Enhancing dual activities in a social network

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- Individuals interact though social networks and social media (friendship, collaboration, ...)
- Their actions are often strategic complements
- Focus here on 'dual' activities and their control by a 'manager'
- ex.: On online social media:
  - Users provide contributions and watch or read the others' contributions and possibly post a rating
  - How can the platform enhance agents' activity?

Questions and objectives

- Build a simple game that exhibits the feedback between actions
- Analyze three types of manager's strategies:
    - allocating a budget to enhance individuals' returns: Who is targeted?
    - increasing the visibility of some contributions: Who is made more visible?
    - providing more or less information to players on others' actions
- Value of information: How much does the manager benefit from the knowledge of the interaction structure?

Related literature: Empirical studies

- On behavior on online social media

  A positive correlation between popularity (attention received) and participation (tendency to contribute) Wu Wilkinson Huberman [2009]

- On Facebook

  Audience is not revealed. Users underestimate their audience Bernstein et al. [2013]. Would showing audience improve attention, hence Facebook ads revenues?

  'Likes' are revealed and non anonymous: People care more about who Likes their posts than how many Likes they receive Scissors et al. [2016]

Related literature: Monopolist strategies

linear response or demand

- 'key player' : suppress a node to minimize activity

  single action Ballester, Calvó-Armengol, Zenou [2006]
  Multiple Activities Chen, Zenou, Zhou [2017]

- pricing with discrimination to maximize profit
  how does the position in a network affects price?

  Bloch and Querou [2013], Candogan, Bimpikis, Ozdaglar [2017]
  Fainmesser and Galeotti [2013] Nie [2017]

Other

Quite different analysis and results in

- Non linear interactions
  Demange [2017]
  binary actions as in adoption/contagion process
  Morris [2003], Domingos and Richardson [2001] in a marketing context, Dodds and Watts [2004] in biology
- Competitive settings
- Dual communication and coordination Calvó-Armengol, Martí, Prat [2015]

Outline

The game

- Each individual has two actions, say attention and contribution
- Individual's payoffs depend on others' actions through exposure
  - Exposure to attention influences contribution
  - Exposure to contribution influences attention

  Exposures are defined by bilateral impacts

- Complementarities in the actions across individuals

Impact and exposures

- n agents, take two actions, ≥ 0
  $a_i = i$'s attention level
  $b_i = i$'s contribution level
- Bilateral Impacts:

  $\alpha_{ji} \geq 0$: *impact* of j' s *attention* on i
  $\beta_{ji} \geq 0$: *impact* of j' s *contribution* on i
- given (a_j) i's exposure to attention:

$$\sum_j \alpha_{ji} a_j$$

given (b_j) i's exposure to contribution:

$$\sum_j \beta_{ji} b_j$$

Payoffs

- $u_i(a_i, b_i, a_{-i}, b_{-i}) =$

$$a_i(x_i + \sum_j \beta_{ji} b_j) - \frac{a_i^2}{2} + b_i(y_i + \sum_j \alpha_{ji} a_j) - \frac{b_i^2}{2}$$

- i's payoff is separable in the two actions
  - return to a_i: x_i + exposure to other contributions
  - x_i positive = i's attention level in isolation

Payoffs-cd

Easy extensions

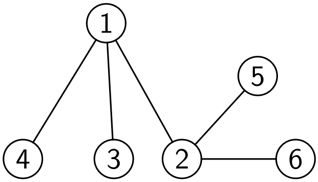
- quadratic cost $c_i \frac{a_i^2}{2} \rightarrow$ scale the parameters
- introduce complementarities in an individuals' actions:
- add a term $\gamma a_i b_i$, with $\gamma > 0$
  marginal benefit of listening is increasing in own's contribution, and symmetrically

Illustration 1: Friendship network or Network of influence

- $g_{ji} = 1$ if j's actions have an impact on i
- if *cumulative* effect:
  i's exposure is the sum of the actions of his friends or influencers

$$\boldsymbol{\alpha} = \frac{1}{c}\boldsymbol{g}, \ \boldsymbol{\beta} = \frac{1}{d}\boldsymbol{g}$$

Example: symmetric friendship, congestion on attention



assume

contributions are like public goods: cumulative effect

congestion on attention: a_i is shared among the followers

$$\begin{aligned} \text{1's exposure to contribution:} \quad & b_2 + b_3 + b_4 \\ \text{1's exposure to attention:} \quad & \frac{a_2}{3} + a_3 + a_4 \end{aligned}$$

Sharing and splitting

- *Sharing* of impacts among followers:
  - a_j = total reading, effort time
    α_{ji} =the proportion of time devoted by j to each of his followers

$$\alpha_{ji} = \frac{1}{j\text{'s out-degree}} \text{ if } i \text{ follows } j$$

  i's exposure to attention : sum of *effective* attention of i's friends

- *Splitting* impacts among influencers:

$$\alpha_{ji} = \frac{1}{i\text{'s in-degree}} \text{ if } i \text{ follows } j$$

  i's exposure =average of actions

- sharing: rows' totals are equal, splitting columns' totals are equal

Illustration 2: two-sided setting

- Two disjoint sets S and T
- the members of one side (students, citizens) listen to the speeches of the members of the other side (teachers, politicians)
- Particular case where each individual takes a single action, a_i for i in S, and b_j for j in T.

Equilibrium

- Standard Nash equilibrium in actions (a_i, b_i)
- Best responses are linear, increasing in exposures: complementarities
- Equilibria are easy to find through iterated reactions if externalities effects are not too strong, Topkis [1979]
- spill-over effects

Equilibrium

- ρ= dominant eigenvalue of matrix $\boldsymbol{\alpha\beta}$
(= to that of $\boldsymbol{\beta\alpha}$ and their transposes)

Let $\rho < 1$. Then an equilibrium exists and is unique given by

$$\boldsymbol{a} = (\mathbb{I}_n - \widetilde{\boldsymbol{\beta}}\widetilde{\boldsymbol{\alpha}})^{-1}(\boldsymbol{x} + \widetilde{\boldsymbol{\beta}}\boldsymbol{y}) \text{ and } \boldsymbol{b} = (\mathbb{I}_n - \widetilde{\boldsymbol{\alpha}}\widetilde{\boldsymbol{\beta}})^{-1}(\boldsymbol{y} + \widetilde{\boldsymbol{\alpha}}\boldsymbol{x})$$

If $\rho \geq 1$, then the game has no equilibrium.

- existence if there are no cycles in the impact structures (then ρ is null) or the costs c and d are high enough

Interpretation

$$\boldsymbol{b} = (\mathbb{I}_n - \widetilde{\boldsymbol{\alpha}}\widetilde{\boldsymbol{\beta}})^{-1}(\boldsymbol{y} + \widetilde{\boldsymbol{\alpha}}\boldsymbol{x})$$

- $\boldsymbol{y} + \widetilde{\boldsymbol{\alpha}}\boldsymbol{x} =$ optimal contributions to the minimal attention levels.
- equilibrium includes all further spill-over effects

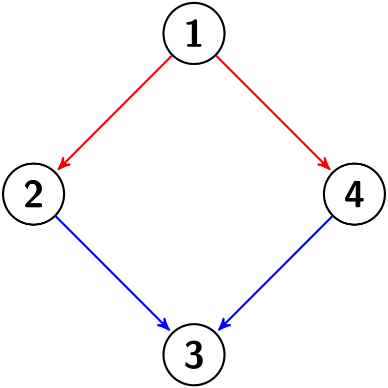
$$\boldsymbol{b} = [\boldsymbol{y} + \widetilde{\boldsymbol{\alpha}}\boldsymbol{x}] + \widetilde{\boldsymbol{\alpha}}\widetilde{\boldsymbol{\beta}}[\boldsymbol{y} + \widetilde{\boldsymbol{\alpha}}\boldsymbol{x}] + ... + (\widetilde{\boldsymbol{\alpha}}\widetilde{\boldsymbol{\beta}})^{(p)}[\boldsymbol{y} + \widetilde{\boldsymbol{\alpha}}\boldsymbol{x}] + ...$$

- $(\boldsymbol{\beta}\boldsymbol{\alpha})_{ji} = \sum_k \beta_{jk}\alpha_{ki}$

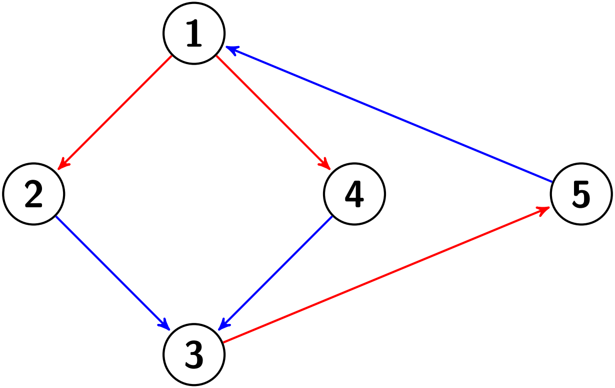
  $=$ sensitivity of i's contribution to j's through attention

  $=$ cross-impact of j's contribution on i's

red arrow from i to j: j's contribution impacts j's attention (β_{ij})
blue arrow from i to j: i's attention impacts j's contribution (α_{ij})



3's sensitivity to 1's contribution is = 2
1's sensitivity to 3's contribution is = 0



spill-over effects

The manager's objective

- The manager aims at improving a weighted sum of actions
  - in education, increasing pupils and teachers' effort
  - on Internet, a platform's profit is increasing in the time users spent on the platform, by selling ads or information to outsiders
- The manager anticipates the full impact of her strategy on actions
- Results stated when the objective is to increase aggregate contribution
- First: consider strategies that modify the returns x_i or y_i

Allocating budgets

- A strategy: allocations (p_i) and (q_i)

  p_i changes i's return to attention x_i into $x_i + p_i$,

  q_i changes i's return to contribution y_i into $y_i + q_i$
- Given endowments $P \geq 0$ and $Q \geq 0$, the strategy is feasible if

$$\sum_i p_i \leq P \text{ and } \sum_i q_i \leq Q.$$

Optimal strategies

A strategy $(\boldsymbol{p}, \boldsymbol{q})$ is *optimal* if it maximizes equilibrium aggregate contribution over all feasible strategies.

i's attention is said to be targeted if $p_i > 0$ and i's contribution if $q_i > 0$.

- The manager accounts for the spill-over effects

Optimal strategies: characterization

$$\begin{aligned} \boldsymbol{K}^{x\to b} &= \boldsymbol{\alpha}(\mathbb{I}_n - \boldsymbol{\beta\alpha})^{-1}\mathbb{1} \quad \text{(indirect index)} \\ \boldsymbol{K}^{y\to b} &= (\mathbb{I}_n - \boldsymbol{\beta\alpha})^{-1}\mathbb{1} \quad \text{(direct index)} \end{aligned}$$

The optimal strategies to increase aggregate contributions allocate

- P among the individuals whose index $K_i^{x\to b}$ is maximal
- Q among the individuals whose index $K_i^{y\to b}$ is maximal
- Improvements $\max_i K_i^{x\to b} P$ and $\max_i K_i^{y\to b} Q$

Extends results for a single action (Demange [2017])

Properties

- Independent of the individual characteristics x_i and y_i (not true under non-linear responses)
- Indices and contributions depend on the impact matrices in a dual way:

  i's indices depend on the sensitivity of others to i's actions, $\boldsymbol{\beta}\boldsymbol{\alpha}$
  i's contributions on how i is sensitive to others' actions, $\widetilde{\boldsymbol{\alpha}}\widetilde{\boldsymbol{\beta}}$

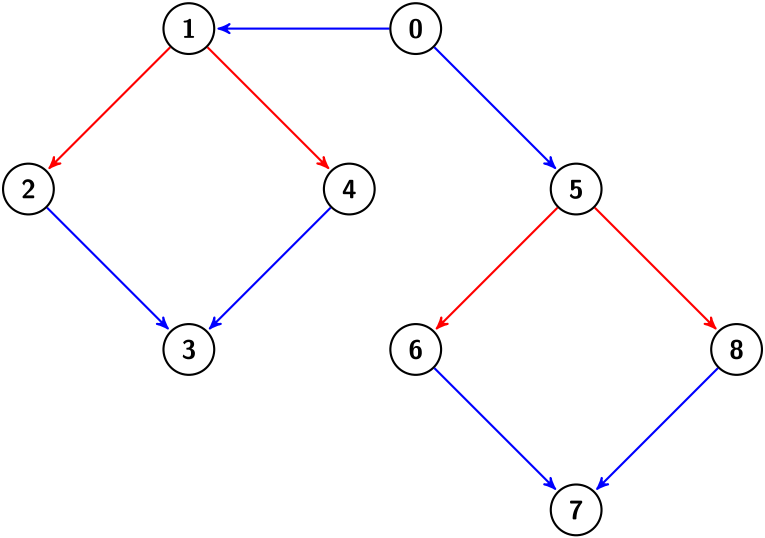
Overall budget

Total amount to allocate on both activities without constraint

Optimal: consider the maximum over the direct and indirect indices

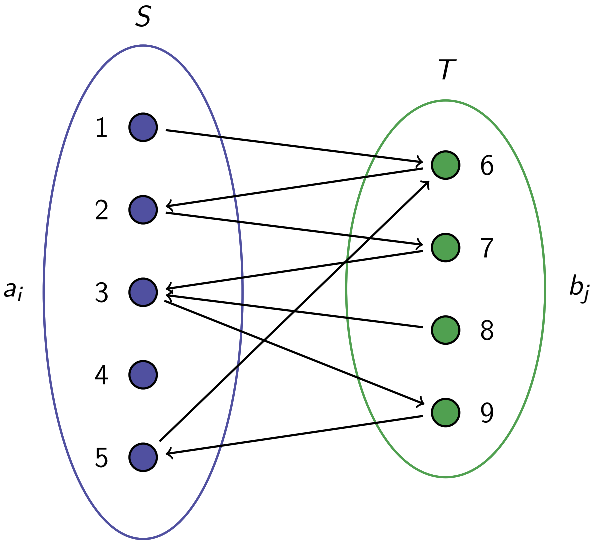
Might be optimal to improve i's attention characteristic x_i if i's attention has a large impact on the js with large direct indices

$$K_i^{x \to b} = \sum_j \alpha_{ij} K_j^{y \to b}$$



Optimal to increase 1's attention to increase aggregate contribution

Impact of heterogeneity: Illustration in a 2-sided setting



Homogeneity in impacts

- $\alpha_{ij} = \alpha$ and $\beta_{ji} = \beta$ for any $i \in S, j \in T$
- Best responses depend on the aggregate action of the other side:

$$a_i = x_i + \beta b_+ \quad \text{and} \quad b_j = y_j + \alpha a_+$$

where

$$a_+ = \textstyle\sum_{i \in S} a_i \quad \text{and} \quad b_+ = \displaystyle\sum_{j \in T} b_j$$

Benchmark-cd

- At equilibrium:

$$a_+ = \frac{x_+ + s\beta y_+}{1 - st\alpha\beta} \quad b_+ = \frac{y_+ + t\alpha x_+}{1 - st\alpha\beta}.$$

- to maximize aggregate action in T: target S if $t\alpha > 1$ otherwise target T
- to maximize aggregate actions, target S if $t\alpha > s\beta$
  i.e. the side that has the highest externality effect on the other

Benchmark-cd

- Heterogeneity on side T. Matrix $\boldsymbol{\beta}$ with same overall total $st\beta$
- The index $K_j^{y \to b}$ when targeting y_j is increasing in j's impact total $\beta_{j+} = \sum_i \beta_{ji}$.
- The maximal index and the increase in aggregate contribution due to one unit is larger than under homogeneity as soon as impact totals differ.

Value of information to the manager

- Value of information: improvement in aggregate contribution due to the knowledge of the impact structures
- Without information, allocate identical amounts to each
- The increase in aggregate contribution for a uniform allocation of P to attention :

$$\frac{1}{n}(\sum_i K_i^{x\to b})]P$$

The value of information is equal to

$$[\max_i K_i^{x\to b} - \frac{1}{n}(\sum_i K_i^{x\to b})]P + [\max_i K_i^{y\to b} - \frac{1}{n}(\sum_i K_i^{y\to b})]Q.$$

The value of information is null and the uniform strategy is optimal if and only if $\alpha\mathbb{1}$ and $\beta\mathbb{1}$ are both proportional to $\mathbb{1}$.

- Null value under equal impact totals of each agent for each action
- Strong condition (typically false for the impact of contributions) Holds under *sharing* for both activities and homogeneous costs

Visibility strategies

- Various tools to discriminate contributions
- Here a *visibility* strategy is described by non-negative (visibility) weights, w_i on i, that sum to n and a positive scaler k.
- i's contributions are presented w_i/w_j times more than j's ones

Feasibility

- $(\boldsymbol{w}, k)$ modifies the paid attention
- for each j, b_j has the same effect as $kw_j b_j$ on the attention paid by others on j
- Overall constraint :

$$k \sum_i w_i b_i \leq \sum_i b_i .$$

Visibility strategies

- $\sum_i (\beta\alpha)_{ji}$ =sensitivity of aggregate contribution to j's
  Let $\gamma^{\max}$ be its maximum.

- If $\gamma^{\max} < 1$, then an optimal visibility strategy sets positive weights on the individuals to which aggregate sensitivity is maximal
  The aggregate contribution is equal to $\frac{\sum_i z_i}{1-\gamma^{\max}}$.
- If $\gamma^{\max} \geq 1$, then aggregate contribution can be made arbitrarily large by feasible visibility strategies.

Dominant eigenvalue ρ of $\beta\alpha$ is less or equal to $\gamma^{\max}$.

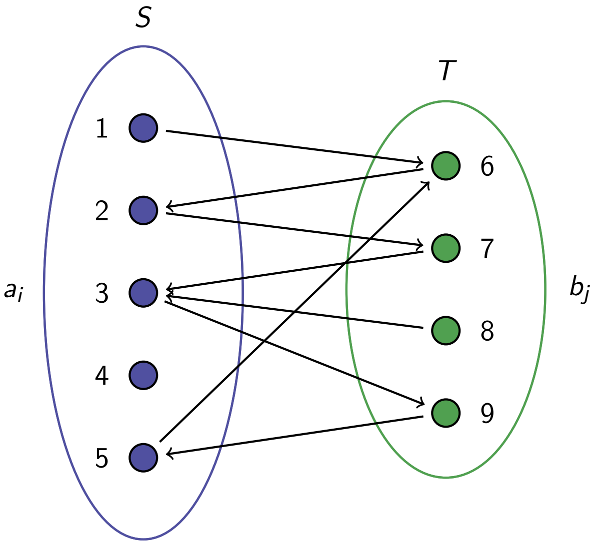
- In general, incentives to make visible a limited number of individuals. those with the largest $\sum_i (\boldsymbol{\beta}\boldsymbol{\alpha})_{ji}$
- Not necessarily those whose contributions have the largest direct impact total i.e. the largest $\sum_i \boldsymbol{\beta}_{ji}$.

Players' information on actions

- What if players only learn aggregate actions?
- Two-sided setting (simpler)
- Each individual in S takes a single action, a_i for i in S

$$u_i(a_i, \boldsymbol{b}) = a_i(x_i + \sum_{j \in T} \beta_{ji} b_j) - \frac{a_i^2}{2}$$

Symmetrically for agents in T


S
T
1
2
3
4
5
6
7
8
9
a_i
b_j

- Assume i learns $b_+ =$ sum of the b_j over T, not the individuals' ones
- Under risk-neutrality

$$E[u_i(a_i, \boldsymbol{b})] = a_i(x_i + \beta_{+i}\frac{b_+}{t}) - \frac{b_i^2}{2}$$

$$\beta_{+i} = \sum_j \beta_{ji} = \text{ impact totals on } i$$

- Similarly for j in T

- Derive equilibrium actions
- Compare the aggregate actions with and without information on players' actions
- Two specific cases:
  - release information on one side only
  - a 'star' in one side

- $a_+^0 = \sum_{i \in S} a_i^0$ and $b_+^0 = \sum_{j \in T} b_j^0$ aggregate actions without information

Let release information on S actions

- Aggregate actions in S and T move in the same direction:

$$a_+ > a_+^0 \Leftrightarrow b_+ > b_+^0$$

- Information leads to an improvement if the impact totals of the agents in S on T, the α_{i+}, are positively correlated
  - with their individual characteristics x_i,
  - and with the impact total on them, the β_{+i}.

ex: Improvement if identical characteristics and agents who are highly impacted (high β_{+i}) tend to have a large impact (high α_{i+}).

Simple case

- $\alpha_{+1} = 2, \alpha_{+2} = 1$ $\beta_{+1} = 1, \beta_{+2} = 2$ up to a cost factor
  same characteristics
  $a_1^0 < a_2^0$ (2 reacts more to b_+^0)
  $\Rightarrow$ if a_1^0, a_2^0 are revealed, decrease in T actions because they react more to a_1 $\rightarrow$ further adjustment downward

T-Star

- Single agent in T, say 1, influences the agents in S.

- Players' information on actions moves all actions in S and T in the same direction
- If individuals characteristics are identical, all actions increase if the impact total of S on the center's contribution is larger than the average one:

$$\alpha_{+1} > \frac{\sum_{j \in T} \alpha_{+j}}{t}$$

and all decrease if the reverse holds.

Concluding remarks

- In a linear model of dual interaction
  - Actions and allocation strategies are explicit, based on dual centrality indices
    May be optimal to target one type of actions to enhance the other
  - The value of information almost always positive, related to the heterogeneity in the interactions
  - Visibility: optimal to make the contributions of the individuals with the largest cross-impact total more visible
  - Not releasing the identity of the players may be optimal for the manager
- Welfare?
- Many issues on the design of interactions on social media

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