apply if the solution to (32) does not feature cross-subsidisation. First, write ω for the positive real root of the following polynomial in X:

$$-4\,\chi\,\theta_L\,(\theta_H-\theta_L)\;((1-\theta_H)+s\,(1-\theta_L))\,(G+sy(1-\theta_L))+2\;(\theta_H-\theta_L)\;X+X^2$$

For an increase in y we obtain:

$$\frac{(1-\theta_L)\;(\theta_H-\theta_L)\;\omega\, s}{(\omega+\theta_H-\theta_L)\,((1-\theta_H)+s\,(1-\theta_L))}$$

For a decrease in G, we obtain an expression of the form K_n/K_d with

$$K_d = ((\theta_H-\theta_L)+\omega)\,((1-\theta_H)+s\,(1-\theta_L))\;(1-\mathbb{E}\,[\theta])>0$$

$$K_n = -\,(\theta_H-\theta_L)\;(1-\theta_L)\,(1-\nu)\,(1+\;s)\,\omega-(\theta_H-\theta_L)^2\nu\;((1-\theta_H)+s\,(1-\theta_L))<0$$

□

E.17. **Proof of lemma 18**

Proof. We shall use the equivalent restatement of reservation utilities as reservation contracts in this proof. For any $s \geq 0$, a single value of C_L saturates simultaneously both constraints in program (54) and is given by $C_{L,\langle 56c\rangle}$, which may be rewritten as follows:

$$C_{L,\langle 56c\rangle} = \bar{C}+(1-\theta_L)\left(y-\bar{R}+\bar{C}\right)s \tag{87}$$

If $\bar{U}_L < \bar{U}_H$ (hence $y-\bar{R}+\bar{C} < 0$ and $\bar{C} < 0$), $C_{L,\langle 56c\rangle}$ is non-positive, and the optimal expectation constraint implies participation for any non-negative C_L, as shown below: imposing

$C_{L,\langle 56c\rangle} = -\Delta_a \leq 0$ and the optimal expectation constraint

$$U_B(\theta_H, R_L, C_L) + sU_B(\theta_L, R_L, C_L) - (1+s)U_B\left(\theta_H, \bar{R}, \bar{C}\right) = \Delta_b \geq 0$$

We see that the participation constraint must hold for any non-negative C_L:

$$U_B(\theta_L, R_L, C_L) - U_B\left(\theta_L, \bar{R}, \bar{C}\right) = \frac{(1-\theta_L)\Delta_b + (\theta_H - \theta_L)\Delta_a + C_L(\theta_H - \theta_L)}{((1-\theta_H) + s(1-\theta_L))} \geq 0 \quad (88)$$

This establishes the first part of lemma 18.

If $\zeta \bar{U}_H \geq \bar{U}_L > \bar{U}_H$, then $\bar{C} < 0$ and $y - \bar{R} + \bar{C} > 0$. $C_{L,\langle 56c\rangle}$ is negative for $s \leq s_M$, where s_M is given by:

$$s_M = \frac{-\bar{C}}{(1-\theta_L)\left(y - \bar{R} + \bar{C}\right)} > 0 \quad (89)$$

If $s > s_M$ or $\bar{U}_L > \zeta \bar{U}_H$, then $C_{L,\langle 56c\rangle}$ is necessarily positive. The threshold functions $\chi_{D,-}$, $\chi_{D,+}$ are implicitly defined by equations $C_{L,\langle 56c\rangle} = C_{L,\langle 56a\rangle}$ and $C_{L,\langle 56c\rangle} = C_{L,\langle 56b\rangle}$, exist and are uniquely defined as long as $C_{L,\langle 56c\rangle} > 0$.

$$\chi_{D,-}(s) = \frac{1}{2}\frac{s(\theta_H - \theta_L)}{((1-\theta_H) + s(1-\theta_L))\theta_H C_{L,\langle 56c\rangle}} \quad (90a)$$

$$\chi_{D,+}(s) = \frac{1}{2}\frac{(\theta_H - \theta_L)}{(1-\theta_L)\theta_H C_{L,\langle 56c\rangle}} \quad (90b)$$

It is readily confirmed that $\chi_{D,+}(s) > \chi_{D,-}(s)$ and that $\chi_{D,+}$ is decreasing:

$$\chi_{D,+}(s) - \chi_{D,-}(s) = \frac{1}{2}\frac{(\theta_H - \theta_L)(1-\theta_H)}{((1-\theta_H) + s(1-\theta_L))(1-\theta_L)\theta_H C_{L,\langle 56c\rangle}} \quad (91a)$$

$$\chi'_{D,+}(s) = \frac{-\left(y - \bar{R} + \bar{C} + (C_{L,\langle 56c\rangle} - \bar{C})\right)(\theta_H - \theta_L)}{2\theta_H(1 + s(1-\theta_L))C_{L,\langle 56c\rangle}{}^2} \quad (91b)$$

Both $\chi_{D,+}$ and $\chi_{D,-}$ converge to 0 as s becomes large. $\chi_{D,-}$ is everywhere decreasing provided that $\bar{C} \leq 0$, otherwise it is increasing in the neighborhood of $s = 0$ and up to some positive value $s = (1-\theta_H)^{1/2}\bar{C}^{1/2}(1-\theta_L)^{-1}\left(y - \bar{R} + \bar{C}\right)^{-1/2}$, then decreasing. These properties can be verified on examination of equation (91c):

$$\chi'_{D,-}(s) = \frac{-(\theta_H - \theta_L)\left((1-\theta_L)^2 s^2\left(y - \bar{R} + \bar{C}\right) - (1-\theta_H)\bar{C}\right)}{2\theta_H(1-\theta_H + s(1-\theta_L))^2\left((1-\theta_L)\left(y - \bar{R} + \bar{C}\right)s + \bar{C}\right)^2} \quad (91c)$$

□

E.18. Proof of lemma 16

Proof. We consider borrower type θ_H and the associated belief t_i, with contract $(R_H, C_H) = (R_i, C_i)$. By lemma 12, there must exist another belief-contract pair indexed by $i+1$, satisfying

$t_{i+1} \leq t_i$ and such that:

$$U_B(t_i, R_i, C_i) = U_B(t_i, R_{i+1}, C_{i+1}) \tag{92a}$$

$$R_i - C_i - R_{i+1} + C_{i+1} = \Delta_a > 0 \tag{92b}$$

In addition, the optimal expectation requirement (46b) must hold as well.

$$\begin{aligned}&[U_B(\theta_H, R_i, C_i) - U_B(\theta_H, R_{i+1}, C_{i+1})]\\&+ s[U_B(t_i, R_i, C_i) - U_B(t_{i+1}, R_{i+1}, C_{i+1})] = \Delta_b \geq 0\end{aligned} \tag{92c}$$

We can now use repeated substitutions and obtain the result

$$\theta_H - t_i = \frac{\Delta_b + s(t_i - t_{i+1})(y - R_i + C_i) + s(t_i - t_{i+1})\Delta_a}{\Delta_a} \geq 0 \tag{93}$$

The same argument, applied to borrower type θ_L, establishes that $\tilde{\theta}_L \leq \theta_L$. It follows that either $t_i = t_{i+1}$ but then $t_i \leq \theta_L < \theta_H$, or $t_i < t_{i+1}$ and inequality (93) is then strict. □

E.19. Proof of lemma 17

Proof. Call (R_T, C_T) a candidate threat contract, constructed so as to satisfy the optimal expectations requirement with equality (a contract with higher amounts of collateral would break such indifference).

$$U_B(t_i, R_L, C_L) = U_B(t_i, R_T, C_T) \tag{94a}$$

$$R_L - C_L - R_T + C_T = \Delta_a \tag{94b}$$

$$sU_B(t_i, R_L, C_L) + U_B(\theta_L, R_L, C_L) = sU_B(0, R_T, C_T) + U_B(\theta_L, R_T, C_T) \tag{94c}$$

A value of collateral C_T satisfying such properties can be found and is higher than C_L, hence nonnegative:

$$C_T = C_L + \frac{\Delta_a((y - R_L + C_L + \Delta_a)s + (1 - \theta_L)\Delta_a)}{(y - R_L + C_L + \Delta_a)s + \Delta_a} \tag{95}$$

□

E.20. Proof of lemma 14

Proof. We show the property holds in the case of three consecutive contracts, or $j = i+2$. Repeated application of the restricted property implies the general result. We show first that the belief T_i induced by the pruned menu lies between $t_i + 2$ and t_i. T_i is defined by the equality:

$$U_B(T_i, R_i, C_i) = U_B(T_i, R_{i+2}, C_{i+2}) \tag{96}$$

Furthermore, by properties (40) and (39), the following relations must hold:

$$- R_{i+1} + C_{i+1} + R_i - C_i =: \Delta_a > 0 \tag{97a}$$

$$- R_{i+2} + C_{i+2} + R_{i+1} - C_{i+1} =: \Delta_b > 0 \tag{97b}$$

$$(1 - t_i)(y - R_{i+1}) - C_{i+1}t_i - (1 - t_i)(y - R_i) + C_i t_i = 0 \tag{97c}$$

$$(1 - t_{i+1})(y - R_{i+2}) - C_{i+2}t_{i+1} - (1 - t_{i+1})(y - R_{i+1}) + C_{i+1}t_{i+1} = 0 \tag{97d}$$

By repeated substitution, we obtain

$$t_i - T_i = (\Delta_b + \Delta_a)^{-1} (\Delta_b (t_i - t_{i+1})) \geq 0 \tag{98a}$$

$$t_{i+2} - T_i = t_{i+2} - t_{i+1} - (\Delta_b + \Delta_a)^{-1} (\Delta_a (t_i - t_{i+1})) \leq 0 \tag{98b}$$

Thus, as announced, the pruned offer induces a belief T_i that is more optimistic than t_i, while the material terms are preserved since the contract (R_H, C_H) is unchanged. Therefore, since the optimal expectation constraint (46b) holds in the initial offer, it holds again in the pruned offer, with the left-hand side increased by $s(y - R_H + C_H)(t_i - T_i)$ and the right-hand side weakly diminished (there are fewer options in the pruned offer).

Finally, the requirement $-\bar{R} + \bar{C} + R_L - C_L \geq 0$ guarantees that the reservation contract is dominated for a borrower with belief T_i: first write

$$- \bar{R} + \bar{C} + R_{i+2} - C_{i+2} = \Delta_c \geq 0 \tag{99a}$$

$$U_B(t_{i+2}, R_{i+2}, C_{i+2}) - U_B\left(t_{i+2}, \bar{R}, \bar{C}\right) = \Delta_d \geq 0 \tag{99b}$$

The following confirms the result:

$$\begin{aligned}(\Delta_b + \Delta_a)\left[U_B(T_i, R_i, C_i) - U_B\left(T_i, \bar{R}, \bar{C}\right)\right] = \\ \Delta_a\Delta_d + \Delta_b\Delta_d + \Delta_c\Delta_a(t_i - t_{i+1}) + \Delta_c\Delta_a(t_{i+1} - t_{i+2}) + \Delta_b\Delta_c(t_{i+1} - t_{i+2}) \geq 0\end{aligned} \tag{100}$$

For the case where $j > i+2$, we may apply the property to i, $i+1$, $i+2$ and prune contract $i+1$, then proceed to prune $i+2$, iteratively. This process defines a strictly decreasing sequence of T_i thresholds that is bounded strictly between t_i and t_j. The condition for contract j $-\bar{R} + \bar{C} + R_j - C_j \geq 0$ and (40) imply that for any $i \leq j$, $-\bar{R} + \bar{C} + R_i - C_i \geq 0$ holds, so the conditions are in place to apply the restricted property at each step. □