Imperfect Targeting and Advertising Strategies[*]

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November 7, 2023

Abstract

The ability to successfully target consumers has been substantially affected by recent developments in digital markets, such as improvements in tracking technologies or GDPR regulation. In this paper, we set up a game-theoretic model to examine the implications of such changes in targeting success on firms' targeting strategies and profits. We explicitly consider that firms can target different consumer groups—i.e., high- or low-valuation consumers—and that targeting is imperfect—i.e., a firm may not reach the intended consumer. We find that a higher targeting success rate has a non-monotone effect on firms' profits—i.e., lowering profits if targeting is rather imprecise, but raising profits if targeting is relatively accurate. If the probability of successful targeting is low, firms target high-valuation consumers (*competition for cherries*). More fine-tuned targeting then amplifies competition and decreases profits. Instead, if targeting is sufficiently precise, more fine-tuned targeting increases profits because firms *segment the market* by targeting different consumer groups. Improvements in the targeting ability also have profound effects on the number of products a firm offers and on the profitability of offering products that appeal not only to a single consumer group. First, although increased targeting success raises the attractiveness of introducing an additional product, it also heats up competition. A firm may therefore optimally reduce the number of products it sells if targeting success increases. Second, if products have a broader appeal, market segmentation is more likely to occur. However, aggregate profits are lower compared to when products are narrow.

Keywords: Targeted advertising, price competition, market segmentation, multi-product firms.

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[*]We are grateful to Nadia Abou Nabout, Paul Belleflamme, Emilio Calvano, Siham El Kihal, Madhav Kumar, José-Luis Moraga González, David Myatt, Johannes Schneider, Nicolas Schutz as well as seminar audiences at the Universities of Bergamo, Bern, Carlos III, Dublin, LUISS Rome, and Toulouse as well as at the IBEO Workshop at Alghero, the Mannheim ICT Conference, and the MaCCI Summer Institute 2021 for their valuable comments and suggestions.

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1 Introduction

The growth of internet usage and the possibility to collect data about users' browsing behavior enable firms to obtain information about consumer preferences at relatively precise levels. This has implications for advertising as it allows targeting of advertising messages in a fine-tuned way.[1] Several recent developments in digital markets have affected this targeting ability, both in a positive and a negative way. For instance, the increased use of artificial intelligence allows firms to analyze and interpret massive amounts of data in a quicker and more accurate way, thereby helping to identify patterns in user data and improving the prediction whether a user will be interested in a product.[2] On the other hand, many browser suppliers, such as Google with its browser Chrome or Mozilla with Firefox, decided to remove third-party cookies or have already done so.[3] This shrinks firms' ability to track users and therefore limits the information they obtain about consumer preferences. Similarly, the introduction of the European Union's General Data Protection Regulation of 2018 or the California Consumer Privacy Act of 2020 lowered the amount of data collection to be used for advertising targeting.[4]

Although firms' targeting ability has substantially improved through the internet, it is generally far from perfect. For instance, a report by Nielsen (2019) provides measures showing that e.g. in the UK in 2018 around 37% of advertising messages do not reach the right audience, and this number varies considerable among industries.[5] A key question for firms is therefore how such changes in advertising success affect their targeting strategy and the profitability of advertising.

An obvious benefit resulting from improved targeting is that advertising is more effective, as it leads to a higher reach and increased conversion rates. At the same time, the targeting ability improves for all firms, which implies that also competitors are more likely to reach the targeted audience, and users are therefore exposed to messages of competing firms with a higher probability. This leads to increased price competition.

Motivated by the recent developments and the countervailing forces of improved targeting, the objective of this paper is to study the implications of changes in the targeting

[1]For instance, the Competition & Market Authority (CMA 2019) states that a defining feature of digital advertising is to "... make use of information not only about the environment (e.g., webpage) in which the ad will appear, but also about the internet user in front of whom the ad will be placed".

[2]See e.g., Sharma (2023).

[3]A third-party cookie collects information about a user who visits a website but the cookie is set up by another website—i.e., a website that a user is not currently on. The browsers Firefox and Safari have phased our third-party cookies some years ago, and Chrome announced that it will do so in 2024.

[4]For detailed studies on the consequences of these regulations see e.g., Johnson et al. (2022) or Peukert et al. (2022).

[5]E.g., in the category computers & electronics, 45% of ads are not on target, whereas for business & consumer services, only 25% do not reach the targeted audience.

ability on firms' targeting strategies. How does a change in targeting success affects the consumer groups that firms target with their products? Does improved targeting raise firms' profits or can the force of increased competition dominate? Can changes in the targeting ability affect the number of products that firms offer?

To answer these questions, we consider the following setup. There are two symmetric firms and two asymmetric consumer groups—i.e., high-valuation and low-valuation consumers. Each firm offers a product and decides which consumer group to target with its product. Firms send advertising messages to inform consumers about their respective products (cf. Butters, 1977). For every consumer, the firm has imperfect information about whether the consumer has high or low valuation. The probability of successfully reaching a consumer of the targeted group is therefore lower than 1. This probability is the same for both firms, which reflects that the success of targeting is determined by the market environment (e.g., the predictive power of cookies for consumer behavior, the possibility of consumers to easily opt out of data sharing, etc.)

In this setup, we first show that the probability of targeting success determines firms' targeting strategies. If this probability is relatively low,[6] then both firms target high-valuation consumers. Some consumers then receive messages from both firms (i.e., a clearinghouse emerges) whereas others are only informed about the product of one firm. Although this leads to competition, both firms choose to target the high-valuation consumers (i.e., compete for cherries) instead of segmenting the market by targeting different consumer groups, because this allows them to set higher prices. If targeting improves, there are two effects: first, each firm reaches more consumers who are interested in its product, which has a positive effect on profits; second, the mass of consumers getting messages from both firms increases, which enhances price competition. We show that the second effect dominates, which implies that profits *fall* as targeting improves.

Instead, if the probability of successful targeting is sufficiently high, firms target different consumer groups. The firm targeting the low-valuation consumer group can then only set a low price. However, this is more profitable than targeting the same consumer group as the rival, because due to the high targeting success, many consumers get messages from both firms, which would lead to fierce competition in the latter case. In such a situation of market segmentation, improved targeting *increases* firms' profits, as there is no competition and only the positive effect of a higher reach prevails.

As a consequence, firms' profits change non-monotonically in the targeting success—i.e., they fall for relatively low success rates, but raise for relatively high success rates. This is in appreciable contrast to existing literature which finds either a monotonic rela-

[6]A relatively low success probability means that the advertising message reaches a consumer who belongs to the intended group with a probability that is only moderately larger than 1/2.

tionship or an inverted u-shape pattern.[7] Interestingly, our result also shows that if the probability to reach the intended consumers is perfect—as is usually the case in clearinghouse models, such as Varian (1980) and Narasimhan (1988)—firms avoid competition by targeting different consumer groups, which implies that a clearinghouse does not emerge.

We then use our framework to study the question how targeting success affects the number of products that firms offer. We show that the two countervailing forces described above also determine how an improvement in targeting affects a firms' product-introduction strategy. An increase in targeting success implies that advertising messages are more likely to reach interested consumers, which makes the introduction of an additional product more profitable. At the same time, if both firms offer the same product, an increase in targeting success enhances competition, which reduces the profitability of offering an additional product. These effects can lead to an inverted u-shape relationship between the *number of products* and *targeting success*. If targeting success is only moderate, consumer reach is relatively low and each firm offers only one product—i.e., the product that targets high-valuation consumers. If targeting becomes better, it pays off for a firm to additionally offer the product that targets low-valuation consumers, thereby becoming a multi-product firm. However, if targeting improves even further, competition for high-valuation consumers gets more intense, which implies that the multi-product firm then optimally withdraws the respective product and sells only the product for low-valuation consumers. Therefore, targeting success does not only influence a firm's targeting strategy, but also the optimal number of products it sells.

An improvement in the targeting ability has also interesting profitability effects in case products appeal more broadly than only to the consumer group to which the product is targeted. In our baseline model, we analyze the situation in which products appeal only to the intended consumer group, whereas the other consumer group is not interested.[8] However, some consumers of the other group may also be interested in the product. We find that this makes market segmentation more likely as a broader appeal is particularly valuable for the firm offering the product for low-valuation consumers. Interestingly, although each product is attractive for more consumers, firms' profits in aggregate are lower. The intuition is that the market is no longer fully segmented if products appeal more broadly, as some high-valuation consumers are also interested in the product for low-valuation consumers, and vice versa. This leads to competition between firms even under market segmentation, and substantially reduces the profit of the firm selling the product mainly intended for high-valuation consumers.

[7] We provide a detailed overview of the related literature in the next section.

[8] This implies that, for instance, a low-valuation consumer is not interested in a product that targets high-valuation consumers, and the respective advertising message would be lost when reaching a low-valuation consumer.

Finally, we study the situation in which firms send advertising messages on publishers—e.g., third-party websites. We show that our main results carry over to this case. In particular, an improvement in targeting (now of the publisher) lowers firms' profits for relatively low values of targeting success, but raises firms' profits if targeting is already relatively good. Moreover, if firms advertise via publishers, additional effects occur, as a firm can now place ads on both publishers, thereby reaching some consumers twice. Although this does not bring additional benefits, it implies that the rival firm does not reach this consumer, which can dampen competitive forces. We show that in case both firms target high-valuation consumers, at least one firm indeed places ads on different publishers, whereas this does not occur if firms target different consumer groups.

The rest of the paper is organized as follows: Section 2 relates our paper to existing literature. Section 3 sets out the basic model. Section 4 presents the equilibria for different advertising costs. Section 5 extends the basic model to analyze, first, the effects of firms' choices to offer multiple products and, second, the equilibrium in case products appeal more broadly. Section 6 analyzes the case in which firms simultaneously choose their targeting strategies and their product prices. Section 7 studies the situation in which firms advertise via publishers. Finally, Section 8 concludes and presents managerial implications. The proofs of the results in Section 4 are in the Appendix. The Online Appendix provides the proofs of all other propositions and some further results.

2 Related literature

Our paper relates to several strands of literature.

First, we contribute to the literature on targeting strategies and their profitability. Iyer et al. (2005) consider a model in which each firm has loyal consumers, but there are also shoppers who buy from the firm offering the cheaper price. Targeting allows a firm to send messages only to specific consumers. They show that this helps firms to reduce competition and save costs, thereby increasing profits.[9] Chen et al. (2001) also study a situation with loyal consumers and shoppers, but each firm can price discriminate between these groups. Firms have imperfect information about the group to which a consumer belongs. They find that profits first increase with an improvement in targetability and then decrease.[10] Bergemann and Bonatti (2011) analyze a model in which targeting leads to a split of a single advertising market into multiple ones. This allows producers of niche

[9] Esteves and Resende (2016) analyze a related model in which each firm offers a higher utility than the rival to consumers in a specific segment, and firms decide which and how many consumers to target. They determine conditions for price discrimination between segments to be profitable for firms.

[10] Belleflamme et al. (2020) analyze a model in which firms can price discriminate between consumers they obtain information about—i.e., profiled consumers—and non-profiled consumers. They find that if firms' profiling abilities are different, they obtain positive profits despite selling a homogeneous good.

products to become active in advertising. They show that small and large firms benefit from targeting, whereas medium-sized firms are worse off because consumer attention migrates to smaller advertising markets.[11] In contrast to these papers, we analyze the case in which firms decide whether to target high- or low-valuation consumers and allow for changes in the targeting success. We show that the effects on profits are very different in that case, as improved targeting lowers profits for *intermediate values* but is beneficial if targeting is already *relatively accurate*.

Some papers focus on firms' choices which consumer segment to target, but—in contrast to our paper—consider perfect targeting. Roy (2000) studies such a framework. Assuming that consumers are distributed on the Hotelling line, he shows that targeting leads to full market segmentation. Galeotti and Moraga-González (2008) analyze a similar demand structure but consider simultaneous targeting and pricing choices. They find that targeting leads to positive profits only if firms are asymmetric. Shaffer and Zhang (1995) study the case in which firms first set regular prices and then decide to target a group of consumers with price discounts and show that this possibility tends to result in a prisoners' dilemma in which firms use targeting but obtain lower profits.[12] Anderson et al. (2022) consider a general framework in which firms first set uniform list prices, and afterward learn their match value with each consumer. Firms can then choose which consumers to target with price discounts. Anderson et al. (2022) determine general conditions under which consumers and firms benefit from targeting and also allow consumers to opt out of targeting.

Our paper also relates to the literature on clearinghouse models, pioneered by Varian (1980) and Narasimhan (1988), in which firms have a captive market segment but also compete for consumers who compare prices.[13] Shelegia and Wilson (2021) provide a generalization of the clearinghouse framework to allow for consumer heterogeneity and study several applications (e.g. different pre-sales activities).[14] Ronayne and Taylor (2021) analyze competition between different sales channels (i.e., direct and competitive channels) and show that this can lead to clearinghouse models with rich asymmetries in the number of consumers. Armstrong and Vickers (2019) consider a general clearinghouse model in which firms can set different prices to consumers in the captive segment and

[11] Esteban et al. (2001) consider a monopoly framework and show that targeting may allow the monopolist to move from mass advertising to specialized advertising, which raises its profit.

[12] Bounie et al. (2021) analyze the situation in which firms can obtain information about consumers from a data broker and may use this information to price discriminate. They find that the data broker strategically withholds information to dampen competition.

[13] See Baye et al. (2006) for a survey of this literature.

[14] Myatt and Ronayne (2023) extend the clearinghouse model by a pre-stage in which firms set list prices and compete thereafter in retail prices that cannot exceed list prices. They show that this leads to a pure-strategy equilibrium with price dispersion.

those that compare prices. They determine conditions for such price discrimination to be beneficial for consumers. Recently, Armstrong and Vickers (2022) provide a general treatment of price competition in clearinghouse models with uniform pricing, allowing for various forms of asymmetries between firms.[15] A common assumption in these papers is that consumers are homogeneous in their ex-ante willingness-to-pay and all advertising messages reach the intended consumers. Instead, in our paper we explicitly allow for imperfect targeting and firms being able to choose to target different consumer groups. We then show that a clearinghouse emerges—i.e., firms target the same consumer group—only if targeting success is moderate.

Finally, our paper also relates to studies analyzing how third parties, such as publishers or advertising networks, affect advertising strategies. Athey and Gans (2010) consider a model of competition between a global and several local outlets and show that targeting helps the global outlet to reduce wasteful impressions, which generally increases profits. Levin and Milgrom (2010) discuss potential problems of targeting by publishers—e.g., making targeting so finely-tuned that only few advertisers are attracted. D'Annunzio and Russo (2020) analyze advertising networks and find that outsourcing the sales of ads tends to be beneficial for publishers as it lowers the number of wasted ads but may hurt advertisers. Johnson et al. (2022) consider competing advertising exchanges and study the incentives to cross-target consumers. They determine how this affects information sharing between firms and therefore firm profitability and consumer welfare. None of these papers considers imperfect targeting and firms' choices which consumer group to target, which is the focus of our paper.

3 The Model

There are two consumer groups a and b, each with a mass of one. All consumers have unit demand for a product and receive zero utility when not buying the product. A consumer in group a has a valuation of $A \geq 0$ for the product, and a consumer in group b has a valuation of $B > A$; hence, consumers in group b are high-valuation consumers and consumers in group a are low-valuation consumers.

There are two firms 1 and 2. All production costs are normalized to zero. Consumers are initially uninformed about firms' offers. However, firms can inform consumers by sending advertising messages. Each firm decides whether to target consumers of group a or b with its product.[16] For every consumer, a firm receives an (imperfect) signal

[15] Chioveanu (2023) analyzes a variant of the model by Armstrong and Vickers (2022) in which firms can price discriminate but products may not be available or suitable for all consumers.

[16] In Section 5.1, we analyze the case in which a firm can offer multiple products and can therefore target consumer group a and b with the corresponding product.

about the group to which the consumer belongs to. The signal is correct with probability $\alpha \in [1/2, 1]$, that is, if a consumer belongs to group $k \in \{a, b\}$, the firm's signal correctly indicates that the consumer is in group k with probability α, but falsely indicates that the consumer is in group $-k$ with probability $1 - \alpha$.[17] This implies that the probability of successfully reaching a targeted consumer is $\alpha \in [1/2, 1]$, that is, if firm $i = 1, 2$ wishes to target consumers of group k, its advertising message reaches the targeted consumer with probability α and a consumer in group $-k$ with probability $1 - \alpha$.[18] The success probability therefore captures in a simple way that a firm obtains information about each consumer, thereby helping the firm to predict the group to which the consumer belongs to. We will refer to $\alpha = 1/2$ as *no targeting*, $\alpha \in (1/2, 1)$ as *imperfect targeting*, and $\alpha = 1$ as *perfect targeting*.

The signals that the two firms receive about a particular consumer are independent of each other, but have the same probability of being correct. This reflects that firms usually do not have the same sources of information as they e.g. monitor different websites visited by the consumer—which implies different consumer tracking—or buy information from different data companies.[19] By contrast, the targeting ability is driven by the technology and the efficiency of data processing and depends on the advertising environment. Due to these considerations, we treat α as exogenous, which allows us to determine the effects of improved targeting possibilities on advertising strategies and profits.

An advertising message informs a consumer about the firm's product. In our baseline model, we assume that only consumers in the targeted group are interested in buying the good.[20] This implies that a message, which does not reach a consumer in the targeted group, is lost as non-targeted consumers are not interested in the firm's product.[21] If the message reaches a consumer in the targeted group (i.e., targeting is successful), the consumer buys the product given that the price is lower than her valuation. In case firms target the same consumers and a targeted consumer receives a message from both firms, the consumer buys from the firm with the lower price. If both firms charge the same

[17] Formally, denoting by $\hat{k} \in \{a, b\}$ the signal a firm receives about a consumer in group k, we have $\text{prob}\Big[\hat{k}|k\Big] = \alpha$ and $\text{prob}\Big[-\hat{k}|k\Big] = 1 - \alpha$.

[18] As the two groups are of equal size and signals are symmetric, the probability for targeting success is indeed α; formally,

$$\text{prob}\Big[k|\hat{k}\Big] = \frac{\text{prob}\Big[\hat{k}|k\Big]}{\text{prob}\Big[\hat{k}|k\Big] + \text{prob}\Big[\hat{k}| - k\Big]} = \frac{\alpha}{\alpha + 1 - \alpha} = \alpha.$$

[19] All our insights still hold if the signals of the two firms are imperfectly correlated.

[20] Targeting is therefore equivalent to sell a product or send an advertising message that only appeals to consumers in the targeted group.

[21] In Section 5.2, we show that our results carry over to the case in which also consumers of the non-targeted group may buy the firm's product.

price, the consumer buys from each firm with equal probability.

We denote by $c \geq 0$ the costs for sending an advertising message to a consumer. To exclude uninteresting cases, we restrict attention to $c \leq \max\{\alpha A, \alpha(1-\alpha)B\} \equiv \hat{c}$. If c was larger then this threshold, either no firm can profitably send messages or it is profitable for only one firm to do so, which would exclude any competition and thereby the interesting effects at work. To present our insights in a simple way, we first analyze the two scenarios in which the costs are (i) equal to (or slightly below) the upper bound $\hat{c}$ and (ii) equal to the lower bound (i.e., $c = 0$). In the first scenario, advertising costs are sufficiently high so that for most parameters each firm profitably sends messages only to consumers in the expected targeted group. In the second scenario, advertising costs are sufficiently low, so that each firm can send messages to consumers in the targeted and the non-targeted group if it wishes to do so. We show that our main insights are conveyed in these two scenarios. We then consider the case with advertising costs being in the full range and show that all our insights extend to this case.

Timing. The timing of the game is as follows: In the first stage, each firm chooses the consumer group it targets with its product and decides to which consumers to send a message. In the second stage, after observing these decisions, each firm sets its price. Finally, consumers observe the advertising messages, inform themselves about the price in case targeting of the message was successful (e.g., by clicking on the ad or visiting the firm's website or store), and make their purchase decisions. The solution concept is subgame perfect Nash equilibrium (SPNE).

This scenario of sequential targeting and pricing decisions is realistic in many industries. In particular, firms' choices which type of product to offer and their respective advertising campaigns are usually long-term decisions and therefore known before firms set their prices. This holds particularly for industries with a strong brand recognition of firms, such as the automobile or perfume and cosmetics industry.

In Section 6, we consider simultaneous targeting and pricing decisions by firms.[22] This timing describes industries with short product life cycles and high turnover, in which product positioning and branding is less important (e.g., particular types of apparel).

4 Analysis

4.1 High Advertising Costs

We start with the case in which c equals (or is slightly below) $\hat{c}$. As consumer group b has a higher valuation than consumer group a, it can never be optimal for both firms to

[22]This implies that the decisions of the first and the second stage are made at the same time.

target consumers of group a. Each firm can then benefit by deviating and targeting group b, as this allows the firm to obtain the monopoly profit from high-valuation consumers. Therefore, in equilibrium, firms either choose to target different consumer groups—e.g., firm 1 targets group a and firm 2 targets group b—or both firms target the high-valuation group b.

In the former case, firms segment the market. Then, in the second stage, there is a (pure-strategy) Nash equilibrium with prices $p_1 = A$ and $p_2 = B$. The expected profits depend on whether a firm sends messages to all consumers or only a subgroup. If each firm sends messages only to consumers in the expected targeted group, the respective profits are $\pi_1 = \alpha A - c$ and $\pi_2 = \alpha B - c$.[23]

If, instead, firms send messages to consumers in both groups, each firm reaches all interested consumers,[24] but incurs higher advertising costs. This leads to expected profits of $\pi_1 = A - 2c$ and $\pi_2 = B - 2c$. The former profit is negative for advertising costs at the upper bound $c = \alpha A$ for all $\alpha > 1/2$. Hence, the strategy of sending messages to all consumers cannot be optimal for the firm that targets group a but only for the firm that targets group b. For the latter firm, sending messages to all consumers is more profitable than sending messages only to consumers in the targeted group if $B(1-\alpha) - c \geq 0$ or $c \leq (1-\alpha)B$.

We next consider the case in which both firms target consumers in group b. For high advertising cost, it is optimal for each firm to send messages only to consumers who are expected to be in the targeted group b.[25] Then, a clearinghouse emerges. In this clearinghouse, three different consumer types exist. First, a mass of α^2 of group-b consumers obtain a message from both firms, as the signals of both firms correctly identified these consumers as belonging to group b. These consumers buy from the firm with the lower price and are referred to as "shoppers". Second, a mass $\alpha(1-\alpha)$ of group-b consumers obtains a message only from firm $i = 1, 2$ and therefore buys from that firm (as long as the firm's price does not exceed the valuation B). These consumers can therefore be considered as "non-shoppers" or loyal consumers of a firm. Finally, a mass $(1-\alpha)^2$ of group-b consumers does not receive any message and therefore does not buy. As firms share the demand of shoppers equally when charging the same price, the

[23]Each firm optimally sends a message to all consumers who are expected to be in the targeted group, and targeting is successful with probability α. In footnote 26 we explain why our assumption that $\hat{c} = \max\{\alpha A, \alpha(1-\alpha)B\}$ ensures that both firms indeed have an incentive to send advertising messages.

[24]The firm reaches a mass α of consumers who are interested in its product through the messages to the expected targeted group and the remaining mass $1-\alpha$ through messages to the other group. Therefore, all consumers become informed, and each firm's demand equals 1.

[25]Indeed, as we show formally in the proof of Lemma 2, the additional profit that a firm can obtain when also informing consumers who are expected to be in the non-targeted group a, is $(1-\alpha)^2 B$, which is strictly lower than $\alpha(1-\alpha)B$—i.e., the value of $\hat{c}$ in this range.

expected profit of firm i as a function of prices is

$$E[\pi_i(p_i, p_{-i})] = \begin{cases} \alpha p_i - c, & \text{if } p_i < p_{-i}; \\ \left(\frac{\alpha^2}{2} + \alpha(1-\alpha)\right) p_i - c, & \text{if } p_i = p_{-i}; \\ \alpha(1-\alpha)p_i - c, & \text{if } p_i > p_{-i}. \end{cases}$$

As is standard in clearinghouse models, each firm has the incentive to either slightly undercut the rival's price to gain the entire demand from shoppers, or set its price equal to B and extract the surplus from non-shoppers. This gives rise to a (symmetric) mixed-strategy equilibrium. Setting $p_i = B$ guarantees a profit of at least $\alpha(1-\alpha)B - c$. As a consequence, undercutting firm $-i$ is profitable for firm i as long as the resulting price is not lower than $p_i = (1-\alpha)B$. Thus, the price range of a potential mixed strategy equilibrium is $[(1-\alpha)B, B]$.

Suppose firm i plays a mixed strategy and draws its price from the cumulative distribution function (cdf) $F(p_i)$ on $[(1-\alpha)B, B]$. Then, firm $-i$ is indifferent between setting any price on $[(1-\alpha)B, B]$ if $\alpha p(1 - F(p)) + \alpha(1-\alpha)pF(p) = \alpha(1-\alpha)B$. Solving for $F(p)$ yields

$$F(p) = \frac{1}{\alpha} - \frac{(1-\alpha)B}{\alpha p}, \tag{1}$$

with $F((1-\alpha)B) = 0$ and $F(B) = 1$.

Moving to the first stage, it is optimal for each firm to target consumers in group b if the resulting profit is larger than the profit from targeting consumers in group a, given that the other firm targets group b. This holds if $\alpha(1-\alpha)B > \alpha A$ or $\alpha < 1 - A/B$.[26] Denoting $\hat{\alpha}_1(A/B) \equiv 1 - A/B$, we obtain the following lemma.

Lemma 1. *The SPNE with advertising costs $c = \hat{c}$ is as follows:*

1. *If $\hat{\alpha}_1(A/B) > 1/2$, then for $\alpha \in [1/2, \hat{\alpha}_1(A/B))$ both firms target consumer group b and there is a symmetric mixed-strategy equilibrium, in which firms set prices in the domain $p_i \in [(1-\alpha)B, B]$. The mixing probability is characterized by the cumulative distribution function $F(p_i)$ in (1). The expected profit is $\pi_i^\star = \alpha(1-\alpha)B - c$.*

2. *For $\alpha \in [max\{1/2, \hat{\alpha}_1(A/B)\}, 1]$, firms segment the market, and prices are $p_i^\star = A$, $p_{-i}^\star = B$, Firms' profits are $\pi_i^\star = \alpha A - c$ and $\pi_{-i}^\star = B - 2c$ if $c \leq (1-\alpha)B$, and $\pi_i^\star = \alpha A - c$ and $\pi_{-i}^\star = \alpha B - c$ if $c > (1-\alpha)B$.*

[26] It follows that for $\alpha < 1 - A/B$, the equilibrium profit of both firms is $\alpha(1-\alpha)B - c$, whereas for $\alpha \geq 1 - A/B$, the equilibrium profit of the firm with the lower profit is $\alpha A - c$. The assumption on $\hat{c}$ then guarantees that profits are (weakly) positive as it implies that $c \leq \alpha(1-\alpha)B$ for $\alpha < 1 - A/B$ and $c \leq \alpha A$ for $\alpha \geq 1 - A/B$.

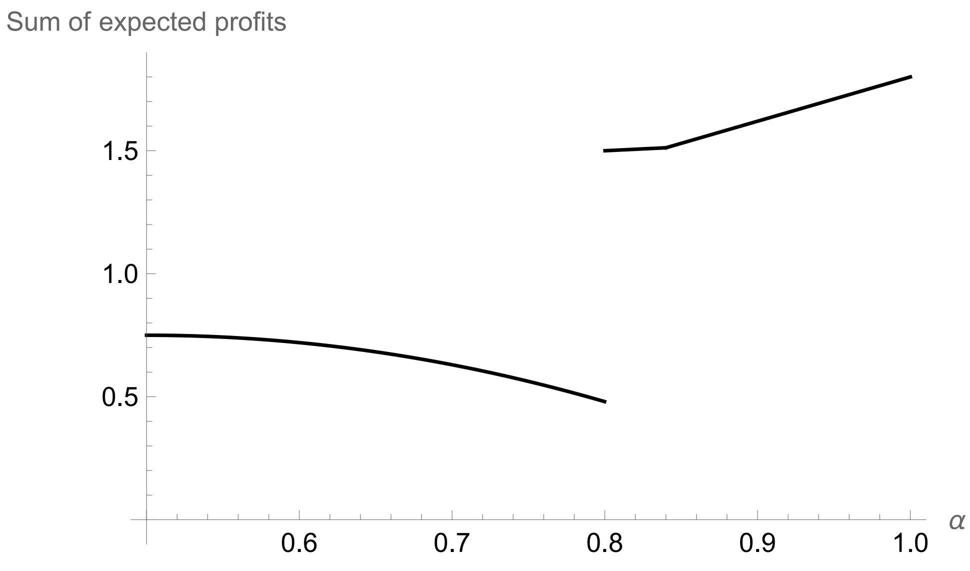
The equilibrium in which both firms target the same consumer group—i.e., the high-valuation group b—only exists if $\hat{\alpha}_1(A/B) > 1/2$ or $A < B/2$. This is intuitive: Although targeting the same consumers intensifies competition, it is profitable to do so if the profit that can be reaped from the other consumer group is sufficiently low. This is indeed the case if consumers in group a have only a low valuation relative to that of consumers in group b.

Having characterized the equilibrium, we can now answer the question how an improvement in targeting affects firms' profits. This can be done by determining how the industry profit (i.e., the sum of the profits of the two firms) changes with α. From Lemma 1, the sum of expected profits is

$$\sum_i E[\pi_i^\star(\alpha)] = \begin{cases} 2\alpha(1-\alpha)B - 2c, & \text{for } \alpha \in [1/2, \hat{\alpha}_1(A/B)) \text{ if } \hat{\alpha}_1(A/B) > 1/2; \\ \alpha A + B - 3c, & \text{for } \alpha \in [\max\{1/2, \hat{\alpha}_1(A/B)\}, 1] \text{ and } c \leq (1-\alpha)B; \\ \alpha(A+B) - 2c, & \text{for } \alpha \in [\max\{1/2, \hat{\alpha}_1(A/B)\}, 1] \text{ and } c > (1-\alpha)B. \end{cases}$$

From the last formula, we immediately obtain the following result.

Proposition 1. *If $A < B/2$, the sum of expected profits is non-monotonic in α. It decreases for $\alpha \in [1/2, \hat{\alpha}_1(A/B))$, jumps upwards at $\alpha = \hat{\alpha}_1(A/B)$, and increases for $\alpha \in [\hat{\alpha}_1(A/B), 1]$.*



The curve represents the sum of expected profits, net of advertising costs to the expected targeted group, as a function of α. The decreasing part arises when both firms target consumer group b, and the increasing part arises when firms segment the market. Parameter values are $A = 0.3$ and $B = 1.5$, which implies $\hat{\alpha}_1(A/B) = 0.8$.

Figure 1: Expected Profits with c slightly below $\hat{c}$

Figure 1 illustrates the non-monotonicity result of Proposition 1. This figure displays the sum of firm's profits, net of the advertising costs to the expected targeted group. The intuition for why the sum of expected profits is non-monotone in the success probability of advertising is as follows: If targeting is rather weak, the probability that a consumer receives an ad from both firms is relatively small. This implies that competition between firms is only moderate even if they both target consumers of the same group. This leads to relatively high average prices. It is then optimal for both firms to target consumers of the high-valuation group. An improvement in targeting has two effects: the direct effect is that it increases the probability that the ad reaches the intended consumers; the indirect effect is that more consumers receive a message from both firms, which intensifies competition. The indirect effect is, however, dominating here, as the messages that reach the intended consumers are already above 50%. Hence, profits fall in the success probability of targeting.

By contrast, if targeting is relatively strong, firms prefer to target different consumer groups to avoid intense competition for high-valuation consumers. Then, only the direct effect of improved targeting exists—i.e., more of the intended consumers receive the message. As a consequence, better targeting leads to an increase in profits.

At the point at which the equilibrium switches from both firms targeting the same consumers to market segmentation—i.e., $\alpha = \hat{\alpha}_1(A/B)$)—an upward jump in industry profits occurs. The reason is as follows: Given that firm $-i$ targets high-valuation consumers, at $\alpha = \hat{\alpha}_1(A/B)$ firm i is indifferent between targeting the high-valuation consumers as well and targeting low-valuation consumers. However, the profit of firm $-i$ is strictly higher if firm i pursues the latter strategy, as there is no competition between the firms in that case.

It is interesting to note that firms decide to enter a clearinghouse only if the interested consumers receive the advertising message with a relatively *low* probability. This is in contrast to previous literature, which considers clearinghouse models with perfect targeting (i.e., consumer receiving an advertising message with probability 1). Our analysis shows that in such a situation firms endogenously shy away from competition if multiple consumer groups are available.

4.2 Low advertising costs

We next consider the scenario in which advertising costs are zero, which implies that firms may send advertising messages to both consumer groups. As in the previous subsection, we start with the case in which firms segment the market and target different consumers. Given that there is no competition, each firm optimally sends a message to all consumers. In the (pure-strategy) Nash equilibrium of the pricing game, firms set prices of $p_i = A$

and $p_{-i} = B$. As in the previous subsection, suppose that firm 1 targets consumers of group a. Then, the corresponding profits are $\pi_1 = A$ and $\pi_2 = B$ (as advertising costs are zero).

Second, we analyze the case in which both firms target consumer group b.[27] With firms potentially sending messages to both groups, three situations can occur. In the first, the two firms send messages to all consumers. Since they are targeting the same consumers and all consumers are informed, this leads to Bertrand competition in the pricing stage and, therefore, zero profits. The second situation is the one in which each firm sends messages only to consumers that the firm expects to belong to group b. The analysis is then the same as above, leading to an (expected) profit of $\alpha(1-\alpha)B$ for each firm.

Finally, the two firms could send messages to different consumers. In particular, one firm sends messages only to consumers it expects to belong to group b and the other firm sends messages to consumers of both groups. This is indeed the only potential equilibrium configuration in which firms differ in the mass of consumers they send messages to because it is never optimal for a firm to send messages only to a subset of consumers of one group.[28] Suppose that firm 1 sends a message only to consumers it expects to belong to group b and firm 2 sends messages to all consumers. Then, following the same arguments as above, firm 1's expected profit as function of prices is

$$E[\pi_1(p_1,p_2)] = \begin{cases} \alpha p_1, & \text{if } p_1 < p_2; \\ \frac{\alpha}{2} p_1, & \text{if } p_1 = p_2; \\ 0, & \text{if } p_1 > p_2, \end{cases}$$

and firm 2's expected profit as function of prices is

$$E[\pi_2(p_1,p_2)] = \begin{cases} p_2, & \text{if } p_2 < p_1; \\ (1-\alpha)p_2 + \frac{\alpha}{2} p_2, & \text{if } p_2 = p_1; \\ (1-\alpha)p_2, & \text{if } p_2 > p_1. \end{cases}$$

Therefore, if both firms target group b and differ in the mass of consumers they send messages to, an asymmetric clearinghouse emerges in which non-shoppers only buy from firm 2. This implies that firm 1 can only obtain a positive profit when charging a lower price than firm 2. As only a mixed-strategy equilibrium exists—due to the undercutting incentive described above—the equilibrium is characterized by a lower expected price of firm 1 than of firm 2.

Despite this fact, the pricing range is the same for both firms and given by $[(1-$

[27] As in the previous case, it can never be an equilibrium that both firms target consumers of group a.

[28] We show this formally in the proof of Lemma 2.

$\alpha)B, B]$. To see this, note that by setting $p_2 = B$, firm 2 guarantees itself a profit of $(1-\alpha)B$. Therefore, undercutting firm 1 is profitable for firm 2 as long as the resulting price is not lower than $p_2 = (1-\alpha)B$. Thus, the price range of firm 2 in a potential mixed strategy equilibrium is $[(1-\alpha)B, B]$. This implies that if firm 1 sets $p_1 = (1-\alpha)B$, it obtains an expected profit of $\alpha(1-\alpha)B$ with a probability of (almost) 1. Given the mixing range of firm 2, firm 1 can never be better off with a lower price. In addition, firm 1 is willing to set prices up to B—although it has no loyal consumers—because the equilibrium mixing distribution of firm 2 entails a mass point at B, with a point mass of $1-\alpha$. We characterize the mixing distributions formally in the proof of Lemma 2.

In this proof, we also show that in case both firms target consumers of group b, this asymmetric equilibrium is indeed the unique equilibrium. In fact, firm 2 cannot profitably deviate and send messages only to consumers in the expected targeted group, as this leads to a lower reach of consumers but does not dampen competition. In addition, firm 1 cannot gain from deviating by sending messages to all consumers, as this makes firms more similar to each other and thereby leads to fiercer competition.

We next turn to the question under which conditions such an equilibrium in which both firms target consumers of group b emerges. As the profits of the two firms are now different in that equilibrium, we need to specify which profit accrues to which firm. To make the equilibrium in which both firms target the high-valuation group most difficult to achieve, we assume that the firm targeting group a under market segmentation—i.e., firm 1—also achieves the lower profit in case firms target the same consumer group. Firm 1 then obtains a higher profit with targeting group b than group a as long as

$$\alpha(1-\alpha)B \geq A \quad \text{or} \quad \alpha \leq \frac{1+\sqrt{1-4\frac{A}{B}}}{2}.$$

Denoting $\hat{\alpha}_2(A/B) \equiv \left(1+\sqrt{1-(4A)/B}\right)/2$, we obtain the following lemma.[29]

Lemma 2. *The SPNE with $c = 0$ is as follows:*

1. *If $\hat{\alpha}_2(A/B) > 1/2$, then for $\alpha \in [1/2, \hat{\alpha}_2(A/B))$, both firms target consumer group b. Firm i sends messages only to consumers it expects to be in the targeted group and firm $-i$ sends messages to all consumers. Both firms set prices in the domain $p_i \in [(1-\alpha)B, B]$. The mixing probabilities are characterized by the cumulative distribution functions $G(p_i) = F(p_i)$ in (1) and*

$$H(p_{-i}) = \begin{cases} \frac{p_{-i}-(1-\alpha)B}{p_{-i}}, & \text{if } p_{-i} \in [(1-\alpha)B, B); \\ 1, & \text{if } p_{-i} = B. \end{cases} \tag{2}$$

[29]The proof of this lemma can be found in the Appendix.

with a mass point of $1-\alpha$ *at* $p_{-i}=B$*. The expected profits are* $\pi_i^\star=\alpha(1-\alpha)B$ *and* $\pi_{-i}^\star=(1-\alpha)B$*.*

2. *For* $\alpha \in [max\{1/2, \hat{\alpha}_2(A/B)\}, 1]$*, in equilibrium, firms segment the market and send messages to all consumers. Prices are* $p_i^\star=A$ *and* $p_{-i}^\star=B$*, and firms' profits are* $\pi_i^\star=A$ *and* $\pi_{-i}^\star=B$*.*

The equilibrium in which both firms target group b now exists only for a smaller range than in the previous subsection. Whereas with $c=\hat{c}$ it existed if $A<B/2$, it now with $c=0$ only exists if $A<B/4$.[30] The reason is that market segmentation is more attractive for $c=0$ as each firm informs all consumers and obtains the maximal demand.

We now again turn to the question how an improvement in targeting affects firms' profits. From Lemma 2, the aggregate expected profits are

$$\sum_i E[\pi_i^\star(\alpha)] = \begin{cases} (1-\alpha^2)B, & \text{for } \alpha\in[1/2,\hat{\alpha}_2(A/B)) \text{ if } \hat{\alpha}_2(A/B)>1/2;\\ A+B, & \text{for } \alpha\in[\max\{1/2,\hat{\alpha}_2(A/B)\},1]. \end{cases}$$

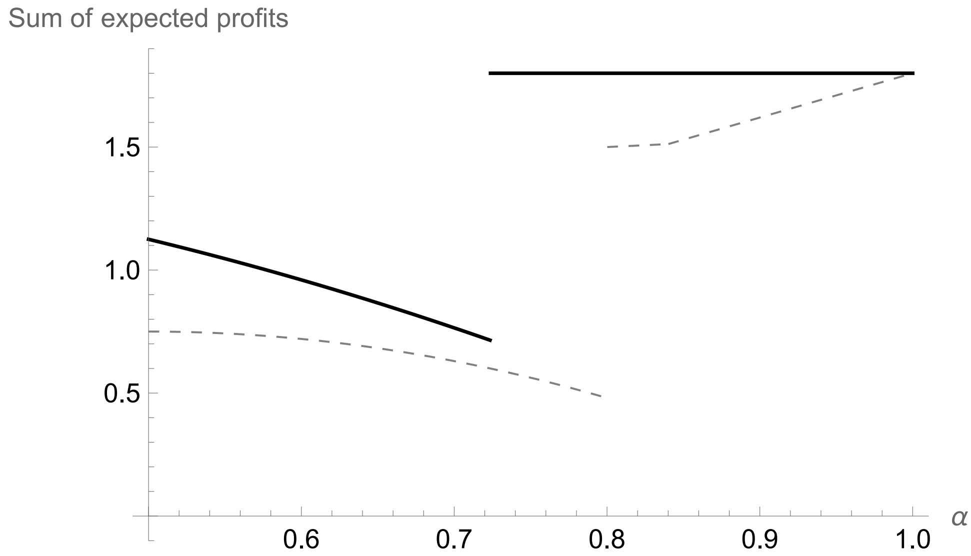
This immediately yields the following result.

Proposition 2. *If* $A<B/4$*, the sum of expected profits is non-monotonic in* α*. It strictly decreases for* $\alpha\in[1/2,\hat{\alpha}_2(A/B))$*, jumps upwards at* $\alpha=\hat{\alpha}_2(A/B)$*, and stays constant for* $\alpha\in[\hat{\alpha}_2(A/B),1]$*, whereby* $\hat{\alpha}_2(A/B)<\hat{\alpha}_1(A/B)$*.*

Interestingly, the proposition shows that the non-monotonicity result can also occur if advertising is costless. This is illustrated in Figure 2.

The result is perhaps surprising as the case with zero advertising costs allows firms to costlessly inform all consumers, which is equivalent to perfect targeting (i.e., all consumers interested in buying the product become informed). However, our result reveals that this is not necessarily the optimal strategy for firms, due to the competitive forces at play. In particular, if the high-valuation group is sufficiently more attractive than the low-valuation group, it is optimal for both firms to target the former group. If both firms would then inform all consumers, this would lead to fierce competition, and therefore low profits. It is instead optimal for one of the firms to *refrain* from such an information strategy and inform only consumers who are expected to be in the targeted group. This creates an asymmetry between firms and allows them to keep prices at a relatively high level. As in the previous subsection, if targeting improves, the firm that sends messages only to the expected targeted group reaches more of the intended consumers, but the competition-enhancing effect dominates, which lowers industry profits. Therefore, imperfect targeting leads to *lower* profits than no targeting.

[30] Indeed, $\hat{\alpha}_2(A/B)<\hat{\alpha}_1(A/B)$ for $A<B/4$.



The solid curve represents the sum of expected profits when advertising costs are equal to zero, whereas the dashed curve represents, for comparison, the sum of expected profits when advertising costs are equal to $\hat{c}$ (net of advertising costs to the expected targeted group). The decreasing part of the solid curve arises when both firms target group b, and the constant part arises when firms segment the market. Parameter values are $A = 0.3$ and $B = 1.5$, which implies $\hat{\alpha}_2(A/B) \approx 0.7236$.

Figure 2: Expected Profits with zero advertising costs

As explained above, the region in which this result occurs is smaller than in the case in which firms can only send messages to the expected targeted group because market segmentation is more attractive when being able to inform all consumers. As can be seen from the figure, with market segmentation, industry profits are independent of α, as the possibility to send messages to all consumers acts as a substitute for improved targeting. Industry profit is then highest, regardless of the targeting success.

Finally, we note that also in case with zero advertising costs, a clearinghouse emerges only if the success of targeting is limited. Instead, if an advertising message reaches the intended consumers with sufficiently high probability, firms prefer to target different consumers and avoid competition.

4.3 Intermediate Advertising Costs

To present our results in a clear way, we focused in the two previous subsections on extreme cases with respect to the advertising costs: either c is equal to or slightly below $\hat{c}$ (i.e., the upper bound), or c is equal to zero. In this section, we now consider the full range of advertising costs $c \in [0, \hat{c}]$.

In each of the two extreme cases, given that B/A is sufficiently large, firms target the same consumer group for *low* values of α, but segment the market by targeting different consumer groups for *high* values of α.[31] Considering intermediate advertising costs, keeping α constant, mainly affects the number of advertising messages that firms send but does not change the equilibrium targeting strategies. Consider, for example, the case with a low value of α. As advertising costs rise, firms move from an equilibrium in which one firm sends advertising messages to all consumers whereas the other firm sends messages only to consumers who the firm expects to be in the targeted group b, to an equilibrium in which both firms send messages only to consumers who are expected to be in the targeted group b. Similarly, if α is large, market segmentation occurs regardless of the value of c, but firms send fewer advertising messages as c rises.

The exception occurs for intermediate values of α. As discussed in the previous subsection, market segmentation then arises already for a smaller value of α when advertising costs are zero as compared to the case when c is close to the upper bound. This implies that, as c rises, the firms move from an equilibrium in which they target different consumers to one in which they both target high-valuation consumers. Small advertising costs are indeed more beneficial to firms when they target different consumer groups, as firms can then send messages to all consumers without fearing competition. Instead, if they target the same consumer group, sending messages to all consumers also intensifies competition. This explains why an increase in advertising costs induces firms to move from segmentation to targeting of the same consumer group for intermediate values of α.

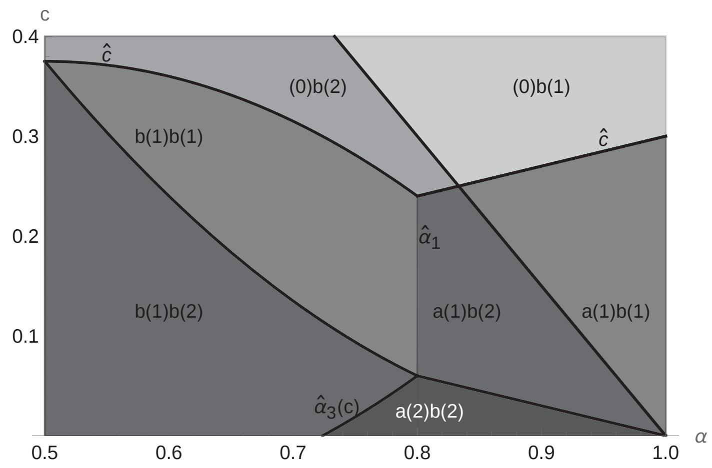
Figure 3 illustrates the different equilibrium regions in an (α, c)-plane if B/A is sufficiently large. In this figure, the notation a(1)b(2), for instance, indicates that in the respective region the equilibrium involves one firm targeting group a and sending messages only to expected group-a consumers—i.e., a(1)—but the other firm targeting group b and sending messages to both consumer groups—i.e., b(2).[32] The upper bound of c (i.e., $c = \hat{c}$) first decreases and then increases in α. As can be seen from the figure, for $c > \hat{c}$, it is optimal for at most one firm to be active.

The figure shows that for values of c in the middle range, four different equilibrium configurations can emerge as we move from $\alpha = 1/2$ to $\alpha = 1$. Instead, at the boundaries of c, only two, respectively, three, configurations occurred—i.e., two in case $c = 0$ and three in case $c = \hat{c}$. Nevertheless, the same non-monotonicity result with respect to the aggregate profits emerges if A is rather small compared to B. Defining $\hat{\alpha}_3((A-c)/B) \equiv \left(1 + \sqrt{1 - 4(A-c)/B}\right)/2$, the following proposition makes this statement precise.

Proposition 3. *1. For $c < A^2/B$, if $A < B/4 + c$, the sum of expected profits is non-monotonic in α. It strictly decreases for $\alpha < \hat{\alpha}_3((A-c)/B)$, jumps upwards at*

[31] As shown above, if B/A is relatively small, segmentation of the market occurs already for $\alpha = 0.5$, that is, the region in which firms target the same consumer group does not exist.

[32] All other equilibrium regions are referred to by the same logic.



Equilibrium regions with advertising costs of $c \in [0, 0.5]$ in an (α, c)-plane. Parameter values are $A = 0.3$ and $B = 1.5$.

Figure 3: Equilibrium regions with intermediate advertising costs

$\alpha = \hat{\alpha}_3((A-c)/B)$, *and increases for* $\alpha > \hat{\alpha}_3((A-c)/B)$, *whereby* $\hat{\alpha}_3((A-c)/B) < \hat{\alpha}_1(A/B)$.

2. *For* $c \geq A^2/B$, *if* $A/B > 1/2$, *the sum of expected profits is non-monotonic in* α. *It strictly decreases for* $\alpha < \hat{\alpha}_1(A/B)$, *jumps upwards at* $\alpha = \hat{\alpha}_1(A/B)$, *and increases for* $\alpha > \hat{\alpha}_1(A/B)$.

Although the equilibrium regime can switch up to three times when moving from no targeting to perfect targeting, there is still only one threshold value of α at which the expected industry profits jump upwards. The intuition for this result is that, at this threshold, one firm switches from targeting consumers in group b to targeting consumers in group a, which has a discontinuous effect on the profit of the other firm. Instead, in the range of α below and above this threshold value, firms only change their number of advertising messages when moving from one equilibrium to another, but not the targeting strategy. This leads to a continuous change in the sum of expected equilibrium profits.

We also note that for all values of c above A^2/B, the threshold is the same as that determined in Section 4.1 where we focused on $c = \hat{c}$. As this threshold does not depend on c, it is a vertical line in the (α, c)-plane. Instead, for $c < A^2/B$, the threshold is strictly increasing in c. This implies that the parameter region in which the sum of expected profit is non-monotonic in α, is lowest at $c = 0$—i.e., the case analyzed in Section 4.2—and becomes larger as $c > 0$.

In summary, the main results derived in the two previous subsections also hold for intermediate advertising costs. Although a larger number of equilibrium configurations can emerge when moving from no to perfect targeting, the non-monotonicity of the equilibrium profits in the targeting ability of firms remains. In addition, the result that firms target different consumer groups when targeting is very effective, thereby avoiding a clearinghouse, also does not depend on the level of the advertising costs.

5 Multi-Product Offers and Broader Attractiveness

We so far analyzed how improved targeting affects firms' product offers and profits. However, improved targeting also interacts with the product portfolio choice of firms and with the number of consumers who are interested in buying the product. In this section, we show that our model allows us to determine the implications of improved targeting on these issues. In Section 5.1, we analyze the case in which firms are able to offer two products, each one appealing to a different consumer group, thereby becoming multi-product firms. In Section 5.2, we study the case in which not only consumers of the targeted group are interested in a firm's product, but, with some probability, also consumers of the other group.

5.1 Multi-Product Firms

If a firm decides to offer two products, it chooses one for consumers of group a and one for consumers of group b. The firm then also sends advertising messages accordingly, that is, it advertises its respective product at least to consumers who are in the expected targeted group. We are particularly interested in the interaction between improved targeting and the number of offered products in equilibrium, thereby determining whether increased targeting success leads to a larger or smaller product portfolio of firms.

To answer this question, we enrich the first stage of the game by allowing each firm to become a multi-product firm. In the modified first stage, each firm can now not only offer either product a or product b—i.e., one product that targets consumers of either group a or group b—but it can also offer two products—i.e., product a *and* product b. Otherwise, the game proceeds as in the baseline model.[33]

We restrict our attention to the case of interest where firms for some parameter range are in competition to each other. As shown in Section 4, this occurs for $A/B \leq 1/2$ as both firms then choose to target consumers in group b for α sufficiently small.

We obtain the following result on the interaction between improved targeting and the equilibrium number of products:

[33]To obtain uniqueness in the number of products, we assume that there are costs of $\epsilon > 0$, with $\epsilon \to 0$, of introducing a product. This avoids the case that a firm always introduces two products but decides to advertise only one of them.

Proposition 4. *For $c \leq A/2$, the equilibrium number of products is (weakly) decreasing in α. Instead, for $c > A/2$, the equilibrium number of products is non-monotonic in α: it is (weakly) increasing in α for α below a threshold value and (weakly) decreasing for α above this threshold.*

The intuition behind this result is as follows: There are two countervailing forces in the interplay between improved targeting and firms' decisions to offer a second product. First, as α increases, the direct effect is that the probability of successfully reaching a consumer interested in the second product gets larger. This implies that, for given level of c, a firm's incentive to offer an additional product is higher. Second, there is again the indirect effect of competition. If firms offer products for the same consumer group, an increase in the targeting probability leads to fiercer competition, which reduces a firm's incentive to offer an additional product. If, for instance, firms offer different products in the single-product case, it pays off less for each firm to step into the rival's territory by offering an additional product.

As the proposition shows, the effects are of different force, dependent on the level of c. For relatively small values of c, the competitive effect always dominates. In this case, each firm offers both products if α is slightly above $1/2$, as advertising costs are low and competition is only moderate. As α increases, competition increases, which implies that there is a threshold value at which one firm optimally withdraws the product intended for the low-value consumer group a. This results in three products in equilibrium. If α increases further, the other firm optimally withdraws its competing product intended to group b. This leads to two products and market segmentation in equilibrium for α close to 1.

Instead, if advertising costs c are large, both firms only offer one product—i.e., the product to group b—for small values of α. As targeting is imprecise, it is not profitable to become a multi-product firm. As α increases, offering a second product becomes profitable, due to the increase in demand resulting from better targeting. However, for α even larger, the competition-enhancing effect starts to dominate. It is then profitable for the multi-product firm to withdraw the competing product. Firms then offer a single product each, leading to market segmentation. The effect of improved targeting on the number of products is therefore non-monotonic.

As a result, improved targeting can reduce the product portfolio of firms due to the competition-enhancing effect. Reducing the number of products allows firms to escape competition and keep prices high. As a result, improved targeting has also interesting implications for the product portfolio choice of firms, and may contribute to firms becoming more specialized.

5.2 Broader Product Attractiveness

In the baseline analysis, we assumed that a message is lost in case it does not reach a consumer of the targeted group. As explained in Section 3, a reason could be that the advertising message or the product is perfectly tailored to (i.e., "made attractive for") its targeted consumers, which implies that non-targeted consumers are not interested in buying the product.[34] In this subsection, we generalize the model by considering the case in which product appeal is broader, that is, also consumers who do not belong to the targeted group may find the product attractive. This allows us to examine the interaction between targeting of advertising messages and the product attractiveness. In particular, we can determine how a broader attractiveness affects firms' profits and whether the result depends on the targeting ability.

Suppose that a non-targeted consumer who receives an advertising message is interested in the product with probability $\kappa \in (0,1)$.[35] This implies that a message that failed to reach its target is only lost with probability $1-\kappa$. Therefore, sending messages to consumers of the expected targeted group now reaches a mass $\alpha + (1-\alpha)\kappa$ of interested consumers (instead of only α). Consistent with the assumption on the targeting probability α, we assume that κ is the same for both firms, but its realization is independent for the two firms.[36]

Solving for the equilibrium in this scenario proceeds along the same lines as in Section 4, but is considerably more involved due to the distinction of more different cases. For exposition purposes, we therefore restrict attention to the case in which advertising costs c are at or slightly below the upper bound $\hat{c}$, that is, the case analyzed in Section 4.1.[37] The findings are summarized in the next proposition, which shows that the non-monotonicity of targeting carries over (Part 1) and that a broader attractiveness can lead to a fall in firms' profits (Part 2).

Proposition 5. *For any $\kappa \in (0,1)$:*

1. *If $A/B \in (0, 1/(2(\kappa+1)))$, the sum of expected profits is non-monotonic in α. There exists a threshold value for α, denoted by $\hat{\alpha}_4(A/B,\kappa)$, such that the sum of expected profits is strictly decreasing for $\alpha < \hat{\alpha}_4(A/B,\kappa)$, has an upward jump at $\alpha = \hat{\alpha}_4(A/B,\kappa)$, and is strictly increasing for $\alpha > \hat{\alpha}_4(A/B,\kappa)$. Instead, for $A/B \in (1/(2(\kappa+1)), 1/2)$, the sum of expected profits is strictly increasing in α.*

[34] An example is an advertising message, which is tailored to family households and therefore targeted to consumers who are expected to belong to this group; however, the message is not interesting for single households.

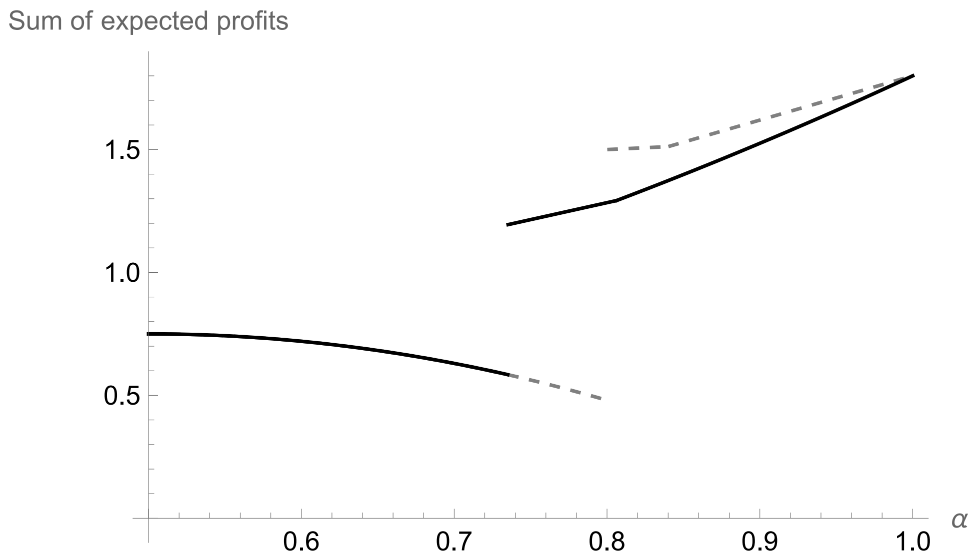
[35] In the main model, we considered the case of $\kappa = 0$.

[36] This assumption is again not crucial, and our main insights carry over to the case of imperfect correlation.

[37] The details can be found in Online Appendix A in the proof of Proposition 5.

2. If $A/B \in (0, 1/(2(\kappa+1)))$, the sum of expected profits is the same for $\kappa > 0$ and for $\kappa = 0$ when $\alpha \leq \hat{\alpha}_4(A/B, \kappa)$, it is strictly larger for $\kappa > 0$ than for $\kappa = 0$ when $\alpha \in (\hat{\alpha}_4(A/B, \kappa), \hat{\alpha}_1(A/B))$, but strictly smaller for $\kappa > 0$ than for $\kappa = 0$ when $\alpha \in (\hat{\alpha}_1(A/B, \kappa), 1)$. Instead, for $A/B \in (1/(2(\kappa+1)), 1/2)$, the sum of expected profits is strictly smaller for $\kappa > 0$ than for $\kappa = 0$.

The first part of the proposition shows that even if product appeal is broader, improved targeting has a non-monotonic effect on firms' expected profits for relatively low values of A/B. The intuition is the same as in the case $\kappa = 0$. For α is small, both firms target consumers of group b. In that region, the effect that improved targeting increases competition dominates the effect that more intended consumers are reached by a message. Instead, for high values of α, market segmentation occurs and the effect of a larger reach dominates.



The sum of expected profits as a function of α for $A/B \in (0, 1/(2(\kappa+1)))$. The decreasing part of the expected profit function arises when both firms target the high consumer type B, and the increasing part arises when firms segment the market. Parameter values are $A = 0.3$, $B = 1.5$, $\kappa = 0.9$ and $\hat{\alpha}_4(A/B, \kappa) = 0.735$, solid line, (resp. $\kappa = 0$ and $\hat{\alpha}_1(A/B, \kappa) = 0.8$, gray dashed line).

Figure 4: Sum of Expected Profits with Broad Tailoring

The region in which the competition-enhancing effect dominates is, however, smaller with broader appeal ($\kappa > 0$) than with narrow appeal ($\kappa = 0$), that is $\hat{\alpha}_4(A/B, \kappa) < \hat{\alpha}_1(A/B)$. Figure 4 illustrates this (i.e., the solid line represents the sum of expected profits with $\kappa > 0$, whereas the dashed line represents these profits with $\kappa = 0$, as in Figure 1). The reason is that a broader product appeal makes segmentation of the market more attractive relative to targeting the same consumer group as the competitor. The profits in the latter equilibrium are not affected by κ due to the fact that firms set prices above A in this equilibrium, which implies that no consumer of the non-targeted group a buys the product. Instead, under market segmentation, a larger κ leads to a raise in the profit of the firm targeting low-valuation consumers, as more of the high-valuation consumers are interested in the firm's product and choose to buy it due to the lower price. Therefore, the region for the equilibrium in which competition for cherries occurs, shrinks.

Instead, if A/B is relatively large, the non-monotonicity result does no longer arise, and the sum of expected profits is strictly increasing in α. This occurs because, in that range, only the segmentation equilibrium exists.

We next explain the second part of the proposition—i.e., how profits differ with broader compared to narrow appeal. Interestingly, as stated in the proposition, despite the fact that broader attractiveness implies that more consumers are interested in the firms' products and market segmentation becomes more likely, the sum of expected profits can be *lower* than with more narrow attractiveness. If A/B is relatively small, this occurs if α is high, as can be seen in the increasing branch in Figure 4. The intuition for this result is again rooted in a competition-enhancing effect. With narrow product appeal, the firms fully segment the market via targeting different consumer groups. This is no longer true when the product appeal is broad because segmentation is less effective. As some consumers of group b are interested in buying the product from the firm that targets group-a consumers, competition occurs for group-b consumers. Therefore, the firm which targets group-b consumers will lower its expected price, leading to a fall in its profit. The proposition shows that this effect dominates the increase in profits of the other firm.

Since market segmentation occurs for a larger range with broad product appeal than with narrow product appeal—and the sum of profits is larger in the segmentation equilibrium than in the equilibrium in which both firms target high-valuation consumers—the overall effect of a broad product appeal on profits is not clear-cut. For intermediate values of α (i.e., $\alpha \in [\hat{\alpha}_4(A/B, \kappa), \hat{\alpha}_1(A/B)]$), a broad appeal increases profits, whereas for high values of α (i.e., $\alpha \in (\hat{\alpha}_1(A/B), 1)$), it decreases profits. As a consequence, the interplay between targeting and the breadth of product appeal shows that a broader appeal is detrimental if targeting is relatively good, but beneficial if targeting is only moderate.

If A/B is relatively large, expected profits always fall as the product attractiveness gets broader. This result occurs for the same reasons as above—i.e., a larger κ intensifies competition by weakening the effectiveness of market segmentation.

In summary, the effect that profits are non-monotonic in the success probability of targeting still occurs if the product appeal is broader. In addition, although a broader product appeal increases the potential demand of each firm, it does not necessarily lead to an increase in profits. Instead, if targeting is sufficiently good, a broader product appeal lowers the effectiveness of segmentation, and reduces profits.

6 Simultaneous Targeting and Pricing Decisions

In the main model, we assume that firms first choose their targeting strategy and then set prices. In this section, we consider the situation with simultaneous targeting and pricing decisions. As explained in Section 3, this describes industries with short product life cycles and high turnover in which the targeting strategy of a firm can be changed relatively quickly.

To simplify the exposition, we focus in the main text on the case of high advertising costs, i.e., $c = \hat{c}$. In Online Appendix C, we derive the equilibrium for all values of $c \in [0, \hat{c}]$.

For $\alpha \leq \hat{\alpha}_1(A/B)$, the equilibrium is the same as in the sequential game. In this equilibrium, both firm target consumer group b, play a mixed pricing strategy, and obtain an expected profit of $\alpha(1-\alpha)B - c$. There is no difference in the outcomes between the sequential and the simultaneous game, as a deviation to target consumer group a leads to the same profit as in the sequential game (due to the fact that products a and b do not compete). Therefore, in this region, equilibrium profits fall in α, as in the sequential game.

Instead, for $\alpha > \hat{\alpha}_1(A/B)$, the market-segmentation equilibrium of the sequential game does no longer exist. The reason is that the firm that targets consumer group a has a profitable deviation to target group b and undercut the rival's price $p = B$. Due to this deviation in the targeting strategy and the price setting, no market-segmentation equilibrium in pure strategies occurs with simultaneous choices. However, there exists an asymmetric mixed-strategy equilibrium.[38] In this equilibrium, one firm does not only mix in its prices, but also in its targeting strategy. In fact, it targets each consumer group with positive probability. When targeting group a, its price is equal to A and otherwise it is mixed. Instead, the other firm targets consumer group b with certainty and mixes prices. The prices are chosen in a way that the expected profit is nevertheless symmetric and equal to $\alpha A - c$.

This allows us to derive the following result:

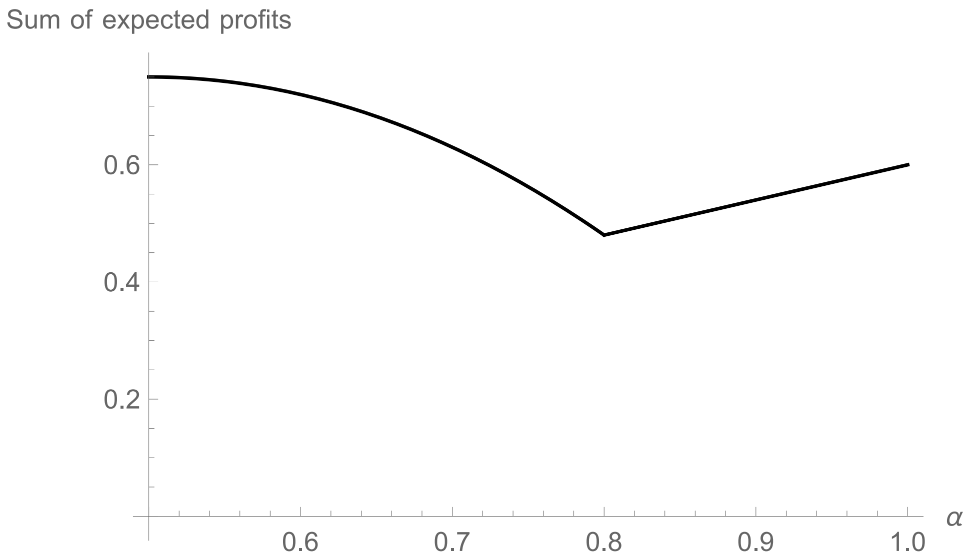
Proposition 6. *If $A/B < 1/2$, the sum of expected profits in the profit-dominant Nash equilibrium*[39] *is non-monotonic in α. It strictly decreases for $\alpha \in [1/2, \hat{\alpha}_1(A/B)]$ and*

[38] We derive this result in the proof of Proposition 6 in Online Appendix B.

[39] As we show in the proof of this proposition, there exists another equilibrium where both firms target consumer groups a and b with the same positive probability but each firm's expected profit is lower than in the two equilibria stated above.

strictly increases for $\alpha \in (\hat{\alpha}_1(A/B), 1]$. *Moreover, the sum of expected profits is lower at* $\alpha = 1$ *than at* $\alpha = 1/2$ *if* $A \leq B/4$.

Figure 5 illustrates this result. In contrast to the simultaneous game, the sum of expected profits is continuous everywhere. The reason is that the change in equilibrium regions at $\alpha = \hat{\alpha}_1$ occurs continuously, that is, for α slightly above $\hat{\alpha}_1(A/B)$, the firm which randomizes in its targeting strategy still chooses to target consumers in group b with a probability close to 1. As α increases, this probability falls and the firm puts more and more weight on targeting group a. However, even at $\alpha = 1$, the firm targets consumer group b with a positive probability, that is, full market segmentation is never achieved



The sum of expected profits as a function of the probability of successfully targeting α. The decreasing part of the expected profit function arises when both firms target the high consumer type B, and the increasing part arises when firms segment the market with a positive probability. Parameter values are $A = 0.3$, $B = 1.5$ and $\hat{\alpha}_1(A/B) = 0.8$.

Figure 5: Expected Profits in the simultaneous game with high advertising costs

It is interesting to note that the sum of expected profits can be higher or lower under perfect targeting ($\alpha = 1$) than under no targeting ($\alpha = 1/2$) depending on whether A is larger or smaller than $B/4$. This occurs because market segmentation is only partial even at $\alpha = 1$.

7 Advertising on Publishers

In this section, we consider an extension of our baseline model in which firms send messages via publishers, i.e., operators of websites, such as news providers, influencers, etc. Consumers visit the websites of publishers, and firms can place ads on these websites. Therefore, targeting now works not directly through the firms but through the publishers who possess (imperfect) information about whether a consumer has high or low valuation. We will show that our main results also hold in that case.

We modify our baseline model as follows: There are two publishers, denoted by x and y. Consumers visit the outlets of both publishers. Each publisher receives a signal whether a consumer belongs to group a or b, where the signals are independent but, as in the baseline model, each signal is correct with probability $\alpha \in [1/2, 1]$. A publisher has one advertising slot per consumer, which can be bought by firm 1 or 2. We denote by j_k the slot on publisher $j \in \{x, y\}$ to advertise to a consumer whom the respective publisher expects to belong to group $k \in \{a, b\}$.

To focus on the interaction between targeting and advertisers' strategies, we restrict our attention to the case at which prices for advertising slots are exogenous and negligibly small. The former simplifies the analysis—i.e., an endogenous advertising price would shift the focus to the game between publishers, which, while in general being interesting, is beyond the scope of this paper—while the latter is motivated by the fact that prices for online ads are usually low. For instance, the average price for an online ad in 2023 is \$0.61 in the U.S. and less than half of this amount in many European countries, such as Germany, the Netherlands, or Norway.[40] Therefore, the expected revenue of firms is in general substantially larger than the price for the advertisement.

The timing of the extended game is as follows: In the first stage, each firm chooses the consumer group it targets with its product (i.e., either a or b). After observing these choices, firms decide about their advertising strategies, that is, which of the four available slots a firm is willing to buy. In the third stage, given the allocation of slots, each firm sets the price for its product. Thereafter, each consumer makes a purchase decision, dependent on the advertising messages she received and the prices of the firms. The solution concept is subgame perfect Nash equilibrium, with the refinement that, if both firms are willing to buy the same slot(s) in the first stage, the equilibrium that gives the higher pay-off to the firm with the lower profit is selected. This assumption is a simple way to ensure competition between firms in case they target the same consumers. In addition, we focus on cases in which both publishers are active, that is, firms buy at least one slot on each publisher.

The main difference to the baseline model is that firms may now place ads on the outlets of both publishers, which implies that a firm can reach the same consumer on different outlets. This was not possible in case firms send messages directly to consumers because a firm then never reaches a consumer twice, but decides whether to advertise to

[40] See e.g. Topdraw (2021) or Smartyads (2023).

consumers it expects to belong to group a, group b, or send messages to all consumers. Reaching a consumers twice implies that the message is lost as the consumer is already informed, but also dampens competition, as the consumer can then not be reached by the rival.

As we show, this possibility gives rise to interesting new advertising configurations that are not possible in the baseline model. Nevertheless, the next proposition states that it does not change our main insights.

Proposition 7. *If advertising occurs via publishers, the following holds:*

1. *If $A/B > 1/3$, the sum of firms' profits is constant in α.*
2. *If $1/3 \geq A/B \gtrapprox 0.277$, the sum of expected profits is decreasing in α for α below a threshold implicitly defined by $\alpha(1-\alpha)/(1-\alpha+\alpha^2) = A/B$, jumps upward at this threshold and then stays constant in α.*
3. *Finally, if $A/B \lessapprox 0.277$, the sum of expected profits is decreasing in α for $\alpha < \breve{\alpha}$, where $\breve{\alpha}$ is implicitly defined by $1 - \breve{\alpha} - \breve{\alpha}^3 = 0$ and approximately equal to 0.682, jumps upward at $\alpha = \breve{\alpha}$, increases for α between $\breve{\alpha}$ and a threshold implicitly defined by $\alpha(1-\alpha^2)/(2-\alpha) = A/B$, jumps upward again at this threshold, and then stays constant in α.*

As in the baseline model, improved targeting lowers firms' aggregate profits for relatively small values of α if A/B is sufficiently low. In that case, both firms target consumer group b, and an increase in α leads to an increase in the mass of overlapping consumers, which makes competition more aggressive. Instead, if A/B is relatively large (i.e., $A/B > 1/3$), market segmentation occurs for all values of α. In that case, one firm buys both slots on publisher x and the other firm both slots on publisher y, which implies that each firm reaches all consumers, and profits do not depend on α, as stated in part *1.* of the proposition. We next explain the statements *2.* and *3.* in more detail, as different equilibrium configurations emerge compared to the baseline model.

First, for α close to 1/2, both firms target group b. In equilibrium, one firm—e.g., firm 1—buys slots x_b and y_a—i.e., the slots to advertise to consumers expected to belong to group b on platform x and to consumers expected to belong to group a on platform y—whereas the other firm buys only slot y_b—i.e., the slot to advertise to consumers expected to belong to group b on platform y. This asymmetric equilibrium is similar to that of Section 4.2.[41] However, in contrast to that equilibrium, firm 1 does not advertise to all

[41] The equivalent to the equilibrium in Section 4.2 would be an allocation in which firm 1 buys slots x_b and x_a. It then reaches all consumers via one platform in the same way as by directly advertising to all consumers.

consumers but reaches some consumers twice, namely a fraction of $\alpha(1-\alpha)$ who platform x expects to belong to group b but platform y expects to belong to group a. Although this implies that firm 1's total reach is smaller, buying slot y_a instead of x_a leads to a smaller overlap with consumers that are also reached by the rival firm 2. In fact, when buying slot y_a, the mass of consumers who receive ads from both firms is α^2, that is, only those consumers that each publisher expects to belong to group b. Instead, if firm 1 would buy slot x_a, it would reach all consumers and thereby also all consumers reached by firm 2, which is a mass of α. This effect of reduced overlap dampens competition and dominates the effect of lower reach. However, similar to the result in Section 4.2, an increase in α also leads to an increase in the number of overlapping consumers, and thereby reduces the sum of expected profits.

Similar to the baseline model, if α is large enough, the profit of firm 2 when targeting group b becomes too small and the firm is better of by targeting group a. As competition does then no longer arise, the sum of profits jumps upward at the threshold value. This explains statement *2.*, which occurs for intermediate values of A/B.

Instead, if A/B is relatively small, a third region occurs for intermediate values of α. In this region, both firms still target group b but the allocation of slots is different compared to the equilibrium above. In particular, all advertising slots are bought by the two firms. However, the equilibrium is still asymmetric, because one firm, say firm 1, buys both slots to advertise to expected group-b consumers—i.e., slots x_b and y_b—whereas firm 2 buys the two slots x_a and y_a. This leads to a mass $2\alpha(1-\alpha)$ of overlapping consumers. This equilibrium emerges as the benefit from targeting consumers of group a is relatively small because A is relatively small, which implies that it pays off for firm 2 to target group b also for intermediate values of α but buy more slots. In this equilibrium, firm 2 balances the trade-off between higher reach and greater overlap. Specifically, the overlap of $2\alpha(1-\alpha)$ is still relatively small compared to other configurations in which firms buy all slots. As α increases, the sum of firms' profits increases, as the overlap between firms' consumers gets smaller. However, there still exists a threshold value for α such that firm 2 prefers to target group a if α is above this value. This is due to the fact that, although the sum of profits is increasing, firm 2's profit falls in α as it reaches fewer consumers who are interested in its product. At this threshold, we move to a segmentation equilibrium, at which there is no competition between firms and aggregate profits jump upward. This explains the third statement in the proposition.

Overall, these results show that our main effects continue to hold in a simple, yet natural, extension where firms advertise via publishers. In addition, a new effect emerges in that case. I.e., reaching the same consumer twice can be profitable despite the fact that the second message does not bring additional benefits, as it prevents the rival from reaching that consumer and therefore lowers price competition. We show how this plays out through different equilibrium configurations as the probability of targeting success changes.

8 Conclusion

The paper analyzes the effects of improved targeting on firms' optimal targeting strategies. An improvement in targeting implies that an advertising message reaches the intended consumer with a greater probability, which is in general beneficial to firms. However, as this holds for both firms, the probability that a consumer receives messages from both firms increases as well, which enhances competition. We show that this trade-off has profound consequences on the optimal targeting strategies.

If the success probability is relatively small, both firms target high-valuation consumers. An increase in targeting then leads to a larger number of consumers who receive messages from competing firms, which lowers expected prices and profits. Instead, if the success probability is rather high, firms target different consumer groups to avoid fierce competition. Improved targeting then increases a firm's reach, and therefore also leads to an increase in profits. Firms' profits in aggregate therefore change non-monotonically in the targeting ability: They first decrease as targeting improves and then increase.

These results also have implications on the optimal number of products that a firm offers. A higher probability of reaching the intended consumer makes it more profitable to introduce an additional product. However, as such a higher probability also leads to fiercer competition, there is a countervailing force. We find that the first effect dominates if advertising costs are rather large, as firms then offer only few products and introducing additional ones has only a moderate effect on competition. Instead, if advertising costs are small, the competition-enhancing effect dominates, and better targeting induces firms to focus on a small number of products.

We show that our results also hold if a firm's product is—with some probability—interesting to multiple consumer groups. Moreover, they also carry over to cases in which targeting strategies are rather short term and in which advertising occurs via publishers (e.g., internet platforms).

An important managerial implication of our analysis is that improved targeting does not only make advertising more efficient, but also affects the competitiveness of an industry and thereby the optimal targeting strategy. In particular, while a natural reaction to an increased reach of an advertising campaign could be to expand the campaign and target high-value consumers, it is important to understand that other firms are likely to pursue the same strategy. As this makes an industry more competitive, a more profitable strategy can be to target a consumer group that promises smaller revenues, but thereby differentiate from competitors. A company can then benefit more from the improved targeting possibilities.

Another implication is that changes in the targeting success may not only influence a firm's targeting strategy, but also affects the profitability of tailoring products to specific consumer groups. Offering a product that appeals to a broader consumer base allows for larger sales, but can also tap into the business of rivals, thereby heating up competition. If targeting is particularly accurate, it can then be more profitable to design products with a more narrow appeal. This helps a company to focus on its core consumer group, thereby segmenting the market.

We conclude by briefly discussing two interesting possibilities for future research that emerge from our analysis. First, the success probability of targeting is determined by the state-of-the-art technology as well the ability of data collection and therefore given for both firms. This is reasonable for our purposes as we wish to analyze the effect of improved targeting on the market outcome. Alternatively, firms may invest in the possibility to target consumers, with higher investment implying better targeting. Firms' targeting abilities could then differ. Analyzing such a model could be a fruitful direction to understand such investment incentives and how the competitive environment shapes these decisions. Second, in the previous section we analyzed the case in which firms send advertising messages via publishers. In order to focus on our main effect, we considered the publishers as passive, that is, they do not set fees for their advertising slots. An interesting, yet potentially challenging, direction for future research is to analyze a game in which publishers actively set fees for their advertising slots (or run auctions to sell these slots), given that firms choose the consumer group to sell to and that publishers' information about consumers is not perfect. This could yield interesting insights into the interplay between publishers' and advertisers' incentives, and how this interplay affects optimal targeting strategies.

Appendix

Proof of Lemma 2. We first characterize the asymmetric mixed-strategy equilibrium in which both firms target consumer group b, firm 1 only sends messages to consumers the firm expects to belong to group b, and firm 2 sends messages to all consumers. Suppose firm 1 plays a mixed strategy and draws its price from the cdf $G(p_1)$ on $[(1-\alpha)B, B]$. Then, from the arguments given in the main text, firm 2 is indifferent between setting any price on $[(1-\alpha)B, B]$ if

$$p(1 - G(p)) + (1-\alpha)pG(p) = (1-\alpha)B.$$

This yields

$$G(p) = \frac{1}{\alpha} - \frac{(1-\alpha)B}{\alpha p},$$

with $G((1-\alpha)B) = 0$ and $G(B) = 1$. Note that $G(p) = F(p)$ in (1).

Suppose next that firm 2 plays a mixed strategy and draws its price from the cdf $H(p_2)$ on $[(1-\alpha)B, B]$. Then, firm 1 is indifferent between setting any price on $[(1-\alpha)B, B]$ if

$$\alpha p \cdot (1 - H(p)) + 0 \cdot H(p) = \alpha(1-\alpha)B.$$

This is equivalent to

$$H(p) = \begin{cases} \frac{p-(1-\alpha)B}{p}, & \text{if } p \in [(1-\alpha)B, B); \\ 1, & \text{if } p = B. \end{cases}$$

with $H\left((1-\alpha)B\right) = 0$ and a mass point of $1-\alpha$ at $p = B$. The resulting expected profits are $\pi_1 = \alpha(1-\alpha)B$ and $\pi_2 = (1-\alpha)B$.

We next show that, given both firms target group b, no firm has an incentive to change its number of messages. First, starting from the situation in which each firm sends a message only to consumers it expects to belong to group b, firm 2's expected profit is strictly increasing if more consumers of the other group receive a message. Indeed, denoting by X the mass of consumers of the other group who receive a message from firm 2, firm 2's expected profit is $(\alpha + X(1-\alpha))(1-\alpha)B$, which rises in X.[42] Therefore, given that firm 1 only sends a message to consumers who are expected to belong to group

[42] It is evident from this formula that the additional profit firm 2 obtains with $X = 1$ as compared to $X = 0$ is $(1-\alpha)^2B$. In case advertising is costly, firm 2 therefore optimally sends messages to all consumers only if $(1-\alpha)^2B \geq c$. It follows that for c slightly below $\hat{c}$ as in Section 4.1, no firm has an incentive to send messages to all consumers because $(1-\alpha)^2B$ is strictly below $\hat{c}$, which equals $\alpha(1-\alpha)B$ in this range.

b, it is optimal for firm 2 to send a message to all consumers.

Second, we turn to firm 1 and show that it is optimal for firm 1 to send a message only to consumers who are expected to belong to group b, given that firm 2 sends messages to all consumers. Suppose that firm 1 instead sends messages to a mass Y of consumers who are expected to belong to group a. The expected profits of the two firms can then be derived in a similar way as in the main text to get

$$E[\pi_1(p_1, p_2)] = \begin{cases} p_1\left(\alpha + Y(1-\alpha)\right), & \text{if } p_1 < p_2; \\ \frac{\alpha+Y(1-\alpha)}{2} p_1, & \text{if } p_1 = p_2; \\ 0, & \text{if } p_1 > p_2, \end{cases}$$

and

$$E[\pi_2(p_1, p_2)] = \begin{cases} p_2, & \text{if } p_2 < p_1; \\ (1-Y)(1-\alpha)p_2 + \frac{\alpha+Y(1-\alpha)}{2} p_2, & \text{if } p_2 = p_1; \\ (1-Y)(1-\alpha)p_2, & \text{if } p_2 > p_1. \end{cases}$$

By the same arguments as in the main text, the lower bound of firm 2's mixing domain is then $(1-Y)(1-\alpha)B$, and firm 1's expected equilibrium profit in the second stage would be $(1-Y)(1-\alpha)\left(\alpha + Y(1-\alpha)\right)B$. Taking the derivative with respect to Y yields $1-2\alpha-2Y(1-\alpha)$, which is strictly negative for all $Y \in [0,1]$ due to the fact that $\alpha > 1/2$. Hence, the asymmetric equilibrium configuration in which firm 1 sends a message only to consumers the firm expects to belong to group b and firm 2 sends a message to all consumers is indeed the equilibrium in case both firms target group b.

The threshold dividing this region from the one in which firms target consumers in different groups is derived in the main text.

□

Proof of Proposition 3. We first consider potential equilibria in which both firms target group b. From Lemmas 1 and 2, we know that for high costs each firm then sends messages only to consumers who are expected to belong to the targeted group and obtain a profit of $\alpha(1-\alpha)B-c$. We denote this configuration by $b(1)b(1)$. Instead, for low advertising costs, one of the firms sends messages to all consumers and obtains a profit of $(1-\alpha)B - 2c$, whereas the other firm sends messages only to expected group-b consumers and obtains the same profit as in configuration $b(1)b(1)$. It follows that the former configuration occurs if $\alpha(1-\alpha)B - c \geq (1-\alpha)B - 2c$ or

$$c \geq (1-\alpha)^2 B,$$

whereas the latter configuration occurs if $c < (1-\alpha)^2 B$. We denote it by $b(1)b(2)$.

As α increases, for each potential equilibrium configuration $b(1)b(1)$ and $b(1)b(2)$, there is a threshold such that the respective equilibrium is no longer valid, as a firm has a profitable deviation to target group a.

We first consider the potential equilibrium configuration $b(1)b(1)$. As shown in Section 4.1, if

$$\alpha > 1 - \frac{A}{B} = \hat{\alpha}_1(A/B), \tag{3}$$

such a deviation to target group a is profitable for one of the firms.[43] The right-hand side of (3) does not depend on c, which implies that this threshold is also valid for lower values of c than $\hat{c}$. Using the result of Lemma 1, if (3) holds, the equilibrium involves one firm targeting group a and sending messages only to expected group-a consumers, whereas the other firm targets group b and sends messages to all consumers if $c \leq (1-\alpha)B$ or only to expected group-b consumers if $c > (1-\alpha)B$. We denote the former configuration by $a(1)b(2)$ and the latter configuration by $a(1)b(1)$. In addition, we need to check whether the firm targeting group-a consumers may also has an incentive to send messages to all consumers. Denoting such a configuration by $a(2)b(2)$, it is easy to check that it can only exist if $c < (1-\alpha)A$.

We can now insert the threshold value for α given by the right-hand side of (3) into $(1-\alpha)A$, which gives A^2/B. Similarly, inserting this threshold value into $(1-\alpha)^2B$—i.e., the boundary between the configurations $b(1)b(1)$ and $b(1)b(2)$—also yields A^2/B. This implies that the boundary between the configurations $b(1)b(1)$ and $b(1)b(2)$ and the boundary between the configurations $a(1)b(2)$ and $a(2)b(2)$ is the same at $\alpha = \hat{\alpha}_1(A/B)$. Therefore, this value of α is indeed the boundary for the equilibrium region $b(1)b(1)$ due to the fact that in the region for which this equilibrium exists, the deviation to target group a *and* send messages only to the expected targeted group is more profitable than the deviation to target group a and send messages to all consumers. Therefore, the equilibrium $b(1)b(1)$ occurs if $c \geq (1-\alpha)^2B$ and $\alpha \leq \hat{\alpha}_1(A/B)$. It follows from this boundary that the equilibrium $b(1)b(1)$ can only occur if $c \geq A^2/B$.

We next turn to the potential equilibrium $b(1)b(2)$. Following the analysis of Section 4.2, the deviation by the firm with the lower profit to target group a and send messages to all consumers is profitable if $\alpha(1-\alpha)B - c < A - 2c$, or

$$\alpha > \frac{1 + \sqrt{1 - 4\frac{A-c}{B}}}{2} = \hat{\alpha}_3((A-c)/B).^{44} \tag{4}$$

Inserting $c = (1-\alpha)^2B$—i.e., the threshold value that separates the configurations

[43]We first focus on the case in which $\hat{\alpha}_1(A/B)$ is larger than 0.5. When writing down the sum of expected profits in each equilibrium region, we also consider the case $\hat{\alpha}_1(A/B) \leq 0.5$.

[44]Again, we focus on the case $\hat{\alpha}_3((A-c)/B) > 0.5$ first.

$b(1)b(2)$ and $b(1)b(1)$—into the right-hand side of (4) yields $\hat{\alpha}_1(A/B)$, which is the same as the right-hand side of (3). In addition, as shown above, it is also the same threshold value as the one that separates the regions for $a(1)b(2)$ and $a(2)b(2)$ at $c = (1-\alpha)A$. It follows that in the region for which the equilibrium $b(1)b(2)$ exists, the deviation to target group a *and* send messages to all consumers is more profitable than the deviation to target group a and send messages only to the expected targeted group; hence, the boundary for the equilibrium region $b(1)b(2)$ is indeed determined by the comparison between $b(1)b(2)$ and $a(2)b(1)$. Therefore, equilibrium $b(1)b(2)$ occurs if $c < (1-\alpha)^2 B$ and $\alpha \leq \hat{\alpha}_3((A-c)/B)$.

Finally, we turn to the region in which α is above the thresholds given by the right-hand sides of (3) and (4). By the arguments above, for $\alpha > \hat{\alpha}_3((A-c)/B)$ and $c < (1-\alpha)A$, the equilibrium is $a(2)b(2)$. Instead, for $c \geq (1-\alpha)A$, the equilibrium is either $a(1)b(2)$ or $a(1)b(1)$. As stated above, the boundary between these regions is $c = (1-\alpha)B$. Therefore, the equilibrium configuration is $a(1)b(2)$ if $\alpha > \hat{\alpha}_1(A/B)$ and $(1-\alpha)A \leq c \leq (1-\alpha)B$. Instead, the equilibrium configuration is $a(1)b(1)$ if $\alpha > \hat{\alpha}_1(A/B)$ and $c > (1-\alpha)B$. The equilibrium configurations can then be displayed as in Figure 3.

We can now determine how the sum of profits changes with α for any value of c. From Figure 3, for all $c \in (0, \hat{c})$, as α increases from 0.5 to 1, at most four different regions emerge. The value of c at which the boundaries of the equilibrium regions $b(1)b(2)$, $a(2)b(2)$, $b(1)b(1)$, and $a(1)b(2)$ intersect is equal to A^2/B. Therefore, for $c \in (0, A^2/B]$, if $\hat{\alpha}_3((A-c)/B) > 0.5$ the equilibrium changes from $b(1)b(2)$ to $a(2)b(2)$ to $a(1)b(2)$ and finally to $a(1)b(1)$, as α increases from 0.5 to 1. Instead, if $\hat{\alpha}_3((A-c)/B) < 0.5$, the configuration $b(1)b(2)$ does not exist. The sum of expected profits is therefore

$$\sum_i E[\pi_i^\star(\alpha)] = \begin{cases} (1-\alpha^2)B - 3c, & \text{for } \alpha \in [0.5, \hat{\alpha}_3((A-c)/B)) \text{ if } \hat{\alpha}_3((A-c)/B)) > 0.5; \\ A + B - 4c, & \text{for } \alpha \in [\max\{0.5, \hat{\alpha}_3((A-c)/B)\}, 1 - c/A); \\ \alpha A + B - 3c, & \text{for } \alpha \in [1 - c/A, 1 - c/B); \\ \alpha(A+B) - 2c, & \text{for } \alpha \in [1 - c/B, 1]. \end{cases}$$

It is easy to see that the sum of expected profit changes non-monotonically in α if $\hat{\alpha}_3((A-c)/B) > 0.5$, which holds if $A < B/4 + c$. In example, the sum falls for $\alpha < \hat{\alpha}_3((A-c)/B)$, jumps upwards at $\alpha = \hat{\alpha}_3((A-c)/B)$, and rises for $\alpha > \hat{\alpha}_3((A-c)/B)$.

Finally, for $c \in (A^2/B, \hat{c}]$, if $\hat{\alpha}_1(A/B) > 0.5$, the equilibrium changes from $b(1)b(2)$ to $b(1)b(1)$ to $a(1)b(2)$ and finally to $a(1)b(1)$ as α increases from 0.5 to 1. Instead, if $\hat{\alpha}(A/B) \leq 0.5$, the configurations $b(1)b(2)$ and $b(1)b(1)$ do not emerge in equilibrium.

The sum of expected profits is therefore

$$\sum_i E[\pi_i^\star(\alpha)] = \begin{cases} (1-\alpha^2)B - 3c, & \text{for } \alpha \in \left[0.5, 1-\sqrt{c/B}\right] \text{ if } \hat{\alpha}_1(A/B) > 0.5; \\ 2\alpha(1-\alpha)B - 2c, & \text{for } \alpha \in \left(1-\sqrt{c/B}, \hat{\alpha}_1(A/B)\right] \text{ if } \hat{\alpha}_1(A/B) > 0.5; \\ \alpha A + B - 3c, & \text{for } \alpha \in (\max\{0.5, \hat{\alpha}_1(A/B)\}, 1-c/B]; \\ \alpha(A+B) - 2c, & \text{for } \alpha \in (1-c/B, 1]. \end{cases}$$

It is again easy to see that the sum of expected profits changes non-monotonically in α for $\hat{\alpha}_1(A/B) > 0.5$.

□

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Imperfect Targeting and Advertising Strategies
Online Appendix (not for publication)

This Online Appendix consists of three sections. In Section A, we provide the proofs of the propositions of Section 5. In Section B, we provide the proofs of the propositions of Sections 6 and 7. Finally, in Section C, we provide the analysis of the game with simultaneous targeting and price setting for the full range of c.

A Proofs of Propositions in Section 5

Proof of Proposition 4. The proof proceeds as follows. Starting from the results obtained in case each firm can offer only one product—i.e., the results of Proposition 3, displayed in Figure 3—we divide the range of c in three different regions. These are the regions $c > A/2$, $\max\{\alpha(1-\alpha)A, A^2/B\} < c \leq A/2$, and $0 \leq c \leq \max\{\alpha(1-\alpha)A, A^2/B\}$. For each region, we determine the incentives of firms to introduce an additional product and the resulting equilibrium. We then show how the equilibrium number of products changes with α in each region.

Region $c > A/2$. We first determine whether it is profitable for one or both firms to introduce product a.[1] If $\alpha A - c \geq 0$ or $c \leq \alpha A$, it is profitable for one firm to introduce product a and advertise only to the expected targeted group. Since $\alpha > 1/2$, this is indeed possible in the parameter range $c > A/2$. However, it is not profitable for a firm to introduce product a and send messages to both consumer groups, as this would lead to a profit of $A - 2c$, which is negative for $c > A/2$. Finally, it can never be profitable that both firms offer product a. By the same result as in Section 4.1, the profit of the firms in case they both send messages to the expected targeting group is $\alpha(1-\alpha)A - c$, which is negative for $c > A/2$ due to the fact that $\alpha > 1/2$. Similarly, by the result of Section 4.2, if firms send a different number of advertising messages, the profit of the firm with the lower number of advertising messages is again $\alpha(1-\alpha)A - c$. This implies that for $A/2 < c \leq \alpha A$, one firms introduces product a (and sends messages only to consumers of the expected targeted group), whereas for $c > \alpha A$, no firm introduces product a.

If $\alpha \leq 1 - A/B$ and $c \leq (1-\alpha)^2 B$, both firms offer product b in case they can offer only one product, and one firm sends messages to all consumers whereas the other firm sends messages only to consumers in the expected targeted group (cf. Section 4.2). Following the notation of the proof of Proposition 3, this equilibrium is denoted by $b(2)b(1)$. As

[1] Here and in what follows we abbreviate the product intended for consumer group $i \in \{A, B\}$ by product i.

just shown, a firm then has the incentive to become a two-product firm—i.e., offer an additional product a—if $\alpha A \geq c$ or $\alpha \geq c/A$.[2] The threshold value c/A is always below $1 - A/B$—i.e., the range in which this comparison is relevant. This is because the upper bound on c is given by $\hat{c} = \max\{\alpha A, \alpha(1-\alpha)B\}$, which equals $A(1 - A/B)$ at $\alpha = 1 - A/B$. Therefore, for $c < \hat{c}$, we have $c/A < 1 - A/B$.

For $\alpha \leq 1 - A/B$ and $c > (1-\alpha)^2 B$, the equilibrium in the case in which firms can only offer one product is $b(1)b(1)$ (cf. Section 4.1). As the introduction of product a is independent of product b, the threshold to become a multi-product firm is again given by $\alpha \geq c/A$.

Therefore, for $\alpha \leq 1 - A/B$, the equilibrium number of products in the market is 2 for $\alpha < c/A$—i.e., both firms only offer product b—and 3 for $\alpha \geq c/A$—i.e., one firm offers both products and the other one product b.

We now turn to the range $\alpha > 1 - A/B$. If $c \leq (1-\alpha)B$ and firms could offer only one product, one firm offers product b and sends messages to all consumers and the other firm offers product a and sends messages only to consumers that it expects to belong to group a—i.e., equilibrium $b(2)a(1)$. The latter firm then has the incentive to additionally offer also product b if $\alpha(1-\alpha)B \geq c$ or

$$\alpha \leq \frac{1}{2} + \frac{\sqrt{B(B-4c)}}{2B}.$$[3]

The right-hand side of the last inequality is strictly above $1 - A/B$. In fact, it is equal to $1 - A/B$ at the upper bound of c given by $A(1 - A/B)$ and strictly decreasing in c.

Finally, for $c > (1-\alpha)B$ the equilibrium is $a(1)b(1)$ in case firms can offer only one product. It is then never optimal for a firm to offer an additional product as the highest additional profit from this action is $\alpha(1-\alpha)B - c$, which is negative for $c > (1-\alpha)B$.

It follows that for $\alpha > 1 - A/B$, the equilibrium number of products in the market is 3 for $\alpha < 1/2 + \sqrt{B(B-4c)}/(2B)$ and 2 if the reverse holds true.

Taken these results together, we obtain that in case $c > A/2$, the equilibrium number of products is 2 for $\alpha < c/A$, it is 3 for $c/A \leq \alpha < 1/2 + \sqrt{B(B-4c)}/(2B)$, and 2 again for $\alpha \geq 1/2 + \sqrt{B(B-4c)}/(2B)$. Therefore, the number of products changes non-monotonically with α.

Region $\max\{\alpha(1-\alpha)A, A^2/B\} < c \leq A/2$. In this parameter range, as in the first one, it is never optimal that both firms introduce product a because $c > \alpha(1-\alpha)A$.

[2] As there is no competition with product b, this incentive is the same regardless of whether considering the firm that sends messages to all consumers or the firm that sends messages only to consumers in the expected targeted group b.

[3] Because $c > A/2$, the firm offering product b does not have an incentive to additionally offer product a.

However, it is optimal for a single firm to introduce product a as $\alpha A - c \geq 0$ due to the fact that $c < A/2$ and $\alpha > 1/2$. Moreover, as $c > A^2/B$, we are still in the range in which the equilibrium $b(2)a(2)$—i.e., firms offer different products but each one sends messages to all consumers—does not arise in case firms can only offer one product (cf. proof of Proposition 3).

We again first consider the parameter range $\alpha \leq 1 - A/B$, in which both firms offer product b in the single-product case. As the introduction of product a is independent of product b, the incentive for a firm to offer product a is the same regardless of the whether the equilibrium is $b(2)b(1)$ or $b(1)b(1)$. In both cases, for $\max\{\alpha(1-\alpha)A, A^2/B\} < c \leq A/2$, it is optimal for one firm to offer product a in addition to product b. Turning to the parameter range $\alpha > 1 - A/B$, the threshold determined in the analysis for the range $c > A/2$ is still relevant as the equilibrium in the single-product case is the same for the regions $c > A/2$ and $\max\{\alpha(1-\alpha)A, A^2/B\} < c \leq A/2$.

It follows that in the range $\max\{\alpha(1-\alpha)A, A^2/B\} < c \leq A/2$, the equilibrium number of products in the market is 3 for $\alpha < 1/2 + \sqrt{B(B-4c)}/(2B)$ and 2 for $\alpha \geq 1/2 + \sqrt{B(B-4c)}/(2B)$. Therefore, the number of products is (weakly) decreasing in α.

Region $c \leq \max\{\alpha(1-\alpha)A, A^2/B\}$. In this region, there are two differences to the previously analyzed regions. First, if $c \leq \alpha(1-\alpha)A$, it is optimal for both firms to offer product a in addition to product b, which implies that in total 4 products are offered in equilibrium. Second, if $c \leq A^2/B$, there is the additional equilibrium $a(2)b(2)$ in the single-product case. We now consider these two differences in turn.

If $c \leq \alpha(1-\alpha)A$, or $\alpha \leq 1/2 + \sqrt{A(A-4c)}/(2A)$, and both firms offer product b in the single-product case, they now both have an incentive to introduce product a in addition. This follows from the analysis of Section 4.1, as the additional profit of each firm from this introduction is at least $\alpha(1-\alpha)A - c$. There can be two equilibria in which each firm offers both products. These equilibria are distinguished by the number of messages that firms send. If $(1-\alpha)^2 A > c$, or $\alpha < 1 - \sqrt{c/A}$, then it pays off for one firm to send messages to all consumers for its additional product a, whereas the other firm sends messages only to consumers of the expected targeted group. Instead, if $\alpha \geq 1 - \sqrt{c/A}$, both firms send messages to consumers only in the expected targeted group. In each equilibrium, the total number of products in the market is 4.

From Proposition 3, the region in which both firms offer only product b in the single-product equilibrium is valid for $\alpha \leq \left(1 + \sqrt{1 - 4(A-c)/B}\right)/2$. Instead, for $\left(1 + \sqrt{1 - 4(A-c)/B}\right)/2 < \alpha \leq 1 - c/A$, the equilibrium $a(2)b(2)$ arises, as long as $c \leq A^2/B$. For $c \leq \alpha(1-\alpha)A$, the same equilibrium distinction as before occurs, that is, both firms offer both products but dependent on whether $(1-\alpha)^2 A$ is larger or smaller than c, they either send a different or the same number of messages. Still, in each equilibrium, the total number of products in the market is 4.

Instead, for $c > \alpha(1-\alpha)A$ (i.e., $A^2/B > \alpha(1-\alpha)B$), the equilibrium in this region involves one firm offering both products and the other firm only product b. The latter firm sends advertising messages for its product to all consumers, whereas the former firm sends messages to all consumers for its product a but only to the expected targeted group for its product b. This occurs because the additional profit that the multi-product firm obtains from introducing product b is $\alpha(1-\alpha)B - c$, which is in this region of α strictly positive due to the fact that $c \leq A^2/B$. Therefore, there are in total 3 products in the market.

We next turn to the region $\alpha > 1-c/A$. In the single-product case, the equilibrium is either $a(1)b(2)$ or $a(1)b(1)$. For $c \leq (1-\alpha)B$, or $\alpha \leq 1-c/B$, the equilibrium is $b(2)a(1)$. Instead, for $1-c/B < \alpha \leq 1$, it is $b(1)a(1)$. If firms can offer both products, they indeed both have the incentive to do so for $c \leq \alpha(1-\alpha)A$, which implies that in equilibrium both firms offer both products.[4] Instead, for $c > \alpha(1-\alpha)A$, the equilibria are the same as those stated in the analysis for the range $c > A/2$. This is, for $c \leq \alpha(1-\alpha)B$, or $\alpha \leq 1/2 + \sqrt{A(A-4c)}/(2A)$, one firm offers both products and the other only product b, whereas for $\alpha > 1/2 + \sqrt{A(A-4c)}/(2A)$, both firms offer only product b.

Relation between the number of products and α. We are now in a position to determine how the equilibrium number of products changes with α in the region $c \leq \max\{\alpha(1-\alpha)A, A^2/B\}$. First, consider the case $c \leq \min\{\alpha(1-\alpha)A, A^2/B\}$, that is, both differences to the previously analyzed regions are relevant. In this region, for $\alpha \in [1/2, 1/2 + \sqrt{A(A-4c)}/(2A)]$, the equilibrium number of products in the market is 4, for $\alpha \in (1/2 + \sqrt{A(A-4c)}/(2A), 1/2 + \sqrt{B(B-4c)}/(2B)]$, it is 3, and for $\alpha \in (1/2 + \sqrt{B(B-4c)}/(2B), 1]$, the equilibrium number of products is 2. Therefore, the equilibrium number of products is (weakly) decreasing in α.

Second, suppose that $\alpha(1-\alpha)A > A^2/B$ and consider the region $A^2/B \leq c < \alpha(1-\alpha)A$. Again, the equilibrium number of products is 4 for $\alpha \in [1/2, 1/2+\sqrt{A(A-4c)}/(2A)]$. From above, the equilibrium number of products is 3 for $\alpha \in (1/2+\sqrt{A(A-4c)}/(2A), 1/2+\sqrt{B(B-4c)}/(2B)]$ and 2 for $\alpha \in (1/2+\sqrt{B(B-4c)}/(2B), 1]$. Therefore, the structure of the equilibrium is the same as in the region $c \leq \min\{\alpha(1-\alpha)A, A^2/B\}$, and the equilibrium number of products is also (weakly) decreasing in α.

Lastly, suppose that $A^2/B > \alpha(1-\alpha)A$ and consider the region $\alpha(1-\alpha)A \leq c < A^2/B$. In this region, the equilibrium with 4 products in total does not occur. Instead, the equilibrium number of products in the market is 3 for $\alpha \in [1/2, 1/2+\sqrt{B(B-4c)}/(2B)]$

[4] Again, dependent on whether $(1-\alpha)^2A$ is above or below c, two different equilibria occur in which firms send different messages, but the overall number of products is the same in both equilibria and equal to 4.

and 2 for $\alpha \in (1/2 + \sqrt{B(B-4c)}/(2B), 1]$, which again implies that it is (weakly) decreasing in α.

□

Proof of Proposition 5. **Nash Equilibria in the Subgames**. We first analyze the Nash equilibria in the different subgames that can occur on the equilibrium path. It follows from Section 4.1 that these are the subgames in which (i) both firms target group b (denoted by subgame bb), (ii) firms target different consumer groups, and each firm sends messages only to consumers in the respective expected targeted group (denoted by subgame ab), and (iii) firms again target different consumer groups, and the firm targeting group b sends messages to all consumers and the firm targeting group a sends messages only to consumers in the expected targeted group (denoted by subgame ab^2). Note that given our assumption that c is close to $\hat{c}$, other subgames do not arise in equilibrium. We denote A/B by $r \in (0, 1/2]$ where convenient and consider $\kappa \in (0, 1]$.

Subgame bb. In the subgame bb, there is no pure-strategy Nash equilibrium because either slightly undercutting the rival's price or increasing the price up to B is always profitable. For α sufficiently close to 1/2, there exists a symmetric mixed-strategy equilibrium in which firms draw prices from an interval between A and B. We denote this equilibrium by bb_1. It resembles the symmetric mixed-strategy equilibrium in Lemma 1.

By setting $p_i = B$, a firm obtains an expected profit of $\alpha(1-\alpha)B - c$. Suppose $p_{-i} > A$. Then, slightly undercutting firm $-i$'s price leads to an expected profit of $\alpha p_i - c$. Hence, the lowest price above A firm i is willing to set is determined by

$$\alpha(1-\alpha)B \leq \alpha p_i,$$

which yields

$$p_i \geq \bar{p}^{bb} \equiv (1-\alpha)B.$$

Next, undercutting firm $-i$'s price by $p_i = A$ (instead of by $p_i = p_{-i} - \epsilon$) yields an expected profit of $(\alpha + (1-\alpha)\kappa)\, A - c$, where $(1-\alpha)\kappa A$ arises because with probability $(1-\alpha)\kappa$ a consumer of the non-targeted group a is reached and interested in firm i's product. Note that, for $\kappa \in (0, 1]$, this profit is (weakly) increasing in α up to a level of A. As $\alpha(1-\alpha)B - c$ is decreasing in α to a level of 0, there exists an α at which both profits are the same. It equals

$$\hat{\alpha}_4(r,\kappa) \equiv \frac{1}{2}\left(1-(1-\kappa)r + \sqrt{(1-(1-\kappa)r)^2 - 4\kappa r}\right). \tag{5}$$

Hence, for $1/2 \leq \alpha \leq \hat{\alpha}_4(r,\kappa)$, $[\bar{p}^{bb}, B]$ constitutes a feasible mixing range for both firms.

It is readily verified that this leads to a mixing probability of $F(p)$ as in (1). Thus, the expected profit of each firm in the equilibrium bb_1 is $\alpha(1-\alpha)B - c$.

If $\alpha > \hat{\alpha}_4(r,\kappa)$, $[\bar{p}^{bb}, B]$ does no longer constitute a feasible mixing range for both firms as it is profitable to undercut $\bar{p}^{bb}$ by A; see above. In the same way as in Section 4.1, we can show that there exists a symmetric mixed-strategy equilibrium in which firms draw prices from an upper interval between A and B and a lower interval below A. We denote this equilibrium by bb_2. The expected profit in this equilibrium is again $\alpha(1-\alpha)B - c$ for each firm.

Subgame *ab*. We next consider the subgame ab in which each firm sends messages only to consumers in its expected target group (group a for firm 1 and group b for firm 2). In this subgame, $p_1 = A$, $p_2 = B$ is the candidate for a pure-strategy Nash equilibrium. We denote this equilibrium candidate by ab_1. In this equilibrium candidate, firm 2 receives an expected profit of $\alpha\left(1-(1-\alpha)\kappa\right)B - c$ because with probability $(1-\alpha)\kappa$ a group-b consumer reached by firm 2 also learns and is interested in product a offered by firm 1. Firm 2's most profitable deviation is to slightly undercut firm 1 leading to an expected profit of $\left(\alpha+(1-\alpha)\kappa\right)(A-\epsilon) - c$. Note that both profits are increasing in α with the former having the steeper slope. It follows that firm 2 does not deviate to a price $p_2 \leq A$ if $\alpha \geq \max\{1/2, \hat{\alpha}_1^{ab}(r,\kappa)\}$, where

$$\hat{\alpha}_1^{ab}(r,\kappa) \equiv \frac{\sqrt{4\kappa^2 r + (\kappa-1)^2(r-1)^2} - (1-\kappa)(1-r)}{2\kappa} \in (0,1) \tag{6}$$

with $r \in (0, 1/2]$ and $\kappa \in (0,1]$. The threshold $\hat{\alpha}_1^{ab}(r,\kappa)$ is strictly increasing in r and κ, respectively, in the feasible range. Note that for $r \leq 1/4$, $\hat{\alpha}_1^{ab}(r,\kappa) \leq 1/2$. This implies that firm 2 may have an incentive to deviate *only for sufficiently large* $r \in (1/4, 1/2]$.

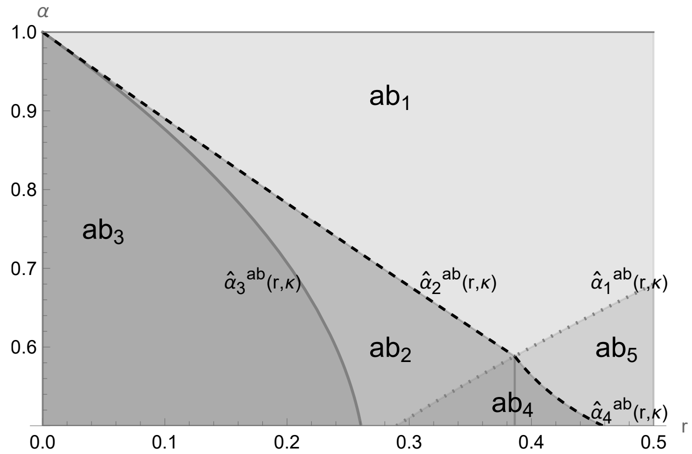
We next consider deviation incentives by firm 1. With $p_1 = A$, $p_2 = B$, firm 1 receives an expected profit of $\left(\alpha+(1-\alpha)\kappa\right)A - c$. Her most profitable deviation is to slightly undercut firm 2 leading to an expected profit of $(1-\alpha)\kappa(B-\epsilon) - c$. The profit in the candidate equilibrium ab_1 is increasing in α, whereas the deviation profit is decreasing. For κ sufficiently large, there exists an $\alpha \geq 1/2$ equal to

$$\hat{\alpha}_2^{ab}(r,\kappa) \equiv \frac{\kappa(1-r)}{\kappa(1-r)+r}$$

that equalizes both profits. $\hat{\alpha}_2^{ab}(r,\kappa)$ is strictly decreasing in r and strictly increasing in κ in the feasible range, respectively. Firm 1 cannot profitably deviate from $p_1 = A$, $p_2 = B$ if α is sufficiently large, i.e. $\alpha \geq \max\{1/2, \hat{\alpha}_2^{ab}(r,\kappa)\}$.

Thus, ab_1 is a pure-strategy Nash equilibrium if $\alpha \geq \max\{1/2, \hat{\alpha}_1^{ab}(r,\kappa), \hat{\alpha}_2^{ab}(r,\kappa)\}$. The expected profits in the equilibrium ab_1 are $\left(\alpha+(1-\alpha)\kappa\right)A - c$ and $\alpha\left(1-(1-\alpha)\kappa\right)B-$

c. Note that for $\kappa > 0$, the equilibrium ab_1 may cease to exist for sufficiently low α. Figure 6 provides an illustration.[5]



Equilibria ab_1 to ab_5 in the subgame ab in the (r, α)-plane at $\kappa = 0.9$. $\hat{\alpha}_1^{ab}(r, \kappa)$ is depicted by the dotted gray line, $\hat{\alpha}_2^{ab}(r, \kappa)$ for low $r \leq 0.386$ and $\hat{\alpha}_4^{ab}(r, \kappa)$ for high r by the dashed black line, and $\hat{\alpha}_3^{ab}(r, \kappa)$ by the solid gray line.

Figure 6: Broader product attractiveness: Equilibria in the Subgame ab

For lower α and sufficiently low r, i.e. if $\hat{\alpha}_1^{ab}(r, \kappa) \leq \alpha < \hat{\alpha}_2^{ab}(r, \kappa)$ (and $\hat{\alpha}_2^{ab}(r, \kappa) > 1/2$), there exists an asymmetric mixed-strategy equilibrium in which firms draw prices from an interval between A and B, and the cdfs of both firms have different mass points. We denote this equilibrium by ab_2. If $p_2 > A$, firm 1 receives $(\alpha + (1 - \alpha)\kappa) A - c$ from setting $p_1 = A$. Setting p_1 above A but below p_2 yields $(1 - \alpha)\kappa p_1 - c$. Hence, the lowest price above A firm 1 is willing to set is given by

$$(\alpha + (1 - \alpha)\kappa) A \leq (1 - \alpha)\kappa p_1.$$

It equals

$$\bar{p}_1^{ab} \equiv \frac{\alpha A}{(1 - \alpha)\kappa} + A. \tag{7}$$

If $p_1 < B$, firm 2' expected profit from setting $p_2 = B$ is $\alpha (1 - (1 - \alpha)\kappa) B - c$. Suppose $p_1 > A$. Then, undercutting firm 1 by $p_2 > A$ leads to an expected profit of $\alpha p_2 - c$. Hence, the lowest price above A firm 2 is willing to set is characterized by

$$\alpha (1 - (1 - \alpha)\kappa) B \leq \alpha p_2,$$

[5]For $\kappa = 0$, it always exists and is the unique equilibrium in this subgame.

which leads to

$$p_2 \geq \bar{p}_2^{ab} \equiv (1-(1-\alpha)\kappa)B.$$

At $\alpha = \hat{\alpha}_2^{ab}(r,\kappa)$ (and $\hat{\alpha}_1^{ab}(r,\kappa) \leq \alpha$), it holds that $B > \bar{p}_1^{ab} > \bar{p}_2^{ab} \geq A$. Therefore, $[\bar{p}_1^{ab}, B)$ constitutes a feasible mixing range for both firms given the other firm mixes in this interval. Furthermore, $\bar{p}_1^{ab} - \bar{p}_2^{ab}$ is increasing in α and there exists an $\alpha < \hat{\alpha}_2^{ab}(r,\kappa)$ such that $\bar{p}_1^{ab} = \bar{p}_2^{ab}$. It equals

$$\hat{\alpha}_3^{ab}(r,\kappa) \equiv \frac{\sqrt{\kappa^2 + (\kappa-1)^2 r^2 + 2(1-3\kappa)\kappa r} + \kappa(2\kappa + r - 1) - r}{2\kappa^2}.$$

Hence, for $\alpha < \hat{\alpha}_3^{ab}(r,\kappa)$, $\bar{p}_1^{ab} < \bar{p}_2^{ab}$ and $[\bar{p}_1^{ab}, B)$ does no longer constitute a feasible mixing range for firm 2. The threshold $\hat{\alpha}_3^{ab}(r,\kappa)$ is strictly decreasing in r in the feasible range. Moreover, $\hat{\alpha}_2^{ab}(r,\kappa)$ and $\hat{\alpha}_3^{ab}(r,\kappa)$ intersect at $r = 0$, and $\hat{\alpha}_2^{ab}(r,\kappa) > \hat{\alpha}_3^{ab}(r,\kappa)$ for $r > 0$.

We continue to characterize equilibrium ab_2. For $\max\left\{1/2, \hat{\alpha}_1^{ab}(r,\kappa), \hat{\alpha}_3^{ab}(r,\kappa)\right\} \leq \alpha < \hat{\alpha}_2^{ab}(r,\kappa)$, firm 2 must be indifferent between setting any price on $[\bar{p}_1^{ab}, B)$ and $p_2 = B$. Suppose firm 1 mixes by $\bar{M}_1(p)$ (replacing p_1 by p). Then, firm 2's expected profit must satisfy

$$\bar{M}_1(p)\alpha\left(1-(1-\alpha)\kappa\right)p + \left(1-\bar{M}_1(p)\right)\alpha p = \alpha\left(1-(1-\alpha)\kappa\right)B,$$

which leads to

$$\bar{M}_1(p) = \frac{p - (1-(1-\alpha)\kappa)B}{(1-\alpha)\kappa p}. \tag{8}$$

$\bar{M}_1(B) = 1$ but $\bar{M}_1(\bar{p}_1^{ab}) > 0$ which implies a mass point at $p_1 = A$. Firm 1 must be indifferent between setting any price on $[\bar{p}_1^{ab}, B)$ and $p_1 = A$. Suppose firm 2 mixes by $\bar{M}_2(p)$ (replacing p_2 by p). Then, firm 1's expected profit must satisfy

$$\bar{M}_2(p)(1-\alpha)^2\kappa p + \left(1-\bar{M}_2(p)\right)(1-\alpha)\kappa p = \left(\alpha + (1-\alpha)\kappa\right)A$$

leading to

$$\bar{M}_2(p) = \frac{(1-\alpha)\kappa p - (\alpha + (1-\alpha)\kappa)A}{(1-\alpha)\alpha\kappa p}. \tag{9}$$

$\bar{M}_2(\bar{p}_1^{ab}) = 0$ but $\bar{M}_2(B) < 1$ which implies a mass point at $p_2 = B$. It follows that, for $\alpha \in \left[\max\left\{1/2, \hat{\alpha}_1^{ab}(r,\kappa), \hat{\alpha}_3^{ab}(r,\kappa)\right\}, \hat{\alpha}_2^{ab}(r,\kappa)\right)$, the mixing probabilities are characterized by the cumulative distribution functions,

$$M_1(p_1) = \begin{cases} \bar{M}_1(\bar{p}_1^{ab}), & \text{if } p_1 = A; \\ \bar{M}_1(p_1), & \text{if } p_1 \in [\bar{p}_1^{ab}, B). \end{cases}$$

$$M_2(p_2) = \begin{cases} \bar{M}_2(p_2), & \text{if } p_2 \in [\bar{p}_1^{ab}, B); \\ 1, & \text{if } p_2 = B. \end{cases}$$

The expected profits in the equilibrium ab_2 are $(\alpha + (1-\alpha)\kappa)\, A - c$ and $\alpha\,(1-(1-\alpha)\kappa)\, B - c$.

For even lower α and sufficiently low r, i.e. if $\alpha \in \left[1/2, \hat{\alpha}_3^{ab}(r,\kappa)\right)$ (given that $\hat{\alpha}_3^{ab}(r,\kappa) > 1/2$), there exists an asymmetric mixed-strategy equilibrium in which firms draw prices from an interval between A and B, where only firm 2's cdf has a mass point on B. We denote this equilibrium by ab_3. For $\alpha < \hat{\alpha}_3^{ab}(r,\kappa)$, it holds that $A < \bar{p}_1^{ab} < \bar{p}_2^{ab} < B$ and the candidate for the mixing range of firms' prices becomes $[\bar{p}_2^{ab}, B)$.[6] By setting the lowest price in this interval, firm 1 receives an expected profit of $(1-\alpha)\kappa\bar{p}_2^{ab} - c$ which equals $(1-\alpha)\kappa(1-(1-\alpha)\kappa)B - c$. Firm 2's expected profit in the mixing range $[\bar{p}_2^{ab}, B)$ is the same as in mixing range $[\bar{p}_1^{ab}, B)$ above. Thus, the expected profits in the equilibrium ab_3 are $(1-\alpha)\kappa(1-(1-\alpha)\kappa)B - c$ and $\alpha\,(1-(1-\alpha)\kappa)\, B - c$. This asymmetric mixed-strategy equilibrium exists for $\alpha \in \left[1/2, \hat{\alpha}_3^{ab}(r,\kappa)\right)$, in case this interval is non-empty.

For high r, i.e. for $r = A/B \in (1/4, 1/2]$, firm 2 has an incentive to set a price below A when targeting is imprecise (low α) and tailoring is broad (high κ) in order to sell to some consumers of target group a. The corresponding critical α-threshold is given by $\hat{\alpha}_1^{ab}(r,\kappa) > 1/2$; cf. (6) from above. There exist two equilibria for $1/2 < \alpha < \hat{\alpha}_1^{ab}(r,\kappa)$ and high κ. We denote them by ab_4 and ab_5. Both equilibria exhibit a mixing range including an interval below A which is derived as follows. When setting $p_2 = B$, firm 2 receives an expected profit of $\alpha\,(1-(1-\alpha)\kappa)\, B - c$, whereas it receives an expected profit of $(\alpha + (1-\alpha)\kappa)\, p_2 - c$ from undercutting p_1 below A. Hence, the lowest price below A firm 2 is willing to set is given by

$$\alpha\,(1-(1-\alpha)\kappa)\, B \leq (\alpha + (1-\alpha)\kappa)\, p_2.$$

[6] Note that if $\hat{\alpha}_3^{ab}(r,\kappa) \geq 1/2$, then $\hat{\alpha}_1^{ab}(r,\kappa)$ is always smaller than $1/2$ except for $r = 1/2$ and $\kappa = 1$, where both are equal to $1/2$. Therefore, $\hat{\alpha}_3^{ab}(r,\kappa) < \hat{\alpha}_1^{ab}(r,\kappa)$ never arises in the relevant range which means that firm 2 never has an incentive to deviate to a price $p_2 \leq A$.

It equals

$$\underline{p}_2^{ab} \equiv \frac{\alpha\,(1-(1-\alpha)\kappa)\,B}{\alpha+(1-\alpha)\kappa}. \tag{10}$$

The lowest price firm 1 is willing to set is derived as follows. First, from setting $p_1 = A$, firm 1 receives (at least) an expected profit of $(\alpha^2+\alpha(1-\alpha)(1-\kappa)+(1-\alpha)^2\kappa)\,A-c$ when undercut by firm 2. Firm 1 receives an expected profit of $(\alpha+(1-\alpha)\kappa)\,p_1-c$ from undercutting p_2 below A. Thus, a first candidate for the lowest price below A firm 1 is willing to set is characterized by

$$\left(\alpha^2+\alpha(1-\alpha)(1-\kappa)+(1-\alpha)^2\kappa\right)A \le (\alpha+(1-\alpha)\kappa)\,p_1.$$

It equals

$$\underline{p}_{1a}^{ab} \equiv \frac{(\alpha^2+\alpha(1-\alpha)(1-\kappa)+(1-\alpha)^2\kappa)\,A}{\alpha+(1-\alpha)\kappa}. \tag{11}$$

Second, from setting $p_1 = B$, firm 1 receives at least an expected profit of $(1-\alpha)^2\kappa B-c$. Therefore, a second candidate for the lowest price below A undercutting p_2 firm 1 is willing to set is given by

$$\underline{p}_{1b}^{ab} \equiv \frac{(1-\alpha)^2\kappa B}{\alpha+(1-\alpha)\kappa}. \tag{12}$$

For given $r \in (1/4, 1/2]$ and $\kappa > 0$, it is readily verified that $\underline{p}_{1b}^{ab}$ is decreasing in α and $\underline{p}_{1b}^{ab}(\alpha=1/2) > \underline{p}_{1a}^{ab}$. Hence, $\underline{p}_{1b}^{ab}(\alpha=1/2) \ge \max\left\{\underline{p}_{1a}^{ab}, \underline{p}_{1b}^{ab}\right\}$. In addition, it holds that $\underline{p}_2^{ab}$ is increasing in α and $\underline{p}_2^{ab} \ge \underline{p}_2^{ab}(\alpha=1/2) \ge \underline{p}_{1b}^{ab}(\alpha=1/2)$. Therefore, $[\underline{p}_2^{ab}, A)$ constitutes a feasible mixing range below A for both firms given the other firm draws prices from this interval.

Hence, for $\alpha \in \left[1/2, \hat{\alpha}_1^{ab}(r,\kappa)\right)$, $[\underline{p}_2^{ab}, A)$ constitutes a feasible mixing range below A for both firms given the other firm draws prices from this interval. The expected profit of firm i is $(\alpha+(1-\alpha)\kappa)\,\underline{p}_2^{ab}-c$ which equals $\alpha\,(1-(1-\alpha)\kappa)\,B-c$ by construction of $\underline{p}_2^{ab}$. Suppose firm i mixes by $\underline{N}(p)$ (replacing p_i by p). Then, firm $-i$'s expected profit must satisfy

$$\underline{N}(p)\left(\alpha^2+\alpha(1-\alpha)(1-\kappa)+(1-\alpha)^2\kappa\right)p+(1-\underline{N}(p))\,(\alpha+(1-\alpha)\kappa)\,p = \alpha\,(1-(1-\alpha)\kappa)\,B,$$

which is equivalent to

$$\underline{N}(p) = \frac{(\alpha(1-\kappa)+\kappa)\,p-\alpha\,(1-(1-\alpha)\kappa)\,B}{2\alpha(1-\alpha)\kappa p}. \tag{13}$$

$\underline{N}(\underline{p}_2^{ab}) = 0$ and $\underline{N}(A) < 1$.

We will show next that whether both firms also draw prices from an upper interval between A and B depends on another critical α-threshold. For $\alpha \in \left[1/2, \hat{\alpha}_1^{ab}(r,\kappa)\right)$, both firms draw prices from $[\underline{p}_2^{ab}, A)$ earning an expected profit of $\alpha\,(1-(1-\alpha)\kappa)\,B - c$. Suppose that, in addition, firm 1 has a mass point at $p_1 = A$ and firm 2 a mass point at $p_2 = B$. Then, firm 1 may have an incentive to slightly undercut firm 2 at $p_2 = B$ whenever α and r are sufficiently low (cf. equilibria ab_2 and ab_3.). Firm 1's deviation profit equals

$$(1-\underline{N}(A))(1-\alpha)\kappa B + \underline{N}(A)(1-\alpha)^2\kappa B - c.$$

It is larger than $\alpha\,(1-(1-\alpha)\kappa)\,B - c$ if and only if

$$\alpha < \hat{\alpha}_4^{ab}(r,\kappa) \equiv \frac{3r + \kappa(1-r) - 1 - \sqrt{(1-\kappa(1-r)-3r)^2 - 4\kappa^2 r(1-2r)}}{2\kappa(1-2r)} \tag{14}$$

Hence, for $\alpha < \hat{\alpha}_4^{ab}(r,\kappa)$, there does not exist an equilibrium where firm 1 sets prices weakly lower than A. For $\alpha \geq \hat{\alpha}_4^{ab}(r,\kappa)$, we show below that such an equilibrium may exist.

We show next that, for $\alpha \in \left[1/2, \min\{\hat{\alpha}_1^{ab}(r,\kappa), \hat{\alpha}_4^{ab}(r,\kappa)\}\right)$, there exists an asymmetric mixed-strategy equilibrium in which both firms draw prices from a lower interval below A and an upper interval between A and B. (We denote this equilibrium by ab_4). We characterize the mixing range and the mixing probabilities of the upper interval simultaneously. Note that $\hat{\alpha}_4^{ab}(r,\kappa) \leq \hat{\alpha}_2^{ab}(r,\kappa)$ and $\hat{\alpha}_4^{ab}(r,\kappa) > \hat{\alpha}_3^{ab}(r,\kappa)$. For $\hat{\alpha}_3^{ab}(r,\kappa) < \alpha < \hat{\alpha}_2^{ab}(r,\kappa)$, the mixing range of the upper interval lies within $[\bar{p}_1^{ab}, B)$; cf. equilibrium ab_2. Denote $[z, B)$ with $z \geq \bar{p}_1^{ab}$ the candidate for the mixing range of the upper interval.

Firm 2 must be indifferent between setting any price on $[z, B]$ and receiving an expected profit of $\alpha\,(1-(1-\alpha)\kappa)\,B - c$. As derived in (8), this requires firm 1 to mix by $\bar{M}_1(p_1)$. Receiving the same expected profit, firm 1 must be indifferent between setting any price on $[z, B)$ and $p_1 = A$. Suppose firm 2 mixes by $\hat{M}_2(p)$ (replacing p_2 by p). Then, firm 1's expected profit must satisfy

$$\hat{M}_2(p)(1-\alpha)^2\kappa p + \left(1-\hat{M}_2(p)\right)(1-\alpha)\kappa p = \alpha\,(1-(1-\alpha)\kappa)\,B$$

leading to

$$\hat{M}_2(p) = \frac{(1-\alpha)\kappa p - \alpha(1-(1-\alpha)\kappa)B}{\alpha(1-\alpha)\kappa p}. \tag{15}$$

$\hat{M}_2(B) < 1$ which implies the existence of a mass point at $p_2 = B$. In order to obtain an atomless mixing probability of firm 2 for $p_2 < B$, it must hold that $\hat{M}_2(z) = \underline{N}(A)$ which determines z:

$$z \equiv \frac{2\alpha r(1-(1-\alpha)k)}{\alpha^2\kappa - \alpha(\kappa r + \kappa + r - 1) + \kappa r}. \tag{16}$$

For $\alpha \in \left[1/2, \min\{\hat{\alpha}_1^{ab}(r,\kappa), \hat{\alpha}_4^{ab}(r,\kappa)\}\right)$, it is readily verified that $z \geq \bar{p}_1^{ab}$.

It follows that, for $\alpha \in \left[1/2, \min\{\hat{\alpha}_1^{ab}(r,\kappa), \hat{\alpha}_4^{ab}(r,\kappa)\}\right)$, the mixing probabilities are characterized by the cumulative distribution functions,

$$N_1(p_1) = \begin{cases} \underline{N}(p_1), & \text{if } p_1 \in [\underline{p}_2^{ab}, A); \\ \bar{M}_1(z), & \text{if } p_1 = A; \\ \bar{M}_1(p_1), & \text{if } p_1 \in [z, B). \end{cases}$$

$$N_2(p_2) = \begin{cases} \underline{N}(p_2), & \text{if } p_2 \in [\underline{p}_2^{ab}, A); \\ \hat{M}_2(p_2), & \text{if } p_2 \in [z, B); \\ 1, & \text{if } p_2 = B. \end{cases}$$

$N_1(p_1)$ contains a mass point at $p_1 = A$ and $N_2(p_2)$ contains a mass point at $p_2 = B$. The expected profit of both firms in the equilibrium ab_4 equals $\alpha\,(1-(1-\alpha)\kappa)\,B - c$.

For even larger r, i.e. for $r \in (1/4, 1/2]$, $\alpha \in \left[\max\left\{1/2, \hat{\alpha}_4^{ab}(r,\kappa)\right\}, \hat{\alpha}_1^{ab}(r,\kappa)\}\right)$ may be satisfied with a non-empty interval. Then, there exists an asymmetric mixed-strategy equilibrium—denoted ab_5—in which both firms draw prices from a lower interval below A, firm 1 has a mass point at $p_1 = A$, and firm 2 has a mass point at $p_2 = B$. As $\alpha > \hat{\alpha}_4^{ab}(r,\kappa)$, firm 1 has no incentive to undercut firm 2 at $p_2 = B$, cf. (14). The expected profit of both firms in the equilibrium ab_5 is $\alpha\,(1-(1-\alpha)\kappa)\,B - c$.

Subgame ab^2. Finally, we consider the subgame ab^2 in which firm 1 sends messages to expected group-a consumers and firm 2 sends messages to all consumers. We show that the equilibria in this subgame resemble those in subgame ab.

In the subgame ab^2, $p_1 = A$, $p_2 = B$ is the candidate for a pure-strategy Nash equilibrium. We denote it by ab_1^2. In equilibrium ab_1^2, firm 2 has a higher reach compared to equilibrium ab_1 and receives an expected profit of $(1-(1-\alpha)\kappa)\,B - 2c$, whereas firm 1 still receives an expected profit of $(\alpha + (1-\alpha)\kappa)\,A - c$. It follows that firm 2 finds it profitable to send messages also to consumers of the expected non-target group if $(1-\alpha)\,(1-(1-\alpha)\kappa)\,B > c$ or, equivalently,

$$\alpha < \tilde{\alpha}(c) \equiv 1 + \frac{\sqrt{1 - \frac{4c\kappa}{B}} - 1}{2\kappa} \in (0,1). \tag{17}$$

Firm 1's most profitable deviation from equilibrium ab_1^2 is to slightly undercut firm 2 leading to the same lower bound $\hat{\alpha}_2^{ab}(r,\kappa)$ on α as in the subgame ab.

Firm 2's most profitable deviation from equilibrium ab_1^2 is to slightly undercut firm 1 leading to an expected profit of $(1+\kappa)(A-\epsilon)-2c$. Thus, firm 2 does not deviate to a price $p_2 \leq A$ if $\alpha \geq \max\{1/2, \hat{\alpha}_1^{ab^2}(r,\kappa)\}$, where

$$\hat{\alpha}_1^{ab^2}(r,\kappa) \equiv 1 - \frac{1-(1+\kappa)r}{\kappa} \in (0,1) \tag{18}$$

with $r \in (0,1/2]$ and $\kappa \in (0,1]$. Note that for $r > 1/(2(1+\kappa))$, $\hat{\alpha}_1^{ab^2}(r,\kappa) > \hat{\alpha}_1^{ab}(r,\kappa)$ such that, for sufficiently large r, firm 2 has a stronger incentive to deviate from the pure-strategy Nash equilibrium in the subgame ab^2 than in the subgame ab.

Overall, ab_1^2 is a pure-strategy Nash equilibrium if $\alpha \geq \max\{1/2, \hat{\alpha}_1^{ab^2}(r,\kappa), \hat{\alpha}_2^{ab}(r,\kappa)\}$. The expected profits are $(\alpha+(1-\alpha)\kappa)\,A-c$ and $(1-(1-\alpha)\kappa)\,B-2c$.

In addition, equilibria ab_2^2 and ab_3^2 are the same as equilibria ab_2 and ab_3 in the subgame ab except for the expected profit of firm 2 to be $(1-(1-\alpha)\kappa)\,B-2c$ instead of $\alpha\,(1-(1-\alpha)\kappa)\,B-c$ due to the higher reach. The reason for the similarity of these equilibria is that optimal prices weakly above A are identical in both subgames.

For sufficiently high r, firm 2 has an incentive to set a price below A when targeting is imprecise (low α) and appeal is broad (high κ) in order to also sell to some consumers of the non-target group a; cf. subgame ab. The corresponding critical α-threshold is $\hat{\alpha}_1^{ab^2}(r,\kappa)$. There exist two equilibria for $1/2 < \alpha < \hat{\alpha}_1^{ab^2}(r,\kappa)$ and high κ. We denote them by ab_4^2 and ab_5^2. Both equilibria exhibit a mixing range including an interval below A. We next derive firms' expected equilibrium profits. From setting $p_2 = B$, firm 2 receives an expected profit of $(1-(1-\alpha)\kappa)\,B-2c$, whereas it receives an expected profit of $(1+\kappa)\,p_2-2c$ from undercutting p_1 below A.

Hence, the lowest price below A firm 2 is willing to set is given by

$$(1-(1-\alpha)\kappa)\,B \leq (1+\kappa)\,p_2.$$

It equals

$$\underline{p}_2^{ab^2} \equiv \frac{(1-(1-\alpha)\kappa)\,B}{1+\kappa}. \tag{19}$$

The lowest price firm 1 is willing to set is derived as follows. First, from setting $p_1 = A$, firm 1 receives at least an expected profit of $\alpha(1-\kappa)A-c$ when being undercut by firm 2. Firm 1 receives an expected profit of $(\alpha+(1-\alpha)\kappa)\,p_1-c$ from undercutting p_2 below A. Thus, a first candidate for the lowest price below A firm 1 is willing to set is characterized by

$$\alpha(1-\kappa)A \leq (\alpha+(1-\alpha)\kappa)\, p_1.$$

It equals

$$\underline{p}_1^{ab^2} \equiv \frac{\alpha(1-\kappa)A}{\alpha+(1-\alpha)\kappa}. \tag{20}$$

Second, from setting $p_1 = B$, firm 1 receives an expected profit of 0.

It is easy to verify that $\underline{p}_1^{ab^2}$ is always lower than $\underline{p}_2^{ab^2}$. Thus, $[\underline{p}_2^{ab^2}, A)$ constitutes a feasible mixing range below A for both firms given the other firm draws prices from this interval. The corresponding equilibrium profits of firm 1 and 2 equal $(\alpha+(1-\alpha)\kappa)\,\underline{p}_2^{ab^2} - c = [(\alpha+(1-\alpha)\kappa)\,(1-(1-\alpha)\kappa)\,B]\,/\,(1+\kappa) - c$ and $(1-(1-\alpha)\kappa)\,B - 2c$. Equilibria ab_4^2 and ab_5^2 exist for $1/2 < \alpha < \hat{\alpha}_1^{ab^2}(r,\kappa)$ and sufficiently high r and κ.

SPNE of the Entire Game. We next determine the SPNE of the entire game and derive the sum of the expected profits as a function of $\alpha \in [1/2, 1]$ for all $r \in [0, 1/2]$ and $\kappa \in (0, 1]$. We begin the analysis by showing that, for sufficiently low r, any of three subgames bb, ab, and ab^2 may be played in equilibrium, whereas otherwise only the two subgames ab and ab^2 may be part of the SPNE.

For sufficiently low r, suppose firm 2 targets consumers of group b and sends messages only to consumers the firm expects to belong to this group or to all consumers. Given our assumption that c is close to $\hat{c}$, firm 1 optimally only sends messages to consumers in its expected targeted group. As shown above, firm 1's expected profit equals $\alpha(1-\alpha)B - c$ in both equilibria in subgame bb and $(\alpha+(1-\alpha)\kappa)A - c$ in all equilibria in subgames ab and ab^2 except for equilibria ab_3 and ab_3^2 where its expected profit is equal to $(1-\alpha)\kappa(1-(1-\alpha)\kappa)B - c$. However, $(1-\alpha)\kappa(1-(1-\alpha)\kappa)B - c$ is always (weakly) lower than $\alpha(1-\alpha)B - c$ which directly rules out the selection of equilibria ab_3 and ab_3^2. Hence, firm 1 prefers to target consumers in the expected targeting group a if and only if $\alpha(1-\alpha)B \leq (\alpha+(1-\alpha)\kappa)\,A$ which is equivalent to $\alpha \geq \hat{\alpha}_4(r,\kappa)$ as in (5). Equilibria ab_2 and ab_2^2 may be selected or not because $\hat{\alpha}_2^{ab}(r,\kappa) - \hat{\alpha}_4(r,\kappa)$ can be positive or negative depending on κ and r. Equilibria ab_2 and ab_2^2 exist for high κ and are selected for sufficiently high r. In addition, Equilibria ab_4 and ab_5 (resp. ab_4^2 and ab_5^2) are always selected when they exist because the expected profit of firm 1 is always higher than $\alpha(1-\alpha)B - c$. Finally, $\hat{\alpha}_4(r,\kappa)$ is decreasing in r. Therefore, from $\hat{\alpha}_4(r,\kappa) \geq 1/2$, we can derive the critical upper bound on r for the existence of the equilibrium bb_1 where both firms target expected group-b consumers. It equals $r \leq 1/(2(k+1))$.

Hence, for $r = A/B \in (0, 1/(2(k+1)))$, the sum of expected profits as a function of

α is given by

$$\sum_i E[\pi_i^\star(\alpha,\kappa)] = \begin{cases} 2\alpha(1-\alpha)B - 2c, & \text{if } \alpha \in [1/2, \hat{\alpha}_4(r,\kappa)]; \\ (\alpha + (1-\alpha)\kappa)\,A + (1-(1-\alpha)\kappa)\,B - 3c, & \text{if } \alpha \in (\hat{\alpha}_4(r,\kappa), 1] \text{ and } \alpha < \tilde{\alpha}(c). \\ (\alpha + (1-\alpha)\kappa)\,A + \alpha\,(1-(1-\alpha)\kappa)\,B - 2c, & \text{if } \alpha \in (\hat{\alpha}_4(r,\kappa), 1] \text{ and } \alpha \geq \tilde{\alpha}(c). \end{cases}$$

It preserves the non-monotonicity in α. It is easy to verify that $\hat{\alpha}_4(r,\kappa) < \hat{\alpha}_4(r,0) = \hat{\alpha}_1(r)$ for all $\kappa > 0$. Furthermore, for c close to $\hat{c}$, $\hat{\alpha}_4(r,\kappa) \leq \tilde{\alpha}(c)$ for $\kappa \geq 0$ where $\tilde{\alpha}(c)$ is the α-threshold below which firm 2 also sends messages to consumers in the expected non-targeted group a (cf. (17)). Then, comparing $\kappa = 0$ and $\kappa > 0$ yields part 2 of Proposition 5 for $r = A/B \in (0, 1/(2(k+1)))$ (cf. Figure 4).

For $r = A/B > 1/(2(k+1))$, the sum of expected profits is *always* strictly increasing in α. It equals

$$\sum_i E[\pi_i^\star(\alpha,\kappa)] = \begin{cases} \left(\frac{\alpha+(1-\alpha)\kappa}{1+\kappa} + 1\right)(1-(1-\alpha)\kappa)\,B - 3c, & \text{if } \alpha \in \left[1/2, \min\{\hat{\alpha}_1^{ab^2}(r,\kappa), \tilde{\alpha}(c)\}\right); \\ 2\alpha\,(1-(1-\alpha)\kappa)\,B - 2c, & \text{if } \alpha \in \left[\tilde{\alpha}(c), \hat{\alpha}_1^{ab}(r,\kappa)\right); \\ (\alpha + (1-\alpha)\kappa)\,A + (1-(1-\alpha)\kappa)\,B - 3c, & \text{if } \alpha \in \left[\max\{\hat{\alpha}_1^{ab^2}(r,\kappa), 1/2\}, \tilde{\alpha}(c)\right] \\ (\alpha + (1-\alpha)\kappa)\,A + \alpha\,(1-(1-\alpha)\kappa)\,B - 2c, & \text{if } \alpha \geq \max\{\hat{\alpha}_1^{ab}(r,\kappa), \tilde{\alpha}(c)\}. \end{cases}$$

Furthermore, it is always strictly decreasing in κ. □

B Proofs of Propositions in Sections 6 and 7

Proof of Proposition 6. If an equilibrium exists in which both firms target consumers of group b, prices and profits in the simultaneous game are the same as those characterized in Section 4.1. This is because these prices constitute the equilibrium in the sequential game in which each firm knew the targeting and messaging strategy of the rival.[7] Hence, there is no profitable deviation from this pricing strategy in the simultaneous game. This equilibrium exists as long as no firm has an incentive to deviate by targeting consumers of group a and charging a price of A. We know that such a deviation is not profitable if $\alpha A \leq \alpha(1-\alpha)B$ or $\alpha \leq \hat{\alpha}_1(A/B)$.

We next determine whether a symmetric mixed-strategy equilibrium where both firms target each group a and b with positive probability and send messages to the consumers in the expected targeted group can exist. When targeting a and setting the price $p_i = A$,

[7]Because c is at $\hat{c}$, it is never profitable for a firm to send messages also to consumers who are not in the expected targeted group, given that the other firm targets group b.

the expected profit of firm i (net of advertising cost) equals

$$\alpha(1-\alpha)\sigma_a A + \alpha(1-\sigma_a)A = \alpha A[1-\alpha\sigma_a],$$

where σ_a is the probability that firm $-i$ targets consumer group a. When targeting b and setting the price $p_i = B$, the expected profit of firm i (net of advertising cost) is

$$\alpha(1-\alpha)(1-\sigma_a)B + \alpha\sigma_a B = \alpha B[1-\alpha(1-\sigma_a)].$$

Both profits are equalized at

$$\sigma_a^* = \frac{A-(1-\alpha)B}{\alpha(A+B)},$$

which is strictly positive if and only if $\alpha > \hat{\alpha}_1(A/B)$. Therefore, for $\alpha \leq \hat{\alpha}_1(A/B)$, a symmetric equilibrium in which both firms randomize in the consumer group they target, does not exist. Instead, for $\alpha > \hat{\alpha}_1(A/B)$, such an equilibrium can exist and, using σ_a^*, the resulting expected profit (net of advertising cost) equals

$$\frac{\alpha AB(2-\alpha)}{(A+B)}.$$

Following the same arguments as e.g. in the proof of Lemma 2, the equilibrium price range when targeting consumer group a is

$$p \in [A(1-\alpha\sigma_a), A] = \left[\frac{(2-\alpha)AB}{(A+B)}, A\right]$$

and the equilibrium price range when targeting group b is

$$p \in [B(1-\alpha(1-\sigma_a)), B] = \left[\frac{(2-\alpha)AB}{(A+B)}, B\right].$$

Both intervals have the same lower bound and the first interval is non-empty for $\alpha > \hat{\alpha}_1(A/B)$.

We now derive the mixing strategy in the lower interval. Suppose firm i targets consumer group a with probability σ_a^* and plays a mixed strategy drawing its price from the cdf $R(p_i)$ on $[(2-\alpha)AB/(A+B), A]$. Then, firm $-i$, when targeting group a, is indifferent between any price on $[(2-\alpha)AB/(A+B), A]$ if

$$\alpha p\left((1-\sigma_a^*) + \sigma_a^*(1-R(p))\right) + \alpha(1-\alpha)p\sigma_a^* R(p) = \frac{(2-\alpha)\alpha AB}{(A+B)},$$

which, using σ_a^* derived above, leads to

$$R(p) = \frac{p(A+B) - (2-\alpha)AB}{p(A-(1-\alpha)B)}$$

with $R((2-\alpha)AB/(A+B)) = 0$ and $R(A) = 1$.

Turning to the mixing strategy in the upper interval, suppose firm i targets consumer group b with probability $(1-\sigma_a^*)$ and plays a mixed strategy drawing its price from the cdf $S(p_i)$ on $[\frac{(2-\alpha)AB}{(A+B)}, B]$. Then, firm $-i$, when targeting group b, is indifferent between any price on $[(2-\alpha)AB/(A+B), B]$ if

$$\alpha p\left(\sigma_a^* + (1-\sigma_a^*)(1-S(p))\right) + \alpha(1-\alpha)p(1-\sigma_a^*)S(p) = \frac{(2-\alpha)\alpha AB}{(A+B)}.$$

This leads to

$$S(p) = \frac{p(A+B) - (2-\alpha)AB}{p(B-(1-\alpha)A)},$$

with $S((2-\alpha)AB/(A+B)) = 0$ and $S(B) = 1$. Given that $c = \hat{c}$, it can never be optimal for a firm to send messages also to consumers who are not in the expected targeted group. This is because, given the mixing ranges, the profit that it can obtain from doing so is $(1-\alpha)(2-\alpha)AB/(A+B) - c$. However, for $c = \hat{c}$, with $\hat{c} = \alpha A$, this is always negative due to the fact that $B > 2A$ in this range (as $\alpha > 1 - A/B$ and $\alpha > 1/2$).

Therefore, for $\alpha > \hat{\alpha}_1(A/B)$ a symmetric equilibrium exists in which both firms obtain an expected profit of $(2-\alpha)\alpha AB/(A+B) - c$.

Next, we consider potential equilibria in which firms play heterogeneous targeting strategies. First, a pure-strategy equilibrium in which firm $-i$ targets group b and firm i targets group a does not exist. With such targeting strategies, firm $-i$ would optimally charge a price of $p_{-i} = B$ and firm i a price of $p_i = A$. However, firm i then has a profitable deviation to target group b but slightly undercut firm $-i$. It follows that in any equilibrium in which firms play heterogeneous targeting strategies, at least one firm must be randomizing in its targeting strategies.

Second, consider an asymmetric equilibrium in which firm $-i$ targets group b with probability one, firm i targets each group with positive probability, and both firms send messages only to consumers in the respective targeted group.[8] In this case, firm i optimally sets $p_i = A$ when targeting group a, as it faces no competition from firm $-i$ then. Therefore, firm i's profit from this strategy is $\alpha A - c$. Suppose that firm $-i$ draws its

[8]For the same reason as laid out in Section 4.1, it cannot be optimal for firm $-i$ to target group a instead of group b, given that the rival firm i is randomizing between targeting groups a and b. This is due to the fact that consumers in group b have a higher valuation.

price from the cdf $I(p_{-i})$, with an upper bound of B. Then, firm i is indifferent between targeting A setting price $p_i = A$ and targeting B setting any price on $[A, B]$ if

$$\alpha p(1 - I(p)) + \alpha(1 - \alpha)pI(p) - c = \alpha A - c.$$

This leads to

$$I(p) = \begin{cases} \frac{p-A}{\alpha p}, & \text{if } p \in [A, B); \\ 1, & \text{if } p = B. \end{cases}$$

with $I(A) = 0$. We must have $(B - A)/(\alpha B) < 1$ as it is equivalent to $A > (1 - \alpha)B$. If instead $A \leq (1 - \alpha)B$, it would never be optimal for firm i to mix between targeting group a and group b, as the profit from targeting group b with probability 1 then exceeds the profit from the randomization strategy. This is due to the fact that, as established in Section 4.1, the profit when both firms target group b with probability 1 is $\alpha(1-\alpha)B-c$, which exceeds $\alpha A - c$ if $A \leq (1 - \alpha)B$. Hence, if the latter inequality holds, both firms target group b with probability 1. In turn, the fact that $(B - A)/(\alpha B) < 1$ if firms play heterogeneous targeting strategies implies that there is a mass point at $p_{-i} = B$ with probability mass $1 - [(B - A)/(\alpha B)] = (A - (1 - \alpha)B)/(\alpha B)$. This establishes that the mixing range of firm $-i$ are prices in $[A, B]$. For the same reason as above, sending messages to both consumer groups can never be optimal for firm $-i$, as the additional profit that it can obtain from this strategy is $(1 - \alpha)A - c$, which is negative for $c = \hat{c} = \alpha A$.

We next turn to the mixing strategy of firm i. Suppose firm i targets consumer group a with probability σ_a and consumer group b with probability $1 - \sigma_a$. In addition, it sets sets $p_i = A$ when targeting consumer group a and plays a mixed strategy drawing its price from the cdf $J(p_i)$ on $[A, B]$ when targeting group b. Then, firm $-i$, which always targets group b, is indifferent between any price on $[A, B]$ if

$$\alpha p\left(\sigma_a + (1 - \sigma_a)(1 - J(p))\right) + \alpha(1 - \alpha)p(1 - \sigma_a)J(p) = \alpha A.$$

This leads to

$$J(p, \sigma_a) = \frac{p - A}{\alpha p(1 - \sigma_a)} = \frac{I(p)}{(1 - \sigma_a)}.$$

Solving for σ_a by using $J(B, \sigma_a) = 1$ yields

$$\sigma_a = \frac{A - (1 - \alpha)B}{\alpha B}$$

and

$$J(p) = \frac{B(p-A)}{(B-A)p}$$

with $J(A) = 0$ and $J(B) = 1$. Note that σ_a is equal to the probability that firm $-i$ sets its price equal to B. This establishes that an asymmetric equilibrium in which one firm uses mixed targeting strategies exists, and the expected profit of each firm is $\alpha A - c$.

Comparing $\alpha A - c$ with the profit of the symmetric equilibrium with mixed targeting strategies, which is $(2-\alpha)\alpha AB/(A+B) - c$, we obtain that the former is larger for $\alpha > \hat{\alpha}_1(A/B)$. Therefore, the asymmetric equilibrium is profit-dominant in the relevant range.

It follows that for $\alpha \leq \hat{\alpha}_1(A/B)$, the unique equilibrium involves both firms targeting consumer group b with probability 1; instead, for $\alpha > \hat{\alpha}_1(A/B)$, the profit-dominant equilibrium is the asymmetric one characterized above. The sum of expected profits as a function of α is therefore given by

$$\sum_i E[\pi_i^\star(\alpha)] = \begin{cases} 2\alpha(1-\alpha)B - 2c, & \text{if } \alpha \in [1/2, \hat{\alpha}_1(A/B)]; \\ 2\alpha A - 2c, & \text{if } \alpha \in (\hat{\alpha}_1(A/B), 1]. \end{cases}$$

The equilibrium profits in ranges $\alpha \in [1/2, \hat{\alpha}(A/B)]$ and $\alpha \in (\hat{\alpha}(A/B), 1]$ are equivalent at $\alpha = \hat{\alpha}_1(A/B)$, which implies that the equilibrium profit is continuous but has a kink at $\alpha = \hat{\alpha}_1(A/B)$. Moreover, it is non-monotonic because it falls with α in the first range but increases with α in the second range. Finally, comparing the profit at $\alpha = 1$ with that at $\alpha = 1/2$, we obtain that the former is larger than the latter if and only if $2A - 2c > 1/2B - 2c$ or $4A > B$.

□

Proof of Proposition 7. We start with the case in which both firms target consumer group b.

First, there can never be an equilibrium in which only one firm buys advertising slots. Suppose to the contrary that this was the case. Then, if the firm obtains 3 or 4 slots (i.e., both slots on platform $j \in \{x, y\}$ and one or both slots on platform $-j$), it has a profitable deviation by not buying the slots on platform $-j$. The reason is that the firm can reach all consumers already via the two slots on platform j, and it is not profitable to buy additional slots on platform $-j$. Instead, if the firm would only buy the two slots on platform j, the other firm can obtain a strictly positive profit by buying the slot to advertise to consumers who are expected to belong to group b on platform $-j$. This is because, from Lemma 2, in the resulting price setting-game, the profit of the firm is $\alpha(1-\alpha)B$, which is strictly positive.

In addition, there does not exist an equilibrium in which both firms are active on different platforms but each of them buys only one advertising slot. To see this, suppose that e.g. firm 1 buys the slot to advertise to the expected consumer group b on platform x—i.e., slot x_b—and firm 2 buys the slot to advertise to the expected consumer group b on platform y—i.e., y_b. The slots to advertise to the expected consumer group a on both platforms—i.e., slots x_a and y_a—are then not taken. From the analysis of Section 4.1, the profit of each firm is then $\alpha(1-\alpha)B$, as the mass of exclusive consumers of a firm is $\alpha(1-\alpha)$. However, following the analysis of Section 4.2, by additional buying slot x_a, firm i could increase its profit to $(1-\alpha)B$. It therefore has a profitable deviation. By a similar argument, there is also a profitable deviation for any other combination in which each firm buys only one advertising slot from different platforms.[9]

Moreover, there can never be an equilibrium in which one firm buys both advertising slots on platform j and the other firm both slots on platform $-j$. Because both firms reach all consumers then, they would obtain zero profits due to Bertrand competition. A firm then has the incentive to deviate and not buy one of the slots. Similarly, it cannot be an equilibrium that a firm buys 3 of the 4 slots and the other firm the remaining slot. The first firm then reaches all consumers already via one platform and therefore has a profitable deviation by not buying the slot on the other platform.

Given that all allocations in the previous three paragraphs are ruled out as potential equilibria, we are left with five equilibrium candidates. These are:

(i) Firm i buys both advertising slots on platform x, and firm $-i$ buys the slot to advertise to consumers expected to belong to group b on platform y.[10] In what follows, we refer to this allocation as $(x_b : i, x_a : i, y_b : -i, y_a : 0)$, where the entry after each colon is the firm that buys the respective advertising slot and a '0' represents that no firm buys this advertising slot.

(ii) Firm i buys the advertising slot to expected group-a consumers on platform x and expected group-b consumers on platform y, whereas firm $-i$ buys the advertising slot to expected group-b consumers on platform x. Using the same notation as above, this allocation can be written as $(x_b : -i, x_a : i, y_b : i, y_a : 0)$.

(iii) Firm i buys the advertising slots to expected group-b consumers on both platforms, whereas firm $-i$ buys the advertising slot to expected group-a consumers only on platform x. This allocation can be represented by $(x_b : i, x_a : -i, y_b : i, y_a : 0)$.

[9]By our assumption to focus on cases with two active platforms, the situation in which firms each buy an advertising slot on only one platform is not possible, as the other platform would then not be active.

[10]Assuming that it is platform x that sells both slots and platform y that sells only one is without loss of generality, as the identity of the platform does not matter due to symmetry.

(iv) Firm i buys the advertising slot to expected group-b consumers on platform x and expected group-a consumers on platform y, whereas for firm $-i$ it is the opposite, that is, firm $-i$ buys the advertising slot to expected group-b consumers on platform y and expected group-a consumers on platform x. This allocation can be stated as $(x_b : i, x_a : -i, y_b : -i, y_a : i)$.

(v) Finally, firm i buys the advertising slots to expected group-b consumers on both platforms, whereas firm $-i$ buys the advertising slots to expected group-a consumers on both platforms. This allocation can be represented by $(x_b : i, x_a : -i, y_b : i, y_a : -i)$.

We next determine the firms' profits in each potential equilibrium candidate. We start with case (i), that is, allocation $(x_b : i, x_a : i, y_b : -i, y_a : 0)$. This allocation leads to a distribution of consumers which resembles that in Section 4.1. The resulting pricing equilibrium is therefore the same as in Section 4.1, leading to profits of $\pi_i = (1-\alpha)B$ and $\pi_{-i} = \alpha(1-\alpha)B$.

We next turn to case (ii), where the allocation is given by $(x_b : -i, x_a : i, y_b : i, y_a : 0)$. As in the previous case, firm i buys two advertising slots and firm $-i$ just one. However, in contrast to the previous case, firm i has its advertising slots now on different platforms. We can write the expected profits of the firms in the different price combinations as follows:

$$E[\pi_i(p_i, p_{-i})] = \begin{cases} \left(\alpha + (1-\alpha)^2\right) p_i, & \text{if } p_i < p_{-i}; \\ \left(1 - \alpha + \frac{\alpha^2}{2}\right) p_i, & \text{if } p_i = p_{-i}; \\ (1-\alpha)p_i, & \text{if } p_i > p_{-i}, \end{cases}$$

and

$$E[\pi_{-i}(p_i, p_{-i})] = \begin{cases} \alpha p_{-i}, & \text{if } p_{-i} < p_i; \\ \left(\alpha(1-\alpha) + \frac{\alpha^2}{2}\right) p_{-i}, & \text{if } p_{-i} = p_i; \\ \alpha(1-\alpha)p_{-i}, & \text{if } p_{-i} > p_i. \end{cases}$$

We can solve for the expected equilibrium profit in a similar way as in Section 4.2. When setting a price of $p_i = B$, firm i obtains a profit of $(1-\alpha)B$. Therefore, the lowest price p_i firm i is willing to set is given by $\left(\alpha + (1-\alpha)^2\right) p_i = (1-\alpha)B$, which yields $p_i = (1-\alpha)B/(1-\alpha+\alpha^2)$. By marginally undercutting this price, firm $-i$ sells to all consumers it reaches and obtains a profit of $\alpha(1-\alpha)B/(1-\alpha+\alpha^2)$. We can determine the respective distribution functions that support these pricing bounds and the resulting profits in the same way as in Section 4.2. Therefore, the expected profits in this equilibrium candidate are $\pi_i = (1-\alpha)B$ and $\pi_{-i} = \alpha(1-\alpha)B/(1-\alpha+\alpha^2)$.

In case (iii), the allocation of slots is again asymmetric—i.e., $(x_b : i, x_a : -i, y_b : i, y_a : 0)$.

We can determine the expected profits in the same way as in the previous case. The mass of consumers that firm i reaches exclusively is α, which implies that firm i's profit from setting a price of B is αB. Instead, the total mass of consumers that firm i reaches is $\alpha(2-\alpha)$. Therefore, the lowest price that firm is willing to set is $B/(2-\alpha)$. Marginally undercutting this price gives a profit of $(1-\alpha)B/(2-\alpha)$ to firm $-i$, as firm $-i$ reaches a mass of $1-\alpha$ consumers. We can again show that this outcome can be supported by a pricing equilibrium in mixed strategies. The expected profits in case (iii) are therefore $\pi_i = \alpha B$ and $\pi_{-i} = (1-\alpha)B/(2-\alpha)$.

The allocation in case (iv) is $(x_b : i, x_a : -i, y_b : -i, y_a : i)$, which implies that it is symmetric and all four slots are bought by firms. The resulting price equilibrium can then be analyzed along very similar lines as in Section 4.1. Each firm has a mass of $\alpha(1-\alpha)$ exclusive consumers. The resulting expected profit in the mixed-strategy equilibrium is $\alpha(1-\alpha)B$, that is, $\pi_1 = \pi_2 = \alpha(1-\alpha)B$.

Finally, we turn to case (v) where the allocation is $(x_b : i, x_a : -i, y_b : i, y_a : -i)$. Again, all four slots are bought by firms, but this time in asymmetric fashion. Writing down the expected profits of the firms for the different price combinations, we obtain

$$E[\pi_i(p_i, p_{-i})] = \begin{cases} \alpha(2-\alpha)p_i, & \text{if } p_i < p_{-i}; \\ \left(\alpha^2 + \alpha(1-\alpha)\right)p_i, & \text{if } p_i = p_{-i}; \\ \alpha^2 p_i, & \text{if } p_i > p_{-i}, \end{cases}$$

and

$$E[\pi_{-i}(p_i, p_{-i})] = \begin{cases} (1-\alpha^2)p_{-i}, & \text{if } p_{-i} < p_i; \\ \left(1-2\alpha+\alpha^2+\alpha(1-\alpha)\right)p_{-i}, & \text{if } p_{-i} = p_i; \\ \alpha(1-2\alpha+\alpha^2)p_{-i}, & \text{if } p_{-i} > p_i. \end{cases}$$

Solving this in the same way as the other cases with asymmetric allocations, we obtain that $\pi_i = \alpha^2 B$ and $\pi_{-i} = \alpha(1-\alpha^2)B/(2-\alpha)$, where the latter is due to the fact that the lowest price in the support of firm i's distribution is $\alpha B/(2-\alpha)$ and firm $-i$ reaches $1-\alpha^2$ consumers.

Having determined the profits in each potential equilibrium candidate, we can now check which of these candidates can be part of a SPNE, with the refinement that, if multiple equilibria exist, the equilibrium that yields the highest profit to the firm with the lower pay-off is played. To do so in the simplest way, we first show that the equilibrium candidate in case (ii) provides a higher profit to the firm with the lower pay-off than the candidates in cases (i), (iii), and (iv), but not (v). We next show that no firm has a profitable deviation in case (ii). This implies that this candidate is indeed an equilibrium in the subgame in which firms choose which slots to buy. It follows that the equilibrium candidates in cases (i), (iii), and (iv) will never be part of a SPNE, given our refinement. We then show that firms have a profitable deviation from the equilibrium candidate in case (v) only if α is sufficiently low, that is, case (v) is an equilibrium only if α is sufficiently large. We finally compare the potential equilibria in cases (ii) and (v) with respect to our selection criterion.

We start with a profit comparison of firm $-i$, which is the firm that receives the (weakly) lower profit in all equilibrium candidates, in the different cases. In case (ii), the firm obtains a profit of $\alpha(1-\alpha)B/(1-\alpha+\alpha^2)$. As the denominator is smaller than 1, this profit is larger than $\alpha(1-\alpha)B$, which is the profit in cases (i) and (iv). Moreover, comparing $\alpha(1-\alpha)B/(1-\alpha+\alpha^2)$ with the profit in case (iii)—i.e., $(1-\alpha)B/(2-\alpha)$—we obtain that the difference between the former and the latter is

$$\frac{(1-\alpha)^2(2\alpha-1}{(2-\alpha)(1-\alpha+\alpha^2)} > 0.$$

Therefore, if the equilibrium candidate in case (ii) is indeed an equilibrium, it will be selected over the candidates in cases (i), (iii), and (iv). Comparing π_{-i} in case (ii) with that in case (v) yields that the difference between the former and the latter is

$$\frac{\alpha(1-\alpha-\alpha^3)(2\alpha-1}{(2-\alpha)(1-\alpha+\alpha^2)},$$

which is positive for $\alpha \in [1/2, 1]$ if and only if $\alpha \leq \breve{\alpha}$, where $\breve{\alpha}$ is implicitly defined by the equation $1-\breve{\alpha}-\breve{\alpha}^3 = 0$ and approximately equal to 0.682. It follows that, if the equilibrium candidates in cases (ii) and (v) are equilibria, the former will be chosen for $\alpha \leq \breve{\alpha}$ and the latter for $\alpha > \breve{\alpha}$, given our selection criterion.

We next analyze whether each firm has a profitable deviation from the equilibrium candidates in cases (ii) and (v), starting with case (ii). The allocation in this candidate is $(x_b : -i, x_a : i, y_b : i, y_a : 0)$. Consider first firm $-i$. If firm $-i$ would deviate by also willing to buy slot y_a, it would get it (as firm i is not willing to buy it). The resulting allocation is then that of case (iv) above, in which each firm's profit is $\alpha(1-\alpha)B$. As this is lower than π_{-i} in case (ii), this deviation is not profitable.

Firm $-i$ can also deviate by willing to buy one or both of the slots that firm i buys in the equilibrium candidate (ii). We now determine which firm would get the respective slot in case both firms are willing to buy it. If firm $-i$ would be willing to buy slot x_a and would get it, the new allocation would be $(x_b : -i, x_a : -i, y_b : i, y_a : 0)$, which is equivalent to the allocation in case (i). However, as just shown, the firm with the lower pay-off receives a lower profit in case (i) than in case (ii). Therefore, the allocation in case (ii) would be selected in that case, which implies that firm $-i$ does not get slot x_a. Similarly, if firm $-i$ would be willing to buy slot y_b and would get it, the new allocation is that of case (iii), where the profit of the firm with the lower pay-off is smaller than in case (ii). Again, this implies that the allocation of case (ii) will be selected and firm $-i$ would not get the slot. Therefore, these deviations are not profitable. In addition, the deviation to buy both slots x_a and y_b is also not profitable. If firm $-i$ would get both these slots, the resulting profit of firm i would be zero, which is lower than the profit firm i or firm $-i$ would obtain by allocating one or both slots to firm i. Hence, the allocation in which firm $-i$ gets all three slots x_b, x_a, and y_b and firm i gets none will not be selected.

Next, we turn to the deviation incentives of firm i. By the argument given above, firm i cannot profitably deviate by buying slot y_a: as the firm reaches all consumers on platform x, it cannot gain by buying this slot on platform y. The firm can also not benefit by not buying one of the two slots it obtains in case (ii). For instance, if the firm would not buy slot x_a, its profit in the resulting price game would be $\alpha(1-\alpha)B$, which is below $(1-\alpha)B$. Therefore, no firm has a profitable deviation from the allocation in case (ii), which implies that this allocation is indeed an equilibrium in case both firms target consumer group b.

We next turn to the allocation in case (v)—i.e., $(x_b : i, x_a : -i, y_b : i, y_a : -i)$. We again first consider a deviation by firm $-i$. Suppose the firm would also bid for a slot that is taken by firm i in the equilibrium candidate, that is, slot x_b or y_b. If firm $-i$ would get this slot, the profit of firm i, which is then the firm that obtains the lower pay-off in the resulting price equilibrium, would be $\alpha(1-\alpha)B$. However, this is strictly lower than $\alpha(1-\alpha^2)/(2-\alpha)$, which is the profit of firm $-i$ in the equilibrium candidate. Therefore, firm $-i$ would not get this slot even when willing to buy it, which implies that such a deviation is not profitable.[11] Suppose next that firm $-i$ deviates by not willing to buy one of the slots x_a or y_a. Its profit in the resulting price game would then be the same as in case (iii) which is equal to $(1-\alpha)/(2-\alpha)B$. The difference between the profit in the candidate equilibrium and that from deviation is $2\alpha - 1 - \alpha^3$. In what follows, we denote the solution that fulfills $\alpha \in [1/2, 1]$ to the equation $2\alpha - 1 - \alpha^3 = 0$ by $\underline{\alpha}$. The left-hand side of this equation is positive if $\alpha \geq \underline{\alpha} \approx 0.618$.

Turning to potential deviations by firm i, we can show in the same way as above that willing to buy additional slots is not profitable as the resulting allocation when getting these slots leads to lower pay-off of firm $-i$ compared to the candidate equilibrium, which implies that firm i would not get these slots. Similarly, deviating by not willing to buy either of the slots lowers firm i's profit. Therefore, firm i has no profitable deviation. It follows that the equilibrium candidate (v) is indeed an equilibrium as long as $\alpha \geq \underline{\alpha}$.

From the above analysis, we have the following result: If both firms target consumer group b, there always exists an equilibrium in which the allocation is as in case (ii), In

[11] Similarly, if firm $-i$ would be willing to buy both slots x_b or y_b, it would not get them as the resulting profit of firm i would then be 0.

addition, for $\alpha \geq \underline{\alpha}$, there also exists an equilibrium in which the allocation is as in case (v). If both equilibria exist, we use our selection criterion, that is, we select the equilibrium in which the firm with the lower pay-off obtains a higher profit. From above, we know that this implies that for $\alpha \leq \check{\alpha}$, the equilibrium with the allocation of case (ii) is selected, whereas for $\alpha > \check{\alpha}$, the equilibrium with the allocation of case (v) is selected. Indeed, due to the fact that $\check{\alpha}$, which is approximately 0.682, is strictly larger than $\underline{\alpha}$, which is approximately 0.618, the allocation of case (v) is indeed an equilibrium for $\alpha > \check{\alpha}$.

Having characterized the equilibrium in case both firms target group b, we next turn to the case in which firm i targets consumer group b and firm $-i$ targets group a. The equilibrium in the sub-game in which firms are deciding which slots to buy is then very simple. One firm buys both slots on platform j and the other firm buys both slots on platform $-j$. Each firm then reaches all consumers, and the resulting profits are $\pi_i = B$ and $\pi_{-i} = A$. It is easy to check that no firm has a profitable deviation from this strategy.

We now turn to the first stage of the game and determine under which conditions both firms target group b or target different consumer groups.[12] Given that one firm, say i, targets group b, firm $-i$ obtains a profit of A from targeting group a. Instead, for $\alpha \in [1/2, \check{\alpha}]$, firm $-i$ obtains a profit of $\alpha(1-\alpha)B/(1-\alpha+\alpha^2)$ when targeting group b. At $\alpha = 1/2$, firm $-i$'s profit from targeting group b is larger than that from targeting group a if and only if $A \leq B/3$. Since $\alpha(1-\alpha)B/(1-\alpha+\alpha^2)$ is strictly decreasing in α for $\alpha > 1/2$, it follows that targeting group b is never optimal if $A > B/3$ in the range $1/2 \leq \alpha \leq \check{\alpha}$. In addition, in the range $\alpha \in (\check{\alpha}, 1]$, the profit from targeting b is $\alpha(1-\alpha^2)B/(2-\alpha)$, which is also strictly below A for $A > B/3$. Therefore, for $A > B/3$, the unique equilibrium in stage 1 is that firms target different consumer groups. The sum of expected profits is then $A + B$.

Instead, for $A \leq B/3$, it is optimal for both firms to target consumer group b for α close to 1/2. As determined above, the profit of firm $-i$ when targeting group b changes from $\alpha(1-\alpha)B/(1-\alpha+\alpha^2)$ to $\alpha(1-\alpha^2)B/(2-\alpha)$ at $\alpha = \check{\alpha}$. If there is an intersection between the former profit and A—i.e., the profit when targeting group a—for a value of α below $\check{\alpha}$, then in first-stage equilibrium both firms target group b for α below this intersection point and target different groups for α above this intersection point. The sum of firm's profits in the former case are

$$(1-\alpha)B + \frac{\alpha(1-\alpha)B}{1-\alpha+\alpha^2} = \frac{(1-\alpha)(1+\alpha^2)B}{1-\alpha+\alpha^2}. \tag{21}$$

Inserting $\check{\alpha}$, which is implicitly defined by $1 - \check{\alpha} - \check{\alpha}^3 = 0$ and approximately equal to

[12] As in Section 4, it is never optimal that both firms target group a.

0.682, into the equation $\alpha(1-\alpha)B/(1-\alpha+\alpha^2) = A$, we obtain that such an intersection point for $\alpha \in [1/2, \breve{\alpha}]$ exists if $A \gtrapprox 0.277B$.

Finally, if $A \lessapprox 0.277B$, in equilibrium both firms target group b for α slightly above $\breve{\alpha}$ and firm $-i$ obtains a profit of $\alpha(1-\alpha^2)B/(2-\alpha)$. It optimally switches to targeting group a if α gets larger than a threshold which is implicitly defined by the solution to the equation $\alpha(1-\alpha^2)B/(2-\alpha) = A$.[13] Denoting this solution by $\tilde{\alpha}$, the sum of firm's profits for $\alpha \in (\breve{\alpha}, \tilde{\alpha}]$ is

$$\alpha^2 B + \frac{\alpha(1-\alpha^2)B}{2-\alpha} = \frac{\alpha\,(1+2\alpha(1-\alpha))\,B}{2-\alpha}. \tag{22}$$

We can now determine how the sum of firms' profits changes with α. Differentiating (21) with respect to α yields $-\alpha^2(3-2\alpha+\alpha^2)/(1-\alpha+\alpha^2)^2 < 0$, whereas differentiating (22) with respect to α yields $2(1-\alpha)(1+5\alpha-2\alpha^2)/((2-\alpha)^2) > 0$. In addition, the difference between (21) and (22) at $\alpha = \breve{\alpha}$ is strictly negative, which implies that the sum of profits jumps upwards at $\alpha = \breve{\alpha}$, given that both firm target group b. Moreover, the sum of profits when firms target different consumer groups (i.e., $B + A$) is strictly larger than (21) for all A between approximately $0.277B$ and $B/3$ in the entire range $\alpha \in [1/2, \breve{\alpha}]$. Similarly, $B + A$ is strictly larger than (22) for all A lower than approximately $0.277B$ in the entire range $\alpha \in (\breve{\alpha}, 1]$. This implies that there is an upward jump in the sum of profits at the point where the equilibrium switches from both firms targeting group b to firms targeting different consumer groups

We can therefore state the following result:

- If $A > B/3$, the sum of firms' profits is constant in α, as it is $B + A$.
- If $B/3 \geq A \gtrapprox 0.277B$, the sum of expected profit is decreasing in α for α below a threshold implicitly defined by $\alpha(1-\alpha)B/(1-\alpha+\alpha^2) = A$, jumps upwards at this threshold, and then stays constant in α.
- Finally, $A \lessapprox 0.277B$, the sum of expected profit is decreasing in α for $\alpha < \breve{\alpha}$, jumps upward at $\alpha = \breve{\alpha}$, increases for α between $\breve{\alpha}$ and a threshold implicitly defined by $\alpha(1-\alpha^2)B/(2-\alpha) = A$, jumps upwards at this threshold, and then stays constant in α.

This proves the statement in the proposition. □

[13]This threshold always exists as the left-hand side goes to 0 as α goes to 1.

C Analysis of the Simultaneous game for All Values of c

In the proof of Proposition 6, we characterized the equilibrium of the simultaneous game for $c = \hat{c}$. By continuity, for c close to $\hat{c}$, the equilibrium is the same.

To analyze the game for c (substantially) smaller than $\hat{c}$, we start again with the case in which both firms target consumers of group b. For lower values of c than $\hat{c}$, firms may have an incentive to send messages to consumers that are not in the expected targeted group. It is profitable to send messages to these consumers only if $c < (1-\alpha)^2 B$. This is because a message is successful (i.e., reaches a consumer who is interested in the product but does not belong to the expected targeted group) only with probability $1-\alpha$ and, given that the lower bound of the mixing region of the rival in the equilibrium $b(1)b(1)$ is $(1-\alpha)B$, the firm sells to the consumer with certainty only when charging a price of (at most) $(1-\alpha)B$. Therefore, the additional revenue when sending a message to a consumer who is not in the expected targeted group is $(1-\alpha)^2 B$, which implies that the equilibrium $b(1)b(1)$ ceases to exist for $c < (1-\alpha)^2 B$.

We next show that there exists a symmetric mixed-strategy equilibrium in which both firms target consumer group b, send messages to both consumer groups with probability σ_2 and only to consumers in the expected targeted group with probability $1-\sigma_2$. Prices are mixed and expected profits are $c(2\alpha-1)/(1-\alpha)$. We denote this equilibrium by $b(1+\sigma_2)b(1+\sigma_2)$.

In such an equilibrium, there is a cumulative distribution function $Q(p)$ for prices p and p_{-i}, such that the following equation holds:

$$E_{p_{-i}}[\pi_i(p_i = p, p_{-i})] = E_{p_{-i}}[\pi_i(p_i = B, p_{-i})]. \tag{23}$$

For $p_i = B$, the expected profit of firm i equals

$$\sigma_2(1-\bar{\sigma}_2)(1-\alpha)B + (1-\sigma_2)(1-\bar{\sigma}_2)\alpha(1-\alpha)B - c(1+\sigma_2),$$

where $\bar{\sigma}_2$ is the mixing probability of firm $-i$. The first term expresses firm i's expected profit when it sends messages to both groups, which happens with probability σ_2. Firm i sells the product at price $p_i = B$ only if the other firm sends only messages to consumers in its expected targeted group b (as otherwise the rival reaches all consumers and sets a lower price) and reaches a non-targeted consumer, which happens with probability $(1-\bar{\sigma}_2)(1-\alpha)$. The second term expresses firm i's expected profit from sending messages only to consumers in the expected targeted group. The product is then sold at price $p_i = B$ to a consumer of the targeted group b (who is reached with probability α) only if the other firm sends only messages to the targeted group b and reaches non-targeted consumers. The corresponding probability is $(1-\bar{\sigma}_2)(1-\alpha)$.

Denote by $\underline{p}$ the lowest price of firm $-i$ at which slightly undercutting this price (above A) yields a weakly larger expected profit than $E_{p_{-i}}[\pi_i(p_i = B, p_{-i})]$. This price is determined by

$$\underline{p}\left(\sigma_2 + (1-\sigma_2)\alpha\right) \geq \left(\sigma_2 + (1-\sigma_2)\alpha\right)(1-\alpha)(1-\bar{\sigma}_2)B,$$

and equals $\underline{p} \equiv (1-\alpha)(1-\bar{\sigma}_2)B$. Solving for the symmetric mixed-strategy equilibrium $b(1+\sigma_2)b(1+\sigma_2)$, suppose firm $-i$ draws prices in the range $p \in [\underline{p}, B]$ from the cumulative distribution function $Q(p)$. Then, firm i is indifferent between any price on $[\underline{p}, B]$ if

$$\begin{aligned} &\left(\sigma_2 + (1-\sigma_2)\alpha\right)(1-\alpha)(1-\bar{\sigma}_2)Q(p)p + \left(\sigma_2 + (1-\sigma_2)\alpha\right)(1-Q(p))p = \\ &\left(\sigma_2 + (1-\sigma_2)\alpha\right)(1-\alpha)(1-\bar{\sigma}_2)B. \end{aligned} \tag{24}$$

Solving for $Q(p)$ given $\sigma_2 = \bar{\sigma}_2$ leads to

$$Q(p) = \frac{p - (1-\alpha)(1-\sigma_2)B}{p\left(\alpha(1-\sigma_2) + \sigma_2\right)}$$

with support $p \in [(1-\alpha)(1-\sigma_2)B, B]$, and $Q(\underline{p}) = 0$ and $Q(B) = 1$.

The mixing probability in the symmetric equilibrium , σ_2^*, is derived as follows. Given $\bar{\sigma}_2$ and c, firm i maximizes its expected profit over σ_2,

$$\max_{\sigma_2} \left(\sigma_2 + (1-\sigma_2)\alpha\right)(1-\alpha)(1-\bar{\sigma}_2)B - c(1+\sigma_2).$$

Its marginal expected profit equals

$$(1-\alpha)^2(1-\bar{\sigma}_2)B - c.$$

Firm i has no incentive to locally or globally deviate in σ_2 when firm $-i$ chooses $\bar{\sigma}_2 = 1 - \frac{c}{(1-\alpha)^2 B}$ where the marginal expected profit equals zero. Thus,

$$\sigma_2^* = 1 - \frac{c}{(1-\alpha)^2 B} \quad \text{for } c \in [0, (1-\alpha)^2 B]$$

is the mixing probability in the candidate symmetric equilibrium. Finally, the candidate symmetric equilibrium is indeed an equilibrium because firm i has no incentive to deviate in σ_2 and p_i jointly. This can be seen from equation (24) as $(\sigma_2 + (1-\sigma_2)\alpha)$ cancels out. Therefore, there is no $\sigma_2 \in [0,1]$ such that any p_i leads to a higher profit than the candidate symmetric equilibrium with $Q(p)$ and σ_2^*. The corresponding expected profit equals $c(2\alpha - 1)/(1 - \alpha)$ and is increasing in α. Furthermore, since $\sigma_2^* > 0$ if and only if $c < (1 - \alpha)^2 B$, the symmetric mixed-strategy equilibrium $b(1 + \sigma_2)b(1 + \sigma_2)$ does not overlap with the symmetric mixed-strategy equilibrium $b(1)b(1)$.

If costs are even lower, we first note that an equilibrium in which both firms target consumer group b and at least one firm sends messages to both consumer groups does no longer exist for two reasons. First, suppose that both firms target consumer group b and send messages to both consumer groups. Then, a firm's expected profit equals zero because of Bertrand competition which arises because all consumers are informed about both prices. Yet, in the simultaneous-move game, there is a profitable deviation to target consumer group a, send messages to both consumer groups, and set $p = A$. Second, suppose that both firms target consumer group b but send messages to a different number of consumer groups—i.e., one firm sends messages only to consumers of the expected targeted group, whereas the other firm sends messages to both consumer groups (cf. $b(1)b(2)$ in in the sequential game). Then, the former firm has a profitable deviation to increase its reach by sending messages also to all consumers and setting $p = B - \epsilon$, with $\epsilon > 0$, but $\epsilon \to 0$. The profit from this deviation equals $\alpha(1 - \alpha)B + (1 - \alpha)B - 2c$ because the deviation exploits that the other firm has a mass point of $1 - \alpha$ at $p = B$ (see (2)). This profit is larger than $\alpha(1 - \alpha)B - c$ for $(1 - \alpha)B > c$, which is fulfilled as we are in the range $c < (1 - \alpha)^2 B$.

However, if $c \leq (1-\alpha)A$, there exists an asymmetric mixed-strategy equilibrium with messages to all consumers where firm $-i$ always targets consumer group b and mixes prices, and firm i targets consumer groups a and b with a positive probability. When targeting consumer group a, firm i sets $p_i = A$ and otherwise it mixes prices. The expected profit for both firms is equal to A.

We now characterize this equilibrium. Suppose firm $-i$ targets B with probability 1, sends messages to all consumers, and plays a mixed strategy drawing its price from the cdf $K(p_{-i})$ on $[A, B]$. Then, firm i is indifferent between targeting group a and setting price $p_i = A$, and targeting group b and setting any price p on $[A, B]$ if

$$p \cdot (1 - K(p)) + 0 \cdot K(p) = A.$$

This leads to

$$K(p) = \begin{cases} \frac{p-A}{p}, & \text{if } p \in [A, B); \\ 1, & \text{if } p = B. \end{cases}$$

with $K(A) = 0$, $(B - A)/(B) < 1$ and a mass point of A/B at $p = B$.

Suppose firm i sends messages to all consumers, targets group a with probability $\bar{\sigma}_A$ and sets $p_i = A$, and targets group b with probability $1 - \bar{\sigma}_A$ and plays a mixed strategy drawing its price from the cdf $L(p_i)$ on $[A, B]$. Then, firm $-i$, always targeting B, is indifferent between any price on $[A, B]$ if

$$p \cdot ((1-\bar{\sigma}_A)(1-L(p)) + \bar{\sigma}_A) + 0 \cdot (1-\bar{\sigma}_A)L(p) = A.$$

This leads to

$$L(p, \bar{\sigma}_A) = \frac{p-A}{p(1-\bar{\sigma}_A)} = \frac{K(p)}{(1-\bar{\sigma}_A)}.$$

Solving for $\bar{\sigma}_A$, such that $L(B, \bar{\sigma}_A) = 1$, yields

$$\bar{\sigma}_A = \frac{A}{B},$$

which implies

$$L(p) = \frac{B(p-A)}{(B-A)p}$$

with $L(A) = 0$ and $L(B) = 1$. Note that $\bar{\sigma}_A$ is equal to the probability that firm $-i$ sets its price equal to B. The strategy of firm i to send messages to all consumers can only be profitable if $c < (1-\alpha)A$, as $(1-\alpha)A$ is the revenue that firm i obtains from consumers who do not belong to the expected targeted group. Therefore, this equilibrium, which we denote by $\bar{\sigma}_A(2)b(2)$, only exists if $c < (1-\alpha)A$.

Finally, we show that firm i does not have an incentive to undercut $p_{-i} = B$ slightly with probability 1. This is satisfied if and only if firm i's expected profit of setting $p_i = B - \epsilon$ is not larger than A. Firm i's expected deviation profit equals

$$0 \cdot (1-\bar{\sigma}_A) + B \cdot \bar{\sigma}_A, \tag{25}$$

where the first term describes firm i's expected profit when firm $-i$ mixes prices on $[A, B)$, which is zero, and the second term that when firm $-i$ sets $p_{-i} = B$. Condition (25) simplifies to A. Therefore, this is not a profitable deviation.

Using profit dominance, in the region $c \in [0, (1-\alpha)A]$—i.e., the region in which the equilibrium $\bar{\sigma}_A(2)b(2)$ exists—we obtain that this equilibrium is indeed selected over the equilibrium $b(1+\sigma_2)b(1+\sigma_2)$ because $A - 2c \geq c(2\alpha-1)/(1-\alpha)$.

We next turn to the boundaries of the different regions. It is straightforward to verify that $(1-\alpha)^2 B \geq (1-\alpha)A$ if and only if $\alpha \leq \hat{\alpha}_1(A/B)$. Therefore, for $\alpha \leq \hat{\alpha}_1(A/B)$, the equilibrium for different values of c is as follows:

- For $c \geq (1-\alpha)^2 B$, the equilibrium is $b(1)b(1)$ and each firm's profit is $\alpha(1-\alpha)B - c$.

- For $(1-\alpha)^2 B > c \geq (1-\alpha)A$, the equilibrium is $b(1+\sigma_2^*)b(1+\sigma_2^*)$ and each firm's profit is $c(2\alpha-1)/(1-\alpha)$.

- For $0 \leq c < (1-\alpha)A$, the equilibrium is $\bar{\sigma}_A(2)b(2)$ and each firm's profit is $A-2c$.

We next turn to the region $\alpha > \hat{\alpha}_1(A/B)$. As shown in the proof of Proposition 6, the equilibrium in which each firm sends messages only to consumers in the expected targeted group and one firm targets group b with certainty, while the other firm targets each group with positive probability only exists if $(1-\alpha)A \leq c$. We denote that equilibrium by $\bar{\sigma}_A(1)b(1)$. Instead, the equilibrium $\bar{\sigma}_A(2)b(2)$ exists only if $(1-\alpha)A > c$. Therefore, for $\alpha > \hat{\alpha}_1(A/B)$, the equilibrium for different values of c is as follows:

- For $(1-\alpha)A \leq c$, the equilibrium is $\bar{\sigma}_A(1)b(1)$ and each firm's profit is $\alpha A - c$.

- For $0 \leq c < (1-\alpha)A$, the equilibrium is $\bar{\sigma}_A(2)b(2)$ and each firm's profit is $A-2c$.

Taking these results together, we obtain that for $c < (1-\alpha)A$, the sum of equilibrium profits is constant in α, as it equals $A-2c$. Instead, for $c \geq (1-\alpha)A$, the sum of equilibrium profits changes non-monotonically with α. The profits increase in α for $\alpha \in [1/2, 1-\sqrt{c/B})$,[14] decrease in α for $\alpha \in [1-\sqrt{c/B}, \hat{\alpha}_1(A/B))$, and then increase again for $\alpha \in [\hat{\alpha}, 1]$. If $c \to \hat{c}$, the first region vanishes,[15] and we obtain the result of Proposition 6.

There are several interesting aspects in the equilibrium with simultaneous decisions. First, in contrast to the case with sequential decisions, the equilibrium profits of firms are always symmetric (despite the fact that asymmetric equilibria arise in some regions). The reason is that, in the case with sequential decisions, choosing an asymmetric targeting and/or advertising strategy allows firms to differentiate, and therefore dampens competition in the pricing stage. In the simultaneous game, this is not possible, as a firm can deviate jointly in its targeting and pricing decision. Therefore, committing to target the low-value consumer group a or sending messages only to the expected targeted group although costs are small, cannot be part of an equilibrium strategy in the simultaneous game, as the respective firm does then have a profitable deviation. Therefore, in any equilibrium firms' profits must be symmetric because gaining advantages by committing to differentiate via equilibrium strategies is not possible. A direct implication is that profits are weakly lower in the simultaneous game. As firms have more deviation possibilities, their ability to dampen competition is reduced, which leads to reduced profits.

With respect to the question how equilibrium profits change with targeting, our result shows that, if advertising costs are sufficiently small, the equilibrium profit does not

[14]The boundary is obtained by solving $c = (1-\alpha)^2 B$ for α.

[15]This is because the equilibrium $b(1+\sigma_2^*)b(1+\sigma_2^*)$ only exists if $c < (1-\alpha)^2 B$ but $\hat{c} = \alpha(1-\alpha)B$, which is strictly larger than $(1-\alpha)^2 B$ only if $\alpha > 1/2$, but $(1-\alpha)^2 B = \alpha(1-\alpha)B$ for $\alpha = 1/2$.

change with α. In this case, each firm always obtains a profit of $A - 2c$. The equilibrium is asymmetric as one firm randomizes in its targeting strategy between both consumer groups, whereas the other chooses consumer group b with certainty. This implies that there is no competition for consumer group a and the resulting revenue from these consumers is A. As advertising costs are small and each firms sends messages to all consumers, α does not matter for profits. Competition for consumers interested in product b is so intense that the expected revenue of each firm is still equal to A.

Instead, if c is in an intermediate range, profits are non-monotonic but change their sign twice—i.e., they are increasing for small values of α, decreasing for intermediate values of α, and increasing again for high values of α. Whereas the second and third region is similar to the case with a sequential timing, the first region only occurs with simultaneous decisions. In this region, each firm targets group b and sends messages to consumers in the expected targeted group but also with some probability to consumers who do not belong to the expected targeted group. The resulting profit of each firm is $c(2\alpha - 1)/(1 - \alpha)$. Interestingly, this profit is increasing in the advertising costs. The intuition is that a higher c lowers the probability with which a firm sends messages to consumers who are not in the expected targeted group, which dampens competition. A higher α also reduces the incentive to send messages to these consumers, which helps to save advertising costs and lowers the competitive pressure. This explains why profits are increasing in α in this range.

Overall, although several new aspects occur with simultaneous as compared to sequential competition, the main effects remain.