Ecosystems and Complementary Platforms*

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Abstract

Motivated by several examples, including Internet of Things patent licensing, we develop a tractable model of multi-product ecosystems, where one or more platforms provide inputs to a set of devices linked through demand-side externalities. Prices depend on each device's Katz-Bonacich centrality in a network defined by the externalities, and we show how the relevant network differs for an ecosystem monopolist, a social planner, or a group of complementary platforms. We use the model to revisit Cournot's analysis of complementary monopolies in a platform setting, and to analyze a partial (one-sided) merger of complementary platforms.

Keywords: Multi-sided Market, Complementary Platforms, Network, Centrality, IoT, Licensing

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1 Introduction

Motivated by the growth and proliferation of digital intermediaries, a growing body of economic theory analyzes pricing by multi-sided platforms. This literature builds upon a series of papers that, for reasons of tractability and exposition, analyze two-sided platforms (Caillaud and Jullien, 2001, 2003; Rochet and Tirole, 2003, 2006; Anderson and Coate, 2005; Armstrong, 2006). In practice, the leading platforms serve a multitude of sides, to the point where many observers describe them as ecosystems. The prior literature has also focused on two types of pricing: monopoly and competition. With the proliferation of platform business models, however, it is natural that some intermediaries find themselves in complementary rather than competitive relationships.

This paper analyzes a model of ecosystems. We assume linear demand for all devices, but allow for an arbitrary number of platforms and sides,[1] as well as a very general specification of the demand externalities among all devices. The model yields answers to a number of novel questions, including: How does a device's position within its ecosystem (network) influence pricing and demand? What are the equilibrium prices charged by complementary platforms that serve overlapping user groups? How does the presence of a complementary intermediary influence decisions to either subsidize or extract value from a particular side of the platform?

For a monopoly platform, the price charged to each side reflects the well-known trade-off between internalizing externalities (subsidizing devices that generate larger positive externalities) and extracting value. These forces are captured by a weighted average of all externalities to/from all other devices, where the weight of each device corresponds to its Katz-Bonacich centrality in the overall demand system. In equilibrium, the output of each device is proportional to its centrality.

We show how the matrix used to compute centrality differs for a monopolist, social planner, and group of complementary platforms. Our analysis reveals that adding complementary platforms leads each platform to place more weight on externality internalization relative to value extraction, such that devices' relative centrality (and equilibrium output) may change.

Using examples, we show how platform complementarity expands the range of equilibrium outcomes relative to the single good case first studied by Cournot (1838, Chapter IX). In particular, the total price charged to a single side of the platform can be less than

[1] Hereafter, we use the terms side and device interchangeably.

the integrated-monopoly benchmark.

To motivate our model, we use the example of patent licensing for the Internet of Things (IoT). Patent holders have traditionally licensed two sides of the cellular network: handsets and base stations. To the extent that handset users value greater coverage (i.e. more base stations) and carrier investments reflect the size of the user base, licensors face a two-sided pricing problem. The emergence of IoT, where connected products include not just phones and networks, but also cars, watches, appliances, eyeglasses, and many other goods, converts this into a many-sided pricing problem. For a monopolist whose patent portfolio covers all devices, our model yields a particularly simple characterization of optimal pricing. Moreover, our framework can be used to analyze the more realistic scenario of multiple patent holders, whose patents are essential for the production of a certain set of devices.

This paper contributes to several strands of literature. First, there is a large literature on pricing by two-sided platforms; early contributions include Anderson and Coate (2005), Armstrong (2006), Caillaud and Jullien (2001, 2003), and Rochet and Tirole (2003, 2006). The literature considers either a monopoly platform or platform competition. For instance, Weyl (2010) studies a monopoly platform with many sides and highlights the role of a Spence distortion. More recently, Tan and Zhou (2021) analyze platform competition in a many-sided market, characterize the symmetric equilibrium prices, and perform comparative statics to find that an increase in the number of platforms can lead to an increase in the prices. Our main contribution to this literature is to show how, for a many-sided platform, the K-B centrality of each side/device plays a crucial role. By characterizing equilibrium pricing in terms of centrality measures, we find that K-B centrality is a natural concept to use in an ecosystem composed of multiple sides, because it captures both *direct* and *indirect* cross-side network effects.[2] Our analysis of complementary platforms is also a contribution to this literature. Van Cayseele and Reynaerts (2011) study the effects of joint ownership in a model where platforms are complementary on the multihoming side but compete on the single-homing side. We analyze a more general model with any number of (strictly) complementary platforms and inter-group network externalities among $n \geq 2$ sides and identify a novel Cournot complementary effect in terms of how the number of platforms distorts the centrality measure of each side.

Second, this paper is related to the literature on pricing in networks in the presence

[2]When there are more than two sides, even if direct externalities between side i and side j are zero, there can be indirect externalities between the two through other sides.

of consumption and price externalities. Building on Ballester et al. (2006)'s approach to network games with strategic complementarities among players, Candogan et al. (2012) and Bloch and Quérou (2013) show that if network effects are symmetric and marginal production costs are constant, a monopolist's optimal prices do not depend on the network structure even if the monopolist is able to price-discriminate. Bloch and Quérou (2013), Chen et al. (2018), Zhang and Chen (2020) and Chen et al. (2022) show that this irrelevance result does not hold in a competitive setting. In that case, firms price-discriminate consumers based on their network positions in terms of Katz-Bonacich centrality. Fainmesser and Galeotti (2016) and Fainmesser and Galeotti (2020) address the same issue in a setting where the network is not perfectly observable, and show that optimal pricing depends on the network configuration as well as firms' knowledge about it. While this literature considers network externalities among consumers, our paper focuses on externalities among products. Another key difference between this literature and our paper is that the former does not consider the case in which firms offer complementary products/inputs.

Third, we contribute to a broader literature, with roots in both management (Adner and Kapoor, 2010; Jacobides et al., 2018) and economics (Rochet and Tirole, 2003; Rysman, 2009), that explores the relationship between platforms and ecosystems. Some authors use the term ecosystem to describe a set of complementary products whose interactions are orchestrated by a single firm, such as Apple, Google, or Amazon (UK Competition and Markets Authority, 2020, p.57). Other authors take a broader industry-level perspective (e.g., Gawer and Cusumano, 2014). Our model highlights a link between the idea of a multi-product ecosystem and the Katz-Bonacich measure of network centrality. And though we do not analyze competition between ecosystems, we offer a tractable framework that represents a first step in that direction, as called for by various competition authorities and commentators (Cremer et al., 2019; Furman et al., 2019; Scott Morton et al., 2019).

Finally, we add to the literature on patent licensing (Katz and Shapiro, 1985; Shapiro, 2001; Lerner and Tirole, 2004; Farrell and Shapiro, 2008) by bringing a novel approach to patent licensing borrowing from the literature on multi-sided platforms. This approach is particularly relevant for a setting with many complementary patent owners and multiple downstream IoT devices all connected by positive demand externalities.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 analyzes the pricing of a monopoly platform. Section 4 analyzes the equilibrium prices when there are several complementary platforms. Section 5 considers partial mergers between overlapping platforms. Section 6 concludes.

2 A Model of Complementary Platforms

This section introduces a model of a multi-product multi-platform ecosystem. There are $n > 1$ devices (indexed by i) and $m \geq 1$ platforms (indexed by k). Each platform supplies an essential input to every device. Each input corresponds to an interface technology that facilitates interaction among end-users. For example, the platforms could represent firms with patents on different parts of the 5G cellular standard, which are licensed to produce IoT devices (phones, watches, cars, appliances, etc.) that are purchased by end-users.

Let p_i^k denote the price charged by platform k to device i. For simplicity, we initially assume all devices are supplied by perfectly competitive downstream markets and normalize marginal costs to zero.[3] As a result, the total price of device i equals the sum of the input prices charged by the m platforms: $p_i = \sum_{k=1}^{m} p_i^k$.

Connectivity among devices creates externalities in demand. Specifically, we assume that demand for device i is given by

$$q_i = \alpha_i - \beta_i p_i + \sum_{j \neq i} \gamma_{ij} q_j. \tag{1}$$

where (α_i, β_i) parameterize the standalone demand for device i, and $\gamma_{ij} \geq 0$ captures the strength of the externality exerted by device j's users on the users of device i. Appendix A provides a micro-foundation for this demand system.

The network externalities, γ_{ij}, may arise from *interactions* among different types of agents, such as buyers and sellers on an exchange, or readers, publishers, and advertisers on a web site. In our 5G licensing example, increased consumption of any one cellular-enabled device in the IoT ecosystem can raise demand for other devices by growing the addressable market of complements (e.g., because my phone can connect with your glasses, or watch, or car). In a multi-product ecosystem, network externalities can also arise because users of application i generate *data* that improves the quality of device j. For example, data from search engines can be used to improve the quality of maps and shopping sites, and vice versa.

[3] In Section 3.5.2, we extend the analysis to the case of downstream market power.

Having introduced the key elements of the model, we can provide a formal definition of ecosystem:

Definition 1 *An ecosystem comprises n devices (or sides) potentially linked through demand externalities, n input markets (one per device), m platforms that provide inputs to the devices, and a group of one or more firms that compete in each device market.*

In principle, each platform might serve its own subset of $n_k \leq n$ devices, and the inputs could be either complements or substitutes. We initially focus on the fully symmetric case where each platform serves every side (i.e., $n_k = n$ for all k), and later consider some simple examples of partial overlap (where $n_k < n$). Throughout, we assume all inputs are complements.

Using matrices, the demand system (1) can be written as

$$\mathbf{q} = \mathbf{a} - \mathbf{B}\mathbf{p} + \mathbf{G}\mathbf{q} \tag{2}$$

where $\mathbf{q}$ is an $n \times 1$ vector of quantities q_i, $\mathbf{p}$ is an $n \times 1$ vector of prices p_i, $\mathbf{a}$ is an $n \times 1$ vector of intercepts α_i, $\mathbf{B}$ is an $n \times n$ matrix of slopes β_i with zero for all off-diagonal elements, and

$$\mathbf{G} = \begin{bmatrix} 0 & \gamma_{12} & \dots & \gamma_{1n} \\ \gamma_{21} & 0 & \dots & \gamma_{2n} \\ \dots & \dots & \dots & \dots \\ \gamma_{n1} & \gamma_{n2} & \dots & 0 \end{bmatrix} \geq 0.$$

Hence, if $\mathbf{I} - \mathbf{G}$ is invertible, the demand system can be written as:

$$\mathbf{q} = (\mathbf{I} - \mathbf{G})^{-1}(\mathbf{a} - \mathbf{B}\mathbf{p})$$

If $\lambda_{\mathbf{G}}$ represents the largest eigenvalue of $\mathbf{G}$, then a sufficient condition for existence and non-negativity of $(\mathbf{I} - \mathbf{G})^{-1}$ is that $\lambda_{\mathbf{G}} < 1$.[4] The eigenvalue $\lambda_{\mathbf{G}}$ reflects the overall strength of network effects in the ecosystem, and if those effects are too large then demand will "explode" given the recursive nature of equation (2).[5] It follows that the sponsors of an ecosystem will generally seek to increase $\lambda_{\mathbf{G}}$, for example by designing interoperability

[4] See Theorem III^* of Debreu and Herstein (1953)

[5] For instance, when $\mathbf{B} = \mathbf{I}$, we can formally write demand as $\mathbf{q} = \mathbf{I}(\mathbf{a} - \mathbf{p}) + \mathbf{G}(\mathbf{a} - \mathbf{p}) + \mathbf{G^2}(\mathbf{a} - \mathbf{p}) + \cdots$, where $\mathbf{G}^L(\mathbf{a} - \mathbf{p})$ is a linear operator on the vector $\mathbf{a} - \mathbf{p}$ that produces a scale transform less than λ^L and a rotation towards the eigenvector associated with $\lambda_{\mathbf{G}}$. Thus, $\mathbf{G}^L(\mathbf{a} - \mathbf{p})$ converges to 0 if and only if $\lambda_{\mathbf{G}} < 1$.

into various devices. Our analysis, however, will take **G** as fixed in order to focus on pricing decisions.

3 A Monopoly Ecosystem

In this section, we analyze monopoly pricing. After introducing key concepts in the familiar setting of a two-sided market, we characterize optimal prices and quantities for a single platform. In our 5G example, the monopoly platform could be a patent pool that licenses essential patents to the producers of n different IoT devices. One could also interpret the platform as a single firm, that controls downstream device prices through both vertical integration (e.g., iPhone and iWatch) and control over essential inputs (e.g., iOS and the AppStore).

3.1 Two devices

To begin simply, suppose there are two devices, and that $\beta_i = 1$ for both of them. In that case, the solution to the demand system specified in (1) is

$$q_i = \frac{\alpha_i - p_i + \gamma_{ij}(\alpha_j - p_j)}{1 - \gamma_{12}\gamma_{21}}.$$

The demand multiplier produced by network effects is $(1 - \gamma_{12}\gamma_{21})^{-1}$, so we require $\gamma_{12}\gamma_{21} < 1$ for stability. A platform monopolist's total profit is $\pi^M = p_1 q_1 + p_2 q_2$, and the first-order condition with respect to p_1 is given by

$$q_1 + p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} = 0$$

or equivalently (after cancelling out the multiplier)

$$\alpha_1 + \gamma_{12}(\alpha_2 - p_2) - \gamma_{21} p_2 = 2p_1. \tag{3}$$

To provide intuition for the monopolist's incentives, we decompose (3) into three parts:

1. Baseline prices: In the absence of demand externalities (i.e., $\gamma_{21} = \gamma_{12} = 0$), the standard monopoly price is given by $p_1 = \alpha_1/2$.

2. Externality internalization (or value creation): The multi-product monopolist internalizes the effect of raising p_1 on demand for device 2. This is captured by the term $\frac{\partial q_2}{\partial p_1} \propto -\gamma_{21} < 0$. Externality internalization leads to lower p_1 through the marginal effect on q_2.

3. Value capture: The positive externality from device 2 to device 1 implies that the value of device 1 is enhanced. Specifically, the constant in the demand for device 1 is boosted by $\gamma_{12}(\alpha_2 - p_2) > 0$. This leads the platform to raise p_1. Value capture occurs not through the marginal effect of changing p_1, but through a level effect (i.e., the level of the constant in the demand).

Throughout the paper, we will use

Definition 2 *Device i is subsidized (respectively, exploited) if p_i is lower (respectively, higher) than its baseline price.*

The two-sided example highlights a tension between *externality internalization* and *value capture*, which creates opposing incentives to reduce or increase the price of each device. We now consider how these forces play out in a more general setting.

3.2 Many devices

Suppose there are n devices and that $\mathbf{B} = \mathbf{I}$. The monopolist maximizes $\Pi^M = \mathbf{p}'\mathbf{q}$, and its system of first-order conditions can be written as

$$(\mathbf{I} - \mathbf{G})^{-1}(\mathbf{a} - \mathbf{p}) - (\mathbf{I} - \mathbf{G}')^{-\mathbf{1}}\mathbf{p} = 0. \tag{4}$$

Appendix B shows that the solution to (4), if one exists, is given by:

$$\mathbf{p}^M = \frac{1}{2}\mathbf{a} + \frac{1}{4}(\mathbf{G} - \mathbf{G}')\left[\mathbf{I} - \left(\frac{\mathbf{G} + \mathbf{G}'}{2}\right)\right]^{-1}\mathbf{a}, \tag{5}$$

where $\mathbf{G}'$ denotes the transpose of $\mathbf{G}$. The first term in (5) is the vector of baseline prices. The second term reflects a tradeoff between value extraction ($\mathbf{G}$), and externality internalization ($\mathbf{G}'$), as in the two-device case. Moreover, the extraction and internalization matrices are both post-multiplied by a set of device-specific weights that is well known in the literature on networks. Specifically, we take from that literature

Definition 3 *The $n \times 1$ vector $\left[\mathbf{I} - \frac{1}{2}\left(\mathbf{G} + \mathbf{G}'\right)\right]^{-1} \mathbf{a} \equiv \mathbf{c}^{KB}$ measures each device's Katz-Bonacich (KB) centrality in the network $\frac{1}{2}\left(\mathbf{G} + \mathbf{G}'\right)$.*

Katz-Bonacich centrality is a commonly used measure of the influence exerted by a particular node in a network.[6] If we define $\overline{\mathbf{G}} = \frac{1}{2}(\mathbf{G} + \mathbf{G}')$, then KB-centrality can be decomposed as $\mathbf{c}^{KB} = \mathbf{a} + \overline{\mathbf{G}}\mathbf{a} + \sum_{t=2}^{\infty} \overline{\mathbf{G}}^{\mathbf{t}}\mathbf{a}$. The term $\overline{\mathbf{G}}\mathbf{a}$ measures direct centrality: the value of all 1-step links to each device, weighted by $\mathbf{a}$. The term $\sum_{t=2}^{\infty} \overline{\mathbf{G}}^{\mathbf{t}}\mathbf{a}$ measures indirect centrality. It is the sum of the value of all t-step links to a device, where $t = 2, 3, 4...$, again weighted by $\mathbf{a}$. Indirect centrality is a geometric sequence that will converge if $\lambda_{\overline{\mathbf{G}}} < 1$. The same condition guarantees that demand is well-behaved.[7] Thus, we have

Theorem 1 *If $\lambda_{\overline{\mathbf{G}}} < 1$, then there exists a unique vector of optimal monopoly prices*

$$\mathbf{p}^{M} = \frac{1}{2}\left[\mathbf{a} + \frac{1}{2}\left(\mathbf{G} - \mathbf{G}'\right)\mathbf{c}^{KB}\right]. \tag{6}$$

Equation (6) shows how demand externalities create a trade-off between value extraction and externality internalization for the monopolist. It can be equivalently written in scalar form as

$$p_i^{M} = \frac{\alpha_i}{2} + \frac{1}{4}\sum_{j \neq i}\left(\gamma_{ij} - \gamma_{ji}\right) c_j^{KB}. \tag{7}$$

This expression reveals that the adjustment to p_i caused by device j is proportional to j's centrality times the *net* externality from j to i (i.e., $\gamma_{ij} - \gamma_{ji}$). Thus, if there are two devices and $\gamma_{12} \neq \gamma_{21}$, then one device will be subsidized and the other exploited. More generally, when $\mathbf{G}$ is symmetric, the value extraction and externality internalization incentives are in perfect balance, leading to

Corollary 1 *For symmetric demand externalities, $\mathbf{G} = \mathbf{G}'$, when $\mathbf{B} = \mathbf{I}$ a monopolist charges the baseline prices $\mathbf{p}^{M} = \frac{1}{2}\mathbf{a}$.*

This corollary is well-known in the context of social networks as well as two-sided platforms (e.g, Candogan et al., 2012; Belleflamme and Peitz, 2018).[8] To solve for demand

[6] Because $\mathbf{a} = \mathbf{1}$ in many applications, some authors refer to $\mathbf{c}^{KB}$ as KB centrality with weight $\mathbf{a}$.

[7] See theorem 10.28 of Zhang (2011).

[8] It is worth noting that asymmetries in $\mathbf{G}$ cannot arise from "ordinary" complementarities rooted in the utility function (Nocke and Schutz, 2017; Amir et al., 2017). Thus, although one might be tempted to interpret $\mathbf{G}$ as a reduced form object that incorporates both complementarities in consumption (e.g. because the devices in an ecosystem work together well) and network externalities, it is only the latter force that gives rise to departures from the baseline prices.

under monopoly pricing, we can substitute the prices from (6) into the demand system (2), which yields

$$(\mathbf{I}-\mathbf{G})\mathbf{q} = \mathbf{a} - \mathbf{p}^M = \frac{1}{2}\mathbf{a} - \frac{1}{4}\left(\mathbf{G}-\mathbf{G}'\right)\mathbf{c}^{KB}.$$

Adding $\frac{1}{2}\mathbf{G}\mathbf{c}^{KB}$ to both sides of the equation and using the definition of $\mathbf{c}^{KB}$, this equality simplifies to

$$(\mathbf{I}-\mathbf{G})\mathbf{q} + \frac{1}{2}\mathbf{G}\mathbf{c}^{KB} = \frac{1}{2}\mathbf{a} + \frac{1}{4}\left(\mathbf{G}+\mathbf{G}'\right)\mathbf{c}^{KB} = \frac{1}{2}\mathbf{c}^{KB}$$
$$\Rightarrow \mathbf{q} = \frac{1}{2}\mathbf{c}^{KB},$$

and we restate this result as

Corollary 2 *For linear demand with monopoly pricing, quantities are proportional to the KB-centrality of each device, with constant of proportionality* $\frac{1}{2}$.

Corollary 2 says that, all else equal, a monopolist sells more of a device when that device is more central in the network defined by $\overline{\mathbf{G}}$. This helps rationalize subsidies for products like search, navigation, and the large platforms' core "Smart Home" devices (i.e., Echo/Alexa, HomePod/Siri, and Nest/Google Assistant). All of these products generate data that can be leveraged across many applications, and interact with many other devices.

3.3 A Link to Armstrong

Armstrong (2006) uses a change of variable to express output in terms of utility for each device

$$u_i = \sum_{j\neq i} \gamma_{ij} q_j - p_i, \tag{8}$$

so quantities are given by $q_i = \alpha_i + u_i$. The platform's profit is $\Pi = \sum p_i q_i$, and its first-order condition with respect to u_i (holding q_j for all $j \neq i$ constant) is therefore

$$\sum_{j\neq i} \gamma_{ij} q_j - u_i - q_i + \sum_{j\neq i} \gamma_{ji} q_j = 0. \tag{9}$$

Rearranging the first-order condition gives the generalized Armstrong pricing rule

$$\frac{p_i + \sum_{j\neq i}\gamma_{ji}q_j}{p_i} = \frac{1}{\varepsilon_i}$$

where $\varepsilon_i = -\frac{\partial q_i}{\partial p_i} / \frac{q_i}{p_i} = p_i / q_i$. The appearance of demand externalities where we would normally observe marginal costs in the Lerner markup rule highlights the marginal effect of reducing p_i on sales of other devices.

Substituting (8) into (9) yields a modified Armstrong pricing formula

$$p_i = \frac{\alpha_i}{2} + \frac{1}{2} \sum_{j \neq i} \left(\gamma_{ij} - \gamma_{ji} \right) q_j \tag{10}$$

that expresses p_i as a function of q_j, an endogenous variable. Our own characterization of the monopoly pricing in (7) takes the same shape, but expresses q_j in terms of the fundamentals. Setting equations (7) and (10) equal to one another reveals, again, that $q_i = c_i^{KB}/2$.

3.4 Examples

To illustrate how a monopolist would price different device ecosystems we consider three examples. For each example, the externality between any pair of devices takes one of three values, $\gamma_{ij} \in \{\mu, \eta, 0\}$. We set all of the demand intercepts $\alpha_i = 1$, and define two parameters $c \equiv \mu + \eta$ and $d \equiv \mu - \eta$.

3.4.1 Star

A star network is defined by $\gamma_{1j} = \eta$ for all $j > 1$; $\gamma_{j1} = \mu$ for all $j > 1$; and $\gamma_{ij} = 0$ for all $i, j > 1$. For this demand system, all of the externalities either originate from or terminate at the "star" device ($i = 1$). In terms of our licensing example, one might think of the star as a smartphone that exhibits bilateral demand externalities with a series of other devices, such as watches, cars, thermostats, eyeglasses, etc. that do not interact with one another. Note that the two-device case is a special case of the star network.

Using (6) and the fact that all peripheral devices ($j > 1$) are symmetric, we can write the monopoly prices as

$$\begin{aligned} p_1^M &= \frac{1}{2} - \frac{1}{4} d(n-1) c_j^{KB} \\ p_j^M &= \frac{1}{2} + \frac{1}{4} d c_1^{KB} \end{aligned}$$

Because the KB-centrality of each device, c^{KB}, is strictly positive, we see that the star device will be subsidized if and only if $d > 0$ (i.e., when its *net* externalities to each peripheral are positive). These price formula also reveal that when the star device is subsidized, the peripherals are exploited, and vice versa. When $d > 0$, the amount of subsidy to the star device is proportionate to $(n-1)$ times the centrality of a peripheral device whereas the amount of exploitation of a peripheral device is proportionate to the centrality of the star.

Appendix C shows that for a more general star network, where demand externalities vary across peripherals, we can derive a similar result: the star device is subsidized if and only if the aggregate externalities that it creates for peripherals exceed the aggregate externalities generated by all peripheral devices to the star.

3.4.2 Hierarchy

Next, consider a "hierarchical" ecosystem of devices, where $\alpha_{ij} = \eta$ for all $i < j$, and $\alpha_{ij} = \mu$ for all $i > j$. When $\mu > \eta$, device 1 generates the most and receives the fewest externalities, device 2 generates the second-most and receives the second-least amount of externalities, and so on. In economic terms, this example corresponds to a setting where some devices clearly produce more externalities than others, but there is no single dominant device or side to the platform.

For this demand system, every non-diagonal element in the matrix $[\mathbf{I} - \overline{\mathbf{G}}]$ equals $-\frac{c}{2}$, and because its inverse exhibits the same symmetry, all devices have the same KB-centrality. Together with (6), this implies that monopoly prices for each device are

$$p_i^M = \frac{1}{2} - \frac{d}{4}\,(n+1-2i)\,c^{KB}.$$

When $d > 0$, a monopolist will subsidize devices that are "higher" in the hierarchy $(i < \frac{n+1}{2})$ and exploit the devices that are "lower" in the hierarchy. For devices near the middle of the hierarchy, which generate and receive similar amounts of externalities, prices will be close to the monopoly baseline. It is also worth emphasizing that in this example, all of the price distortions reflect the trade-off between externality internalization and value extraction, as captured by $[\mathbf{G} - \mathbf{G}']$, given that every device has the same KB-centrality.

3.4.3 Ring

As a final example, we consider a demand system with "circular" externalities represented by $\alpha_{ij} = \mu$ if $i = j - 1$ (or $i = n$ and $j = 1$); $\alpha_{ij} = \eta$ if $i = j + 1$ (or $i = 1$ and $j = n$); and otherwise $\alpha_{ij} = 0$. In this example, each device has two neighbors, one of which receives μ and creates η, while the other receives η and creates μ for the focal device. Although we are not aware of any actual ecosystems that exhibit this type of circularity, the example remains useful for developing intuition.

As in the previous example of a hierarchical demand system, all devices in the ring have the same KB-Centrality. Moreover, each row in $[\mathbf{G} - \mathbf{G}']$ has exactly one entry equal to d, one equal to $-d$, and the rest equal to zero. Therefore, applying (6) reveals that

$$p_i^M = \frac{1}{2} + \frac{1}{4}(dc^{KB} - dc^{KB}) = \frac{1}{2}.$$

The monopoly platform sponsor selects baseline prices in this example because, although $\mathbf{G}$ is not symmetric, the ring structure implies that the *net* externalities produced by each device are zero.

3.5 Generalizations

This sub-section generalizes the analysis in two dimensions: introducing heterogeneity in β_i, and allowing for imperfect competition among downstream device producers.

3.5.1 Device-specific Demand

Thus far, we have assumed linear demand and equal slopes ($\mathbf{B} = \mathbf{I}$). Both assumptions can be relaxed as long as we retain the linear structure of the network externalities. In particular, suppose demand for each device is given by the function $q_i(p_i)$ and that $\frac{\partial q_i}{\partial p_j} = 0$ for all $i \neq j$. This implies that demand for device i can be approximated using the first-order terms of a Taylor expansion: $\beta_i = q_i'(p_i)$ and $\alpha_i = q_i(p_i) - q_i'(p_i)p_i$. The monopolist's system of first-order conditions in a neighborhood of any profit maximizing price vector can therefore be written as

$$(\mathbf{I} - \mathbf{G})^{-1}[\mathbf{a} - \mathbf{B}\mathbf{p}] - \mathbf{B}'(\mathbf{I} - \mathbf{G}')^{-1}\mathbf{p} = 0$$

and Appendix B shows that the solution to this system is

$$\mathbf{p}^M = \frac{1}{2}\mathbf{B^{-1}a} + \frac{1}{4}\left(\mathbf{B^{-1}GB} - \mathbf{G}'\right)\mathbf{c}^{KB(\mathbf{B})} \tag{11}$$
$$\text{for } \mathbf{c}^{KB(\mathbf{B})} \equiv \left[\mathbf{I} - \left(\frac{\mathbf{B^{-1}GB} + \mathbf{G}'}{2}\right)\right]^{-1}\mathbf{B^{-1}a}.$$

Equation (11) resembles (6), but with two changes. First, the intercepts $\mathbf{a}$ (and hence, the baseline prices) are scaled by $\mathbf{B}^{-1}$, so baseline prices are lower for devices with larger β_i. Second, the value extraction matrix $\mathbf{G}$ is replaced by $\mathbf{B^{-1}GB}$. Thus, when $\mathbf{B} \neq \mathbf{I}$, a symmetric $\mathbf{G}$ no longer implies that the monopolist will charge the baseline price for each device. To understand the latter change, note that the externality internalization matrix $\mathbf{G}'$ does not change with $\mathbf{B}$. The value extraction matrix, on the other hand, reflects the incentive to raise prices when demand grows larger. This incentive to extract more rent depends upon *both* price elasticities and network effects.

Using Armstrong's approach, as described above, yields an element-wise version of equation (11)

$$p_i = \frac{\alpha_i}{2\beta_i} + \frac{\sum_{j\neq i}\left(\frac{\gamma_{ij}}{\beta_i}\beta_j - \gamma_{ji}\right)\frac{q_j}{\beta_j}}{2} = \frac{\alpha_i}{2\beta_i} + \frac{\sum_{j\neq i}\left(\frac{\gamma_{ij}}{\beta_i} - \frac{\gamma_{ji}}{\beta_j}\right)q_j}{2}.$$

The second part of this equality shows that the inbound externality from device j to device i is discounted by β_i whereas the outbound externality from device i to device j is discounted by β_j. So, even if all of the γ_{ij} are identical, devices with relatively large (small) β_i will be subsidized (exploited). Because the elasticity of demand of device i increases with β_i, devices with relatively large (small) elasticities will be subsidized (exploited). Moreover, comparing the first part of the equality with (11) reveals that

$$\frac{c_j^{KB(\mathbf{B})}}{2} = \frac{q_j}{\beta_j}.$$

3.5.2 Downstream Market Power

Now suppose that for each device, there are $l_i \geq 1$ symmetric downstream producers that compete à la Cournot. Each downstream firm sells a single device.[9] To distinguish the

[9] If downstream firms produce multiple devices, they can also engage in platform pricing to internalize externalities among devices. This is an interesting topic for future research.

upstream input prices from the downstream device prices, let r_i be the price (royalty) charged by the platform to device i. We continue to use p_i for the downstream price of device i.

Given r_i and the output of all other devices, $\mathbf{q}_{-i}$, each producer of device i selects its output. For instance, firm $i1$ chooses a quantity q_{i1} to maximize $(p_i - r_i)q_{i1}$, where

$$p_i = \frac{\alpha_i + \sum_{j \neq i} \gamma_{ij} q_j - (q_{i1} + \sum_{k \neq 1} q_{k1})}{\beta_i}.$$

From the first-order condition, and using symmetry, we find that each firm's equilibrium output $q_{i1} = ... = q_{il_i} = \tilde{q}_i$ is given by

$$\alpha_i + \sum_{j \neq i} \gamma_{ij} q_j - l_i \tilde{q}_i - \beta_i r_i - \tilde{q}_i = 0.$$

This implies that

$$q_i = l_i \tilde{q}_i = L_i \left[\alpha_i - \beta_i r_i + \sum_{j \neq i} \gamma_{ij} q_j \right] \tag{12}$$

where $L_i \equiv \frac{l_i}{l_i+1}$. Note that as l_i goes to infinity for all i, the demand system (12) converges to (1), the input demand under perfect downstream competition. We can therefore state

Theorem 2 *If each device i is produced by $l_i \geq 1$ symmetric downstream firms that compete à la Cournot, then the unique vector of optimal prices for an ecosystem monopolist are given by (11) after replacing $(\alpha_i, \beta_i, \gamma_{ij})$ with $(L_i\alpha_i, L_i\beta_i, L_i\gamma_{ij})$.*

This result can be extended to the case of m symmetric platforms that we analyze in Section 4. Henceforth, unless otherwise noted, we assume perfect downstream competition and use p_i to denote both the input and the device price.[10] Also, for ease of exposition, we now return to assuming that $\mathbf{B} = \mathbf{I}$ unless otherwise noted.

3.6 Welfare Comparisons

We conclude our analysis of an ecosystem monopolist by comparing its welfare properties to two alternatives: first-best pricing as implemented by a social planner, and pricing each device at marginal cost.

[10] In Appendix H we provide additional insights into the effects of downstream competition by focusing on the special case of a star network.

3.6.1 First-Best

For the utility functions in Appendix A that rationalize our system of linear demand functions, we show in Appendix D that social welfare is equal to

$$W = \sum_i (\alpha_i q_i - \frac{q_i^2}{2}) + \sum_i \sum_{j \neq i} \gamma_{ij} q_i q_j. \tag{13}$$

Differentiating with respect to q_i implies that at the social optimum, it must hold that

$$\alpha_i - q_i + \sum_{j \neq i} (\gamma_{ij} + \gamma_{ji}) q_j = 0 \tag{14}$$

Substituting (1) for q_i in this expression, and putting the result in matrix form, we have the following relationship between welfare-maximizing prices and quantities

$$\mathbf{p^W} = -\,\mathbf{G'q^W} \tag{15}$$

Finally, substituting equilibrium demand from (2) into this expression shows that

Theorem 3 *If $\lambda_{2\overline{G}} < 1$, welfare-maximizing prices are given by*

$$\mathbf{p^W} = -\,\mathbf{G'}[\mathbf{I} -\,(\mathbf{G} + \mathbf{G'})]^{-1}\mathbf{a} \tag{16}$$

At first-best prices, the output of each device is $\mathbf{q^W} = [\mathbf{I} -\,(\mathbf{G} + \mathbf{G'})]^{-1}\mathbf{a}$, which is the KB centrality vector in the network $\mathbf{G} + \mathbf{G'}$.

For intuition, it is helpful to compare the welfare-maximizing prices in (16) to the monopoly prices in (6). Absent externalities, the "baseline prices" of $\mathbf{p}^W$ are equal to the marginal costs, which we normalized to zero. When externalities are present, the monopolist faces a tradeoff between surplus extraction and internalizing externalities, as captured by the term $(\mathbf{G} - \mathbf{G'})\,\mathbf{c}^{KB}$. The social planner, on the other hand, cares only about internalizing externalities; extracting surplus is a pure transfer. Consequently, only the externality internalization matrix $-\mathbf{G'}$ is multiplied by a centrality vector, which implies that a social planner subsidizes all devices.

Finally, the matrix that a social planner uses to compute centrality is different from the one used by a monopoly platform. In Appendix D we show that the different centrality measures reflect the fact that a social planner cares about the social marginal surplus from expanding output, whereas a monopoly platform cares about its marginal profit. Note that despite having these different objectives, the monopolist and social planner give the same relative weight to each of the matrices $\mathbf{G}$ and $\mathbf{G}'$ when computing the centrality measures.

3.6.2 Pricing at Marginal Cost

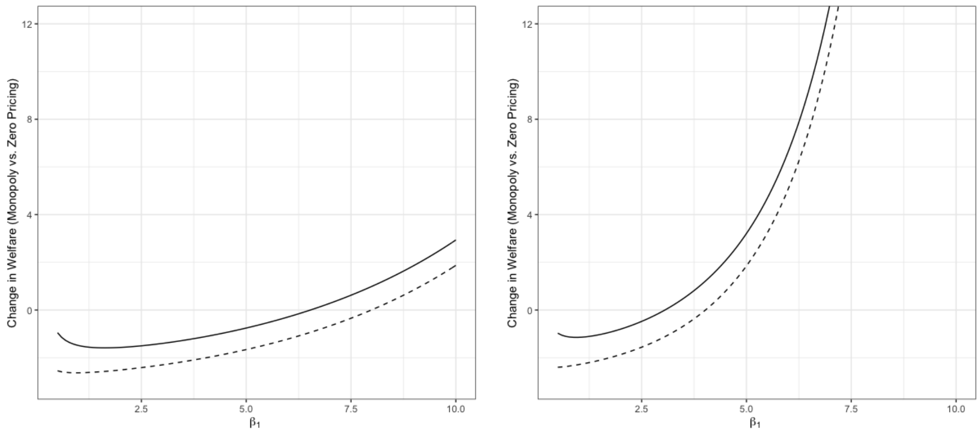
We have seen that a social planner sets all prices below marginal cost, whereas a monopolist may set some prices below marginal cost in order to capture value from others. This raises the question of whether an ecosystem monopolist may be preferable to a decentralized ecosystem where all devices are priced at marginal cost. For instance, in our 5G IoT licensing example, this is equivalent to asking whether a monopolistic patent licensing platform can produce more *static* welfare (i.e., ignoring innovation incentives) than a setting where no party holds IP rights. In this section, we use an example based on the star network to show that this is possible: welfare under monopoly can exceed welfare under marginal cost pricing because the latter fails to internalize network externalities.

For the star example, recall that η (μ) represents the inbound (outbound) externality from the star device from (to) a peripheral and that $d = \mu - \eta$. As a first step, we can show that[11]

Lemma 1 *If $\alpha_i = 1$ for all i, and $\beta_j = 1$ for all peripherals (i.e., $j > 1$), then a star device (peripheral device) is subsidized (exploited) if and only if $\mu > \frac{\eta}{\beta_1}$.*

When $d = 0$ and $\beta_1 = 1$, the ecosystem monopolist will choose the same positive baseline price for every device, so welfare under monopoly must be lower than under zero pricing. If we increase β_1, however, Lemma 1 says that a monopolist will subsidize the star device. (A larger β_1 leads to a lower price on the star, and that in turn reduces the marginal benefit of inbound relative to outbound externalities.) To see whether there is a threshold level of β_1, beyond which an ecosystem monopolist generates more welfare than zero pricing, we fix d and use equation (13) to compute welfare at different values of β_1. These calculations are summarized in Figure 1.

[11] The proof of this result immediately follows from (H.1) with $L = 1$ in Appendix H.



(a) d=0 (b) d=0.3

Figure 1: Monopoly vs. Zero pricing

Welfare = Solid line, Consumer Surplus = Dashed Line

The figures show that even when $d = 0$, so externalities are symmetric, monopoly pricing can dominate zero-pricing if β_1 is sufficiently large. As d increases from zero to 0.3, outbound externalities become relatively larger and the threshold value of β_1 declines. We summarize these findings in

Theorem 4 *An ecosystem monopolist that internalizes downstream externalities may produce more welfare and consumer surplus than marginal cost (zero) pricing of each device.*

Thoerem 4 is particularly interesting when the ecosystem monopolist is a licensing platform, as in our 5G IoT example. The standard argument for granting temporary monopoly power to a patent holder is based on the trade-off between ex ante innovation incentives and ex post market power. An implicit assumption behind this argument is that each patent is associated with a single product. The theorem suggests that things could change dramatically when a patent (or a bundle of complementary patents) is associated with a multi-product ecosystem. In that case, it is possible that the usual dynamic trade-off no longer exists, because the ecosystem monopolist generates higher welfare (and consumer surplus) than the zero-price equilibrium that occurs without any intellectual property.

4 Complementary Platforms

We now consider a model with m platforms that supply perfectly complementary inputs to each of the n devices. In a licensing context, these inputs could represent a portfolio of IP rights held by m distinct licensors that are essential for the production of all n devices. This is roughly the situation faced by participants in the licensing market for 5G Standard Essential Patents (SEPs), where the m platforms correspond to patent owners such as Ericsson, Nokia, Qualcomm, Samsung or Huawei, and the n devices correspond to various "Internet of Things" devices.[12] Prior literature has analyzed the complementary monopolies problem in SEP licensing (e.g., Shapiro, 2001; Geradin et al., 2008), but not in a setting with downstream externalities among licensed products.

4.1 Many devices and many platforms

It is useful to define the parameter $\sigma = \frac{1}{m+1}$. Recall that p_i^k is the price charged by platform k to device i, and $p_i = \sum_{k=1}^{m} p_i^k$. Let $\mathbf{p}^k = \left(p_1^k, p_2^k, ..., p_n^k\right)'$ represent the vector of prices charged by platform k. Maintaining the assumption that the downstream market is perfectly competitive, platform k's profit is given by

$$\mathbf{\Pi}^k = \mathbf{p}^{k\prime}(\mathbf{I}-\mathbf{G})^{-1}(\mathbf{a}-\mathbf{p}).$$

To solve for the symmetric equilibrium prices charged by all platforms to each device, we differentiate this expression with respect to $\mathbf{p}^k$ and aggregate the system of first-order conditions. These computations, found in Appendix B, show that the vector of prices charged by each of the m platforms is

$$\mathbf{p}^* = \sigma\mathbf{a} + \sigma^2(\mathbf{G}-\mathbf{G}')\left[\mathbf{I} - \frac{1}{m+1}\mathbf{G} - \frac{m}{m+1}\mathbf{G}'\right]^{-1}\mathbf{a} \tag{17}$$

The first term in (17) equals $\mathbf{a}/(m+1)$. This is the price charged by each one of m independent monopolists in Cournot's famous complementary monopolies problem. Henceforth, we refer to these as the Cournot baseline.

The second term in (17) contains the matrix $(\mathbf{G}-\mathbf{G}')$. As in the monopoly case, this matrix reflects a tradeoff between value capture (through $\mathbf{G}$) and externality inter-

[12] In actual 5G licensing, most of the licensing "platforms" have made commitments to license their patents on Fair Reasonable and Non-Discriminatory (FRAND) terms. In our analysis, we simply treat each platform as a monopoly input supplier, thereby ignoring any FRAND pricing constraints.

nalization (through $\mathbf{G}'$). The second term differs from the monopoly formulas for subsidy/exploitation, however, in the device-specific weights that post-multiply $(\mathbf{G} - \mathbf{G}')$. We therefore introduce

Definition 4 *The $n \times 1$ vector $[\mathbf{I} - \mathbf{G}^m]^{-1} \mathbf{a} \equiv \mathbf{c}^{KB,m}$ measures each device's Katz-Bonacich centrality in the network $\mathbf{G}^m \equiv \sigma \mathbf{G} + (1 - \sigma)\mathbf{G}'$.*

We refer to the i^{th} component of $c^{KB,m}$ as device i's KB-m centrality. Compared to the monopoly case, the network used to calculate KB-m centrality places more weight on externality internalization. Intuitively, as we add more monopoly input suppliers, the value-capture incentive declines because each firm's residual demand curve shifts inward (i.e. the demand intercept for any single firm shifts from α_i to $\alpha_i - \sum_{j \neq k} p_i^k$). The internalization incentive, however, remains unchanged because it reflects a marginal effect and not the level of demand. Thus, as m increases, the network used to compute KB-m centrality places increased weight on internalization. We summarize the general expression for symmetric equilibirum pricing in

Theorem 5 *If $\lambda_{\mathbf{G}^m} < 1$, then the unique vector of symmetric equilibrium prices charged by each of m complementary platforms is given by*

$$\mathbf{p}^* = \sigma \left[\mathbf{a} + \sigma \left(\mathbf{G} - \mathbf{G}' \right) \mathbf{c}^{KB,m} \right]. \tag{18}$$

For a symmetric demand system, the second term in (18) disappears, so we have

Corollary 3 *If the network externalities are symmetric (i.e., $\mathbf{G} = \mathbf{G}'$), then equilibrium prices are equal to the Cournot baseline $\mathbf{p}^* = \sigma \mathbf{a}$.*

We can also solve for the equilibrium quantity vector, using the approach described above for the monopoly case. This reveals that $\mathbf{q} = \sigma \mathbf{c}^{KB,m}$, which we restate as

Corollary 4 *For linear demand with m complementary platforms, the equilibrium quantities are proportional to the KB-centrality of each device $\mathbf{c}^{KB,m}$, with constant of proportionality $\sigma = \frac{1}{m+1}$.*

To illustrate the distortion in KB-m centrality caused by the Cournot complements effect, we can revisit the example of a hierarchical network introduced in Section 3.4.2. For a monopoly platform, every non-diagonal element in the matrix $[\mathbf{I} - \overline{\mathbf{G}}]$ equals $-\frac{c}{2}$, so all devices have the same KB-centrality. The same logic holds for a social planner. With

$m > 1$ complementary platforms, however, the KB-m centrality measures associated with $\mathbf{G}^m \equiv \sigma\mathbf{G} + (1-\sigma)\mathbf{G}'$ have a strict ranking. In particular, when $\mu > \eta$, centrality is strictly decreasing with i, as each platform puts more weight on the outbound externalities produced by each device.[13]

4.2 Double Marginalization

With multiple platform sponsors, the baseline prices suffer from double marginalization. That is, aggregate input prices exceed the monopoly benchmark ($m\sigma\mathbf{a} = \frac{m\mathbf{a}}{m+1} > \frac{\mathbf{a}}{2}$), so the m platforms would profit from a coordinated price reduction. In the context of patent licensing, double marginalization is often called *royalty stacking*, and it is frequently offered as a justification for joint licensing programs (e.g. through patent pools or "licensing platforms" such as Avanci).[14]

As we have just seen, however, the prices charged by complementary platforms will also reflect incentives to internalize demand externalities. To illustrate how this may alter standard Cournot results we analyze a "super-star" example, where the star device ($j = 1$) has greater demand *and* produces larger externalities than the peripherals. In particular, suppose that device 1 (the star) generates an externality μ to each peripheral, each peripheral generates η to the star, and the demand intercepts are $\alpha_1 > \alpha_{i>1} = 1$. Recall that $c = \mu + \eta$ and $d = \mu - \eta$.

In appendix F, we use equation (18) to compute the equilibrium prices for m symmetric platforms, which are

$$p_1^* = \alpha_1\sigma - \sigma^2\Delta d(n-1)\left[(1+\alpha_1(\eta+\sigma d)\right]$$
$$p_k^* = \sigma + \sigma^2\Delta d\left[\alpha_1 + (n-1)(\mu - \sigma d)\right]$$

where $\Delta^{-1} = 1 - \frac{(n-1)}{4}(c^2 - (1-2\sigma)^2 d^2) > 0$. These prices imply that the star device is subsidized and the peripherals exploited if and only if $d > 0$.

The fundamental Cournot result is that increasing m leads each supplier to charge a lower price, $\sigma = \frac{1}{m+1}$, but still generates a higher total downstream cost $\frac{m}{m+1} = 1 - \sigma$. We would like to know whether this intuition remains true for every device in the complementary platforms setting. It turns out the answer is no. For the super-star example, we can provide sufficient conditions for the price of a peripheral to fall when

[13] See Appendix E for a formal proof of this claim.
[14] See https://www.avanci.com for more details.

moving from one to two complementary platforms. In particular, we show that

Theorem 6 *For a symmetric star network with $d > 0$, baseline demand $\alpha_1 > \alpha_k = 1$ for $k = 2 \ldots n$, and $m \geq 1$ strictly complementary platforms supplying each device*

- *The total price of the star device mp_1^* increases with m*
- *If $\alpha_1 > \frac{6}{d} + \frac{4}{3}(n-1)d$, then the total price of a peripheral device, mp_k^*, is smaller when $m = 2$ than when $m = 1$.*

Proof. See appendix F. ■

The intuition for the first part of this result is that double marginalization raises the baseline price of the star device, and reduces at the same time the incentive for any single platform to subsidize that device to internalize externality. These two effects work together, so the total price of the star device increases with m by more than in the simple Cournot model without platform externalities.

For the peripheral devices, increasing the number of platforms increases the baseline price but reduces the amount of exploitation. More precisely, the double marginalization that increases with m reduces the subsidy to the star device, and thereby the positive externality from the star device to the peripheral devices, which in turn puts downward pressure on the peripheral prices as m increases. This is opposite to the upward pressure from the increase in the baseline prices. In general, we might expect the change in baseline prices to dominate the downward pressure, because that factor has a first-order impact on p_k^*, whereas the change in subsidy/exploitation has only a second-order effect (i.e., the former is proportional to a change in σ and the latter a change in σ^2). As the demand intercept of the star device increases, however, the externality from the star device to the peripherals becomes more important. The above result shows that when α_1 is large enough, the downward pressure dominates the increase in the baseline price so that overall peripheral device prices are lower when $m = 2$ than when $m = 1$.[15]

Theorem 6 shows that the basic pricing externality analyzed by Cournot over 100 years ago remains present in a platform setting. At the same time, it is possible that at least *some* prices fall when there are more complementary platforms, contradicting the comparative static results that Cournot derived for a single downstream device.

[15] We note that the sufficient condition provided in Theorem 6 is not a tight bound. For instance, if we set $n = 3$, $\mu = \frac{1}{2}$ and $\eta = \frac{1}{4}$, the theorem says that when $\alpha_1 > 18$ then the total price of the peripherals will decline, but numerical calculations show that $\alpha_1 > 5$ will suffice. Our point is simply that when demand for the star device is large enough, it is possible to reverse the standard Cournot pricing result for the peripheral device.

5 Partial Merger

Thus far, we have assumed that each of the m platforms provides a necessary input to all of the n devices. One can imagine, however, that platforms trade assets such that different platforms are only partially overlapping (i.e., not all devices require an input from every platform). For example, in licensing, one two-sided platform might sell all of its cellular infrastructure patents to a second platform in order to focus on handset producers.

As a final step in our analysis, we study a partial merger in a setting with two devices and two platforms, assuming perfect downstream competition and $\beta_1 = \beta_2 = 1$. Initially, there are two fully-overlapping platforms. We study how prices and outputs change when platform S (seller) transfers all of its "device 2 assets" to platform B (buyer). This scenario is illustrated in Figure 2. Before the partial merger, both devices have a double-marginalization problem and both platforms can internalize demand externalities. After the partial merger, the double-marginalization problem for device 2 is eliminated, but S supplies only device 1 and in that sense is no longer a platform.

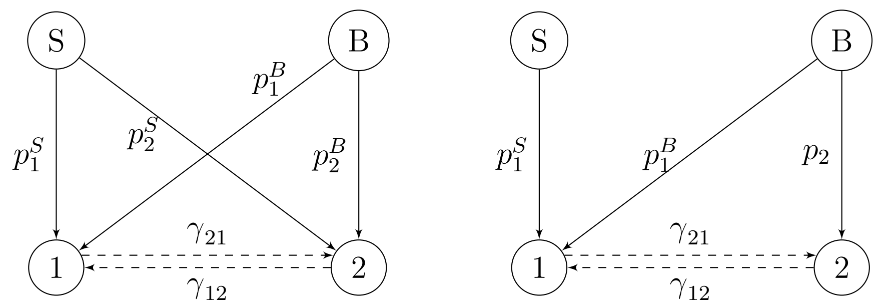


Figure 2: Pre-Merger (Left) and Post-Merger (Right) Pricing

The prices charged by each platform before the merger are characterized in equation (18), and reduce to the following: for $k \in \{S, B\}$

$$
\begin{aligned}
p_1^{k*} &= \frac{1}{3}\alpha_1 - \frac{2(3c\alpha_1 - d\alpha_2 + 6)}{3(36 - 9c^2 + d^2)}d, \\
p_2^{k*} &= \frac{1}{3}\alpha_2 + \frac{2(6\alpha_1 + 3c + d)}{3(36 - 9c^2 + d^2)}d.
\end{aligned}
\tag{19}
$$

where $c \equiv \gamma_{12} + \gamma_{21}, d \equiv \gamma_{21} - \gamma_{12}$. As noted above, device 2 is subsidized if and only if $d < 0$. Thus, we can define two types of partial merger:

Definition 5 *A partial merger between two platforms takes place on the value capture side if $\gamma_{21} > \gamma_{12}$; the partial merger takes place on the value creation side if $\gamma_{21} < \gamma_{12}$.*

When $\gamma_{21} > \gamma_{12}$, both platforms subsidize device 1 and exploit device 2 ex ante, so the merger takes place on the value capture side. Conversely, when $\gamma_{21} < \gamma_{12}$, the merger takes place on the value creation side. In both cases, double marginalization in p_2 is eliminated (which might be particularly helpful if device 2 is subsidized), at the cost of removing any ex ante subsidies from platform S.

In Appendix G, we solve for the post-merger prices of each device. They are:

$$\begin{aligned} p_1^{**} &= \frac{2}{3}\alpha_1 + \frac{c-3d}{12}\alpha_2 - \frac{(3c-d)(2\alpha_1 + c\alpha_2)}{6(12-3c^2+d^2)}d \\ p_2^{**} &= \frac{1}{2}\alpha_2 + \frac{2\alpha_1 + c\alpha_2}{12-3c^2+d^2}d \end{aligned} \tag{20}$$

The merger eliminates the double marginalization for device 2, but also leaves S without any *skin in the game* and induces it to exploit device 1 no matter the externality structure.

Figure 3 shows how the partial merger impacts prices, output and welfare for different levels of externality $(\gamma_{12}, \gamma_{21})$.[16] When $\alpha_1 = \alpha_2 = 1$, it is always the case that p_1 increases, p_2 declines, and the merger is good for social welfare. The left column in Figure 3 shows results when $\alpha_1 = \alpha_2 = 0.3$. For a merger on the value capture side, p_1 increases and q_1 declines, even though it is socially efficient to subsidize device 1. This mis-alignment can lead to welfare-destroying mergers when d is sufficiently large (though such mergers are not generally profitable). By contrast, for this demand system, partial mergers on the value creation side are always good for welfare. We do see, however, that when $d \approx -1$ the partial merger can lead to increased prices and reduced output of device 2. This happens for the same reason we explored in Theorem 6: the reduction in device 2 subsidies from S, who no longer has skin in the game, are larger than the reduction in baseline prices from eliminating double marginalization.

[16]Code for producing the underlying simulation results is available upon request.

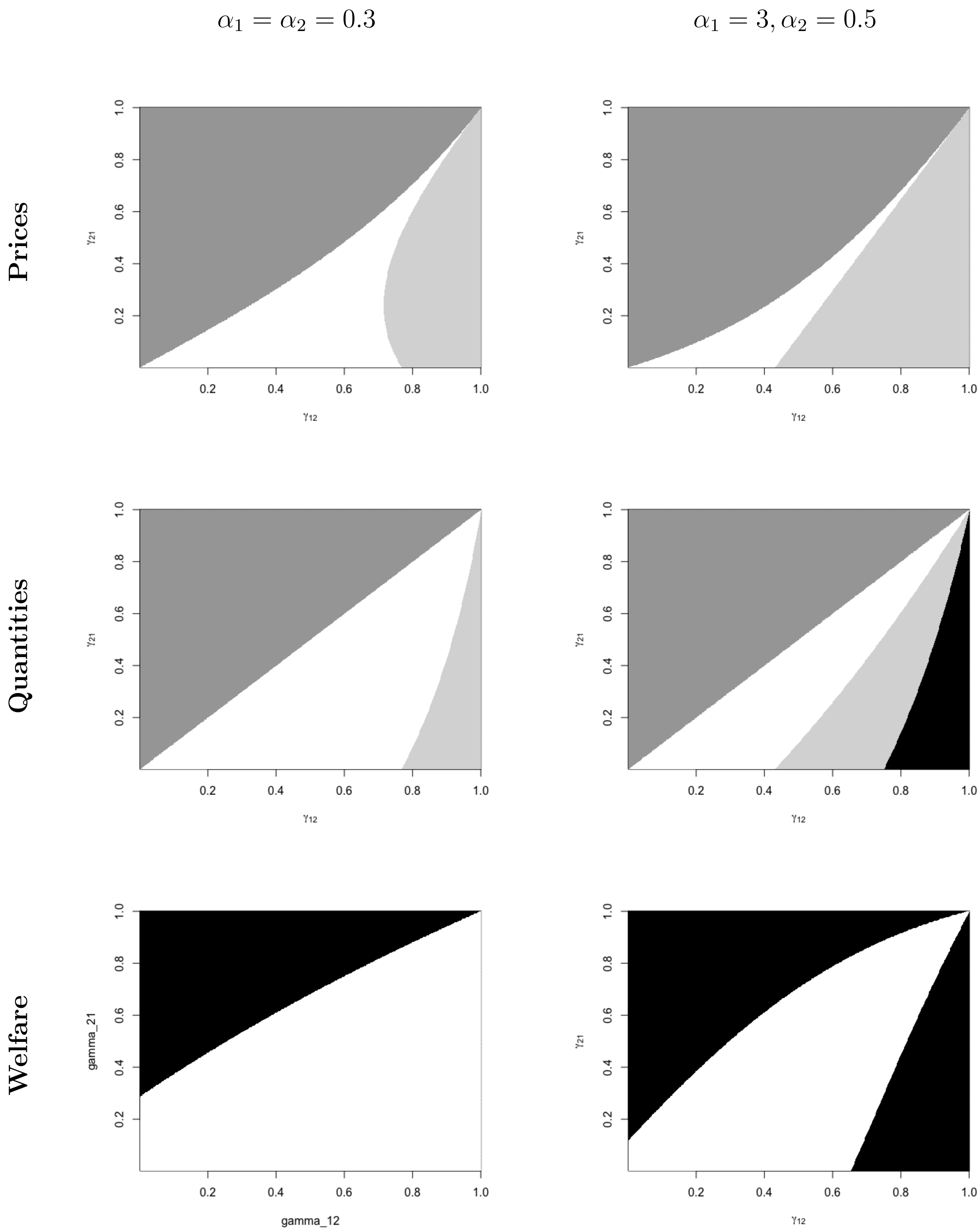


Figure 3: Effects of Partial Merger

White= P↓, Q↑, Welfare↑; Black=P↑, Q↓, Welfare↓
Light Gray=$p_1 \downarrow, p_2 \uparrow$; Dark Gray=$p_1 \uparrow, p_2 \downarrow$

The right column in Figure 3 simulates a series of equilibria for the asymmetric demand system where $\alpha_1 = 3$ and $\alpha_2 = 0.5$. Partial mergers on the value capture side are qualitatively similar to the previous case, except that they reduce social welfare for a larger range of parameter values. For mergers on the value creation side, however, we observe some cases where the loss of S's subsidies for the value creating device is so great that output falls on both sides, and social welfare declines. The merger is never profitable when it reduces output of both devices. However, for this demand system there are some partial mergers on the value creation side that lead to an increase in joint profits and a reduction in consumer surplus.

We summarize the results of this exercise as

Theorem 7 *Partial mergers create a trade-off between solving double-marginalization on one side of a platform, and increasing incentives for value extraction on the other side. They can be good or bad for social welfare. Partial mergers are more likely to harm welfare when demand externalities are very asymmetric, so that* $|d| = |\gamma_{21} - \gamma_{12}|$ *is large.*

6 Conclusions

We develop a tractable model of complementary multi-sided platforms, and use it to study a number of questions. Our first set of results show how a monopolist prices its "ecosystem" of inter-related products. We illustrate the link between Katz-Bonacich centrality and monopoly pricing, and use several examples to show how ecosystem pricing responds to the structure of demand externalities. We also compare welfare under a monopoly platform and marginal cost (zero) pricing. When network externalities are present, a monopolist that internalizes network effects may outperform zero prices: a result that has interesting implications for patent licensing of platform technologies.

We then use our model to study how pricing changes when complementary platforms serve overlapping user groups. The key insight emerging from this analysis is that adding complementers leads any single platform to place increasing weight on externality internalization (relative to value extraction) in its pricing decisions. We find that this expands the range of outcomes for the total price charged to any single side/device, such that it is possible to overturn the Cournot intuition that complementary monopolists charge a higher combined price than an integrated monopoly.

Finally, we use our model to study a partial merger that leaves complementary monopolies on just one side of a two-sided platform. This type of transaction produces a novel tradeoff between eliminating a double marginalization problem on the merging side of the platform, but leaving one platform with no ability to internalize downstream externalities. We find that this type of partial (vertical) merger can yield higher prices and lower output on all sides of the platform.

Our theoretical framework might be extended in several directions. A key simplifying assumption throughout the analysis is linearity of both demand and the downstream network externalities. We show how the former assumption can be relaxed, but have not considered a more general (nonlinear) specification of the network effects. The other obvious extension is to analyze platform competition. In particular, future research might characterize the link between pricing and network centrality measures when two or more platforms compete on one or more sides of a many-sided ecosystem.

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Appendices

A Micro-foundations for demand

Consider a unit-mass of heterogeneous consumers indexed by $\theta \in [0,1]$. Denote p_i the price of device i and N_i the mass of consumers buying device i. We assume that the utility of consumer $\theta \in [0,1]$ is given by

$$u^\theta = \sum_i u_i^\theta$$

where

$$u_i^\theta = a_i^\theta - p_i + \sum_{j \neq i} \gamma_{ij} N_j$$

is the utility obtained by the consumer from using device i. The parameter $\gamma_{ij} \geq 0$ captures the network externality exerted by the users of device j on the users of device i.

For the sake of simplicity, we assume that $a_1^\theta, a_2^\theta, ..., a_n^\theta$ are not correlated for any $\theta \in [0,1]$ and that a_i^θ is uniformly distributed over an interval $[\underline{a}_i, \overline{a}_i]$ where $\underline{a}_i < \overline{a}_i$. The assumption that $a_1^\theta, a_2^\theta, ..., a_n^\theta$ are not correlated for any $\theta \in [0,1]$ implies that there are no complementarities between the devices at the individual level. In other words, network externalities are the only source of complementarities.

For given expectations N_j, $j \neq i$, the demand for device i is

$$\begin{aligned} q_i &= \Pr[u_i^\theta \geq 0] \\ &= \Pr[a_i^\theta \geq p_i - \sum_{j \neq i} \gamma_{ij} N_j] \\ &= \frac{\overline{a}_i - p_i + \sum_{j \neq i} \gamma_{ij} N_j}{\overline{a}_i - \underline{a}_i} \end{aligned}$$

over the range of prices for which this expression is between 0 and 1.

It is sufficient to define $\alpha_i \equiv \frac{\overline{a}}{\overline{a}_i - \underline{a}_i}$ and $\beta_i \equiv \frac{1}{\overline{a}_i - \underline{a}_i}$, so that we obtain

$$q_i = \alpha_i - \beta_i p_i + \sum_{j \neq i} \gamma_{ij} N_j$$

which in a fulfilled expectation equilibrium, where $q_j = N_j$, is identical to the the demand system in equation (1).

Importantly, the above microfoundation can be extended to the case in which each consumer may only be interested in a subset of devices. This follows easily from our assumption that $a_1^\theta, a_2^\theta, ..., a_n^\theta$ are not correlated for any $\theta \in [0,1]$.

B Derivation of Optimal Prices

Define $\mathbf{V} = \mathbf{I} - \mathbf{G}$, and let m represent the number of platforms. Recall that p_i^k is the price charged by platform k to device i, and $p_i = \sum_{k=1}^m p_i^k$. Let $\mathbf{p}^k = \left(p_1^k, p_2^k, ..., p_n^k\right)'$ represent the vector of prices charged by platform k and $\mathbf{P} = (p_1, p_2, ..., p_n)'$ represent the vector of total prices (input costs) collectively charged by the m platforms to each device. Maintaining the assumption that the downstream market is perfectly competitive, platform k's profit is given by

$$\mathbf{\Pi}^k = \mathbf{p}^{k\prime}\mathbf{V}^{-1}\left(\mathbf{a} - \mathbf{BP}\right).$$

The first-order condition associated with the maximization of $\mathbf{\Pi}^k$ with respect to $\mathbf{p}^k$ is

$$\mathbf{V}^{-1}\left(\mathbf{a} - \mathbf{BP}\right) - \mathbf{B}'\left(\mathbf{V^{-1}}\right)'\mathbf{p}^k = \mathbf{0}.$$

In a symmetric equilibrium, we have $\mathbf{p}^k = \mathbf{p}^*$ for all platforms k, so that

$$\mathbf{V}^{-1}\left(\mathbf{a} - m\mathbf{Bp}^*\right) - \mathbf{B}'\left(\mathbf{V^{-1}}\right)'\mathbf{p}^* = \mathbf{0},$$

which (after some matrix manipulation) leads to

$$\begin{aligned}
\mathbf{p}^* &= \left[m\mathbf{V^{-1}B} + \mathbf{B}'(\mathbf{V^{-1}})'\right]^{-1}\mathbf{V^{-1}a} \\
&= \left[m\mathbf{B} + \mathbf{VB}'(\mathbf{V^{-1}})'\right]^{-1}\mathbf{a}
\end{aligned}$$

Pre-multiplying each side of this expression by $\mathbf{B}$ and rearranging yields

$$\begin{aligned}
\mathbf{Bp}^* &= \left[\left(m\mathbf{B} + \mathbf{VB}'(\mathbf{V^{-1}})'\right)\mathbf{B}^{-1}\right]^{-1}\mathbf{a} \\
&= \left[m\mathbf{I} + \mathbf{VB}'\left(\mathbf{BV}'\right)^{-1}\right]^{-1}\mathbf{a} \\
&= \left[(m+1)\mathbf{I} + \left(\mathbf{VB}' - \mathbf{BV}'\right)\left(\mathbf{BV}'\right)^{-1}\right]^{-1}\mathbf{a}
\end{aligned}$$

Defining $\sigma = \frac{1}{m+1}$, and applying the formula $(\mathbf{X}+\mathbf{Y})^{-1} = \mathbf{X}^{-1} - \mathbf{X}^{-1}\left(\mathbf{X}^{-1}+\mathbf{Y}^{-1}\right)\mathbf{X}^{-1}$, this becomes

$$\begin{aligned}
\mathbf{Bp}^* &= \left[\sigma\mathbf{I} - \sigma^2\left(\sigma\mathbf{I} + \mathbf{BV}'\left[\mathbf{VB}' - \mathbf{BV}'\right]^{-1}\right)^{-1}\right]\mathbf{a} \\
&= \sigma\mathbf{a} - \sigma^2\left(\sigma\left[\mathbf{VB}' - \mathbf{BV}' + (m+1)\mathbf{BV}'\right]\left[\mathbf{VB}' - \mathbf{BV}'\right]^{-1}\right)^{-1}\mathbf{a} \\
&= \sigma\mathbf{a} - \sigma^2\left[\mathbf{VB}' - \mathbf{BV}'\right]\left(\frac{\mathbf{VB}' + m\mathbf{VB}'}{m+1}\right)^{-1}\mathbf{a}
\end{aligned}$$

Finally, by substituting $\mathbf{V} = \mathbf{I} - \mathbf{G}$, we can solve for the vector of prices

$$\begin{aligned}
\mathbf{Bp}^* &= \sigma\mathbf{a} + \sigma^2\left(\mathbf{GB} - \mathbf{B}'\mathbf{G}'\right)\left(\mathbf{B} - \frac{\mathbf{GB}' + m\mathbf{BG}'}{m+1}\right)^{-1}\mathbf{a} \\
\mathbf{p}^* &= \sigma\left(\mathbf{I} + \sigma\left(\mathbf{B}^{-1}\mathbf{GB} - \mathbf{G}'\right)\left[\mathbf{I} - \frac{\mathbf{B}^{-1}\mathbf{GB} + m\mathbf{G}'}{m+1}\right]^{-1}\right)\mathbf{B}^{-1}\mathbf{a} \qquad \text{(B.1)}
\end{aligned}$$

When $m = 1$ (so $\sigma = \frac{1}{2}$) and $\mathbf{B} = \mathbf{I}$, equation (B.1) simplifies to the monopoly pricing formula in Theorem 1. For a monopoly with demands having different elasticity (i.e., $\mathbf{B} \neq \mathbf{I}$), it is easy to see that equation (B.1) is equivalent to (11), given the definition of $\mathbf{c}^{KB(\mathbf{B})}$. Finally, for $m > 1$ and $\mathbf{B} = \mathbf{I}$, equation (B.1) provides the equilibrium pricing for symmetric complementary platforms, as in Theorem 5.

C Generalized Star Network

A star network is defined by

$$\mathbf{G} = \begin{pmatrix} 0 & \boldsymbol{\eta}' \\ \boldsymbol{\mu} & \mathbf{0} \end{pmatrix},$$

where $\boldsymbol{\mu}' = (\mu_2, ..., \mu_n)$ and $\boldsymbol{\eta}' = (\eta_2, ..., \eta_n)$.

It is useful to define two vectors $\mathbf{c} \equiv \boldsymbol{\mu} + \boldsymbol{\eta}$ and $\mathbf{d} \equiv \boldsymbol{\mu} - \boldsymbol{\eta}$, and to let d_k represent the element in the k^{th} row of $\mathbf{d}$. The elements of $\mathbf{c}$ are (weakly) positive, and correspond to the *total* externalities between a pair of devices, whereas d_k might take either sign and represents the *net* externality from the star to device k. Using these definitions, we can prove that monopoly prices for a star network are given by:

$$p_1^M = \frac{1}{2} - \frac{1}{4}\Delta\mathbf{d}'(\mathbf{1} + \frac{1}{2}\mathbf{c} + \frac{1}{4}(\mathbf{cc}'\mathbf{1} - \mathbf{1c}'\mathbf{c})); \qquad \text{(C.1)}$$

$$p_k^M = \frac{1}{2} + \frac{1}{4}\Delta d_k(1 + \frac{1}{2}\mathbf{c}'\mathbf{1}), \text{ for } k = 2, ..., n, \tag{C.2}$$

where $\Delta = (1 - \mathbf{c}'\mathbf{c}/\mathbf{4})^{-1}$.

In the special case where $c_j = c$ for all $j \geq 2$, the monopoly prices are

$$p_1^M = \frac{1}{2} - \frac{1}{4}\Delta(\Sigma_{k=2}^n d_k)(1 + \frac{1}{2}c)$$

$$p_k^M = \frac{1}{2} + \frac{1}{4}\Delta d_k(1 + \frac{1}{2}c(n-1)))$$

The first equation reveals that the star device is subsidized if and only if $\Sigma_{k=2}^n d_k > 0$. That is, a necessary and sufficient condition for subsidizing the star device in this example is that aggregate externalities generated by the star to all peripheral devices exceed aggregate externalities generated by all peripheral devices to the star. The second equation indicates that a peripheral device k is subsidized if and only if $d_k < 0$, which implies that it creates stronger externalities for the star than vice versa.

D Social welfare

Let q_i denote the demand for device i and denote $\tilde{p}_i = p_i - \sum_{j \neq i} \gamma_{ij} q_j = \sum_{k=1}^m p_i^k - \sum_{j \neq i} \gamma_{ij} q_j$ the "externality-adjusted" price of device i. Recall that $q_i = \alpha_i - \tilde{p}_i$.

Aggregate consumer surplus is given by

$$\begin{aligned} CS &= \sum_i \int_{\tilde{p}_i}^{\alpha_i} (\alpha_i^\theta - \tilde{p}_i) d\alpha_i^\theta \\ &= \sum_i \left(\int_{\tilde{p}_i}^{\alpha_i} \alpha_i^\theta d\alpha_i^\theta - q_i \tilde{p}_i \right). \end{aligned}$$

Since

$$\int_{\tilde{p}_i}^{\alpha_i} \alpha_i^\theta d\alpha_i^\theta = \frac{1}{2}(\alpha_i^2 - \tilde{p}_i^2) = \frac{1}{2}(\alpha_i - \tilde{p}_i)(\alpha_i + \tilde{p}_i) = \frac{q_i}{2}(2\alpha_i - q_i)$$

we get

$$CS = \sum_i (\alpha_i q_i - \frac{q_i^2}{2} - q_i \tilde{p}_i) = \sum_i (\alpha_i q_i - \frac{q_i^2}{2} - q_i p_i) + \sum_i \sum_{j \neq i} \gamma_{ij} q_i q_j.$$

Therefore, social welfare is given by

$$W = \sum_i (\alpha_i N_i - \frac{q_i^2}{2}) + \sum_i \sum_{j \neq i} \gamma_{ij} q_i q_j$$

which is equivalent to equation (13) in the paper.

The welfare maximizing prices, as shown in the paper, are

$$\mathbf{p}^{\mathbf{W}} = -\,\mathbf{G}'[\mathbf{I} -\, (\mathbf{G} + \mathbf{G})']^{-1}\mathbf{a}$$

The matrix to compute the centrality measure used by the social planner is different from the one used by a monopoly platform. While the social planner cares about the social marginal surplus, a monopoly platform cares about its marginal profit. The social marginal surplus can be expressed by rewriting (14) in a matrix form as

$$\underbrace{\mathbf{G} - [\mathbf{I} - (\mathbf{G} + \mathbf{G}')]\mathbf{Q}}_{\text{social marginal surplus}} = \mathbf{0},$$

while the marginal profit is obtained from the first-order condition of the monopolist's profit, $\mathbf{\Pi}^M$=$[\mathbf{G} - (\mathbf{I} - \mathbf{G})\mathbf{Q}]'\mathbf{Q}$, with respect to $\mathbf{Q}$:

$$\underbrace{\mathbf{G} - 2\left[\mathbf{I} - \frac{(\mathbf{G} + \mathbf{G}')}{2}\right]\mathbf{Q}}_{\text{marginal profit}} = 0$$

Comparing the social marginal surplus and the marginal profit shows why the matrix to compute the centrality measure used by the social planner is different from the one of the monopolist.

E Hierarchical Network with Complementary Platforms

Lemma 2 *Consider a hierarchical network $\boldsymbol{G}$, where $\alpha_{ij} = \eta$ for all $i < j$, $\alpha_{ij} = \mu > \eta$ for all $i > j$, and $\alpha_{ii} = 0$ for all i. The KB-centrality measures associated with $\mathbf{G}^m \equiv \sigma\mathbf{G} + (1-\sigma)\mathbf{G}'$ are strictly decreasing with i.*

Proof. Denote n the number of devices. The KB-centrality vector associated with G^m

is:

$$\mathbf{c_n} = (\mathbf{I} - \mathbf{G^m})^{-1}\mathbb{1}$$

Define $\mathbf{A_n} = \mathbf{I} - \mathbf{G^m}$ and note that $\mathbf{A_n} = \begin{pmatrix} 1 & -\epsilon\mathbb{1}' \\ -\tilde{\epsilon}\mathbb{1} & \mathbf{A_{n-1}} \end{pmatrix}$, where $\epsilon = \sigma\eta + (1-\sigma)\mu$, $\tilde{\epsilon} = \sigma\mu + (1-\sigma)\eta$. From this observation it follows that

$$\mathbf{c_n} = \Delta_n^{-1} \begin{pmatrix} 1 & \epsilon\mathbb{1}'\mathbf{A_{n-1}^{-1}} \\ \tilde{\epsilon}\mathbf{A_{n-1}^{-1}}\mathbb{1} & \Delta_n \mathbf{A_{n-1}^{-1}} + \epsilon\tilde{\epsilon}\mathbf{A_{n-1}^{-1}}\mathbb{1}\mathbb{1}'\mathbf{A_{n-1}^{-1}} \end{pmatrix} \begin{pmatrix} 1 \\ \mathbb{1}_{n-1} \end{pmatrix} = \quad \Delta_n^{-1} \begin{pmatrix} 1 + \epsilon\Sigma c_{n-1,j} \\ (\tilde{\epsilon}+1)\mathbf{c_{n-1}} \end{pmatrix}$$

where $\Delta_n^{-1} = 1 - \epsilon\tilde{\epsilon}\Sigma c_{n,j}$. Notice that when there are $m(>1)$ platforms, we have $\sigma < \frac{1}{2}$ and $\epsilon \geq \tilde{\epsilon}$. Let us show recursively that the centrality device i is strictly decreasing with i. Starting with $n = 2$, it follows from $\mu > \eta$ and $\sigma > 1/2$ that $c_1^2 > c_2^2$.Now suppose that KB-centrality decreases with i for a given number of devices n. The following sequence of inequalities holds: $c_{n+1,1} = \Delta_{n+1}^{-1}[1 + \epsilon \sum c_{n,j}] > \Delta_{n+1}^{-1}[1 + \tilde{\epsilon} \sum c_{n,j}] > c_{n+1,2}(= \Delta_{n+1}^{-1}[1 + \tilde{\epsilon}c_{n,1}]) > c_{n+1,3}(= \Delta_{n+1}^{-1}[1 + \tilde{\epsilon}c_{n,2}]) > ... > c_{n+1,n+1}(= \Delta_{n+1}^{-1}[1 + \tilde{\epsilon}c_{n,n}])$. Thus, KB-centrality decreases with i for a number $n+1$ of devices. This concludes the proof. $\blacksquare$

F Proof of Theorem 6

This proof proceeds in three steps. We start by deriving the equilibrium prices for the super-star example. The second step analyzes the comparative statics for the star device as m increases (the first bullet point in the Theorem). The third step derives sufficient conditions for the the total price of the peripheral to fall when moving from $m = 1$ to $m = 2$ (the second bullet point).

Step 1: Derivation of Equilibrium Prices

For the super-star example, we have

$$\mathbf{G} = \begin{pmatrix} 0 & \boldsymbol{\eta}' \\ \boldsymbol{\mu} & \mathbf{O} \end{pmatrix} \quad \mathbf{G}' = \begin{pmatrix} 0 & \boldsymbol{\mu}' \\ \boldsymbol{\eta} & \mathbf{O} \end{pmatrix}$$

where $\boldsymbol{\eta}$ and $\boldsymbol{\mu}$ are $(n-1)\times 1$ column-vectors with each element equal to η or μ respectively.

Then, according to Theorem 5, we have

$$\mathbf{p} = \sigma\mathbf{a} + \sigma^2(\mathbf{G} - \mathbf{G}')(\mathbf{I} - \mathbf{G}^m)^{-1}\mathbf{a} \tag{F.1}$$

where

$$\mathbf{I} - \mathbf{G}^m = \begin{pmatrix} 1 & -(\sigma\boldsymbol{\eta}' + (1-\sigma)\boldsymbol{\mu}') \\ -(\sigma\boldsymbol{\mu} + (1-\sigma)\boldsymbol{\eta}) & \mathbf{I} \end{pmatrix} \equiv \begin{pmatrix} 1 & -\boldsymbol{\epsilon}' \\ -\tilde{\boldsymbol{\epsilon}} & \mathbf{I} \end{pmatrix}$$

To ensure the existence of KB-m centrality we assume that

$$\lambda_{\mathbf{G}^m} < 1 \iff \sqrt{\boldsymbol{\epsilon}'\tilde{\boldsymbol{\epsilon}}} \leq \frac{\mathbf{c}'\mathbf{c}}{4} < 1$$

so that, according to Katz (1953) the KB-m centrality for the star network is given by

$$\mathbf{c}^{KB,m} = (\mathbf{I} - \mathbf{G}^m)^{-1}\mathbf{a} = \begin{pmatrix} \Delta & \Delta\boldsymbol{\epsilon}' \\ \Delta\tilde{\boldsymbol{\epsilon}} & \mathbf{I} + \Delta\tilde{\boldsymbol{\epsilon}}\boldsymbol{\epsilon}' \end{pmatrix}\mathbf{a}$$

for

$$\Delta^{-1} = 1 - \boldsymbol{\epsilon}'\tilde{\boldsymbol{\epsilon}} = 1 - \frac{\mathbf{c}'\mathbf{c} - (1-2\sigma)^2\mathbf{d}'\mathbf{d}}{4} \tag{F.2}$$

Finally, substituting $\mathbf{c}^{KB,m}$ into (F.1) and using $\mathbf{a}' = (\alpha_1, 1, \ldots, 1)$ we can derive the equilibrium prices

$$p_1^* = \sigma\alpha_1 - \sigma^2(n-1)d\Delta\left[1 + \alpha_1(\mu + \sigma d)\right] \tag{F.3}$$

$$p_k^* = \sigma + \sigma^2 d\Delta\left[\alpha_1 + (n-1)(\mu - \sigma d)\right] \tag{F.4}$$

Step 2: Comparative Statics for Star Device

The total price for the star device is mp_1^*. The baseline price $m\sigma\alpha_1 = \frac{m\alpha_1}{m+1}$ increases with m, so it is sufficient to show that the total subsidy is decreasing. From (F.3), the total subsidy is equal to

$$m\sigma^2(n-1)d\Delta\left[1 + \alpha_1(\mu + \sigma d)\right].$$

It is easy to show that both $m\sigma^2$ and $(\mu+\sigma d)$ are decreasing with m. Equation (F.2) implies that Δ^{-1} increases with m, so Δ is also decreasing and this implies that the total subsidy is decreasing. Thus, the total price of the star device, mp_1^*, is increasing with the number of complementary platforms m.

Step 3: Sufficient Conditions for Peripheral Price Decline

Denote Δ^M and Δ^D the values of Δ for $m=1$ and $m=2$, respectively. Using (F.4), we can write the equilibrium total price for a peripheral device where there are one or two platforms, respectively, as

$$\begin{aligned} p_k^M &= \frac{1}{2} + \frac{1}{4}\Delta^M d[\alpha_1 + (n-1)(\mu - \frac{1}{2}d)], \\ p_k^D &= 2p_k^* = \frac{2}{3} + \frac{2}{9}\Delta^D d[\alpha_1 + (n-1)(\mu - \frac{1}{3}d)]. \end{aligned}$$

From $\Delta^M > \Delta^D$ it follows that

$$p_k^D - p_k^M < \frac{1}{6} + \Delta^M d[-\frac{1}{36}\alpha_1 - \frac{1}{36}(n-1)\mu + \frac{11}{216}(n-1)d].$$

This implies that

$$p_k^D - p_k^M < \frac{1}{6} + d\Delta^M[-\frac{1}{36}\alpha_1 - \frac{1}{72}(n-1)d + \frac{11}{216}(n-1)d].$$

because $\mu > \frac{c}{2} > \frac{d}{2}$. Thus, a sufficient condition for $p_k^D < p_k^M$ is that

$$\frac{1}{36}\alpha_1 - \frac{1}{27}(n-1)d > \frac{1}{6d\Delta^M}$$

or, equivalently,

$$\alpha_1 > \frac{6}{d\Delta^M} + \frac{4}{3}(n-1)d.$$

A sufficient condition for the above inequality to hold and, therefore for $p_k^D < p_k^M$ to hold as well, is

$$\alpha_1 > \frac{6}{d} + \frac{4}{3}(n-1)d$$

because $\frac{1}{\Delta^M} < 1$.

G Partial Merger

The demand system in this setting is

$$q_1 = \frac{\alpha_1 - p_1 + \gamma_{12}(\alpha_2 - p_2)}{1 - \gamma_{12}\gamma_{21}}$$

$$q_2 = \frac{\alpha_2 - p_2 + \gamma_{21}(\alpha_1 - p_1)}{1 - \gamma_{12}\gamma_{21}}$$

Firm S's post-merger maximization problem is

$$\pi_B = \max_{p_1^1} p_1^1 q_1$$

,

while firm B's post-merger maximzation problem is

$$\pi_M = \max_{p_1^2, p_2} p_1^2 q_1 + p_2 q_2$$

The corresponding first-order conditions are given by

$$2p_1^1 = \alpha_1 - p_1^2 + \gamma_{12}\alpha_2 - \gamma_{12}p_2 \tag{G.1}$$

$$2p_1^2 = \alpha_1 - p_1^1 + \gamma_{12}\alpha_2 - (\gamma_{21} + \gamma_{12})p_2 \tag{G.2}$$

$$2p_2 = a_2 + \gamma_{21}\alpha_1 - \gamma_{21}p_1^1 - (\gamma_{21} + \gamma_{12})p_1^2 \tag{G.3}$$

We then have:

G.1 - G.2 $\Rightarrow$

$$p_1^1 - p_1^2 = \gamma_{21}p_2$$

G.1 + G.2 $\Rightarrow$

$$p_1^1 + p_1^2 (\equiv p_1) = \frac{2}{3}\alpha_1 + \frac{2}{3}\gamma_{12}\alpha_2 - \frac{2\gamma_{12} + \gamma_{21}}{3} p_2$$

Therefore, we can express p_1^1 and p_1^2 as functions of p_2:

$$p_1^1 = \frac{1}{3}[\alpha_1 + \gamma_{12}\alpha_2 + (\gamma_{21} - \gamma_{12})p_2] \tag{G.4}$$

$$p_1^2 = \frac{1}{3}[\alpha_1 + \gamma_{12}\alpha_2 - (\gamma_{21} + 2\gamma_{12})p_2] \tag{G.5}$$

Finally, combining G.3, G.4 and G.5, we get the following post-merger equilibrium prices:

$$p_1^{**} = \frac{2}{3}\alpha_1 + \frac{c - 3d}{12}\alpha_2 - \frac{(3c - d)(2\alpha_1 + c\alpha_2)}{6(12 - 3c^2 + d^2)}d$$

$$p_2^{**} = \frac{1}{2}\alpha_2 + \frac{2\alpha_1 + c\alpha_2}{12 - 3c^2 + d^2}d$$

where $c \equiv \gamma_{21} + \gamma_{12}$ and $d \equiv \gamma_{21} - \gamma_{12}$.

H Downstream Market Power in a Star Network

We now examine the effect of downstream market power in a star network by considering a monopoly platform (i.e., $m = 1$). Precisely, we assume that there are l symmetric downstream firms competing à la Cournot in the star device whereas there is perfect downstream competition in peripheral devices. Let $L \equiv \frac{l}{l+1}$.

As before η (μ) represents the inbound (outbound) externality to the star device from a peripheral one (from the star device to a peripheral one). We set $\alpha_i = \beta_i = 1$ for all peripheral devices $i > 1$ while maintaining the general notation (α_1, β_1) for the star device. Note that introducing the downstream market power in the star device reduces only the inbound externalities to the star device from η to $L\eta$ but has no impact on the outbound externalities from the star device.

Denote $\lambda = \mu - \frac{\eta}{\beta_1}$ and $\sigma = \mu + \frac{\eta}{\beta_1}$. Let k be a generic indicator for a peripheral device. Then, we find that the monopoly platform pricing is given by

$$r_1 = \frac{1}{2}\frac{\alpha_1}{\beta_1} - \frac{1}{4}n\lambda c_k, r_k = \frac{1}{2} + \frac{1}{4}L\beta_1\lambda c_1. \tag{H.1}$$

where c_1 (c_k) is the centrality measure of the star device (a peripheral device). This pricing formulae show that whether a device is subsidized or not by the platform is not affected by the downstream market power. Given c_k, the price chosen by the platform for the star device is not affected either whereas, given c_1, the downstream market power reduces the amount of subsidization or exploitation of peripheral devices.

When we study the centrality measure and the equilibrium quantity of each device, we find

$$c_1 = \Delta(\frac{\alpha_1}{\beta_1} + \frac{1}{2}n\sigma), c_k = \Delta(1 + \frac{1}{2}L\alpha_1\sigma), \tag{H.2}$$

$$q_1 = \frac{1}{2}L\beta_1 c_1, q_k = \frac{1}{2}c_k,$$

where

$$\Delta^{-1} = 1 - \frac{1}{4}nL\beta_1\sigma^2.$$

The downstream market power in the star device has no impact on the centrality measure of the star device but reduces the centrality measure of peripheral devices, which lowers the output of peripheral devices. This has to do with the fact the downstream market power reduces only the inbound externalities from peripheral devices to the star device, which makes peripheral devices less central. In addition, the downstream market power reduces the output of the star device associated with a given centrality measure of the star device.

Finally, when we study the downstream price of the star device which takes into account the downstream market power, we find

$$p_1 = \frac{1}{2}\frac{\alpha_1}{\beta_1} + \frac{1}{4}(n\lambda c_k + 2(1-L)c_1).$$

where c_k and c_1 are also functions of L (see (H.2)). We find that the price of the star device increases with the downstream market power.