Firm Heterogeneity, Industry Dynamics and Climate Policy*

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Abstract

We develop a dynamic general equilibrium model with heterogeneous firms to investigate the interaction between climate policy, industry dynamics, and the elasticity of substitution between clean and dirty energy. A central economic force is the selection between firms that differ in their flexibility to switch from dirty to clean energy and the endogenous exit of least flexible firms in response to climate policy. The exit of inflexible firms improves the average elasticity among active firms in our model, which is consistent with the data, leading to a substantial reduction in the optimal carbon tax and welfare costs.

Keywords: Climate policy, directed technical change, industry dynamics.
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1 Introduction

The transition from fossil fuels to renewable energy takes a central role in the policy visions to halt the progress of global warming (IPCC, 2018). As a crucial parameter that governs the transition process, the degree of substitutability between clean and dirty energy has received much attention as a main determinant of important outcomes such as induced innovation in green technologies (e.g., Acemoglu et al., 2012; Fried, 2018), the relative efficacy of different climate policy instruments (Lemoine, 2017; Greaker et al., 2018; Hart, 2019), and the overall economic costs of climate change mitigation (Golosov et al., 2014). Despite its prominence, microfoundations behind the elasticity of substitution between different types of energy are hitherto lacking in the literature. For instance, is the substitution elasticity changing over time and if so, what are the drivers behind the evolution? Or, is there heterogeneity across firms and sectors in the elasticity and what are the policy implications of heterogeneity in this dimension?

In this paper, we tackle these questions by developing a microfounded general equilibrium model of directed technical change that allows us to study the interplay of climate policy and the elasticity of substitution. Two stylized facts motivate our research question. First, using micro data from the French manufacturing sector with rich variation in energy prices and consumption at the firm level, we empirically document that the elasticity of substitution between clean and dirty energy has been increasing in the past two decades. This observation stands in contrast to the common assumption of a constant and exogenous elasticity of substitution in the directed technical change framework. In particular, it is noteworthy that the evolution has been concurrent with climate policy becoming more stringent over the same time period in France.

Second, we find empirical evidence of heterogeneity in the elasticity of substitution across firms, which points to differences in their ability to cope with climate policy or the rising relative price of dirty energy. This is again in contrast to the standard approach in the relevant macro-environmental literature of assuming the same degree of energy substitutability for all firms or sectors. In light of these stylized facts, our aim is to provide a theoretical framework that rationalizes these novel empirical patterns by introducing industry dynamics and understand their policy implications. Specifically, our model builds on the large macroeconomic literature of directed technical change and the environment (e.g., Acemoglu et al., 2012), but extends these models by incorporating firm heterogeneity in the elasticity of substitution between clean and dirty energy and endogenous exit and entry. These margins are critical for our investigation of how the average substitution elasticity across firms evolves endogenously over time interacting with climate policy.

We consider an economy with two segments: industrial production and energy services. In the industrial sector, incumbent firms produce differentiated products by combining clean and dirty energy and incurring a fixed cost of operation in terms of low-skilled labor. Forward-looking potential entrants make optimal entry decisions based on their expected lifetime profits relative to the fixed cost of entry. Firms are heterogeneous with respect to their elasticity of substitution between clean and dirty energy, which affects their ability to cope with climate policy. Due to the presence of the fixed cost, firms may exit if their operating costs increase and profits drop below a certain threshold, which we characterize. Energy services in our model consist of the clean and dirty energy sector and provide energy inputs to the heterogeneous firms in the industry. Moreover, innovation takes place in the clean and dirty energy sector, improving their productivity.

The interplay of firm heterogeneity, industry dynamics and climate policy in our model endogenizes the average elasticity among active firms and replicates the empirical observation of its growth over time. Climate policy, or the consequent increase in the relative price of fossil-based energy, induces least flexible firms to exit the market, while simultaneously allowing the entry of firms that are relatively more capable of substituting clean for dirty energy and therefore have lower operating costs and sufficiently high expected profits to cover the sunk entry costs. The higher average elasticity of substitution brought about by industry dynamics creates larger demand shifts to clean energy in the industry, as well as stronger innovation response in the clean energy sector.

For the quantitative analysis, we calibrate the parameters of our model to match key moments implied by the model with their empirical counterparts in micro and macro data. It is important for our model to capture the relationships between energy prices, industry dynamics, and production and innovation in the energy sector. Thus, we target moments in the data that capture these features to discipline our parameters. For instance, a crucial empirical moment for our model to match is the average elasticity of substitution between clean and dirty fuels among firms that we estimate from our micro data. The model performs well and matches the targeted and non-targeted moments closely, suggesting that the model's fit is reasonably strong.

Using our calibrated model, we compute a set of counterfactual stationary equilibria to understand the impact of industry dynamics and the endogenous change in the average elasticity of substitution on optimal climate policy. We compare two equilibria that achieve the same policy goal of carbon neutrality but one where the channel of industry dynamics is at play (the endogenous model) and the other where this channel is shut off (the exogenous model).

Three main findings emerge. First, the higher average elasticity of substitution arising from industry dynamics lowers the optimal carbon tax that achieves carbon neutrality in the new equilibrium by 48 percent. The sizeable impact of the substitution elasticity on the optimal carbon tax we observe here echoes the findings of Acemoglu et al. (2012), although the main difference is that the change in the elasticity of substitution is endogenously induced by dynamic industry response in our model. The exit of least flexible firms increases the average elasticity of substitution among active firms by 10 percent in the new equilibrium. The higher elasticity enables larger demand shifts to clean energy in the industry, lowering the required size of the tax to achieve the same emissions reduction target and reducing welfare costs.

Second, climate policy and industry dynamics lead to a structural change in the economy: as inflexible firms exit the market, essential resources (labor) reallocate to the clean energy sector, as demand for clean energy is now higher.

The relative market size of the clean energy sector increases much more in the endogenous model compared to the exogenous model, absorbing the freed labor from the industry. Third, industry dynamics also strengthen innovation response. This is again primarily driven by the higher average elasticity of substitution due to the selection effects, which creates stronger demand response in the industry and incentivizes innovation in the clean sector.

Next, we investigate the implications of different policy instruments, namely, a carbon tax and a research subsidy to clean innovation, in the presence of endogenous industry dynamics. To this end, we compute the size of research subsidies that achieve the same policy objective in the counterfactual equilibrium. Research subsidies achieve carbon neutrality by increasing the relative productivity of clean technologies by 3 times more than the carbon tax does in meeting the same policy objective. This is because subsidies provide indirect price incentives by improving clean technologies, while the tax generates direct price incentives for the energy-consuming industrial firms. We find that industry dynamics continue to be important. Without the endogenous exit of inflexible firms and the average elasticity of substitution among active firms adjusting in response to the policy, the optimal subsidy is 21 percentage points larger in the exogenous model.

Our paper is most closely related to the literature on directed technical change and climate in general equilibrium.[1] Our theoretical framework extends the literature in noteworthy dimensions. To the best of our knowledge, this paper is the first to endogenize the elasticity of substitution between clean and dirty energy, which is a key structural parameter in the large literature on climate change and growth (Hart, 2019; Meng, 2021). As a result, the model is able to reproduce the empirical observation that the average substitution elasticity among firms is growing over time, which is a feature not captured by previous models and has strong policy implications as our quantitative exercises highlight. This is achieved by allowing energy-consuming firms to be heterogeneous in their ability to substitute clean for dirty fuels which induces

[1] See, for example, Acemoglu et al. (2012); Bretschger and Smulders (2012); Golosov et al. (2014); Fried (2018); Greaker et al. (2018); Borissov et al. (2019); Hart (2019).

endogenous exit and entry in response to climate policy, in contrast to most models in the literature featuring a single elasticity of substitution parameter (constant and exogenous through the CES aggregate production function).

Particularly related to our paper are Hassler et al. (2021) that show the degree of substitutability between energy and other inputs (labor and capital) can be significantly higher in the long run with endogenous technical change, and Jo and Miftakhova (2024) that explore the impact of a time-varying elasticity of substitution between clean and dirty energy on climate policy. Yet, neither paper explicitly models industry dynamics through endogenous entry and exit of firms that animate our analysis.

Lastly, our paper is related to the growing group of papers that emphasize the structure and dynamics of the industries in the context of environmental regulation (Ryan, 2012; Fowlie et al., 2016; Miller et al., 2017; Leslie, 2018), but is distinguished by our framework that marries the issue of industry dynamics to directed technical change and by our focus on how dynamic industry response affects the average elasticity of substitution among firms, which is a critical determinant of the cost of environmental policy.

The rest of the paper is organized as follows. Section 2 provides stylized empirical facts that motivate our research question and modelling approach. Section 3 presents the model. Section 4 describes our data and quantitative analysis. Section 5 presents results from our quantitative exercise. The last section concludes.

2 Stylized facts

We begin by providing two stylized facts that motivate our research question, namely, (1) growth in the elasticity of substitution between clean and dirty energy and its positive correlation with the stringency of climate policy and (2) firm heterogeneity in the elasticity of substitution. We use micro data between 1995 and 2015 from the French manufacturing industry that accounts for approximately 20 percent of the total energy consumption and is the second largest source of greenhouse gas emissions after the transportation sector in France (Citepa, 2023). The data comes from two main sources: the *Enquête sur les Consommations d'Énergie dans l'Industrie* (EACEI) that provides information on energy consumption and expenditure by fuel and *Fichier approché des résultats d'Esane* (FARE) that contains information on financial variables.

The consumption of different fuels is aggregated to a clean and a dirty bundle for each firm in order to investigate the elasticity of substitution between the two types of energy. Following earlier studies estimating the elasticity (Papageorgiou et al., 2017; Jo, 2020), we add up electricity, steam and renewables into the clean bundle and all other types (natural gas, petroleum products, etc.) into the dirty bundle.[2] We similarly aggregate expenditures on each fuel to the clean and dirty bundle and calculate the unit costs for clean and dirty energy by dividing the expenditure measures by the corresponding consumption measures. Detailed descriptions of the data are relegated to Section 4.1.

2.1 Energy substitutability and the stringency of climate policy

To examine whether the elasticity of substitution between clean and dirty energy has evolved over time, we estimate the elasticity separately for each year between 1995 and 2015 using the following specification:

$$\ln\left(\frac{b_{jt}}{g_{jt}}\right) = \sigma \ln\left(\frac{p_{g,jt}}{p_{b,jt}}\right) + \theta_s + \mu_r + \delta_t + \epsilon_{jt} \tag{1}$$

where g_{jt} and b_{jt} are clean and dirty energy consumption of firm j in year t, respectively, and $p_{g,jt}$ and $p_{b,jt}$ reflect their respective unit costs. The coefficient of interest σ therefore captures the response of the ratio of dirty to clean

[2] The clean bundle mostly consists of electricity (over 99 percent) which is generated by exceptionally low-carbon mix in France with nuclear and renewables accounting for over 90 percent between 1995 and 2015 (IEA, 2022). Furthermore, electrification in all sectors with simultaneous decarbonisation of electricity generation is a main agenda for climate policy in most countries, which makes substitution between electricity and non-electricity the key dimension of energy transition beyond France (Holland et al., 2022).

energy demands to a change in the relative prices of clean and dirty energy which corresponds to the definition of the elasticity of substitution.[3] θ_s and μ_r denote industry and region fixed effects, respectively. The specification additionally controls for firm characteristics that might be correlated with flexibility in input choices such as productivity and firm size. We exploit cross-sectional variation in energy consumption and unit prices across firms to capture long-run substitution that involves large-scale capital adjustments and estimate the long-run elasticity of substitution which corresponds more closely to the interpretation of this parameter in the theoretical literature (Apostolakis, 1990; Arnberg and Bjørner, 2007).[4]

Firm-level energy prices are likely to be endogenous due to omitted variable bias such as demand and productivity shocks that may simultaneously affect energy input choices and unit prices.[5] Thus, we follow previous studies (e.g., Linn, 2008; Jo, 2020; Marin and Vona, 2021) and construct instruments based on national energy prices which are plausibly independent of firm-specific unobservables. The instrument for the price of clean energy $\tilde{p}_{gjt}$ is constructed as follows:

$$\tilde{p}_{g,jt} = p_{g,j0} \times \prod_{s=1}^{t} (1 + \gamma_s^g) \tag{2}$$

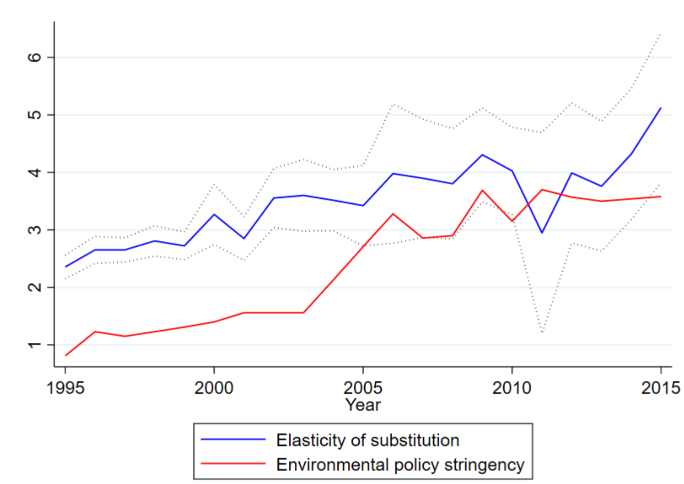
where $p_{g,j0}$ denotes the price of clean energy in firm j in the pre-sample period ($t = 0$) and γ_t^g captures the growth rate of the national average price of clean energy between $t - 1$ and t. Intuitively, the instrument applies the national growth rates in the price of clean energy to firm-specific pre-sample prices.

[3]The estimating equation can also be derived from the industrial firm's cost minimization problem in our model presented in the next section (see equation (6)).

[4]Moreover, energy substitution is likely to be more limited in the short run due to the prevalence of forward contracts for industrial consumers who tailor their capital equipment accordingly. Jo (2020) confirms that as expected, exploiting time-variation within firms yields estimates that are smaller in magnitude than estimates based on cross-sectional variation as they do not capture non-instantaneous adjustments in capital stock.

[5]For instance, there might be productivity shocks at the firm level that may affect energy demand and the unit price of energy. That is, to the extent that firms take into account their factor-specific productivity when choosing inputs, a positive productivity shock in the use of green energy, for instance, would affect the relative input ratio by changing the demand for green energy, which in turn may affect the price ratio through changes in quantity discounts (i.e., lower unit price due to higher demand for green energy).

Figure 1: Evolution of the average elasticity of substitution and the stringency of environmental policy



Note: Cross-sectional IV estimates of the elasticity of substitution for each year with 95 percent confidence intervals. The Environmental Policy Stringency (EPS) index comes from the OECD.

The pre-sample period is set to the first year of the dataset (1994) and the estimation sample runs from the following year (1995). The instrument for the price of dirty energy is similarly constructed. We use these two instruments to instrument for the log price ratio in equation (1).

The IV estimates and the 95 percent confidence intervals are graphically reported in Figure 1. We observe a clear upward trend in the average elasticity of substitution among manufacturing firms: the estimated elasticity of substitution more than doubles over the 20-year period, increasing from just above 2 in 1995 to over 5 in 2015.[6] Furthermore, it is noteworthy that the increase in the substitution elasticity was concurrent with climate policy strengthening

[6] Jo and Miftakhova (2024) similarly document that the elasticity has been increasing with the penetration of clean energy in manufacturing industry. However, numerical studies such as Wiskich (2019) and Stöckl and Zerrahn (2023) show that the substitutability may in fact decrease at very high shares of clean energy that are not empirically observed yet, implying an inverted U-shape relationship between clean energy penetration and the substitution elasticity. Our analysis focuses on the increase in the substitution elasticity that is empirically relevant today and attempts to rationalize these empirical patterns in the analysis of optimal climate policy by focusing on the role of industry dynamics.

over the same time period as measured by the Environmental Policy Stringency (EPS) from the OECD.[7] This stylized fact challenges the standard assumption of a constant elasticity of substitution between clean and dirty inputs in the directed technical change framework.

2.2 Firm heterogeneity in the elasticity of substitution

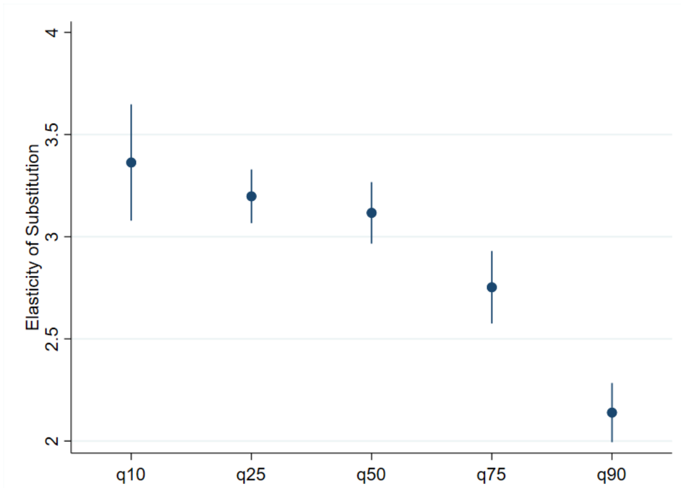
Next, we examine the presence of heterogeneity in the elasticity of substitution across firms which affects their ability to cope with climate policy that raises the relative price of dirty energy. For the purpose, we estimate equation (1) by quantile regressions. This approach allows us to examine heterogeneity in firms' substitution capacity across different quantiles of the conditional distribution of the relative dirty energy consumption, $\ln(b_{jt}/g_{jt})$.[8]

Figure 2 shows substantial heterogeneity in the elasticity across firms. Firms at or below the 50th percentile (cleaner firms) are associated with stronger energy substitutability, while those in the 75th and 90th percentile (dirtier firms) display lower degrees of substitutability. The estimates of the elasticities associated with the 10th and 90th percentile firms are statistically different at 1 percent level. We explore firm heterogeneity more broadly along dimensions other than the relative dirty energy consumption and report further evidence in Online Appendix (Figure OA1). This exercise provides evidence for the presence of heterogeneity in energy substitutability across firms which has not received much attention in the literature. In the next sections, we develop a theoretical framework that rationalizes these novel empirical patterns by focusing on the role of industry dynamics and explore their policy implications.

[7] The EPS measure aggregates selected environmental policy instruments, primarily related to climate and air pollution into a composite index (Botta and Koźluk, 2014). Moreover, carbon pricing as a single policy has also been increasing in its stringency in France (OECD, 2021a).

[8] We additionally control for year fixed effects in these regressions.

Figure 2: Heterogeneity in the elasticity of substitution across firms



Note: Estimates of the elasticity of substitution between clean and dirty energy from quantile regressions. Lower quantiles refer to firms with a lower relative share of dirty energy (green firms) and higher quantiles refer to firms with a higher relative share of dirty energy (brown firms).

3 Model

This section presents our theoretical framework that consists of the industrial sector and the energy sector and characterizes the economy's stationary balanced growth equilibrium.[9]

[9] We consider an aggregate industrial sector and focus on firm heterogeneity in the elasticity of substitution rather than sectoral heterogeneity. The goal is to examine the channels of industry entry/exit dynamics that generate selection effects leading to an increase in the average elasticity of substitution among surviving firms. The insights of our model will go through in a more involved model with higher granularity within the industrial sector, given the presence of firm heterogeneity within sectors as shown in Figure 2. Sector-level elasticities of substitution will evolve through the selection channel, affecting the average elasticity of substitution across sectors.

3.1 Industry

3.1.1 Final good technology

The final good Y is produced competitively using the output of a continuum of intermediate firms in a Dixit-Stiglitz form with the elasticity of substitution $\epsilon > 1$:

$$Y = \left[\int_{j \in J} y_j^{\frac{\epsilon - 1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon - 1}}. \tag{3}$$

The demand for each differentiated intermediate good y_j and the price index for good Y are given by profit maximization as:

$$y_j = A p_j^{-\epsilon}, \qquad A \equiv Y P^{\epsilon} \qquad \text{and} \qquad P = \left[\int_{j \in J} p_j^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}} \tag{4}$$

where p_j is the price of intermediate good y_j. We choose the final good as the numeraire and set its price to unity, i.e., $P = 1$ and $A = Y$.

3.1.2 Intermediate good production

Production of intermediate goods by firms entails a fixed cost of operation f_p in terms of unskilled labor. Once the fixed cost is covered, each intermediate firm j produces their output by combining clean (g_j) and dirty (b_j) energy inputs according to the CES production function:

$$y_j = \varphi_j \left(a_j g_j^{\frac{\sigma_j - 1}{\sigma_j}} + (1 - a_j) b_j^{\frac{\sigma_j - 1}{\sigma_j}} \right)^{\frac{\sigma_j}{\sigma_j - 1}}$$

where $\varphi_j > 0$ denotes the productivity parameter, $a_j \in (0, 1)$ the distribution parameter and $\sigma_j > 0$ the firm-specific elasticity of substitution between clean and dirty inputs.[10]

In order to distinguish firms by their elasticities of substitution, the CES production function has to be normalized to a benchmark point. This is be-

[10]We do not have capital as an input for the sake of simplicity. Introducing capital in a Cobb-Douglas fashion would add complexity without generating new insights.

cause the productivity parameter φ_j and the distribution parameter a_j are intrinsically linked to the elasticity of substitution (see, for example, Klump and de La Grandville (2000) and León-Ledesma et al. (2010) for relevant theoretical discussions), which makes it difficult to differentiate firms only by the substitution elasticity while holding other parameters constant.[11] Thus, with benchmark values of input demands, output and input prices denoted as $\{g_0, b_0, y_0, p_{g0}, p_{b0}\}$, the normalized production function reads:

$$y_j = y_0 \left[\kappa_0 \left(\frac{g_j}{g_0} \right)^{\frac{\sigma_j - 1}{\sigma_j}} + (1 - \kappa_0) \left(\frac{b_j}{b_0} \right)^{\frac{\sigma_j - 1}{\sigma_j}} \right]^{\frac{\sigma_j}{\sigma_j - 1}},$$

where $\kappa_0 = p_{g0}g_0/(p_{g0}g_0 + p_{b0}b_0)$ is the expenditure share of the clean input at the point of normalization. Since firms differ only with respect to their elasticity of substitution once normalized, we drop j and index firms from now on by σ; for instance, we refer to y_j as $y(\sigma)$.

With p_g and p_b representing the input prices and τ a tax on the dirty input, the variable cost of production $c_\sigma(p_g, p_b)$ reads:

$$c(\sigma) = c_0 \left[\kappa_0 \left(\frac{p_g}{p_{g0}} \right)^{1-\sigma} + (1 - \kappa_0) \left(\frac{p_b + \tau}{p_{b0}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{5}$$

with $c_0 \equiv (p_{g0}g_0 + p_{b0}b_0)/y_0$ represents the benchmark variable cost. By Shephard's lemma, the demand for each input can be written as:

$$\begin{aligned} g(\sigma) &= y(\sigma) \left(\kappa_0 \frac{c(\sigma)}{p_g} \right)^{\sigma} \left(\frac{g_0}{y_0} \right)^{1-\sigma}, \\ b(\sigma) &= y(\sigma) \left((1 - \kappa_0) \frac{c(\sigma)}{p_b + \tau} \right)^{\sigma} \left(\frac{b_0}{y_0} \right)^{1-\sigma}. \end{aligned} \tag{6}$$

[11] In macroeconomics literature, this is particularly important when examining the effects of variation in the elasticity of substitution on economic growth over time or across countries (Klump et al., 2012). As the substitution elasticity varies, the 'dimensional constants' (the productivity and distribution parameters) also vary in the CES function, making it hard to isolate the impact of varying elasticities of substitution. de La Grandville (1989) and Klump and de La Grandville (2000) among others have emphasized the importance of normalizing CES functions as a way to deal with this dimensional problem when analyzing the theoretical consequences of variation in the elasticity of substitution.

The following lemma establishes that a firm with a higher elasticity of substitution is able to produce at a lower cost compared to another firm producing the same amount of output with a lower elasticity of substitution.

Lemma 1. *All else equal, the variable cost of production $c(\sigma)$ is decreasing in σ.*

Proof. See Appendix A1. $\square$

The Dixit-Stiglitz structure in (3) supports monopolistic competition in the supply of each differentiated variety $y(\sigma)$. Profit maximization of intermediate firms implies that each firm charges a price that includes a constant markup over its variable cost $c(\sigma)$:

$$p(\sigma) = \frac{\epsilon}{\epsilon - 1} c(\sigma). \tag{7}$$

Firm revenue is then given by $p(\sigma)y(\sigma) = Y p(\sigma)^{1-\epsilon}$ (see (4)) and firm profit is written as:

$$\pi(\sigma) = (Y/\epsilon)p(\sigma)^{1-\epsilon} - w_{li} f_p, \tag{8}$$

where w_{li} represents the wage rate of (unskilled) labor in the industrial segment. Note that since the cost of production is declining in the elasticity of substitution according to the lemma above, a higher σ is associated with cheaper production, a lower market price, and higher revenues and profits (because demand is elastic with $\epsilon > 1$), holding all else constant.

3.1.3 Firm entry and exit

There is a large pool of ex-ante identical potential entrants. Entry into the market entails an initial investment $f_e > 0$ in terms of unskilled labor, which is thereafter sunk. Once the entry cost is paid, firms then draw their substitution elasticity parameter σ from a common distribution $\phi(\sigma)$ with a positive support $(0, \infty)$; we denote with $\Phi(\sigma)$ its cumulative distribution.[12] An entrant

[12] We view this set up of firms drawing their level of elasticity of substitution as firms facing uncertainty about their potential for input substitution once production begins. For

with a bad draw of σ may exit immediately and never produce. If a firm remains in the market and produces, it faces an idiosyncratic shock that forces it to exit the market at the constant rate δ. As in Melitz (2003), the specification of a common distribution $\phi(\sigma)$ and the exit rate δ exogenously determine the shape of the equilibrium distribution of the substitution elasticity and the ex ante survival probabilities. However, the simple model is nevertheless able to endogenously determine the range of substitution elasticities for surviving firms and therefore the average elasticity of substitution, which are critical margins in our model that interact with climate policy.

With r representing the discount rate, a firm's value at the time of entry $v(\sigma)$ is equal to the discounted expected lifetime profits:

$$v_t(\sigma) = \max\left\{0, \sum_{s=0}^{\infty}\left(\prod_{u=0}^{s-1}\frac{1}{1+r_{t+s-u}}\right)(1-\delta)^s \pi_{t+s}(\sigma)\right\}.$$

With growth stemming from technological innovation in the energy sector of our economy (will be introduced in the next section), profits in (8) grow with the aggregate technology Q (see Appendix A2), which in turn grows at the constant rate of g in a stationary equilibrium. Thus, we normalize profits by Q such that $\tilde{\pi}(\sigma) \equiv \pi(\sigma)/Q$ is constant. The equilibrium stationary value of a firm then reads as:

$$\tilde{v}(\sigma) = \max\left\{0, \sum_{s=0}^{\infty}\left(\frac{(1-\delta)(1+g)}{1+r}\right)^s \tilde{\pi}(\sigma)\right\} = \max\left\{0, \frac{1}{\omega}\tilde{\pi}(\sigma)\right\} \tag{9}$$

where $\omega \equiv \frac{r+\delta-(1-\delta)g}{1+r}$ is the augmented discount factor that incorporates the probability of exogenous destruction and growth.

Since firms face fixed costs of operation and profits increase in σ, an entering firm with a bad draw of σ may immediately exit if the profit level were negative. This decision defines a cutoff level elasticity of substitution σ^* that solves $\tilde{\pi}(\sigma^*) = 0$, the zero cutoff profit condition, below which firms exit and

example, there can be uncertainty about the best-practice technologies at the time of firm establishment or about the conditions of energy supply contracts due to poor management.

do not produce. The distribution of active firms conditional upon successful entry is then endogenously determined by σ^* as:

$$\psi(\sigma) = \begin{cases} \frac{\phi(\sigma)}{1-\Phi(\sigma^*)} & \text{if } \sigma \geq \sigma^* \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

with $1-\Phi(\sigma^*)$ being the ex-ante probability of successful entry. Subsequently, the average elasticity of substitution of active firms as a function of the threshold σ^* is given by:

$$\bar{\sigma}(\sigma^*) = \int_{\sigma^*}^{\infty} \sigma\psi(\sigma)d\sigma. \tag{11}$$

The last two equations reveal how the shape of the equilibrium distribution of elasticities is tied to the exogenous ex-ante distribution $\phi(\sigma)$ while allowing the range of elasticities $\psi(\sigma)$ and hence the average elasticity of substitution levels in the economy $\bar{\sigma}(\sigma^*)$ to be endogenously determined.

All active firms (other than the cutoff firm that makes a zero profit) earn positive profits, which implies that the average profit $\bar{\pi}$ must be positive. The expectation of future positive profits is in fact the only reason why entrants consider paying the sunk cost of entry. The present value of the average profit flows should then exactly cover this sunk cost. There would be unbounded entry if it exceeds the sunk cost and no firm would want to enter if it is below the sunk cost. From (9), we derive the free entry condition:

$$\frac{1-\Phi(\sigma^*)}{\omega}\bar{\pi} = w_{li}f_e \tag{12}$$

where the average profit reads $\bar{\pi} = \int_{\sigma^*}^{\infty} \pi(\sigma)\psi(\sigma)d\sigma$.

3.1.4 Industry aggregates

The zero cutoff profit condition and the free entry condition jointly determine the cutoff elasticity level σ^*:

$$J(\sigma^*) = \omega f_e/f_p, \qquad J(\sigma^*) \equiv \int_{\sigma^*}^{\infty} \left[\left(\frac{c(\sigma)}{c(\sigma^*)}\right)^{1-\epsilon} - 1\right]\phi(\sigma)d\sigma. \tag{13}$$

Although there is no closed form solution for σ^*, it can be computed numerically given the exogenous distribution $\phi(\sigma)$ and input prices.[13] The following proposition characterizes how the cutoff elasticity of substitution relates to the size of the tax on dirty inputs.

Proposition 1. *All else equal, the cutoff level of elasticity of substitution σ^* is non-decreasing in the tax on the dirty input τ.*

Proof. See Appendix A1. □

A higher tax on the dirty input increases the cost of production $c(\sigma)$, which lowers the profits in (8). According to lemma 1, firms with lower levels of σ that cannot easily switch to the relatively cheaper clean input will experience a larger increase in their operating costs and a larger decrease in profits. For some of these firms (close to the cutoff level of elasticity of substitution), the fall in profits can be substantial enough to make them unable to meet the fixed cost of operation and exit the market, pushing up the survival cutoff level of the elasticity of substitution.

Once σ^* is determined, we can characterize all aggregate variables such as aggregate revenue and profit as well as the mass of active firms in the industry (see Appendix A2 for details). With M denoting the mass of active firms, the aggregate demand for the clean and dirty inputs, respectively, reads:

$$\begin{aligned} G &= \int_{\sigma^*}^{\infty} g(\sigma) M \psi(\sigma) d\sigma = M\bar{g}, \\ B &= \int_{\sigma^*}^{\infty} b(\sigma) M \psi(\sigma) d\sigma = M\bar{b} \end{aligned} \tag{14}$$

where $g(\sigma)$ and $b(\sigma)$ are defined in (6) and $\bar{g}$ and $\bar{b}$ denote averages.

In addition, let M_e denote the mass of potential entrants. In a stationary equilibrium, the additional value from the mass of successful entrants must exactly replace the changing value of incumbents according to the augmented

[13] We note that while σ is non-negative by definition, $J(\sigma^*)$ is technically defined on $(-\infty, \infty)$, which implies that σ^* may also be negative. Restricting the parametric space such that $J(0) > \omega f_e / f_p$ holds, ensures that σ^* is non-negative.

discount factor ω that accounts for the probability of exogenous exit as well as growth:

$$(1 - \Phi(\sigma^*))\, M_e = \omega M. \tag{15}$$

The aggregate labor employed in the industry L_i is distributed between the labor used by the entrants L_e (both successful and unsuccessful) and the labor employed by incumbents L_p, which leads to: $L_i = L_p + L_e = M f_p + M_e f_e$. From (12) and (15), we note that the aggregate industry profit Π_I exactly covers the aggregate entry cost incurred by entrants: $\Pi_i = w_{li} L_e$.

3.2 Energy sector

We now turn to the energy sector that supplies the two energy inputs used by intermediate goods producers.

3.2.1 Energy inputs

Clean and dirty energy inputs are produced competitively and are available to every industrial firm of the economy. Following the tradition of the directed technical change framework (Hémous and Olsen, 2021), the production function for each of the two inputs combines unskilled labor and a unit mass of machines in a constant returns to scale fashion:

$$\begin{aligned} G &= L_g^{1-\alpha_g} \int_0^1 x_{gi}^{\alpha_g} q_{gi}^{1-\alpha_g} di, \\ B &= L_b^{1-\alpha_b} \int_0^1 x_{bi}^{\alpha_b} q_{bi}^{1-\alpha_b} di. \end{aligned} \tag{16}$$

Variable q_{ki}, with $k \in \{g, b\}$, denotes the technology embodied in machine x_{ki} and $\alpha_k \in (0,1)$ is the factor share of machines in sector k. A representative producer of input k maximizes profits by choosing labor L_k and machines x_{ki}, while taking prices (the wage rate and the price of machines) and the level of machine-embodied technology as given. Note that labor is mobile between the industrial sector and the energy sector, which is an important margin for our quantitative analysis we present in the next section. Labor market clearing requires that $L_g + L_b \leq L - L_i$, where L is the fixed exogenous supply of unskilled workers in the economy and L_i is the aggregate labor employed in the industrial sector. The profit maximization problem of a representative input producer is described in detail in Appendix A2.

3.2.2 Machines

There exists a unit mass of machine producers in each energy sector. The machine producers sell their machines to the energy input producers in their specific sectors. A machine x_{ki} costs one unit of the final good to produce. The market for machines is monopolistically competitive, such that the machine producers earn positive profits. In addition, each sector-specific machine producer hires scientists at the market wage for scientists w_{sk} to innovate on the embodied technology of their machines. The government may subsidize part of the wage for scientists in the clean sector to encourage innovation in clean technologies. The evolution of technology for machine producer i in sector k is:

$$q_{kit} = q_{kt-1}\left(1 + \gamma s_{kit}^{\eta}\left(\frac{Q_{t-1}}{q_{kt-1}}\right)^{\xi}\right), \qquad k \in \{g, b\}. \tag{17}$$

Note that time subscript t is introduced to make the state dependence in the evolution of technology explicit: technology in sector k builds on the existing level of technology q_{kt-1}. s_{kit} denotes the number of scientists hired by machine producer i in sector k in period t. Parameter η captures the degree of diminishing returns to scientific research and γ addresses efficiency in innovation. We allow cross-sector spillovers in innovation by the parameter $\xi \in [0, 1]$, following Fried (2018). This is to incorporate the empirical observation that innovation has been taking place in both sectors, rather than only in one sector.[14] Thus, the specification captures the intuition that if sector k is relatively backward, then there are many ideas from the other sector that have

[14] In France, for example, all energy sources (fossil fuels, nuclear, and renewables) show active R&D activities measured by non-zero expenditure since the 2000s (IEA, 2019).

not yet been applied in sector k. This "low-hanging fruit" situation increases the productivity of research in sector k.

Variable q_k denotes the technology level in sector k:

$$q_{kt} = \int_0^1 q_{kit} di. \tag{18}$$

The aggregate technology Q_t defined as the average of the technologies in the two sectors grows at the constant rate of g on a balanced growth path.

Each machine producer chooses the quantity of machines, the machine price, and the number of scientists to maximize her profits. She takes the existing levels of technology as given. Scientist market clearing requires that $S_{gt}+S_{bt} \leq S$ where S is the fixed exogenous supply of scientists in the economy and S_{kt} is the number of scientists in sector k in period t. Appendix A2 discusses the profit maximization problem of machine producers in detail.

3.3 Household

The representative household is inhabited by L workers, S scientists, a unit mass of intermediate goods producers and a unit mass of machine producers in each energy input sector. The relative supplies of workers and scientists are fixed. Additionally, we assume that both workers and scientists are mobile across sectors so that they can switch sectors without incurring adjustment costs (again, low-skilled labor is mobile across economic segments, i.e., the industry and the energy sector, as well as between the two energy sectors). We assume constant relative risk aversion (CRRA) preferences for the household with the utility function $U(C) = C^{(1-\theta)}/(1-\theta)$, where C is household consumption and $1/\theta$ captures the intertemporal elasticity of substitution. The representative household's budget constraint is given by:

$$C = w_{li}L_i + w_{lg}L_g + w_{lb}L_b + w_{sg}S_g + w_{sb}S_b + \Pi_g + \Pi_b + T - S \tag{19}$$

where Π_g and Π_b are aggregate profits earned by machine producers in the clean and dirty energy sector, respectively.[15] T denotes non-distorting lump-sum transfers, which in equilibrium are $T = \tau B$, where τ is the carbon tax on the dirty energy consumption. Finally, S is the budget for subsidies for clean innovation raised by a lump-sum tax on the representative household, which equals $w_{sg} S_g v$ in equilibrium where v denotes the share of the wage subsidized by the government.

The aggregate resource constraint implies the final good can be consumed or used for production of machines:

$$Y = C + \int_0^1 (x_{gi} + x_{bi}) di. \tag{20}$$

3.4 Equilibrium

A *stationary decentralized equilibrium* consists of prices for intermediate goods ($p(\sigma)$), prices for energy inputs (p_g, p_b), prices for machines (p_g^x, p_b^x), wages for low-skilled workers and scientists ($w_{li}, w_{lg}, w_{lb}, w_{sg}, w_{sb}$), the cutoff level of elasticity of substitution (σ^*), a mass of active firms (M), a mass of potential entrants (M_e), allocation of low-skilled workers (L_i, L_g, L_b), allocation of scientists (S_g, S_b), energy inputs choices ($g(\sigma), b(\sigma)$), and machines (x_{gi}, x_{bi}) such that (i) $g(\sigma), b(\sigma)$ and $p(\sigma)$ maximize the intermediate goods producers' profits; (ii) x_{gi}, x_{bi} and L_g, L_b maximize the energy input producers' profits; (iii) p_g^x, p_b^x, S_g, S_b, x_{gi}, x_{bi} maximize the machine producers' profits; (iv) L_i, L_g, L_b and S_g, S_b maximize the representative household's utility; (v) σ^* is given by (13); (vi) M and M_e satisfy (15); (vii) $p(\sigma)$ clear the intermediate goods market; ($viii$) p_g, p_b clear the energy input markets; (ix) p_g^x, p_g^x clear the machine markets; and (x) w_{li}, w_{lg}, w_{lb} and w_{sg}, w_{sb} clear labor markets for low-skilled workers and scientists, respectively.

Although the equilibrium is relatively complex, all equilibrium objects can be written in closed form, given the cutoff level of elasticity of substitution and

[15] Note that profits earned by active intermediate goods producers in the industrial segment of the economy exactly cover the aggregate entry costs incurred by entrants, and therefore do not enter the household's budget constraint as income.

energy input prices, which we compute numerically. We use this computation in the method of moments procedure outlined in the next section.

4 Quantitative analysis

To quantify the interplay between firm heterogeneity, industry dynamics and climate policy, we calibrate our model using micro and macro data between 1995 and 2015 for France. Following the literature, we directly calibrate a group of parameters from the data series. Next, we jointly calibrate the remaining parameters to match moments implied by our model to their empirical counterparts. We describe our data sources and estimation procedures in the next sections.

4.1 Data

We define the industrial segment of our model (the final and intermediate goods production) as manufacturing and obtain key moments relevant for the industry such as the average elasticity of substitution between clean and dirty energy and entrants' share of employment from the same micro data used in Section 2. The micro data comes from two main sources. The first dataset is the EACEI collected by the French National Institute of Statistics and Economic Studies (Insee) that provides plant-level information on energy use and expenditures by fuel. It covers a representative sample of manufacturing plants with at least 20 employees. The second dataset, the FARE, is also administered by Insee and contains information on key firm-level characteristics such as industry, employees, date of creation and cessation, and financial information for the universe of businesses operating in France.[16]

To merge the two datasets, we aggregate plant-level information from the EACEI to the firm-level. Since the EACEI covers only a sample of manufacturing plants (although representative in all covered sectors), we only keep firm-year pairs for which all plants of a firm were surveyed in the EACEI to

[16] FARE replaced Fichier de comptabilité unifié dans SUSE (FICUS) in 2008.

ensure that the aggregation of energy use and expenditure is comprehensive at the firm level. The final dataset covers around 13,000 firms in 19 manufacturing industries for the period between 1995 and 2015.

We aggregate the consumption of different sources of energy to a clean and a dirty bundle for each firm, with the clean bundle including electricity, steam and renewables and the dirty bundle consisting of all other fuels (natural gas, petroleum products, etc.). Fuel composition of each energy bundle is provided in Table OA1. The unit cost of each bundle is constructed by dividing the expenditure measures (that are similarly aggregated to a clean and dirty bundle) by the corresponding consumption measures. The variation in the unit costs of energy across firms is largely driven by different fuel mix within each bundle and quantity discounts in the French context (Marin and Vona, 2021). Table OA2 provides key descriptive statistics by industry.

The energy sector is split into clean and dirty sectors. The clean sector corresponds to the production and distribution of electricity, over 90 percent of which is generated by low-carbon sources (on average 76 percent by nuclear and 14 percent by renewables between 1995 and 2015) (IEA, 2022). The dirty sector corresponds to mining and quarrying in the data that comprises mining of coal and lignite, extraction of crude petroleum and natural gas, mining of metal ores, other mining and quarrying, and mining support service activities. The data sources of sector-level moments are summarized in Table OA3.

4.2 External calibration

One period is assumed to be five years. We set the discount rate to 1.5 percent and the inverse of the intertemporal elasticity of substitution to $\theta = 2$. We take the elasticity of substitution between different intermediate products to be $\epsilon = 2.9$. The labor share in the dirty energy sector $1 - \alpha_b$ is set to 0.26, which corresponds to the average share of personnel costs in total operating costs in mining and quarrying (Eurostat, 2016). We normalize the workforce to unity. During our sample period (1995 - 2015), on average 0.8 percent of workers were engaged in research activities in France (OECD, 2021b). Thus,

Table 1: External parameter values

Parameter	Description	Value
r	Discount rate	0.015
θ	Inverse of the intertemporal elasticity of substitution	2
ϵ	Elasticity of substitution between goods	2.9
α_b	Machine share in dirty energy	0.74
ξ	Cross-sector spillover	0.65
η	Diminishing returns	0.78
L	Number of workers	1
S	Number of scientists	0.008

we set the number of scientists to 0.008.

The parameter ξ determines the strength of the cross-sector spillover in innovation. All else equal, weaker spillover (small ξ) will strengthen the effect of directed technical change with all innovation occurring in one sector. On the other hand, stronger spillover (large ξ) will lead to a stable interior balanced growth path where innovation occurs in both clean and dirty energy sector. In France, the clean energy sector is relatively larger and accounts for a large share of total research expenditures in the energy sector (IEA, 2020), which implies that a small ξ will rapidly lead to a corner solution where innovation only occurs in the clean sector. Thus, we choose a conservative benchmark value of 0.65 for the parameter. The level of diminishing returns to innovation parameter η is set to 0.78 (Fried, 2018). The impact of these two parameters (ξ and η) on our results is explored in the sensitivity analysis in Section OA3 of Online Appendix. Table 1 collates the values of the parameters discussed so far.

Finally, for parameters that normalize the CES production function across intermediate goods producers, we assume $g_0 = b_0 = y_0 = 1$ without loss of generality. p_{g0} and p_{b0} are set to 0.67 and 0.33, respectively, according to the average unit prices of clean and dirty energy among French manufacturing firms based on the EACEI data.[17] These values lead to the distribution pa-

[17] We rescale the actual average prices (0.929 euro per TOE for clean and 0.473 euro per TOE for fossil energy) so that they add up to one. Given $g_0 = b_0 = y_0 = 1$, this ensures

rameter κ_0 and the variable unit cost c_0 of 0.67 and 1, respectively, according to their definitions.

4.3 Method of moments

The remaining parameters $\{\alpha_g, \delta, f_e, f_p, \gamma\}$ as well as the parameters that form the exogenous distribution $\phi(\sigma)$ are jointly calibrated using the quantitative implications of our model. The distribution $\phi(\sigma)$ is assumed to follow the Gamma distribution defined on a positive support $(0, \infty)$ with shape parameter a and scale parameter b. The resulting mean and the variance of the distribution is ab and ab^2, respectively. For the first four parameters of the model and those of $\phi(\sigma)$, we use the method of moments approach that chooses the parameter vector so as to minimize the distance between several key moments implied by our model and the corresponding moments in the data. The approach iteratively searches across sets of parameter values for $\alpha_g, \delta, f_e, f_p, a$ and b until the model's moments are as close as possible to the empirical moments. Additionally, we target the annualized growth rate of GDP per capita of 2 percent on the balanced growth path, which pins down the innovation efficiency parameter γ.

It is important for our model to capture the relationships between energy prices, industry dynamics, and production and innovation in the energy sector. Thus, our central moments are the average elasticity of substitution between clean and dirty energy, the labor cost share in manufacturing, firm entry measured by entrants' share of employment, the relative market size of the clean energy sector compared to the dirty energy sector in terms of employment, and R&D expenditure in green technologies as a share of total research expenditure in the energy sector.

In calibrating the parameters of the exogenous distribution $\phi(\sigma)$, we note that the average elasticity of substitution we observe from the data — the point estimate — is estimated from the sample of surviving firms. However, the

internal consistency of the normalization parameters: $y_0 = \frac{p_{g0}}{\kappa_0} g_0$ and $p_{b0} = \frac{(1-\kappa_0) y_0}{b_0}$ (León-Ledesma et al., 2010).

Table 2: The average elasticity of substitution between clean and dirty energy

	(1) IV	(2) IV + Heckman's two-step
$\log \frac{p_{git}}{p_{bit}}$	2.909 (0.146)	2.864 (0.146)
Observations	54,309	54,309

Note: Estimated coefficients and standard errors in parenthesis. Both regressions include industry, region, and year fixed effects. Appendix OA2 explains in detail the Heckman's two step procedure in column (2). Standard errors are clustered at the firm level.

underlying exogenous distribution should also capture firms that exit or cannot enter the market due to their low elasticities and its mean (ab) is likely to be lower than the mean estimated from surviving firms only. To operationalize the idea, we note from Figure 2 that dirtier firms tend to display lower levels of flexibility in energy input choices. Based on this observation, we make the assumption that the dependent variable $\ln(b_{jt}/g_{jt})$ in equation (1) is truncated from above: firms that are very dirty and thus likely to have low elasticities of substitution are not observed in the data. This assumption allows us to approach the task of uncovering the mean of the underlying distribution as a sample selection problem (also known as incidental truncation) and apply the popular Heckman's two-step procedure. Table 2 reports estimates from this exercise. Column (1) reports the IV estimate of the elasticity of substitution that does not account for the selection bias. In column (2), we apply the Heckman's two-step procedure and find a slightly smaller estimate of 2.864 as expected from our model.[18] Appendix OA2 explains the estimation process in detail. We use the estimate in column (1) as our target moment for the average elasticity of substitution among active firms and the one in column (2) as our target moment for the mean of the underlying distribution $\phi(\sigma)$.

[18]The IMR term has a positive coefficient (SD) of 2.757 (1.032) which is consistent with the revealed upward bias in the estimate in column (1) and statistically significant at 1 percent level.

Table 3: Internal parameter values

Parameter	Description	Value
a	Shape parameter of $\phi(\sigma)$	2.352
b	Scale parameter of $\phi(\sigma)$	1.218
α_g	Machine share in clean energy	0.875
f_e	Fixed cost of entry	0.389
f_p	Fixed cost of operation	0.019
δ	Exogenous rate of destruction	0.086
γ	Scientist efficiency	7.646

Table 3 summarizes our parameter estimates from the method of moment procedure. The shape parameter a and the scale parameter b of $\phi(\sigma)$ are calibrated to 2.35 and 1.22, respectively. The resulting standard deviation of the distribution is 1.87. The calibrated α_g is 0.88, which is consistent with green energy technologies such as nuclear and solar, being highly capital intensive. The model predicts a sizable fixed-cost advantage for operating firms: their fixed cost of operation is around 5 percent of the entrants' fixed cost. The exogenous rate of destruction is calibrated to approximately 1.7 percent per year (8.6 percent over a five-year period). The scientist efficiency parameter $\gamma = 7.65$ matches the targeted 2 percent long-run annual growth rate.

While all parameters are calibrated jointly by the targeted moments, the two average elasticities of substitution (one that relates to all firms and the other relating to active firms only) and the entrant share primarily help pin down the parameters of the exogenous distribution. Given the target mean of the distribution, the difference between the two substitution elasticities and the share of entrants determine the variance of the distribution through the cutoff level of elasticity of substitution and the probability of successful entry. These moments also discipline other internal parameters associated with the industry, f_e, f_p, and δ. For instance, a higher rate of exogenous destruction δ requires the probability of successful entry to be also higher, which lowers the cutoff

Table 4: Targeted moments of model and data

Moments	Model	Data
Average elasticity of substitution, active firms ($\bar{\sigma}$)	2.909	2.909
Average elasticity of substitution, all firms (ab)	2.864	2.864
Labor cost share in manufacturing	0.231	0.231
Entrant share	0.063	0.063
Share of R&D in green	0.869	0.869
Market size of clean relative to dirty	5.612	5.612

level of elasticity of substitution and consequently its average.[19] The market size of the clean relative to dirty energy sector and the research expenditure moment pin down the labor share in clean energy parameter $1 - \alpha_g$. All else equal, a higher market size of clean relative to dirty energy is associated with a higher labor share in clean energy. It also raises the share of R&D in clean technologies by raising the profitability in innovation in that sector.

4.4 Goodness of fit

Table 4 reports the values of the moments we target and the predicted values from our model, which are very closely matched. Table 5 shows the key equilibrium objects in our baseline economy. These objects will be used for comparison in our policy experiment in the next section. The table shows that the cutoff level of elasticity of substitution (σ^*) below which firms are unable to survive in the market is 0.37. This results in the average substitution elasticity among active firms ($\bar{\sigma}$) of 2.91 which closely matches the empirical target. We note that the clean energy sector is more productive than the dirty energy sector in the baseline economy ($q_g/q_f = 9.88$), consistent with a large share of scientists working in the clean sector ($S_g/S = 0.87$). For ease of comparison in the next section, we normalize baseline welfare to 100.

We also assess the performance of our model by comparing its implications

[19] This is because in a stationary equilibrium, the additional value from successful entrants must exactly replace the change in the value of incumbents due to the exogenous destruction and growth (see (15)).

Table 5: Baseline economy

σ^*	$\bar{\sigma}$	M	L_g/L_f	q_g/q_f	S_g/S	Tax	Wel
0.37	2.91	17.88	5.61	9.88	0.87	-	100.00

for several non-targeted moments, namely, the relative size of the manufacturing industry, the labor cost share in the clean energy sector as well as the clean-to-dirty capital ratio in the energy sector.[20] Table OA7 shows that the values of these non-targeted moments are comparable across the model and the data, suggesting that the model's fit is reasonably strong. The model predicts the relative size of manufacturing in the entire industry fairly well (0.80 in the model and 0.91 in the data). The labor cost share in the clean energy sector is also comparable but somewhat lower in the model (0.13 as opposed to 0.19 in the data). The model performs well in replicating the clean energy sector being much larger than the dirty energy sector and predicts the clean-to-dirty capital ratio to be 16.27, although it struggles to match the exact ratio of 24.06 in the data.

5 Results

5.1 Carbon tax and industry dynamics

We compute a set of counterfactual stationary equilibria to understand and quantify the effects of dynamic industry response to climate policy. We consider two economies on the same baseline balanced growth path but in one model, which we refer to as the endogenous model, the cutoff level of elasticity of substitution changes in response to climate policy. This affects firms' entry

[20]The relative size of manufacturing is measured by the share of employment in manufacturing in the industry that comprises manufacturing, mining and quarrying, electricity, gas, steam and air conditioning supply, and water supply, using data from Eurostat (Eurostat, 2016). The data on labor cost shares in the clean energy sector and the amount of capital in the clean and dirty energy sector are also available from the same database. The clean-to-dirty capital ratio is measured by the ratio of gross investment in tangible goods in the clean energy sector to the same measure in the dirty energy sector.

and exit decisions and the average substitution elasticity between clean and dirty energy among active firms as described in our theoretical model. In the other model, which we refer to as the exogenous model, this channel is shut off. Thus, the cutoff as well as the average elasticity of substitution is fixed at the baseline level.

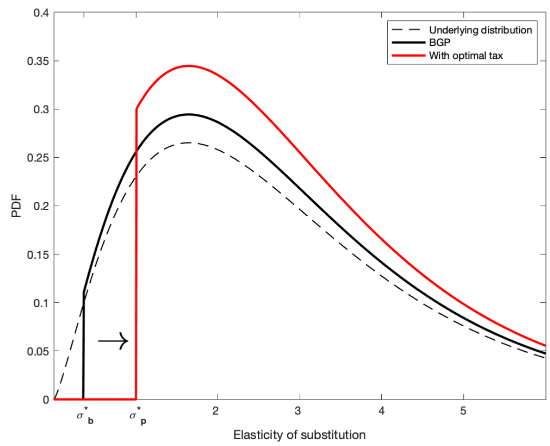
Our goal is to compare the size of the carbon tax that achieves carbon neutrality in the new equilibrium across these two economies in line with France's long-term climate policy of reaching net zero emissions. It translates to a 76.5 percent reduction in emissions from the baseline growth path, which we take to be 2015 in our model.[21]

We find that the carbon tax required to achieve the policy goal is 223 and 114 EUR per tCO_2 (in 2015 EUR) in the exogenous and endogenous model, respectively.[22] The optimal tax in the endogenous model is 48 percent lower compared to the one in the exogenous model. The difference is driven by the change in the equilibrium distribution of the elasticity of substitution induced by the selection channel. In the endogenous model, firms with limited energy substitutability that cannot easily switch to the relatively cheaper clean energy experience a larger increase in operating costs, hence a larger decrease in profits. Some of these firms (close to the cutoff elasticity of substitution) are forced to exit the market when they can no longer cover the fixed cost of operation. Figure 3 graphically shows the impact of the carbon tax on the equilibrium distribution of firms. In response to the tax, the cutoff level of elasticity of substitution σ^* is pushed to the right and firms with degrees of energy substitutability between σ_b^* and σ_p^* are forced to exit the market in the new equilibrium. Consequently, the average elasticity of substitution among

[21] Conforming to the European Green Deal in the European Union approved in 2020, France aims to achieve carbon neutrality by 2050 which we consider sufficiently long-run, corresponding to our new counterfactual equilibria. Achieving carbon neutrality entails reducing emissions at least by 80 percent compared to 1990 level in addition to investments in carbon sinks and the utilization of carbon capture and storage (OECD, 2021c). Given emissions reduced by 2015, this long-term goal implies a further 76.5 percent reduction in emissions from the 2015 level (Eurostat, 2022).

[22] For reference, the explicit carbon price in 2021 was 29 EUR per tCO_2 and the fuel excise taxes (i.e., implicit carbon prices) amounted to 64 EUR per tCO_2 in France (OECD, 2021a).

Figure 3: The impact of the optimal carbon tax on the distribution of the substitution elasticity among active firms



active firms is higher in the long run.

The sizeable impact of the elasticity of substitution on the optimal carbon tax is similar in spirit to the findings of Acemoglu et al. (2012). The higher substitution elasticity in the industry where demand for energy inputs are determined increases the effectiveness of the carbon tax by shifting the demand for clean energy by a larger margin in response to the same size of the tax. This in turn lowers the required tax that achieves the same emissions reduction target in the endogenous model. In addition, the larger shifts in demand lead to stronger incentives to innovate in the clean sector where the demand is now higher. Over time, more innovation in the clean sector reduces the relative price of clean energy, which induces further shifts in demand towards clean energy among industrial firms.

The higher effectiveness of the carbon tax in the endogenous model can also be demonstrated by applying the optimal tax from the exogenous model (223 EUR per tCO_2) to the endogenous model. We find that dynamic industry response increases the percent reduction in emissions by close to 11 percentage points (88 percent reduction compared to the 77 percent reduction in the exogenous model). This exercise illustrates that failing to account for dynamic industry response to climate policy and the subsequent change in the average

Table 6: The impact of the optimal carbon tax

σ^*	$\bar{\sigma}$	M	L_g/L_f	q_g/q_f	S_g/S	Tax	Wel
Panel A: Baseline							
0.37	2.91	17.88	5.61	9.88	0.87	-	100.00
Panel B: Counterfactual, endogenous							
1.01	3.19	17.28	17.54	39.23	0.95	114	98.03
Panel C: Counterfactual, exogenous							
0.37	2.91	18.51	16.30	35.91	0.95	223	88.17

energy substitutability among firms can lead to a substantial overestimation of the optimal carbon tax to meet a policy goal.

Table 6 provides more details on the mechanisms behind the effect of endogenous industry dynamics. Panel A reproduces the equilibrium objects on the balanced growth path without policy in Table 5 for ease of comparison. Panel B and C show how the same objects change in the counterfactual equilibrium with the optimal carbon tax in the endogenous and exogenous model, respectively. In the endogenous model, the cutoff level of elasticity of substitution σ^* increases from 0.37 in the baseline to 1.01 in the endogenous model (174 percent increase), pushing up the average elasticity of substitution among active firms $\bar{\sigma}$ by approximately 10 percent. As a result of the higher survival cutoff, the mass of active firms M falls by 3 percent in the endogenous model which illustrates the strength of selection effects arising from industry dynamics.

The equilibrium objects relevant for the energy sector show that the production and innovation response is generally stronger in the endogenous model compared to the exogenous model. For example, the relative market size (measured in labor) of clean energy (L_g/L_f) increases by 22 percentage points more in the endogenous model than in the exogenous model. The difference is partly driven by the low-skilled labor (previously employed to cover the fixed cost of operation) released from the manufacturing industry with the exit of inflexible firms, which is now going into the clean energy sector where the demand is higher. This finding is consistent with empirical evidence on the reallocation of labor (Walker, 2011) or creation of green jobs in response to environmental regulation (Vona et al., 2018; Popp et al., 2020).

Furthermore, the economy in the endogenous model features the relative technology in the clean sector (q_g/q_f) and the share of scientists working in clean technologies (S_g/S) that are 33 and 0.4 percentage points higher, respectively, compared to the economy in the exogenous model. This stronger innovation response in the endogenous model is induced by larger demand shifts toward clean energy in the industry facilitated by the higher average elasticity of substitution in the new stationary equilibrium.

Finally, we study how the carbon tax affects welfare in the endogenous and exogenous model.[23] We measure the welfare effects using the consumption equivalent variation (CEV). The CEV is the percent increase in consumption, C, that a household would need in the counterfactual equilibrium with climate policy to ensure the same discounted utility between the new equilibrium and the baseline equilibrium.

We find that industry dynamics reduces welfare costs of climate policy by close to 10 percentage points in reaching the same policy objective. The carbon tax affects welfare in two different channels. First, it generates distortionary costs, which increase in the size of the tax and reduce welfare. The optimal tax is smaller in the endogenous model due to industry dynamics, lowering distortionary costs and contributing to welfare. Second, the tax shifts innovation to the clean sector which is more productive than the dirty sector in the French context when the tax is implemented. Shifting energy production to the more productive clean sector therefore increases the aggregate growth rate in the new stationary equilibrium. The shifts are larger in the endogenous model compared to the exogenous model, which leads to a higher aggregate growth rate and raises welfare in the new equilibrium.

[23] We do not model environmental damage that negatively affects welfare as our goal is to quantify the channel of industry dynamics. Accounting for the costs of environmental damage will improve welfare in our analysis of climate policy.

5.2 Subsidy to clean research and industry dynamics

Next, we compare the implications of different policy instruments, namely, a carbon tax and a subsidy to clean innovation, in the presence of endogenous industry dynamics. To do so, we compute the size of a research subsidy required to achieve the same policy goal as before in the counterfactual equilibrium and compare the implications of key equilibrium objects associated with the two policy instruments.

The optimal research subsidy that achieves carbon neutrality in the new equilibrium is 65 percent in our endogenous model. In other words, 65 percent of the scientists' wage working in the clean energy sector is to be subsidized (financed through a lump-sum tax on the representative household) if the economy were to achieve the policy objective with subsidies instead of a tax. Given that all policy instruments operate through price incentives that shift demand toward clean energy, the relatively large optimal subsidy is primarily due to subsidies providing only indirect price incentives by advancing clean technologies which lower the price of the clean input over time, rather than directly affecting final energy prices as in the case of a carbon tax (Fischer and Newell, 2008). Another factor that determines the size of required research subsidies in our context is cross-sector spillovers in innovation. This makes the effect of directed technical change weaker as research directed to clean technologies also benefits dirty technologies through spillovers, dampening the price effect induced by the increasing productivity in the clean relative to dirty sector.[24]

Table 7 reports equilibrium objects associated with the two different policy instruments that achieve the same emissions reduction target in the long run. The baseline and tax outcomes in Panel A and B are identical to the values in Table 6, but reproduced for convenience. Panel C shows how the objects change when subsidies are implemented instead.

[24] Similarly, we observe in the sensitivity analysis that a weaker cross-sector spillover (smaller ξ) leads to lower optimal taxes by strengthening the innovation response in the energy sector. However, the effect of industry dynamics on the size of the carbon tax required to achieve carbon neutrality is similar to the baseline calibration.

Table 7: Comparison of tax and subsidy

σ^*	$\bar{\sigma}$	M	L_g/L_f	q_g/q_f	S_g/S	Policy	Wel
Panel A: Baseline							
0.37	2.91	17.88	5.61	9.88	0.87	-	100.00
Panel B: Counterfactual with a carbon tax							
1.01	3.19	17.28	17.54	39.23	0.95	114	98.03
Panel C: Counterfactual with clean research subsidies							
1.02	3.20	17.35	14.00	105.18	0.98	0.65	101.59

In Panel B and C, we observe that the two policies lead to changes of similar magnitudes in the equilibrium objects relevant for the industry $(\sigma_*, \bar{\sigma}, M)$. The average elasticity of substitution goes up by 10 percent in response to the tax and the research subsidy. The difference is more pronounced when we look at equilibrium objects relevant for the energy sector. We find that, without direct price incentives, the relative productivity of clean technologies in the economy with research subsidies has to grow by more than 3 times as much in order to achieve the same policy target, compared to the economy with the carbon tax.

Regarding welfare effects of these different policy instruments, we find that implementing research subsidies leads to higher welfare gains compared to using the tax. This is because subsidies lead to larger shifts in production and innovation from dirty to clean energy, which was more advanced in the baseline economy. Consequently, the aggregate growth rate in the new stationary equilibrium with research subsidies is higher (2.07 percent) than the growth rate in the new equilibrium with the carbon tax (2.05 percent), contributing to welfare gains.

The role of endogenous industry dynamics remains important with research subsidies. Turning back to the exogenous model, the optimal size of the research subsidy that meets the policy goal is 86 percent as opposed to 65 percent in the endogenous model. Without the endogenous exit of inflexible firms and the average elasticity of substitution among active firms adjusting to the policy, achieving carbon neutrality requires a much larger shift to clean innovation through higher research subsidies (Table OA8). Despite the higher aggregate growth rate in the new equilibrium (2.08 percent) due to a larger increase in the relative productivity of clean technologies needed to achieve the policy objective, welfare is lower in the exogenous model (91.27) compared to the endogenous model, as distortionary costs become very high as subsidies grow larger.

6 Conclusion

In this paper we build a microfounded general equilibrium model of directed technical change to study the relationship between climate policy, industry dynamics, and the average elasticity of substitution between clean and dirty energy. Our model reproduces the empirical observation that firms are on average becoming more capable of substituting clean for dirty energy over time through the force of industry dynamics and firm heterogeneity.

We use the model to examine the effects of endogenous industry dynamics on the effectiveness of different policy instruments. We find that accounting for dynamic industry response to climate policy is crucial in the analyses of optimal climate policy: taking into account industry dynamics leads to a 48 percent lower optimal carbon tax that achieves carbon neutrality in the new stationary equilibrium. Our model also reveals that climate policy can free up resources (labor) from the least flexible firms that exit the market as a consequence of climate policy, which is then reallocated to the clean energy sector. Further, we find that larger demand shifts to clean energy due to a higher average elasticity of substitution among active firms lead to stronger innovation response in the energy sector with a larger increase in clean innovation.

Several follow-up research questions are left for future research. First, our analysis can be made richer by allowing the firms to invest in improving their capability to substitute clean for dirty energy over time. Moreover, exploring the determinants of the elasticity of substitution at the firm level would also be informative. In our model, firms randomly draw their level of elasticity of substitution from an exogenous distribution, as in Melitz (2003) where firms randomly draw their productivity levels. However, some knowledge in the determinants of firms' capability of energy substitution at the firm level will be useful in providing firms with the right incentives that could lead to an optimal level of substitution capability.

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Appendix

A1 Proofs

Proof of Lemma 1: The proof follows La Grandville et al. (2017) (p.111). One can show that

$$\text{sign}\left\{\frac{\partial \log c(\sigma)}{\partial \sigma}\right\} = \text{sign}\Bigg\{ H\left(\kappa_0\left(\frac{p_g}{p_{g0}}\right)^{1-\sigma} + (1-\kappa_0)\left(\frac{p_b+\tau}{p_{b0}}\right)^{1-\sigma}\right)\kappa_0 H\left(\left(\frac{p_g}{p_{g0}}\right)^{1-\sigma}\right) - (1-\kappa_0)H\left(\left(\frac{p_b+\tau}{p_{b0}}\right)^{1-\sigma}\right)\Bigg\},$$

with $H(z) \equiv z \log z$ being a convex function. The negative sign of $\frac{\partial \log c(\sigma)}{\partial \sigma}$ follows from the definition of convexity. $\square$

Proof of Proposition 1: First note that $J(.)$ is monotonically decreasing with $\frac{dJ(\sigma^*)}{d\sigma^*} < 0$ and $\lim_{\sigma^* \to \infty} \frac{dJ(\sigma^*)}{d\sigma^*} = 0$, given $\epsilon > 1$ and lemma 1. Moreover, $\frac{dJ(\sigma^*)}{d\tau} > 0$ for $\epsilon > 1$ since $\frac{dc(\sigma)}{d\tau} < \frac{dc(\sigma^*)}{d\tau}$ for $\sigma > \sigma^*$ according to lemma 1. Then, $\frac{d\sigma^*}{d\tau} \geq 0$ follows from (13) and the monotonicity of $J(.)$. $\square$

A2 Characterization of the equilibrium

Industry aggregates Once σ^* is determined by (13), we can characterize the distribution of all firm performance measures such as revenue and profit. Let M denote the mass of active firms in the industrial segment of the economy. The aggregate revenue, profit and market value are, respectively:

$$R_i = Y = \left(\int_{\sigma^*}^{\infty} y(\sigma)^{\frac{\epsilon-1}{\epsilon}} M\psi(\sigma)d\sigma\right)^{\frac{\epsilon}{\epsilon-1}} = M^{\frac{\epsilon}{\epsilon-1}}\bar{y}, \tag{A1}$$

$$\Pi_i = \int_{\sigma^*}^{\infty} \pi(\sigma)M\psi(\sigma)d\sigma = M\bar{\pi}, \tag{A2}$$

$$V_i = M\bar{v}, \tag{A3}$$

with average profit $\bar{\pi}$ and firm market value $\bar{v}$ defined before. In equation (A1), $\bar{y} \equiv \left(\int_{\sigma^*}^{\infty} y(\sigma)^{\frac{\epsilon-1}{\epsilon}} \psi(\sigma) d\sigma\right)^{\frac{\epsilon}{\epsilon-1}}$ is the average firm output and $M^{\frac{\epsilon}{\epsilon-1}}$ measures gains from specialisation in the use of intermediates, a common feature in the endogenous growth literature (Grossman and Helpman, 1991). The aggregate price index P (set to unity) is given by:

$$P \equiv 1 = \left(\int_{\sigma^*}^{\infty} p(\sigma)^{1-\epsilon} M \psi(\sigma) d\sigma\right)^{\frac{1}{1-\epsilon}} = M^{\frac{1}{1-\epsilon}} \bar{p} \implies \bar{p} = M^{\frac{1}{\epsilon-1}}, \tag{A4}$$

where $\bar{p} = \left(\int_{\sigma^*}^{\infty} p(\sigma)^{1-\epsilon} \psi(\sigma) d\sigma\right)^{\frac{1}{1-\epsilon}}$. In turn, with (4), (A4), the aggregate variable cost of production is:

$$K_i = \int_{\sigma^*}^{\infty} c(\sigma) y(\sigma) M \psi(\sigma) d\sigma = \frac{\epsilon - 1}{\epsilon} Y. \tag{A5}$$

The industry's balance equates profits with revenues minus variable and overhead costs:

$$\Pi_i = Y - K_i - w_{li} L_p. \tag{A6}$$

Also from (12) and (15), we derive $\Pi_i = w_{li} L_e$. This implies that the aggregate profits from active firms in (A6) exactly cover the aggregate entry costs incurred by entrants. Combining the two expressions, we derive the total unskilled labor employed in the industrial segment of the economy:

$$L_i = \frac{Y/\epsilon}{w_{li}}. \tag{A7}$$

From (8), (A6), (A7), and (12), the mass of active industrial firms is written as:

$$M = \frac{L_i}{\frac{\omega}{1-\Phi(\sigma^*)} f_e + f_p}. \tag{A8}$$

Input producers' optimization problem The energy input producer chooses labor and machines to maximize profits taking prices as given:

$$\max_{L_{ki},x_{ki}} p_k L_{ki}^{1-\alpha_k} \int_0^1 x_{ki}^{\alpha_k} q_{ki}^{1-\alpha_k} di - w_{lk} L_{ki} - \int_0^1 p_{ki}^x x_{ki} di. \tag{A9}$$

where p_k is the market price of energy input k and p_{ki}^x is the price of machine i in sector $k \in \{g, b\}$. The demand for machines is then:

$$x_{ki} = \left(\alpha_k \frac{p_k}{p_k^x}\right)^{\frac{1}{1-\alpha_k}} L_k q_{ki} \tag{A10}$$

where $1/(1-\alpha_k)$ captures the price elasticity of demand for machines. This implies that the equilibrium production level of each energy input is written as:

$$\begin{aligned} G &= \left(\alpha_g \frac{p_g}{p_g^x}\right)^{\frac{\alpha_g}{1-\alpha_g}} L_g q_g, \\ B &= \left(\alpha_b \frac{p_b}{p_b^x}\right)^{\frac{\alpha_b}{1-\alpha_b}} L_b q_b. \end{aligned} \tag{A11}$$

Finally, the inverse demand function for low-skilled labor reads:

$$w_{lk} = (1-\alpha_k)\left(\frac{\alpha_k}{p_k^x}\right)^{\frac{\alpha_k}{1-\alpha_k}} (p_k)^{\frac{1}{1-\alpha_k}} q_k, \qquad k \in \{g, b\}, \tag{A12}$$

The two expressions above, (A11) and (A12), suggest that on the balanced growth path (BGP), G and B as well as the wage rate w_{lk} will grow with the aggregate technology Q (since q_g and q_b, and subsequently Q, grow at the same constant rate g on BGP). Note also the wage for low-skilled labor w_{lk} is the same as the wage in the industrial sector of the economy in equilibrium.

Machine producers' optimization problem The machine producers produce machines to sell to the energy input producers. As mentioned in the main text, each machine costs one unit of the final good to produce. Each machine producer chooses price, quantity of machines, and the number of scientists to maximize profits. The optimization problem is given by:

$$\max_{p_{ki}^x, x_{ki}, s_{ki}} p_{ki}^x x_{ki} - x_{ki} - w_{sk} s_{ki} \tag{A13}$$

which is subject to the evolution of technology (17) and the demand for machines (A10). The optimization of the machine producer in the clean energy sector yields:

$$w_{sg} = \eta\gamma\alpha_g(1-\alpha_g)s_{git}^{\eta-1}\left(\frac{Q_{t-1}}{q_{gt-1}}\right)^{\xi}\left(\frac{q_{gt}}{q_{git}}\right)^{\alpha_g}\frac{q_{gt-1}}{q_{gt}}p_g G, \tag{A14}$$

$$x_{gi} = \left(\alpha_g^2 p_g\right)^{\frac{1}{1-\alpha_g}} L_g q_{gi}, \tag{A15}$$

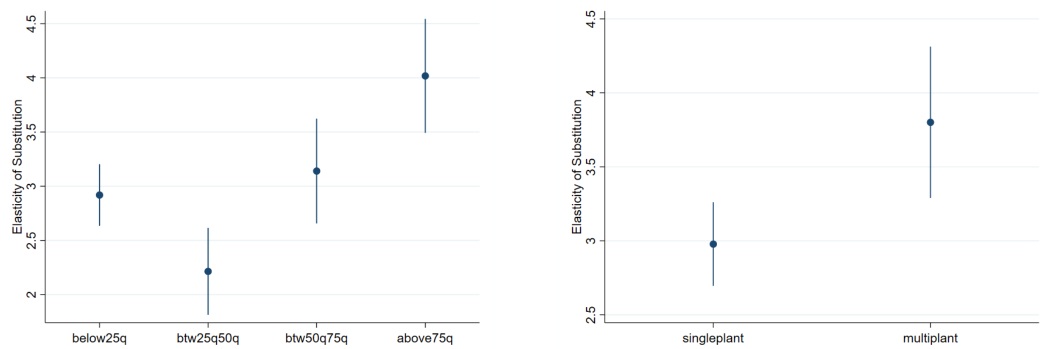
where for (A14) we employed (A11). The optimization problem of the machine producer in the dirty sector is similar. With the usual assumption of symmetry across firms, each firm in sector $k \in \{g, b\}$ has the same level of technology $q_{ki} = q_k$ (such that in (A14) $q_k/q_{ki} = 1$), sales $x_{ki} = X_k$, profits $\pi_{ki} = \Pi_k$, and scientific labor $s_{ki} = S_k$.

Equation (A14) sets the marginal cost of scientific labor equal to its marginal benefit in innovation. Note that in equilibrium, $w_{sg} = w_{sb}$ holds, implying a no-arbitrage condition for active research in both sectors. Equation (A15) combined with the inverse demand function for machines gives $p_k^x = 1/\alpha_k$.

Online Appendix (not for publication)

OA1 Additional tables and figures

Figure OA1: Heterogeneity in the elasticity of substitution between clean and dirty energy: Along other dimensions



Note: The left panel plots elasticity estimates by productivity quantiles (higher quantiles include more productive firms). The right panel plots elasticity estimates by multi-plant status. Each estimate comes from a separate regression (1) by sub-group divided by either productivity levels or multi-plant status.

Table OA1: Fuel composition in clean and dirty energy aggregate

Clean energy aggregate	Mean	SD
Electricity	.996	.047
Steam	.004	.047

Dirty energy aggregate	Mean	SD
Natural gas	.627	.457
Heavy fuel oil	.213	.372
Butane/propane	.121	.295
Heating oil	.025	.142
Other gas	.007	.080
Coke	.005	.062
Coal	.002	.040
Petroleum coke	.000	.019

Table OA2: Descriptive statistics: EACEI and FARE

	Rates of Growth						
	G/B	P_g/P_d	G/E	B/E	P_g	P_d	Rev/E
Industry	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Steel	-0.017	-0.035	0.013	-0.008	-0.000	0.032	0.083
Metals	-0.017	-0.029	0.012	-0.010	0.001	0.030	0.022
Non-metalic	-0.038	-0.027	-0.017	0.008	0.015	0.045	0.105
Plasters, lime, cement	0.072	-0.038	0.050	-0.033	0.009	0.050	0.079
Ceramic	0.051	-0.031	0.022	-0.014	0.003	0.035	0.048
Glass	0.046	-0.031	0.018	-0.019	-0.000	0.032	0.041
Fertilizer	0.005	-0.039	0.003	-0.002	-0.002	0.037	0.062
Other minerals	0.115	-0.029	0.028	-0.020	0.008	0.040	0.014
Plastic	-0.113	-0.034	0.006	-0.009	-0.002	0.034	-0.018
Pharmaceutical	0.012	-0.028	0.011	-0.010	0.002	0.032	-0.015
Steel processing	0.024	-0.026	0.010	-0.010	-0.000	0.027	0.030
Machinery	0.038	-0.029	0.012	-0.012	-0.004	0.029	0.020
Electronics	0.037	-0.028	0.005	-0.006	0.000	0.032	0.019
Transport equipment	0.045	-0.028	0.010	-0.009	-0.003	0.026	0.027
Shipbuilding	0.062	-0.036	0.013	-0.015	-0.010	0.029	0.032
Textile	0.018	-0.030	0.009	-0.007	0.002	0.040	0.033
Paper	0.046	-0.030	0.011	-0.010	-0.001	0.029	0.004
Rubber products	-0.102	-0.030	0.004	-0.005	0.000	0.032	0.024
Plastic products	0.067	-0.027	0.002	-0.004	0.002	0.030	0.031

Notes: Calculated for 1995-2015. E in column (3) denotes total energy consumption (in kTOE). Rev/E in column (7) refers to energy efficiency measured by output per unit energy consumption. Other notations are equivalent to those in the main text.

Table OA3: Data sources for macro moments

Description	Source
Share of personnel costs in total operating costs	Eurostat
Employment in manufacturing, clean and energy	Eurostat
Gross investment in tangible goods in clean and dirty sector	Eurostat
Scientists per 1000 workers	OECD
Fuel specific R&D expenditure	IEA

Note: All the data from Eurostat come from Annual detailed enterprise statistics for industry (SBS_NA_IND_R2). Labor cost share and investment in the clean and dirty sector are available for 2009-2015. Employment in manufacturing and energy sector is available for 2008-2020. Scientists per 1000 workers and fuel specific R&D expenditure is available for 1995-2015.

OA2 Uncovering the mean of the exogenous distribution $\phi(\sigma)$

In our model, firms draw their substitution elasticity parameter σ from a common distribution $\phi(\sigma)$ with a positive support $(0, \infty)$. We assume $\phi(\sigma)$ follows the Gamma distribution which is also defined on a positive support $(0, \infty)$ and has two parameters: shape parameter a and scale parameter b. Since firms with a bad draw of σ immediately exit the market without producing, they are not observed in the data. Thus, the average elasticity of substitution estimated from the sample of active firms is likely to be larger than the average substitution elasticity of the underlying distribution.

To operationalize the idea, we note from Figure 2 that dirtier firms tend to display lower levels of substitution elasticities. Based on this observation, we make the assumption that the dependent variable $\ln(b_{jt}/g_{jt})$ in equation (1) is truncated from above: firms that are very dirty and thus likely to have low elasticities of substitution are not observed in the data. Formulating the problem at hand as a sample selection problem (also known as incidental truncation) allows us to apply the popular Heckman's two-step model to correct for the selection bias and recover the average elasticity of the underlying distribution.

The first step is to study selection into the sample by estimating survival probabilities. To begin, we identify exiting firms as those exist in a given year but not in the following year within our sample period. Using this variation, we estimate survival probabilities by Probit by using age and age square as instruments for selection that are likely to affect survival but do not enter (1), our second-stage equation. Following Wooldridge (2010), we also include the instruments $\tilde{p}_{gjt}$ and $\tilde{b}_{gjt}$ of the second stage equation in the selection equation.[25]

Table OA4 reports the Probit estimation of survival probabilities. Nor surprisingly, age has a positive coefficient, implying that older, established firms are more likely to survive in the next period. Age square shows a negative coefficient, pointing to a nonlinear relationship between age and survival probabilities. Together, they are also strongly jointly significant with $p-value = 0.001$. We construct the Inverse Mill's Ratio (IMR) term from this estimation. In the second step, we estimate equation (1) using the same instruments developed in Section 2 with the IMR term as an additional control. Importantly, we treat firms that would exit in the next period as having exited in the current period already and hence treat their information on energy consumption and prices in that year as missing (or truncated) and estimate the second step only on the set of firm-year observations that continue to survive in the next period.

The average elasticities of substitution with and without correcting for the selection bias are presented in Table 2. As expected, the estimate in column (2) from the Heckman's two-step procedure is slightly smaller than the estimate in column (1) that does not account for the selection bias. The IMR term has a positive coefficient (SD) of 2.757 (1.032) which is consistent with the revealed upward bias in the estimate in column (1) and statistically significant at 1 percent level. We use these two estimates as target moments for the average elasticity of substitution among active firms and all firms in the method of moment procedure.

[25] See Section 19.6.2 for details.

Table OA4: Probit estimation of survival probabilities

Variables	Estimate
Age	0.008
	(0.002)
Age square	-0.000
	(0.000)
$\tilde{p}_{gjt}$	0.068
	(0.047)
$\tilde{p}_{bjt}$	0.023
	(0.046)
Number of observations	55,596

Note: Estimated coefficients and standard errors in parenthesis. The regression includes industry, region, and year fixed effects. Standard errors are clustered at the firm level.

OA3 Sensitivity analysis and nontargeted moments

We examine the robustness of our results to the parameters ξ and η that are neither internally calibrated by method of moments nor directly come from the data series. We try values of ξ and η that are 15 percent smaller and larger than their baseline values. A smaller (larger) ξ implies weaker (stronger) cross-sector spillovers in innovation, while a smaller (larger) η implies returns to research diminish more quickly (slowly). For each set of alternative parameterization, we recalibrate the model to match model moments with their empirical targets as close as possible.

As expected, stronger cross-sector spillovers (higher ξ) weaken the innovation response in the energy sector as innovation in the clean sector also benefits the dirty energy sector, which increases the optimal carbon tax required to achieve carbon neutrality (Table OA5). This lowers the impact of industry dynamics: the difference in the optimal tax between the endogenous and exogenous model is 42 percent (compared to 48 percent in the baseline). In contrast, weaker cross-sector spillovers (lower ξ) strengthens the innovation response, lowering the optimal tax in the endogenous model. The stronger

Table OA5: Sensitivity analysis 1: cross-sector spillover

σ^*	$\bar{\sigma}$	M	L_g/L_f	q_g/q_f	S_g/S	Tax	Wel
Panel A: Baseline with stronger cross-sector spillover (higher ξ)							
0.40	2.91	19.75	5.61	7.38	0.87	-	100.00
Panel A.1: Counterfactual, endogenous							
1.00	3.15	18.93	17.72	24.73	0.95	131	92.69
Panel A.2: Counterfactual, exogenous							
0.40	2.91	20.14	16.57	23.04	0.95	228	84.42
Panel B: Baseline with weaker cross-sector spillover (lower ξ)							
0.25	2.91	15.65	5.61	14.29	0.87	-	100.00
Panel B.1: Counterfactual, endogenous							
0.96	3.29	15.06	17.08	69.72	0.95	81	101.83
Panel B.2: Counterfactual, exogenous							
0.25	2.91	16.43	15.70	61.83	0.95	212	89.76

innovation response adds to the force of industry dynamics, increasing the impact of industry dynamics: the carbon tax in the endogenous model is 61 percent lower than the tax in the exogenous model.

For a similar reason, the tax is also slightly lower when returns to research diminish more slowly with a higher η due to stronger innovation response as shown in Table OA6. The stronger innovation response strengthens the impact of industry dynamics, raising the difference in the optimal tax to 55 percent. On the other hand, the channel of industry dynamics is weakened when price signals from innovation are weaker, which pushes up the optimal tax in the endogenous model (Panel B.1). The impact of industry dynamics is also lower, 42 percent, compared to the baseline.

Table OA6: Sensitivity analysis 2: diminishing returns

σ^*	$\bar{\sigma}$	M	L_g/L_f	q_g/q_f	S_g/S	Tax	Wel
Panel A: Baseline with lower diminishing returns (higher η)							
0.37	2.91	15.27	5.61	13.92	0.87	-	100.00
Panel A.1: Counterfactual, endogenous							
1.03	3.20	14.98	17.37	67.08	0.95	96	104.89
Panel A.2: Counterfactual, exogenous							
0.37	2.91	16.07	16.07	60.23	0.95	217	93.22
Panel B: Baseline with higher diminishing returns (lower η)							
0.37	2.91	20.88	5.61	7.01	0.87	-	100.00
Panel B.1: Counterfactual, endogenous							
0.98	3.17	19.88	17.74	22.90	0.95	131	91.53
Panel B.2: Counterfactual, exogenous							
0.37	2.91	21.23	16.56	21.33	0.95	228	83.303

Table OA7: Non-targeted moments of model and data

Moments	Model	Data
Industry size	0.795	0.908
Labor cost share in clean	0.125	0.191
Capital in clean relative to dirty	16.271	24.064

Table OA8: Comparison of tax and subsidy in the exogenous model

σ^*	$\bar{\sigma}$	M	L_g/L_f	q_g/q_f	S_g/S	Policy	Wel
Panel A: Baseline							
0.37	2.91	17.88	5.61	9.88	0.87	-	100.00
Panel B: Counterfactual with a carbon tax							
0.37	2.91	18.51	16.30	35.91	0.95	223	88.17
Panel C: Counterfactual with clean subsidies							
0.37	2.91	18.64	10.46	35.91	0.99	0.86	91.27