Payments, Velocity, Prices, and Output

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Introduction

Hume (1752), Mill (1848), Fisher (1911)

$$Mv = Py$$

Modern literature focuses on select aspects

Introduction

Hume (1752), Mill (1848), Fisher (1911)

$$Mv = Py$$

Unified perspective, multiple means of payment, goods

- Structural change in payments
- Transmission of monetary policy, financial shocks
- Effects on production
- Price indices
- Consumption Euler equation
- (In-)efficiency of portfolio, velocity choices

Perspectives on $Mv = Py$

- Baumol (1952), Tobin (1956): $M^d v = Py$
  Partial equilibrium
- Sidrauski (1967), NK model: $M^d / P = f(i, y)$
  No payments perspective, no explicit velocity
- Robertson (1933), Clower (1967), Lucas (1982): $M^d \cdot 1 \geq Py$
  Fixed velocity

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◦ This paper: $\{M, v\} \leftrightarrow \{p, y\}$, $P(p)$ aggregation
  General equilibrium, payments perspective, endog velocity
  Portfolios reflect liquidity needs, $i - \Delta$, convenience

The Model

Agents, Goods, Assets

Households, infinitely lived, homogeneous

- $U = \sum \beta^t \mathbb{E}[u(C_t)]$

Goods $j \in \mathcal{J}$

- $C \equiv \text{CES}(\{c_j\}_{j\in\mathcal{J}}),\ \{p_j\}_{j\in\mathcal{J}},\ \{e_j\}_{j\in\mathcal{J}}$

Assets $s \in \mathcal{S}$, stores of value, means of payment

- $\{I_s\}_{s\in\mathcal{S}}$

Timing

Timing in each period (Lucas, 1982)

i. Uncertainty about returns, endowments resolved

ii. Portfolios rebalanced

iii. Trading subject to liquidity constraints
Competition

Household Program: Portfolio Choice

Household chooses $\{a_{s0}, \{a_{sj}\}_{j\in\mathcal{J}}\}_{s\in\mathcal{S}}$ s.t.

$$w = \sum_{s\in\mathcal{S}} \left(a_{s0} + \sum_{j\in\mathcal{J}} a_{sj} \right) + \text{outlays} - \text{dividends}$$

Typical household chooses $\{\bar{a}_{s0}, \{\bar{a}_{sj}\}_{j\in\mathcal{J}}\}_{s\in\mathcal{S}}$

Household Program: Trading

Inflows of means of payment at rates $\bar{\alpha}_s$, $\sum_{s \in \mathcal{S}} \bar{\alpha}_s = \sum_{j \in \mathcal{J}} p_j e_j$

Liquidity constraints

$$a_{sj} + \bar{\alpha}_s (1 - \delta_s) \sigma_{sj} \geq p_j c_j \pi_{sj}$$

acquired at portfolio choice stage

acquired during trading period

(inverse) velocity choice $\delta_s \in (0, 1]$

Lower δ_s (higher velocity) increases liquidity, at cost $\tau(\delta; \ldots)$

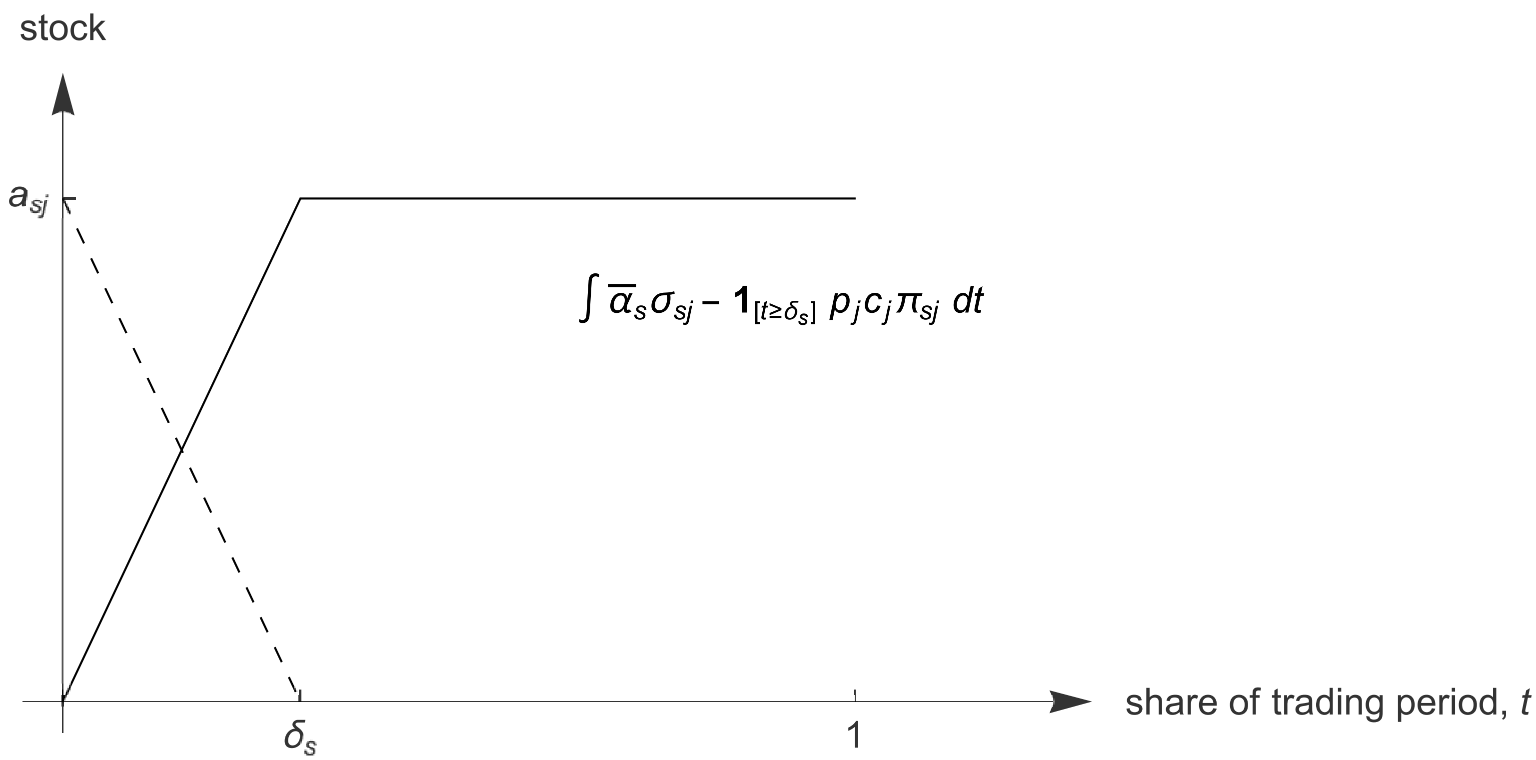

Figure 1: Stock of means of payment s for purchases of good j during the trading stage.

Household Program: Convenience

Data leakage depending on (s, j) combination

$$l_{sj} \equiv p_j c_j \pi_{sj}$$
$$k'_{sj} = k_{sj}(1-\gamma_{sj}) + l_{sj}$$

Costs $\phi(l, k; \ldots)$ increasing, convex in l, increasing in k

Household Program: Continuation Wealth

$$w' = \sum_{s\in\mathcal{S}} a_{s0} I_s + \sum_{s\in\mathcal{S},j\in\mathcal{J}} a_{sj} J_s + \sum_{s\in\mathcal{S},j\in\mathcal{J}} \bar{a}_{sj}(I_s - J_s) + T'$$
$$+ \sum_{s\in\mathcal{S}} \left(\bar{\alpha}_s - \sum_{j\in\mathcal{J}} p_j c_j \pi_{sj} \right) K_s$$

$$I_s \equiv 1 + i_s, \;\; J_s \equiv 1 + i_s - \Delta_s, \;\; K_s \equiv 1 + \frac{i_s - \Delta_s}{2}$$

Refunded payment service provider profits

Injections by transfer

Household Program: Optimality Conditions (Disregarding Risk)

$$\begin{aligned}
a_{s0} &: \lambda = \beta\lambda' I_s \\
a_{sj} &: \lambda \geq \lambda\xi_{sj} + \beta\lambda' J_s, \;\; a_{sj} \geq 0 \\
\delta_s &: -\lambda\frac{\partial\tau_s}{\partial\delta_s}\bar{\alpha}_s \geq \lambda\bar{\alpha}_s \sum_{j\in\mathcal{J}} \sigma_{sj}\xi_{sj}, \;\; \delta_s \leq 1 \\
c_j &: \frac{\partial u(C)}{\partial C}\frac{\partial C}{\partial c_j} = p_j\lambda\left(\sum_{s\in\mathcal{S}} \pi_{sj}(\xi_{sj} + \varphi_{sj}) + \beta\frac{\lambda'}{\lambda}\sum_{s\in\mathcal{S}} \pi_{sj}K_s\right) \\
\sigma_{sj} &: \xi_{si} = \xi_{sj} = \xi_s \\
\pi_{sj} &: \xi_{sj} + \varphi_{sj} + \beta\frac{\lambda'}{\lambda}K_s = \xi_{tj} + \varphi_{tj} + \beta\frac{\lambda'}{\lambda}K_t
\end{aligned}$$

$$
\begin{aligned}
a_{s0} &: \lambda = \beta \lambda' I_s \\
a_{sj} &: \lambda \geq \lambda \xi_{sj} + \beta \lambda' J_s, \;\; a_{sj} \geq 0 \\
\delta_s &: -\frac{\partial \tau_s}{\partial \delta_s} \bar{\alpha}_s \geq \bar{\alpha}_s \xi_s, \;\; \delta_s \leq 1 \\
c_j &: \frac{\partial u(C)}{\partial C} \frac{\partial C}{\partial c_j} = p_j \lambda \left(1 + \varphi_{sj} + \beta \frac{\lambda'}{\lambda} (K_s - J_s) \right)
\end{aligned}
$$

Service Providers

Services for households

- Velocity, cost $\tau(\delta;\ldots)$
- Convenience (leakage), cost $\phi(l,k;\ldots)$
- Management of means of payment, unit cost $I_s - J_s$

Social costs

- Households work for service providers, $u(C)$ $-$ effort
- Social costs $= \lambda \left(\tau + \phi + \sum_{s\in\mathcal{S},j\in\mathcal{J}} a_{sj}\beta\frac{\lambda'}{\lambda}(I_s - J_s) \right)$

Equilibrium

Symmetry

$$a = \bar{a},\;\; \delta = \bar{\delta},\;\; \sigma = \bar{\sigma},\;\; \pi = \bar{\pi}$$
$$\sigma_{sj} = a_{sj} / \sum_{i \in \mathcal{J}} a_{si}$$
$$\pi_{sj} = a_{sj} / (\delta_s p_j c_j)$$
$$\bar{\alpha}_s = \sum_{j \in \mathcal{J}} \bar{a}_{sj} / \bar{\delta}_s$$

Market clearing

$$e_j = c_j$$
$$0 = \bar{a}_{s0} + \sum_{j \in \mathcal{J}} \bar{a}_{sj}$$

Nominal anchor

Analysis

Interest Rates, Liquidity, and Velocity

$$\xi_s = \beta \frac{\lambda'}{\lambda}(I_s - J_s)$$
$$\xi_s > 0 \;\Rightarrow\; \delta_s < 1 \qquad \text{all viable assets}$$

Intermediation margin

$\Rightarrow$ Liquidity value (CIA, $J_s = 1$)

$\Rightarrow$ Velocity (Baumol (1952), Tobin (1956), $\sim$ Sidrauski (1967))

Liquidity $\perp$ convenience

Interest Rates, Liquidity, Convenience, and Prices

\$ cost of consuming good j

$$q_j \equiv p_j \underbrace{\left(\xi_s + \varphi_{sj} + \beta\frac{\lambda'}{\lambda}K_s\right)}_{\Phi_j} = p_j\left(1 + \varphi_{sj} + \beta\frac{\lambda'}{\lambda}(K_s - J_s)\right)$$

$$Q \equiv \left(\tilde{J}^{-1}\sum_{j\in\mathcal{J}} q_j{}^{1-\eta}\right)^{\frac{1}{1-\eta}} \quad \text{vs.} \quad P \equiv \left(\tilde{J}^{-1}\sum_{j\in\mathcal{J}} p_j{}^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

Conventional CIA has $q_j = p_j, Q = P$

Approximate inflation wedge

$$\frac{dQ}{Q}\Big|_{P=1} = \sum_j \underbrace{\frac{X_j}{\Phi_j}}_{\approx\ \text{share}_j} \left(d\varphi_{sj} - \underbrace{\frac{1+\Delta_s}{2I_s^2}}_{\approx 1/2} di_s + \underbrace{\frac{1}{2I_s}}_{\approx 1/2} d\Delta_s \right)$$

(Robust) implications

- i_s from 2% to 6% $\Rightarrow$ $Q/P \downarrow$ by 2%
- Δ_s from 1% to 3% $\Rightarrow$ $Q/P \uparrow$ by 1%
- φ_{sj} from 1% to 2% $\Rightarrow$ $Q/P \uparrow$ by 1% $*$ share$_j$

Interest Rates, Liquidity, Convenience, and Consumption

$$U_j = \beta\, U_j' \; \frac{p_j}{p_j'} \; I_s \; \frac{\Phi_j}{\Phi_j'}$$

Transaction costs affect inter temporal substitution

Approximate effects on $I_s \Phi_j / \Phi_j'$

$$i_s \;\Rightarrow 1 - 1/2, \;\; i_s' \;\Rightarrow 1/2$$

$$\Delta_s \;\Rightarrow 1/2, \;\; \Delta_s' \;\Rightarrow -1/2$$

$$\varphi_{sj} \;\Rightarrow 1, \;\; \varphi_{sj}' \;\Rightarrow -1$$

Financial distress $\approx \Delta_s, \varphi_{sj} \uparrow$

Determinacy

Along BGP

- Indeterminate portfolio if $\phi = 0$
- Indeterminate portfolio if $\varphi_{sj} = \text{constant}_{sj}$
- Indeterminate portfolio if φ_{sj} depends on $l_{sj} + l_{tj}, \ldots$
- Determinacy if ϕ strictly convex, separable across (sj)

Constrained Inefficiency

Benchmark

- Planner subject to payment technology
- Minimizes $\lambda \left(\tau + \phi + \sum a_{sj} \beta \frac{\lambda'}{\lambda} (I_s - J_s) \right)$
- Internalizes *circular flow* of monies

Equilibrium

- Velocity choice constrained efficient
- Portfolio choice constrained inefficient

Production

Production subject to utility cost rather than endowment

Payments shape production

$$v_j = \lambda p_j \sum_{s \in \mathcal{S}} \bar{\pi}_{sj} \left\{ \xi_s (1 - \delta_s) + \beta \frac{\lambda'}{\lambda} K_s \right\}$$

In CIA ($\delta_s = 1$, $K_s = 1$), $v_j = \lambda p_j$

Unlike in CIA, relative producer costs $\neq$ relative goods price

Conclusion

$Mv = Py$

- Multiple M, liquidity needs, $i - \Delta$, convenience
- Endogenous v

Findings

- Payments shape p, P vs. Q
- Euler equation
- Constrained inefficiency
- Payments shape production

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