Goofing off at work and optimal income taxation

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Introduction

- The standard OT model assumes that effort is unobservable and then uses time endowment minus effort as “leisure”.
- This procedure does not differentiate between different types of leisure (or, equivalently, treat different types of leisure as perfect substitutes).
- We revisit the OT problem in a setting where different types of leisure are imperfect substitutes.
- In particular, we consider two different types of leisure: "traditional" leisure, i.e. leisure time outside the workplace, and "goofing at work".
- We consider both the case when the only available policy instrument is a nonlinear income tax and the case when it is possible to levy a nonlinear tax that jointly depends on income and time spent at work.

- With respect to the assumptions on individual preferences, we consider both the case when agents' preferences are homogeneous and the case when agents' preferences are heterogeneous due to a heterogeneity in the taste for goofing off (to capture what Firouz has labelled a "**Helmuth's effect**").

- We highlight that:
  - formula-wise, the expression for optimal marginal income tax rates is not affected by the possibility to condition the tax liability also on time spent at the workplace;
  - however, simulations show that marginal income tax rates are substantially higher (and the average tax profile more progressive) when the tax liability is allowed to depend also on time spent at the workplace;
  - the push-up effect on marginal income tax rates is magnified when the Helmuth's effect is at work;
  - welfare gains are substantial.

Preliminaries

- Time endowment, H, is spent on work effort e, goofing off at work, g, and leisure at home, l:

$$e + g + l = H$$

- g and l "produce" a good called "leisure" and denoted by d.
- d is produced according to production function:

$$d = \Psi(g, l; w),$$

  where w denotes the productivity/skills equal to "wage".

- Preferences are additively separable in consumption, c, and d:

$$u = \rho(c) + \digamma(d) = \rho(c) + \digamma(\Psi(g, l; w)).$$

- We also assume that

$$F(\Psi(g,l;w)) = a(w)\varphi(g) + \psi(l)$$

so that

$$u = \rho(c) + a(w)\varphi(g) + \psi(l)$$

with

$$\begin{aligned} a(w) &> 0, a'(w) \leq 0 \\ \rho'(\cdot) &> 0, \varphi'(\cdot) > 0, \psi'(\cdot) > 0, \\ \rho''(\cdot) &< 0, \varphi''(\cdot) < 0, \psi''(\cdot) < 0. \end{aligned}$$

- Neither e nor g are publicly observable. Their sum, time spent at work, is. Denote time spent at work by L, we have

$$L = e + g.$$

- Income is equal to $I = we$ and is observable.

- Substituting $L - e$ for g, $H - L$ for l, and I/w for e, we have:

$$v\left(c, \frac{I}{w}, L\right) \equiv \rho(c) + a(w)\varphi\left(L - \frac{I}{w}\right) + \psi(H - L).$$

- Observe that:

  – the separability property between (c, g, l) does not extend to (c, I, L). In particular, MRS_{cI} is not independent of L; nor is MRS_{cL} independent of I;

  – for a given value of I, and thus e, choosing L is tantamount to the individual deciding as how he wants to divide his remaining time, $H - e$, between leisure at home and goofing off at work;

  – v_L can take both positive as well as negative. Suppose L is close to I/w which means g is small. We would expect that $v_L > 0$. On the other hand, if L is close to H, l is small and $v_L < 0$.

- We have ⇒

$$
\begin{aligned}
v_c &= \rho'(c) > 0, \\
v_I &= -\frac{a(w)}{w}\varphi'\left(L - \frac{I}{w}\right) < 0, \\
v_L &= a(w)\varphi'\left(L - \frac{I}{w}\right) - \psi'(H - L).
\end{aligned}
$$

- ⇒

$$
\begin{aligned}
v_{cc} &= \rho''(c) < 0, \\
v_{II} &= \frac{a(w)}{w^2}\varphi''\left(L - \frac{I}{w}\right) < 0, \\
v_{LL} &= a(w)\varphi''\left(L - \frac{I}{w}\right) + \psi''(H - L) < 0, \\
v_{cI} &= v_{cL} = 0, \\
v_{IL} &= -\frac{a(w)}{w}\varphi''\left(L - \frac{I}{w}\right) > 0.
\end{aligned}
$$

Geometrical representation

(c, I) **space**

- Indifference curves in (c, I) space (conditional on the values of L and w):

$$v(c, I; Lw) = \rho(c) + a(w)\varphi\left(L - \frac{I}{w}\right) + \psi(H - L) = v^*$$

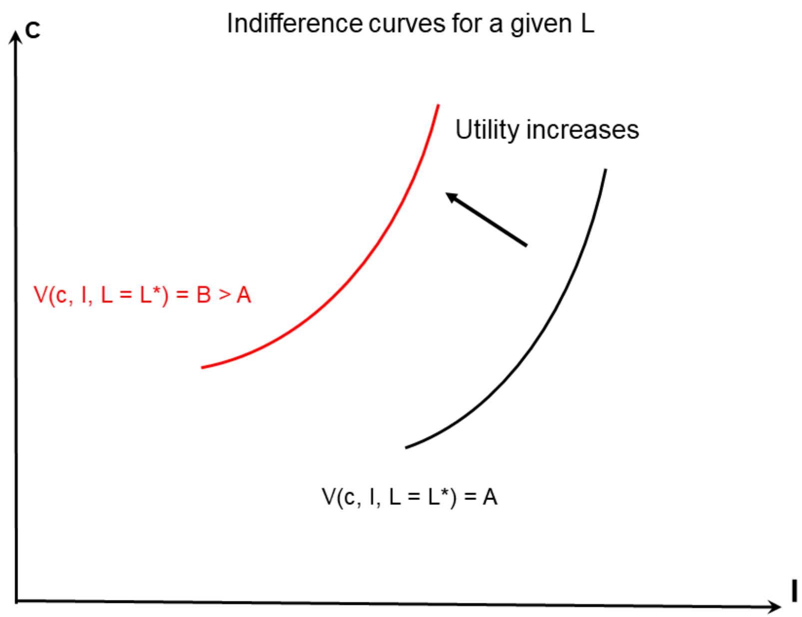
- The MRS between c and I, for a given value L and w, is defined as

$$MRS_{cI}(c, I; Lw) = -\frac{v_I}{v_c} = \frac{a(w)}{w\rho'(c)}\varphi'\left(L - \frac{I}{w}\right) > 0.$$

- $\Rightarrow$

$$\frac{\partial MRS_{cI}}{\partial I} = -\frac{a(w)}{w^2}\frac{\varphi''(L - I/w)}{\rho'(c)} > 0.$$

$\Rightarrow$ positively-sloping and convex.

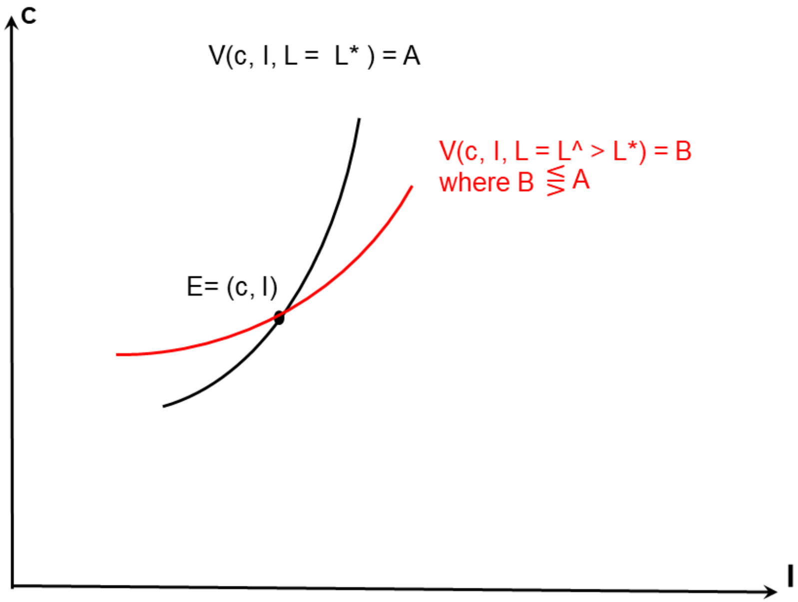

Indifference curves for a given L
c
Utility increases
V(c, I, L = L*) = B > A
V(c, I, L = L*) = A
I

- $MRS_{cI}(c, I, L; w)$ as L increases,

$$\frac{\partial}{\partial L}\left(-\frac{v_I}{v_c}\right) = \frac{a(w)}{w\rho'(c)}\varphi''\left(L - \frac{I}{w}\right) < 0,$$

where

$$\frac{\partial}{\partial L}v(c, I, L; w) = a(w)\,\varphi'\left(L - \frac{I}{w}\right) - \psi'(H - L) \lesseqgtr 0.$$


c
V(c, l, L = L*) = A
V(c, l, L = L^ > L*) = B
where B ⋚ A
E= (c, l)
l

Geometrical representation

(c, L) **space**

- Indifference curves in (c, L) space (conditional on the values of I and w):

$$v(c, L; I, w) = \rho(c) + a(w)\varphi\left(L - \frac{I}{w}\right) + \psi(H - L) = v^*$$

- The MRS between c and L, for a given value of I and w, is defined as

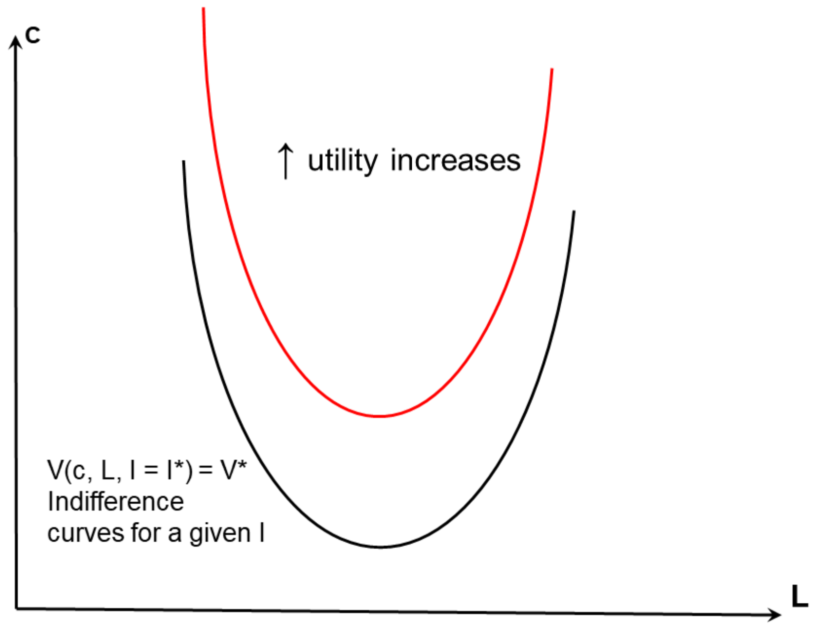
$$MRS_{cL}(c, L; I, w) = -\frac{v_L}{v_c} = -\frac{a(w)\varphi'\left(L - \frac{I}{w}\right) - \psi'(H - L)}{\rho'(c)} \lesseqgtr 0,$$

which is negative for small values of L turning positive as L increases.

- $\Rightarrow$

$$\frac{\partial MRS_{cL}}{\partial L} = -\frac{a(w)\varphi''\left(L - \frac{I}{w}\right) + \psi''(H - L)}{\rho'(c)} > 0$$

U-shaped.

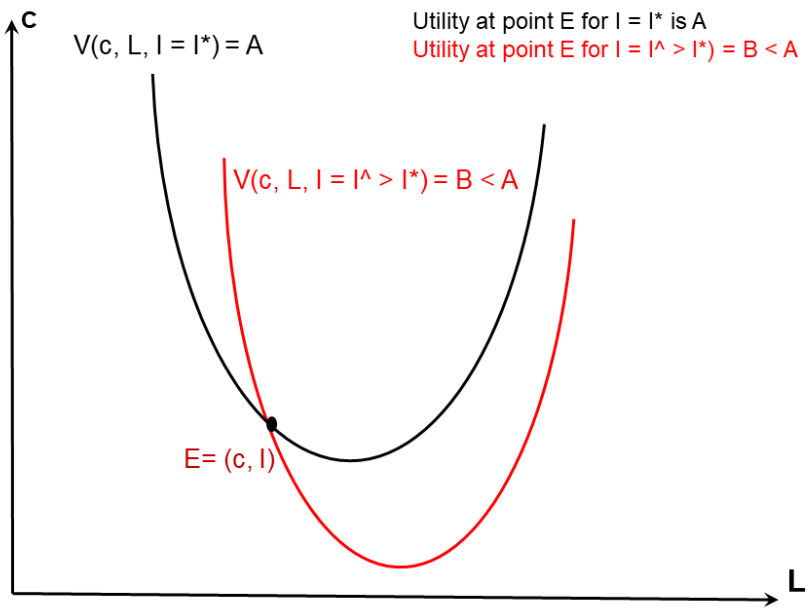

c
↑ utility increases
V(c, L, l = l*) = V*
Indifference
curves for a given l
L

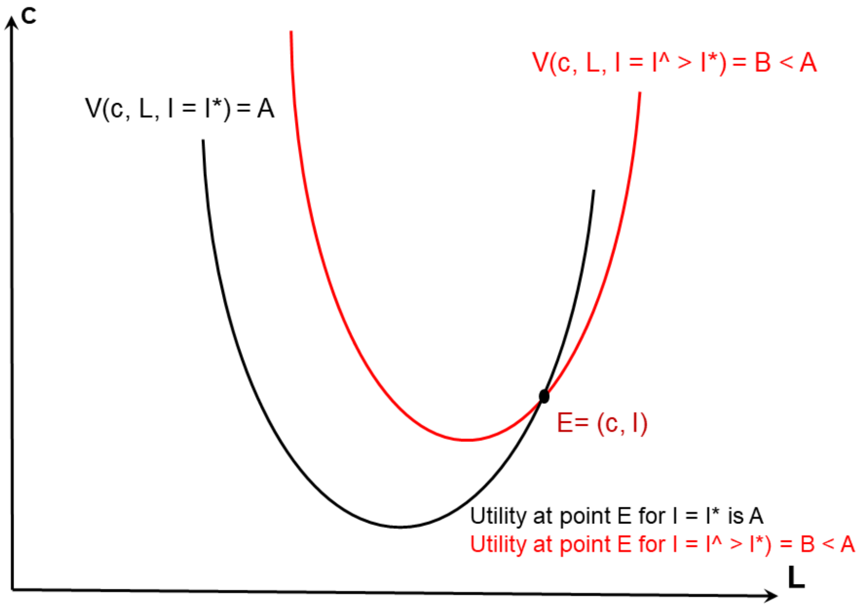
- $MRS_{cL}(c, L, I; w)$ as I increases⇒

$$\frac{\partial MRS_{cL}}{\partial I} = \frac{a(w)\,\varphi''\left(L - \frac{I}{w}\right)}{w\rho'(c)} < 0$$

and

$$\frac{\partial}{\partial I} v(c, L, I; w) = -\frac{a(w)}{w}\varphi'\left(L - \frac{I}{w}\right) < 0.$$


C
V(c, L, I = I*) = A
Utility at point E for I = I* is A
Utility at point E for I = I^ > I*) = B < A
V(c, L, I = I^ > I*) = B < A
E= (c, l)
L


c
V(c, L, l = l^ > l*) = B < A
V(c, L, l = l*) = A
E= (c, l)
Utility at point E for l = l* is A
Utility at point E for l = l^ > l*) = B < A
L

Laissez-faire

- Max

$$v(c, I, L, w) \equiv \rho(c) + a(w)\varphi\left(L - \frac{I}{w}\right) + \psi(H - L),$$

st $c = I$.

- FOC yields the laissez-faire solution

$$\begin{aligned} a(w)\varphi'\left(L - \frac{I}{w}\right) &= w\rho'(c), \\ a(w)\varphi'\left(L - \frac{I}{w}\right) &= \psi'(H - L), \\ c &= I. \end{aligned}$$

- The second equation tells us that at the laissez-faire solution, the individual equalizes the marginal utilities of g and l.

Income monotonicity in laissez-faire

- Totally differentiating the LF solution wrt w setting $dI/dw = dc/dw \Rightarrow$

$$\frac{dI}{dw} = \frac{dc}{dw} = \frac{\frac{c}{w^2} - \frac{\rho'}{\psi''} - \frac{\varphi'}{a(w)\varphi''}\left[\frac{a(w)}{w} - a'(w)\right]}{\frac{1}{w} + \frac{w\rho''}{\psi''} + \frac{w\rho''}{a(w)\varphi''}};$$

with $a'(w) \leq 0$, we have income monotonicity in the LF.

- For dL/dw we have that

$$\frac{dL}{dw} = \frac{\rho' + w\frac{dc}{dw}\rho''}{-\psi''} = -\frac{1}{\psi''}\left[\frac{\rho' + c\rho'' + \frac{wa'(w)}{a(w)}\frac{\rho''}{\varphi''}w\varphi'}{1 + \left(\frac{1}{a(w)\varphi''} + \frac{1}{\psi''}\right)w^2\rho''}\right],$$

which is of ambiguous sign (unless utility is assumed to be linear in consumption).

- Similarly,

$$\begin{aligned}\frac{de}{dw} &= \frac{d}{dw}\left(\frac{c}{w}\right) = \frac{-c}{w^2} + \frac{1}{w}\frac{dc}{dw} \\ &= -\frac{1}{w}\left[\frac{(c\rho'' + \rho')\left(\frac{1}{a(w)\varphi''} + \frac{1}{\psi''}\right) - \frac{a'(w)\varphi'}{a(w)\varphi''}}{\frac{1}{w} + \left(\frac{1}{a(w)\varphi''} + \frac{1}{\psi''}\right)w\rho''}\right],\end{aligned}$$

which is again of ambiguous sign (except for QL preferences).

Mechanism designer problem with two-types

- The mechanism designer offers two bundles $\left(c^h, L^h, I^h\right)$ and $\left(c^\ell, L^\ell, I^\ell\right)$ to the h-type and the ℓ-type which are found as the solution to

$$
\begin{aligned}
\Gamma &= v\left(c^h, \frac{I^h}{w^h}, L^h; w^h\right) + \delta v\left(c^\ell, \frac{I^\ell}{w^\ell}, L^\ell; w^\ell\right) \\
&\quad + \lambda\left[v\left(c^h, \frac{I^h}{w^h}, L^h; w^h\right) - v\left(c^\ell, \frac{I^\ell}{w^h}, L^\ell; w^h\right)\right] \\
&\quad + \mu\left[\pi^h\left(I^h - c^h\right) + \pi^\ell\left(I^\ell - c^\ell\right) - \overline{R}\right]
\end{aligned}
$$

- FOCS are:

$$v_c^h = \mu \pi^h / (1 + \lambda), \tag{1}$$
$$v_I^h = -\mu \pi^h / (1 + \lambda), \tag{2}$$
$$v_L^h = 0, \tag{3}$$
$$\delta v_c^\ell = \lambda v_c^{h\ell} + \mu \pi^\ell, \tag{4}$$
$$\delta v_I^\ell = \lambda v_I^{h\ell} - \mu \pi^\ell, \tag{5}$$
$$\delta v_L^\ell = \lambda v_L^{h\ell}. \tag{6}$$

This system of equation, plus the incentive compatibility and resource constraints, determine the optimal values of the two bundles $\left(c^h, I^h, L^h\right)$ and $\left(c^\ell, I^\ell, L^\ell\right)$ that needs to be implemented by a tax system.

The tax system

- *Marginal tax rates*: To determine the marginal tax rates, with respect to income and labor supply, consider the problem of an individual facing the income tax function $T = T(I, L)$. The individual w maximizes $v(c, I, L; w)$ subject to $c = I - T(I, L)$.

- The Lagrangian associated with this problem is

$$\Omega = v(c, I, L; w) + \eta [I - T(I, L) - c].$$

- From the focs one gets:

$$\frac{\partial T(I, L)}{\partial I} = 1 + \frac{v_I}{v_c}, \quad (7)$$

$$\frac{\partial T(I, L)}{\partial L} = \frac{v_L}{v_c}. \quad (8)$$

MTRs for the h-type

- Standard no-distortion-at-the-top result:

$$\frac{\partial T\left(I^h, L^h\right)}{\partial I^h} = 1 + \frac{v_I^h}{v_c^h} = 0,$$
$$\frac{\partial T\left(I^h, L^h\right)}{\partial L^h} = \frac{v_L^h}{v_c^h} = 0.$$

- It follows from $v_L^h / v_c^h = 0$ that, at the optimum, the following relationships hold for our specification of preferences

$$a\left(w^h\right)\varphi'\left(g^h\right) - \psi'\left(l^h\right) = 0.$$

MTRs for the ℓ-type

- One gets:

$$\begin{aligned}
\frac{\partial T\left(I^{\ell}, L^{\ell}\right)}{\partial I^{\ell}} &= 1+\frac{v_I^{\ell}}{v_c^{\ell}}=\frac{\lambda v_c^{h\ell}}{\mu\pi^{\ell}}\left(\frac{v_I^{h\ell}}{v_c^{h\ell}}-\frac{v_I^{\ell}}{v_c^{\ell}}\right) \\
&= \frac{\lambda v_c^{h\ell}}{\mu\pi^{\ell}}\left(MRS_{Ic}^{\ell}-MRS_{Ic}^{h\ell}\right), \\
\frac{\partial T\left(I^{\ell}, L^{\ell}\right)}{\partial L^{\ell}} &= \frac{v_L^{\ell}}{v_c^{\ell}}=\frac{\lambda v_c^{h\ell}}{\mu\pi^{\ell}}\left(\frac{v_L^{h\ell}}{v_c^{h\ell}}-\frac{v_L^{\ell}}{v_c^{\ell}}\right) \\
&= \frac{\lambda v_c^{h\ell}}{\mu\pi^{\ell}}\left(MRS_{Lc}^{h\ell}-MRS_{Lc}^{\ell}\right).
\end{aligned}$$

- **Marginal income tax.** We have

$$MRS^{\ell}_{Ic} - MRS^{h\ell}_{Ic} \equiv \frac{v^{h\ell}_I}{v^{h\ell}_c} - \frac{v^{\ell}_I}{v^{\ell}_c}$$

$$= \frac{\frac{a\left(w^{\ell}\right)}{w^{\ell}}\varphi'\left(L^{\ell} - \frac{I^{\ell}}{w^{\ell}}\right) - \frac{a\left(w^{h}\right)}{w^{h}}\varphi'\left(L^{\ell} - \frac{I^{\ell}}{w^{h}}\right)}{\rho'\left(c^{\ell}\right)} > 0,$$

where the sign follows from concavity of $\varphi(\cdot)$ and $a'(w) \leq 0$.

- Consequently,

$$\frac{\partial T\left(I^{\ell}, L^{\ell}\right)}{\partial I^{\ell}} > 0;$$

that is, income should be taxed at the margin and distorted downward consistent with the traditional result.

• **Marginal tax on** L. Turning next to the marginal tax on L, we have

$$MRS_{Lc}^{h\ell} - MRS_{Lc}^{\ell} \equiv \frac{v_L^{h\ell}}{v_c^{h\ell}} - \frac{v_L^{\ell}}{v_c^{\ell}} =$$

$$\frac{a\left(w^h\right)\varphi'\left(L^\ell - \frac{I^\ell}{w^h}\right) - \psi'\left(H - L^\ell\right)}{\rho'\left(c^\ell\right)}$$

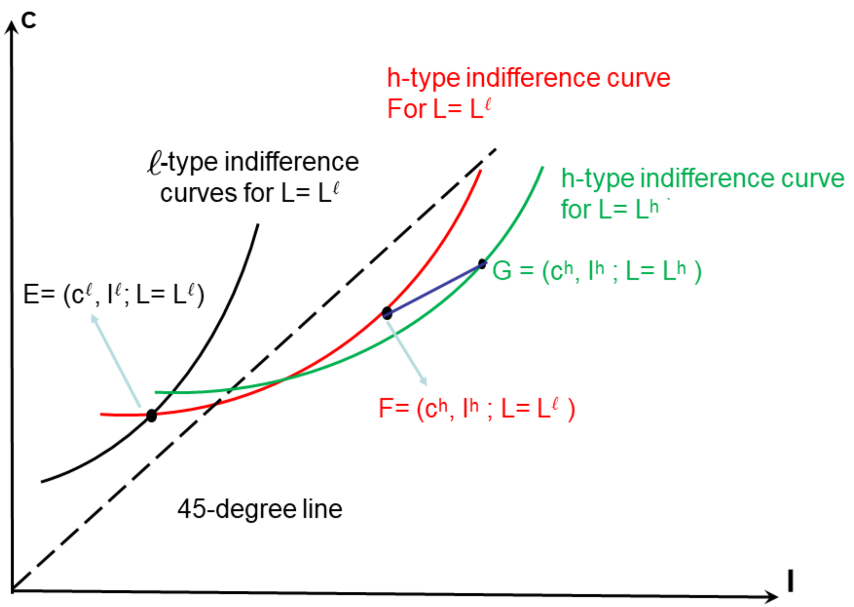
$$-\frac{a\left(w^\ell\right)\varphi'\left(L^\ell - \frac{I^\ell}{w^\ell}\right) - \psi'\left(H - L^\ell\right)}{\rho'\left(c^\ell\right)}$$

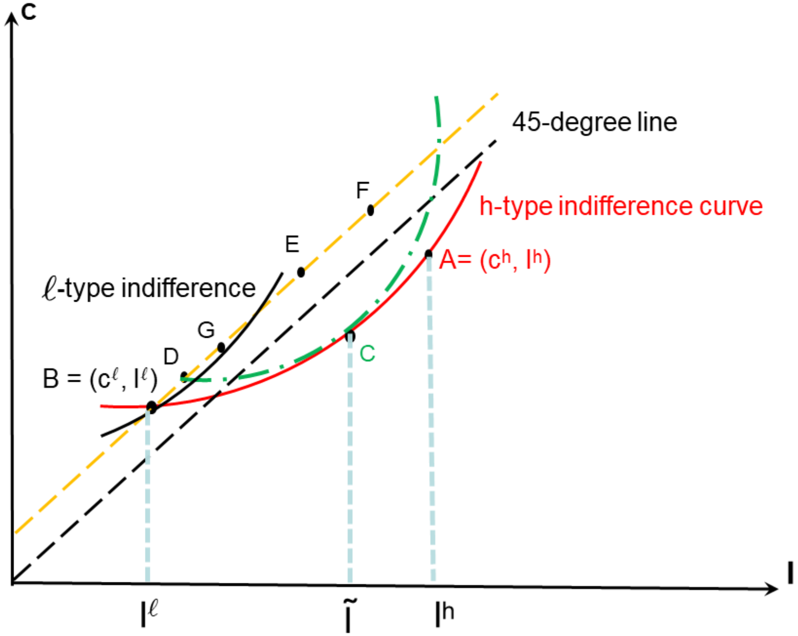
$$= \frac{a\left(w^h\right)\varphi'\left(L^\ell - \frac{I^\ell}{w^h}\right) - a\left(w^\ell\right)\varphi'\left(L^\ell - \frac{I^\ell}{w^\ell}\right)}{\rho'\left(c^\ell\right)} < 0,$$

where again the sign follows from concavity of $\varphi(\cdot)$ and $a'(w) \leq 0$.

- $\Rightarrow$
$$\frac{\partial T\left(I^{\ell}, L^{\ell}\right)}{\partial L^{\ell}} < 0;$$
that is, L **should be subsidized** and distorted upwards.

- Given income, L determines the division of the remaining time between leisure at home and goofing off at work. Increasing L^{ℓ} increases the goofing off time. Since $g^{h\ell} > g^{\ell}$, this is more detrimental (or provides a smaller benefit) for a mimicker than for a low-skilled agent.


c
h-type indifference curve
For L= L^ℓ
ℓ-type indifference
curves for L= L^ℓ
h-type indifference curve
for L= L^h
G = (c^h, I^h ; L= L^h)
E= (c^ℓ, I^ℓ; L= L^ℓ)
F= (c^h, I^h ; L= L^ℓ)
45-degree line
I


c
45-degree line
F
h-type indifference curve
E
A= (c^h, l^h)
ℓ-type indifference
G
D
C
B = (c^ℓ, l^ℓ)
l
l^ℓ
l̃
l^h

Continuum case

- w is continuously distributed over the interval $[\underline{w}, \overline{w}]$ with distribution $F(w)$ and density $f(w)$.

- Let $\widetilde{w}$ denote reported w, and define

$$V(w) \equiv v(c(w), I(w), L(w); w) = \max_{\widetilde{w}} v(c(\widetilde{w}), I(\widetilde{w}), L(\widetilde{w}); w),$$

  where $v(c(w), I(w), L(w); w) = \rho(c(w)) + a(w)\varphi\left(L(w) - \frac{I(w)}{w}\right) + \psi(H - L(w))$.

- From the envelope theorem

$$\dot{V}(w) = \frac{dV(w)}{dw} = v_w(c(w), I(w), L(w); w).$$

Government problem

- Define problem $\mathcal{P}$ as

$$\max_{V(w),c(w),I(w),L(w)} \int_{\underline{w}}^{\overline{w}} \Gamma\left[V\left(w\right)\right] f\left(w\right) dw, \tag{9}$$

$$\text{s.t.} \quad \dot{V}\left(w\right) = v_w\left(c\left(w\right), I\left(w\right), L\left(w\right); w\right), \tag{10}$$

$$V\left(w\right) = v\left(c\left(w\right), I\left(w\right), L\left(w\right); w\right), \tag{11}$$

$$\int_{\underline{w}}^{\overline{w}} \left[I\left(w\right) - c\left(w\right)\right] f\left(w\right) dw \geq \overline{R}. \tag{12}$$

- T_I and T_L are of opposite signs and the tax formulas can be expressed as

$$\frac{T_I}{1-T_I} = A(w) \times B(w) \times C(w) \times \rho'(c) \tag{13}$$

$$T_L = D(w) \times B(w) \times C(w), \tag{14}$$

where

$$A(w) \equiv 1 - \frac{I(w)}{w}\frac{\varphi''}{\varphi'} - \frac{wa'(w)}{a(w)};$$

$$B(w) \equiv \frac{1}{1-F(w)} \int_{w}^{\overline{w}} \left[\frac{1}{\rho'(c)} - \frac{\Gamma'}{\mu}\right] dF; \quad C(w) \equiv \frac{1-F(w)}{wf(w)};$$

$$D(w) \equiv wa'(w)\varphi' + a(w)\frac{I(w)}{w}\varphi'' = \left[\frac{wa'(w)}{a(w)} + \frac{I(w)}{w}\frac{\varphi''}{\varphi'}\right] a(w)\varphi'.$$

- Furthermore, from (13)-(14) it follows that

$$\begin{aligned}
\frac{T_L}{(1-T_I)\,w} &= [1-A(w)] \times B(w) \times C(w) \times \rho'(c)\,, \\
\frac{T_L/w}{T_I} &= \frac{1-A(w)}{A(w)} = \frac{1}{w}\frac{v_{wL}}{v_{wI}}, \\
T_I + \frac{T_L}{w} &= B(w) \times C(w) \times \rho'(c)\,,
\end{aligned}$$

where:

- $\frac{T_L}{(1-T_I)w} = 1 - \frac{\psi'(l)}{a(w)\varphi'}$ provides a measure of the wedge between l and g (since we have that $\frac{T_L}{(1-T_I)w} = 1 - \frac{\psi'(l)}{a(w)\varphi'}$);
- $T_I + T_L/w$ provides a measure of the wedge between l and c (since we have that $T_I + T_L/w = 1 - \frac{\psi'(l)}{w\rho'(c)}$).

- The tax rule for marginal income tax rates remains formally the same irrespective of whether the tax liability can be conditioned on both I and L or only on I. However, **one would expect higher marginal income tax rates to arise when** $T(I, L)$. This is driven by the fact that: i) L is subsidized under a tax system $T(I, L)$, and ii) for given I, c and w, MRS_{cI} is declining in L, i.e. an increase in L has a flattening effect on the shape of the indifference curves in the (I, c)-space.

- Comparing the formulas with those obtained for the case when preferences are homogeneous, i.e. $a'(w) = 0$, **one would expect marginal income tax rates to be higher under preference heterogeneity**. In the case of the formula for T_L the comparison is less clearcut.

Numerical simulations

- Same wage data as Mankiw et al. (2009) and Bastani (2015).
- For the "traditional" case where l and g are treated as perfect substitutes, we use the utility function $U = 1.7 \ln c - l^{-1}$.
- For the case where l and g are imperfect substitutes, we consider the following cases:
  - case a): $U = 2.5 \ln c - \left[\frac{4}{5} l^{1/2} + \frac{1}{5} g^{1/2}\right]^{-2}$, implying that the elasticity of substitution between l and g is constant and equal to 2. In this case, under an income tax, the ratio l/g does not vary with I.
  - case b): $U = 1.785 \ln c - \frac{l^{-0.9}}{0.9} - 0.285 g^{-0.2}$. In this case, also under an income tax, the ratio l/g varies with I.
  - case c): $U = 1.785 \ln c - \frac{l^{-0.9}}{0.9} - a(w)\, 0.285 g^{-0.2}$, with $a(w) > 0$ and $a'(w) \leq 0$ (**Helmuth's effect**).