

*January 2014*

## Heterogenous switching costs

Gary Biglaiser (University of North Carolina, Chapel Hill)  
Jacques Crémer Toulouse School of Economics (GREMAQ, CNRS and IDEI)  
Gergely Dobos European Commission,(DG Competition)

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Gary Biglaiser  
University of North Carolina, Chapel Hill

Jacques Crémer	Gergely Dobos
Toulouse School of Economics	European Commission,
(GREMAQ, CNRS and IDEI)	(DG Competition)

January 24, 2014

\*We would like to thank Andrew Clausen, Philipp Kircher, George Mailath, Curt Taylor, Paul Klemperer, and the participants at seminars where this paper has been presented, especially those of the Economics department at the University of Cambridge.

## **Abstract**

We consider a simple two period model where consumers have different switching costs. Before the market opens, there was an incumbent who sold to all consumers. We identify the equilibrium both with Stackelberg and Bertrand competition and show how the presence of low switching cost consumers benefits the incumbent, despite the fact that it never sells to any of them.

JEL:D43,L13

keywords: switching cost.

# 1 Introduction

A firm which, in the past, has acquired a clientele composed of consumers with low switching cost will be forced, in the presence of entry, to charge low prices for fear of losing its consumers. By contrast, a firm which in the past would have acquired a clientele composed of high switching cost consumers will be able to charge a high price. Foreseeing this, high switching costs clients will have incentives to “follow” low switching costs clients. All of this will influence the pricing strategies of firms and, as a matter of consequence, the equilibrium distribution of clients. In this paper we explore these links, which have been for the most part ignored in the economic literature.<sup>1</sup>

In order to do so, we build a simple two period model. There is a continuum of consumers who need to buy one unit of a good in each of two periods. Changing supplier induces a switching cost and they try to minimize the total discounted sum of the prices which they pay and of the switching costs that they incur. These consumers can be of two types: those with low switching costs ( $\sigma_L$ ) and those with high switching costs ( $\sigma_H$ ). There are also two types of firms: at the beginning of period 1, an Incumbent firm which has sold to all consumers in the past is present and a sufficient number of potential entrants in each period.

If consumers all had the same switching cost, they could make their purchasing decisions without taking into account the choices of other consumers. On the other hand, when switching costs differ, a firm’s future price will depend on the type of consumers to which it sold to in the past. Therefore, rational consumers take into account which consumers they expect to purchase from each firm when making their purchasing decision. This creates an externality across consumer types; as we will see, at equal prices, high switching cost consumers would prefer to purchase from a firm whose clientele is mostly composed of low switching cost consumers.

We demonstrate that this externality has a significant effect on an incumbent firm’s pricing decision, subsequent market equilibrium market shares as well as profits. For example, if  $\sigma_L/\sigma_H$  is small enough the incumbent firm will price so that it will *always* lose some of the high switch cost consumers in equilibrium; it finds it too expensive to “keep” all of these consumers. Furthermore, while the incumbent will never sell to any low switching cost

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<sup>1</sup>Switching cost models were introduced in the economics literature by Klemperer (1987) (see the surveys of the theoretical literature in Klemperer (1995), Annex A of National Economic Research Associates (2003), and Farrell and Klemperer (2007), and the discussion of policy implications in National Economic Research Associates (2003, especially Annex C)). The paper the closest to ours is Taylor (2003). We discuss the literature in more detail in section 5.

consumers their presence is important for its profits which increase as their number increases and as their switching cost decrease. We are even able to identify circumstances where, for intermediate values of  $\sigma_L/\sigma_H$  the profits of the incumbent is smaller in the two period model than in the one period model (see Corollary 4). Finally, we demonstrate that for a large class of parameters the negative consequence for the incumbent of a decrease in the switching costs of high switching cost consumers can be overwhelmed by an equal decrease in the switching cost of the low type consumers.

In a companion paper, Biglaiser, Crémer, and Dobos (2013), we study an infinite horizon model where some consumer have positive switch cost and others have no switching cost. We demonstrate that in the stationary equilibrium the incumbent gains from having the zero switching cost consumers in the market because they get in the way of entrants trying to attract consumers with positive switching costs. Since a firm will never make profits off the zero switching cost consumers, their presence will not affect a firm's price and the effect that we identify in the current paper is not present.<sup>2</sup>

We present the model in the next section, section 2. Then, as a benchmark, in section 3, we briefly discussed the equilibrium when all consumers have the same switching cost. The bulk of our analysis is presented in section 4: we characterize and show of existence of the equilibrium. We present a brief discussion of the literature in section 5 and some concluding remarks in section 6.

## 2 Model

We consider a two period model of switching cost, with initially one incumbent, which we will call the Incumbent, and free entry in every period. The firms produce identical goods and the cost of production is equal to 0.<sup>3</sup> There is a continuum of consumers with mass normalized to 1. Consumers have a perfectly inelastic demand for one unit of the good, and therefore buy one unit in each period — their aim is to minimize the cost of acquiring these units. There are two types of consumers: high switching cost (HSC) consumers, who

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<sup>2</sup>We did not conduct the analysis where both types of consumers had positive switching costs in the infinite horizon model because of the difficulties in defining and proving that stationary equilibria exists in such a setting.

<sup>3</sup>In order to attract consumers, firms will charge negative prices. Our model yields exactly the same results with strictly positive marginal cost: the negative price must simply be interpreted as a discount below marginal cost. In markets where negative prices are impossible, for instance because consumers would buy goods they have no intention of consuming, we would need to assume that the marginal cost is large enough compared to the switching cost.

represent a fraction  $\alpha \in (0, 1)$  of the population, have a switching cost equal to  $\sigma_H$ , while low switching cost (LSC) consumers, who form a fraction  $1 - \alpha$  of the population, have a switching cost equal to  $\sigma_L$ , with  $\sigma_H > \sigma_L > 0$ . The switching cost is incurred every time a consumer changes from one supplier to another. It reflects industry wide similarities or compatibilities between products, rather than idiosyncrasies of specific sellers.

In period 0, and, maybe, in previous periods, the consumers have bought from the Incumbent, firm  $I$ . We do not study the process by which firm  $I$  became the incumbent, but only the continuation game after entry is possible. In general, at least some of the incumbency rents which we identify would have been dissipated in the competition to become the incumbent.

For simplicity, in the main text we assume that there is free entry, *i.e.*, that there is a large number<sup>4</sup> of potential entrants which can enter the market at zero cost, in periods 1 and 2. It is easy to see that the proofs hold with minor changes if there are only two potential entrants in each period.<sup>5</sup> In period 2 the incumbent(s) are all the firms that sold to a strictly positive mass of consumers in each period. These incumbents include the Incumbent, if, as he clearly will in equilibrium, “keep” some of its consumers, and the *successful* or *active* period 1 entrants, that is those who attracted consumers. The main focus of our study is the following “Bertrand” game:

**Stage 1:** The incumbent(s) and the entrants set prices;

**Stage 2:** The consumers choose from which firm to buy.

All of our qualitative results also hold true, and are sometimes easier to establish, in the “Stackelberg” version of this game:

**Stage 1:** The incumbent(s) sets price(s);

**Stage 2:** The entrants set their prices;

**Stage 3:** The consumers choose from which firm to buy.

The discount factor, common to all the firms and all the consumers, is  $\delta \in (0, 1]$ .

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<sup>4</sup>Technically, a denumerable set. This is equivalent to a zero profit condition for entrants.

<sup>5</sup>As we will prove, consumers always buy from one of the lowest price entrants. Therefore Bertrand competition among the entrants forces them to use strategies which yield zero expected profits: in any equilibrium in which this would not be the case, at least one of the entrants would have incentives to undercut the other(s). The free entry assumption simplifies the exposition by making the discussion of this undercutting unnecessary.

Note that following much of the literature, we assume that only short term contracts are used and that consumers' switching costs do not depend on the firm from which they purchase.

We solve for the subgame perfect equilibrium, assuming that the equilibrium strategy of each firm depends only on the number (measure) of consumers of each type which purchased its product in the previous period. This constraint on the strategies only binds in the second period, and implies that a consumer can change supplier in the first period without affecting second period prices.

### 3 One switching cost

Before turning to the analysis of the model described above, it may be worth sketching the analysis of the case where all consumers have the same switching cost:  $\sigma_H = \sigma_L \stackrel{\text{def}}{=} \sigma$  (see Biglaiser et al. (2013) for a more detailed description).

If there is only one period, at equilibrium the Incumbent would clearly charge  $\sigma$  and “keep” all the consumers. Because there is a mass 1 of consumers, its profit would also be equal to  $\sigma$ .

Turning to the two period model, in equilibrium, whether in the Bertrand or Stackelberg models, all second period incumbents (that is all firms that have sold a positive amount of the good in the first period) charge  $\sigma$ , and make profits equal to  $\sigma$  times the mass of their first period customers. Therefore, the lower bound of the prices that entrants can charge in the first period and not make negative profits is  $-\delta\sigma$ . Consumers know that all incumbents will charge  $\sigma$  in the second period. Hence, firm  $I$  will be able to “keep” its customers only by charging a price less than or equal to  $-\delta\sigma + \sigma$ . It is straightforward to show that it indeed charges this price and “keeps” all its customers, under Bertrand or Stackelberg competition. Hence its discounted profit is  $-\delta\sigma + \sigma + \delta\sigma = \sigma$ . The profits are the same in the intertemporal model as in the static model: a firm gets only one bite of the switching costs!

### 4 Analysis

Except in subsection 4.5, we assume that  $\sigma_L$  is small, more precisely,

$$\sigma_L < \frac{\alpha\delta}{1+\delta}\sigma_H, \tag{1}$$

which implies

$$\sigma_L < \alpha\sigma_H. \tag{2}$$

In a one period model, entrants will always charge a price of 0. Thus, the Incumbent would charge  $\sigma_H$ , sell to all the HSC consumers and to no LSC consumer, and make a profit equal to  $\alpha\sigma_H$  instead of selling to all consumers at a price of  $\sigma_L$  for a profit of  $\sigma_L$ . (In subsection 4.5, we study environments where inequality (1) does not hold.)

Turning to the two period model, we will show that equilibrium profits are the same in the Bertrand and Stackelberg model. We present these profits and some comparative statics in Proposition 1 and Corollary 1. After some comments and some interpretation, we prove Proposition 1 for the Stackelberg model in 4.1 and for the Bertrand model in 4.2. It turns out that, despite the fact that profits are the same in both model, consumer surplus and welfare differ; we discuss these differences in 4.4.

The following proposition states our main result.

**Proposition 1.** *If inequality (1) is satisfied, the equilibrium profit of the Incumbent is*

$$\Pi = \sigma_H \left[ \frac{\alpha\sigma_H - \sigma_L}{\sigma_H - \sigma_L} (1 + \delta - \alpha\delta) \right] \quad (3)$$

*under either Stackelberg or Bertrand competition.  $\Pi$  is greater than the one period profit,  $\alpha\sigma_H$ , and smaller than the discounted value of a flow of one period profit,  $\alpha\sigma_H(1 + \delta)$ .*

The presence of LSC buyers enables the Incumbent to generate higher profits than it would receive in the one period model,  $\alpha\sigma_H$ . Furthermore, as  $\alpha$  converges to 1,  $\Pi$  converges to  $\sigma_H$ , the one period profit. This is also the case if  $\sigma_L$  converges to  $\alpha\delta\sigma_H/(1 + \delta)$ , the right hand side of (1).<sup>6</sup>

As a consequence of proposition 1, we have the following corollary.

**Corollary 1.** *Under the hypotheses of Proposition 1:*

1. *The profit  $\Pi$  of the Incumbent is increasing in  $\alpha$  and  $\sigma_H$  and decreasing in  $\sigma_L$ ;*
2. *If  $\alpha < (\sigma_L + \sigma_H)/2\sigma_H$ , which is always satisfied if  $\alpha < 1/2$ , then an equal increase in  $\sigma_H$  and  $\sigma_L$  leads to a decrease in  $\Pi$  ( $\partial\Pi/\partial\sigma_L + \partial\Pi/\partial\sigma_H < 0$ ).*
3. *If  $\sigma_L < \alpha^2\delta\sigma_H/(1 + \delta)$ , then a small increase in the number of LSC consumers increases the profits of the Incumbent. For small enough  $\eta$ , if  $\sigma_H$  is increased by  $\eta$  and  $\sigma_L$  reduced by  $\eta(1 - \alpha)/\alpha$  so that the market average switching cost stays constant, then  $\Pi$  increases.*

Without surprise, when  $\alpha$  or  $\sigma_H$  increase, the profit of the Incumbent increases. To understand why an increase in  $\sigma_L$  decreases profits, we note

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<sup>6</sup>As detailed in Corollary 4, when  $\sigma_L$  becomes larger than  $\alpha\delta\sigma_H/(1 + \delta)$  the profit in the one period model becomes larger than the profit in the two period model.



first that the Incumbent will always price in such a way that it sells to no LSC consumer. Let us assume, only for expository purposes, that only one entrant attracted customers in the first period, and let  $\gamma' > 0$  be the proportion of the HSC customers that it attracted. Because the LSC consumers are the most eager to switch suppliers, the entrant must also have attracted all of them. Therefore, its second period profit is  $\alpha\gamma'\sigma_H$  if it charges  $\sigma_H$ , and  $(\alpha\gamma' + (1 - \alpha))\sigma_L$  if it charges  $\sigma_L$ . If it has attracted the proportion  $\gamma$  of HSC customers such that

$$\begin{aligned} \alpha\gamma\sigma_H &= (\alpha\gamma + (1 - \alpha))\sigma_L \\ \iff \gamma &= \frac{1 - \alpha}{\alpha} \frac{\sigma_L}{\sigma_H - \sigma_L} = \frac{1 - \alpha}{\alpha} \left( \frac{\sigma_H}{\sigma_H - \sigma_L} - 1 \right), \quad (4) \end{aligned}$$

it will be indifferent between charging  $\sigma_L$  and  $\sigma_H$ . From (4), it is straightforward that an increase in  $\sigma_L$  leads to an increase in  $\gamma$ : the benefits of ‘keeping’ the LSC customers increases, thus the number of HSC consumers attracted in the first period must increase if the entrant is to be kept indifferent between its two plausible second period strategies. In equilibrium, in the first period a proportion  $\gamma$  of HSC consumers purchase from the entrant: if fewer than this proportion did so, the entrant would charge a low price in the second period, and be very attractive to HSC customers.<sup>7</sup> Therefore when  $\sigma_L$  increases, the Incumbent loses more customers, which explains the result. The fact that a strictly positive measure of HSC consumers can purchase from an entrant without affecting the price it charges in the second period is the key to the positive externality that the LSC consumers provide the HSC consumers and the resulting loss to the Incumbent.

Whether an equal increase in both  $\sigma_H$  and  $\sigma_L$  will increase or decrease the profit of the Incumbent will therefore depend on the relative strengths of two opposing effects, which, by (3), can be determined by evaluating the change in  $\sigma_H(\alpha\sigma_H - \sigma_L)$ . Adding  $\eta$  to both  $\sigma_H$  and  $\sigma_L$  and taking the derivative for  $\eta = 0$ , we obtain result ii) in Corollary 1: the negative consequences for the Incumbent of an increase in  $\sigma_L$  swamps the positive consequences of an equal increase in  $\sigma_H$  when  $\alpha$  is small enough.<sup>8</sup> (As explained in section 5, we obtain these “counterintuitive” comparative statics for reasons different than the rest of the literature.)

<sup>7</sup>As we will see shortly, the entrant mixes between  $\sigma_H$  and  $\sigma_L$  in the second period.

<sup>8</sup>It is easy to prove by computing the value of the derivative of

$$(1 + \eta)\sigma_H \left[ \frac{\frac{\alpha}{1+\eta}\sigma_H - \sigma_L}{\sigma_H - \sigma_L} (1 + \delta - \frac{\alpha}{1+\eta}\delta) \right] = \frac{\sigma_H}{\sigma_H - \sigma_L} (\alpha\sigma_H - (1 + \eta)\sigma_L) (1 + \delta - \frac{\alpha}{1+\eta}\delta).$$

with respect to  $\eta$  for  $\eta = 0$ .

Note that part 3 of the corollary requires a  $\sigma_L$  smaller than the upper bound authorized by equation (1). Indeed, when  $\sigma_L$  is small entrants do not want to attract LSC customers, and an increase in their number makes them less aggressive. On the other hand, when  $\sigma_L$  is larger, LSC customers become valuable enough to entrants that an increase in their number makes them more aggressive.

Much empirical research is focussed on changes of the average switching cost in the market. Routine calculations demonstrate that if  $\sigma_H$  is increased by  $\eta$  and  $\sigma_L$  is reduced by  $\eta(1-\alpha)/\alpha$ , so as to maintain the average switching cost but increase the variance of the switching costs, then the Incumbent's profit is increasing in  $\eta$ . *An increase in the variance of the switching cost increases the profits of the Incumbent.*

We now turn to the proof of the Proposition 1.

#### 4.1 Proof of Proposition 1 for the Stackelberg model

In period 2 of the Stackelberg model, all the firms which sold strictly positive amounts in period 1 (the “period 2 incumbents”) announce their prices first, followed by the entrants. The following lemma is straightforward.

**Lemma 1.** *In period 2, (i) incumbents charge  $\sigma_H$  if the proportion of HSC consumers in their first period clientèle is strictly greater than  $\sigma_L/\sigma_H$ ; (ii) they charge  $\sigma_L$  if this proportion is strictly smaller than  $\sigma_L/\sigma_H$ ; (iii) they charge  $\sigma_H$  or  $\sigma_L$ , maybe mixing between the two, if it is equal to  $\sigma_L/\sigma_H$ .*

If the firm from which it purchased in period 1 charges  $\sigma_H$  in period 2, a LSC consumer will choose to purchase from a period 2 entrant at a price of 0. Hence, his total period 2 cost will always be exactly  $\sigma_L$ , whatever he does in period 1.

Let us call  $p_I$  the price charged by the Incumbent in period 1 and  $p_E$  the lowest price charged by any entrant in that period. We have established the following lemma.

**Lemma 2.** *In the first period, LSC consumers purchase from one of the lowest price entrants if  $p_E + \sigma_L < p_I$  and from the Incumbent if  $p_E + \sigma_L > p_I$ . If  $p_E + \sigma_L = p_I$  they will either purchase from one of the lowest priced entrants or from the Incumbent.*

Effectively, LSC customers minimize their cost in each period.

Because it is strictly more expensive for HSC than for LSC consumers to switch firms, no HSC consumer switches suppliers in the first period unless all the LSC consumers switch. This implies that the proportion of HSC consumers

in the first period clientèle of the Incumbent will be at least equal to  $\alpha$ , and therefore proves part (i) of Lemma 3. Part (ii) is straightforward.

**Lemma 3.** *In period 2, (i) the first period Incumbent charges  $\sigma_H$ ; (ii) successful first period entrants charge  $\sigma_H$  if the proportion of HSC consumers is strictly greater than  $\sigma_L/\sigma_H$ ; they charge  $\sigma_L$  if this proportion is strictly smaller than  $\sigma_L/\sigma_H$  and either  $\sigma_H$  or  $\sigma_L$  if it is exactly equal to  $\sigma_L/\sigma_H$ .*

Define a *successful* first period entrant as an entrant who attracts a strictly positive mass of consumers. We leave the proof of the following easy lemma to the reader.

**Lemma 4.** *If HSC consumers purchase from an entrant in the first period, they purchase from one of the lowest price entrants and so do all the LSC consumers. The proportion of HSC consumers in the first period clientèle of all first period successful entrants either (i) is less than or equal to  $\sigma_L/\sigma_H$  — in which case in period 2 they all charge  $\sigma_L$  with probability 1; (ii) is greater than or equal to  $\sigma_L/\sigma_H$  — in which case in period 2 they all charge  $\sigma_H$  with probability 1; (iii) is equal to  $\sigma_L/\sigma_H$  — in which case in period 2 they all charge  $\sigma_L$  and  $\sigma_H$  with the same probabilities.*

Going back to period 1 for the Stackelberg model, these three lemmas enable us to prove the following lemma, which describes the continuation payoff of the Incumbent as a function of the price it charges in the first period.

**Lemma 5.** *For a given price,  $p_I$ , charged by the Incumbent in the first period:*

1. *if  $p_I < (1 - \delta)\sigma_L$ , the Incumbent sells to all consumers in period 1 and to all HSC consumers (at price  $\sigma_H$ ) in period 2. Its profit is  $p_I + \delta\alpha\sigma_H$ .*
2. *if  $p_I \in ((1 - \delta)\sigma_L, (1 - \delta)\sigma_H)$ , the Incumbent sells to all HSC consumers in both periods and to no LSC consumer in either period. Its profit is  $\alpha(p_I + \delta\sigma_H)$ .*
3. *if  $p_I \in ((1 - \delta)\sigma_H, (1 - \alpha\delta)\sigma_H)$ , there exists  $\gamma \in (0, 1)$  such that the Incumbent sells to  $\alpha(1 - \gamma)$  HSC consumers at price  $p_I$  in period 1 and at price  $\sigma_H$  in period 2, while its sales to LSC consumers are equal to 0 in both periods. Its profit is  $\alpha(1 - \gamma)(p_I + \delta\sigma_H)$ .*
4. *if  $p_I > (1 - \alpha\delta)\sigma_H$ , the Incumbent has zero sales in both periods.*

From Lemma 5, the profits of the Incumbent are increasing on the intervals  $(-\infty, (1 - \delta)\sigma_L)$ ,  $((1 - \delta)\sigma_L, (1 - \delta)\sigma_H)$  and  $((1 - \delta)\sigma_H, (1 - \alpha\delta)\sigma_H)$ . Given the restrictions that we have imposed on  $\sigma_L/\sigma_H$ , it is easy to check that it is maximized on the union of these intervals for  $p_I$  smaller than and ‘very close to’  $(1 - \alpha\delta)\sigma_H$ . Therefore, the only equilibrium of the game

has the Incumbent charging  $(1 - \alpha\delta)\sigma_H$  in the first period with the continuation equilibrium described in point 3, yielding the profits described by equation (3). For the HSC consumers to be indifferent between purchasing from the Incumbent and an entrant in period 1, it must be the case that any entrant who attracts consumers in period 1 charges  $\sigma_H$  in period 2 with probability

$$q \stackrel{\text{def}}{=} \frac{\alpha\sigma_H - \sigma_L}{\sigma_H - \sigma_L}. \quad (5)$$

This proves Proposition 1 for the Stackelberg model.

## 4.2 The proof of Proposition 1 in the Bertrand model

In this subsection, we prove that if there is an equilibrium, then equation (3), and therefore Proposition 1, hold. We postpone the proof of the existence of an equilibrium to subsection 4.3.

Notice first that Lemmas 2 to 4 also hold in the case of Bertrand equilibria — the proofs are exactly the same.

Because of free entry, period 2 entrants choose a price equal to 0. By Lemma 4, as in the Stackelberg case, a period 2 incumbent charges  $\sigma_L$  or  $\sigma_H$  depending on whether the proportion of its HSC customers in period 1 was less or greater than  $\sigma_L/\sigma_H$ , and, clearly, the period 1 Incumbent will charge  $\sigma_H$  in period 2. Furthermore, this implies that, as in the Stackelberg case, in period 1 LSC consumers will optimally behave as if they were myopic, switching to one of the lowest price entrants if the difference between its price and the Incumbent's price is greater than  $\sigma_L$  and not switching if this difference is smaller than  $\sigma_L$ . It also implies that any HSC consumer who does not buy from the Incumbent in period 1 also buys from one of the lowest priced entrants. Indeed, any other entrant would attract only HSC customers, and therefore charge  $\sigma_H$  in period 2.

We are now ready to study the pricing behavior of the firms in period 1. The next set of claims, hold for any set of parameters and are not restricted to the case when  $\sigma_L < \frac{\alpha\delta}{1+\delta}\sigma_H$ . We begin by Claim 1 which describes the behavior of entrants.

**Claim 1.** *In period 1, any active entrant charges a price in  $[-\delta\alpha\sigma_H, -\delta\sigma_L]$ .*

*Proof.* Any entrant who has attracted consumers in period 1 will charge at least  $\sigma_L$  in period 2. Therefore, competition and free entry will ensure that in period 1 no entrant which charges more than  $-\delta\sigma_L$  attracts a positive measure of customers with positive probability. If the lower priced entrants charge prices strictly smaller than  $-\delta\alpha\sigma_H$  and attract consumers their aggregate profit is negative: they attract all the LSC consumers; therefore the

proportion of HSC consumers in their clientèle is at most  $\alpha$  and their aggregate second period profit at most  $\alpha\sigma_H$ .  $\square$

Thus, an entrant will never price less than the present discounted value,  $\delta\alpha\sigma_H$ , of the period 2 profit if it attracted all the consumers in period 1 and then only kept the HSC customers in period 2. On the other hand, an entrant's price cannot exceed  $-\delta\sigma_L$  in order to attract any consumer; it would be under cut by another entrant. The next two claims describe properties of the Incumbent's first period demand function.

**Claim 2.** *If the Incumbent charges a price strictly greater than  $\sigma_H(1 - \delta)$  in period 1, then it sells to at most  $(1 - \gamma)\alpha$  HSC customers.*

*Proof.* Assume  $p_I > (1 - \delta)\sigma_H$ . Because  $-\delta\sigma_L + \sigma_L < \sigma_H(1 - \delta)$ , Claim 1 implies that all the LSC consumers, who optimally act myopically in the first period, purchase from entrants. If in the aggregate the entrants attract a proportion of HSC customers smaller than  $\gamma$ , at least one of them will have a proportion of period 1 HSC customers strictly smaller than  $\sigma_L/\sigma_H$  and therefore charge  $\sigma_L$  with probability 1 in period 2. HSC customers would find this entrant more attractive than the Incumbent as  $(-\delta\sigma_L + \sigma_H) + \delta\sigma_L < \sigma_H(1 - \delta) + \delta\sigma_H$ , which establishes the contradiction.  $\square$

**Claim 3.** *If in period 1 the Incumbent charges a price strictly smaller than  $\sigma_H(1 - \alpha\delta)$ , then it sells to at least  $(1 - \gamma)\alpha$  HSC customers.*

*Proof.* If the Incumbent sold to fewer than  $(1 - \gamma)\alpha$  consumers, at least one of the successful entrants would attract enough HSC customers in the first period to charge  $\sigma_H$  in the second period; by Claim 1, these HSC customers would incur total discounted costs equal to at least  $-\delta\alpha\sigma_H + \sigma_H + \delta\sigma_H$ , which is strictly larger than the total discounted costs that they would incur from buying from the Incumbent in both periods.  $\square$

Claims 2 and 3 show that for  $p_I \in (\sigma_H(1 - \delta), \sigma_H(1 - \alpha\delta))$ , the Incumbent sells to exactly  $(1 - \gamma)\alpha$  customers. This implies the following claim.

**Claim 4.** *The Incumbent will never choose a first period price in  $(\sigma_H(1 - \delta), \sigma_H(1 - \alpha\delta))$ .*

Furthermore, this allows us to put a lower bound on the profit of the Incumbent.

**Claim 5.** *By choosing  $p_I$  below but 'close to'  $\sigma_H(1 - \alpha\delta)$ , the Incumbent can guarantee itself discounted profits arbitrarily close to  $(1 - \gamma)\alpha\sigma_H(1 - \alpha\delta + \delta)$ .*

*Proof.* It sells to at least  $(1 - \gamma)\alpha$  HSC consumers at price  $(1 - \alpha\delta)\sigma_H$  in period 1 and at price  $\sigma_H$  in period 2.  $\square$

Letting  $\underline{b}_I$  denote the lower bound on the Incumbent's prices, we find

**Claim 6.**  $\underline{b}_I \leq \sigma_H(1 - \alpha\delta)$ .

*Proof.* The Incumbent makes strictly positive profits. This implies that  $p_E$  is not strictly smaller than  $p_I - \sigma_H$  with probability 1. However, if  $\underline{b}_I > \sigma_H(1 - \alpha\delta)$  an entrant could charge a price in  $(-\alpha\delta\sigma_H, \underline{b}_I + \sigma_H)$  and obtain strictly positive expected profits. In the states of nature where it is not the lowest priced entrant, it would attract no consumers and make a profit equal to 0. When it is the lowest price entrant, which would happen with strictly positive probability by Claim 1, it would undercut the other entrants and also undercut the Incumbent by more than  $\sigma_H$ ; its discounted profit would be strictly positive, which establishes the contradiction.  $\square$

Up to now, the only restriction on  $\sigma_L/\sigma_H$  which we have used is that it is smaller than  $\alpha$ . We now restrict the analysis of the cases where condition (1) ( $\sigma_L < \alpha\delta\sigma_H/(1 + \delta)$ ) holds. This enables us to prove the following claim and by implication Proposition 1.

**Claim 7.** *If condition (1) holds, then at equilibrium  $\underline{b}_I > \sigma_H(1 - \delta)$ .*

*Proof.* The Incumbent never sells to any LSC consumers. Indeed, to do so it would have to choose a price of at most  $(1 - \delta)\sigma_L$  and its profit would be at most  $(1 - \delta)\sigma_L + \delta\alpha\sigma_H$ . When (1) holds this profit is less than the profit which it can guarantee by selling to  $(1 - \gamma)\alpha$  consumers (see Claim 5). Indeed, because

$$1 - \gamma = \frac{1}{\alpha} \left( 1 - \frac{(1 - \alpha)\sigma_H}{\sigma_H - \sigma_L} \right)$$

we have

$$\begin{aligned} & (1 - \delta)\sigma_L + \delta\alpha\sigma_H - [(1 - \gamma)\alpha\sigma_H(1 - \alpha\delta + \delta)] \\ &= (1 - \delta)\sigma_L + \delta\alpha\sigma_H - \sigma_H(1 - \alpha\delta + \delta) \left( 1 - \frac{(1 - \alpha)\sigma_H}{\sigma_H - \sigma_L} \right) \end{aligned} \quad (6)$$

$$< \sigma_H \left[ \frac{(1 - \delta)\alpha\delta}{1 + \delta} + \delta\alpha - (1 - \alpha\delta + \delta) \left( 1 - \frac{1 - \alpha}{1 - \frac{\alpha\delta}{1 + \delta}} \right) \right] \quad (7)$$

$$\begin{aligned} &= \sigma_H \left[ \frac{(1 - \delta)\alpha\delta}{1 + \delta} + \delta\alpha - (1 - \alpha\delta + \delta) \frac{\alpha}{1 + \delta - \alpha\delta} \right] \\ &= \frac{\alpha\sigma_H}{1 + \delta} (-1 + \delta) < 0, \end{aligned}$$

where the inequality in (7) stems from (1) and from the fact that the right hand side of (6) is decreasing in  $\sigma_L$

By Claim 2, if the Incumbent chooses  $p_I > \sigma_H(1 - \delta)$ , at least  $\alpha\gamma$  HSC consumers buy from a period 1 entrant. Thus, the highest profit the Incumbent could make while selling to all HSC consumers in period 1 is  $\sigma_H(1 - \delta) + \delta\sigma_H = \alpha\sigma_H$ . Using Claim 5, the Incumbent can improve its profit by charging a price larger than  $\sigma_H(1 - \delta)$ , since equation (1) is equivalent to  $\alpha\sigma_H < (1 - \gamma)\alpha\sigma_H(1 - \alpha\delta + \delta)$ .  $\square$

Claims 4, 6 and 7 imply  $\underline{b}_I = \sigma_H(1 - \alpha\delta)$  whenever (1) holds. By Claim 2 this implies that the discounted profit of the Incumbent is bounded above by

$$(1 - \gamma)\alpha\sigma_H(1 - \alpha\delta + \delta\sigma_H) = (1 - \gamma)\alpha(\underline{b}_I + \delta\sigma_H).$$

By Claim 5, this quantity is also a lower bound on the profit, and this proves Proposition 1.

### 4.3 What do Bertrand equilibria look like?

The reasoning of 4.2 is sufficient to prove equation (3), but a) leaves open the question of existence of equilibrium and b) does not provide much intuition about the equilibrium strategies of the agents. In this subsection, we tackle both of these issues by describing explicitly one equilibrium of the Bertrand game.

In all equilibria the Incumbent and the entrants use mixed strategies in period 1. For simplicity, we present an equilibrium where there is only one<sup>9</sup> active entrant, who chooses its price  $p_E$  in  $[-\alpha\delta\sigma_H, -\delta\sigma_L]$ , while the Incumbent chooses  $p_I$  in  $[\sigma_H(1 - \alpha\delta), -\delta\sigma_L + \sigma_H]$  and at least one other entrant charges  $-\delta\sigma_L$  with probability 1. Then, all LSC customers buy from the active entrant, and, depending on the difference between  $p_I$  and  $p_E$ , either all or a fraction  $\gamma$  of HSC customers purchase from the entrant:

- if  $p_I - p_E \geq \sigma_H$ , then all HSC consumers buy from the entrant, who therefore charges  $\sigma_H$  in the second period — its second period profit is  $\alpha\sigma_H$ ;
- if  $p_I - p_E < \sigma_H$ , a proportion  $\gamma$  purchases from the entrant, who in the second period uses a mixed strategy: he chooses prices  $\sigma_L$  and  $\sigma_H$  with probabilities such that the HSC customers are indifferent between

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<sup>9</sup>Our equilibrium is also an equilibrium if there are several active entrants and they each choose a mixed strategy such that the distribution of the minimum of the prices they charge is the function  $G_E$  defined below. In other words, there are many different payoff equivalent equilibria. The key to all equilibria is that there is free entry and that all entrants make zero expected profits in equilibrium.

switching and not switching suppliers in period 1 — its second period profit is  $(\alpha\gamma + (1 - \alpha))\sigma_L = \alpha\gamma\sigma_H$  (in the states of nature where its second period price is  $\sigma_H$ , all the LSC customers switch to a period 2 entrant).

Therefore, in equilibrium, a proportion at least equal to  $\gamma$  of the HSC consumers purchase from the entrant in period 1.

The entrant chooses  $p_E$  according to the following distribution  $G_E$ , which has a mass point at  $-\delta\sigma_L$ :

$$G_E(p_E) = \begin{cases} \frac{p_E + \alpha\delta\sigma_H}{p_E + (1 + \delta)\sigma_H} & \text{if } p_E \in [-\alpha\delta\sigma_H, -\delta\sigma_L), \\ 1 & \text{if } p_E = -\delta\sigma_L. \end{cases} \quad (8)$$

Then, for any  $p_I \in [\sigma_H(1 - \alpha\delta), \sigma_H - \delta\sigma_L)$ , the Incumbent's expected discounted profit is

$$G_E(p_I - \sigma_H) \times 0 + (1 - G_E(p_I - \sigma_H)) \times (1 - \gamma)(p_I + \delta\sigma_H) = (1 - \gamma)\sigma_H(1 + \delta - \alpha\delta). \quad (9)$$

To understand why the Incumbent will not find it profitable to choose a price outside of the interval  $[\sigma_H(1 - \alpha\delta), \sigma_H - \delta\sigma_L)$ , we check for possible deviations. A) It is not profitable for the Incumbent to choose  $p_I \geq \sigma_H - \delta\sigma_L$ , as this implies  $p_I - p_E \geq \sigma_H$  with probability 1, and no sales! B) To show that it is not profitable to choose  $p_I < \sigma_H(1 - \alpha\delta)$ , we proceed in two steps. a) First, note that by charging  $\sigma_H(1 - \alpha\delta)$ , the Incumbent sells to a proportion  $1 - \gamma$  of HSC customers. By Claim 2, to sell more the Incumbent must choose  $p_I \leq \sigma_H(1 - \delta)$ , which implies that as long as it does not sell to LSC customers, its profit,  $\alpha(p_I + \delta\sigma_H)$ , is at most  $\alpha\sigma_H$  and therefore smaller than  $(1 - \gamma)\sigma_H(1 + \delta - \alpha\delta)$  by (1). b) Second, in order to sell to LSC customers, the Incumbent needs to make their total costs, over both periods, less than  $\sigma_L$ , which is the upper bound of their cost if they switch to the entrant in the first period. Given that they will switch in period 2 when it charges  $\sigma_H$ , this necessitates  $p_I \leq (1 - \delta)\sigma_L$ , which leads to profits  $p_I + \delta\alpha\sigma_H$  smaller than the profits when using the equilibrium strategy.

Similarly, in our equilibrium the Incumbent chooses  $p_I$  according to the distribution

$$G_I(p_I) = \frac{p_I - \sigma_H(1 - \alpha\delta)}{p_I - \sigma_H(1 - \alpha\delta) + ((1 - \alpha) + \alpha\gamma)(\sigma_H - p_I - \delta\sigma_L)}.$$

Then, the profit of the active entrant is

$$G_I(p_E + \sigma_H) \times (p_E + \delta\sigma_L)(1 - \alpha + \alpha\gamma) + (1 - G_I(p_E + \sigma_H)) \times (p_E + \delta\alpha\sigma_H) = 0$$



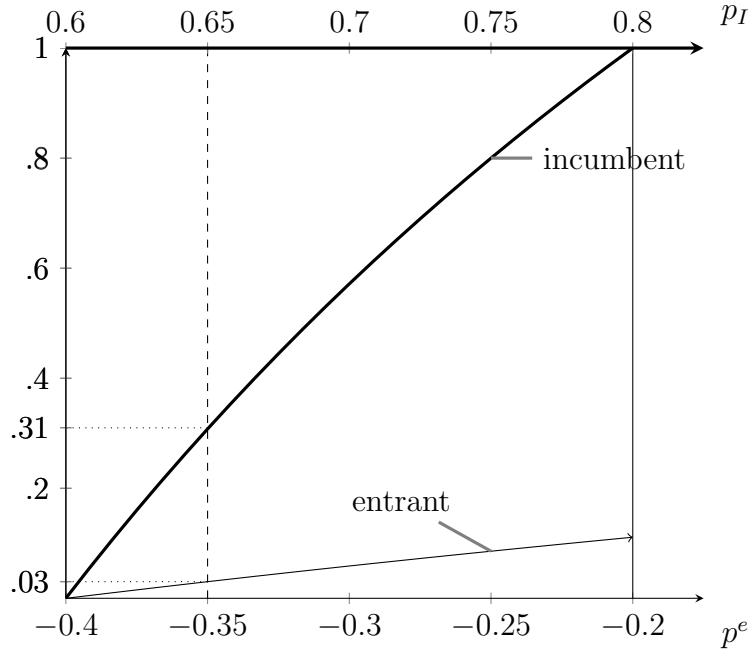


Figure 1: This figure represents the probability distributions in the mixed strategies of the Incumbent and the entrant with  $\sigma_H = 1$ ,  $\sigma_L = .2$ ,  $\alpha = .4$  and  $\delta = 1$ , which implies  $\gamma = 37.5\%$ . For instance, reading along the vertical dashed line, if  $p_E = -0.35$ , we obtain  $G(p_E) \approx 0.03$ , which implies that if the Incumbent chooses  $p_I = 0.65 = -0.35 + \sigma_H$ , then it loses all its HSC customers with probability 3% and sells to a proportion  $1 - \gamma$  of them with probability 97%. Similarly, if the entrant chooses  $p_E = -0.35$ , it sells to a proportion  $\gamma$  of HSC customers with a probability 31% and to all of them with probability 69%.

when it chooses a price in  $[-\alpha\delta\sigma_H, -\delta\sigma_L]$ , and smaller than or equal to 0 when it chooses a price outside of this interval (the presence of another “inactive” entrant who charges  $-\delta\sigma_L$  is crucial for this last point).

In all equilibria  $p_I$  will be distributed according to  $G_I$  and  $p_E$ , interpreted as the lower bound of the prices of the active entrants, will be distributed according to  $G_E$ . We will let the interested reader convince himself of this fact.

Figure 1 shows the equilibrium strategies with  $\sigma_H = 1$ ,  $\sigma_L = .2$ ,  $\alpha = .4$  and  $\delta = 1$ .

## 4.4 Comparing Stackelberg and Bertrand equilibria

We found a pure strategy equilibrium in the Stackelberg model, but only mixed strategy equilibria in the Bertrand model. By Proposition 1, we saw that the equilibrium profits were the same. On the other hand, consumer utilities and welfare will differ across the two models. We first take up consumer utilities:

**Corollary 2.** *The expected utility of the HSC customers is lower in the Bertrand equilibrium than in the Stackelberg equilibrium, while the expected utility of the LSC customers is higher in the Bertrand than in the Stackelberg equilibrium.*

*Proof.* We first prove the results for the HSC consumers.

In period 1 of the Stackelberg equilibrium, some HSC consumers purchase from the Incumbent at price  $p_I = \sigma_H(1 - \alpha\delta)$ . These consumers pay  $\sigma_H$  in period 2, and their total discounted cost is  $-\sigma_H(1 + \delta - \alpha\delta)$ .

In the Bertrand equilibrium, there are only mixed strategy equilibria, where the Incumbent's first period price is strictly greater than  $\sigma_H(1 - \alpha\delta)$ , which is the price it charges in the Stackelberg equilibrium. In the states of nature when  $p_I \leq p_E + \sigma_H$ , the Incumbent sells to some HSC consumers. In period 2, these consumers will pay  $\sigma_H$  as in the Stackelberg equilibrium; therefore, in these states of nature, the utility of HSC consumers is smaller in the Bertrand model. In the states of nature where  $p_I > p_E + \sigma_H$ , none of the HSC consumers purchase from the Incumbent. In period 2, they pay  $\sigma_H$  to the entrant from which they purchased in period 1. Because this entrant charged at least  $-\alpha\delta\sigma_H$  in period 1, the total cost of the HSC consumers is at least  $\sigma_H(1 + \delta - \alpha\delta)$ , which is also their utility in the Stackelberg equilibrium. Therefore, the HSC consumers are never better off and sometimes strictly worse off in the Bertrand equilibrium.

Now, we prove the result for the LSC consumers. In both equilibria, the period 2 cost of the LSC consumers is  $\sigma_L$ . In period 1 of both equilibria, the LSC consumers purchase from one of the lowest priced entrant. In the Stackelberg equilibrium, they pay  $-\delta\sigma_L$  with probability 1. In the Bertrand equilibrium they never pay more than this amount, and pay strictly less with strictly positive probability, which proves the result.  $\square$

In the first period, entrants charge less in the Bertrand than in the Stackelberg equilibrium. This leads to lower costs for the LSC consumers. However, this decrease in price is not enough to compensate the HSC consumers for the higher second period prices and for the fact that they will do more switching in period 1.

Now, we turn to the difference in welfare between the two equilibria. Since the firms offer identical products and all consumers make purchases each period, the only difference in welfare is due to switching costs.

**Corollary 3.** *Welfare is lower in the Bertrand than in the Stackelberg equilibrium.*

*Proof.* In the Stackelberg equilibrium all LSC customers switch in period 1 and a fraction  $\gamma$  of the HSC customers switch. In period 2, the LSC customers switch with probability  $q$  — see (5). Thus, the aggregate welfare loss due to switching is  $\alpha\gamma\sigma_H + (1 - \alpha)\sigma_L(1 + \delta q)$ .

In the Bertrand equilibrium, when  $p_I > p_E + \sigma_H$  all the consumers switch in period 1 and all the LSC customers switch in period 2, for an aggregate welfare loss equal to  $\alpha\sigma_H + (1 - \alpha)\sigma_L(1 + \delta)$ , which is strictly greater than the loss in the Stackelberg equilibrium.

When  $p_I \leq p_E + \sigma_H$  a fraction  $\gamma$  of the HSC customers and all the HSC customers switch in period 1, as in the Stackelberg equilibrium. In period 2, none of the HSC customers switch, again as in the Stackelberg equilibrium, and the LSC customers all switch with probability

$$1 - \frac{\sigma_H + p_E - p_I}{\delta(\sigma_H - \sigma_L)}.$$

Since this probability is strictly greater than  $q$ , the LSC customers switch more in period 2 in the Bertrand equilibrium. Hence, there is always at least as much switching in the Bertrand as in the Stackelberg equilibrium.  $\square$

Corollary 3 is a consequence of the fact that in the first period, in some states of nature, all the HSC customers will purchase from an entrant, which is not possible in the Stackelberg equilibrium. Furthermore, when all HSC consumers switch supplier in period 1, the LSC customers will switch in period 2, to avoid paying  $\sigma_H$ .

## 4.5 Equilibrium with large $\sigma_L$

For completeness, we now turn to a discussion of the equilibrium when equation (1) does not hold. Proofs and more details can be found in the web appendix of this paper.

If  $\sigma_L$  is very large, *i.e.*,  $\sigma_L \geq \alpha\sigma_H$ , then everything happens as if all the consumers were LSC consumers: the Incumbent charges  $\sigma_L(1 - \delta)$  in period 1,  $\sigma_L$  in period 2, and sells to all consumers in every period. Its total discounted profit is  $\sigma_L$  the same profit as in the one period model with free entry and the same profit when all consumers are identical, see Biglaiser et al. (2013).

This leaves open the question of the cases when  $\sigma_L/\sigma_H \in (\delta\alpha/(1+\alpha), \alpha)$ . If  $\sigma_L/\sigma_H \in (x_C, \alpha)$ , where  $x_C$  is the solution of (10) below, then there exists a pure strategy equilibrium where in period 1 the Incumbent sells to all the HSC consumers at a price  $\sigma_H(1-\delta)$  and the entrants sell to all the LSC consumers at a price  $-\delta\sigma_L$ . In period 2, the Incumbent charges  $\sigma_H$  and ‘keeps’ all the HSC consumers — its profit over both periods is therefore  $\alpha\sigma_H$ . The best alternative strategy for the Incumbent would be to charge  $\sigma_H - \delta\sigma_L$  in period 1, and sell to a proportion  $1 - \gamma$  of the HSC consumers. This strategy becomes more attractive as  $\sigma_L$  decreases, and dominates when  $\sigma_L/\sigma_H < x_C$ , where  $x_C \in (\delta\alpha/(1+\alpha), \alpha)$  is the solution of

$$x_C(1 + \delta + \alpha\delta - \alpha) = \delta(\alpha + x_C^2). \quad (10)$$

Claim A.3 in the web appendix shows that the mixed strategy equilibrium described in 4.3 for the case  $\sigma_L < \alpha\delta\sigma_H/(1+\delta)$  also holds when  $\sigma_L/\sigma_H \in (\delta\alpha/(1+\delta), x_C)$ . The only difference stems from the fact that the profit of the Incumbent in the one period game is greater than the equilibrium profit in the two period game. As a consequence, we need to check that deviations where the Incumbent would retain all the HSC customers are not profitable. This yields the following corollary.

**Corollary 4.** *The equilibrium described in 4.3 is also an equilibrium when  $\sigma_L/\sigma_H \in (\delta\alpha/(1+\delta), x_C)$ . The profit of the Incumbent in this equilibrium is smaller than its profit in the equilibrium of the one period game.*

Before commenting on this result, it is worth emphasizing that we have shown that there exists *one* equilibrium of the dynamic game which yields profits smaller than static profit, *not* that all equilibria of the dynamic game satisfy this property.<sup>10</sup>

In the static model, all HSC consumers always purchase from the Incumbent at price  $\sigma_H$ ; the LSC consumers impose no externality on the HSC consumers, since there is no future price to take into account. In the dynamic game, entrants are more aggressive than in the static game and also more attractive to the HSC consumers because they charge less than the Incumbent in the second period (at least if not too many HSC consumers change suppliers). When  $\alpha$  is large enough, the entrants charge a sufficiently low price and attract sufficiently many HSC consumers that the Incumbent would be better off if it had only one period with less aggressive competitors to accumulate its profits. We believe that this is the first time that this phenomenon has been identified in the literature.

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<sup>10</sup>We conjecture, but have been unable to prove, that all equilibria of the dynamic game do satisfy the property.

## 5 Literature

Taylor (2003) has the closest model to ours in the literature. The main aim of his paper is to analyze dynamic competition between firms in a subscription model where consumers draw switching costs in each period from identical, independent distributions (in subscription models, introduced in the literature by Chen (1997), firms can discriminate between their current consumers and the consumers of other firms). With enough firms (in most of his analysis Taylor assumes free entry) competition drives the profits obtained from attracting consumers from other firms to zero.

In section 5 of his paper, Taylor examines a two period model where two types of consumers draw their switching cost (as before, independently in each period) from different distributions. In some sense our model is a specialization of Taylor’s model with degenerate distributions. His focus is on describing the equilibrium strategies of the firms and of the consumers. As in our models some HSC consumers want to “hide” among LSC consumers. He interprets this as an attempt by HSC consumers to acquire a reputation as LSC consumers. In our model consumers are anonymous and there is no way to discriminate between them; they have no way and no incentives to acquire a reputation. Yet, we obtain the same result. The explanation is simple: HSC consumers have incentives to mix with LSC consumers, as long as not too many of them do so. Our second, and maybe more important, contribution is that we focus on the profit of the Incumbent and derive clean results comparing static and dynamic models. In particular, we are able to show how increasing the number of LSC consumers protects the Incumbent from the attempts by entrants to steal away some of the HSC consumers.

To the best of our knowledge, Taylor’s section 5 is the only model in the literature where consumers have persistent differences between their switching costs.

For instance, Dubé, Hitsch, and Rossi (2009) have studied an infinite horizon model where consumers have random utility and firms sell differentiated products; their focus is on empirics, and, through the use of simulation methods, they provide numerical examples where prices fall when switching costs increase: as in our model, the increase in switching costs makes firms more aggressive as attracting consumers become more valuable. They assume that all consumers have the same switching costs.

A number of authors have constructed model versions of the model of Dubé et al. (2009) designed for theoretical exploration. For instance, Cabral (2013) analyzes an infinite horizon subscription model of competition between two producers. The relative value that the consumers attach to the goods produced by the two consumers is independent from period to period.

He shows that an increase in switching cost from a small level leads to a decrease in the price, for the same reasons as in Dubé et al. (2009). All consumers have the same switching cost.<sup>11,12</sup> Somaini and Einav (2013) develop an overlapping generation model where consumers have identical switching costs that share some features of Cabral.<sup>13</sup>

It may also be worthwhile noting that most of the literature assumes a fixed number of firms, whereas our results hold true with free entry, and therefore more intense competition.<sup>14</sup> Hence, the Incumbent has no incentive to invest in the acquisition of new customers, on which it can only make zero profits — indeed, in equilibrium, the Incumbent does not try to “recover” the consumers that it has lost to other firms. Our comparative statics are entirely the consequence of the heterogeneity of switching costs, and of the fact that low switching cost customers protect the Incumbent from entry.

## 6 Conclusion

In this conclusion we provide some remarks about our modelling choices and about possible extensions.

Due to free entry, all our results hold if firms are allowed to discriminate on the basis of the past purchasing history of consumers — that is if we transformed our model in a subscription model. Indeed, an incumbent will compete for new consumers with entrants, and will therefore not be able to generate any profits on that market.

In our previous paper, Biglaiser et al. (2013) we studied a model similar to the model of this paper, with an infinite horizon, while assuming  $\sigma_L = 0$ . In that case, consumers allocate themselves in different firms (a firm will choose a “high” price as soon as its clientèle contains one HSC consumer). Studying the case where  $\sigma_L$  is strictly positive enables us to focus on the way in which firms accumulate a mixture of clients types. However, we have not been able to solve the model of the present paper with an infinite number of

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<sup>11</sup>Arie and Grieco (2013) also provide a theoretical analysis of Dubé et al. (2009), again which consumers who all have the same switching costs. Unlike Cabral (and like Dubé et al.), they assume that the consumers are myopic. Our consumers are forward looking.

<sup>12</sup>In a subscription model with forward looking consumers based on Chen (1997), Bouckaert, Degryse, and Provoost (2008) also show that higher switching costs can lead to lower profits. In their model, consumers are ex-ante identical and learn of their switching costs only after making their first purchase.

<sup>13</sup>Arie and Grieco (2013) also provide a theoretical analysis of Dubé et al. (2009) but, unlike Cabral (and like Dubé et al.), they assume that the consumers are myopic. (Our consumers are forward looking.)

<sup>14</sup>As mentioned above, Taylor (2003) also analyses free entry.

periods.

It would be of great interest to extend the model to more than two types of consumers, and in particular to an infinite number. This raises a very interesting issue about the type of clientèle that a firm would accumulate. Let the switching costs be distributed over  $[\underline{\sigma}, \bar{\sigma}]$ . Would firms sell to consumers whose switching costs are allocated over the whole interval  $[\underline{\sigma}, \bar{\sigma}]$ , some having a relatively higher proportion of customers with lower switching costs? Alternately, would some firms would have consumers allocated over the whole interval with others concentrating on higher switching cost consumers?

Network effects have sometimes been compared to social switching costs because they make it more difficult for groups of consumers to switch from one platform to another. In order to test this parallelism Biglaiser and Crémer (2014) study dynamic competition in a model with an incumbent and free entry and with heterogenous network effects. They show that there are similarities but also substantial differences<sup>15</sup> between the two setups (see Crémer and Biglaiser (2012) for a preliminary discussion of some of these results and comparisons between the two setups).

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<sup>15</sup>From a modelling viewpoint, the main difference is probably that with network effects the migration from one platform to the other is the outcome of a coordination game between consumers. In order to develop the model one needs to take a stance on the coordination failures that might arise. See Biglaiser, Crémer, and Veiga (2014) for a more detailed study of these failures.

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# Appendix

## Proof of Lemma 5

In this Appendix, we prove Lemma 5. We begin by establishing three claims; the first one is part 4 of the lemma.

**Claim A.1.** *If  $p_I > (1 - \alpha\delta)\sigma_H$ , the Incumbent has zero sales in both periods.*

*Proof.* If  $p_I > (1 - \alpha\delta)\sigma_H$ , a unique lowest price entrant who would charge  $p_E \in (-\alpha\delta\sigma_H, p_I - \sigma_H)$  would make strictly positive profits equal to  $p_E + \delta\alpha\sigma_H$ , as it would attract all the consumers in period 1. Free entry prevents this, and therefore in the continuation game, one or several entrants must charge  $-\alpha\delta\sigma_H$ , and attract all the consumers while making zero profits.  $\square$

**Claim A.2.** *If  $p_I < (1 - \alpha\delta)\sigma_H$ , no entrant attracts enough HSC consumers in period 1 that it finds it optimal to charge  $\sigma_H$  with probability 1 in period 2.*

*Proof.* Assume that entrant  $\tilde{e}$  attracted a large enough proportion of HSC customers that it found it optimal to charge  $\sigma_H$  in period 2. Because LSC consumers always find it strictly more profitable to switch suppliers than do HSC consumers, the Incumbent would have no LSC customers and, therefore, HSC customers can guarantee themselves a second price of  $\sigma_H$  by “staying with” the Incumbent. Therefore, entrant  $\tilde{e}$  must have chosen a period 1 price  $p_{\tilde{e}} \leq p_I - \sigma_H < -\delta\alpha\sigma_H < -\delta\sigma_L$  (the last inequality is a consequence of equation (2)). By lemma 2, all the entrants that attract consumers in the first period also charge  $p_{\tilde{e}}$ . They attract all the LSC consumers and, at best, some HSC consumers. The sum of their profits is therefore smaller than  $\max\{p_{\tilde{e}} + \delta\sigma_L, p_{\tilde{e}} + \delta\alpha\sigma_H\} < 0$ , which establishes the contradiction.  $\square$

**Claim A.3.** *If  $p_I < (1 - \delta)\sigma_H$ , all HSC consumers purchase from the Incumbent in period 1.*

*Proof.* By Claim A.2, any period 1 entrant who has attracted consumers in period 1 charges  $\sigma_L$  with positive probability in period 2. Therefore, its second period profit will be  $\sigma_L$  times the mass of consumers it attracted in the first period and, by free entry, its period 1 price must be  $-\delta\sigma_L$ . The total discounted cost for a HSC consumer who would purchase from a period 1 entrant would therefore be at least  $(-\delta\sigma_L + \sigma_H) + \delta\sigma_L = \sigma_H$  (it would be greater if in period 2 the entrant charged  $\sigma_H$  with a strictly positive probability). If the consumer purchases from the Incumbent, his total cost is  $p_I + \delta\sigma_H < \sigma_H$ , which establishes the claim.  $\square$

Parts 1 and 2 of the lemma follow immediately from Claim A.3.

If  $p_I \in ((1 - \delta)\sigma_H, (1 - \alpha\delta)\sigma_H)$ , HSC consumers prefer to purchase from an entrant if its period 2 price is  $\sigma_L$  and from the Incumbent if the entrant's period 2 price is  $\sigma_H$ . Therefore, there can be an equilibrium only if the entrants play a mixed strategy in period 2, which is feasible only if in period 1 they attract a proportion  $\gamma > 0$  of the HSC consumers. This establishes part 3 of the lemma and completes the proof.

# Web Appendix

In this Appendix, which is not intended for publication, we prove the results discussed in subsection 4.5. Note that we are less ambitious than when (1) holds: we are only trying to identify one equilibrium for each value of  $\sigma_L/\sigma_H$ , not to characterize all the equilibria. We present the results under the form of three claims, starting with the largest value of  $\sigma_L/\sigma_H$ .

**Claim WA.1.** *If  $\sigma_L/\sigma_H > \alpha$ , then the two period Bertrand game has a unique equilibrium in which the Incumbent charges  $\sigma_L(1 - \delta)$  in period 1 and  $\sigma_L$  in period 2. It sells to all consumers and its profits are  $\sigma_L$ .*

As in the one period model, when  $\sigma_L > \alpha\sigma_H$  the Incumbent and the entrant act as if there were only LSC customers in the economy. We leave the proof of the claim to the reader.

For  $\sigma_L/\sigma_H \in (x_C, \alpha)$ , with  $x_C$  defined by (10), we establish the following claim:

**Claim WA.2.** *If  $\sigma_L/\sigma_H \in [x_C, \alpha]$ , there exists a pure strategy equilibrium in which the Incumbent, whose profits are  $\alpha\sigma_H$ , sells to the HSC consumers in both periods, at prices respectively equal to  $\sigma_H(1 - \delta)$  and  $\sigma_H$ . All LSC customers purchase from entrants at price  $-\delta\sigma_L$  in period 1 and at price  $\sigma_L$  in period 2.*

*Proof.* We show that the strategies described in the claim form an equilibrium. The LSC customers are clearly better off switching in period 1. The strategy of the HSC customers is a best response to the strategy of the other agents as they are indifferent between purchasing from the Incumbent in both periods and switching to an entrant in the first period — in both cases their total discounted costs are equal to  $\sigma_H$ .

This indifference of HSC consumers implies that the Incumbent would lose at least a proportion  $\gamma$  of its customers if it increased its period 1 price. It is straightforward to see that, under these circumstances, its most profitable increase in price is to  $\sigma_H - \sigma_L$ . This deviation is unprofitable as long as

$$\begin{aligned} \alpha\sigma_H &\geq (1 - \gamma)(\sigma_H - \delta\sigma_L + \delta\sigma_H) \\ &\iff \frac{\sigma_L}{\sigma_H} [1 + \delta + \alpha\delta - \alpha] \geq \delta [\alpha + (\sigma_L/\sigma_H)^2], \end{aligned}$$

and therefore, as long as  $\sigma_L/\sigma_H \in [x_C, \alpha]$ . A small decrease in period 1 price obviously decreases the profits of the Incumbent. A decrease to  $\sigma_L(1 - \delta)$  allows it to sell to all consumers in period 1, but decreases its profits. Finally, it is easy to show that the entrants strategy is indeed a best response to the strategies of the other agents.  $\square$

For the remaining set of parameters, we can prove the following claim.

**Claim WA.3.** *The equilibrium described in 4.3 is also an equilibrium when  $\sigma_L/\sigma_H \in [\alpha\delta/(1 + \delta), x_C]$ .*

*Proof.* The proof is exactly the same as when  $\sigma_L/\sigma_H \leq \alpha\delta/(1 + \delta)$ , except that we need to be a bit more careful when showing that the Incumbent does not gain by deviating to  $p_I$  in  $(\sigma_L(1 - \delta), \sigma_H(1 - \delta))$  and possibly selling to all the HSC consumers. The Incumbent sells to all the HSC customers if

$$p_I + \delta\sigma_H \leq p_E + \sigma_H + \delta\sigma_L \iff p_I \leq p_E + (1 - \delta)\sigma_H + \delta\sigma_L.$$

This implies that if  $p_I \in [(1 - \alpha\delta - \delta)\sigma_H + \delta\sigma_L, (1 - \delta)\sigma_H]$ , the Incumbent sells to all the HSC consumers with probability strictly between  $1 - \gamma$  and  $1$  — in the other states of nature, it sells to a proportion  $1 - \gamma$  of them. Using the mixing probability of the entrant, see equation (8), the Incumbent's profit for prices in  $[(1 - \alpha\delta - \delta)\sigma_H + \delta\sigma_L, (1 - \delta)\sigma_H]$  is

$$\begin{aligned} & (p_I + \delta\sigma_H)\alpha [G_E(p_E) + (1 - G_E(p_E))(1 - \gamma)] \\ &= (p_I + \delta\sigma_H)\alpha \left[ G_E(p_I - (1 - \delta)\sigma_H - \delta\sigma_L) \right. \\ & \quad \left. + (1 - G_E(p_I - (1 - \delta)\sigma_H - \delta\sigma_L))(1 - \gamma) \right] \\ &= \frac{(p_I + \delta\sigma_H)\alpha}{p_I + 2\delta\sigma_H - \delta\sigma_L} \\ & \quad \times \left[ p_I - \sigma_H(1 - \delta - \alpha\delta) - \delta\sigma_L + (1 - \gamma)\sigma_H(1 + \delta - \alpha\delta) \right]. \end{aligned}$$

The first and the second terms are both increasing in  $p_I$ . So, the maximal profit from this deviation is at a price of  $p_I = \sigma_H(1 - \delta)$ . At this price the Incumbent only keeps all the HSC customers when the entrant is pricing at  $-\delta\sigma_L$ , where there is an atom in the distribution of its prices. It is straightforward to see that the profit is less than the putative equilibrium profit. Finally, it is straightforward to show that a deviation that keeps the LSC customers cannot improve profits.  $\square$