Platform Transaction Taxes and Freemium Pricing*

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Abstract

We study transaction taxes in markets where sellers adopt second-degree price discrimination by offering multiple versions of their product. We show that an ad valorem tax that targets the top version can alleviate the typical distortions in these markets, increasing consumer surplus and welfare. This targeting is natural with sellers that adopt "freemium" pricing, as in the case of digital goods like mobile apps. We explore the implications of these findings for platforms where these products are traded. In these markets, integration between platform and sellers can reduce welfare and consumer surplus. Moreover, selling devices that give consumers access to the marketplace (e.g., smartphones) and/or advertising induces the platform to set a higher transaction tax. By contrast, selling a product that competes with third-party sellers on the marketplace should induce the platforms to set a lower tax.

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1 Introduction

Most business-to-consumer transactions are subject to taxes. Traditional examples include VAT and excise duties. Taxes are also commonly applied by digital marketplace platforms, including the app stores run by Google and Apple.[1] These taxes have recently drawn increasing attention as a key part of the agency model, which is typical of digital platforms. In this model, platforms act as retailers, let sellers set final prices and extract a percentage of each sale (Johnson, 2017; Foros et al., 2017). The role of these transaction taxes in digital markets is quite controversial. While platforms maintain that these fees are key to the viability of their services, many sellers claim that taxes unfairly squeeze their profits and force them to raise prices, harming consumers as well. Moreover, because some platforms sell their own products on the marketplace, regulators have raised concerns that transaction taxes may be anti-competitive, by putting third-parties at a disadvantage.[2]

In this paper, we examine the effects of transaction taxes in digital marketplaces and analyze the implications for the strategy of digital platforms. We contribute to the growing literature on this topic (reviewed in Section 2) by considering the sophisticated pricing policies commonly adopted by suppliers of apps, including, e.g., mobile apps in music and video streaming, gaming, and data storage. Notably, these suppliers often adopt a form of second-degree price discrimination called *freemium* pricing, where the basic version of a product is free of charge (though users may pay, e.g., by having to see ads), whereas a monetary price applies for the premium version.[3] In this context, even if the transaction tax applies formally to all versions of the product, effectively it targets only the premium one. Because the price and features of the premium and base version are interdependent, the seller's response to the tax is far from obvious when it adopts freemium pricing.[4]

[1] Apple and Google currently charge 30 percent of the price consumers pay to download apps, initial subscriptions or in-app purchases, and 15 percent for repeated subscriptions.

[2] This is one of the main concerns regarding possible anticompetitive behavior by platforms reported by the Dutch Competition Authority (ACM, 2019, chpt. 3 and 4), in the context of the mobile app market. Similar concerns were raised in a recent antitrust lawsuit against Google brought by multiple US States. See https://www.courtlistener.com/docket/60042641/522/state-of-utah-v-google-llc/.

[3] As reported by ACM (2019), freemium accounts for more than 90% of revenue from the games category of apps in the App Store. See also http://tinyurl.com/5aj4cdtj.

[4] Besides freemium pricing, second-degree price discrimination can take several forms, such as offering a product in different sizes (e.g., soft drinks in 0.5 and 2 litre bottles), quality (e.g., first- and second-class train tickets) or functionalities (e.g., a car with different engine types and features). The analysis of the paper also applies to these more general instances of second-degree price discrimination, when governments set ad valorem taxes. Although ad valorem taxes on a product are typically not conditioned on its version, there are many examples of tax rates that depend on the (characteristics of) the version sold. For instance, some countries apply different tax rates to business- and economy-class flight tickets. See, e.g., the UK Air Passenger Duty: https://www.gov.uk/guidance/rates-and-allowances-for-air-passenger-duty. Furthermore, road taxes often depend on the weight, size, engine displacement and power of vehicles. See https://en.wikipedia.org/wiki/Road_tax for several examples of road taxes throughout the world. Although our analysis focuses on digital marketplaces, its insights can be applied to these markets as well.

Conventional wisdom suggests that taxing a product should result in a higher equilibrium price and in lower supply, imposing a burden on consumers and producers. Our first result is that the effects of an ad valorem transaction tax that targets only one version of the product sold by a price-discriminating firm may be counterintuitive. Specifically, a tax on the top version can alleviate the typical distortions imposed by the seller, which originates from the classic rent extraction vs. efficiency trade-off (Laffont and Martimort, 2002). We show that the tax produces a "double dividend" by raising welfare, consumer surplus and revenue for the platform at the same time. Instead, a unit tax does not have the same effect.

To understand the above claim, consider a monopolist offering two versions of its product, intended for two types of consumers that differ in their marginal utility from the product. The seller must ensure that consumers self-select on the intended version. This incentive compatibility constraint limits the amount of revenue that can be extracted from the high types, who receive an information rent. Facing this constraint, the seller (i) sets the quality/quantity intended for the high types at the efficient level, and (ii) distorts the quality/quantity of the low version, so that the low types' marginal utility exceeds the marginal cost, to reduce the information rent (Maskin and Riley, 1984). By taking away a percentage of the revenue extracted from the high types, an ad valorem tax on the top version makes generating revenue from the low types relatively more attractive to the seller. Hence, the incentive to distort their allocation diminishes. The drawback is that the tax distorts the allocation intended for the high type. However, starting from the laissez-faire equilibrium, this second distortion is small in magnitude, so consumer and total surplus increase. This result is robust to including competition among third-party sellers and more than two consumer types.

We then focus on a marketplace platform that distributes digital goods (e.g., apps). To incorporate freemium pricing, we extend our basic model allowing the seller to charge a monetary price for the top version and a non-monetary price for the low version (consumers can pay with their attention, by being exposed to ads, or with their personal data). In this setting, even though it formally applies to all versions, the platform's ad valorem transaction tax effectively targets the premium version that is the only one exchanged for a price on the marketplace. By the same mechanism presented above, consumer surplus and welfare can increase with the tax.

In an extension, we endogenize the number of consumers and suppliers that join the platform, and we show that they may increase with the tax as well. This is because, although the tax reduces the per-consumer profit of each supplier, consumers get more surplus from interacting with each supplier.

The above findings contribute to the study of the agency model and, more generally, of vertical relations in digital markets. In our basic setting, the platform is purely an intermediary connecting consumers to the seller and collecting a percentage of its revenue. A platform integrated with the seller would just maximise the sum of their profits, that is equal to the seller's profit without tax. However, this vertically integrated entity would set a transaction tax reducing consumer surplus and welfare, because the latter are *higher* when the platform applies its ad valorem tax than with no tax at all. This contrasts with the prescription that vertical integration increases welfare due to the otherwise imperfect coordination between upstream and downstream firms (Tirole, 1988).

Next, we consider the incentives of the platform when setting its fees, considering the interaction between transaction taxes and other sources of revenue. We first let the platform sell a device essential to accessing the marketplace (e.g., a smartphone). By effectively charging consumers for access to the market, the platform internalizes the effect of the tax on their surplus. As established above, this effect is positive, which induces the platform to set a higher tax. This is in contrast to standard settings, where access and transaction charges are substitutes (see, e.g., Oi, 1971). We find a similar result when the platform captures part of the advertising revenue the seller makes from the free version. This setting is meant to capture the fact that some platforms (e.g., Google) also operate as intermediaries in the online advertising market (see the recent report by CMA, 2020). The tax induces the seller to raise the quality of the free version and, thus, the non-monetary price (volume of ads) that consumers are willing to tolerate.

We then address the role of transaction taxes on hybrid platforms that sell their own products on the marketplace. For example, Apple and Google offer music and video streaming apps that compete with third-party ones sold on their marketplaces. As mentioned, a major concern is that platforms may use taxes to force third-party sellers to raise prices and weaken competition to their own product. We find that, given freemium pricing, the platform prefers a lower transaction tax than if it had no competing product to sell. The intuition is that, as argued above, a marginal increase in the tax induces the third-party seller to adjust its offer such that consumers get more surplus. Hence, the tax increases the competitive pressure on the platform's product from the seller, as long as the latter adopts the fremium model.

The remainder of the paper is organized as follows. Section 2 provides a review of the literature. Section 3 introduces a basic model of price discrimination to illustrate the main mechanism behind our findings. In Section 4, we introduce a more articulated version of the model that more closely fits the case of digital marketplaces hosting sellers that adopt freemium pricing. In Section 5 we consider a few extensions. First, we endogenize the number of sellers and consumers in the market (Section 5.1). Then, we study the pricing strategies of the platform, focusing on the interaction between transaction taxes and the sale of devices or ads (Section 5.2) and the platforms' own products (Section 5.3). Section 6 concludes.

2 Literature

There is an extensive literature studying price discrimination (Tirole, 1988; Laffont and Martimort, 2002; Stole, 2007). Recently, this literature has focused on price discrimination on digital platforms. Lin (2020) and Jeon et al. (2022) study second-degree discrimination, focusing on how the platform's incentives and ability to screen participants on one side depend on the externalities generated on the other side. de Corniere et al. (2023) study third-degree price discrimination by a platform hosting different types of sellers. Wang and Wright (2017) show that ad valorem taxes allow the platform to efficiently price discriminate across goods with different costs and values, unlike unit taxes. We consider the effects of transaction taxes when the sellers on the platform, rather than the platform itself, engage in price discrimination.

Our study also contributes to the growing literature studying digital marketplace platforms. A branch of this literature studies the agency model, which is typical of these platforms. The literature has found that this model may perform better than other vertical arrangements (such as wholesale) in terms of welfare and consumer surplus (Johnson, 2017; Foros et al., 2017). However, due to the usual lack of coordination between upstream and downstream firms, welfare and consumer surplus would be higher with an integrated firm. Our findings confirm the welfare-superiority of ad valorem taxes/fees in vertical relations compared to unit taxes, but we show that when sellers adopt freemium pricing, consumer and total surplus (welfare) can be *higher* when the platform applies its ad valorem tax than with no tax at all, that is, than with a vertically integrated firm. Another branch of this literature focuses on the relation between transaction taxes and other sources of revenue for the platform, such as the sale of devices and/or ads (see, e.g., Etro, 2021; Gaudin and White, 2021). Unlike previous papers, we find that, when one accounts for freemium pricing by sellers, transaction and access charges can be complementary. Moreover, the platform tends to set a higher transaction tax if it can collect revenue from ads.

Our paper also contributes to the literature studying hybrid marketplace platforms (Hagiu et al., 2020, 2022). Anderson and Bedre-Defolie (2021) consider monopolistic competition among sellers and a platform that provides a range of competing products. They find that, compared to a pure marketplace one, a hybrid platform may set higher transaction taxes to steer consumers towards its products. Focusing on the case of freemium sellers, we show that the competitive pressure from third-party sellers may increase with the tax, meaning that the tax is lower when the platform is hybrid. Tremblay (2022) shows that, when entering a market, a platform tends to reduce transaction fees applied to other sellers in that market. Competition from the platform reduces the sellers' output and, hence, their willingness to pay. This result is fairly consistent with our findings, though our setting, and the mechanism behind our results, are different.

More generally, this paper makes also a contribution to the literature on the incidence of indirect taxes, which is a classical topic in economics (Fullerton and Metcalf, 2002). Many previous studies have looked at tax incidence in imperfectly competitive markets, focusing mostly on firms that adopt linear pricing (Anderson et al., 2001; Delipalla and Keen, 1992; Weyl and Fabinger, 2013). A fundamental result in this literature is that taxes depress supply and increase prices, with very few exceptions. These include Cremer and Thisse (1994), who show that taxation can increase welfare in a vertically differentiated oligopoly, and Carbonnier (2014), who studies price-dependent tax schedules. Only a handful of studies have investigated the effects of taxation in markets with second-degree price discrimination. Laffont (1987) and Cheung (1998) consider tax rates that apply uniformly to all quantities supplied by a nonlinear pricing monopolist. Jensen and Schjelderup (2011) and D'Annunzio et al. (2020) study taxation when firms apply multi-part tariffs. In this paper, we consider firms that adopt versioning and freemium pricing. Our main results stem from considering the effects of taxes that target only some versions. McCalman (2010) considers a similar type of differentiation when analyzing trade tariffs applied on a foreign monopolist seller. Tariffs have conventional effects on equilibrium quantities and prices in his model, although they can increase domestic welfare (that is, when the surplus of the seller is ignored).

3 A basic model

To demonstrate the mechanism underlying our main results, we begin the analysis by considering a standard model of second-degree price discrimination. Consider a monopolist (the "seller") providing a single good. There are two types of consumers, differing in their preference for this good. This preference is captured by the parameter θ_i with $i = H, L$ (where H stands for "high" and L for "low"). A consumer of type i gets the following net utility from consuming the good

$$U_i(p,q) \equiv u(q,\theta_i) - p,\ i = H,L,$$

where p is the price and q is the quality or the quantity of the bundle (Mussa and Rosen, 1978; Maskin and Riley, 1984). We assume that $\theta_H > \theta_L$, $\frac{\partial u}{\partial q} > 0$, $\frac{\partial^2 u}{\partial q^2} < 0$, $\frac{\partial u}{\partial \theta} > 0$ and $\frac{\partial^2 u}{\partial q \partial \theta} > 0$. The parameter θ_i is private information. The total number of consumers is normalized to one and $v \in (0,1)$ is the share of type-H consumers. The cost of providing one unit of q to the seller is c. We assume the seller offers to consumers two bundles, (q_i, p_i), to choose from, each intended for one consumer type. These bundles can be interpreted as two versions of the product differing in their features (e.g., a phone with small and large memory or processing power) or size (e.g., a regular and a supersize package), and sold at different prices. For concreteness, in the following we will refer to q as the quality of the bundle.

We assume each version of the product is subject to a different ad valorem tax rate, t_i, $i = H, L$. t_i can be interpreted as a tax imposed by a government or as a transaction fee imposed by an upstream provider or a downstream distributor (e.g., a platform). Consistently with the type of transaction taxes encountered on digital marketplaces, we allow the tax rates to differ by version because we are interested in the effects of taxes that target only a subset of the versions provided by the seller. As we shall explain, this is the case even if tax rates are not formally differentiated across versions, but the seller gives one version away for free (see Section 4). Moreover, as explained in the Introduction, tax rates imposed by governments are sometimes differentiated by the version of the product on sale.

The seller's problem is

$$\max_{q_H, p_H, q_L, p_L} \quad \pi = v\left(\left(1-t_H\right) p_H - c q_H\right) + (1-v)\left(\left(1-t_L\right) p_L - c q_L\right), \tag{1}$$

$$s.t. \quad u\left(q_H, \theta_H\right) - p_H \geq u\left(q_L, \theta_H\right) - p_L, \tag{2}$$

$$u\left(q_L, \theta_L\right) - p_L \geq u\left(q_H, \theta_L\right) - p_H, \tag{3}$$

$$u\left(q_H, \theta_H\right) - p_H \geq 0, \tag{4}$$

$$u\left(q_L, \theta_L\right) - p_L \geq 0, \tag{5}$$

where (2) and (3) are the incentive compatibility constraints, while (4) and (5) are the participation constraints for H and L-type consumers, respectively (we normalize the utility from no consumption to zero). In the following, we denote the equilibrium variables (given the tax rates) with the superscript e.

The social welfare function, obtained as the sum of consumer surplus, the seller's profit and tax revenue, boils down to total surplus in this market

$$W = v\left(u_H - c q_H\right) + (1-v)\left(u_L - c q_L\right). \tag{6}$$

In Appendix C, we provide extensions with more than two types of consumers and competing sellers. These extensions show the robustness of the main results derived below.

3.1 Equilibrium and effects of transaction taxes

Following standard steps (Laffont and Martimort, 2002), one can show that only (2) and (5) are binding at the allocation that solves the seller's problem. Therefore, we ignore (3) and (4), and set

$$p_H = u\left(q_H, \theta_H\right) - u\left(q_L, \theta_H\right) + u\left(q_L, \theta_L\right), \quad p_L = u\left(q_L, \theta_L\right). \tag{7}$$

Hence, we can rewrite the seller's problem in (1) as

$$\max_{q_H,q_L} \quad \pi = v\left((1-t_H)\left(u_H - u_{HL} + u_L\right) - cq_H\right) + (1-v)\left((1-t_L)\,u_L - cq_L\right), \tag{8}$$

where $u_i \equiv u(q_i, \theta_i)$, $i = H, L$, $u_{HL} \equiv u(q_L, \theta_H)$ and $u_{LH} \equiv u(q_H, \theta_L)$. In the above expression, $u_{HL} - u_L$ represents the high types' information rent, which the seller must grant to prevent them from choosing the version intended for the low types. There is, instead, no rent left to the low types. We thus get the following expressions for consumer surplus

$$CS_H = u_{HL} - u_L, \qquad\qquad CS_L = 0. \tag{9}$$

The equilibrium qualities, q_i^e, solve the following system of equations

$$\frac{\partial \pi}{\partial q_H} = v\left(\frac{\partial u_H}{\partial q_H}(1-t_H) - c\right) = 0, \tag{10}$$

$$\frac{\partial \pi}{\partial q_L} = v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L}\right)(1-t_H) + (1-v)\left(\frac{\partial u_L}{\partial q_L}(1-t_L) - c\right) = 0. \tag{11}$$

Setting taxes aside ($t_H = t_L = 0$), these equations indicate that the seller offers an efficient version to the high types, in the sense that the marginal utility these consumers get from quality equals the marginal cost. Given $\frac{\partial u_{HL}}{\partial q_L} > \frac{\partial u_L}{\partial q_L}$, the seller distorts the version intended for the low types. This is to reduce the information rent to the high types.

Let us now study the effects of the two tax rates. We show in Appendix A.1 that

$$\frac{\partial q_H^e}{\partial t_H} < 0, \quad \frac{\partial q_L^e}{\partial t_H} > 0, \quad \frac{\partial q_H^e}{\partial t_L} = 0, \quad \frac{\partial q_L^e}{\partial t_L} < 0. \tag{12}$$

Interestingly, the quality of the version intended for the low type *increases* with the tax rate that targets the high version. To see why, recall that the seller distorts q_L downward to reduce the information rent of the high types. The tax t_H takes part of the revenue earned from selling to these consumers away, without affecting the revenue from the L-version directly. Hence, the tax reduces the incentive to distort q_L (see (11)). By the same token, the tax induces a downward distortion in q_H. However, as we shall see, starting from the laissez-faire equilibrium, the magnitude of this latter distortion is of second order. The effects of the tax on the L-version are quite different: the quality of the L-version goes down, whereas the quality of the H-version is unaffected.

The effects of taxes on prices mirror those on quality. Starting from (7) and given (12), we have

$$\frac{\partial p_H^e}{\partial t_H} < 0, \quad \frac{\partial p_L^e}{\partial t_H} > 0, \quad \frac{\partial p_H^e}{\partial t_L} > 0, \quad \frac{\partial p_L^e}{\partial t_L} < 0. \tag{13}$$

Notably, the tax on the H-version reduces the price of this version. To see why, note that, while q_H^e decreases, q_L^e increases, which raises the high type's information rent. On the other hand, the tax increases the price of the L-version by an amount equal to the increased willingness to pay of the low types. Thus, the high types benefit from the tax applied to version H, while the net surplus of the low types remains equal to zero (see (9)). Overall, therefore, consumer surplus *increases* with t_H.

The effects of the tax on the L-version are quite different: p_L^e goes down, but p_H^e increases, because the information rent goes down. Therefore, this tax makes no consumer better off. We summarize the above discussion in the following.

Proposition 1. *An ad valorem tax targeting the H-version reduces the quality and price of this version, and increases the quality and price of the L-version. Moreover, this tax increases the surplus of H-consumers, leaving the L-types' unchanged. By contrast, a tax targeting the L-version reduces the quality and price of this version, while raising the price of the H-version without affecting its quality. Consumer surplus decreases with this tax.*

Finally, consider the effects of the tax rates on welfare. Proposition 1 summarizes the effects on consumer surplus. Also, the seller's profit decreases with either tax rate. Differentiating (6) and given the first-order conditions of the seller's problem, we obtain[5]

$$\frac{\partial W}{\partial t_H} = v\frac{\partial q_H^e}{\partial t_H}\frac{\partial u_H}{\partial q_H}t_H + (1-v)\frac{\partial q_L^e}{\partial t_H}\left(\frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)(1-t_H) + \frac{\partial u_L}{\partial q_L}t_L\right), \tag{14}$$

$$\frac{\partial W}{\partial t_L} = (1-v)\frac{\partial q_L^e}{\partial t_L}\left(\frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)(1-t_H) + \frac{\partial u_L}{\partial q_L}t_L\right). \tag{15}$$

It is useful to evaluate the above derivatives at the laissez-faire equilibrium, where $t_L = t_H = 0$ to assess the effects of introducing a tax. Given (12), we obtain

$$\left.\frac{\partial W}{\partial t_H}\right|_{t_L=t_H=0} = v\frac{\partial q_L^e}{\partial t_H}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0, \qquad \left.\frac{\partial W}{\partial t_L}\right|_{t_L=t_H=0} = v\frac{\partial q_L^e}{\partial t_L}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) < 0. \tag{16}$$

These expressions show that welfare increases with the introduction of a subsidy on the L-version or a *tax* on the H-version. As shown above, the tax on the H-version reduces the distortion that the seller imposes on the L-version, while the distortion on the H-version is of second-order.

[5] In these expressions, the derivatives of utility with respect to q_i are evaluated at the equilibrium values, q_i^e, given t_i.

Proposition 2. *Starting from the no-tax equilibrium, welfare increases with an ad valorem tax targeting the H-version or a subsidy targeting the L-version.*

In Appendix B we consider other tax instruments (unit taxes and an ad valorem tax that applies with the same rate to all versions), showing that they have different implications for consumer surplus and welfare.

A caveat regarding the analysis is that, given it induces a reduction in q_H^e and an increase in q_L^e, a sufficiently high t_H might result in these two quantities being equal. At that point, the seller could not implement a price schedule such that $p_H > p_L$, because the constraint (2) would be violated. In response, the seller might stop serving the low-type consumers and offer a single version that targets the high-types (setting $p_H = u_H$). Let this threshold tax level be $\bar{t}_H$. Characterizing this threshold is quite cumbersome. However, qualitatively, there is little loss in ignoring it. One can always find a possibly small, but strictly positive t_H, such that the seller serves both types and the above effects apply.

4 Transaction taxes on platforms with a freemium seller

We now adapt the model to more closely fit the market for mobile apps. In this market, most app sellers adopt the "freemium" sales strategy, providing two versions of their product: a basic version available for free (consumers "pay" for it with their attention and/or their personal data), and a premium one, for which consumers pay some money, possibly via repeated in-app purchases. The market for mobile apps is primarily hosted by platforms, like the App Store and Google Play, which charge sellers a percentage of every sale they make to consumers.

Accordingly, the model we now introduce has two main differences from the basic one presented above: first, we allow the seller to charge a non-monetary price for its good, focusing on the case where the low version is free. Second, we consider an ad valorem tax set by the platform that is not differentiated by product version and applies to all the monetary transactions with consumers. As will become clear shortly, if the low version is free, the tax effectively targets only the top version. Our first objective is to establish that the effects of transaction taxes we have shown in Proposition 1 and 2 carry through to the freemium setting. In the next Section, we are going to consider the implications of these effects for the behavior of marketplace platforms.

4.1 Effects of an ad valorem transaction tax with a freemium seller

We modify the model of Section 3, assuming that, for each version of its product, the seller can charge a monetary price, p_i, a non-monetary price, x_i, or both. We interpret the non-monetary price as the volume of ads, but it could also represent the volume of personal data collected from consumers using the product. We assume consumers sustain a disutility $\alpha_i > 0$ for every non-monetary unit paid. Thus, the utility of a type-i consumer is

$$U_i(p,q,x) = u(q,\theta_i) - p - \alpha_i x,\ i = H,L. \tag{17}$$

The seller earns a revenue r_i for every unit of non-monetary price (e.g., the advertising rate) on version i and pays to the platform an ad valorem tax, t, when consumers pay a monetary price on the marketplace. Given these assumptions, and using the same notation for utility as in the previous section, the seller's problem is

$$\max_{q_H,p_H,x_H,q_L,p_L,x_L} \quad \pi = v\left((1-t)\,p_H + r_H x_H - cq_H\right) + (1-v)\left((1-t)\,p_L + r_L x_L - cq_L\right), \tag{18}$$

$$s.t. \quad u_H - p_H - \alpha_H x_H \geq u_{HL} - p_L - \alpha_H x_L, \tag{19}$$

$$u_L - p_L - \alpha_L x_L \geq u_{LH} - p_H - \alpha_L x_H, \tag{20}$$

$$u_H - p_H - \alpha_H x_H \geq 0, \tag{21}$$

$$u_L - p_L - \alpha_L x_L \geq 0. \tag{22}$$

For simplicity, we assume there is perfect correlation between the parameters θ and α. We also assume that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L}$ holds. In Appendix A.2 we show that under this assumption, the usual constraints (19) and (22) bind at the solution to this problem, so we have

$$\begin{aligned} p_H + \alpha_H x_H &= u_H - u_{HL} + \alpha_H x_L + u_L - \alpha_L x_L, \\ p_L + \alpha_L x_L &= u_L. \end{aligned} \tag{23}$$

Consequently, we can rewrite the seller's problem as

$$\begin{aligned} \max_{q_H,x_H,q_L,x_L} \quad \pi = v\left((1-t)\left(u_H - \alpha_H x_H - u_{HL} + \alpha_H x_L + u_L - \alpha_L x_L\right) + r_H x_H - cq_H\right) + \\ + (1-v)\left((1-t)\left(u_L - \alpha_L x_L\right) + r_L x_L - cq_L\right). \end{aligned} \tag{24}$$

Given the linearity of the objective in x_H and x_L, the solution is such that

$$\begin{cases} x_L = \frac{u_L}{\alpha_L},\ x_H = \frac{1}{\alpha_H}\left(u_H - u_{HL} + \frac{\alpha_H}{\alpha_L}u_L\right) & \text{if} \quad \alpha_L - \frac{v}{1-v}(\alpha_H - \alpha_L) \leq \frac{r_L}{(1-t)}, \text{ and } \alpha_H \leq \frac{r_H}{(1-t)} \\ x_L = 0,\ x_H = \frac{1}{\alpha_H}(u_H - u_{HL} + u_L) & \text{if} \quad \alpha_L - \frac{v}{1-v}(\alpha_H - \alpha_L) > \frac{r_L}{(1-t)}, \text{ and } \alpha_H \leq \frac{r_H}{(1-t)} \\ x_L = \frac{u_L}{\alpha_L},\ x_H = 0 & \text{if} \quad \alpha_L - \frac{v}{1-v}(\alpha_H - \alpha_L) \leq \frac{r_L}{(1-t)}, \text{ and } \alpha_H > \frac{r_H}{(1-t)}, \\ x_L = 0,\ x_H = 0 & \text{if} \quad \alpha_L - \frac{v}{1-v}(\alpha_H - \alpha_L) > \frac{r_L}{(1-t)}, \text{ and } \alpha_H > \frac{r_H}{(1-t)}. \end{cases}$$

In words, the seller offers version i for free if and only if the revenue r_i is large enough compared to the disutility α_i. Although the model contemplates many possible cases, to concentrate on the most relevant one we assume henceforth that $\frac{r_H}{(1-t)} < \alpha_H$ and $\alpha_L - \frac{v}{1-v}(\alpha_H - \alpha_L) \leq \frac{r_L}{(1-t)}$. Given these conditions, we have freemium pricing: the high quality version is offered for a monetary price and free of ads (i.e., $p_H > 0$ and $x_H = 0$), while the low quality version is offered for free with ads (i.e., $p_L = 0$ and $x_L > 0$).[6] We can therefore write the expressions for consumer surplus in this setting as

$$CS_H = u_{HL} - \frac{\alpha_H}{\alpha_L}u_L, \qquad CS_L = 0. \tag{25}$$

Note that the condition $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L}$ guarantees that CS_H is strictly positive.
Considering the freemium model, we can rewrite the seller's problem as

$$\max_{q_H, q_L} \quad \pi = v\left((1-t)\left(u_H - u_{HL} + \frac{\alpha_H}{\alpha_L}u_L\right) - cq_H\right) + (1-v)\left(\frac{u_L r_L}{\alpha_L} - cq_L\right). \tag{26}$$

The above expression (which is the counterpart to (8) in this setting) shows that, when the seller adopts the freemium strategy, the transaction tax effectively targets the top version of the product. Therefore, the effects on prices, qualities and consumer surplus are the same as in Proposition 1. The intuition is that a tax on monetary transactions with consumers makes collecting revenue from such transactions relatively less profitable to the seller. If the seller adopts a freemium strategy, generating revenue from the "top" version of its product is thus less profitable, which gives the seller an incentive to reduce the distortion on the "low" version and increase its quality. We can therefore state the following

Proposition 3. *If the seller adopts the freemium model, the quality and price of the H-version decrease with the ad valorem transaction tax charged by the platform, whereas the quality of the*

[6] See Sato (2019) and Zennyo (2020) for previous studies on freemium as a form of price discrimination. Unlike these papers, we are not interested in studying the profitability of freemium per se, but in the implications of platform taxes when the app sellers adopt this pricing strategy. Accordingly, we assume that sufficient conditions hold such that freemium pricing arises in equilibrium.

L-version increases. Moreover, while the profit of the seller decreases, consumer surplus increases.

As discussed at the end of Section 3, there is an upper bound on the level of the tax, beyond which the the implementability condition, $q_H > q_L$, fails. Let the threshold $\bar{t} > 0$ denote the level of the tax such that the values of q_H^e and q_L^e that solve problem (26) are equal. In the analysis that follows, we concentrate on tax levels such that $t < \bar{t}$ where sellers implement second-degree discrimination even when subject to the tax. Note also that raising the tax up to $\bar{t}$ is not necessarily in the platform's interest. This incentive is even lower when the platform can recover part of consumer surplus from app purchases via an access charge (like the price of an essential device). We return to this point in Section 5.2.

To conclude this part of the analysis, we again consider the welfare effects of the tax. Welfare is the sum of platform's profits, sellers' profits and consumer surplus. Assuming the platform sustains no cost, its profit is given by

$$\pi_P = vtp_H^e = vt\left(u_H^e - u_{HL}^e + \frac{\alpha_H}{\alpha_L}u_L^e\right). \tag{27}$$

Intuitively, starting from the zero tax level, the platform's profit increases with t, while the seller's profit decreases. Social welfare in this setting is

$$W = v\left(u_H - cq_H\right) + (1-v)\left(\frac{u_L r_L}{\alpha_L} - cq_L\right). \tag{28}$$

Differentiating this expression and using the first-order conditions of the seller's problem, we obtain

$$\frac{\partial W}{\partial t} = \frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L}\right) - t\left(\frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L}\right) - \frac{\partial q_H}{\partial t}\frac{\partial u_H}{\partial q_H}\right). \tag{29}$$

Given that $\frac{\partial q_L}{\partial t} > 0$ and $\frac{\partial u_{HL}}{\partial q_L} - \frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L} > 0$, this derivative is positive when $t = 0$. Hence, having no transaction tax would not be socially optimal, but this does not imply that the tax rate chosen by the platform is optimal. Let the welfare-maximising tax rate, t^W, be such that (29) equals zero. The equilibrium tax rate, t^e, maximises the platform's profit in (27). Assuming an interior solution, this rate satisfies the following first-order condition

$$\frac{\partial \pi_P}{\partial t} = \left(u_H^e - u_{HL}^e + \frac{\alpha_H}{\alpha_L}u_L^e\right) - t\left(\frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L}\right) - \frac{\partial q_H}{\partial t}\frac{\partial u_H}{\partial q_H}\right) = 0. \tag{30}$$

As we show in Appendix A.3 the comparison between t^W and t^e is not straightforward.

Proposition 4. *When the seller adopts the freemium model, welfare increases with the platform's tax if the latter is within the range* $\left[0, t^W\right]$.

4.2 Implications for the analysis of vertical structures

4.2.1 Vertical integration vs. separation

The seller-platform structure we consider is an example of the agency model adopted in vertical relations (Johnson, 2017; Foros et al., 2017). Propositions 3 and 4 show that, given freemium pricing, welfare and consumer surplus increase with the tax applied by the platform if the tax is within the range $\left[0, t^W\right]$, although the seller's profit decreases. Hence, the fact that the platform does not internalize the effect of its policy on the supplier's profit can improve efficiency and consumer surplus. This finding contrasts with the common prescription that, by removing the frictions caused by the lack of coordination between suppliers and retailers, vertical integration increases market performance (Tirole, 1988).

In the setting we consider, integration between platform and seller may in fact reduce market performance. The vertically integrated firm would maximize the sum of seller and platform profit, given by (18) and (27) respectively. This sum boils down to the profit the seller earns gross of the tax. The firm would therefore implement the same prices and quality levels as when $t = 0$.

Remark. When the seller adopts freemium pricing and the platform taxes monetary transactions, vertical integration between the seller and the platform can reduce efficiency and consumer surplus.

4.2.2 Ad valorem vs. unit transaction taxes

We have assumed the transaction tax is ad valorem, consistently with the taxes applied by platforms like Apple and Google. Previous literature has compared ad valorem to unit fees in vertical relations, often finding ad valorem ones to be superior in terms of welfare (see, e.g., Wang and Wright, 2017; Gaudin and White, 2021).[7] To address this question, in Appendix A.4 we study the effects of a unit transaction tax in the freemium model. If the fee is proportional to q_i and differentiated by version, the seller's profit is $\pi = v\left(p_H - (c + \tau_H) q_H\right) + (1 - v)\left(p_L - (c + \tau_L) q_L\right)$.[8] Clearly, unit taxes are smilar to an increase in the cost of production. Accordingly, we find that the quality of each version

[7] This finding is consistent with the literature on commodity taxation, which shows that ad valorem taxes are less distortionary than unit taxes in imperfectly competitive markets (Delipalla and Keen, 1992; Anderson et al., 2001; Auerbach and Hines, 2002).

[8] A natural application is the case where q is the quantity of the good. Otherwise, implementing this tax would require measuring "units of quality".

decreases with the respective unit fee, so that the effect on consumer and total surplus is negative (a similar result would apply if the fee was uniform across versions, i.e. $\tau_H = \tau_H = \tau$).

Alternatively, we could model unit fees as proportional to each bundle sold to consumers, implying that profits are $\pi = v(p_H - cq_H - \tau_H) + (1-v)(p_L - cq_L - \tau_L)$. Given our assumptions, these fees would not have any effect on the equilibrium variables.

5 Extensions

We now propose several extensions to the model of Section 4. We begin from an extension where the number of consumers and sellers on the platform is endogenous. We then analyze the platform's incentives when the platform has other sources of revenues (from devices, ads, or apps) on top of the tax on the monetary transactions of the seller.

5.1 Endogenous number of consumers and sellers

We relax the assumption that the number of consumers and sellers on the platform is exogenous in Appendix A.5. For simplicity, we assume consumers interact with monopolistic sellers, each providing a different product. Given the number of sellers, the tax increases consumer surplus (see Proposition 3), which gives consumers a greater incentive to visit the platform. Hence, although the tax reduces the profit of each seller for a given number of consumers, its overall effect on sellers can be positive, because the negative effect on per-consumer profit may be more than compensated by the expansion in the number of consumers. On the other hand, the net effect on consumers can be negative if many sellers abandon the platform. Therefore, the transaction tax may result in either a smaller or larger total number of consumers and sellers joining the platform.

5.2 Platforms selling devices and ads

Taxes on monetary transactions in the app stores are typically one of several sources of revenue to the main platforms in the sector. For instance, platforms like Google are also involved in the sale of ads displayed on the ad-funded versions of the apps, acting as intermediaries between advertisers and app sellers. Also, Apple and Google sell the devices needed to access the marketplace (e.g., smartphones and tablets), in addition to imposing transaction taxes. Previous literature has suggested that access and transaction charges should be substitute instruments from the platform's perspective (see, e.g., Etro, 2021; Gaudin and White, 2021). This is because the platform can recover consumer surplus via the access price and this surplus decreases with the tax. In this section, we consider how the incentives to set the transaction tax are affected when the platform has these other sources of revenue.

5.2.1 Device sales

Suppose that consumers need a device, sold by the platform at price p_D, to access the marketplace. Consumers get an intrinsic utility d from owning the device. We normalize the consumers' outside option and the marginal cost of the device to zero (without loss). We assume consumers do not observe their own preference parameter for the seller's product, θ, prior to acquiring the device and observing the products available on the marketplace. Moreover, consumers have rational expectations regarding the seller's product and can thus compute their expected surplus. The timing is as follows: at stage 1, the platform sets t and p_D. At stage 2, the seller sets the price and quality of its products and consumers decide whether to buy the device. At stage 3, consumers who bought the device observe the seller's products and decide which version, if any, to buy.

We solve the model in Appendix A.6 and here provide an overview of the results. At stage 3, the solution to the seller's problem is the same as in Section (4), because consumers do not observe the sellers' products until they have bought the device, implying that the seller takes the size of the market as given when designing and pricing such products at stage 2. Given (25), the expected consumer surplus from the seller's product at stage 3 is $E(CS) = v\left(u^e_{HL} - \frac{\alpha_H}{\alpha_L}u^e_L\right)$, where $u^e_i \equiv (q^e_i, \theta_i)$ and $u_{HL} \equiv u(q^e_H, \theta_L)$. The platform recovers this surplus through the price of the device by setting

$$p_D = d + v\left(u^e_{HL} - \frac{\alpha_H}{\alpha_L}u^e_L\right).$$

The price of the device is equivalent to an access charge and is set at the highest level such that consumers buy the device. When choosing t, therefore, the platform maximises the following

$$\pi_P = d + v\left(u^e_{HL} - \frac{\alpha_H}{\alpha_L}u^e_L\right) + tvp^e_H. \tag{31}$$

By Proposition 3, consumer surplus (expression (25)) increases in t. Hence, the second term in (31) increases with t. Therefore, comparing to 27, one can show that the equilibrium tax must be *larger* than when the platform does not sell any device. From the platform's perspective, transaction and access charges are *complements* rather than substitutes: they both increases when the tax increases. The intuition is that, due to the device charge working as an access charge, the platform internalizes the effect of the tax on consumer surplus.

Proposition 5. *When sellers adopt the freemium model, if the platform can charge consumers for* *access to the marketplace (e.g., by selling an essential device), it sets a higher transaction fee than in the absence of this source of revenue.*

Note that via p_D the platform internalizes the loss consumers would suffer if the seller decided to serve only the H-types. Thus, as anticipated, violating the constraint $t < \bar{t}$ would not necessarily be beneficial to the platform when it sells an essential device.[9]

5.2.2 Ad sales

Suppose now that the platform obtains an exogenous revenue r_P for every ad that the seller displays on its app. An example is Google, that controls the Play Store and is also the largest online advertising intermediary. The seller's revenue per ad, r_L, is to be interpreted as net of the advertising intermediation fees and is the same as in Section 4.[10] As in the baseline model, the platform sets t first and the seller moves next.

Proceeding backwards, the solution to the seller's problem is as in our baseline model. Given $x_L^e = \frac{u_L^e}{\alpha_L}$, the profit of the platform is

$$\pi_P = tvp_H^e + r_P(1-v)\left(\frac{u_L^e}{\alpha_L}\right). \tag{32}$$

The platform accounts for the effect of the transaction tax on the advertising revenue. As established in Proposition 3, the quality of the ad-funded L-version of the seller's product, q_L^e, increases with t. Consequently, u_L^e increases, and so does the L-consumers' willingness to pay for the bundle. The ad revenue therefore *increases* with t. Hence, the platform sets a higher level of t when it also act as an advertising intermediary than in the baseline model (see Appendix A.6).

Proposition 6. *When sellers adopt the freemium model, if the platform captures some of the revenue from advertising on the apps, it sets a higher transaction fee than in the absence of this source of revenue.*

As in the previous section, we note that if the platform recovers some of the ad revenue generated on low-type consumers, its incentives to set $t \geq \bar{t}$ would be even weaker.

[9] To see this, recall that $p_H^e = u_H^e - u_{HL}^e + \frac{\alpha_H}{\alpha_L}u_L^e$ if the seller serves both types, so (31) can be written as $\pi_P = tvu_H^e + d + v(1-t)\left(u_{HL}^e - \frac{\alpha_H}{\alpha_L}u_L^e\right)$. By contrast, if the seller serves only the H types, it sets $p_H^e = u_H^e$ (but the quality level q_H^e does not change). Hence, the platform cannot set p_D above d, and its total profit is $\pi_P = tvu_H^e + d$.

[10] App suppliers generally rely on an intermediary to sell their ad space to advertisers. In the baseline model, we implicitly assume this intermediary is unrelated to the marketplace platform. We here consider the case where the platform and the intermediary are integrated. To focus on the effects of interest, we assume the supplier's net revenue per ad is the same in the two scenarios. As pointed out in recent market studies, the way large ad intermediaries set their fees is fairly complex and obscure (see, e.g., CMA, 2020). This issue is beyond the scope of our investigation.

5.3 Transaction taxes on hybrid platforms

It is common for platforms to sell their own products on the marketplace they host. The literature has referred to these platforms as hybrid platforms (see, e.g., Anderson and Bedre-Defolie, 2021; Hagiu et al., 2020, 2022). For example, Apple and Google provide apps that compete with (possibly well established) third-party ones in, e.g., video and music streaming, office utilities and cloud storage. Third-party apps sold for a monetary price are subject to the transaction tax, which potentially puts them at a disadvantage with respect to the platform's own products. Many sellers complain that these fees squeeze their margins, implying that they are forced to raise prices at the disadvantage of consumers (ACM, 2019). Furthermore, platforms often make their own products prominent on their marketplaces and/or devices.[11] In this section, we investigate how being a hybrid platform affects the incentives to set the transaction tax.

Consider the model of Section 4, but assume the platform provides a product that competes with the third-party one. We assume that a share $s \in [0,1]$ of "loyal" consumers only buys the seller's product, if any. The other consumers obtain the same utility (see (17)) from either product. The distribution of θ is independent of whether consumers are loyal or not. We assume that all consumers observe the platform's product at no cost, because it is prominent. By contrast, consumers must incur a small search cost to observe the seller's product (but they have rational expectations).[12] The platform has the same production cost as the seller and obtains the same advertising rate on this product, r_i^P. As in previous sections, we concentrate on the freemium scenario where platform and seller charge no monetary price for the basic version of their product.[13]

The timing is as follows. At stage 1, the platform sets t and the characteristics (p_i^P, x_i^P and q_i^P) of its product. At stage 2, the seller sets the features (p_i, x_i and q_i) of its own product. At stage 3, consumers land on the marketplace and observe the platform's product. Unless they are loyal, they decide whether and which version to buy from the platform or search the third-party one. Finally, at stage 4, non-loyal consumers who searched and loyal ones observe the third-party product and decide which version of this product to buy, if any.

We describe the main findings here and relegate the analysis to Appendix A.7. In equilibrium, only loyal consumers search and buy the third-party product. The values of p_i^e, x_i^e and q_i^e chosen by the seller are the same as in Section 4.[14] These consumers obtain the surplus given in (25). The

[11] For instance, Apple and Google pre-install some of their own apps on smartphones and tablets running the respective operating systems.

[12] We ignore the search cost for loyal consumers for simplicity and without loss.

[13] We conduct the analysis assuming that t is low enough so that both versions of the product are sold and all consumers are served.

[14] The seller is effectively a monopolist for all consumers that search its product and for the loyal ones. It treats the share of consumers that search as given, because consumers observe the features of its product only after searching.

non-loyal consumers buy from the platform in equilibrium, and its profit function can be written as

$$\pi_P = (1-s)\left(v\left(u_H^P - CS_H^e - cq_H^P\right) + (1-v)\left(u_L^P r_L/\alpha_L - cq_L^P\right)\right) + t\left(svp_H^e\right).$$

The last term in the sum is the tax revenue, whereas the other terms capture the profit the platform gets from providing its own products (transaction fees from the H-version and advertising fees from the L-version). The revenues from the H- version is constrained by the (expected) surplus, CS_H^e,that the H-types would get from the seller's product. To attract the non-captive consumers, the platform must ensure they get the same surplus they would get from the third-party product, conditional on their type. Proposition 3 states that the tax makes the seller's product *more* attractive to consumers and thus more competitive with the platform's own product: CS_H^e increases with t. Therefore, we find that the equilibirum tax rate is *smaller* than the rate the platform would choose if it did not sell its own product.

Proposition 7. *If the seller adopts the freemium model, the tax chosen by a hybrid platform is lower than the tax that a pure marketplace platform (that does not compete with the seller) would choose.*

As a final remark, we note that setting the tax so high that only H-types are served (i.e. a tax above $\bar{t}$) would enable the platform to charge more for the H-version of its product, because the seller would only serve the H-types and extract their entire surplus, so that $CS_H^e = CS_L^e = 0$. However, this does not necessarily increase the platform's total profit, because the revenue generated by the tax may decrease. Note also that it cannot be optimal for the platform to set t so high that the third-party seller is foreclosed, because $t = \bar{t}$ is sufficient to ensure that consumers get zero surplus from the seller's product. Hence, raising the tax further would not make the platform's product more profitable.

6 Conclusions

We have studied transaction taxes in markets where sellers implement second-degree price discrimination by offering different versions of their product. We have shown that ad valorem taxes that target the top version can alleviate the typical distortions in these markets, increasing consumer surplus and welfare.

We provide an application of this effect to markets where sellers adopt the freemium pricing strategy (offering the basic version for free and the premium version for a price) and using

Therefore, the seller's problem is almost identical to that in the baseline model.

intermediaries to sell their goods. Our main example is the market for mobile apps where, even if the transaction tax imposed by the intermediary is not formally differentiated, it targets the top version sold on the marketplace. Our findings imply that vertical separation between the platform and the seller increases both consumer surplus and welfare. Moreover, differently from ad valorem taxes, unit taxes decrease the quality of each version, with negative effects on consumer and total surplus.

We explored the implications of the above findings for the choice of the platform to set transaction taxes. We have considered different scenarios, where the platform has other sources of revenues, on top of transaction taxes. More specifically, we have considered the interaction between transaction taxes and the sale of devices, ads and the platforms' own products that compete with third-party ones on the marketplace.

We conclude by briefly discussing an extension of the analysis that may be an interesting topic for further research. Some of the markets where firms apply second-degree price discrimination are subject to specific excise taxes, intended to control the consumption of unhealthy products. Examples include sugary food and beverages, alcohol and tobacco products. Although we have not considered the implications of health-related consumption externalities in the paper, we note that, by inducing sellers to reduce the size of the largest packages, the differentiated taxation we analyze here may result in an additional social benefit if we consider the largest package to be more harmful. However, these differentiated taxes also increase the size of the package intended for the low types. Hence, an analysis of the effects of differentiated taxation in these specific markets should consider the characteristics of different groups of consumers to capture the effects of these relative changes.

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Appendix

A Proofs of results not given in the text

A.1 Establishing the signs of the derivatives in (12) and (13)

By totally differentiating the first-order conditions of the monopolist's problem in(10) and (11), we find that

$$\frac{\partial q_i}{\partial t_i} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial t_i} - \frac{\partial^2 \pi}{\partial q_i \partial t_j}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \quad \frac{\partial q_j}{\partial t_i} = -\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial t_i} - \frac{\partial^2 \pi}{\partial q_i \partial t_i}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \; i,j = H,L, \; j \neq i.$$

where $H \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0$, $\frac{\partial^2 \pi}{\partial q_j^2} < 0$, $\frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial t_L} = 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial t_H} = \frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0$, $\frac{\partial^2 \pi}{\partial q_H \partial t_H} = \frac{\partial u_H}{\partial q_H} > 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial t_L} = -\frac{\partial u_L}{\partial q_H} < 0$. Hence, we have

$$\text{sgn}\left(\frac{\partial q_H}{\partial t_H}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial u_H}{\partial q_H}\right) < 0$$

$$\text{sgn}\left(\frac{\partial q_L}{\partial t_H}\right) = \text{sgn}\left(-\frac{\partial^2 \pi}{\partial q_H^2}\frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\right) > 0$$

$$\frac{\partial q_H}{\partial t_L} = 0$$

$$\text{sgn}\left(\frac{\partial q_L}{\partial t_L}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}\frac{\partial u_L}{\partial q_L}\right) < 0.$$

Let us now compute the derivatives of the equilibrium prices $p_H = u_H - u_{HL} + u_L$ and $p_L = u_L$ with respect to t_i, $i = H, L$. Taking into account that $\frac{\partial u}{\partial q} > 0$ and $\frac{\partial^2 u}{\partial q \partial \theta} > 0$, we have

$$\frac{\partial p_H}{\partial t_H} = \frac{\partial u_H}{\partial q_H}\frac{\partial q_H}{\partial t_H} < 0, \qquad \frac{\partial p_L}{\partial t_H} = \frac{\partial u_L}{\partial q_L}\frac{\partial q_L}{\partial t_H} > 0.$$

$$\frac{\partial p_H}{\partial t_L} = -\frac{\partial q_L}{\partial t_L}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0, \qquad \frac{\partial p_L}{\partial t_L} = \frac{\partial u_L}{\partial q_L}\frac{\partial q_L}{\partial t_L} < 0.$$

A.2 Binding constraints in problem (18)

As a first step, we show that (19) and (22) imply that (21) holds. Constraint (22) can be rewritten as $p_L \leq u(q_L, \theta_L) - \alpha_L x_L$. Setting p_L at the upper bound of this constraint gets the right hand side of (19) as close as possible to zero. Hence, if $u(q_L, \theta_H) - u(q_L, \theta_L) + \alpha_L x_L - \alpha_H x_L \geq 0$, constraint (21) must be implied by (19). Given the linearity of the problem in x_L, we can anticipate that either $x_L = 0$ or $x_L = u(q_L, \theta_L)/\alpha_L$ holds at the solution. In the former case, $u(q_L, \theta_H) - u(q_L, \theta_L) + \alpha_L x_L - \alpha_H x_L \geq 0$ is satisfied because $u(q_L, \theta_H) > u(q_L, \theta_L)$ by assumption. In the latter case, the constraint boils down to $u(q_L, \theta_H) - \alpha_H u(q_L, \theta_L)/\alpha_L \geq 0$, which is satisfied given the assumption that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L}$. Summing up, we can ignore constraint (21) and anticipate that (19) must be binding at the solution of (18).

In the second step, we show that (19) being binding implies that (20) is slack and can be ignored. Given the linearity of the problem, we can anticipate that if (19) binds, either $x_H = 0$ or $x_H = \frac{u(q_H, \theta_H) - u(q_L, \theta_H) + p_L + \alpha_H x_L}{\alpha_H}$ hold. Suppose first that $x_H = 0$, so that $p_H = u(q_H, \theta_H) - u(q_L, \theta_H) + u(q_L, \theta_L)$. Plugging these expressions in the right hand side of (20) we get after some rearrangements: $u(q_H, \theta_L) - u(q_L, \theta_L) - (u(q_H, \theta_H) - u(q_L, \theta_H))$. This expression is strictly negative by assumption, which implies that (20) is slack. Suppose now that $x_H = \frac{u(q_H, \theta_H) - u(q_L, \theta_H) + p_L + \alpha_H x_L}{\alpha_H}$ and $p_H = 0$. Plugging these expressions in (20) we get

$$u(q_L, \theta_L) - p_L - \alpha_L x_L \geq u(q_H, \theta_L) - \frac{\alpha_L}{\alpha_H}\left(u(q_H, \theta_H) - u(q_L, \theta_H) + p_L + \alpha_H x_L\right).$$

Suppose the solution is such that $x_L = 0$ and $p_L = u(q_L, \theta_L)$. The above constraint can then be written after some rearrangements as

$$0 \geq u(q_H, \theta_L) - \frac{\alpha_L}{\alpha_H}\left(u(q_H, \theta_H) - u(q_L, \theta_H) + u(q_L, \theta_L)\right).$$

The last term in brackets on the right hand side is positive. Hence, given the assumption that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L} \iff \frac{\alpha_L}{\alpha_H} > \frac{u_L}{u_{HL}}$, the constraint must hold if it holds when $\frac{\alpha_L}{\alpha_H} = \frac{u_L}{u_{HL}}$. Plugging this expression in the constraint, we have after some rearrangements that

$$\frac{u(q_H, \theta_H)}{u(q_L, \theta_H)} \geq \frac{u(q_H, \theta_L)}{u(q_L, \theta_L)},$$

which holds strictly by our assumptions on utility. Finally, suppose that the solution is such that $x_L = u(q_L, \theta_L)/\alpha_L$ and $p_L = 0$. The constraint (20) can then be written as

$$0 \geq u(q_H, \theta_L) - u(q_L, \theta_L) - \frac{\alpha_L}{\alpha_H}\left(u(q_H, \theta_H) - u(q_L, \theta_H)\right).$$

The last term in brackets on the right hand side is positive. Hence, given the assumption that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L} \iff \frac{\alpha_L}{\alpha_H} > \frac{u_L}{u_{HL}}$, the constraint must hold if it holds when $\frac{\alpha_L}{\alpha_H} = \frac{u_L}{u_{HL}}$. Plugging this expression in the constraint, we have after some rearrangements that

$$\frac{u(q_H, \theta_H)}{u(q_L, \theta_H)} \geq \frac{u(q_H, \theta_L)}{u(q_L, \theta_L)},$$

which holds strictly by our assumptions on utility.

A.3 Comparing equilibrium and welfare-optimal platform tax

To perform the comparison, assume that both the welfare function, given in (28), and the platform's revenue, $vt\left(u_H - u_{HL} + \frac{\alpha_H}{\alpha_L}u_L\right)$ are concave in t and thus admit a unique interior solution. The equilibrium level of the tax, t^e, satisfies the following first-order condition

$$\left(\frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L}\right) - \frac{\partial q_H}{\partial t}\frac{\partial u_H}{\partial q_H}\right)t^e - \left(u_H - u_{HL} + \frac{\alpha_H}{\alpha_L}u_L\right) = 0.$$

Evaluating the first-order derivative of the welfare function with respect to t,

$$\left(\frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L}\right) - \frac{\partial q_H}{\partial t}\frac{\partial u_H}{\partial q_H}\right)t - \frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L}\right),$$

in t^e, we obtain that this derivative is positive if and only if

$$\left(u_H - u_{HL} + \frac{\alpha_H}{\alpha_L} u_L\right) > \frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L}\right) \iff \frac{q_L}{t^e}\left(-t^e\left(\frac{p_H}{q_L}/\frac{\partial p_H}{\partial q_L}\right) - \frac{t^e}{q_L}\frac{\partial q_L}{\partial t}\right) > 0.$$

where the second inequality follows from noting that $\frac{\partial p_H}{\partial q_L} = -\frac{\partial u_{HL}}{\partial q_L} + \frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L} < 0$. The comparison therefore depends on the elasticity of p_H with respect to q_L, and the elasticity of q_L with respect to t.

A.4 Unit transaction tax

Suppose the seller is subject to unit taxes, denoted by τ_i, $i = H, L$. The profit function is

$$\pi = v\left(p_H + r_H x_H - (c + \tau_H) q_H\right) + (1 - v)\left(p_L + r_L x_L - (c + \tau_L) q_L\right). \tag{33}$$

The seller maximizes this function subject to (19)-(22). We find that (19) and (22) are binding, hence prices are (23). Replacing these prices in

(33), we solve it with respect to (q_L, q_H, x_L, x_H). Assuming the freemium model holds, we assume $\alpha_L - \frac{v}{1-v}(\alpha_H - \alpha_L) \leq \frac{r_L}{(1-t)}$ and $\alpha_H > \frac{r_H}{(1-t)}$ hold, implying that $x_L = \frac{u_L}{\alpha_L}$ and $x_H = 0$. Hence, profits can be rewritten as $\pi = v\left(u_H - u_{HL} + \frac{\alpha_H}{\alpha_L} u_L - (c + \tau_H) q_H\right) + (1 - v)\left(\frac{r_L}{\alpha_L} u_L - (c + \tau_L) q_L\right)$.

The equilibrium qualities solve the following system of equations

$$\frac{\partial \pi}{\partial q_H} := v\left(\frac{\partial u_H}{\partial q_H} - c - \tau_H\right) = 0, \tag{34}$$

$$\frac{\partial \pi}{\partial q_L} := v\left(\frac{\alpha_H}{\alpha_L}\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{HL}}{\partial q_L}\right) + (1 - v)\left(\frac{r_L}{\alpha_L}\frac{\partial u_L}{\partial q_L} - c - \tau_L\right) = 0. \tag{35}$$

The effect of either tax rate is thus similar to that of an increase in the cost of the respective version. As we show below, we get

$$\frac{\partial q_L^e}{\partial \tau_L} < 0, \qquad \frac{\partial q_H^e}{\partial \tau_L} = 0, \qquad \frac{\partial q_H^e}{\partial \tau_H} < 0, \qquad \frac{\partial q_L^e}{\partial \tau_H} = 0. \tag{36}$$

The effect of these taxes on consumer surplus, and welfare, can only be negative. Intuitively, similar results apply with a uniform unit tax rate, i.e., $\tau_L = \tau_H = \tau$.

Proof. By totally differentiating the first-order conditions of the monopolist's problem in(34) and (35), we find that

$$\frac{\partial q_i}{\partial \tau_i}=-\frac{\frac{\partial^2\pi}{\partial q_j^2}\frac{\partial^2\pi}{\partial q_i\partial\tau_i}-\frac{\partial^2\pi}{\partial q_i\partial\tau_j}\frac{\partial^2\pi}{\partial q_H\partial q_L}}{H},\quad \frac{\partial q_j}{\partial \tau_i}=-\frac{\frac{\partial^2\pi}{\partial q_i^2}\frac{\partial^2\pi}{\partial q_j\partial\tau_i}-\frac{\partial^2\pi}{\partial q_i\partial\tau_i}\frac{\partial^2\pi}{\partial q_H\partial q_L}}{H},\ i,j=H,L,\ j\neq i.$$

where $H\equiv\frac{\partial^2\pi}{\partial q_L^2}\frac{\partial^2\pi}{\partial q_H^2}-\left(\frac{\partial^2\pi}{\partial q_H\partial q_L}\right)^2>0,\quad \frac{\partial^2\pi}{\partial q_j^2}<0,\quad \frac{\partial^2\pi}{\partial q_i^2}<0$ by second order conditions. Moreover, $\frac{\partial^2\pi}{\partial q_H\partial q_L}=0$, $\frac{\partial^2\pi}{\partial q_H\partial\tau_L}=0$ and $\frac{\partial^2\pi}{\partial q_L\partial\tau_H}=0$, $\frac{\partial^2\pi}{\partial q_H\partial\tau_H}=-v$ and $\frac{\partial^2\pi}{\partial q_L\partial\tau_L}=-(1-v)$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial\tau_H}\right)=\operatorname{sgn}\left(\frac{\partial^2\pi}{\partial q_L^2}v\right)<0\qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial\tau_L}\right)=\operatorname{sgn}\left(\frac{\partial^2\pi}{\partial q_H^2}(1-v)\right)<0.$$

$$\frac{\partial q_L}{\partial\tau_H}=0,\frac{\partial q_H}{\partial\tau_L}=0$$

Consider now a uniform unit tax $\tau_L=\tau_H=\tau$. By totally differentiating the first-order conditions of the monopolist's problem in(34) and (35), we find that

$$\frac{\partial q_i}{\partial \tau}=-\frac{\frac{\partial^2\pi}{\partial q_j^2}\frac{\partial^2\pi}{\partial q_i\partial\tau}-\frac{\partial^2\pi}{\partial q_i\partial\tau}\frac{\partial^2\pi}{\partial q_H\partial q_L}}{H},\quad \frac{\partial q_j}{\partial \tau}=-\frac{\frac{\partial^2\pi}{\partial q_i^2}\frac{\partial^2\pi}{\partial q_j\partial\tau}-\frac{\partial^2\pi}{\partial q_i\partial\tau}\frac{\partial^2\pi}{\partial q_H\partial q_L}}{H},\ i,j=H,L,\ j\neq i.$$

where $\frac{\partial^2\pi}{\partial q_H\partial q_L}=0$, $\frac{\partial^2\pi}{\partial q_H\partial\tau}=-v$ and $\frac{\partial^2\pi}{\partial q_L\partial\tau}=-(1-v)$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial\tau}\right)=\operatorname{sgn}\left(\frac{\partial^2\pi}{\partial q_L^2}v\right)<0,$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial\tau}\right)=\operatorname{sgn}\left(\frac{\partial^2\pi}{\partial q_H^2}(1-v)\right)<0.$$

A.5 Endogenous number of consumers and sellers

Let the number of consumers and sellers that join the platform be n_c and n_s, respectively. Each seller provides a different product category, so each is a monopolist in its own category. Each consumer interacts with all sellers available on the platform. Sellers are symmetric in terms of their production cost. Therefore, each seller's profit function (conditional on joining the platform) is given by

$$\pi=n_cv\left((1-t)\,p_H+r_Hx_H-cq_H\right)+n_c\,(1-v)\left((1-t)\,p_L+r_Lx_L-cq_L\right).$$

The timing is as follows: at stage 1, the platform sets t. At stage 2, sellers and consumers decide whether to join the platform and each seller sets p_i, q_i and x_i, for $i = H, L$. At stage 3, consumers observe the features of the products available from each seller and decide which to buy, if any. We assume each seller takes n_c as given when deciding whether to join the platform and choosing the values of p_i, q_i and x_i, $i = H, L$. This is because consumers do not observe the features of the products available on the platform prior to joining (but have rational expectations). We denote the values that these variables take in equilibrium with superscript e. Consumers derive the same utility from consuming each good of each seller on the platform and observe their type only after joining. Consumer take n_s as given when deciding whether to join the platform.

To capture the fact that sellers and consumers differ in the opportunity cost of joining the platform, we assume that $n_s = \phi_s(\pi)$, where $\phi_s(.)$ is an increasing and continuously differentiable function. Denoting the surplus that each consumer expects to get when joining the platform by $E(CS)$, we assume that $n_c = \phi_c(n_s E(CS))$, where $\phi_c(.)$ is an increasing and continuously differentiable function.

We solve the model backwards. At stage 3, consumers obtain the same surplus from each seller as in the baseline model, i.e., $CS_H^e = u_{HL}^e - \frac{\alpha_H}{\alpha_L} u_L^e$ and $CS_L^e = 0$. To see why, consider that at stage 2, each seller faces the same problem as in (26), except that the total number of consumers is n_c. Since this number is taken as given, the solution is identical to (26) and we get the same values of q_i^e, p_i^e and x_i^e. Each consumer expects to obtain the surplus $n_s E(CS) = n_s(vCS_H^e + (1-v)CS_L^e) = n_s v CS_H^e$, whereas each seller gets the profit in (26), that we denote by π^e after replacing for the equilibrium values q_H^e and q_L^e (given t), and multiplied by n_c. Hence, we have $n_s = \phi_s(n_c \pi^e)$ and $n_c = \phi_c(n_s v CS_H^e)$. Starting from these expressions, we can write the following derivatives

$$\frac{\partial n_s}{\partial t} = \phi_s' n_c \frac{\partial \pi^e}{\partial t} < 0, \qquad \frac{\partial n_c}{\partial t} = \phi_c' n_s v \frac{\partial CS_H^e}{\partial t} > 0,$$

$$\frac{dn_s}{dt} = \frac{\partial n_s}{\partial t} + \phi_s' \frac{dn_c}{dt} \pi^e, \qquad \frac{dn_c}{dt} = \frac{\partial n_c}{\partial t} + \phi_c' v CS_H^e \frac{dn_s}{dt}.$$

Combining the above derivatives and rearranging, we obtain

$$\frac{dn_s}{dt} = \frac{\phi_s'\left(n_c \frac{\partial \pi^e}{\partial t} + \phi_c' n_s v \frac{\partial CS_H^e}{\partial t} \pi^e\right)}{1 - \phi_c' \phi_s' v CS_H^e \pi^e}, \qquad \frac{dn_c}{dt} = \frac{\phi_c'\left(n_s v \frac{\partial CS_H^e}{\partial t} + \phi_s' n_c \frac{\partial \pi^e}{\partial t} v CS_H^e\right)}{1 - \phi_c' \phi_s' v CS_H^e \pi^e}.$$

These derivatives show that, if the number of consumers is much less responsive than the number of sellers (i.e., $\phi_s' \to 0$), then $\frac{dn_s}{dt} \to 0$, whereas $\frac{dn_c}{dt} \to \phi_c' n_s v \frac{\partial CS_H^e}{\partial t} > 0$. By contrast, if the number of sellers is much less responsive than the number of consumers, we have $\frac{dn_c}{dt} \to 0$, whereas $\frac{dn_s}{dt} \to \phi_s' n_c \frac{\partial \pi^e}{\partial t} < 0$. Furthermore, both total derivatives can be positive, provided the magnitude of $\frac{\partial \pi^e}{\partial t}$ is small enough compared to that of $v\frac{\partial CS_H^e}{\partial t}$.

A.6 Analysis with device-selling platform and ad-selling platform

A.6.1 Device sales (Section 5.2)

We solve the model by backward induction. Consider Stage 3. Let $n \in [0,1]$ be the number of consumers who bought the device at Stage 2. At Stage 3, these consumers observe θ and select the version of the product based on the same utility function as in (17). Consider now Stage 2. Each consumer gets an expected payoff equal to $d + E(CS) - p_D$ when buying the device, where $E(CS)$ is the ex-ante expected surplus a consumer gets from the seller's products. It follows that all consumers buy the device, i.e., $n = 1$, if and only if $d + E(CS) \geq p_D$, whereas $n = 0$ otherwise. The latter scenario cannot be optimal to the platform, so we restrict attention to $p_D \leq d + E(CS)$ and $n = 1$.

Given that the share of high types, v, is the same as in the baseline model, the profit of the seller is isomorphic to (18), except that it is multiplied by n. Note that the seller takes n as given, because consumers make their decision whether to buy the device prior to observing the values of the variables p_i, q_i and x_i, for $i = H, L$. It follows that the solution to the seller's problem, given p_D and t, is the same as in the model of Section 4. Hence, at stage 3, H-type consumers get the same surplus as in (25) and their expected surplus at stage 2 is $E(CS) = v\left(u_{HL}^e - \frac{\alpha_H}{\alpha_L}u_L^e\right)$, where $u_i^e \equiv (q_i^e, \theta_i)$ and $u_{HL} \equiv u(q_H^e, \theta_L)$ and the superscript e denotes the values chosen by the seller in equilibrium (given t).

Finally, consider Stage 1. The solution to the platform's problem must be such that $p_D = d + E(CS)$. When choosing t, therefore, the platform maximises (31). Compare this expression to (27), and notice that $u_{HL}^e - \frac{\alpha_H}{\alpha_L}u_L^e$ increases with t. Hence the derivative of (31) with respect to t is everywhere greater than the derivative of (27) (recall that we focus on the $0 \leq t < \bar{t}$ interval). Thus, one can apply the results of Milgrom and Shannon (1994) to conclude that the equilibrium level of the tax must be higher when the platform sells the device than when it does not.

A.6.2 Ad sales (Section 5.2.2)

Compare the platform's profit in expression (32) to (27) and notice that $r_P(1-v)\left(\frac{u_L^e}{\alpha_L}\right)$ increases with t, while the term tvp_H^e is identical in the two expressions. Hence the derivative of (31) with respect to t is everywhere greater than the derivative of (27) (recall that we focus on $0 \leq t < \bar{t}$ interval). Thus, one can apply the results by Milgrom and Shannon (1994) to conclude that the equilibrium level of the tax must be higher when the platform sells the ads than when it does not.

A.7 Analysis with hybrid platform

The game is described in Section 5.3. At stage 3, consumers can buy the platform's product or search the seller's. Consumers of type i who search expect to get the surplus $CS_i^e - \sigma$, where σ is the search cost and CS_i^e is the surplus conditional on the equilibrium values of p_i, x_i and q_i (that the seller chooses at stage 2, given t), that we shall denote with the superscript e. Recall that consumers have a rational expectation about this surplus, but they need to search to observe the characteristics of the seller's product. The search cost is small, i.e., $\sigma \to 0$, and thus omitted in the expressions that follow. In equilibrium, no loyal consumers buy the platform's product, while the non-loyal search it if and only if $CS_i^e \geq CS_i^P$. Therefore, all consumers of type i are available to the seller if $CS_i^e \geq CS_i^P$, while only a share s is available otherwise.

Consider now stage 2. The seller chooses p_i, x_i and q_i, given t, p_i^P, x_i^P, q_i^P, and the share of consumers that is available. Non-loyal consumers do not observe the equilibrium values of p_i, x_i and q_i prior to searching, but only have a rational expectation about such values. Hence, the seller treats the shares of consumers that are available as given when choosing these variables (since the loyal consumers only buy the seller's product by definition). Let these shares be S_H and S_L among, respectively, high- and low-type consumers. We have $S_H = vs$ if $CS_H^e < CS_H^P$, and $S_H = v$ otherwise. Similarly, $S_L = s(1-v)$ if $CS_L^e < CS_L^P$, and $S_L = 1-v$ otherwise. The constraints faced by the seller are the same as in the baseline model (i.e., (19)-(22)). Given the same the values of r_i, the seller adopts the freemium pricing scheme, with prices set as in expression (23). Specifically, $p_H = u_H - u_{HL} + \frac{\alpha_H}{\alpha_L} u_L$ and $x_L = u_L r_L / \alpha_L$ must hold, where $u_i \equiv (q_i, \theta_i)$ and $u_{HL} \equiv u(q_H, \theta_L)$. Hence, the seller's problem reduces to

$$\max_{q_H, q_L} \quad \pi = S_H \left((1-t) \left(u_H - u_{HL} + \frac{\alpha_H}{\alpha_L} u_L \right) - c q_H \right) + S_L \left(\frac{u_L r_L}{\alpha_L} - c q_L \right). \tag{37}$$

Note that if $S_H = vs$ and $S_L = (1-v)s$, or if $S_H = v$ and $S_L = (1-v)$, the objective is isomorphic (up to a multiplicative constant) to (26), so the two problems must have the same solution. In words, the seller faces the same problem as in the baseline model when either all consumers or only the captive ones search.

Whenever $t < \bar{t}$, the seller serves both consumer types, high-type consumers get a surplus $CS_H^e = u_{HL}^e - \frac{\alpha_H}{\alpha_L} u_L^e$ in equilibrium, where $u_i^e \equiv (q_i^e, \theta_i)$ and so on, whereas low type consumers get $CS_L^e = 0$. If $t \geq \bar{t}$, the seller only sells a single version of its product, targeting the high-types and sets $p_H = u_H$, so that $CS_H^e = CS_L^e = 0$. Consumers would of course obtain the same levels of expected surplus if t was so large that the seller simply dropped out of the market. Observe that the solution to the seller's problem only depends on the platform's decisions at stage 1 through t and the surpluses CS_i^P (which affect the shares S_H and S_L).

Focus now on stage 1. We assume that the platform wants to serve all consumer types with its product. The platform's problem is therefore

$$\max_{t,q_H^P,p_H^P,x_H^P,q_L^P,p_L^P,x_L^P} \quad \pi_P = (1-S_H)\left(p_H^P + r_H x_H^P - c q_H^P\right) + (1-v)(1-S_L)\left(p_L^P + r_L x_L^P - c q_L^P\right) + t S_H p_H.$$

with $S_H = vs$ and $S_L = (1-v)s$ (i.e., only the loyal consumers buy from the seller). The platform must satisfy the following constraints

$$u\left(q_H^P,\theta_H\right) - p_H^P - \alpha_H x_H^P \geq u\left(q_L^P,\theta_H\right) - p_L^P - \alpha_H x_L^P, \tag{38}$$

$$u\left(q_L^P,\theta_L\right) - p_L^P - \alpha_L x_L^P \geq u\left(q_H^P,\theta_L\right) - p_H^P - \alpha_L x_H^P, \tag{39}$$

$$u\left(q_H^P,\theta_H\right) - p_H^P - \alpha_H x_H^P \geq \max\left(0, CS_H^e\right), \tag{40}$$

$$u\left(q_L,\theta_L\right) - p_L - \alpha_L x_L \geq \max\left(0, CS_L^e\right). \tag{41}$$

The first two constraints are incentive compatibility constraints. The last two constraints are participation constraints: each (non-loyal) consumer type must receive at least the surplus it can expect to get by searching the third-party seller's product. Following standard procedures, and noting that $\max\left(0, CS_H^e\right) = CS_H^e$, while $\max\left(0, CS_L^e\right) = 0$, we have

$$\begin{aligned} p_H^P + \alpha_H x_H^P &= \min\left(u_H^P - u_{HL}^P + u_L^P + \alpha_H x_L^P - \alpha_L x_L^P, CS_H^e\right), \\ p_L^P + \alpha_L x_L^P &= u_L^P, \end{aligned}$$

where $u_i^P \equiv \left(q_i^P,\theta_i\right)$ and $u_{HL}^P \equiv u\left(q_H^P,\theta_L\right)$. Note that in this setting the incentive compatibility constraint for the H-type is not necessarily binding in equilibrium, because the third-party seller's product tightens the participation constraints. Assuming that $x_H^P = 0$ and $x_L^P = u_L^P/\alpha_L$ as in the baseline model, we have

$$\begin{aligned} p_H^P &= \min\left(u_H^P - u_{HL}^P + \frac{\alpha_H}{\alpha_L} u_L^P, u_H^P - CS_H^e\right), \\ \alpha_L x_L^P &= u_L^P. \end{aligned} \tag{42}$$

Suppose that the H-type's incentive compatibility constraint binds, that is, that $u_H^P - u_{HL}^P + \frac{\alpha_H}{\alpha_L} u_L^P \leq u_H^P - CS_H^e$, so that $p_H^P = u_H^P - u_{HL}^P + \frac{\alpha_H}{\alpha_L} u_L^P$. The platform's problem would then reduce to

$$\max_{t,q_H^P,q_L^P} \quad (1-s)\left(v\left(u_H^P - u_{HL}^P + \frac{\alpha_H}{\alpha_L} u_L^P - c q_H^P\right) + (1-v)\left(\frac{u_L^P r_L}{\alpha_L} - c q_L^P\right)\right) + stvp_H^e.$$

Since p_H^e does not depend on q_i^P, the pair $\left(q_{H0}^P, q_{L0}^P\right)$ that solves this problem must be the same as the pair solving (37) when $t=0$ (and $S_H = vs$ and $S_L = (1-v)s$ hold). Hence, conditional on $t=0$, the surplus of the high types, $u_{HL0}^P - \frac{\alpha_H}{\alpha_L} u_{L0}^P$ equals CS_H^e. However, by Proposition 3, CS_H^e increases with t, for any $0 \le t < \bar{t}$. Hence, for any $0 < t < \bar{t}$, we have $u_{HL0}^P - \frac{\alpha_H}{\alpha_L} u_{L0}^P < CS_H^e$, so $u_H^P - u_{HL}^P + \frac{\alpha_H}{\alpha_L} u_L^P > u_H^P - CS_H^e$ must hold. That is, the H-type's participation constraint binds. Finally, when $t \ge \bar{t}$, $CS_H^e = 0$, so $u_H^P - u_{HL}^P + \frac{\alpha_H}{\alpha_L} u_L^P \le u_H^P - CS_H^e$ must hold. This is because the seller only serves the H-types in this case, and extract all their surplus.

Summing up, we can write the platform's problem as

$$\max_{q_H^P, q_L^P, t} \quad \pi_P = \begin{cases} (1-s)\left(v\left(u_H^P - CS_H^e - cq_H^P\right) + (1-v)\left(\frac{u_L^P r_L}{\alpha_L} - cq_L^P\right)\right) + stvp_H & \text{if } 0 \le t < \bar{t}, \\ (1-s)\left(v\left(u_H^P - u_{HL}^P + \frac{\alpha_H}{\alpha_L} u_L^P - cq_H^P\right) + (1-v)\left(\frac{u_L^P r_L}{\alpha_L} - cq_L^P\right)\right) + stvp_H & \text{if } t \ge \bar{t}. \end{cases} \tag{43}$$

Let us first focus on the case where $0 \le t < \bar{t}$. Comparing the platform's profit in expression (43) to (27), and noting that CS_H^e increases with t, while $\left(q_H^P, q_L^P\right)$ do not depend on it. Note also that the term tvp_H^e is identical in the two expressions, for any t. Hence the derivative of (43) with respect to t is everywhere smaller than the derivative of (27). Thus, one can apply the results by Milgrom and Shannon (1994) to conclude that the equilibrium level of the tax must be smaller when the platform sells its own product than when it does not (again, conditional on the solution being such that $0 \le t < \bar{t}$).

However, we cannot exclude the possibility that the platform prefers a tax such that $t \ge \bar{t}$ when selling its own product, and a tax such that $0 \le t < \bar{t}$ when it is a pure marketplace. This is because there is a discrete increase in the revenue from selling the product when t reaches the level $\bar{t}$, compared to when $0 < t < \bar{t}$, as established above. Nevertheless, by setting $t \ge \bar{t}$ the platform already ensures that no consumer gets a positive surplus when buying from the seller, because the latter only serves the high types and captures all their surplus (setting $p_H = u_H$), so that $CS_H^e = 0$. Therefore setting t to a level such that the seller makes zero net profit, and exits the market, cannot be optimal: the platform would then earn the same profit from the sale of its product as in the second row of (43), but forgo the tax revenue $tvsp_H^e$.

B Other tax instruments

We briefly study the effects of unit taxes and of a uniform ad valorem tax rate applied to all versions in the basic model of Section 3.

B.1 Unit taxes

Suppose the seller is subject to unit taxes, denoted by τ_i, $i = H, L$. The profit function is

$$\pi = v\left(p_H - (c + \tau_H)\, q_H\right) + (1 - v)\left(p_L - (c + \tau_L)\, q_L\right). \tag{44}$$

The seller maximizes this function subject to (2)-(5). The equilibrium qualities solve the following system of equations

$$\frac{\partial \pi}{\partial q_H} := v\left(\frac{\partial u_H}{\partial q_H} - c - \tau_H\right) = 0, \tag{45}$$

$$\frac{\partial \pi}{\partial q_L} := v\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{HL}}{\partial q_L}\right) + (1 - v)\left(\frac{\partial u_L}{\partial q_L} - c - \tau_L\right) = 0. \tag{46}$$

The effect of either tax rate is thus similar to that of an increase in the cost of the respective version. As we show below, we get

$$\frac{\partial q_L^e}{\partial \tau_L} < 0, \qquad \frac{\partial q_H^e}{\partial \tau_L} = 0, \qquad \frac{\partial q_H^e}{\partial \tau_H} < 0, \qquad \frac{\partial q_L^e}{\partial \tau_H} = 0. \tag{47}$$

The effect of these taxes on consumer surplus, and welfare, can only be negative. Intuitively, similar results apply with a uniform unit tax rate, i.e., $\tau_L = \tau_H = \tau$.

Proof. We solve the seller's problem following the same steps as in Section 3: constraints (2) and (5) are binding, meaning that equilibrium prices are as given in (7). Replacing these prices in (44), and using again the shorthand notation $u_i \equiv u(q_i, \theta_i)$, $i = H, L$, and $u_{HL} \equiv u(q_L, \theta_H)$, we get

$$\pi = v\left(u_H + u_{HL} - u_L - (c + \tau_H)\, q_H\right) + (1 - v)\left(u_L - (c + \tau_L)\, q_L\right). \tag{48}$$

By totally differentiating the first-order conditions of the monopolist's problem in(45) and (46), we find that

$$\frac{\partial q_i}{\partial \tau_i} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial \tau_i} - \frac{\partial^2 \pi}{\partial q_i \partial \tau_j}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \qquad \frac{\partial q_j}{\partial \tau_i} = -\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial \tau_i} - \frac{\partial^2 \pi}{\partial q_i \partial \tau_i}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \; i, j = H, L, \; j \neq i.$$

where $H \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0$, $\frac{\partial^2 \pi}{\partial q_j^2} < 0$, $\frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau_L} = 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau_H} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau_H} = -v$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau_L} = -(1 - v) < 0$. Hence, we have

$$\text{sgn}\left(\frac{\partial q_H}{\partial \tau_H}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2} v\right) < 0$$

$$\frac{\partial q_L}{\partial \tau_H} = 0, \frac{\partial q_H}{\partial \tau_L} = 0$$

$$\text{sgn}\left(\frac{\partial q_L}{\partial \tau_L}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}(1-v)\right) < 0.$$

Consider now a uniform unit tax $\tau_L = \tau_H = \tau$. By totally differentiating the first-order conditions of the monopolist's problem in(45) and (46), we find that

$$\frac{\partial q_i}{\partial \tau} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial \tau} - \frac{\partial^2 \pi}{\partial q_i \partial \tau}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \quad \frac{\partial q_j}{\partial \tau} = -\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial \tau} - \frac{\partial^2 \pi}{\partial q_i \partial \tau}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \; i,j = H,L, \; j \neq i.$$

where $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau} = -v$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau} = -(1-v)$. Hence, we have

$$\text{sgn}\left(\frac{\partial q_H}{\partial \tau}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2} v\right) < 0,$$

$$\text{sgn}\left(\frac{\partial q_L}{\partial \tau}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}(1-v)\right) < 0.$$

B.2 Uniform ad valorem tax

Consider now a uniform ad valorem tax on the two bundles, i.e., $t_H = t_L = t$. It can be shown (see proof below) that

$$\frac{\partial q_L^e}{\partial t} < 0, \qquad \frac{\partial q_H^e}{\partial t} < 0. \tag{49}$$

To grasp the intuition, replace $t_H = t_L = t$ in the first-order conditions of the seller's problem, (10) and (11), and divide both expressions by $1-t$, to obtain

$$\frac{\partial \pi}{\partial q_H} := \frac{\partial u_H}{\partial q_H} - \frac{c}{1-t} = 0, \tag{50}$$

$$\frac{\partial \pi}{\partial q_L} := \left(v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L}\right) + (1-v)\frac{\partial u_L}{\partial q_L}\right) - (1-v)\frac{c}{1-t} = 0. \tag{51}$$

The way the tax affects the seller's decision is again tantamount to an increase in the cost of production, with a negative effect on consumer surplus and welfare. However, it is important to note that these conclusions hold under the assumption that both versions of the good are sold at a positive (monetary) price, unlike in the freemium scenario we consider in Section 4.

Proof. Consider profits in (1) with $t_L = t_H = t$. The first-order conditions of the monopolist's problem are

$$\frac{\partial \pi}{\partial q_H} := \frac{\partial u_H}{\partial q_H}(1-t_H) - c = 0, \tag{52}$$

$$\frac{\partial \pi}{\partial q_L} := v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L}\right)(1-t_H) + (1-v)\left(\frac{\partial u_L}{\partial q_L}(1-t_L) - c\right) = 0. \tag{53}$$

By totally differentiating the above first-order conditions of the monopolist's problem with respect to a uniform tax t, we find that

$$\frac{\partial q_i}{\partial t} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial t} - \frac{\partial^2 \pi}{\partial q_i \partial t}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H},$$

where $H \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0$, $\frac{\partial^2 \pi}{\partial q_j^2} < 0$, $\frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial t} = -v\frac{\partial u_H}{\partial q_H} < 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial t} = -\left(\frac{\partial u_L}{\partial q_L} - v\frac{\partial u_{HL}}{\partial q_L}\right) < 0$. Hence,

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H \partial t}\right) = \operatorname{sgn}\left(-v\frac{\partial u_H}{\partial q_H}\right) < 0,$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial^2 \pi}{\partial q_H^2}\frac{\partial^2 \pi}{\partial q_L \partial t}\right) = \operatorname{sgn}\left(-\left(\frac{\partial u_L}{\partial q_L} - v\frac{\partial u_{HL}}{\partial q_L}\right)\right) < 0.$$

Furthermore, the introduction of a small ad valorem tax has negative effects on welfare

$$\left.\frac{\partial W}{\partial t}\right|_{t=0} = \frac{\partial q_H}{\partial t}v\left(\frac{\partial u_H}{\partial q_H} - c\right) + \frac{\partial q_L}{\partial t}(1-v)\left(\frac{\partial u_L}{\partial q_L} - c\right) < 0.$$

C Robustness checks (online)

C.1 More than two types

We consider a setting with three types of consumers. Specifically, we assume the preference parameter θ can take three different values, $\theta \in \{\theta_H, \theta_M, \theta_L\}$, with $\theta_H > \theta_M > \theta_L$. At equilibrium, only the incentive compatibility constraints ensuring that a higher type does not mimic the type immediately below are binding, while the participation constraint is binding for the L-type. As a result, the seller distorts the quantity (or quality) of the bundle intended for all types except the highest one. Types H and M receive an information rent, unlike L-type consumers.

We find that an ad valorem tax applied to the H-version reduces the distortion on the M- and L-versions, by inducing the seller to increase q_i for both types. Furthermore, the tax on the H-version reduces the price of that version, but raises the price of the L-version (the effect on the price of the M-version is ambiguous). The logic behind this result is the same as above: the tax makes collecting revenue from the H-types less attractive to the seller, and so reduces the incentive to relax their incentive compatibility constraint. There is also a knock-on effect on the L-version: its price increases, but q_L increases. The reason is that an increase in q_M relaxes the incentive compatibility constraint that applies to the M-type and, hence, reduces the need to distort q_L for the seller. The resulting effect on welfare is positive.

Proof. We assume there are three types of consumers, characterized by the preference parameter $\theta \in \{\theta_H, \theta_M, \theta_L\}$, with $\theta_H > \theta_M > \theta_L$. Let v_H, v_M and v_L be the shares of consumers of type H, M and L, respectively, with $v_H + v_M + v_L = 1$. Furthermore, to avoid "bunching" of types we assume that $\frac{v_L}{v_M} < \frac{v_L + v_M}{v_H}$, i.e., that the distribution of types satisfies the monotone hazard rate property (Laffont and Martimort, 2002, p.90). The model is otherwise identical to our baseline setup.

The seller offers to consumers three bundles, (q_i, p_i), each intended for one type. These bundles must satisfy six incentive constraints (two for each type)

$$u(q_i, \theta_i) - p_i \geq u(q_j, \theta_i) - p_j, \quad i, j = L, M, H \quad i \neq j,$$

and three participation constraints (one per each type)

$$u(q_i, \theta_i) - p_i \geq 0, \quad i = L, M, H.$$

Following standard steps (Laffont and Martimort, 2002), one can show that, in equilibrium, there are two binding incentives constraints (the ones such that a higher type want to mimic a lower type) and one binding participation constraint (the one of low types). From these binding constraints we derive the equilibrium prices. Hence, the seller maximizes the following problem

$$\max_{(q_i,p_i)} \quad \pi = \sum_{i=L,M,H} v_i\left(\left(1-t_i\right)p_i - cq_i\right), \tag{54}$$

$$s.t. \quad p_H = u_H + u_M + u_L - u_{ML} - u_{HM}, \tag{55}$$

$$p_M = u_M + u_L - u_{ML}, \tag{56}$$

$$p_L = u_L, \tag{57}$$

where $u_i \equiv u\left(q_i, \theta_i\right)$ for each $i = L,M,H$, and $u_{ij} \equiv u\left(q_j, \theta_i\right)$ for each $i,j = L,M,H$ with $i \neq j$. Hence, we derive the following first-order conditions

$$\frac{\partial \pi}{\partial q_H} := v_H\left(\frac{\partial u_H}{\partial q_H}\left(1-t_H\right) - c\right) = 0, \tag{58}$$

$$\frac{\partial \pi}{\partial q_M} := v_H\left(\frac{\partial u_M}{\partial q_M} - \frac{\partial u_{HM}}{\partial q_M}\right)\left(1-t_H\right) + v_M\left(\frac{\partial u_M}{\partial q_M}\left(1-t_M\right) - c\right) = 0, \tag{59}$$

$$\frac{\partial \pi}{\partial q_L} := v_H\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right)\left(1-t_H\right) + v_M\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right)\left(1-t_M\right) + v_L\left(\frac{\partial u_L}{\partial q_L}\left(1-t_L\right) - c\right) = 0. \tag{60}$$

Totally differentiating the above equations and taking into account that cross-profits derivatives are zero ($\frac{\partial^2 \pi}{\partial q_i \partial q_j} = 0$ for $i,j = L,M,H$ with $i \neq j$), we find that

$$\frac{\partial q_H}{\partial t_H} = -\frac{\begin{vmatrix} \frac{\partial^2 \pi}{\partial q_L^2} & \frac{\partial^2 \pi}{\partial q_L \partial q_M} & \frac{\partial^2 \pi}{\partial q_L \partial t_H} \\ \frac{\partial^2 \pi}{\partial q_M \partial q_L} & \frac{\partial^2 \pi}{\partial q_M^2} & \frac{\partial^2 \pi}{\partial q_M \partial t_H} \\ \frac{\partial^2 \pi}{\partial q_H \partial q_L} & \frac{\partial^2 \pi}{\partial q_H \partial q_M} & \frac{\partial^2 \pi}{\partial q_H \partial t_H} \end{vmatrix}}{H} = -\frac{\frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_M^2}\frac{\partial^2 \pi}{\partial q_H \partial t_H}}{H} \leq 0,$$

where H is the determinant of the Hessian matrix, which is negative by second order conditions, $\frac{\partial^2 \pi}{\partial q_i^2} < 0$ for $i = L,M,H$ also by second order conditions, and $\frac{\partial^2 \pi}{\partial q_H \partial t_H} = -v_H \frac{\partial u_H}{\partial q_H} < 0$. Following similar steps, we find that the derivatives of q_M and q_L with respect to t_H are, respectively, such that

$$\operatorname{sgn}\left(\frac{\partial q_M}{\partial t_H}\right) = \operatorname{sgn}\left(-v_H\left(\frac{\partial u_M}{\partial q_M} - \frac{\partial u_{HM}}{\partial q_M}\right)\right) \geq 0, \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial t_H}\right) = \operatorname{sgn}\left(-v_H\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right)\right) \geq 0 \ .$$

These signs follow from the assumption that $\frac{\partial^2 u}{\partial q \partial \theta} > 0$. This establishes that the effect of the ad valorem tax applied to the H-bundle is such that the quantity of the other two bundles increases, reducing the distortion applied by the seller.

Similarly, the derivatives of the equilibrium quantities with respect to t_M and t_L are such that

$$\frac{\partial q_H}{\partial t_M}=0, \qquad \operatorname{sgn}\left(\frac{\partial q_M}{\partial t_M}\right)=\operatorname{sgn}\left(-v_M\frac{\partial u_M}{\partial q_M}\right)\leq 0, \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial t_M}\right)=\operatorname{sgn}\left(-v_M\left(\frac{\partial u_L}{\partial q_L}-\frac{\partial u_{ML}}{\partial q_L}\right)\right)\geq 0,$$

$$\frac{\partial q_H}{\partial t_L}=0, \qquad \frac{\partial q_M}{\partial t_L}=0, \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial t_L}\right)=\operatorname{sgn}\left(-v_L\frac{\partial u_L}{\partial q_L}\right)\leq 0.$$

C.2 Duopoly

We consider a setting with two horizontally differentiated and symmetric sellers. In accordance with previous literature (Spulber, 1989; Stole, 2007), we assume there is perfect correlation between a consumers' preference for a seller and the marginal utility from consuming the good. Specifically, we consider two "high" consumer types, each with a strict preference for one of the two sellers (i.e., these consumers get zero utility from consuming the good supplied by the other). Moreover, we consider two "low" types that have a weak preference for one of the sellers (i.e., they get a smaller, but positive, utility from consuming from the least preferred one). Compared to the high types, the low types get a smaller marginal utility from consuming the good from their preferred vendor.

We show that the bundles offered by the duopolists in equilibrium are similar to those offered by a monopolist. Specifically, each seller serves only the two types (high and low) that have a preference for its good, and offers a distorted bundle to the low types. Unlike in a monopoly, each seller must leave the low types with some surplus to avoid such types switching to the rival. As a result, the price charged to the high types must decrease as well. Therefore, competition forces the sellers to extract less surplus from consumers overall. However, the equilibrium values of q_i satisfy the same first-order conditions as in (10) and (11). Hence, the effects of taxation in this setting are the same as in the monopoly setting.

Proof. We consider two symmetric sellers, indexed by $s\in\{1,2\}$ and four consumer types, indexed by $i\in\{H_1,L_1,H_2,L_2\}$, differing in (i) their intensity of preferences for the good and (ii) their preference for the two sellers. The utility when buying from seller s is $u_s(q,\theta_i)-p$, where p is the price and θ_i is the preference parameter. Let v_i be the share of consumers of type i, with $\sum_{i=H_1,L_1,H_2,L_2}v_i=1$, and assume that each consumer buys from at most one seller. We assume the utility function satisfies the following conditions:

$$u_1(q,\theta_{H_1})>u_1(q,\theta_{L_1})>u_1(q,\theta_{L_2})>u_1(q,\theta_{H_2})=0, \quad \forall q>0,$$

$$u_2(q,\theta_{H_2}) > u_2(q,\theta_{L_2}) > u_1(q,\theta_{L_1}) > u_1(q,\theta_{H_1}) = 0, \quad \forall q > 0,$$
$$\frac{\partial u_1}{\partial q}(q,\theta_{H_1}) > \frac{\partial u_1}{\partial q}(q,\theta_{L_1}) > \frac{\partial u_1}{\partial q}(q,\theta_{L_2}) > \frac{\partial u_1}{\partial q}(q,\theta_{H_2}) = 0, \quad \forall q > 0,$$
$$\frac{\partial u_2}{\partial q}(q,\theta_{H_2}) > \frac{\partial u_2}{\partial q}(q,\theta_{L_2}) > \frac{\partial u_2}{\partial q}(q,\theta_{L_1}) > \frac{\partial u_2}{\partial q}(q,\theta_{H_1}) = 0, \quad \forall q > 0.$$

These conditions imply a perfect correlation between the preference for one seller and the intensity of preference for the good it supplies (Spulber, 1989). For simplicity, we assume only the "low" types are willing to buy from either seller, whereas the "high" types do not get any utility from buying from their least preferred seller.

Let (q_i, p_i) denote the bundle that a seller proposes to consumers of type i. Given the condition that consumers self-select on the intended bundle, there is no loss in proceeding under the assumption that seller 1 only offers bundles intended for the couple of consumer types that prefer its product, i.e., H_1 and L_1, whereas seller 2 only serves H_2 and L_2. We are now going to state the constraints that the sellers face regarding each type of consumer. Considering a seller s, we have the following incentives and participation constraints that apply to the H_s-bundle:

$$u_s(q_{H_s},\theta_{H_s}) - p_{H_s} \geq u_s(q_{L_s},\theta_{H_s}) - p_{L_s},\ s = 1,2, \tag{61}$$

$$u_s(q_{H_s},\theta_{H_s}) - p_{H_s} \geq u_{s'}\left(q_{L_{s'}},\theta_{H_s}\right) - p_{L_{s'}},\ s,s' = 1,2,\ s' \neq s, \tag{62}$$

$$u_s(q_{H_s},\theta_{H_s}) - p_{H_s} \geq u_{s'}\left(q_{H_{s'}},\theta_{H_s}\right) - p_{H_{s'}},\ s,s' = 1,2,\ s' \neq s, \tag{63}$$

$$u_s(q_{H_s},\theta_{H_s}) - p_{H_s} \geq 0,\ s = 1,2. \tag{64}$$

Constraint (61) must hold in order for H_s types not to choose the bundle offered to L_s consumers by the same seller . The next two constraints, (62) and (63), must hold to avoid that H_s types buy any of the bundles offered by the other seller, s'. Finally, (64) must hold for H_s types to prefer the bundle intended for them to not participating in the market at all.

Symmetrically, the constraints that apply to the L_s-bundle are as follows

$$u_s(q_{L_s},\theta_{L_s}) - p_{L_s} \geq u_s(q_{H_s},\theta_{L_s}) - p_{H_s},\ s = 1,2, \tag{65}$$

$$u_s(q_{L_s},\theta_{L_s}) - p_{L_s} \geq u_{s'}\left(q_{L_{s'}},\theta_{L_s}\right) - p_{L_{s'}},\ s,s' = 1,2,\ s' \neq s, \tag{66}$$

$$u_s(q_{L_s},\theta_{L_s}) - p_{L_s} \geq u_{s'}\left(q_{H_{s'}},\theta_{L_s}\right) - p_{H_{s'}},\ s,s' = 1,2,\ s' \neq s, \tag{67}$$

$$u_s(q_{L_s},\theta_{L_s}) - p_{L_s} \geq 0,\ s = 1,2. \tag{68}$$

Constraint (65) must hold in order for L_s types not to choose the bundle offered to H_s consumers by seller s. The next two constraints, (62) and (63), must hold to avoid that L_s types buy from the other seller. Finally, (64) must hold for L_s types to prefer the bundle intended for them to not participating in the market at all.

Given the differentiated ad valorem tax rates we consider in the baseline setting, the problem of seller s is

$$\max_{q_{H_s}, p_{H_s}, q_{L_s}, p_{L_s}} \quad \pi = v_{H_s}\left[(1-t_H)\, p_{H_s} - c q_{H_s}\right] + v_{L_s}\left[(1-t_L)\, p_{L_s} - c q_{L_s}\right],\ s = 1,2, \tag{69}$$

subject to constraints (61)-(68).

We are now going to solve seller s's problem characterized above, focusing on symmetric equilibria. Our first step is to establish which constraints are going to be binding in equilibrium to determine equilibrium prices. Given $u_{s'}\left(q_{L_{s'}}, \theta_{H_s}\right) = u_{s'}\left(q_{H_{s'}}, \theta_{H_s}\right) = 0$, constraints (62) and (63) cannot be binding, because of the participation constraints in (64). Furthermore, given (68), and that $u_s\left(q_{L_s}, \theta_{H_s}\right) > u_s\left(q_{L_s}, \theta_{L_s}\right)$, constraint (64) cannot be binding either. Hence, the equilibrium must be such that (61) is binding. We have

$$p_{H_s} = p_{L_s} + u_s\left(q_{H_s}, \theta_{H_s}\right) - u_s\left(q_{L_s}, \theta_{H_s}\right),\ s = 1,2. \tag{70}$$

Given (70), we can write the constraints (65), after some rearrangements, as

$$u_s\left(q_{H_s}, \theta_{H_s}\right) - u_s\left(q_{L_s}, \theta_{H_s}\right) \geq u_s\left(q_{H_s}, \theta_{L_s}\right) - u_s\left(q_{L_s}, \theta_{L_s}\right),\ s = 1,2,$$

which must hold strictly by the assumption that $\frac{\partial u_s}{\partial q}\left(q, \theta_{H_s}\right) > \frac{\partial u_s}{\partial q}\left(q, \theta_{L_s}\right)$. Hence, these constraints cannot be binding. Consider now the constraints (67). These can be rewritten, using (70) and after a few rearrangements as

$$u_{s'}\left(q_{H_{s'}}, \theta_{H_{s'}}\right) - u_{s'}\left(q_{L_{s'}}, \theta_{H_{s'}}\right) - p_{L_s} \geq u_{s'}\left(q_{H_{s'}}, \theta_{L_s}\right) - u_s\left(q_{L_s}, \theta_{L_s}\right) - p_{L_{s'}},\ s = 1,2.$$

In a symmetric equilibrium (where $p_{L_s} = p_{L_{s'}}$ and $q_{L_s} = q_{L_{s'}}$), this inequality must hold strictly by the assumption that $\frac{\partial u_{s'}}{\partial q}\left(q, \theta_{H_{s'}}\right) > \frac{\partial u_s}{\partial q}\left(q, \theta_{L_s}\right) > \frac{\partial u_{s'}}{\partial q}\left(q, \theta_{L_s}\right)$. Therefore, the only constraints that can be binding are (66) and (68). We have

$$p_{L_s} = u_s\left(q_{L_s}, \theta_{L_s}\right) - max\left(0, u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - p_{L_{s'}}\right)\ s, s' = 1,2, s' \neq s. \tag{71}$$

Given (70) and (71), we can therefore write the the problem of seller s as

$$\max_{q_{H_s},q_{L_s}} \quad \pi_s = v_{H_s}\left[(1-t_H)\left(u_s\left(q_{L_s},\theta_{L_s}\right) - max\left(0, u_{s'}\left(q_{L_{s'}},\theta_{L_s}\right) - p_{L_{s'}}\right) + u_s\left(q_{H_s},\theta_{H_s}\right) - \left(q_{L_s},\theta_{H_s}\right)\right) - cq_{H_s}\right] \tag{72}$$
$$+v_{L_s}\left[(1-t_L)\left(u_s\left(q_{L_s},\theta_{L_s}\right) - max\left(0, u_{s'}\left(q_{L_{s'}},\theta_{L_s}\right) - p_{L_{s'}}\right)\right) - cq_{L_s}\right], \quad s,s' = 1,2,s' \neq s.$$

Observe that $u_{s'}\left(q_{L_{s'}},\theta_{L_s}\right) - p_{L_{s'}}$ does not depend on q_{H_s} nor on q_{L_s}. The first-order conditions of this problem are

$$\frac{\partial \pi}{\partial q_{H_s}} := \frac{\partial u_s\left(q_{H_s},\theta_{H_s}\right)}{\partial q_{H_s}}(1-t_H) - c = 0 \; s = 1,2, \tag{73}$$

$$\frac{\partial \pi}{\partial q_{L_s}} := v_{H_s}\left(-\frac{\partial u_s\left(q_{L_s},\theta_{H_s}\right)}{\partial q_{L_s}} + \frac{\partial u_s\left(q_{L_s},\theta_{L_s}\right)}{\partial q_{L_s}}\right)(1-t_H) + v_{L_s}\left(\frac{\partial u_s\left(q_{L_s},\theta_{L_s}\right)}{\partial q_{L_s}}(1-t_L) - c\right) = 0 \; s = 1,2. \tag{74}$$

The key observation is that these equations have the same form as (10) and (11), which implies that the effects of taxation must be also be the same, and so are the implications for optimal policy.