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Contracting and Regulation under the threat of duplication*

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Abstract

We study infrastructure investment by an incumbent, when a local government may contract with the incumbent and subsidies him to improve its market offers. Without asymmetries of information between the incumbent and the local authorities, or when entry is not possible, allowing local subsidies to the incumbent improves efficiency. We then highlight the complex interaction between the subsidies to the incumbent, and the subsidies to entry, under asymmetric information. Complexity arises due to the interaction between beliefs and actions, resulting in a multiplicity of equilibria. This may create hold-up problems and under-investment.

At last, we discuss the implications for the choice of a national price-cap in the context of a regulated industry. We show that the regulator must avoid an excessively tight regulation that depresses profit margins, but also an excessively lenient regulation that generates hold-up.

Preliminary - Comments welcome

1 Introduction

For the last twenty years, many industries with large infrastructure have been either privatized and subject to regulation or simply opened to competition. This move toward a less centralized way of managing infrastructure-intensive industries has led the newly privatized firms to make their choices on the basis of financial return more than on the basis on the social benefits of these activities. Many public authorities have then tried to influence their

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choice, either by keeping some stakes in these firms or by directly having an impact on the market conditions. In particular, even in industries regulated by independent bodies, some public intervention occurred through the imposition of rules, by contracting with the dominant firm or by promoting the creation of new firms competing with the incumbent. The objective of this paper is to study the consequences of this intervention and to draw some normative guidelines on the reasonable scope of public intervention.

The leading example motivating our research is the telecommunication industry, even if many themes developed below could be extended to other sectors as transports or energy. In this quickly evolving sector, new investments are needed quite often and firms decide to engage the necessary funds on the prospect of making some positive profits. In recent Guidelines focused on the State Aid to boost the deployment of broadband networks [1] the European Commission insisted on the care one should take when intervening in those markets. In particular, "*it must be ensured that State aid does not crowd out market initiative in the broadband sector. If State aid ...were to be used in areas where market operators would normally choose to invest or have already invested, this could affect investment already made by broadband operators on market terms and significantly undermine the incentives of market operators to invest in the first place.* In France or United Kingdom, some regional governments have already chosen to intervene in those markets, either by subsidizing the incumbent operator or by using public funds to create some competition at the expenses of the incumbent. Note that this intervention may happen in different circumstances. When public intervention happens in areas where no private investment ever takes place, it can be justified by some equity considerations as "regional cohesion". When it happens in a dense area where private investment can be easily developed, public intervention is much harder to defend, except when publicly controlled firm behaves as private investors [2]. In areas where a very limited number of firms are willing to invest, the risk of public intervention is, according to the EC Guidelines, that *"subsidies for the construction of an alternative network can distort market dynamics"*. It seems clear that, especially in the latter case, some thoughts are now needed to determine where the limits to direct public intervention in markets should be put.

In the present paper, we analyze contractual agreements whereby local authorities subsidize an incumbent firm to change the access conditions to the infrastructure when such contracts are allowed, and study the implications for the investment decisions and the optimal rules a national regulator must set to promote social efficiency.[3] To do so, we consider a model where a firm, the historical operator referred to as incumbent, must decide whether to invest in a new technology in a specific area called a district. Our set-up encompasses

[1] "Community Guidelines for the application of State aid rules to rapid deployment of broadband networks", European Commission, 19.5.2009

[2] See the case of the Fibre-to-the-Home broadband access network in Amsterdam and the European Commission Decision allowing such intervention

[3] Our companion paper, Jullien-Pouyet-Sand-Zantman (2009) proposes an analysis on the same topic focusing less on the contractual issues and more or the general question of whether some public intervention should be allowed of not.

both the case of an unregulated monopoly and the case where the firm's price is constrained by a price-cap fixed ex-ante by a national regulator. The incumbent decides to invest or not considering not only the price he will be able to fix but also the possible hazards that may appear ex-post. In our model, these hazards take the form of a public intervention made by the local authorities. This intervention can take several forms and we focus on two possibilities: local authorities may decide to duplicate existing infrastructure by building a competing infrastructure, or they may contract with the incumbent to obtain the service at a negotiated price in exchange of a subsidy.

We start by considering different forms of public local intervention when there is symmetric information between the parties on the demand parameter of the district. Whether this negotiation occurs before the investment stage or after this investment, once local authorities can duplicate the private investment it is optimal to allow contracting between the parties. The reason is that under symmetric information, negotiation does not distort the investment decision and generates an allocation that is ex-post efficient.

The main part of the analysis is devoted to the case where at the negotiation stage, the incumbent has some private information on the state of the demand. We first consider the case where local authorities are not allowed to duplicate (or just not willing to do so). In this case, they will propose a contract to the incumbent taking into account the outside option profit obtained when rejecting the offer. The optimal negotiated price depends then positively on the level of realized demand while the incumbent always obtains more profit than its "reservation" profit.

When duplication is possible, the analysis changes since duplication acts as a threat that influences the incumbent's incentives to accept or reject the contract. More precisely, consider the situation when the local authorities propose a contract to the incumbent to reduce the access price but cannot commit on their behavior (duplicate or not) when the offer is turned down. With this lack of commitment,[4] the belief hold by the local authorities on the incumbent information following a refusal plays a major role not only for the decision to duplicate but also for the decision of the incumbent to accept the offer. The reason is that the profit upon rejection of the offer depends on the likelihood of duplication. We then identify two possible scenarios corresponding to two possible equilibria. In the first one, the incumbent accepts all type of contract and this may lead to the full expropriation of the incumbent profits and thus dramatically undermines the investment process. In the other one, some profit is left to the incumbent and we identify the possible contracts and the precise behavior of the parties in this case. At the end, we discuss the optimal ex-ante behavior of the regulator and the optimal way to induce efficient investment.

A few articles have discussed the regulation process when different possible market structures are possible. In the literature focusing on the regulation of infrastructure, the standard trade-off is between granting a generous access to the essential facility (or promoting competition at the upstream level) and recouping the cost of investment. For example, Dana

[4]This commitment problem can be viewed as a particular type of ratchet affect. See Freixas-Guesnerie-Tirole (1985) or Laffont-Tirole (1988).

and Spier (1994) made one of the first contributions where the modes of production and the market structure are endogenous. In the present paper, there are more tools on the regulation side and the private value parameter has some common value aspect. Even closer to our paper is the article of Caillaud and Tirole (2004) highlighting a conflict between social optimality and financial viability. Caillaud and Tirole showed that when the demand is private information of the incumbent, it is difficult to elicit this information since when demand is high, the tensions between the planner' objective and incumbent incentive are too strong to be compatible. We also have a similar conflict between local authorities and incumbent through a common value element but the potential competitor is a public agency. Moreover, we analyze the role of the national regulator in mitigating this risk by choosing the regulation rules. On the more technical side, our model bears some similarity with Lewis-Sappington (1988) analysis of regulation when the demand is unknown. Nevertheless, the introduction on a type-dependent outside option (see Jullien 2000 for a general analysis of this problem) modifies many conclusions of their work. The presence of an outside option can also be found in a recent contribution of Auriol-Picard (2008) where the standard regulation model is used to analyze outsourcing. In this paper, the authorities can propose a contract to a monopoly to change the market price in exchange of transfers. By contrast, in our paper, the contract is proposed under the threat of duplication and the initial price may be a regulated price.

Section 2 presents the model and the benchmark without negotiation. Section 3 describes the outcome of negotiation when there is symmetric information between the local authorities and the incumbent. Section 4 characterizes the equilibrium contracts and allocation when negotiation occurs under asymmetric information but with no possibility of duplication. Section 5 introduces this possibility and analyzes the various equilibria and the form of the contracts proposed by the local authorities. Section 6 discusses the optimal ex-ante regulation rule in this context while the last section concludes. Most of the proofs are relegated to an Appendix.

2 The Model

There is one representative geographical zone, called the 'district', characterized by its level of (ex-post) demand for services denoted by θ, distributed on $[\underline{\theta}, \overline{\theta}]$ according to a strictly positive density $g(.)$ and a cdf $G(.)$. We assume that the Monotone Hazard Rate Property is satisfied, i.e. $\frac{G(\theta)}{g(\theta)}$ strictly increasing with θ.

Customers located on the district may benefit from a new service provided by a set of identical firms. The provision of this service requires access to an up-to-date network. An incumbent operator, denoted by I, has the possibility to upgrade its existing network at a cost $c > 0$ to allow the provision of the service.[5]

Access to the infrastructure network is set on a nondiscriminatory basis and the unit price is denoted by a. Service providers are assumed to behave competitively with a constant

[5]This cost may depend on the observable characteristics of the district such as its density for example.

marginal cost normalized to zero, so that the final retail price they charge to customers is always equal to the access price $p = a$. The demand is then $\theta D(a)$ and we assume $D(0) > 0$. The corresponding consumers surplus is denoted by $\theta W(a)$, with $W'(a) = -D(a)$. Let $\varepsilon(a) = -aD'(a)D(a)$ be the price elasticity of the demand. We assume that $\varepsilon(a)$ is increasing and $\varepsilon(0) = 0$. As it will always be optimal to set the price below the monopoly level, we restrict attention to access price a such that $\varepsilon(a) \leq 1$.

For conciseness, we assume that $\underline{\theta}W(0) > c$, which ensures that there is scope for efficient investment at all levels of θ.

The access price to the incumbent network once it is build and there is no local interference is denoted r. For the most part, this level is fixed and it can be either the profit maximizing price $r^m = \arg\max_a aD(a)$or a price-cap $r \leq r^m$ set by the regulator. In section 6, we will discuss the choice of $r$but up to this point, it will be considered as a parameter.

Instead of relying on the incumbent, a local authority L may decide to build its own network.[6] L's cost is given by k, which we assume to be known by all parties. We assume that the local authority is a priory less efficient than the incumbent operator to upgrade the network: $k \geq c$. The local authority's objective is to maximize the surplus of its constituency.

If both the incumbent and the local authority build an upgraded network, there is Bertrand competition on the wholesale market for access. We refer to the coexistence of the two infrastructures as duplication.

To get an first intuition on the problems to come, assume that L can only decide to bypass or not the existing network but cannot bargain with the firm, neither before nor after I makes its investment decision. The timing is as follows

1. Incumbent operator I decides whether to upgrade the network. The value of the demand θ is then realized and publicly observed.

2. Local authority L decides whether to build a competing network.

If the local authority chooses to duplicate, then it decides the term of access to the local public network newly created, and there is Bertrand competition between the local public network and the incumbent. If it doesn't, then the service might be provided by the firms using the incumbent's network only at an access price of r.

Notice that the incumbent invests prior to observing θ, while the local authority can postpone this decision after the realization of the level of demand. We view this asymmetry as the result of the different organization of each institution. The incumbent is a national operator which decides on its investment policy at the national level. This reduces the cost of investment but it requires more planning in advance than in the case of a local authority which only invests locally.

Assuming I has upgraded its network and θ has been realized, the local authority is

[6] Equivalently, L could subsidize the entry of a competitor.

willing to duplicate the network if, and only if:

$$\theta W(0) - k \geq \theta W(r). \tag{1}$$

When duplication occurs, competition leads to price access at marginal cost, i.e. $a = 0$.[7] Therefore, L's decision to duplicate the incumbent's network (1) rewrites as follows:

$$\theta \geq \hat{\theta} \equiv \frac{k}{W(0) - W(r)}.$$

where the threshold level of demand $\hat{\theta}$ decreases with r.[8]

Let us now turn on to the decision faced by the incumbent at the investment stage of the game with this possibility of duplication. The incumbent makes positive profit only when it is not duplicated. Thus, it invests provided that:

$$G(\hat{\theta})\mathbb{E}\{\theta \mid \theta \leq \hat{\theta}\} r D(r) \geq c.$$

To make the analysis interesting, we assume that there exists some prices that induce investment with duplication:[9]

Assumption 1 $a_D = \inf \left\{ a \mid G(\hat{\theta}(a))\mathbb{E}\{\theta \mid \theta \leq \hat{\theta}(a)\} a D(a) = c \right\}$ *exists.*

3 Contracting under symmetric information

We now tackle the following issue: When, and under which conditions, should the local authority and the incumbent be allowed to modify the terms of the provision of the network services? This is a broad question, which may be refined according to several dimensions.

First, the nature of the contractual agreement between the incumbent and the local authority has to be determined. The local authority may be allowed to contract bilaterally with the incumbent only to implement a lower access price on an existing infrastructure - this amounts to saying that the local authority is allowed to buy back the incumbent's infrastructure. Or, it may be allowed to contract with the incumbent in order to provide a new infrastructure, instead of using its own and more costly technology. Hence, this first dimension can be related to the timing of the local authority's intervention: the local authority may intervene either before or after the incumbent has decided whether to undertake the investment.

Second, since the local authority has the possibility to deploy unilaterally an infrastructure, the credibility of the local authority's decision is of tantamount importance. For in-

[7] Note that $a = 0$ also maximizes the local authorities' welfare.

[8] A similar reasoning shows that the local authority invests if the incumbent does not upgrade its network if, and only if $\theta \geq \frac{k}{W(0)}$.

[9] See our companion paper on that point.

stance, when contracting with the incumbent, the local authority may be able to commit to invest if the negotiation with the incumbent breaks down; or, it may lack such a commitment capability.

The third dimension deals with the informational structure of the various actors, which may bear on the level of the demand or the local authority's opportunity cost of building a network.

In this section, we focus on the full information case and look at two situations. In the first one, the local authority contracts ex ante, i.e. before the incumbent's decision to invest or not has been taken. In the second situation, the local authority contracts ex post, i.e. after the investment decision.

Let us first characterize briefly the social optimum. Given that the incumbent's cost to build the network is lower than the local authority's, and given that the local authority can use public funds to finance the infrastructure, the social optimum is characterized as follows: The incumbent builds the network on behalf on the local authority, which sets an access price equal to 0 and exactly compensates the incumbent for its cost ($t = c$). The level of welfare would then be $\theta W(0) - c$, which has been assumed to be positive.[10]

3.1 Ex ante contracting under the threat of duplication

We start with the situation in which the local authority may contract with the incumbent before it has decided to invest. That is, the chronology of events is the same as in the initial timing except that we add stage 0.5 just before stage 1:

Stage 0.5 Before stage 1, the local authority can offer a contract to the incumbent on a take-it-or-leave-it basis. This contract stipulates whether I undertakes the investment, a transfer T from L to I, and the access price a_L that will prevail on the new infrastructure. The incumbent firm then accepts or rejects.[11]

Of course, the contract offer is enforced only if accepted by I.

The outcome of the game depends on the expectations about the local authority's investment decision at the last stage of the game (stage 2). Recall that the critical demand level above which the local authority finds it worthwhile to duplicate is $\hat{\theta}$. Thus, if $\theta \geq \hat{\theta}$ (respectively, $\theta \leq \hat{\theta}$) then the local authority invests (respectively, does not invest) when the incumbent has rejected its contractual offer and has invested.

Therefore, at the investment stage, if the contract has been refused, the incumbent invests if, and only if:

$$\Pi = G(\hat{\theta})\mathbb{E}\{\theta \mid \theta \leq \hat{\theta}\}rD(r) \geq c, \tag{2}$$

[10] Note that any transfer $t \geq c$ would lead to the same total welfare level, but with different gains for the incumbent and the local authority.

[11] We could allow the local authority to make a transfer contingent on whether it duplicates the infrastructure without changing the results.

Consider now the contracting stage (stage 0.5). The local authority can offer any contract provided the firm's net expected profit if it accepts is at least $\max\{\Pi - c, 0\}$. Since there is no asymmetric information, it is optimal for L to offer the contract that maximizes the local welfare (profit plus consumer surplus) given the investment possibilities. Thus, the optimal contract always induces investment by I and a zero price. At stage 2, the optimal policy for the local authority consists of choosing a_L and T such the previous constraint is binding. More precisely, since the optimal retail price is 0, it is optimal to set $a_L = 0$.

Proposition 1 *Consider the case of ex ante contracting under symmetric information between the local authority and the incumbent. The local authority always subsidizes the incumbent to undertake the investment on its behalf. With respect to the case of no negotiation, the local authority's contractual intervention always improves welfare.*

Proof. See the Appendix. ■

3.2 Ex post contracting under symmetric information

We now consider a setting in which the local authority "moves" after the incumbent's decision to invest or not and after the realization of the level of demand is commonly known. That is, the following stage 1.5 is added:

Stage 1.5 Between stages 1 and stage 2, the local authority can offer a contract to the incumbent on a take-it-or-leave-it basis. This contract stipulates a transfer T from L to I, the access price a_L that will prevail on the new infrastructure and possibly rules concerning duplications by L. The incumbent firm then accepts or rejects.

As before, at stage 2 of the game, if the incumbent has invested and turned down the contract proposed by the local authority, there may be duplication if $\theta \geq \hat{\theta}$.

At stage 1.5, the local authority's offer depends on the state of the demand. If $\theta > \hat{\theta}$, duplication is a credible threat, which would drive the incumbent profit to zero. In this case, the local authority can expropriate the incumbent by proposing a contract with $a_L = 0$ and $T = 0$. If $\theta < \hat{\theta}$, it is common knowledge that the local authority will not duplicate if the incumbent rejects the contract offer. Thus, the optimal contract it can offer is such that $a_L = 0$ and $T = \theta r D(r)$.

At stage 1, the incumbent anticipates that his investment is expropriated when $\theta \geq \hat{\theta}$. For a a given level of regulated price r, he then decides to invest if, and only if:

$$G(\hat{\theta})\mathbb{E}\{\theta \mid \theta \leq \hat{\theta}\} r D(r) \geq c. \tag{3}$$

The following proposition, whose proof is omitted, immediately obtains.

Proposition 2 *Consider the case of ex-post contracting under symmetric information between the local authority and the incumbent. The local authority always subsidizes the incumbent to reduce it price. With respect to the case of banned negotiation, the local authority's contractual intervention always improves welfare.*

Thus when negotiation occurs prior to the duplication stage but under symmetric information, the final allocation is efficient.

In the following sections, we study the impact of asymmetric information on the level of demand. As said before, the value of the demand has an impact not only on the incumbent profit but also on the type of contract proposed by the local authority. In the next section, we study the special case where the local authority can propose ex-post a contract to the incumbent but is not able to duplicate the infrastructure. Afterwards, we will study the general case and detail the different possible scenarios.

4 Contracting under asymmetric information with no duplication

>From now on, we focus on the more relevant case in which asymmetric information impedes the relationship between the incumbent and the local authority at the time of contracting. In this section, we assume that the local authority cannot duplicate the incumbent's network, putting aside for a moment the credibility of this assumption. More precisely, events unfold as follows:

1. The incumbent firm decides to invest or not in the district. The state of demand θ is then realized and privately revealed to the incumbent.
2. The local authority offers a contract to the incumbent. The incumbent firm then accepts or rejects.
3. Two possibilities arise depending on the incumbent firm's decision to accept or not the local authority's contract:
   (a) If the contract has been accepted, then the terms of the contracts are enforced.
   (b) Otherwise, the standard price r applies.

The subgame composed of stages 2 and 3 has a flavor of a game of contracting between a Principal (the local authority) and an Agent (the incumbent firm), the latter having superior knowledge of the state of the demand. With a slight abuse of notation, we will often call θ the type of the incumbent, even if stricto sensu this parameter is related to the demand. Last, but not least, one should remark that the reservation utility if the incumbent refuses the contract depends on its type (see Jullien (2000) for a general analysis of this case). This informational structure with type-dependent reservation utility will shape the optimal contract from the local authority's point of view.

The contract C proposed by the local authority may be seen as a menu of couples $\{a, T\}$or equivalently a tariff $T^{\mathcal{C}}(.)$ defined for $0 \leq a \leq r$. In what follows, we use the notations $a^{\mathcal{C}}(.)$ and $\pi^{\mathcal{C}}(.)$ to denote the price that would set the incumbent and its profit if it accepts the contract C. This is given by:

$$\pi^{\mathcal{C}}(\theta) = \max_{0 \leq a \leq r} \theta a D(a) + T^{\mathcal{C}}(a), \quad (4a)$$

$$a^{\mathcal{C}}(\theta) = \arg \max_{0 \leq a \leq r} \theta a D(a) + T^{\mathcal{C}}(a). \quad (4b)$$

The price $a^{\mathcal{C}}(\theta)$is non-decreasing and such that $\dot{\pi}^{\mathcal{C}}(\theta) = a^{\mathcal{C}}(\theta) D(a^{\mathcal{C}}(\theta))$. From the revelation principle we can identify a contract C with a profile $\left(a^{\mathcal{C}}(.), \pi^{\mathcal{C}}(.)\right)$.

In equilibrium the incumbent may not accept the offer. We then use $(a(\theta), \pi(\theta))$ to refer to the allocation effectively implemented, accounting for acceptance or rejection by the incumbent. If the local authority's offer is rejected by the incumbent, the tariff in place is r and the incumbent with type θ obtains:

$$\pi^R(\theta) = \theta r D(r).$$

Thus, the incumbent rejects the contract C if $\pi^{\mathcal{C}}(\theta) \leq \pi^R(\theta)$. The final profit $\pi(\theta)$ obtained when contract C is offered is thus given by:

$$\pi(\theta) = \max\left(\pi^{\mathcal{C}}(\theta), \pi^R(\theta)\right).$$

Notice that the slope of the incumbent's rent with respect to θ when the contract is accepted is always smaller than the slope of its profit when the initial price is implemented (over the relevant range of access price):

$$0 \leq \dot{\pi}^{\mathcal{C}}(\theta) = a^{\mathcal{C}}(\theta) D(a^{\mathcal{C}}(\theta)) \leq \dot{\pi}^R(\theta) = r D(r), \quad (5)$$

since $a^{\mathcal{C}}(\theta) \leq r \leq r^m$. A similar argument can be used to prove that $\pi^{\mathcal{C}}(\theta)$ is convex in θ; by contrast, $\pi^R(\theta)$ is linear in θ.

Let us now derive the contract proposed by the local authority. Notice that L will always offer a contract such that $\pi^{\mathcal{C}}(\theta) = \pi^R(\theta)$ for at least one type.[12] This along with property (5) implies that there exists a critical type θ^p defined by

$$\theta^p \equiv \min\left\{\theta \in [\underline{\theta}, \overline{\theta}] \mid \pi^{\mathcal{C}}(\theta) \leq \pi^R(\theta)\right\}. \quad (6)$$

[12] Otherwise, it would be possible to reduce the transfer provided to the incumbent without affecting its incentives.

such that an incumbent with a type $\theta > \theta^p$ will keep the initial price r and obtain profit $\pi(\theta) = \pi^R(\theta)$,while the incumbent with a type $\theta < \theta^p$ accepts the contract and obtains profit $\pi(\theta) = \pi^{\mathcal{C}}(\theta)$. Without loss of generality, we assume that the contract is rejected by types strictly above θ^p.

Neglecting the sufficient condition for incentive compatibility [13], and using routine computations, the problem faced by the local authority is then to choose θ^p and the allocation $(a(.), \pi(.))$ for θ below θ^p, which can be stated as follows:

$$\max_{\{a(.),\pi(.),\theta^p\}} \int_{\underline{\theta}}^{\theta^p} [\theta W(a(\theta)) + \theta a(\theta) D(a(\theta)) - \pi(\theta)]\, g(\theta) d\theta + \int_{\theta^p}^{\bar{\theta}} \theta W(r) g(\theta) d\theta$$

$$\begin{aligned} \text{subject to } \forall \theta \in [\underline{\theta}, \theta^p] : \dot{\pi}(\theta) &= a(\theta) D(a(\theta)), \\ \pi(\theta^p) &= \pi^R(\theta^p), \\ 0 \le a(\theta) &\le r. \end{aligned}$$

Notice that we do not include the constraint $\pi(\theta) \ge \pi^R(\theta)$ since it is implied by the last constraint. The optimal contract is derived formally in the Appendix. Intuitively, notice that using:

$$\pi(\theta) = \pi(\theta^p) - \int_{\theta}^{\theta^p} a(x) D(a(x)) dx,$$

we can rewrite the objective as:

$$\int_{\underline{\theta}}^{\theta^p} \left[\theta W(a(\theta)) + \left(\theta + \frac{G(\theta)}{g(\theta)}\right) a(\theta) D(a(\theta))\right] g(\theta) d\theta - G(\theta^p)\, \pi(\theta^p) + \int_{\theta^p}^{\bar{\theta}} \theta W(r) g(\theta) d\theta.$$

This expression can be interpreted as a ‘virtual surplus’ where $G(\theta)/g(\theta)$ represents the informational rents to be paid to lower types for incentive compatibility purposes when reducing the price of type θ. This virtual surplus is maximized at the price $a^*(\theta)$ solution of $-\theta D(a(\theta)) + [\theta + G(\theta)/g(\theta)][a(\theta)D'(a(\theta)) + D(a(\theta))] = 0$, or:

$$\varepsilon(a^*(\theta)) = \left[1 + \frac{\theta g(\theta)}{G(\theta)}\right]^{-1}.$$

Notice that with no information asymmetry, the optimal price is zero. When asymmetric information generates incentive problems, $a^*(\theta)$ is increasing with θ starting from its first-best value $(a^*(\underline{\theta}) = 0)$.

Lemma 1 *Define* $\theta^* \equiv \min\{\bar{\theta}, a^{*-1}(r)\}$. *The optimal contract* $\mathcal{C}^*$ *proposed by the local authority is such that:*

- *For* $\theta \in [\underline{\theta}, \theta^*]$, *the negotiated access price is* $a^*(\theta)$ *and the incumbent accepts.*

[13] We show in the Appendix that it is implied by MLRP and the assumption on the elasticity of demand.

- *For $\theta \in]\theta^*, \overline{\theta}]$, the negotiated access price is r and the incumbent is indifferent between accepting or refusing the contract.*

- *The incumbent's profit $\pi^*(\theta)$ is such that $\pi^*(\theta) - \pi^R(\theta)$ is decreasing for $\theta \in [\underline{\theta}, \theta^*]$ and equal to 0 for $\theta \in [\theta^*, \overline{\theta}]$.*

Proof. See the Appendix. ■

To focus on the intuition underlying the previous lemma, let us neglect for a while the outside option of the incumbent. It is therefore analogous to a situation in which a regulator wants to control the price fixed by a monopolist when the latter has some private information on the demand parameter. In such a context, Lewis-Sappington (1988) showed that, with the cost structure considered in our setting, the local authority could achieve the first-best by setting a regulated price equal to zero.

Accounting now for the incumbent's outside option, the latter can refuse the contract and keep the initial price r, so new incentives emerge. Low-demand firms want to mimic high-demand ones in order to pretend having a high outside option. Paradoxically, incumbents knowing that the demand is low will be able to command some informational rents (defined as $\pi - \pi^R$). For those types, the price proposed by the local authority will be close to the first-best access price while, as the demand level increases, the contractual price tends to r.

The contract may induce partial participation. Indeed, the optimal threshold θ^* is such that the optimal tariff proposed by the local authority is equal to the initial price r. Therefore, for the low-type incumbent, the local authority chooses giving up some informational rents in return for a smaller tariff while for the high-type incumbent, no change is made compared to the initial access price.

The case developed in this section corresponds to the situation where duplication is never an option. The local authority has no credibility as a potential competitor and, thus, the incumbent doesn't fear any expropriation. Rather, the incumbent anticipates a positive informational rent from its negotiation with the local authority. Thus, negotiation induces the following intuitive trade-off. On the one hand, some public subsidies may be used to decrease the access price, thereby boosting consumers surplus. On the other hand, negotiating with a better informed incumbent leads to giving up some costly rents. Since the feasible set of the local authority problem includes no contract (with $a = r$ and $T = 0$ for all types), it is immediate that negotiation is Pareto-improving. Thus not only does renegotiation improve the allocation but, by shifting some rents from the local authority to the incumbent, it also allows to reduce further the ex-ante distortion on the access price and to improve the incumbent's incentives to invest. We thus conclude:

Proposition 3 *When the local authority can not duplicate private infrastructures, it is strictly optimal to allow ex-post contracting with the incumbent.*

5 Contracting under Asymmetric Information and the Threat of Duplication

5.1 General Analysis

We now turn to the general analysis of the game by allowing the possibility of duplication. Introducing this new possibility leads to change stage 3.b into the new stage.

Stage 3.b: If the incumbent has rejected the contract, then the local authority decides to duplicate the infrastructure or not.

A distinctive feature of our model is that once the local authority has offered a contract to the incumbent firm, it may decide to duplicate the network after the incumbent's choice to accept the contract or not. This lack of commitment on the principal's side raises several new challenges. First, the decision to duplicate will depend on the incumbent acceptance of the contract, which itself depends not only on the contract proposed but also on the likelihood of duplication. This chicken-and-eggs problem opens the door to the possibility of multiple equilibria. Second, since the Principal makes a second move after the Agent's decision to participate or not, the validity of the Revelation Principle is put into question. We nevertheless keep the same form of contract as above and assume that the local authority offers the incumbent a (nonlinear) transfer $T(.)$ for $0 \leq a \leq r$.[14]

As in section 4, the main part of the analysis lies in the description of the game that starts at stage 2. An equilibrium of this game is described by three elements

- An offer made by the local authority, denoted $\mathcal{C}$ at the equilibrium.
- The incumbent decision to accept or not. Note that this decision will depend on the incumbent type θ.
- The decision to duplicate or not if the contract has been refused.

In what follows we identify a contract $\mathcal{C}$ with the profile of profit and price $\left(a^{\mathcal{C}}(.), \pi^{\mathcal{C}}(.)\right)$ that is implemented when the incumbent accepts the offer. We restrict to offers that induces non-negative profit for the firm, $\pi^{\mathcal{C}}(\theta) \geq 0$.[15] Note also that for any optimal contract, we must have $\min_\theta \left(\pi^{\mathcal{C}}(\theta) - \pi^{R}(\theta)\right) \leq 0$ as there is no reason to leave unnecessary rents to all possible types.

For the sake of exposition we assume that the offer is rejected whenever the incumbent is indifferent between accepting and rejecting and anticipates that rejection triggers no duplication, unless stated otherwise.[16] For a contract $\mathcal{C}$ and for θ^p defined by equation (6), agents

[14] Whether this restriction on the space of the available contracts entails a loss of generality is left for future research.

[15] In full generality, one may want to consider offers with negative profit for some types and positive profit for other types. In fact, this offer is never an equilibrium since we can show (proof available on request) that it is always dominated for the same out-equilibrium belief by an offer with $T = (0, 0)$.

[16] This is without loss of generality but the proof would be more tedious if we allowed the incumbent to accept when indifferent in this case.

with type above θ^p will refuse the contract if they anticipate no duplication, and those with type less than θ^p will accept it. The following lemma describes the set of possible class of equilibria, where $\hat{\theta}$ is defined in section 2:

Lemma 2 *In the subgame following a contract offer $\mathcal{C}$,*

- *If $E[\theta \mid \theta \geq \theta^p] > \hat{\theta}$, then there a unique continuation equilibrium where $\mathcal{C}$ is accepted by all.*
- *If $E[\theta \mid \theta > \theta^p] \leq \hat{\theta} \leq \bar{\theta}$, then there are two continuation equilibria.*
  1. *Either, the contract $\mathcal{C}$ is accepted by all;*
  2. *or, the contract $\mathcal{C}$ is accepted by types $\theta < \theta^p$, and no duplication follows a rejection.*
- *If $\bar{\theta} < \hat{\theta}$, there a unique continuation equilibrium where $\mathcal{C}$ is accepted by types $\theta < \theta^p$, and no duplication follows a rejection.*

The previous lemma characterizes the different possible classes of equilibria. As it appears, for some range of initial prices, there is a multiplicity of continuation equilibria which of course will give rise to a multiplicity of equilibria in the full game. As a preliminary remark, the continuation equilibrium is unique if $\hat{\theta} < E(\theta)$ or if $\hat{\theta} > \bar{\theta}$. Indeed, given that $E(\theta \mid \theta > \theta^p) \geq E(\theta)$, the above lemma shows that when the former condition is satisfied, an offer C is accepted by all types of incumbent so that L will offer $C = \{0, 0\}$. Similarly, in the latter case duplication is never a credible threat and thus, the local authority offers the contract C^* defined in the previous section.

We thus restrict attention to the case where $E(\theta) \leq \hat{\theta} \leq \bar{\theta}$. Given that $\hat{\theta}$decreases with r, this corresponds a range $r \in [\underline{a}, \bar{a}]$ where

$$\underline{a} \equiv W^{-1}\left(W(0) - \frac{k}{\bar{\theta}}\right) \text{ and } \bar{a} \equiv W^{-1}\left(W(0) - \frac{k}{E(\theta)}\right)$$

solve respectively the conditions $\hat{\theta} = \bar{\theta}$ and $\hat{\theta} = E(\theta)$. To avoid triviality we assume that $\underline{a} < r^m$.

Before providing a full characterization of the set of equilibria, it is useful to highlight particular equilibria that brings relevant economic insights and will ease the treatment of the general case.

5.2 The minimal rent scenario

One of the possible case consists in having an equilibrium in which the transfer proposed by L is always reduced to the minimum. This case is called "the minimal rent scenario" and consists in selecting particular continuation equilibria in case of multiplicity, according to the following criterion:

Minimal Rent Scenario: *For any r such that $\hat{\theta} \leq \bar{\theta}$, all contracts are accepted.*

The interpretation is that the local authority anticipates the contract offer should be accepted, and interprets a rejection as a signal that demand is high which induces duplication. From the viewpoint of the local authority, the optimal contract at stage 4, which ensures that the incumbent participates, is $a^L = 0$ and $\pi(\theta) = 0$ when $\hat{\theta} \leq \bar{\theta}$. [17] As a consequence, anticipating that it will be expropriated from any investment it undertakes, the incumbent decides not to invest at all. Therefore, the regulator will try to avoid being in this situation. Formally,

Proposition 4 *In the "minimal rent scenario",*

- *If $\hat{\theta} > \bar{\theta}$, the incumbent invests and the local authority proposes $\mathcal{C}^*$.*
- *If $\hat{\theta} \leq \bar{\theta}$, there is no investment by the incumbent and the local authority invests if and only if $E(\theta) > \frac{k}{W(0)}$.*

5.3 The maximal rent scenario

In the previous subsection, the equilibrium selection procedure could lead to an situation with the absence of investment due to an "unfair contract" offer by the local authority. Consider now the case where $E(\theta) < \hat{\theta}$ and take any contract offer such that $\pi(\theta) \leq \pi^R(\theta)$ for all θ. The equilibrium where the the contract $a^L = 0$ and $\pi(\theta) = 0$ is proposed and accepted by all still exists but there is another equilibrium where all types refuse the contract and no duplication occurs. If this equilibrium prevails, the local authority will never propose a contract such that the profit is less than the outside option profit for all types. Therefore, the incumbent will have strictly positive rents at the equilibrium, either by accepting the contract or by applying the initial price r. Formally, we define an equilibrium in the maximal rent scenario as one that satisfies the following condition.

Maximal Rent Scenario: *For any contract $\mathcal{C}$:*

i) *If $E[\theta \mid \theta \geq \theta^p] < \hat{\theta}$, the contract is accepted by types $\theta < \theta^p$ only and there is no duplication after rejection.*

ii) *If $E[\theta \mid \theta \geq \theta^p] > \hat{\theta}$, the contract is accepted by all types.*

Notice that we do not impose restriction when there is equality between $E[\theta \mid \theta \geq \theta^p]$ and $\hat{\theta}$[18]. With the restrictions implied by the maximal rent scenario, the local authority has to choose between a contract inducing full participation and a contract with partial participation. A contract inducing partial participation necessarily implements the price r

[17] The indifference between acceptation and refusal can be broken as usual by offering a very small positive transfer $\varepsilon > 0$ instead of 0.

[18] This is done to guarantee the existence of an equilibrium in all cases.

for types θ above θ^p. By contrast, a contract with full participation allows to reduce the price for the high types but implies that higher transfers (so higher profits) should be left to the incumbent. In fact, those two types of contract are associated to different profiles of profit. For what follows we define:

Definition: For $r \in [\underline{a}, \bar{a}]$, $\tilde{\theta}$ is the type such that $E[\theta \mid \theta > \tilde{\theta}] = \hat{\theta}$**.**

This type $\tilde{\theta}$is the level θ^p that would make L indifferent between duplicating or not. It is clearly smaller that $\hat{\theta}$, and it decreases from $\bar{\theta}$ to $\underline{\theta}$ on the relevant range of r.

Take first a contract inducing partial participation. Since in this case, we have $E[\theta \mid \theta > \theta^p] \leq \hat{\theta}$, it is clear that $\theta^p \leq \tilde{\theta}$. So, for any type θ below θ^p, $\pi(\theta) \leq \pi^R(\tilde{\theta})$. Therefore, in a contract with partial participation, the final allocation is such that $\pi(\theta) \leq \max\{\pi^R(\tilde{\theta}), \pi^R(\theta)\}$. Let us consider now a contract with full participation. In this case, $\theta^p \geq \tilde{\theta}$ so $\pi(\tilde{\theta}) \geq \pi^R(\tilde{\theta})$. It is then direct to see that, for any type θ, $\pi(\theta) \geq \min\{\pi^R(\tilde{\theta}), \pi^R(\theta)\}$. To sum up, a contract with partial participation is a contract where the profits are bounded above whereas a contract with full participation is a contract where the profits are bounded below.

Even if the allocations generated by the two possible contracts are "characterized" by different constraints on the profile of profit, they share some common property. More precisely, for any contract and equilibrium allocation $(a(.), \pi(.))$ generated by this contract, we have $\pi(\tilde{\theta}) \geq \pi^R(\tilde{\theta})$. In the sequel, we will use this common property and show that that the two previous contracts are the two possible implementations of a general allocation problem with this property.

Lemma 3 *Consider the maximal rent scenario and take any allocation $(a(\theta), \pi(\theta))$ that is it is feasible and incentive compatible. Then there exists a contract offer by L and a continuation equilibrium that implements this allocation if and only if $\pi(\tilde{\theta}) \geq \pi^R(\tilde{\theta})$.*

Proof. see appendix. ■

The previous lemma provides a set of allocations that may be obtained in equilibrium. We can thus search for the preferred allocation of L within the set of allocations that satisfies the properties of the lemma. Formally, it amounts to find the solution to the following problem :

$$P$$

$$\max_{\{a(.)\pi(.)\}} \mathbb{E}_\theta \left\{\theta W(a(\theta)) + \theta a(\theta) D(a(\theta)) - \pi(\theta)\right\},$$

$$\begin{aligned}
\text{subject to } \forall\theta, \dot{\pi}(\theta) &= a(\theta)D(a(\theta)) \\
\dot{a}(\theta) &\geq 0, \\
\pi(\theta) &\geq 0 \\
\pi\left(\tilde{\theta}\right) &\geq \pi^R\left(\tilde{\theta}\right) \\
a(\theta) &\in [0, r].
\end{aligned}$$

Using $\pi(\theta) = \pi\left(\tilde{\theta}\right) + \int_{\tilde{\theta}}^{\theta} a(s) D(a(s)) ds$, we can rewrite the objective as the expectation of the virtual surplus:

$$\int_{\underline{\theta}}^{\tilde{\theta}} \left(\theta W(a(\theta)) + \left(\theta + \frac{G(\theta)}{g(\theta)}\right) a(\theta) D(a(\theta))\right) g(\theta) d\theta$$
$$+ \int_{\tilde{\theta}}^{\bar{\theta}} \left(\theta W(a(\theta)) + \left(\theta + \frac{G(\theta) - 1}{g(\theta)}\right) a(\theta) D(a(\theta))\right) g(\theta) d\theta - \pi\left(\tilde{\theta}\right).$$

Using results in Jullien (2000, Theorem 3 and 4), we can state the following proposition.

Proposition 5 *Assume that $E(\theta) \leq \hat{\theta} < \bar{\theta}$. Then the solution $\mathcal{C}^A$ to the program (P) is unique and such that $\pi\left(\tilde{\theta}\right) = \pi^R\left(\tilde{\theta}\right)$ and there exists $\alpha^A \leq r$ and $\theta^A < \tilde{\theta}$ such that:*

- $a^A(\theta) = \min\left(a^*(\theta), \alpha^A\right)$,
- $\alpha^A = a^*\left(\theta^A\right)$,
- *If $\tilde{\theta} \leq E(\theta)$ then $\theta^A = \underline{\theta}$ (and $\alpha^A = 0$),*
- *If $\tilde{\theta} > E(\theta)$ then $\underline{\theta} < \theta^A \leq \theta^*$, and*

$$\int_{\theta^A}^{\tilde{\theta}} \left(\frac{G(\theta)}{g(\theta)} - (\theta + \frac{G(\theta)}{g(\theta)})\varepsilon(\alpha^A)\right) g(\theta) d\theta$$
$$+ \int_{\tilde{\theta}}^{\bar{\theta}} \left(\frac{G(\theta) - 1}{g(\theta)} - (\theta + \frac{G(\theta) - 1}{g(\theta)})\varepsilon(\alpha^A)\right) g(\theta) d\theta \geq 0 \tag{7}$$

with equality if $\theta^A < \theta^*$.

Proof. See appendix. ■

The equilibrium offer thus includes bunching on the top, all types of firms above some level are offered the same rate α^A. The LHS of condition (7) is the mean virtual surplus evaluated at α^A on the bunching interval. After some integration by part the condition can be rewritten as

$$\left(1 - \varepsilon(\alpha^A)\right)\left(\tilde{\theta} - \theta^A G\left(\theta^A\right) - \int_{\theta^A}^{\bar{\theta}} \theta g(\theta) d\theta\right) - \varepsilon(\alpha^A) \int_{\theta^A}^{\bar{\theta}} \theta g(\theta) d\theta \geq 0. \tag{8}$$

This take value $\tilde{\theta} - E(\theta)$ at $\theta^A = \underline{\theta}$ since $\varepsilon(0) = 0$ and is negative at $\tilde{\theta}$.

There may be two types of solutions. If $\theta^A < \theta^*$, price are uniformly below the level r, and the prof it function is as shown in Figure 1. Otherwise the solution corresponds to $\mathcal{C}^*$, with profit as in Figure 2.

Clearly the solution of the program generates an expected welfare for L as least as large as the equilibrium welfare. It is thus an equilibrium allocation if L can uniquely implement

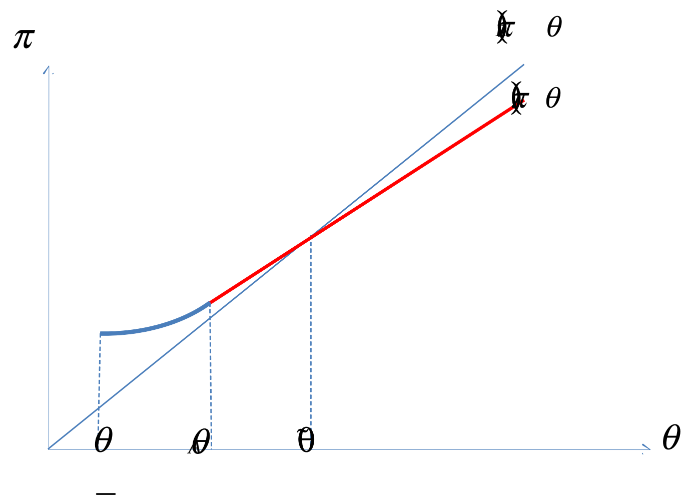

Figure 1: Full participation

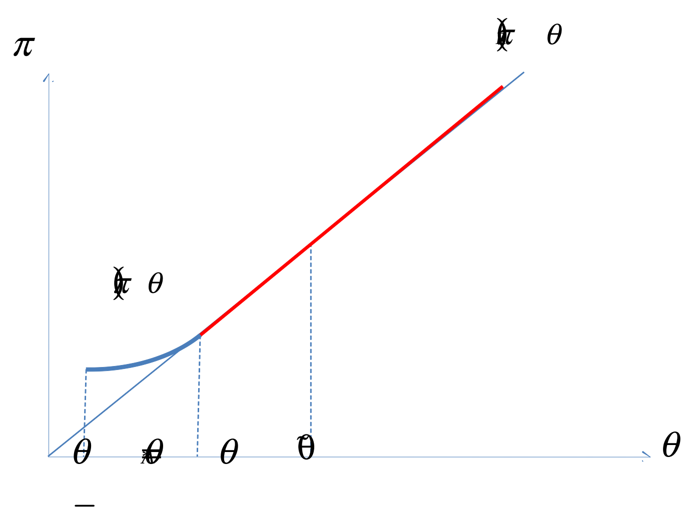

Figure 2: Partial participation

this allocation by the choice of an adequate contract proposal, or at least approximate it. The next proposition confirms that this is indeed the case.

Proposition 6 *Under the maximal rent scenario, the continuation-equilibrium allocation exists and implements the solution of the program P.*

Proof. Let us first show that any equilibrium is such that $\pi = \pi^A$. Suppose it is not the case. From the previous proposition, L's surplus S is such that $S(\pi) < S(\pi^A)$. Then consider an contract offer $\mathcal{C} = \{\pi^A + \varepsilon\}$ is accepted by all and generated a surplus $S(\pi^A) - \varepsilon > S(\pi)$. So π cannot be an equilibrium. Moreover, lemma 3 showed that offering $\pi^A(\theta)$ is an equilibrium. ■

Therefore, in the maximal rent scenario, the solution $\mathcal{C}^A$ can be implemented either by a full participation contract or by a partial participation contract $\mathcal{C}^*$. In the latter case, it means that $\theta^A = \theta^* < \tilde{\theta}$ and the incumbent with type $\theta \geq \theta^*$ would obtain utility $\pi^R(\theta)$ and set price r. In the case of partial participation we then have $\theta^A \leq \inf\left(\theta^*, \tilde{\theta}\right)$. Notice that the price profile is fully characterized by α^A. Surprisingly whenever allowing duplication, the final prices decreases with the initial price r.

Corollary 1 *θ^A and α^A decreases with r if $\theta^A \in]\underline{\theta}, \theta^*[$*

Proof. See appendix. ■

To understand this result, one need to realize that under the maximal rent scenario with $\alpha^A < r$, the local authority always implements an allocation that is accepted by all under the threat of duplication. Thus the price chosen at the initial stage is never implemented. In this context, increasing the rate increases the credibility of the threat of duplication.

>From the corollary there must exist some threshold above which $\alpha^A < r$ so that the local authority chooses a contract accepted by all instead of the same contract as in the case where it is committed not to duplicate ($\mathcal{C}^*$). The question is then when the shift occurs.

Proposition 7 *In the "maximal rent scenario", there exists $\hat{a} \geq \underline{a}$ such that L offers $\mathcal{C}^A = \mathcal{C}^*$ if $r \leq \hat{a}$, and $\mathcal{C}^A \neq \mathcal{C}^*$ with $\alpha^A < \hat{a}$ if $r > \hat{a}$. Moreover $\hat{a} = \underline{a}$ if and only if $\theta^*(\underline{a}) \geq \bar{\theta}$.*

Proof. See appendix. ■

For a given initial price, when the incumbent invests, the expected local welfare is clearly higher in this scenario than in the case where duplication is forbidden.

5.4 Profit comparison

To see the impact of the various settings on the equilibrium of the whole game, it is necessary to get a clear understanding of the profit values and therefore of the incentives to invest ex-ante. First note that in the minimal rent scenario, the firm ends up with a null profit. Since any investment would be totally expropriated by the local authority, there is little hope to induce investment in this case.

The most intricate part of the analysis concerns the maximal rent scenario. Recall that the solution $\mathcal{C}^A$ to the general problem could be implemented by two types of contract, one of them being the same as in the case where no duplication is allowed (denoted $\mathcal{C}^*$). For future reference, denote by $E(\pi_r^A)$ the expected profit generated by the solution $\mathcal{C}^A$ with an initial price of r and $E(\pi_r^*)$ the profit generated by the solution $\mathcal{C}^*$ with the same initial price. Note that it has been shown before that for $r < \hat{a}$, the contract proposed by the local authorities does not change when duplication is allowed. Therefore $E(\pi_r^*) = E(\pi_r^A)$ in this case.

More generally, using the (IC) condition in any contract, we know that

$$\pi(\theta) = \pi\left(\tilde{\theta}\right) + \int_{\tilde{\theta}}^{\theta} a\left(s\right) D\left(a\left(s\right)\right) ds, \tag{9}$$

so for any profile of effective price, we can write the expected profit as

$$E(\pi_r) = \pi\left(\tilde{\theta}\right) - \int_{\underline{\theta}}^{\tilde{\theta}} G\left(\theta\right) a(\theta) D(a(\theta)) d\theta + \int_{\tilde{\theta}}^{\bar{\theta}} \left(1 - G\left(\theta\right)\right) a(\theta) D(a(\theta)) d\theta.$$

We show in appendix that for r close to $\hat{a}$, allowing the local authority to duplicate raises the expected profit of the firm. On the other hand, for large values of r such that $\tilde{\theta} < E\left(\theta\right)$, the reverse holds. The following proposition directly follows

Proposition 8 *There exist two threshold values v_1 and v_2 with $\hat{a} < v_1 \leq v_2$ such that*

1. *for $r \leq v_1$, $E\left(\pi_r^A\right) \geq E\left(\pi_r^*\right)$*
2. *for $r \geq v_2$, $E\left(\pi_r^*\right) > E\left(\pi_r^A\right)$*

Proof. See appendix. ■

We see that there is no simple profit comparison between the situation where duplication is allowed and the situation where it is banned.

5.5 The general case.

So far we have restricted to two possible scenarios. For the sake of completeness let us discuss the general case. To do so, we analyze one more time the subgame starting after the incumbent has chosen to invest or not. >From lemma 2, the multiplicity issue arises only if $\hat{\theta}$ lies between $E\left(\theta\right)$ and $\bar{\theta}$, which we assume here.

An equilibrium of the game then consists into i) an investment decision by the incumbent, ii) a contract offered by L in stage 2 (conditional on the incumbent investing), iii) a mapping from contracts to stage 3 allocations $\left(a\left(.\right), \pi\left(.\right)\right)$, such that individual decisions are optimal at each stage and the mapping associates to each contract a continuation equilibrium allocation of stage 3.

$>$From lemma 2, in any equilibrium a contract $\mathcal{C}$ is either accepted by all types (FP), or rejected by type $\theta > \theta^P$ (PP). Given that, the stage 3 mapping can be summarized by the type of continuation equilibrium, namely FP or PP. Any mapping $\mathcal{M}$ from the set of contracts into $\{FP, PP\}$ consistent with lemma 2 generates an equilibrium provided that there exists an offer at stage 2 that is optimal for L faced to the selection of stage 3 continuation equilibria induced by this mapping. The incumbent then invests in this equilibrium if it expects a non-negative profit from doing so.

There is an infinity of potential mappings $\mathcal{M}$ generating an infinity of optimal contract offers for L at stage 2, and thus an infinity of equilibria.

For any allocation $(a(.), \pi(.))$ of final prices and profits (net of investment costs), the expected surplus of the local authorities is

$$S = \mathbb{E}_\theta \left\{\theta W(a(\theta)) + \theta a(\theta) D(a(\theta)) - \pi(\theta)\right\}.$$

Conditional on the incumbent investing, the local authorities stage 2 equilibrium surplus is the maximal surplus achievable given that contracts are accepted by all types if $\mathcal{M}(\mathcal{C}) = FP$ while others are rejected for $\theta > \theta^p$. The minimal rent scenario maximizes the pre-image of FP under all scenarii, while the maximal rent scenario minimizes it. In order to derive the set of equilibrium allocations of the full game, it suffices to notice that the local authorities payoff is minimized when the set of contracts inducing full participation ($\mathcal{S}(\mathcal{C}) = FP$) is maximal, thus under the maximal rent scenario. To see that, notice that if L offers the contract $\left(a^A, \pi^A\right)$ then the incumbent accepts for all relevant types θ under any mapping different from the maximal rent scenario. One immediate consequence is that S cannot be smaller than under the maximal rent scenario. This implies the following characterization, where we define S^A as the surplus obtained by L under the maximal rent scenario if the incumbent invests :

Proposition 9 *Suppose that $\bar{\theta} > \hat{\theta}$. There exists an equilibrium where I invests and the final allocation is $(a(.), \pi(.))$ if and only if the allocation is incentive compatible, I expected profit is at least c and $L's$ expected surplus is at least S^A.*

Proof. The necessity is immediate because an offer $\mathcal{C}^A$ is accepted (by relevant types) for any section $\mathcal{M}$ of continuation equilibria.

For sufficiency consider stage 2 when the incumbent has invested. Let S be $L's$ surplus if $(a(.), \pi(.))$ is implemented and assume that $S \geq S^A$. Define the contract $\bar{\mathcal{C}} = (a(.), \pi(.))$. We build the equilibrium by choosing the stage 3 selection mapping as follows:

i) $\mathcal{M}\left(\bar{\mathcal{C}}\right) = FP$: if L offers $\mathcal{C}$ and the firm rejects then L duplicates;

ii) If $E[\theta \mid \theta \geq \theta^p] > \hat{\theta}$, then $\mathcal{M}(\mathcal{C}) = FP$;

iii) For any other offer, $\mathcal{M}(\mathcal{C}) = PP$.

Thus the stage 3 continuation equilibrium coincides with the maximal rent scenario for all contracts except $\bar{\mathcal{C}}$. This implies that the maximal surplus that L can expect by offering $\mathcal{C} \neq \bar{\mathcal{C}}$ is S^A. Thus it is optimal to offer $\bar{\mathcal{C}}$. Then condition i) ensures that the incumbent accepts for all θ. Finally $\hat{\theta} < \bar{\theta}$ implies that duplication is credible, $E(\pi(\theta)) \geq c$ ensures that the incumbent invest. ■

Thus one can characterize all the equilibrium allocations by using the maximal rent scenario surplus.

6 Optimal ex-ante regulation

6.1 Presentation

In the previous sections, we have taken the price r as given. As in Auriol-Picard (2008), this price can be the standard monopoly price. Indeed, in our model, this price does not depend on the level of the realized demand. Therefore, even if it is fixed ex post, one can easily anticipate its precise value ex ante, i.e., before any party has taken any action (investment or contracts).

In this section, we look at another possible case, namely that this price be set by a national regulator. Indeed, in many network industries, the access price for infrastructure is constrained by a price-cap chosen by a regulator on a national basis. In our context, this price must maximize social welfare taking into account the impact of local authority intervention on the incumbent incentives to invest.

To study the choice of ex ante regulation, we consider three cases in turn. First the situation of symmetric information at the contracting stage (as in section 3). Then we will look at the optimal regulatory price when the level of demand is private information of the incumbent but without considering the possibility of duplication (as in section 4). At last, we will discuss the general case with both asymmetric information on the demand and threat of duplication.

Since the price r is endogenous, we use subscript r to highlight variable that are directly affected by r.

6.2 Ex ante regulation with contracting under complete information

We discuss here the possibility of an ex ante regulated choice of r in the situation developed in section 3.2. Even if the negotiation takes place under complete information, there is still some uncertainty not only at the regulation stage but also at the investment stage. At this latter stage, the incumbent does not know the ex post level demand θ but knows that there will be duplication (or possible hold-up) if $\theta \geq \hat{\theta}_r$. Since $\hat{\theta}_r$ is an decreasing function of r, the higher r, the larger the probability of duplication. The ex-ante social welfare function can be written as

$$S=\int_{\underline{\theta}}^{\hat{\theta}_r}[\theta W(r)+\theta rD(r)]\,f(\theta)d\theta+\int_{\hat{\theta}_r}^{\overline{\theta}}[\theta W(0)-k]\,f(\theta)d\theta-c$$

The first derivative with respect to r is such that

$$\frac{dS}{dr}=\frac{d\hat{\theta}_r}{dr}[\hat{\theta}_rD(r)r]f(\hat{\theta}_r)+\int_{\underline{\theta}}^{\hat{\theta}_r}[\theta rD'(r)]\,f(\theta)d\theta$$

Since $\hat{\theta}_r$ and $D(r)$ decrease with r, this derivative is always negative. Nevertheless, the incumbent must be induced to invest so r cannot be below the level a_D defined in section 2.

When the national regulator can choose the access price, he chooses the level that just induces the incumbent to invest, i.e., r is set equal to a_D.

6.3 Ex ante regulation with contracting under asymmetric information without duplication

Consider now the second case when duplication is not possible but the level of demand is only known by the incumbent at the contractual stage. In this case, the choice of r has no impact on the probability of duplication but rather on the rents left to the incumbent at the contracting stage and so on its incentives to invest ex ante. More precisely, we have seen in section 4 that negotiation leads to the contact $\mathcal{C}^*$ where low-type incumbents (incumbent with type less than θ_r^*) are proposed a contract with a negotiation access price $a^*(\theta)$ while high-type incumbent will choose the initial price r. Since transfers do not matter for computing social welfare, the latter (in expected terms) is given by

$$S=\int_{\underline{\theta}}^{\theta_r^*}[\theta W(a^*(\theta))+\theta a^*(\theta)D(a^*(\theta))]\,f(\theta)d\theta+\int_{\theta_r^*}^{\overline{\theta}}[\theta W(r)+\theta rD(r)]f(\theta)d\theta-c$$

In lemma 1, θ_r^* has been defined as $\min\{\overline{\theta},a^{*-1}(r)\}$. If $\theta_r^*=\overline{\theta}$ then a change in r has only an impact on the transfers given to the incumbent and therefore does not modify expected social welfare. On the contrary, if $\theta_r^*=a^{*-1}(r)$, increasing r has an impact of expected social welfare. More precisely,

$$\frac{dS}{dr}=\int_{\theta_r^*}^{\overline{\theta}}[\theta rD'(r)]f(\theta)d\theta<0$$

As in the previous case, the optimal regulated price is the lowest price compatible with investment by the incumbent. Let r^I denote the minimal access price inducing investment when no negotiation takes place and all incumbents propose the same access price.

Lemma 4 *Define $r=r^*$ as the minimal regulated tariff that induces investment when contract $\mathcal{C}^*$ is anticipated by the firm. Then, r^* is the optimal regulated tariff without duplication and $E(\theta)\,r^*D(r^*)<c$.*

Proof. We have shown that expected social welfare is decreasing with r. Note also that, we have for any r: $\mathbb{E}\{\pi^*(\theta)\} \geq \mathbb{E}\{\pi^R(\theta)\}$. Moreover, using the fact that $a^*(\underline{\theta}) = 0 < r$, this inequality becomes strict. At last, one can easily show that $\mathbb{E}\{\pi^*(\theta)\}$ and $\mathbb{E}\{\pi^R(\theta)\}$ are both increasing is r. The result then directly follows. ■

Thus when duplication is forbidden or not credible, allowing local authorities to subsidize the incumbent not only induces a reduction of prices through local intervention but also allows the national regulator to reduce the price-cap. Thus it is clearly optimal to allow these negotiations.

6.4 Ex ante regulation with contracting under asymmetric information and threat of duplication

In this last case, the task of a national regulator who aims at controlling access prices while inducing private investment is rather complex. Not only he must anticipate the type of equilibrium played in the sub-game following his choice but the schedule of rents in one of the equilibrium is heavily influenced by the tariff set.

Consider first the minimal rent scenario. In this case, the only situation where investment is induced is when $\hat{\theta}_r > \overline{\theta}$ which, as we have seen above, occurs when r is below the threshold $\underline{a}$. Then

Proposition 10 *Assume that the minimal rent scenario emerges with duplication and local transfers allowed. If $r^* \geq \underline{a}$, then there is no regulated price that may induce investment and regulation is useless. If $r^* < \underline{a}$, then the optimal regulated price is $r = r^*$.*

Proof. The optimal access price is the minimal price that avoids duplication (or hold-up) and induce investment. The first condition leads to $r < \underline{a}$ and the second $r \geq r^*$. Therefore if $r^* \geq \underline{a}$, no regulation may avoid duplication in case of investment so regulation is useless. If $r^* < \underline{a}$, the national regulator should choose the lowest price, i.e., $r = r^*$. ■

We will study now the maximal rent scenario. Remind that $\bar{a}$ is such that $\hat{\theta}_{\bar{a}} = E(\theta)$. For any regulated tariff above $\overline{a}$, the contract offered by the local authorities leads to the null profit for the firm. Therefore, under the maximal rent scenario, only regulated tariff less this level should be considered. Welfare writes as

$$S = \mathbb{E}_\theta \left\{\theta W(a(\theta)) + \theta a(\theta) D(a(\theta))\right\}.$$

and that in the case where negotiation is allowed the final price takes the form

$$a(\theta) = \min\left\{a^*(\theta), \alpha_r^A\right\},$$

where $\alpha_r^A = r$ if $r \leq \hat{a}$ and $\alpha_r^A < \hat{a}$ if $r > \hat{a}$, from proposition 7. Thus α_r^A reaches a maximum at $r = \hat{a}$.

$>$From the regulator's perspective, the only objective is to decrease effective price, i.e. the price implemented after negotiation, and at the same time ensuring that investment takes place. As different types of contracts can be proposed in the maximal rent scenario, one must discuss the level of price necessary to induce investment with the different contracts.

Suppose first that $r^* \geq \hat{a}$ and there exists $r^A \in]\hat{a}, \overline{a}[$ such that $E[\pi^A_{r^A}] \geq c$. Then we know that the effective access price is less than the regulated tariff r^A since it is at most equal to $\alpha^A_{r^A}$. Therefore the optimal regulated price is the highest r^A such that the firm breaks makes a positive profit which also yields the smallest value of $\alpha^A_{r^A}$.

Suppose then that $r^* \leq \hat{a}$ and that there exists $r^A \in]\hat{a}, \overline{a}[$ such that $E[\pi^A_{r^A}] \geq c$. Then the choice is between r^* and some value of $r > \hat{a}$ for which we know that the effective access price is less than the regulated tariff since it is at most equal to $\alpha^A_{r^A}$. Of course choosing $r > \hat{a}$ is optimal only if it reduces the final price $\alpha^A_{r^A}$ below r^*.

We can then derive the following proposition.

Proposition 11 *Assume that the maximal rent scenario emerges with duplication and local transfers allowed. Let $\bar{r}^A$ be the highest $r \in [\hat{a}, \overline{a}[$ such that $E[\pi^A_r] \geq c$.*

1. *If $r^* \geq \hat{a}$, then $\bar{r}^A$ is the optimal regulated tariff.*
2. *If $r^* < \hat{a}$, then*
   - *(a) if $r^* \leq \alpha^A_{\bar{r}^A}$, r^* is the optimal regulated tariff;*
   - *(b) if $r^* \geq \alpha^A_{\bar{r}^A}$, $\bar{r}^A$ is the optimal regulated tariff.*

Given that the final price is a monotonic function of α^A, the proposition shows that under the maximal rent scenario, once the price-cap r is adjusted to reflect the change in expected profit of the firm, allowing the local authority to subsidize the incumbent leads to prices that are uniformly lower, except for small r^*.

Surprisingly, too high a price-cap r $(r > \bar{r}^A)$ may result in no investment as it exacerbates the threat of duplication.

To conclude this section, notice that allowing for more general scenarii would yield a larger set of possible outcome. Indeed, proposition 9 implies that, provided that $r^* < \bar{a}$, any price-cap r and contract $C = (a(.), \pi(.))$ that yields expected profit larger than c and expected local surplus S larger than $S^A_r$is an equilibrium of the game where a national regulator set $r$in a first stage. To see that this is an equilibrium, it suffices to notice that the national regulator would have no incentive to set a different price-cap if doing so would always induce the minimal rent scenario (and thus no investment). Proposition 9 then shows that C is indeed an equilibrium allocation of the continuation game. Thus local subsidies may result in lower or larger final prices.

7 Conclusion

Our work emphasizes the dual nature of local intervention in infrastructures. Local government may subsidize entry of new competitors or may subsidize incumbent firms to improve their market offers. Our companion paper analyzes the first form of intervention, pointing to potential benefits and issues, and discussing some remedies. This paper focused on the second dimension.

Without surprise, we found that albeit asymmetries of information between the incumbent and the local authorities, allowing local governments to subsidize incumbent firms improves efficiency. The conclusion extends to situations with asymmetric information provided that entry of new competitors is not possible due to technological or regulatory barriers to entry. At this stage we should point that this result relies on the implicit assumption that there is no externality between local collectivities or regions. [19]

Our main contribution has highlighted the complex interaction between the subsidies that a local government may offer to an incumbent, and its ability to subsidize entry of competitors (being private or public). This interaction may result in very inefficient outcomes where hold-up problems are exacerbated and generate underinvestment by efficient incumbents. While it may also result in improved efficiency, our analysis suggests that it introduces element of complexities for national regulators that may be difficult to resume. Complexity may first arise due to the difficulty in predicting the final outcome, given the multiplicity of equilibria. Second, we have shown that, contrarily to the standard regulatory environment, following a lenient regulatory policy is not sufficient to induce sufficient private investment. The freedom local authorities have in choosing their mode of intervention makes lenient regulation as risky as tight regulation.

When investment would not occur without a subsidy, then clearly allowing for local subsidy raises efficiency by fostering investment. This is the case in telecommunication for the so-called "white zone" where no operator wish to invest. In other cases, faced to this complexity and potential regulatory failure, one may try to limit intervention to one type of policy by either preventing entry subsidies or preventing subsidies to incumbent. Any of these solutions requires that the incumbent be well identified and corresponds to an asymmetric regulation which may apply only to restricted situations with little innovation. One alternative policy that may be worth exploring is to limit local intervention to the case of wholly owned public subsidiaries that compete on equal footing with private firms, as a mean to commit to a particular and restricted type of intervention.

In many cases, a single incumbent is not well identified (for instance for new generation broadband mobile communications). In this case, it seems tantamount to distinguish ex-post between a subsidy to reduce price or to raise quality and a subsidy to entry. To address these issues, one would need to extend the model by considering for imperfect competition between

[19] See our companion paper on this issue

several firms. While we believe our insights would extend to this case, new phenomena may arise. In particular ex-post negotiation between the local authority and firm A may generate a hold-up problem for firm B. Moreover ex-post competition for subsidies may create a global hold-up problem and discourage investment.

Appendix

Proof of Proposition 1. Assume first that $\Pi < c$. In this case, there is no private investment without public subsidies. When L can not contract with I, social welfare is given by

$$\int_{\underline{\theta}}^{\bar{\theta}} \max\{[\theta W(0) - k] f(\theta) d\theta, 0\}.$$

With contracting, if L offers a transfer $T = c$ when it is optimal to do so, social welfare is given by

$$\int_{\underline{\theta}}^{\bar{\theta}} [\theta W(0) - c] f(\theta) d\theta.$$

Since $c < k$ and $\underline{\theta} W(0) > c$, contracting is clearly socially optimal. Moreover, in this case, social welfare is equal to consumer's welfare so L's choices are socially optimal.

Assume now that $\Pi > C$. Then, even when no contract has been signed ex-ante, I decides to build the new infrastructure knowing that it will be duplicated whenever $\theta > \hat{\theta}$. When no contract is signed ex ante, L's utility is given by

$$\int_{\underline{\theta}}^{\hat{\theta}} \theta W(r) f(\theta) d\theta + \int_{\hat{\theta}}^{\bar{\theta}} [\theta W(0) - k] f(\theta) d\theta.$$

When L proposes a contract with a transfer $T = \Pi = \int_{\underline{\theta}}^{\hat{\theta}} \theta r D(r) f(\theta) d\theta$, L's utility is

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta W(0) f(\theta) d\theta - \int_{\underline{\theta}}^{\hat{\theta}} \theta r D(r) f(\theta) d\theta.$$

Contracting is optimal for L if and only if

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta W(0) f(\theta) d\theta \geq \int_{\underline{\theta}}^{\theta} \theta [W(r) + rD(r)] f(\theta) d\theta + \int_{\hat{\theta}}^{\bar{\theta}} [\theta W(0) - k] f(\theta) d\theta.$$

Since $\theta W(0) \geq \max\{\theta[W(r) + rD(r)], \theta W(0) - k\}$, it is optimal for L to propose ex ante a contract to I. Let us look at last at social welfare in this case. In this absence of contract, social welfare is given by

$$\int_{\underline{\theta}}^{\hat{\theta}} \theta [rD(r) + W(r)] f(\theta) d\theta - c + \int_{\hat{\theta}}^{\bar{\theta}} [\theta W(0) - k] f(\theta) d\theta.$$

while with a contract between L and I, it is given by

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta W(0) f(\theta) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} \theta r D(r) f(\theta) d\theta - c.$$

Contracting is socially optimal if and only if

$$\int_{\underline{\theta}}^{\overline{\theta}} \theta W(0) f(\theta) d\theta \geq \int_{\underline{\theta}}^{\theta} \theta W(r) f(\theta) d\theta + \int_{\hat{\theta}}^{\overline{\theta}} [\theta W(0) - k] f(\theta) d\theta.$$

Ex-ante contracting is then even more interesting from a social point of view than from L's point of view. ■

Proof of Lemma 1. We solve this problem in two steps, first by looking at the interval $[\theta, \theta^p]$ and then optimizing with respect to θ^p.

The first part is solved using Pontryagin Principle. We define the Hamiltonian of the problem, with μ the co-state variable, as:

$$H = [\theta W(a(\theta)) + \theta a(\theta) D(a(\theta)) - \pi(\theta)] \, g(\theta) + \mu(\theta) \, [a(\theta) D(a(\theta))],$$

Using the sufficient theorems for concave objectives derived by Seierstad and Sydsaeter (1977), the following conditions must hold:

- $a(.)$ should maximize H so the first-order condition is:

$$a(\theta) \in \arg\max_{a \in [0,r]} \theta W(a) + \left(\theta + \frac{\mu(\theta)}{g(\theta)} \right) a D(a).$$

- $\dot{\mu} = -\frac{\partial L}{\partial \pi} = g(\theta)$.
- $\dot{\pi} = a(\theta) D(a(\theta))$.
- $\mu(\underline{\theta}) = 0$.

Our strategy to solve the problem is to conjecture a solution and then verify that it satisfies the sufficient condition. Since $a(\theta)$ must be increasing from the SOC but less than the profit maximizing price, the rent $\pi(.)$ will be increasing so strictly positive except at the boundary of the interval of firms that contract with the local authority. The conditions stated above imply $\mu(\theta) = G(\theta)$. Quasi-concavity then implies that $a(\theta) = \min\{a^*(\theta), r\}$ where $a^*(\theta)$ is implicitly defined by:

$$\theta a^*(\theta) D'(a^*(\theta)) + \frac{G(\theta)}{g(\theta)} [D(a^*(\theta)) + a^*(\theta) D'(a^*(\theta))] = 0 \Leftrightarrow \varepsilon(a^*(\theta)) = \frac{\frac{G(\theta)}{g(\theta)}}{\theta + \frac{G(\theta)}{g(\theta)}}.$$

Since $\varepsilon(a)$ and $\frac{\frac{G(\theta)}{g(\theta)}}{\theta + \frac{G(\theta)}{g(\theta)}}$ are non-decreasing, $a^*(\theta)$ is non-decreasing and the SOC holds. If $a^*(\theta^p) \leq r$, the preceding solution holds for all types. If $a^*(\theta^p) > r$, then the contract only applies to incumbents with type θ such that $a^*(\theta) \leq r$. Note that for the types above , we can equivalently assume that no contract is proposed or that a contract with $a = r$ and $t = 0$ is proposed to all incumbent reporting a type such that $a^*(\theta) > r$.

The second part of the proof consists in optimizing with respect to the cut-off type θ^p. Using classical results in dynamic control (see Seierstad-Sydsaeter (1987, chapiter 5, Theorem 17)), this cut-off is such that

$$\max_{\theta^T} \int_{\underline{\theta}}^{\theta^T} \left[\theta W(a^*(\theta)) + \theta a^*(\theta) D(a^*(\theta)) - \pi(\theta)\right] g(\theta) d\theta + \int_{\theta^T}^{\bar{\theta}} \theta W(r) g(\theta) d\theta$$

The first derivative is given by:

$$\theta^T W(a^*(\theta^T) + \theta^T a^*(\theta^T) D(a(\theta^T)) - \pi(\theta^T) - \theta^T W(r)$$

Since by continuity, $\pi(\theta^T) = \theta^T r D(r)$, it can be written as

$$\theta^T \left[W(\hat{a}(\theta^T) + \hat{a}(\theta^T) D(a(\theta^T)) - W(r) - rD(r)\right]$$

Using the fact that $W(a) + aD(a)$ is monotonic, the objective is then quasi-concave with the first derivative positive up to θ^T such that $a^*(\theta^T) = r$ and negative after. Therefore, if $a^*(\bar{\theta}) < r$, then $\theta^* = \bar{\theta}$. Otherwise, it is defined by $a^{*-1}(r)$. ■

Proof of Lemma 2. Consider first the case where $E[\theta \mid \theta > \theta^p] > \hat{\theta}$. It is easy to see that there must exist an equilibrium such that all agents accept and that induce duplication in case of out-of equilibrium refusal. Indeed, by choosing wisely the out-of-equilibrium belief following contract refusal (for example $\bar{\theta}$), duplication follows any refusal so all offers are accepted.

Let us show now that there is no other type of equilibrium. First, it is clear that an incumbent with type less than θ^p has no reason to refuse since its profit is greater with the contract with without, even if the local authority duplicates. Let us now look at the incumbents whose types are greater than $\hat{\theta}$ - the high types. Suppose now that some high type refuse the contract. It is easy to show, for incentives compatibility reasons, that the set of those refusing types should be connected. Therefore, the expected value of this set is at least as high as $E[\theta \mid \theta > \theta^p]$ and so always greater than $\hat{\theta}(a)$ so there will be duplication by the local authority and the incumbent will get zero profit. Therefore, when $E[\theta \mid \theta > \theta^p] > \hat{\theta}$, the only possible equilibrium is such that all incumbents accept and there is duplication in case of out-of-equilibrium refusal.

Suppose now that $E[\theta \mid \theta > \theta^p] < \hat{\theta} \leq \bar{\theta}$. There is now two possible classes of equilibria. The first one has the same feature as above. But there is also the possibility that only part of the type accept the contract. Indeed, if all types greater than θ^p refuse the contract, the local authority has no incentive to duplicate. ■

Proof of lemma 3. We have seen above that any contract under the maximal rent scenario corresponds to an allocation that is feasible, incentive compatible and $\pi(\tilde{\theta}) \geq \pi^R(\tilde{\theta})$. Let us then show that the condition is sufficient.

Let us assume first that $\pi(\tilde{\theta}) > \pi^R(\tilde{\theta})$. Then, consider the contract $\mathcal{C} = (a(\theta), \pi(\theta))$ with full participation and duplication out-of-equilibrium. This contract with full participation implements the allocation. Indeed, since $\pi(\tilde{\theta}) > \pi^R(\tilde{\theta})$, for all $\theta \leq \tilde{\theta}$, $\pi(\theta) > \pi^R(\theta)$ so $\theta^P > \tilde{\theta}$. It is then direct to see that the contract is accepted by all and so implements the initial allocation.

Let us now assume that $\pi(\tilde{\theta}) = \pi^R(\tilde{\theta})$ in the allocation. We define the cut-off τ by $\tau = \max\{\theta \mid a(\theta) < r\}$.
Consider the case where $\tau > \tilde{\theta}$ then the arguments are similar as above. Indeed, since $a(\theta)$ is increasing by incentive compatibility, then for all $\theta < \tilde{\theta} < \tau$, we have $a(\theta) < r$. Since the slope of $\pi(\theta)$ is increasing with $a(\theta)$, for $\theta < \tilde{\theta}$, $\pi(\theta) > \pi^R(\theta)$ so $\tilde{\theta} = \theta^P$ and the contract with full participation implements the allocation.
Now, consider the case where $\tau < \tilde{\theta}$. Notice that in this case, we have $a(\theta) = r$ for all $\theta > \tau$ which implies that $\pi(\theta) = \pi^R(\theta)$ for all these types. Also, for the same reason as above, we have for $\theta < \tau$, $\pi(\theta) > \pi^R(\theta)$. As before, any contract that replicates the allocation for $\theta < \tau$ implements the allocation. For example, consider the contract $\mathcal{C}$ with profit schedule $\pi^C(\theta) = \pi(\theta)$ for all type. Then, $\theta^P = \tau$, $\mathcal{C}$ is accepted by types less then τ and the allocation is implemented.
Finally, consider the case where $\tau = \tilde{\theta}$. Then, combining the above reasoning, when the contract $\pi^C(\theta) = \pi(\theta)$ is offered for all types, then there are two continuation equilibria: one where all types accept and rejection triggers duplication and another one where only types below $\tilde{\theta}$ accept and there is no duplication in case of rejection. Thus the first equilibrium implements $\pi(\theta)$. ■

Proof of proposition 5. Notice that $\pi(\tilde{\theta}) = \pi^R\left(\tilde{\theta}\right)$ is the only binding constraint. From Jullien, Theorems 3 and 4 (adapting the constraint $q \geq 0$ to $a \leq r$), the solution is characterized by (where in Theorem 4)

- $a\left(\theta\right)$ is continuous;
- $\gamma^*\left(\theta\right) = 0$ for $\theta < \tilde{\theta}$ and $\gamma^*\left(\theta\right) = 1$ for $\theta \geq \tilde{\theta}$
- There exists θ^A such that:
  - $a(\theta) = a^*\left(\theta\right) < r$ if $\theta < \theta^A$, $a\left(\theta\right) = \alpha^A \leq r$ if $\theta \geq \theta^A\left(r\right)$
  - When evaluated at $\alpha^A = a^*(\theta^A)$, if $\alpha^A > 0$ for all $\tau \in \left[\theta^A, \bar{\theta}\right]$

$$\int_{\theta^A}^{\tau} \frac{\partial}{\partial a}\left(\frac{G(\theta) - \gamma^*\left(\theta\right)}{g(\theta)}W(\alpha^A) + (\theta + \frac{G(\theta) - \gamma^*\left(\theta\right)}{g(\theta)})\alpha^A D(\alpha^A)\right) g(\theta)d\theta \geq 0 \quad \text{(A1)}$$

  - and if $\alpha^A < r$, for all $\tau \in \left[\theta^A, \bar{\theta}\right]$

$$\int_{\tau}^{\bar{\theta}} \frac{\partial}{\partial a}\left(\frac{G(\theta) - \gamma^*\left(\theta\right)}{g(\theta)}W(\alpha^A) + (\theta + \frac{G(\theta) - \gamma^*\left(\theta\right)}{g(\theta)})\alpha^A D(\alpha^A)\right) g(\theta)d\theta \leq 0. \quad \text{(A2)}$$

We write

$$
\begin{aligned}
&\frac{\partial}{\partial a}\left(\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)}W(a)+(\theta+\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)})aD(a)\right) \\
=\ &\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)}D(a)+(\theta+\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)})aD'(a) \\
=\ &D(a)\left(\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)}-(\theta+\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)})\varepsilon(a)\right).
\end{aligned}
$$

Consider the case where $\underline{\theta}<\theta^{A}<\bar{\theta}$. Continuity implies that $\alpha^{A}=a^{*}(\theta^{A})>0$. Then

$$
\begin{aligned}
\frac{G(\theta)}{g(\theta)}-(\theta+\frac{G(\theta)}{g(\theta)})\varepsilon(\alpha^{A}) &> 0 \text{ for } \theta>\theta^{A} \text{ as } \alpha^{A}<a^{*}(\theta) \\
\frac{G(\theta)-1}{g(\theta)}-(\theta+\frac{G(\theta)-1}{g(\theta)})\varepsilon(\alpha^{A}) &< 0 \text{ for } \theta>\theta^{A} \text{ as } \varepsilon(\alpha^{A})<1
\end{aligned}
$$

Thus the LHS of $(A1)$ is quasi concave in τ, while the LHS of $(A2)$ is quasi convex. The condition $(A1)$ then holds for all τ if it holds for $\tau=\bar{\theta}$:

$$
\int_{\theta^{A}}^{\bar{\theta}}\left(\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)}-(\theta+\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)})\varepsilon\left(\alpha^{A}\right)\right)g(\theta)d\theta\geq 0;
$$

while the condition $(A2)$ holds for all τ if

$$
\int_{\theta^{A}}^{\bar{\theta}}\left(\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)}-(\theta+\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)})\varepsilon\left(\alpha^{A}\right)\right)g(\theta)d\theta\leq 0.
$$

Thus we have condition (7). Notice that this implies $\theta^{A}<\tilde{\theta}$, since otherwise the integral is negative.

Similarly if $\alpha^{A}=r$ we have

$$
\int_{\theta^{*}}^{\bar{\theta}}\left(\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)}-(\theta+\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)})\varepsilon\left(r\right)\right)g(\theta)d\theta\geq 0;
$$

and for $\alpha^{A}=0$ we have

$$
\int_{\underline{\theta}}^{\bar{\theta}}\left(\frac{G(\theta)-\gamma^{*}(\theta)}{g(\theta)}\right)g(\theta)d\theta=\tilde{\theta}-E\left(\theta\right)\leq 0.
$$

■

Proof of corollary 1. θ^{A} is implicitly defined by

$$
\left(1-\varepsilon(\alpha^{A}\right)\left(\tilde{\theta}-\theta^{A}G\left(\theta^{A}\right)-\int_{\theta^{A}}^{\bar{\theta}}\theta g\left(\theta\right)d\theta\right)-\varepsilon(\alpha^{A})\int_{\theta^{A}}^{\bar{\theta}}\theta g(\theta)d\theta=0
$$

The derivative of the LHS with respect to r is simply

$$(1-\varepsilon(\alpha^A))(\frac{d\tilde{\theta}}{dr})<0 \tag{A3}$$

since $\frac{d\tilde{\theta}}{dr}<0$. Let us now look at the partial derivative with respect to θ^A.

$$-\frac{da^*(\theta)}{d\theta}\frac{d\varepsilon}{da}\left[\tilde{\theta}-\theta^A G\left(\theta^A\right)\right]+\left(1-\varepsilon(\alpha^A)\right)\left[-G(\theta^A)\right]+\varepsilon(\alpha^A)\theta^A g(\theta^A)$$

By definition,

$$\varepsilon(a^*(\theta^A))=\frac{G(\theta^A)}{G(\theta^A)+\theta^A g(\theta^A)}$$

and $\alpha^A=a^*\left(\theta^A\right)$ .Therefore, it is direct to see that

$$\left(1-\varepsilon(\alpha^A)\right)\left[-G(\theta^A)\right]+\varepsilon(\alpha^A)\theta^A g(\theta^A)=0$$

The partial derivative with respect to θ^A is simply given by

$$-\frac{da^*(\theta)}{d\theta}\frac{d\varepsilon}{da}\left[\tilde{\theta}-\theta^A G(\theta^A)\right]<0$$

since $\tilde{\theta}>\theta^A$, a^* in increasing and the elasticity of demand is also increasing (by assumption).

Since both partial derivatives of the equilibrium equation defining θ^A are negative, it is direct to state the first result, i.e. θ^A decreasing with r. Moreover, by definition, $\alpha^A(r)=a^*(\theta^A(r))$. Since a^* is an increasing function, the above result implies that $\alpha^A(r)$ is decreasing.
■

Proof of proposition 7. In this proof, we make explicit the relationship between the various thresholds $(\theta^A,\theta^*,\tilde{\theta}....)$ and the initial price r. Notice first that for r converging to $\underline{a}$, θ^A converges to θ^*. Indeed, consider the inequality (7). Using the definition $\tilde{\theta}(r)$, it is direct that $\tilde{\theta}(\underline{a})=\bar{\theta}$. Therefore, if $\tau=\theta^A(\underline{a})$, then at $r=\underline{a}$, inequality (7) converges to

$$\int_\tau^{\bar{\theta}}\left(\frac{G(\theta)}{g(\theta)}-(\theta+\frac{G(\theta)}{g(\theta)})\varepsilon(a^*(\tau))\right)g(\theta)d\theta\geq 0.$$

If the LHS is strictly positive, then the optimal contract coincides with $\mathcal{C}^*(r)$ and $\theta^*(r)<\bar{\theta}$. If it is equal to zero, it means that θ^A converges to $\tau=\overline{\theta}$ so the solution converges to $\mathcal{C}^*(r)$. Therefore, at least for $r\leq\hat{a}$, the contract implementing the equilibrium allocation is a partial participation contract. We will now look at two different cases.

Suppose that $\theta^*(\underline{a})\geq\bar{\theta}$. Then $\theta^*(r)>\tilde{\theta}(r)$ for $r>\underline{a}$. and the solution of the program P is implemented by a contract with full participation.

Suppose now that $\theta^*(\underline{a})<\bar{\theta}$. We have seen that for r larger but close to $\underline{a}$, the optimal contract is $\mathcal{C}^*(r)$. Since $\theta^*(r)>\underline{\theta}$ is non-decreasing and $\tilde{\theta}(r)$ is decreasing with value $\underline{\theta}$ when

$\hat{\theta}(r) = E(\theta)$, $\theta^*(r) \geq \tilde{\theta}(r)$ for r large enough, implying that $\theta^A(r) < \theta^*(r)$. More precisely, let us define r_1 such that $\theta^*(r_1) = \tilde{\theta}(r_1)$.

1. $r \geq r_1$. Since $\theta^A(r) < \tilde{\theta}(r)$ and the monotonicity of $\theta^*(r)$ and $\tilde{\theta}(r)$, we have $\theta^A(r) < \theta^*(r)$. Therefore, from proposition 5, it is clear that the solution of P is implemented by a contract with full participation.

2. $r \in [\underline{a}, r_1]$. Let us consider the LHS of inequality (7) at $\theta^A = \theta^*$.

$$\int_{\theta^*(r)}^{\tilde{\theta}(r)} \left(\frac{G(\theta)}{g(\theta)} - (\theta + \frac{G(\theta)}{g(\theta)})\varepsilon(r) \right) g(\theta) d\theta$$
$$+ \int_{\tilde{\theta}(r)}^{\bar{\theta}} \left(\frac{G(\theta) - 1}{g(\theta)} - (\theta + \frac{G(\theta) - 1}{g(\theta)})\varepsilon(r) \right) g(\theta) d\theta.$$

For $r = \underline{a}$, $\tilde{\theta} = \bar{\theta}$ so the LHS is positive. For $r = r_1$, $\theta^* = \tilde{\theta}$ so the expression is negative. Moreover, it is monotonic and decreasing with respect to r. Indeed, differentiating with respect to r and using $\tilde{\theta}(r) > \theta^*(r)$ and $\varepsilon'(p) > 0$ leads to

$$\begin{aligned} & -\varepsilon'(r) \left(\int_{\theta^*(r)}^{\tilde{\theta}(r)} (\theta g(\theta) + G(\theta)) d\theta + \int_{\tilde{\theta}(r)}^{\bar{\theta}} (\theta g(\theta) + G(\theta) - 1) d\theta \right) + (1 - \varepsilon(r)) \frac{\partial \tilde{\theta}(r)}{\partial r} \\ = \; & -\varepsilon'(r) \left([\theta G(\theta)]_{\theta^*(r)}^{\tilde{\theta}(r)}) d\theta + [\theta (G(\theta) - 1)]_{\tilde{\theta}(r)}^{\bar{\theta}} \right) + (1 - \varepsilon(r)) \frac{\partial \tilde{\theta}(r)}{\partial r} \\ = \; & -\varepsilon'(r) \left(\tilde{\theta}(r) - \theta^*(r) G(\theta^*(r)) \right) + (1 - \varepsilon(r)) \frac{\partial \tilde{\theta}(r)}{\partial r} < 0 \end{aligned}$$

Therefore, there exists $\hat{a} \in [\underline{a}, r_1]$ such that for $r > \hat{a}$, $\theta^A < \theta^*$ and the solution is implemented with full participation and for $r \leq \hat{a}$, it is implemented by a contact with partial participation ■

Proof of proposition 8. The proof follws directly from the following lemma/ ■

Lemma 5 *Consider the maximal rent scenario. Then*

1. *for $r > \hat{a}$ but close, $E(\pi^A) > E(\pi^*)$.*

2. *If $\tilde{\theta} < E(\theta)$ then $E(\pi^A) < E(\pi^*)$.*

Proof of lemma 5. First, let us define $\psi(\alpha, r)$ as

$$\begin{aligned} \psi(\alpha, r) \;=\; & (rD(r) - \alpha D(\alpha)) \tilde{\theta}(r) + \alpha D(\alpha)) \left(\theta^*(\alpha) G(\theta^*(\alpha)) + \int_{\theta^*(\alpha)}^{\bar{\theta}} \theta g(\theta) d\theta \right) \quad \text{(A4)} \\ & - \int_{\underline{\theta}}^{\theta^*(\alpha)} G(\theta) a^*(\theta) D(a^*(\theta)) d\theta \end{aligned}$$

A direct computation then shows that $E\left(\pi_r^A\left(\theta\right)\right)=\psi\left(\alpha^A\left(r\right),r\right)$

Moreover, we can show that $E\left(\pi^*\left(\theta\right)\right)=\psi\left(r,r\right)$. Indeed, it is direct to show that we have

$$E\left(\pi_r^*\left(\theta\right)\right)=\pi^R\left(\theta^*\right)-\int_{\underline{\theta}}^{\theta^*}G\left(\theta\right)a^*(\theta)D(a^*(\theta))d\theta+\int_{\theta^*}^{\bar{\theta}}\left(1-G\left(\theta\right)\right)rD(r)d\theta.$$

Using $\pi^R\left(\theta^*\right)=\pi^R\left(\tilde{\theta}\right)+rD\left(r\right)\left(\theta^*-\tilde{\theta}\right)$ we have

$$\begin{aligned}E\left(\pi^*\left(\theta\right)\right) &= rD\left(r\right)\tilde{\theta}-\int_{\underline{\theta}}^{\theta^*}G\left(\theta\right)a^*(\theta)D(a^*(\theta))d\theta+rD\left(r\right)\left(\theta^*-\tilde{\theta}+\int_{\theta^*}^{\bar{\theta}}\left(1-G\left(\theta\right)\right)d\theta\right)\\ &= rD\left(r\right)\tilde{\theta}-\int_{\underline{\theta}}^{\theta^*}G\left(\theta\right)a^*(\theta)D(a^*(\theta))d\theta+rD\left(r\right)\left(-\tilde{\theta}+\theta^*G\left(\theta^*\right)+\int_{\theta^*}^{\bar{\theta}}\theta g\left(\theta\right)d\theta\right)\\ &= \psi\left(r,r\right).\end{aligned}$$

Then, comparing the profits amounts to determining the sign of $\psi\left(\alpha^A,r\right)-\psi\left(r,r\right)$

Consider the first derivative of ψ .

$$\begin{aligned}\frac{\partial\psi\left(\alpha,r\right)}{\partial\alpha} &= G\left(\theta^*\right)\alpha D(\alpha)+\alpha D(\alpha))\left(-G\left(\theta^*\left(\alpha\right)\right)\right)\\ &\quad+\frac{\partial\alpha D(\alpha)}{\partial\alpha}\left(\tilde{\theta}\left(r\right)-\theta^*\left(\alpha\right)G\left(\theta^*\left(\alpha\right)\right)-\int_{\theta^*(\alpha)}^{\bar{\theta}}\theta g\left(\theta\right)d\theta\right)\\ &= \frac{\partial\alpha D(\alpha)}{\partial\alpha}\left(-\tilde{\theta}\left(r\right)+\theta^*\left(\alpha\right)G\left(\theta^*\left(\alpha\right)\right)+\int_{\theta^*(\alpha)}^{\bar{\theta}}\theta g\left(\theta\right)d\theta\right)\end{aligned}$$

Notice that

$$\frac{\partial}{\partial\alpha}\left(-\tilde{\theta}\left(r\right)+\theta^*\left(\alpha\right)G\left(\theta^*\left(\alpha\right)\right)+\int_{\theta^*(\alpha)}^{\bar{\theta}}\theta g\left(\theta\right)d\theta\right)=\frac{\partial\theta^*\left(\alpha\right)}{\partial\alpha}G\left(\theta^*\left(\alpha\right)\right)>0.$$

Hence ψ is quasi-convex in α, implying that $\psi\left(\alpha^A\left(r\right),r\right)>\psi\left(r,r\right)$ whenever $\frac{\partial\psi(r,r)}{\partial\alpha}<0$

$$\tilde{\theta}\left(r\right)-\theta^*\left(r\right)G\left(\theta^*\left(r\right)\right)-\int_{\theta^*(r)}^{\bar{\theta}}\theta g\left(\theta\right)d\theta>0.$$

At $r=\hat{a}$ we have $\alpha=\hat{a}$ and from equation (8)

$$\tilde{\theta}\left(r\right)-\theta^*\left(r\right)G\left(\theta^*\left(r\right)\right)-\int_{\theta^*(r)}^{\bar{\theta}}\theta g\left(\theta\right)d\theta=\frac{\varepsilon\left(r\right)}{1-\varepsilon\left(r\right)}\int_{\theta^*(r)}^{\bar{\theta}}\theta g\left(\theta\right)d\theta>0.$$

Hence for r close to $\hat{a}$, $\frac{\partial\psi(\alpha,r)}{\partial\alpha} < 0$ for all $\alpha \in [\alpha(r), r]$ which implies that $\psi\left(\alpha^A(r), r\right) > \psi(r,r)$.

>From the above reasoning, we also have $\psi\left(\alpha^A(r), r\right) < \psi(r,r)$ when $\frac{\partial\psi\left(\alpha^A(r),r\right)}{\partial\alpha} > 0$ or

$$\tilde{\theta}(r) < \theta^*\left(\alpha^A\right) G\left(\theta^*\left(\alpha^A\right)\right) + \int_{\theta^*(\alpha^A)}^{\bar{\theta}} \theta g(\theta)\, d\theta.$$

This holds if $\tilde{\theta}(r) < E(\theta)$ and $\alpha^A = 0$. This implies that $\psi\left(\alpha^A(r), r\right) = \psi(0,r) < \psi(r,r)$. ■

Proof of proposition 7.

$$\begin{aligned} E\left(\pi^A(\theta)\right) &= \pi^R\left(\tilde{\theta}\right) - \int_{\underline{\theta}}^{\theta^A} G(\theta)\, a^*(\theta) D(a^*(\theta)) d\theta \\ &\quad + a_1 D(a_1)\left(-\int_{\theta^A}^{\tilde{\theta}} G(\theta)\, d\theta + \int_{\tilde{\theta}}^{\bar{\theta}} (1 - G(\theta))\, d\theta.\right) \end{aligned}$$

$$E(\pi^*(\theta)) = \pi^*\left(\tilde{\theta}\right) - \int_{\underline{\theta}}^{\tilde{\theta}} G(\theta)\, a^*(\theta) D(a^*(\theta)) d\theta + \int_{\tilde{\theta}}^{\bar{\theta}} (1 - G(\theta))\, a^*(\theta) D(a^*(\theta)) d\theta.$$

$$\begin{aligned} \frac{\partial E\left(\pi^A(\theta)\right)}{\partial r} &= \frac{\partial \pi^R\left(\tilde{\theta}\right)}{\partial r} - a_1 D(a_1)\frac{\partial\tilde{\theta}}{\partial r} \\ &\quad + \frac{\partial}{\partial a}(a_1 D(a_1))\left(-\int_{\tau}^{\tilde{\theta}} G(\theta)\, d\theta + \int_{\tilde{\theta}}^{\bar{\theta}} (1 - G(\theta))\, d\theta\right)\frac{\partial a_1}{\partial r} \end{aligned}$$

Since

$$(1 - \varepsilon(a_1))\left(\int_{\theta^A}^{\tilde{\theta}} G(\theta)) d\theta + \int_{\tilde{\theta}}^{\bar{\theta}} (G(\theta) - 1)\, d\theta\right) = \varepsilon(a_1)\int_{\theta^A}^{\bar{\theta}} \theta g(\theta) d\theta$$

then

$$-\int_{\tau}^{\tilde{\theta}} G(\theta)\, d\theta + \int_{\tilde{\theta}}^{\bar{\theta}} (1 - G(\theta))\, d\theta < 0$$

$$\frac{\partial E\left(\pi^A(\theta)\right)}{\partial r} < \frac{\partial \pi^R\left(\tilde{\theta}\right)}{\partial r} - a_1 D(a_1)\frac{\partial\tilde{\theta}}{\partial r}$$

Moreover

$$E(\pi^*(\theta)) = \frac{\partial \pi^*\left(\tilde{\theta}\right)}{\partial r} - a^*(\tilde{\theta}) D(a^*(\tilde{\theta}))\frac{\partial\tilde{\theta}}{\partial r}$$

Thus $a_1 D(a_1) < a^*(\tilde{\theta}) D(a^*(\tilde{\theta}))$ implies that

$$\frac{\partial E\left(\pi^A\left(\theta\right)\right)}{\partial r} < \frac{\partial E\left(\pi^*\left(\theta\right)\right)}{\partial r} \quad if \quad \frac{\partial \pi^R\left(\tilde{\theta}\right)}{\partial r} \leq \frac{\partial \pi^*\left(\tilde{\theta}\right)}{\partial r}.$$

This holds if $\theta^*\left(r\right) \leq \tilde{\theta}\left(r\right)$ since then $\pi^*\left(\tilde{\theta}\right) = \pi^R\left(\tilde{\theta}\right)$.

Suppose that $\theta^* > \tilde{\theta}$ then

$$\frac{\partial \pi^R\left(\tilde{\theta}\right)}{\partial r} = rD\left(r\right)\frac{\partial \tilde{\theta}}{\partial r} + \frac{\partial rD\left(r\right)}{\partial r}\tilde{\theta}$$

$$\begin{aligned} \pi^*\left(\tilde{\theta}\right) &= \pi^R\left(\theta^*\right) - \int_{\tilde{\theta}}^{\theta^*} a^*(\theta)D(a^*(\theta))d\theta. \\ \frac{\partial \pi^*\left(\tilde{\theta}\right)}{\partial r} &= a^*(\tilde{\theta})D(a^*(\tilde{\theta}))\frac{\partial \tilde{\theta}}{\partial r} + \frac{\partial rD\left(r\right)}{\partial r}\theta^* \end{aligned}$$

Since $\frac{\partial \tilde{\theta}}{\partial r} < 0$, $a^*(\tilde{\theta})D(a^*(\tilde{\theta}))\frac{\partial \tilde{\theta}}{\partial r} > rD\left(r\right)\frac{\partial \tilde{\theta}}{\partial r}$. Since $\theta^* > \tilde{\theta}$, $\frac{\partial rD(r)}{\partial r}\theta^* > \frac{\partial rD(r)}{\partial r}\tilde{\theta}$. Thus again

$$\frac{\partial \pi^*\left(\tilde{\theta}\right)}{\partial r} > \frac{\partial \pi^R\left(\tilde{\theta}\right)}{\partial r}$$

The conclusion is that $\frac{\partial E\left(\pi^A(\theta)\right)}{\partial r} < \frac{\partial E(\pi^*(\theta))}{\partial r}$ on the range $\theta^A < \theta^*$. Since $E\left(\pi^A\left(\theta\right)\right) = E\left(\pi^*\left(\theta\right)\right)$ for $r \leq \hat{a}$, we conclude that $E\left(\pi^A\left(\theta\right)\right) < E\left(\pi^*\left(\theta\right)\right)$ on $r > \hat{a}$. ■

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