A General Solution to the Quasi Linear Screening Problem

G. Carlier, X. Dupuis, J.C. Rochet and J. Thanassoulis

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Motivation

Dissatisfaction with policy implications of optimal tax theory.
When optimal labor taxation is possible:

- Chamley (1986) and Judd (1985): capital taxes should be zero.
- Diamond-Mirrlees (1971): commodity taxes should be zero
- Atkinson-Stiglitz (1976): consumption taxes should be zero.

At odds with common sense and common practice in many countries.

Motivation (continued)

These results rely on assumption that heterogeneity is only about labor productivities:

- Saez (2002): taxing capital income is optimal when more productive people have a higher propensity to save.
- Mirrlees (1976): very restrictive to assume only one source of adverse selection, assumed only for tractability.
- Need to consider multidimensional adverse selection.

Cremer (2003): "Because of the technical difficulties raised by multidimensional screening models, the specification of tractable taxation models has long been neglected"

Multidimensional Tax Models

Different approaches in the literature:

- Assume special preferences or technology so that multidimensionality can be eliminated: Kleven et al.(2009) on taxation of couples, Chone and Laroque (2010) on labor taxation, Beaudry et al. (2009) on employment subsidies.
- Assume government can only tax **total** income: Rothschild et al. (2013), (2016), Jacquet et al. (2013).
- Compute the gradient of social welfare with respect to the different taxes: allows to analyze impact of tax reforms: Golosov et al. (2014). Not valid when there is **bunching**: very frequent in multidimensional problems.

Multidimensional Extensions 2

- Purely numerical approach: Tarkiainen and Tuomala (1999), (2007), Judd et al (2017) but no guarantee that the algorithms converge.
- Cremer et al. (2001), (2003): 2×2 models (two dimensions of heterogeneity and two possible values for each parameter). Also Boadway et al. (2002). Only illustrative: cannot be calibrated to real data.

This paper

- We provide a numerical algorithm that can solve almost any discrete quasi-linear multidimensional screening problem.
- Based on a primal-dual algorithm used in medical imaging (Chambolle Pock 2011).
- Extremely flexible: all types of discrete distributions can be dealt with.
- We illustrate this algorithm by solving:
    1. Generic monopolist price discrimination problem.
    2. Optimal taxation of labor and savings incomes when individuals differ in two dimensions.

Why are unidimensional screening problems so simple?

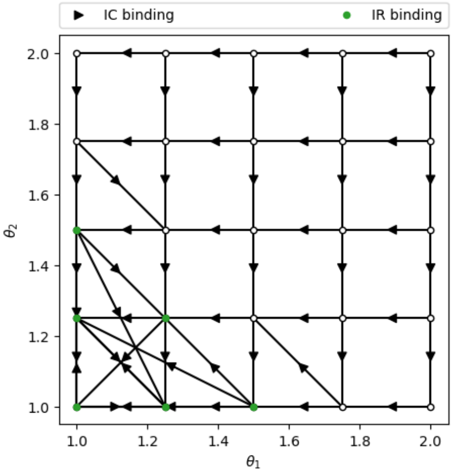
- Magic trick: Single Crossing Property (SCP).
- With SCP, individuals are always ranked in the same way.
- Ex: more productive people always get a higher income.
- "Local downward" IC constraints are always binding.
- Informational rents computed by adding incremental utilities from "above".
- Second best allocations maximize "virtual surplus": surplus minus informational rents.
- Simple analytical formula for virtual surplus: smooth function of allocation.

Why are multidimensional screening problems so difficult?

- In multiple dimension (or when SCP does not hold) the ranking of individuals varies with the allocation.
- Non local or "transverse" incentive compatibility can be binding.
- Informational rents are non differentiable functions of allocations.
- No simple expression is available.
- No way to compute virtual surplus analytically.

An Illustration of the Difficulties

Binding constraints, two dimensional monopoly price discrimination problem:



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How our algorithm overcomes these difficulties

- Using the characterization of implementable allocations in Rochet (1987), we transform the maximization problem of the principal problem into a maxmin problem involving the Lagrange multipliers of IC constraints.
- We adapt the primal dual algorithm of Chambolle Pock (2009) that was designed for medical imaging problems.
- At each iteration, the algorithm adjusts the allocation (primal) and the Lagrange multipliers (dual).
- Convergence is guaranteed under mild regularity conditions.
- The limit is a local solution (global if the problem is convex).

Application: taxation of savings and labor incomes

- Simple extension of Mirrlees (1971): individuals differ in their initial endowments e and disutilities of working x .
- Consume at two dates $t = 1, 2$, quasi linear preferences:

$$V_i = u(C_i^1) + C_i^2 = u(e_i - s_i) + Rs_i + (w - x_i)l_i - T(s_i, l_i)$$

- Tax $T(s_i, l_i)$ only depends on observable decisions of agent i : savings s_i and labor supply $0 \leq l_i \leq 1$.
- R: return on savings and w: unit wage.
- Both are exogenous and uniform across agents.

Model (continued)

Government maximizes a weighted sum of a Rawlsian objective and utilitarian welfare:

$$W = \alpha min_j V_j + (1 - \alpha) \sum_i f_i V_i,$$

under the constraint that tax revenue covers public expenditure G

Economic question: should savings be taxed more heavily for employed or unemployed people?

Separable Taxation

- In the one dimensional case (when endowments or labor costs are publicly observable), the optimal tax is separable.
- Total tax = tax on labor income plus tax on savings income.
- Participation in the labor force ($l > 0$) only depends on labor cost x.
- It is independent of initial endowment e.

The two dimensional case

- Optimal tax on savings may depend on employment status: $T_1(s)$ for employed, $T_0(s)$ for unemployed.
- Employed individuals choose savings $s_1(e)$ that solve

$$v_1(e) \equiv \max_s u(e-s) + Rs - T_1(s).$$

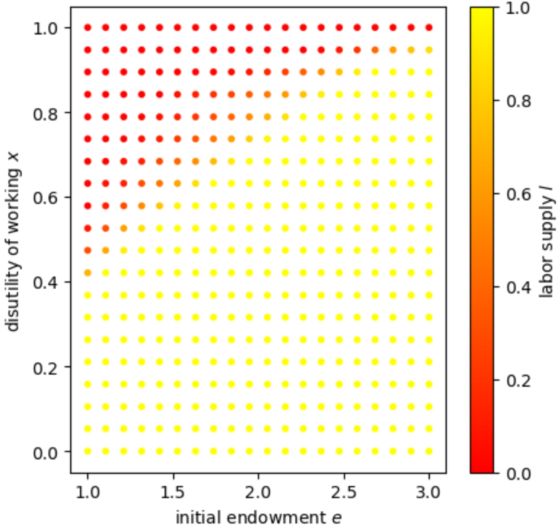
- Unemployed individuals choose savings $s_0(e)$ that solve

$$v_0(e) \equiv \max_s u(e-s) + Rs - T_0(s).$$

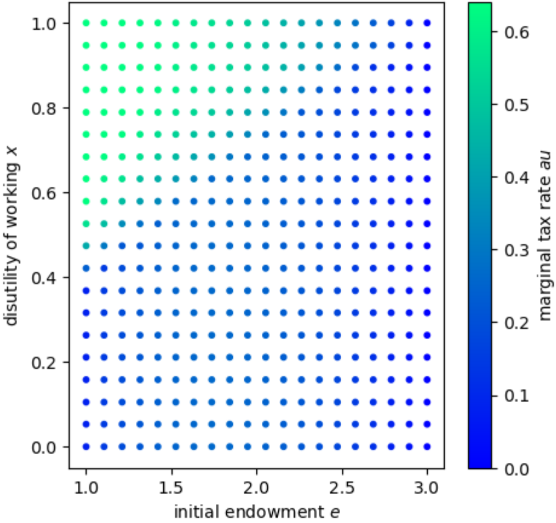
- Indirect utility is

$$V(e,x) = \max[v_0(e), v_1(e) + w - x].$$

The solution



The solution (continued)



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The solution (end)

- Employment decision depends on initial endowments:

$$l = 1 \iff x < w + v_1(e) - v_0(e) = x^*(e).$$

- Critical labor cost $x^*(e)$ increases in e
- This implies that marginal tax rate on savings must be higher for unemployed individuals.
- This is meant to encourage labor force participation.

Conclusion

- This example is only illustrative.
- We do not mean to derive serious policy implications.
- We just want to illustrate the power of our algorithm, which is easy to use and extremely flexible.
- It is publicly available at https://github.com/x-dupuis/screening-algo.
- We hope it will be adopted by the optimal tax community to solve the multidimensional screening problems they find interesting.