App Platform Model*

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Abstract

We model a software application platform where apps are complements, which differ in their quality and consumers differ in their benefit from app quality. The platform sells devices to consumers and collects a percentage commission on in-app purchases. App developers choose to develop an app (incurring a fixed cost) and their in-app purchase price. Consumers decide to buy a device and how much to purchase from each app. We characterize the platform's equilibrium allocations and fees implementing them. The platform sets excessive fees on both sides, distorting consumer and app surpluses. A cap regulation on commissions increases app entry and app surplus, but lowers consumer participation and can harm consumers. Allowing apps to use a third-party payment channel effectively mirrors a commission cap, and so can harm consumers while benefiting apps. If the platform introduces its own apps replacing some high-quality apps, it sets a lower commission and higher device fee resulting in lower consumer participation, more app entry and higher profits for the remaining apps. We also study the implications of different app financing models (ad-financed, subscription-based).

Keywords: App store, quality, antitrust policy, regulation

JEL Codes: D42, L12, L13, L40, H25

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1 Introduction

Software application platforms like Apple (iOS) and Google (Android) run app stores where app developers and consumers interact (app usage and in-app purchases). Consumer spending for apps (from in-app purchases and paid apps) was $171 billion in 2023 (Statista, May 8, 2024), $\$35.8$ billion in the second quarter of 2024, 68.7% of which was generated in Apple iOS Statista, August 30, 2024. In-app purchases, which include features, upgrades and subscription within an app, account for 96.5% of total consumer spending for apps (Statista, September 6, 2024).[1] Consumers typically use devices of only one system due to high switching costs, gains from "single-homing" (interoperability within each system), and learning costs. On the other hand, most apps are present on both app stores, that is, apps are "multi-homing".[2] App stores have therefore monopoly power over apps in providing access to their locked-in consumers, "Competitive bottlenecks", Armstrong (2006).

Policy makers around the world have been concerned about app store rules that prevent 3rd-party apps from competing effectively and favor platform-owned apps. They argue that consumers are harmed due to reduced entry of innovative apps since apps cannot escape high commissions, which are around 30% on in-app purchase transactions of iOS and Google, and go down to around 15% for subscription revenues after the first year of subscription (Statista, October 8, 2024). Following a complaint by Spotify, the European Commission (EC) fined Apple by $\$2$ billion for abusing its dominant position via excessive commissions imposed on in-app purchases of iOS devices (the EC Press Release, March 4, 2024). In March 2024 the US DOJ has opened a series of antitrust law suits against Apple, see U.S and Plaintiff States v. Apple Inc.[3] Recently, the EC's Digital Markets Act and Digital Services Act introduced ex-ante regulations forcing gatekeeper app stores (iOS and Android) to allow third-party payment processors and competing app stores on their systems.[4] As a response to DMA obligations, in August 2024, Apple lowered its commission fee on in-app purchases within Apple App store to 17% and introduced a "Core Technology Fee" of 0.50 Euros for each new app installation after the first million installs if the app chooses to use an alternative app store or an alternative payment channel within iOS.

We develop a model of a software application platforms incorporating their important peculiarities. Apps are complements. They differ in their quality and consumers differ in their unit benefit

[1] Consumer spending for apps account for 33.2% of total mobile app market value in 2022, whereas the remaining share, 66.8%, was from mobile ads (Statista, October 22, 2024)

[2] According to the Competition and Markets Authority Mobile Ecosystems Market Study (2022) 85% of the top 5000 apps multihome.

[3] Epic Games sued Apple (The Verge, September 12, 2021) and Google (New York Times, December 11, 2023) for preventing its third-party payment processor.

[4] The US congress proposed the Open App Markets Act bill (February, 2022) which includes similar ex-ante regulations on app store platforms of Apple and Google.

from app quality. Consumers choose whether to buy a device from which they access apps and spend time on them during which they make micro payments via the app-store (in-app purchases). Apps decide whether to develop an app at some fixed cost and then choose their in-app purchase. More apps and higher level of in-app purchases (due to lower price for micro-payments, better quality of apps, or higher WTP for quality) make consumers more likely to buy the device and spend time on other apps on the device. Thus, network effects generate complementarity across apps.

The platform first sets its app commission and consumer device fee. Next, given the platform's fees, consumers and apps make their participation decisions. Apps on board set their in-app purchase price and consumers on-board choose their in-app purchase amount for each app. We solve the platform's problem as choosing the marginal consumer type and the marginal app type. We then characterize the device fee and app commission implementing the equilibrium level of participation on each side. This is like the insulating tariff equilibrium approach of Weyl (2010).

Our model incorporates standard network effects: more apps attract more consumers and vice versa. Different from the literature we incorporate vertical differentiation between apps, so each consumer-app transaction value depends on the consumer's willingness-to-pay for the app as well as the app's quality. Hence, we have heterogenous transaction externalities across different consumers and different apps. The marginal types of consumers and apps not only pins down the volume of transactions, but also determine the value of transaction per participant on each side. Our model can therefore be used to evaluate the performance of the platform's choices not only on prices to consumers and apps, but also on the quality of the marginal app and the purchase proclivity of the marginal consumer, and thereby the total transaction value generated on the app store.

We show that the platform's fees are excessive on each side of the market and they lead to too low consumer engagement and too low app participation compared to the level that maximizes consumer surplus or the level that maximizes app surplus. We use the model to study different policy interventions that previous regulations, antitrust cases, law suits and antitrust investigations have implemented or considered.

We first investigate the implications of a cap regulation on app commission to inform the current regulatory debate about capping app commissions. We find that the platform rebalances its business model toward setting a higher device fee, which makes the marginal consumer worse off. Due to the cap, more apps join the platform for a given level of consumer participation. As the platform raises its device fee, leading to fewer consumers on-board, apps' gains from the fee cap are reduced. We illustrate that a cap regulation increases total app surplus. But it lowers consumer surplus when consumer losses from a higher device fee are higher than the gains of infra-marginal consumers from increased app participation. This is the case if consumer and app types are uniformly distributed over a unit interval.

The second application is a binding cap on device price. The platform lowers its device fee encouraging more consumers to have a device. However, it raises its app commission lowering the amount of apps on the platform and app surplus. We illustrate that a binding device fee cap increases consumer surplus when consumer gains from a lower device fee are higher than the losses of infra-marginal consumers from decreased app participation (the case with uniformly distributed types).

The third application of the model determines the effects of allowing apps to transact on a third-party payment processor. This direction is motivated by recent cases against Apple in the EU and the US, in which plaintiffs have complained about the excessive commissions (up to 30%) exacted on on-platform micro-payments and subscription fees. We show that if apps are allowed to transact on a third-party payment channel, while consumers incur an inconvenience cost from such transactions, the platform cannot charge a commission above a certain threshold, since otherwise apps would set the on-platform price so high that all transactions would be diverted to the third-party payment channel. The threshold on the platform commission is increasing in consumers' inconvenience cost of transacting off-platform. Hence, if this threshold is binding, this shifts rents from the platform to app developers, increasing app participation. The platform re-balances its revenue by raising device fee and so lowering consumer participation. This mimics the implications of a cap on commissions.

The fourth application is to allow for a hybrid platform which offers its own apps in the app store while replacing some high quality (infra-marginal) apps with in-house ones. The platform can do this by giving exclusive rights to its own apps and denying platform access to any competing app performing the same function. In this case the platform captures all the profit (rent) on its apps, so it wants to direct more traffic to them. This induces the platform to reduce the app commission, ceteris paribus, because it becomes more important to get on-board more third-party apps to attract more consumers, and less important to "tax" the infra-marginal apps' profits. This reasoning might also suggest that the platform would also want to reduce the device fee for consumers for the reason that it now has more benefits from consumers being on-board. On the other hand, increased app participation and so increased consumer participation demand leads the platform to raise the device fee. Although not too much because the platform now cares more about consumer traffic due to its own app revenue. Resolving these countervailing forces, we show a strong neutrality result if the platform sets the same in-app purchase price as third-party apps: consumer participation in equilibrium stays the same as without in-house apps. This means the forces described above lead to a higher device fee such that it totally undoes the reduction in commission, and the marginal consumer stays the same. Then aggregate consumer surplus rises because all infra-marginal consumers have strictly positive gains and the marginal consumer is indifferent. Thus, hybrid model on an app platform helps consumers and (remaining) third party app developers. This is in stark contrast to the result on hybrid platforms in marketplaces as elaborated in Anderson and Bedre Defolie (2023), who model differentiated competition between third-party products and the platform-owned products in a marketplace, whereas here apps are complements, so the hybrid platform lowers third-party commission to benefit from more apps attracting more consumer traffic. We also show that when the hybrid platform chooses its own-app prices, it sets a lower price than third-party apps since it can capture consumers expected transaction surplus via device fee. In that case, we show that the hybrid platform induces a lower level of consumer participation and higher level of app entry at the margin compared to the platform without own apps.

In the extensions we study different financing models for apps: ad-financed apps and subscription-based apps. We show that the benchmark model with in-app purchases can capture an ad-financed app model, where in-app purchase price becomes ad nuisance cost times the amount of ad per unit of usage. If an app pays the same commission over in-app purchases and over ad revenues, the app prefers ad-financed model if and only if its revenue per ad is higher than the nuisance cost per ad. Lowering the revenue per ad of all apps (e.g., by new data tracking policy of iOS), leads to less app participation and higher device fee, and so harms apps and consumers. If apps charge only subscription fee, there is a hold-up problem; apps extract all surplus of the marginal consumer. Anticipating this the platform cannot charge positive device fee. There are fewer consumers on the platform. Apps prefer subscription based financing if and only if app participation increases. This is the case with uniformly distributed types.

1.1 Related literature

We contribute to the literature on two-sided markets (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006) and recent literature studying software application platforms (Etro, 2021, 2023; Teh and Wright, 2023; Tirole and Bisceglia, 2023; Jeon and Rey, 2024) by providing a model capturing unique features of app platforms. We differ from the previous work through the interaction of two key aspects. First, we model a continuum of ex-ante heterogenous users on each side on their unit transaction value: each buyer has a personal willingness-to-pay for a given seller quality and each seller generates idiosyncratic revenues from interacting with a given buyer type. Hence, each transaction has a unique value proportional to the product of the seller quality and buyer benefit of the matched pair, and so unit externalities between the two sides differ for different matches. Second, the platform uses asymmetric pricing tools: a device fee to consumers and a percentage commission on revenues of sellers. The platform's pricing determines the marginal type (participation margin) on each side. In each app market the app quality and its pricing determine the level of app consumption per consumer (usage margin). Our model thereby generates the level of externality between the two sides endogenously in equilibrium. The characterization of equilibrium allocations and prices therefore differs substantially from existing work. The platform always charges a positive device fee to consumers and a positive commission to apps (as long as app development cost is low enough), which are above the levels that maximize consumer surplus and app surplus. A regulation that caps commission increases app surplus but can lower consumer surplus.

Recent literature on app platforms (Etro, 2021, 2023; Teh and Wright, 2023; Jeon and Rey, 2024) uses the competitive bottleneck setting of Armstrong (2006) to study how platform competition affects outcome variables, like fees, consumer surplus, app surplus and total welfare.[5] In these models, a monopoly platform captures the transaction surplus of consumers via a device fee since consumers do not know their transaction values ex-ante and they differ ex-ante only in their tastes for the platforms. Thus, the platform would choose the "right" app commission from the viewpoint of consumers. This is in stark contrast to our paper which shows that the platform sets excessive fees on both sides since it cannot capture transaction surplus of consumers or sellers due to ex-ante heterogeneity of users in their transaction benefit or quality. We thereby provide a different reason for excessively high fees set by a platform.

We study the implications of cap regulation on seller commissions as does the literature on access fee regulation in two-sided markets (Bedre-Defolie and Calvano, 2013; Gomes and Mantovani, 2024; Tirole and Bisceglia, 2023). Bedre-Defolie and Calvano (2013) and Gomes and Mantovani (2024) illustrate rationales for cap regulation on seller fees when sellers are not allowed to price their products/services differently on and off the platform (price-parity-clauses). In contrast, in our setting where apps can use an alternative payment channel, we allow apps to pass through commissions on within platform app pricing and show that this reduces app commission that the platform can charge, like a cap on commissions, which might be harmful to consumers. Different from Tirole and Bisceglia (2023) we allow for downward sloping demand within each app market per device-holder (consumer) and we have an elastic margin of participation on each side. Thus, allowing for ad-valorem fees on app side enables the platform to capture more value from higher value transactions and capping commissions lowers consumer participation as the platform rebalances its revenue sources and raises the device fee as a reaction to the cap. The cap always increases app surplus and lowers the platform profit, and can also lower total consumer

[5]Jeon and Rey (2024) show that platform competition leads to excessively high commissions (compared to the consumer surplus maximizing level and the total welfare maximizing level) when raising one platform's app commission lowers the rival's app base. This happens when app developers incur the development cost once and can post their apps on the competing platform (multi-home) without additional development cost. Teh and Wright (2023) show that in general seller fees are too high when there are negative spillovers between the platforms, that is, increasing the seller commissions lowers the attractiveness of the rival platform for buyers (see the latter paper for other sources of spillovers between competing platforms).

surplus. This is stark contrast to the result of Tirole and Bisceglia (2023), who show that capping access fees increases efficiency and total welfare when zero-lower-bound constraint is binding on the consumer side (when the platform wants to subsidize consumer participation, but cannot do so due to ZBL).[6]

Another recent literature studies the implications of a hybrid business model of a platform in which the platform's own products compete against third-party products (Hagiu et al., 2022; Zennyo, 2022; Anderson and Bedre Defolie, 2023; Etro, 2023; Tirole and Bisceglia, 2023). Like this literature, we study the hybrid mode implications. Different from them we focus on the case when the platform's apps complement third-party apps. We show that app commissions are lower when the platform's own apps entry replace some high-quality third-party apps. This benefits remaining apps but can harm consumers. These results differ from Anderson and Bedre Defolie (2023), who show that hybrid platform mode leads to higher third-party seller commissions, higher consumer prices and lower variety of products on the platform.

2 Monopoly problem

There is a monopoly platform enabling interactions between a mass of app developers and a mass of consumers. App developers differ in their quality and consumers differ in their marginal willingness-to-pay for app quality. App type x is a random draw from $[0, \overline{x}]$ with pdf $g(x)$ and cdf $G(x)$. Consumer type b is a random draw from $[0, \overline{b}]$ with pdf $f(b)$ and cdf $F(b)$. App developers incur a fixed cost $K > 0$ if they post their app on the platform and have zero marginal costs (apps are digital products). The platform incurs production cost of $C > 0$ per device and has zero marginal costs of enabling in-app transactions. The platform charges a (fixed) device fee S to consumers and a percentage fee τ over revenue generated by app developers. Each app is a monopolist in its market and consumers are willing to consume in all app markets. This consumption could be interpreted as time spent on an app or in-app purchases (micro-payments). Consumers pay a price p per unit consumption to the app developer and also incur an exogenous cost, $\lambda > 0$, which captures an intrinsic cost of a transaction. As we show below, a decrease in λ shifts the demand for in-app transactions up.

Consumer type b's utility from purchasing q units of app of quality x at price p is

$$U(b, x, q, p) = 2\sqrt{bxq} - (\lambda + p)q. \tag{1}$$

The timing of the interactions is the following

[6] Note that in Tirole and Bisceglia (2023) consumers always have zero surplus in their benchmark as they have homogenous benefits from using the platform and using an app.

1. The platform chooses a percentage app fee, τ, and a device fee, S.
2. App developers choose to enter the platform and incur a fixed cost K upon entry. Simultaneously, consumers decide whether to participate and pay the device fee.[7]
3. App developers on the platform choose their price p for micro payments.
4. Consumers on the platform decide how much to consume.

We look for a Subgame Perfect Nash Equilibrium of this game.

2.1 Preliminaries and assumptions

Let $\tilde{b}$ denote the marginal consumer type and all consumers with $b \geq \tilde{b}$ hold a device and $\tilde{x}$ denote the marginal app type and all apps with $x \geq \tilde{x}$ develop an app and post it on the platform. We characterize $\tilde{b}$ and $\tilde{x}$ in the equilibrium analysis. We define two important objects that we use in the analysis. The expected app quality on the platform, $Q(\tilde{x})$, is

$$Q(\tilde{x}) = \int_{\tilde{x}}^{\overline{x}} xg(x)dx \qquad \text{with} \qquad Q'(\tilde{x}) = -\tilde{x}g(\tilde{x}) < 0. \tag{2}$$

Thus, the expected app quality decreases in the marginal app type. Higher $\tilde{x}$ implies fewer of apps joining the platform and those who join are of higher quality. The expected value of a consumer participating the platform, $B(\tilde{b})$, is

$$B(\tilde{b}) = \int_{\tilde{b}}^{\overline{b}} bf(b)db \qquad \text{with} \qquad B'(\tilde{b}) = -\tilde{b}f(\tilde{b}) < 0. \tag{3}$$

The expected consumer value decreases in the marginal consumer type. Higher $\tilde{b}$ implies fewer consumers joining the platform and those who join have higher valuation.

We make the following assumptions on the distribution functions to guarantee a unique solution to the platform's optimization problem:

Assumption 1 *(i) $F(x)$ satisfies the Monotone Hazard Rate Property (MHRP): $\frac{f(b)}{1-F(b)}$ is non-decreasing. (ii) $G(x)$ satisfies the MHRP.*

Lemma 6 in the Appendix shows that Assumption 1 implies log-concavity of $B(\tilde{b})$ and $Q(\tilde{x})$.

[7] The participation decisions of apps and consumers could also be modelled sequentially. This would not change the equilibrium analysis given that each app and each consumer is inconsequential for the equilibrium level of participation on either side of the market.

3 Equilibrium analysis

In Stage 4, in a given app market with in-app purchase price p and app type x , consumer type b chooses her optimal consumption level by maximizing her utility:

$$\max_q U(q,b,x,p) = \max_q \left[2\sqrt{bxq} - (\lambda + p)q\right],$$

which gives the transaction demand of consumer type b for app type x as:

$$q^*(b,x,p) = \frac{bx}{(\lambda+p)^2}. \tag{4}$$

Better apps (higher x) enjoy more in-app purchase demand from every consumer type and higher WTP consumers (higher b types) consume more from each app type. A decrease in λ or a lower in-app purchase price p shifts the demand for in-app transactions up for any type of consumer and in every app market. The indirect utility from transacting with app type x is then

$$v(b,x,p) = \frac{bx}{\lambda+p}. \tag{5}$$

In Stage 3, given τ and the consumers participating on the platform, those app developers that joined the platform choose their price. Recall that apps are in independent markets and each app developer is a monopolist. App developer type x chooses its price p by maximizing its profit from transactions:

$$\pi(p,x,\tilde{b},\tau) = (1-\tau)p\int_{\tilde{b}}^{\bar{b}} q^*(b,x,p)f(b)db = (1-\tau)xB(\tilde{b})\frac{p}{(\lambda+p)^2},$$

where the marginal consumer type is $\tilde{b}$. The first-order condition gives the equilibrium app price:

$$\frac{d\pi}{dp} = (1-\tau)xB(\tilde{b})\left(\frac{(\lambda+p^*)^2 - 2p^*(\lambda+p^*)}{(\lambda+p^*)^4}\right) = 0 \qquad \text{or} \qquad p^* = \lambda.$$

Thus, each xb interaction is priced at λ. The equilibrium app price does not depend on the app commission, since apps have zero marginal cost. Each app takes the consumer participation level given when pricing, and so ignores consumer traffic to the platform when pricing its app. As we illustrate in the hybrid app platform analysis, Section 5.6, the platform would like to induce a lower in-app price than the independent app developers.

In Stage 2, given S and the set of apps participating on the platform, consumers decide whether to participate (buy a device). Each x generates equilibrium benefit $\frac{bx}{2\lambda}$. Hence, the aggreage expression for the indirect gross utility for type b consumer is

$$V(b,\tilde{x}) \;=\; \frac{b}{2\lambda}Q(\tilde{x}),$$

where $\tilde{x}$ is the marginal app type participating and $Q(\tilde{x})$ is the expected app quality on the platform with $Q'(\tilde{x}) < 0$, (2). The indifferent consumer type, $\tilde{b}$, has zero net surplus from participating: $V(\tilde{b},\tilde{x}) = S$, so the marginal consumer type $\tilde{b}$ is a function of the device fee S and the marginal app type $\tilde{x}$:

$$PC_C : \frac{\tilde{b}}{2\lambda}Q(\tilde{x}) = S. \tag{6}$$

We refer to this condition as the "Participation Condition of Consumers" or PC_C as it pins down the set of consumers joining the platform, that is, the measure of consumers with types above $\tilde{b}$, $1 - F(\tilde{b})$.

The expected gross profit of the app type x from joining the platform is

$$\pi(x,\tilde{b},\tau) = (1-\tau)\int_{\tilde{b}}^{\bar{b}} \frac{bx}{4\lambda} f(b)db = (1-\tau)\frac{x}{4\lambda}B(\tilde{b}),$$

where $B(\tilde{b})$ is the expected value of a consumer participating the platform (3). The indifferent app type $\tilde{x}$ has zero net profit from joining: $\pi(\tilde{x},\tilde{b},\tau) = K$, so the marginal app $\tilde{x}$ is a function of the app fee τ, the marginal consumer type $\tilde{b}$ and the cost of app development K:

$$PC_A : (1-\tau)\frac{\tilde{x}}{4\lambda}B(\tilde{b}) = K. \tag{7}$$

We refer to this condition as the "Participation Condition of Apps"or PC_A as it pins down the mass of apps joining the platform as $1 - G(\tilde{x})$.

Using the participation condition of consumers (6) we express the device fee as a function of the marginal consumer type $\tilde{b}$ and marginal app type $\tilde{x}$. Note that (6) gives $\tilde{b}$ as an increasing function of $\tilde{x}$ since $Q'(\tilde{x}) = -\tilde{x}g(\tilde{x}) < 0$. The pink curve in Figure 1 illustrates (6) for device fee $S = 0.3$ and commission $\tau = 0.15$ assuming uniformly distributed types over $[0,1]$, and parameter values $K = \lambda = 0.21$.[8] The upward-sloping curve means higher app participation (lower $\tilde{x}$) generates more consumer participation (lower $\tilde{b}$). There must be some apps on board to induce positive consumer participation when the device fee S is positive. Hence, the consumer participation curve starts from a positive marginal consumer type. A higher device fee shifts consumer participation

[8] In the figure $\tilde{b}$ is a convex function of $\tilde{x}$. In general we have this property if and only if $Q''(\tilde{x}) = -[g(\tilde{x}) + \tilde{x}g'(\tilde{x})] < 0$, that is, $g(x)$ is inelastic: $-\tilde{x}\frac{g'(\tilde{x})}{g(\tilde{x})} < 1$. This holds when $g(x)$ is uniformly distributed over $[0,1]$.

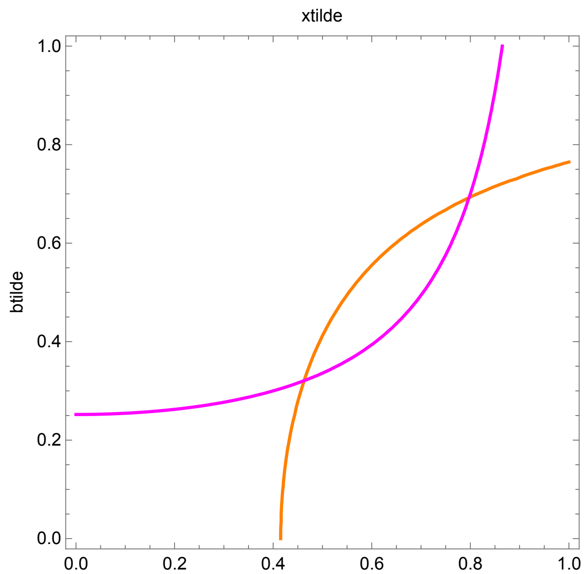

Figure 1: App PC (orange curve) and Consumer PC (pink curve) for uniformly distributed $\{b, x\} \sim [0, 1]$, given $\tau = 0.15, S = 0.3$, for parameter values $K = \lambda = 0.21$.

curve (6) up, leading to fewer consumers for any app entry.

Similarly, using the participation condition of apps (7) we express the app commission as a function of the marginal consumer type $\tilde{b}$ and marginal app type $\tilde{x}$. Note that (7) gives $\tilde{x}$ as an increasing function of $\tilde{b}$ since $B'(\tilde{b}) = -\tilde{b}f(\tilde{b}) < 0$. The orange curve in Figure 1 illustrates (7) for device fee $S = 0.3$ and commission $\tau = 0.15$ assuming uniformly distributed types over $[0, 1]$, and parameter values $K = \lambda = 0.21$.[9] The upward sloping curve means higher consumer participation (lower $\tilde{b}$) generates more app participation (lower $\tilde{x}$). There must be some consumers on board to induce a positive app participation due to the app development cost. Hence, the app participation curve starts from a positive marginal app type. A higher app development cost shifts the app participation curve (7) right, leading to less app entry for any consumer participation level.

This representation of participation conditions (6) and (7) is key because we are able to convert the platform's problem of choosing τ and S to an equivalent problem of choosing the marginal app $\tilde{x}$ and the marginal consumer type $\tilde{b}$ to deliver the equilibrium analysis.

In Stage 1, anticipating the participation condition of consumers $(6) : V(\tilde{b}, \tilde{x}) = S$, and the participation condition of apps $(7) : \pi(\tilde{x}, \tilde{b}, \tau) = K$, the platform chooses its fee for consumers,

[9]In the figure $\tilde{x}$ is a convex function of $\tilde{b}$. In general we have this property if and only if $B''(\tilde{b}) = -[f(\tilde{b}) + \tilde{b}f'(\tilde{b})] < 0$, that is, $f(b)$ is inelastic: $-\tilde{b}\frac{f'(\tilde{b})}{f(\tilde{b})} < 1$. This holds when $f(b)$ is uniformly distributed over $[0, 1]$.

S, and its commission for apps, τ. The platform's profit is

$$\begin{aligned}\Pi(S,\tau,\tilde{x},\tilde{b}) &= (S-C)(1-F(\tilde{b}))+\tau p^{*}\int_{\tilde{x}}^{\overline{x}}\int_{\tilde{b}}^{\overline{b}} q^{*}(b,x,p^{*})f(b)g(x)dbdx,\\ &= (S-C)(1-F(\tilde{b}))+\tau\frac{1}{4\lambda}B(\tilde{b})Q(\tilde{x}).\end{aligned}$$

We re-write the latter problem by substituting S and τ from the marginal agents' conditions, (6) and (7), namely $S=\frac{1}{2\lambda}\tilde{b}Q\left(\tilde{x}\right)$ and $\tau=1-\frac{4\lambda K}{B(\tilde{b})\tilde{x}}$:

$$\begin{aligned}\Pi(\tilde{x},\tilde{b}) &= \left(\frac{1}{2\lambda}\tilde{b}Q\left(\tilde{x}\right)-C\right)(1-F(\tilde{b}))+\left(1-\frac{4\lambda K}{B(\tilde{b})\tilde{x}}\right)\frac{1}{4\lambda}B(\tilde{b})Q(\tilde{x}),\\ &= \left(\frac{1}{2\lambda}\tilde{b}(1-F(\tilde{b}))+\frac{B(\tilde{b})}{4\lambda}-\frac{K}{\tilde{x}}\right)Q(\tilde{x})-C(1-F(\tilde{b})).\end{aligned} \tag{8}$$

We analyze the platform's problem by considering an equivalent problem where the platform chooses $\{\tilde{x},\tilde{b}\}$ to maximize its profit (8). Participation conditions (6) and (7) show that a given $\{\tilde{x},\tilde{b}\}$ allocation can be implemented by a unique fee combination $\{S,\tau\}$ as illustrated in Figure 1. Once we determine equilibrium choices for the marginal consumer and marginal app types, we back out the associated pair of $\{S,\tau\}$ that delivers the high-participation equilibrium (the lower intersection point of participation curves on Figure 1) since we show below that the higher intersection of the participation curves in Figure 1 is unstable and the lower intersection point Pareto dominates the high one. This approach is similar to Weyl (2010)'s "insulated tariff equilibrium".[10] The first-order condition of (8) with respect to the marginal consumer type is

$$FOC_{\tilde{b}}:\frac{\partial\Pi}{\partial\tilde{b}}=\left[1-F(\tilde{b})-\tilde{b}f(\tilde{b})+\frac{1}{2}B'(\tilde{b})\right]\frac{Q(\tilde{x})}{2\lambda}+Cf(\tilde{b})=0.$$

We have $B'(\tilde{b})=-\tilde{b}f(\tilde{b})$, so the condition becomes

$$FOC_{\tilde{b}}:\frac{\partial\Pi}{\partial\tilde{b}}=\left[1-F(\tilde{b})-\frac{3}{2}\tilde{b}f(\tilde{b})\right]\frac{Q(\tilde{x})}{2\lambda}+Cf(\tilde{b}) = 0. \tag{9}$$

Let $\tilde{b}^{*}(\tilde{x})$ denote $\tilde{b}$ satisfying (9).

The first-order condition for the equilibrium marginal app type is

$$FOC_{\tilde{x}}:\frac{\partial\Pi}{\partial\tilde{x}}=\left[\frac{1}{2\lambda}\tilde{b}(1-F(\tilde{b}))+\frac{B(\tilde{b})}{4\lambda}-\frac{K}{\tilde{x}}\right]Q'(\tilde{x})+\left(\frac{K}{\tilde{x}^{2}}\right)Q(\tilde{x})=0. \tag{10}$$

[10] Note that we can just as well consider the platform's problem as choosing the levels of Q and B, or participations $1-F(\tilde{b})$ and $1-G(\tilde{x})$. To fix the ideas the platform's equilibrium choices are the marginal types on both sides.

Let $\tilde{x}^*(\tilde{b})$ denote $\tilde{x}$ satisfying (10).

We proceed via a series of properties of the first-order conditions, the platform's problem, and of the supporting subscription price and commission rate. We then illustrate these with uniform type distributions.

We first show the strict quasi-concavity of the platform's profit:

Lemma 1 *The platform's profit (8) is strictly quasi-concave in $\tilde{b}$ and $\tilde{x}$.*

Hence, the solution to (9) and (10) at which the remaining second-order condition holds: $\frac{d^2\Pi}{d\tilde{b}^2}\frac{d^2\Pi}{d\tilde{x}^2} > \left(\frac{d^2\Pi}{d\tilde{b}d\tilde{x}}\right)^2$, characterizes the platform's equilibrium choices. To understand better the key properties of the equilibrium allocations, we document first some important properties of $\tilde{b}^*(\tilde{x})$ satisfying (9):

Lemma 2 *For any $\tilde{x} \in [0, \overline{x}]$, $\tilde{b}^*(\tilde{x}) > 0$ and $\tilde{b}^*(\tilde{x})$ is strictly increasing in C.*

i. If $C > 0$, $\tilde{b}^*(\tilde{x})$ is strictly increasing in $\tilde{x}$. If $C = 0$, $\tilde{b}^*(\tilde{x}) \equiv \tilde{b}_o$ is constant in $\tilde{x}$.

iii. $\tilde{b}^*(\tilde{x})$ is a convex function of $\tilde{x}$ if $g(\tilde{x})$ is inelastic, which is the case for $g(\tilde{x})$ uniform.

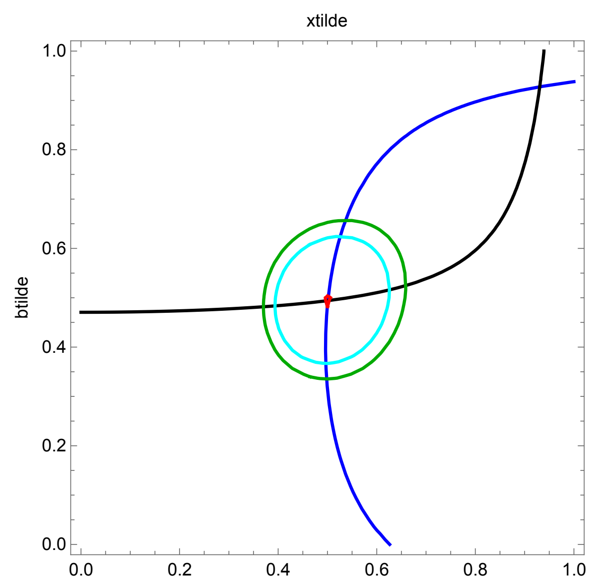

Figure 2: Isoprofit curves (green curves), the platform's equilibrium choice (the red point) is the intersection of $\tilde{b}^*(\tilde{x})$ (black curve) and $\tilde{x}^*(\tilde{b})$, (blue curve) for uniformly distributed $\{b, x\} \sim [0, 1]$, and parameter values $C = K = \lambda = 0.21$.

The black curve in Figure 2 illustrates $\tilde{b}^*(\tilde{x})$ as an increasing and convex function of $\tilde{x}$ when consumer types b are uniformly distributed over $[0, 1]$. Increasing C shifts $\tilde{b}^*(\tilde{x})$ up, resulting in higher marginal consumer type for any given $\tilde{x}$. As shown in Lemma 2 these properties of $\tilde{b}^*(\tilde{x})$ apply more generally.

We next document some important properties of $\tilde{x}^*(\tilde{b})$ satisfying (10):

Lemma 3 *For any $\tilde{b} \in [0, \overline{b}]$, $\tilde{x}^*(\tilde{b}) > 0$ and $\tilde{x}^*(\tilde{b})$ is strictly increasing in K.*

i. $\frac{d\tilde{x}^*(\tilde{b})}{d\tilde{b}} < 0$ *for* $\tilde{b} < \tilde{b}_o$ *and* $\frac{d\tilde{x}^*(\tilde{b})}{d\tilde{b}} > 0$ *for* $\tilde{b} > \tilde{b}_o$ *with* $\frac{d\tilde{x}^*(\tilde{b}_o)}{d\tilde{b}} = 0$.

ii. $\tilde{x}^*(\tilde{b})$ *is a convex function of* $\tilde{b}$ *if* $f(\tilde{b})$ *is inelastic, which is the case for* $f(\tilde{b})$ *uniform.*

The blue curve in Figure 2 illustrates $\tilde{x}^*(\tilde{b})$ as a convex U-shaped function of $\tilde{b}$ when app types x are uniformly distributed over $[0, 1]$. Increasing K shifts $\tilde{x}^*(\tilde{b})$ up, resulting in a higher marginal app type for any given $\tilde{b}$. As shown in Lemma 3 these properties of $\tilde{x}^*(\tilde{b})$ apply more generally.

Using Lemmas 1, 2 and 3, we characterize the platform's equilibrium choices $(\tilde{b}^*, \tilde{x}^*)$:

Proposition 1 *The platform implements a marginal agent types given by (9) and (10) with $\tilde{b}^* > 0$ and $\tilde{x}^* > 0$ where*

i. $\tilde{b}^*(\tilde{x}) = \tilde{b}_o$ *if* $C = 0$ *and* $\tilde{b}^*$ *is strictly increasing in* C *and* K.

ii. $\tilde{x}^*$ *is strictly increasing in* C *and* K.

iii. $0 < \frac{d\tilde{b}^*(\tilde{x})}{d\tilde{x}} < \frac{1}{\frac{d\tilde{x}^*(\tilde{b})}{d\tilde{b}}}$

Using (9) and (10) we show that the slope condition in Proposition 1(ii) holds if and only if the remaining second-order condition is satisfied: $\frac{d^2\Pi}{d\tilde{b}^2}\frac{d^2\Pi}{d\tilde{x}^2} - \left(\frac{d^2\Pi}{d\tilde{b}d\tilde{x}}\right)^2 > 0$. Lemmas 2 and 3 imply that when $C > 0$ the equilibrium is on an upward sloping part of both first-order conditions. This

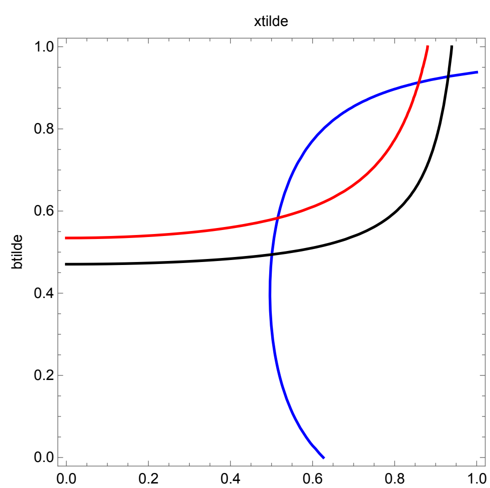

Figure 3: The equilibrium is the intersection of $\tilde{b}^*(\tilde{x})$ (black curve) and $\tilde{x}^*(\tilde{b})$ (blue curve) for uniformly distributed $\{b, x\} \sim [0, 1]$, and $C = K = \lambda = 0.21$. $\tilde{b}^*(\tilde{x})$ shifts up (the red curve) when $C = 0.4$ leading to the new equilibrium (intersection of the red and blue curves).

combined with the slope condition deliver intuitive comparative statics for the equilibrium. Increasing C shifts $\tilde{b}^*(\tilde{x})$ upward resulting in a higher $\tilde{b}^*$ and $\tilde{x}^*$ in equilibrium (Figure 3). Similarly, increasing K shifts $\tilde{x}^*(\tilde{b})$ right resulting in a higher $\tilde{b}^*$ and $\tilde{x}^*$ in equilibrium (Figure 4).

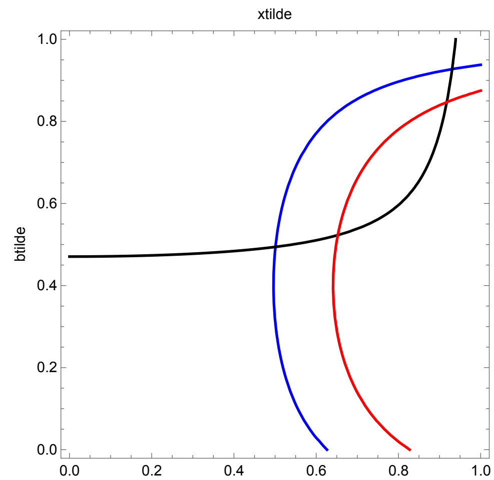

Figure 4: The equilibrium is the intersection of $\tilde{b}^*(\tilde{x})$ (black curve) and $\tilde{x}^*(\tilde{b})$ (blue curve) foruniformly distributed $\{b, x\} \sim [0, 1]$, and $C = K = \lambda = 0.21$. $\tilde{x}^*(\tilde{b})$ shifts right (the red curve) when $K = 0.4$ leading to the new equilibrium (intersection of the red and black curves).

Convexity conditions of $\tilde{b}^*(\tilde{x})$ and $\tilde{x}^*(\tilde{b})$ (given by Lemmas 2(ii) and 3)(ii) ensure that the first-order conditions intersect at most at two points. The lower intersection point of the first-order conditions, which satisfies the slope condition given in Proposition 1(iii), is therefore the global maximum for the platform's problem

In the uniform example illustrated in Figure 2 the black increasing and convex curve is $\tilde{b}^*(\tilde{x})$ (9) and the blue U-shaped curve is $\tilde{x}^*(\tilde{b})$ (10). The uniform distribution satisfies the convexity conditions of $\tilde{b}^*(\tilde{x})$ and $\tilde{x}^*(\tilde{b})$, and therefore these curves have two intersection points. The lower one (the red point) satisfies the slope condition, so gives the unique equilibrium allocations $(\tilde{b}^*, \tilde{x}^*)$. Figures 3 and 4 illustrate the comparative statics with respect to C and K respectively.

The platform's equilibrium choices $(\tilde{b}^*, \tilde{x}^*)$ are uniquely implemented by (7) and (6):

Corollary 1 *The equilibrium prices are* $S^* = \frac{\tilde{b}^*}{2\lambda} Q(\tilde{x}^*)$ *and* $\tau^* = 1 - \frac{4\lambda K}{B(\tilde{b}^*)\tilde{x}^*}$. *The equilibrium device fee is positive,* $S^* > 0$, *and app commission is positive if and only if* $K < \frac{B(\tilde{b}^*)\tilde{x}^*}{4\lambda}$.

As shown in footnotes 8 and 9 the participation curves are convex if and only if distribution functions are inelastic ($f(b) + bf'(b) \geq 0$ and $g(x) + xg'(x) \geq 0$). In that case, the app PC and the consumer PC intersect at most twice. As noted earlier, the lower intersection is the stable one and Pareto dominates the intersection with lower participation on both sides. Figure 5 illustrates the participation curves implemented by the equilibrium fees in our example. The platform's equilibrium choice of marginal types corresponds to the lowest intersection of the app participation curve (orange curve) and the consumer participation curve (the purple curve).

Corollary 1 shows that the equilibrium device fee is positive irrespective of the externalities between consumers and apps, and irrespective of the device cost for the platform. This result

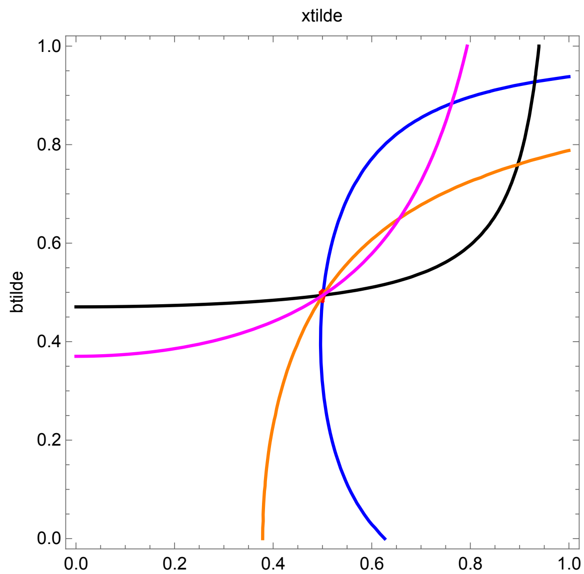

Figure 5: The platform's equilibrium choice is at the lower intersection of the App PC (orange curve) and the Consumer PC (pink curve), for uniformly distributed $\{b, x\} \sim [0, 1]$, and parameter values $C = K = \lambda = 0.21$.

differs from the literature which finds that the equilibrium fee to each side of the market can be negative if the degree of positive externalities that side imposes on the other are sufficiently higher than the cost of serving that side, which is also known as opportunity cost pricing, see for example Rochet and Tirole (2006) and Armstrong (2006).

The shapes of the various relations in Figures 1 through 5 for the uniform distribution represent well the key properties of the general distribution specification. Pulling together the various properties noted above we have the following take-aways. For $C = 0$ the consumer-side optimality condition ($\tilde{b}^* (\tilde{x})$) is flat and goes through the minimum of the app-side optimality condition ($\tilde{x}^* \left(\tilde{b}\right)$), which intersection is therefore the equilibrium. For $C > 0$, the consumer-side curve is increasing so the equilibrium intersection is thus on the up-sloping part of the app-side curve, with $\tilde{b}^* (\tilde{x})$ cutting $\tilde{x}^* \left(\tilde{b}\right)$ from above to deliver the comparative static properties.

In the next section we define surpluses of users (consumers and apps) and total welfare, and assess the performance of the platform's choices from the view point of consumers and apps, their total surplus, and total welfare.

4 Consumer Surplus, App Surplus, and Welfare

For given marginal types $(\tilde{x}, \tilde{b})$ (i.e., given participation levels on each side) and device fee S consumer surplus from the app store is the sum of transaction surplus of consumers minus device fee aggregated over those consumer types who join the platform:

$$CS(S,\tilde{x},\tilde{b}) \;=\; \int_{\tilde{b}}^{\overline{b}} \left(\frac{1}{2\lambda} b Q(\tilde{x}) - S\right) f(b)db.$$

Similar to the platform's profit, we re-write consumer surplus by substituting the device fee (6) that induces $\tilde{b}$ given $\tilde{x}$:

$$CS(\tilde{x},\tilde{b}) \;=\; \frac{1}{2\lambda}\left[B(\tilde{b}) - \tilde{b}(1-F(\tilde{b}))\right] Q(\tilde{x}), \tag{11}$$

which increases in consumer participation, $\frac{\partial CS}{\partial \tilde{b}} < 0$, since $B'(\tilde{b}) < 0$, and increases in app participation, $\frac{\partial CS}{\partial \tilde{x}} < 0$ since $Q'(\tilde{x}) < 0$.

For given marginal types $(\tilde{x},\tilde{b})$ and the app commission τ, app surplus is the revenue of each app from in-app transactions minus the development cost of the app, aggregated over apps that join the platform:

$$AS(\tau,\tilde{x},\tilde{b}) \;=\; \int_{\tilde{x}}^{\overline{x}} \left((1-\tau)\frac{x}{4\lambda}B(\tilde{b}) - K\right) g(x)dx = (1-\tau)\frac{1}{4\lambda}B(\tilde{b})Q(\tilde{x}) - K(1-G(\tilde{x})).$$

As before, we re-write app surplus by substituting the app commission (7) that induces $\tilde{x}$ given $\tilde{b}$:

$$AS(\tilde{x}) \;=\; K\left(\frac{Q(\tilde{x})}{\tilde{x}} - (1-G(\tilde{x}))\right). \tag{12}$$

Analogous to consumer surplus, app surplus decreases in the marginal app type:

$$AS'(\tilde{x}) = K\left(\frac{Q'(\tilde{x})\tilde{x} - Q(\tilde{x})}{(\tilde{x})^2} + g(\tilde{x})\right) = -K\frac{Q(\tilde{x})}{(\tilde{x})^2} < 0. \tag{13}$$

In contrast to consumer surplus, *the marginal app type $\tilde{x}$ is a sufficient statistic for app surplus*.[11] This difference arises from the asymmetry of the fee structure on the two sides: consumers pay a fixed subscription fee and apps pay a percentage commission over their in-app purchase revenues.

Total user surplus is the sum of consumer surplus and app surplus: $TUS = CS + AS$, that is, total welfare minus the platform profit:

$$TUS(\tilde{x},\tilde{b}) \;=\; \frac{3}{4\lambda}B(\tilde{b})Q(\tilde{x}) - K(1-G(\tilde{x})) - C(1-F(\tilde{b})) - \Pi(\tilde{x},\tilde{b}) \tag{14}$$

Now consider how the platform's choices compare to those maximizing the total user surplus,

[11] This is an important property for later because app surplus is ensured to rise whenever $\tilde{x}$ falls, while $\tilde{b}$ falling is not sufficient to conclude that consumer surplus rises.

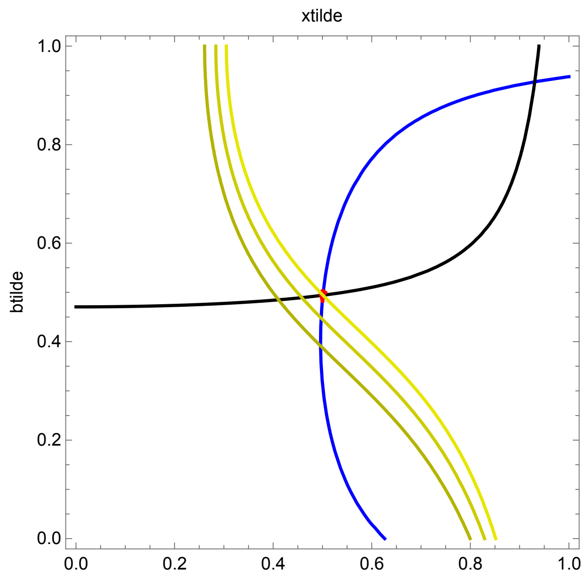

Figure 6: Iso-TUS curves (light green curves), the platform's equilibrium choice (the red point) for uniformly distributed types $\{b, x\} \sim [0, 1]$, and parameter values $C = K = \lambda = 0.21$.

TUS, given in (14). Given that consumer suplus (11) increases in participation on both sides, app surplus (12) increases in participation on the app side, without any constraint, both surpluses are maximized with full participation on both sides. The platform's optimal pricing therefore induces too low consumer participation and too low app entry compared to the allocations that maximize total user surplus.

Figure 6 shows the iso-TUS curves (light green curves); the total user surplus increases as move towards the south-west direction. The lightest green TUS curve passes through the equilibrium and so it represents the level of the total user surplus at the equilibrium choices of the platform for parameters $C = K = \lambda = 0.21$. Darker green iso-TUS curves correspond to a larger total user surplus.

Total welfare is the sum of the TUS and the platform profit:

$$W(\tilde{x}, \tilde{b}) = \frac{3}{4\lambda} B(\tilde{b})Q(\tilde{x}) - K(1 - G(\tilde{x})) - C(1 - F(\tilde{b})). \tag{15}$$

Optimizing gives the welfare maximizing allocations and supporting prices are calculated using the participation condition of buyers and sellers. The following summarizes the welfare results:

Proposition 2 *Taking as given the equilibrium app pricing, $p^* = \lambda$, the welfare maximizing marginal consumer type and the marginal app type are respectively*

$$\tilde{b}^W = \frac{4C\lambda}{3Q(\tilde{x}^W)}, \qquad \tilde{x}^W = \frac{4K\lambda}{3B(\tilde{b}^W)}.$$

which are supported by fees below cost: $S^W = \frac{2C}{3} < C$ *and* $\tau^W = -2$*, and the platform makes an operating loss.*

Hence, the fees that sustain the welfare maximizing allocations require subsidizing consumers, $S^W = \frac{2C}{3} < C$, as long as $C > 0$, and subsidizing apps, $\tau^W = -2 < 0$. From the values in Proposition 2 we never have all users on board at the (constrained) optimum if there is a positive device cost and if there is a positive cost of app development. Following the objectives of the policy makers in the digital markets, for example, the EC's Digital Markets Act and Digital Services Act, we focus on total user surplus as the main objective of the social planner and study potential policy interventions that could lead to higher total user surplus, ignoring the profits of the platform. A policy that increases participation on both sides or only on one side of the market can achieve this. We next analyze the effectiveness of such policies.

5 Applications of the model for policy questions

We use our framework to address four policy questions. The first two have been discussed among regulators and anti-trust agencies as interventions to increase total user surplus from app stores: 1) Introducing a cap on third-party app commission (τ in our model), 2) Allowing third-party payment processor on the platform. Related to the first question, as a third intervention we analyze the implications of a cap on device fee and compare its performance to the one of a cap on third-party app commission. In the fourth application we discuss the implications of the app store's own apps replacing the infra-marginal third-party apps (hybrid platform). To simplify the analysis of applications we assume zero fixed cost for device, $C = 0$, in this section. We present an important comparative statics property that we use in the analysis below:

Lemma 4 *A lower app development cost K induces the platform to raise its app commission and device fee. More apps enter and the same set of consumers buy the device.*

When development costs are lower, more apps enter, and so more consumers buy the device if the platform collects the same commission and the same device fee. The platform raises its app commission to capture more surplus from apps, but not so much as to offset the reduction in app cost, so this leads to more app entry. The platform raises its device fee to fully capture the marginal consumer's additional surplus arising from more apps. As a result, consumer participation stays the same.

5.1 Cap on commission τ

Consider a binding cap on third-party app commission, $\overline{\tau} < \tau^*$. The app entry condition gives us a relationship between the marginal app type, $\tilde{x}$, and the marginal consumer type, $\tilde{b}$, for a fixed

$\tau = \overline{\tau}$ as $\overline{\tau} = 1 - \frac{4\lambda K}{B(\tilde{b})\tilde{x}}$. In the unconstrained problem (without the cap), holding $\tilde{x}$ fixed, when the platform changes $\tilde{b}$, it changes τ (higher $\tilde{b}$ would reduce $B(\tilde{b})$ and so reduce τ). But now with the binding cap, the platform has to change $\tilde{x}$ when it moves $\tilde{b}$ so that the commission stays at $\overline{\tau}$, that is, $B(\tilde{b})\tilde{x}$ must stay constant. Higher $\tilde{b}$ requires higher $\tilde{x}$ to keep τ constant since $B'(\tilde{b}) < 0$. The platform maximizes its profit with respect to the marginal types, $(\tilde{x}, \tilde{b})$, subject to the cap constraint:

$$\max_{\{\tilde{x},\tilde{b}\}} \left[\frac{1}{2\lambda}\tilde{b}(1 - F(\tilde{b})) + \frac{B(\tilde{b})}{4\lambda} - \frac{K}{\tilde{x}}\right] Q(\tilde{x}) \qquad \text{s.t.} \qquad \overline{\tau} = 1 - \frac{4\lambda K}{B(\tilde{b})\tilde{x}}.$$

The constraint determines the marginal app type as an implicit function of the marginal consumer type via $\frac{d\tilde{x}^{\overline{\tau}}}{d\tilde{b}} = -\frac{B'(\tilde{b})\tilde{x}}{B(\tilde{b})} > 0$ (since $B'(\tilde{b}) < 0$). The platform therefore has a single choice variable, $\tilde{b}$. The first-order condition with respect to $\tilde{b}$ is

$$\begin{aligned} \frac{d\Pi}{d\tilde{b}} &= \frac{1}{2\lambda}\left[(1 - F(\tilde{b})) - \frac{3}{2}\tilde{b}f(\tilde{b})\right] Q\left(\tilde{x}\right) \\ &+ \left(\frac{1}{2\lambda}\left[\tilde{b}(1 - F(\tilde{b})) + B(\tilde{b})\frac{1}{2} - \frac{K}{\tilde{x}}\right] Q'(\tilde{x}) + \frac{K}{\tilde{x}^2}Q(\tilde{x})\right)\frac{d\tilde{x}^{\overline{\tau}}}{d\tilde{b}} = 0. \end{aligned} \tag{16}$$

Proposition 3 *A binding cap regulation on third-party app increases app participation and lowers consumer participation on the platform. The commission cap increases app surplus.*

The cap on commissions reduces the platform's commission from apps, which increases app entry, and so increases consumer gross surplus from device. The platform raises its device fee to capture more revenues from consumers. This in turn results in lower consumer participation.

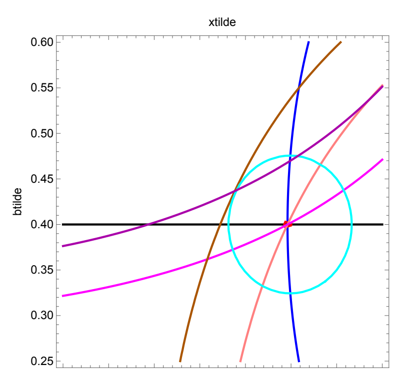

Figure 7: A binding τ cap (App PC) moves the equilibrium to the light green isoprofit (tangent to the brown), leading to lower consumer participation (new Consumer PC). The figure is drawn for $\{b, x\} \sim U[0, 1]$, and parameter values $C = 0, K = \lambda = 0.21$.

Figure 7 illustrates how a binding cap on τ affects the platform's equilibrium choices. The brown curve represents the app participation under a binding cap on τ shifting the app participation curve, the orange curve, up-left: for any level of consumer participation, there is more app entry. The constrained equilibrium point corresponds to the platform iso-profit which is tangent to the brown curve, the light green iso-profit. Hence, the cap induces a higher level of app participation (lowers $\tilde{x}^*$) and a lower level of consumer participation (higher $\tilde{b}^*$).

Recall that app surplus is independent of the marginal consumer type and is decreasing in the marginal app type (see 12). App surplus therefore increases due to the cap. On the other hand, consumer surplus is decreasing in both the marginal consumer type and the marginal app type (see 11). The cap raises the marginal consumer and lowers the marginal app, and therefore its effect on consumer surplus is in general unclear. Given that the cap raises the marginal consumer type, those consumer types that are excluded lose from the cap. The new marginal type and also those that are slightly above the new marginal type also lose due to the cap, since they were making strictly positive surplus before, but now they get zero or nearly zero surplus. However, consumers who value app quality a lot (inframarginal consumers) benefit more from app entry than the marginal type. Thus, their gains from more apps are higher than their losses from the increased device fee. Their surplus increases due to the cap. Whether total consumer surplus increases or decreases due to the commission cap depends in general on the distribution of consumer types (comparing the gains of high types from increased quality and the losses of low types from increased price) and also on the distribution of app types (how much lowering cap increases the participation of apps). We next show that for uniform distribution the cap harms consumers in aggregate:

Corollary 2 *Suppose consumer and app types, (b, x), are uniformly distributed over $[0, 1]$. A binding cap regulation on third-party app commission lowers aggregate consumer surplus.*

This shows that a binding cap on commissions can have an unforeseen consequence of lowering consumer surplus via raising consumer device fee that the platform sets.

The cap lowers $\tilde{x}$ by $\Delta\tilde{x} = \tilde{x} - \tilde{x}^C$ and raises $\tilde{b}$ by $\Delta\tilde{b} = \tilde{b}^C - \tilde{b}$. Using (11) we define the change in consumer surplus due to the cap as

$$\Delta CS = CS(\tilde{x}^C, \tilde{b}^C) - CS(\tilde{x}^*, \tilde{b}^*)$$

By adding and subtracting $CS(\tilde{x}^C, \tilde{b}^*)$ to the right hand side, we rewrite the consumer surplus

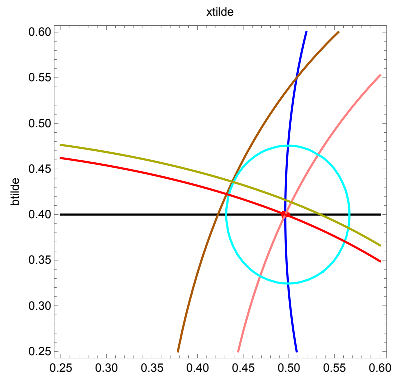

Figure 8: A binding τ cap reduces consumer surplus from the red iso-consumer surplus curve to the dark green iso-consumer surplus curve. The figure is drawn for $\{b, x\} \sim U[0, 1]$, and parameter values $C = 0, K = \lambda = 0.21$.

change as

$$\begin{aligned}\Delta CS &= \int_{\tilde{b}^*}^{\tilde{b}^{C*}} \frac{\partial CS(\tilde{x}^{C*}, \tilde{b})}{\partial \tilde{b}} d\tilde{b} - \int_{\tilde{x}^{C*}}^{\tilde{x}^*} \frac{\partial CS(\tilde{x}, \tilde{b}^*)}{\partial \tilde{x}} d\tilde{x}, \\ &= -\frac{1}{2\lambda} Q(\tilde{x}^{C*}) \int_{\tilde{b}^*}^{\tilde{b}^{C*}} (1 - F(\tilde{b})) d\tilde{b} - \frac{(1 - F(\tilde{b}^*))}{2\lambda} \left[\frac{B(\tilde{b}^*)}{1 - F(\tilde{b}^*)} - \tilde{b}^* \right] \left(Q(\tilde{x}^*) - Q(\tilde{x}^{C*}) \right).\end{aligned}$$

where the first term captures losses of consumers from the raised device fee and these losses are proportional to the app quality at the new equilibrium, $Q(\tilde{x}^{C*})$. The second term captures the gains of consumers from increased app participation and these gains are proportional to the difference between the average and the marginal consumer at the original equilibrium level of consumer participation: $(1 - F(\tilde{b}^*)) \left[\frac{B(\tilde{b}^*)}{1-F(\tilde{b}^*)} - \tilde{b}^* \right]$. Thus, consumers are worse off by the cap when the losses (the first term) are greater in absolute value than the gains (the second term). The comparison of consumer losses and gains depends on the distribution of consumer types, $f(\tilde{b})$, and app types, $g(\tilde{x})$. Figure 8 illustrates how a binding cap on τ reduces consumer surplus in the case of the uniform distribution. The red and the dark green curves are iso-consumer surplus curves.Consumer surplus increases towards south-west direction (towards having more consumers and more apps participating). The red iso-consumer surplus goes through the initial equilibrium point, so it represents the level of consumer surplus at the initial equilibrium choices of the platform. The dark green curve goes through the new equilibrium (constrained equilibrium with the binding cap on τ). As the figure illustrates, a cap on app commission lowers consumer surplus.

5.2 Cap on device fee S

Consider a binding cap on the device fee: $\overline{S} < S^*$. The consumer participation condition that relates the marginal app type, $\tilde{x}$, and the marginal consumer type, $\tilde{b}$, for a fixed S at $\overline{S}$ is: $\overline{S} = \frac{\tilde{b}}{2\lambda}Q(\tilde{x})$. In the unconstrained problem (without the cap), holding $\tilde{x}$ fixed, the platform would implement a higher $\tilde{b}$ by increasing S. But now with the binding cap, the platform has to change $\tilde{x}$ when it moves $\tilde{b}$ so that the device fee stays at $\overline{S}$. Higher $\tilde{b}$ requires higher $\tilde{x}$ to keep S constant since $Q'(\tilde{x}) < 0$. The platform maximizes its profit with respect to the marginal types, $(\tilde{x}, \tilde{b})$, subject to the cap constraint:

$$max_{\{\tilde{x},\tilde{b}\}} \left[\frac{1}{2\lambda}\tilde{b}(1-F(\tilde{b})) + \frac{B(\tilde{b})}{4\lambda} - \frac{K}{\tilde{x}}\right] Q(\tilde{x}) \qquad \text{s.t.} \qquad \overline{S} = \frac{\tilde{b}}{2\lambda}Q(\tilde{x})$$

The constraint determines the marginal app type as an implicit function of the marginal consumer type and we have $\frac{d\tilde{x}^{\overline{S}}}{d\tilde{b}} = -\frac{Q(\tilde{x})}{Q'(\tilde{x})\tilde{b}} > 0$, since $Q'(\tilde{x}) < 0$. The platform therefore has a single choice variable, $\tilde{b}$. The first-order condition with respect to $\tilde{b}$ is

$$\begin{aligned} \frac{d\Pi}{d\tilde{b}} &= \frac{1}{2\lambda}\left[(1-F(\tilde{b})) - \frac{3}{2}\tilde{b}f(\tilde{b})\right] Q(\tilde{x}) \\ &\quad + \left(\frac{1}{2\lambda}\left[\tilde{b}(1-F(\tilde{b})) + B(\tilde{b})\frac{1}{2} - \frac{K}{\tilde{x}}\right] Q'(\tilde{x}) + \frac{K}{\tilde{x}^2}Q(\tilde{x})\right)\frac{d\tilde{x}^{\overline{S}}}{d\tilde{b}} = 0. \qquad (17) \end{aligned}$$

Proposition 4 *A binding cap regulation on consumer device fee increases consumer participation and decreases app participation to the platform. The cap lowers app surplus.*

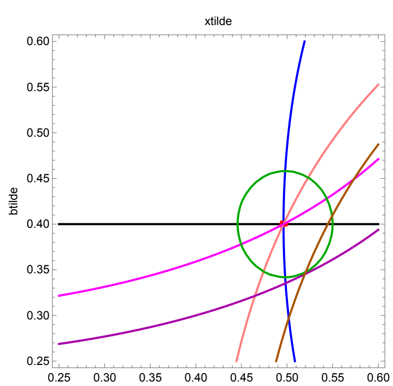

Figure 9: A binding S cap (Consumer PC) moves the equilibrium to the dark green isoprofit (tangent to the PCC), leading to lower app participation (new App PC). The figure is drawn for $\{b, x\} \sim U[0, 1]$, and parameter values $C = 0, K = \lambda = 0.21$.

Figure 9 illustrates how a binding cap on S affects the platform's equilibrium choices. The dark purple curve represents the consumer participation under a binding cap on S shifting the consumer participation curve, the purple curve, down right: for any level of app entry, there is more consumer participation. The constrained equilibrium point corresponds to the platform iso-profit which is tangent to the dark purple curve, the dark green iso-profit curve. Hence, the cap induces a higher level of consumer participation (lowers $\tilde{b}^*$) and a lower level of app participation (raises $\tilde{x}^*$). Given that app surplus decreases with lower app participation (see 12), the S-cap lowers app surplus. As in the case of τ cap, the effect of the S-cap on consumers is in general unclear since consumer surplus increases due to more consumer participation, yet consumer surplus decreases due to less app entry, and balancing consumers gains against consumers loses depend on the distributions of consumer and app types. We show that consumers are better off with a cap on device fee for uniformly distributed types:

Corollary 3 *Suppose consumer and app types,* (b, x)*, are uniformly distributed over* $[0, 1]$*. A binding cap regulation on consumer device fee increases consumer surplus.*

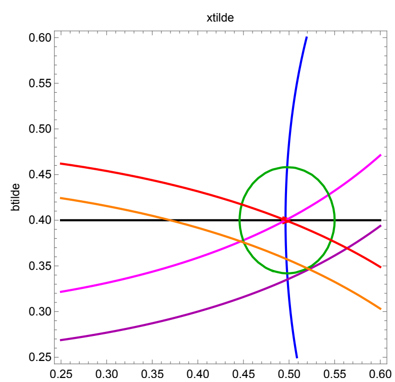

Figure 10: A binding S cap increases consumer surplus from the red iso-consumer surplus to the orange iso-consumer surplus. The figure is drawn for $\{b, x\} \sim U[0, 1]$, and parameter values $C = 0, K = \lambda = 0.21$.

Figure 10 illustrates how a binding cap on S increases consumer surplus. The red and the orange curves are iso-consumer surplus curves. Consumer surplus increases towards south-west direction (towards having more consumers and more apps participating). The red iso-consumer surplus goes through the initial equilibrium point, so it represents the level of consumer surplus at the unconstrained equilibrium choices of the platform. The orange curve goes through the new equilibrium (constrained equilibrium with the binding cap on S). As the figure illustrates, a cap on device fee increases consumer surplus.

Figure 11 illustrates an example where a binding cap on τ can increase the total user surplus by the same amount via a binding cap on S, however the platform profit is higher with the S-cap

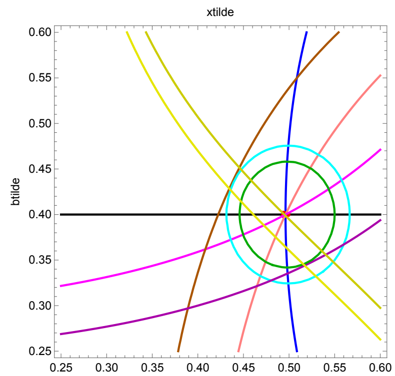

Figure 11: Example of achieving the same total user surplus increase by a binding τ cap and a binding S cap, where the platform has higher profits with the S-cap than with the τ-cap. The figure is drawn for $\{b, x\} \sim U[0, 1]$, and parameter values $C = 0, K = \lambda = 0.21$.

than with the τ-cap. The brown curve represents the app participation curve with the cap and the tangent of the brown curve with the light green iso-profit is the new equilibrium point with the cap. The dark purple curve represents the consumer participation curve with the S-cap and the tangent of the dark purple curve with the dark green iso-profit is the new equilibrium point with the S-cap. Yellow curves are iso-TUS curves where on each curve the total user surplus is the same. At the new equilibrium (with either the τ cap or the S cap) the total user surplus is higher than the equilibrium without any cap, since total user surplus increases when an iso-TUS curve moves towards the south-west direction. The dark green iso-profit is closer to the unconstraint equilibrium profit than the light green iso-profit. Hence, we conclude that a S-cap can be a more effective tool to raise total user surplus than a τ-cap.

5.3 Allowing an alternative payment system for app developers

Now suppose that app developers can shift transactions outside the platform by choosing a different price for transactions outside the platform, p^{off}, than the price for transactions on (inside) the platform, p^{on}. Suppose that consumers incur an inconvenience cost of $c > 0$ (per transaction) if they transact outside the platform, but these costs are not too high to keep the alternative channel as a viable option (see the exact condition below). Suppose that when apps are indifferent between participating to the platform or not, they join the platform. To understand the impact of introducing an alternative payment option for apps, let us first consider the simplest version of our model where both consumers and apps are homogenous.

5.4 Homogenous consumers and homogenous apps

Suppose that consumers have the same benefit b and apps have the same quality x. Assume also that there are efficiency gains from app development: $K \leq \frac{bx}{4\lambda}$. If there is no alternative payment option for an app, given in-app price p, the solution of consumers' in-app purchases is the same as before: $q^*(b, x, p) = \frac{bx}{(\lambda+p)^2}$, and the app developer on the platform sets the same price: $p^* = \lambda$. Given the app is on the platform, the indirect utility of the consumer from joining the platform is

$$V(b, x) = 2\sqrt{bxq^*(b, x, p^*)} - (\lambda + p^*)q^*(b, x, p^*) = \frac{bx}{2\lambda}.$$

Given the consumer joins the platform, the profit of the app developer is

$$\pi(x, b, \tau) = (1 - \tau)\frac{bx}{4\lambda} - K.$$

The platform sets the highest (S, τ) ensuring that the app developer and the consumer both join the platform: $\pi(x, b, \tau^*) - K = 0$, or $\tau^* = 1 - \frac{4\lambda K}{bx}$, $S^* = V(b, x) = \frac{bx}{2\lambda}$. Hence, consumer surplus and app developer surplus are both zero at the equilibrium fees. The platform captures the entire surplus from in-app purchases after leaving the app-developer just enough revenue to cover its fixed cost:

$$\Pi(x, b) = \frac{3bx}{4\lambda} - K.$$

Now consider the case where the app developer has its alternative payment channel and the consumer faces an inconvenience cost $c > 0$ for each transaction on the app developer's payment system rather than transacting on the platform, but c is not too high, $\frac{bx}{4(\lambda+c)} > K$, to ensure that the alternative channel is a viable option. If the consumer chooses to transact on the platform, her demand is, see (4): $q^*(b, x, p^{on}) = \frac{bx}{(\lambda+p^{on})^2}$. If the consumer chooses to transact on the app's alternative channel, she maximizes her utility

$$U(b, x, q, p^{off}) = 2\sqrt{bxq} - (\lambda + c + p^{off})q, \tag{18}$$

and so her demand is $q^*(b, x, p^{off}) = \frac{bx}{(\lambda+c+p^{off})^2}$. Note that the inconvenience cost of transacting outside the platform lowers consumer demand for transaction for any given level of price, p^{off}. Thus, consumer has lower demand for off-platform transactions than the demand on the platform. The consumer would transact on the app's payment channel if and only if her net utility from doing so is higher than the net utility from transacting on the platform:

$$V^{on}(x, b, p^{on}) = \frac{bx}{\lambda + p^{on}} \leq V^{off}(x, b, p^{off}) = \frac{bx}{\lambda + c + p^{off}}, \qquad or \qquad p^{on} \geq p^{off} + c.$$

Now consider the pricing options of the app developer. If the app developer sets $p^{on} \geq p^{off} + c$, it diverts all transactions to its alternative channel and captures entire revenue from app transactions by avoiding commissions: $\pi^{off}(x,b) = p^{off} \frac{bx}{(\lambda+c+p^{off})^2} - K$. In this case, the optimal off-platform price would be $p^{off} = \lambda + c$, which would then lead to profit of $\pi^{off}(x,b) = \frac{bx}{4(\lambda+c)} - K$. If the app developer sets $p^{on} < p^{off} + c$, it keeps all transactions on the platform and captures revenue from app transactions after paying the platform commission: $\pi^{on}(x,b) = (1-\tau)p^{on} \frac{bx}{(\lambda+p^{on})^2} - K$. In this case, the optimal on-platform price would be $p^{on} = \lambda$, which would lead to profit of $\pi^{on}(x,b) = (1-\tau)\frac{bx}{4\lambda} - K$. The app developer prefers to keep all transactions on the platform if and only if $\tau \leq \frac{c}{c+\lambda} \equiv \overline{\tau}$ and its revenues cover the fixed cost: $(1-\overline{\tau})\frac{bx}{4\lambda} \geq K$ or $\frac{bx}{4(\lambda+c)} \geq K$, which we assume to be the case. The app developer then chooses $p^{on} = \lambda$ and p^{off} sufficiently high, $p^{off} > \lambda - c$. Consumer indirect utility from transactions on the platform will be the same as without the alternative payment system: $V(b,x) = \frac{bx}{2\lambda}$. Anticipating this, the platform sets $\tau = \overline{\tau}$ and captures the same fee from consumers, $S^* = \frac{bx}{2\lambda}$. Hence, given the inconvenience of transaction off-platform is not too high relative to the app development cost, $\frac{bx}{4(\lambda+c)} > K$, the alternative payment system shifts app profits from the platform to the app developer, but does not affect the equilibrium prices or consumer surplus. The platform's profit decreases to $\Pi(x,b) = \frac{bx}{2\lambda}\left(\frac{2\lambda+3c}{2\lambda+2c}\right) < \frac{3bx}{4\lambda} - K$ given that $\frac{bx}{4(\lambda+c)} > K$, and the app developer's profit increases from zero to its outside option: $\pi(x,b) = \frac{bx}{4(\lambda+c)} - K$. The following summarizes the effects of introducing an alternative payment channel for the app when consumers and apps are homogenous:

Proposition 5 *Suppose consumers have the same benefit from app transactions and apps have the same quality. Allowing third-party apps to have an alternative payment channel does not affect the equilibrium prices or consumer surplus, but it shifts transaction revenues of apps from the platform to app developers.*

5.5 Model with heterogenous consumers and heterogenous apps

Now we return to our model with heterogeneity of consumers and apps. Different from the previous analysis, there is elastic consumer participation and elastic app participation with heterogeneity on both sides. Apps do not internalize the effect of p^{on} or p^{off} on the consumer traffic to the platform (they take $\tilde{b}$ as given). When they decide whether to divert sales to their payment channel, they do not account for how the inconvenience costs of transacting outside the platform rather than on the platform would affect consumer participation to the platform.

Following similar steps as above, we show that in each app market, consumers prefer to transact off the platform if and only if $p^{on} \geq p^{off} + c$. Each app developer decides whether to divert transactions to its own channel by comparing its expected maximum profit in each case. If the app diverts sales to its own channel, it sets $p^{on} \geq p^{off} + c$ and $p^{off} = \lambda + c$, which leads to the app profit of $\pi^{off}(x,b) = \int_{\tilde{b}}^{\overline{b}} \frac{bx}{4(\lambda+c)} f(b)db - K$. If the app keeps transactions on the platform, it sets $p^{off} > p^{on} - c$ and $p^{on} = \lambda$, which leads to the app profit of $\pi^{on}(x,b) = (1-\tau)\int_{\tilde{b}}^{\overline{b}} \frac{bx}{4\lambda} f(b)db - K$. Hence, as before, if $\tau > \overline{\tau} = \frac{c}{c+\lambda}$, all apps again choose $p^{off} = \lambda + c$ and p^{on} high enough so that all users of the platform choose to transact outside the platform. If $\tau \leq \overline{\tau} = \frac{c}{c+\lambda}$, apps choose $p^{on} = \lambda$ and p^{off} high enough that all platform users choose to transact on the platform. Given this, the platform's problem is the same as its problem subject to a τ cap at $\overline{\tau} = \frac{c}{c+\lambda}$. Proposition 3 then implies that if this cap is binding, that is, if $\tau^* > \overline{\tau}$, the marginal app type decreases in equilibrium, leading to higher app surplus, and consumer subscription fee increases in equilibrium (the marginal consumer type increases). This leads to lower consumer surplus in the case of uniform distribution, Corollary 2. The following summarizes the effects of introducing an alternative payment channel for apps in our model:

Proposition 6 *When apps can divert in-app purchases to their alternative payment channel where consumers face an additional transaction cost of $c > 0$, the platform has to set its commission below $\overline{\tau} = \frac{c}{c+\lambda}$. If this cap is binding, it lowers consumer participation, increases app participation and increases app surplus. Otherwise, the platform's equilibrium choices are the same as Proposition 1.*

5.6 Hybrid app store

App stores typically include some apps from the store-owner. For example, Apple Music, Apple TV, Apple Arcade are Apple-owned apps and similarly Google owns various types of apps including maps, cloud, TV, Docs, YouTube, etc. We here show how ownership structure of the app store impacts performance and pricing (in-app pricing, commission rates, and device prices). We do not attempt to model how many or which apps are in-house. Clearly, if there is no impediment, the app-store owner would like to operate all the extant apps, although doing so would create an expropriation problem such that future developers would not wish to invest, etc. We concentrate on exogenous ownership structure for simplicity via the expedient thought experiment of replacing some of the infra-marginal third-party apps by in-house apps.

Let the platform introduce its own apps at the same stage as choosing a commission for third-party apps and the device price.

5.6.1 Exogenous app pricing of the platform at $p^h = \lambda$

First, assume that the platform prices its apps just like the other app suppliers. So it sets $p^h = \lambda$ (regardless of τ). From this assumption we draw some strong and clear conclusions. We will then look at the platform's optimal pricing of its apps to show how this modulates our conclusions.

Recall that $Q(\tilde{x}) = \int_{\tilde{x}}^{\overline{x}} xg(x)dx$ is the expected app "quality" on the platform (2). We now define $Q^P(\tilde{x}) = \int_{\tilde{x}}^{\overline{x}} xg^p(x)dx$ as the expected quality of the platform's apps, with $g^p(x) \leq g(x)$ as the density of the platform-owned apps. That is, at any quality level x the platform replaces a fraction $\frac{g^p(x)}{g(x)}$ of the existing apps without changing the quality of the replaced apps. We assume that $g^p(x) = 0$ in the neighborhood of $\tilde{x}$ in order to guarantee that the platform apps are infra-marginal ones: we do not consider the entry of marginal platform apps.

The platform does not impact the marginal app entering the platform, which is done by third party apps. The marginal consumer's entry decision is still characterized by $S = \frac{1}{2\lambda}\tilde{b}Q(\tilde{x})$ (see (6)) because the total quality is the same even under replacement and the total consumer surplus from the app store is still $CS(S, \tilde{x}, \tilde{b}) = \frac{1}{2\lambda}B(\tilde{b})Q(\tilde{x}) - S(1 - F(\tilde{b}))$. The marginal app's entry decision is still made by third party apps and characterized by (7) as $\tau = 1 - \frac{4\lambda K}{B(\tilde{b})\tilde{x}}$.

The platform's profit changes since in addition to collecting τ commission on all third-party apps, it gets additional $(1-\tau)$ fraction of the revenue from its own apps. This is effectively a transfer of rents from the third-party apps to the platform due to app replacement. Now the platform's profit is

$$\begin{aligned} \Pi^h(S,\tau,\tilde{x},\tilde{b}) &= S(1-F(\tilde{b})) + \tau\lambda \int_{\tilde{x}}^{\overline{x}} \int_{\tilde{b}}^{\overline{b}} \frac{bx}{4\lambda^2} f(b)g(x)dbdx + (1-\tau)\lambda \int_{\tilde{x}}^{\overline{x}} \int_{\tilde{b}}^{\overline{b}} \frac{bx}{4\lambda^2} f(b)g^p(x)dbdx \\ &= S(1-F(\tilde{b})) + \frac{1}{4\lambda}\tau B(\tilde{b})Q(\tilde{x}) + \frac{1}{4\lambda}(1-\tau) B(\tilde{b})Q^P(\tilde{x}). \end{aligned} \tag{19}$$

Compared to the platform's profit in the benchmark, the last term on each line is the additional revenue from platform-owned apps. As before, by substituting S and τ from the marginal agents' conditions ($S = \frac{1}{2\lambda}\tilde{b}Q(\tilde{x})$ and $\tau = 1 - \frac{4\lambda K}{B(\tilde{b})\tilde{x}}$) we rewrite the hybrid platform profit as

$$\begin{aligned} \Pi^h(\tilde{x},\tilde{b}) &= \frac{1}{2\lambda}\tilde{b}Q(\tilde{x})(1-F(\tilde{b})) + \frac{1}{4\lambda}\left(1 - \frac{4\lambda K}{B(\tilde{b})\tilde{x}}\right) B(\tilde{b})Q(\tilde{x}) + \frac{1}{4\lambda}\left(\frac{4\lambda K}{B(\tilde{b})\tilde{x}}\right) B(\tilde{b})Q^P(\tilde{x}) \\ &= \frac{1}{2\lambda}\left[\tilde{b}(1-F(\tilde{b})) + \frac{1}{2}\left(B(\tilde{b}) - \frac{4\lambda K}{\tilde{x}}\right)\right] Q(\tilde{x}) + \frac{K}{\tilde{x}}Q^P(\tilde{x}) \end{aligned} \tag{20}$$

The derivative of the platform's profit with respect to $\tilde{b}$ is the same as before, namely $1 - F(\tilde{b}^*) - \frac{3}{2}\tilde{b}^* f(\tilde{b}^*) = 0$, see (3) for $C = 0$, so the equilibrium level of the marginal consumer is the same as before, $\tilde{b} = \tilde{b}^*$.

To study how the platform changes $\tilde{x}$, we take the derivative of the platform profit (20) with respect to $\tilde{x}$. Without in-house apps, the last term, $\frac{K}{\tilde{x}}Q^P(\tilde{x})$, was absent. Furthermore, we show above that $\tilde{b} = \tilde{b}^*$, so the derivative of the first term is the same as without in-house apps. But the derivative of the last term is negative. This means the hybrid platform allows more apps, $\tilde{x}^{h*} < \tilde{x}^*$. To implement this it should set $\tau^{h*} < \tau^*$, given that $\tilde{b} = \tilde{b}^*$, and also $S^{h*} > S^*$. Figure

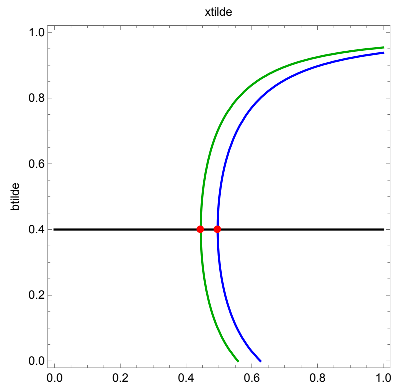

Figure 12: When the platform owns some inframarginal apps and sets the same price, $p^h = p^* = \lambda$, the equilibrium moves to the intersection of the green curve (the new optimality condition with respect to $\tilde{x}$) and the black line (the same optimality condition with respect to $\tilde{b}$), resulting in more app entry (lower $\tilde{x}$) and the same amount of consumer participation. The figure is drawn for $\{b, x\} \sim U[0, 1]$, and parameter values $C = 0, K = \lambda = 0.21$ and assuming that the platform-owned apps replace $1/4$ of third-party apps.

12 illustrates the benchmark optimality conditions when the platform does not own any apps for $\{b, x\} \sim U[0, 1]$, and parameter values $C = 0, K = \lambda = 0.21$: the blue curve is the optimality condition with respect to the marginal app type and the black line is the optimality condition with respect to the marginal consumer type. Their intersection corresponds to the benchmark equilibrium allocation. In the case of hybrid platform, the figure assumes that the platform replaces a quarter of infra-marginal third-party apps with its own apps while pricing them the same as the third-party apps, $p^h = p^* = \lambda$. In that case, the platform's optimality condition with respect to the marginal app type shifts left (the green curve) and the optimality condition with respect to the marginal consumer type does not change. As a result, at the new intersection point, the platform induces higher participation of apps (lower $\tilde{x}$) and the same participation level of consumers (the same $\tilde{b}$) compared to the case of no platform ownership of apps.

Infra-marginal consumers are better off since there is more app entry. Consumers are better off in aggregate – indeed, all but the marginal consumer are strictly better off. All active app developers are better off, since they now engage with more subscribers to the platform.[12] The following summarizes the results of the hyrbid platform with exogenous pricing

Proposition 7 *If the app platform replaces infra-marginal third-party apps with its own apps and prices own apps like third party apps, the platform hosts more apps by lowering its third-party app commission and sells the same number of devices by compensating the greater app variety with an offsetting device fee. This increases the total consumer surplus along with the rents accruing to*

[12]Those apps which are replaced by the platform-owned apps are worse off because they earned rents before being "removed."

the remaining third party apps.

When the platform's apps replace some of the infra-marginal third-party apps, the platform lowers its third-party app commission compared to the benchmark (without platform-owned apps). The presence of own apps tilts the business model away from collecting revenues via third-party commissions since platform ownership means the platform is less concerned by losing revenue on apps and more interested in expanding the app base. Bringing in more apps is now more desirable because it has a positive impact on its own apps' profit (due to complementarity of apps) and the platform prices out this with its subscription price rise.

The result that consumers and remaining apps are better off with the hybrid platform is in stark contrast to Anderson and Bedre Defolie (2023). Two key differences from the latter paper are that the remaining apps are complements to the platform-owned apps (no competition between the platform's apps and remaining third-party apps) and apps do not pass-through commissions on consumer prices as they sell digital goods with zero marginal cost.

Positive implications of platform's ownership of apps on consumers may be reversed once the platform's in-app price is endogenized. We next analyze this case.

5.6.2 Endogenous app pricing

We now allow the platform price its apps' micro-payments optimally. We denote the price of the platform's apps by p_P, and the density of third-party apps by $g^{3P}(x)$, and the density of platform apps by $g^P(x)$ so that, in terms of the benchmark model notation, we have $g^{3P}(x)+g^P(x) = g(x)$. We again have $g^P(x) = 0$ in the neighborhood of $\tilde{x}$ (by assumption). We also define the expected quality of third-party apps as $Q^{3P}(\tilde{x}) = \int_{\tilde{x}}^{\overline{x}} xg^{3P}(x)dx = Q(\tilde{x}) - Q^P$.

Recall that each app market is independent, so the platform's own app pricing does not affect pricing of remaining third-party apps. Thus, each third-party app again sets its price $p^* = \lambda$. Any (b, x) match generates an output $\frac{bx}{4\lambda^2}$ for a third-party app (given $p^* = \lambda$ for these apps), and so a revenue of $\frac{bx}{4\lambda}$. On the other hand, any (b, x) match generates a revenue of $\frac{p_P}{(\lambda+p_P)^2}bx$ for a platform-owned app. Hence the profit of the platform is now the sum of subscription revenue, commission on third-party sales, and own-app revenues (cf. (19)):

$$\begin{aligned}\Pi^h(S,\tau,\tilde{x},\tilde{b},p_P) &= S(1-F(\tilde{b}))+\tau\lambda\int_{\tilde{x}}^{\overline{x}}\int_{\tilde{b}}^{\overline{b}}\frac{bx}{4\lambda^2}f(b)g^{3P}(x)dbdx+\frac{p_P}{(\lambda+p_P)^2}\int_{\tilde{x}}^{\overline{x}}\int_{\tilde{b}}^{\overline{b}}bxf(b)g^P(x)dbdx \\ &= S(1-F(\tilde{b}))+\frac{1}{4\lambda}\tau B(\tilde{b})Q^{3P}(\tilde{x})+\frac{p_P}{(\lambda+p_P)^2}B(\tilde{b})Q^P. \end{aligned} \tag{21}$$

From the participation condition of apps (7) we have $\tau = 1 - \frac{4\lambda K}{B(\tilde{b})\tilde{x}}$ and consumer participation condition (6) now becomes $S = \frac{1}{2\lambda}\tilde{b}\left(Q^{3P}(\tilde{x}) + \frac{2\lambda}{(\lambda+p_P)}Q^P\right)$, noting that the surplus from a (b, x)

match generates a consumer surplus $\frac{bx}{(\lambda+p_P)}$ for a platform app. Substituting these expressions into the profit function (21) enables us to write the platform profit as

$$\begin{aligned}\Pi^h(\tilde{x},\tilde{b},p_P) &= \frac{1}{2\lambda}\tilde{b}\left(Q^{3P}(\tilde{x})+\frac{2\lambda}{(\lambda+p_P)}Q^P\right)(1-F(\tilde{b}))+\frac{B(\tilde{b})Q^{3P}(\tilde{x})}{4\lambda}\left(1-\frac{4\lambda K}{B(\tilde{b})\tilde{x}}\right)+\frac{p_P B(\tilde{b})Q^P}{(\lambda+p_P)^2} \\ &= \left(\tilde{b}(1-F(\tilde{b}))+\frac{B(\tilde{b})}{2}-\frac{2\lambda K}{\tilde{x}}\right)\frac{Q^{3P}(\tilde{x})}{2\lambda}+\left(\tilde{b}(1-F(\tilde{b}))+\frac{p_P B(\tilde{b})}{\lambda+p_P}\right)\frac{Q^P}{\lambda+p_P}. \qquad (22)\end{aligned}$$

We next show that the platform apps have a lower price than third-party apps:

Lemma 5 *The hybrid platform prices own apps above its marginal cost and cheaper than third-party apps:* $0 < p_P^* < p^* = \lambda$.

The platform prices its apps below the monopoly level because it can exact a form of two-part tariff – not directly within the app, but with linear pricing in the app and the second part embodied in the device fee collected from consumers. The platform captures the marginal consumer's surplus from transactions via the device fee and collects in-app purchase revenues of its own apps. Marginal cost pricing, setting $p_P^* = 0$, would maximize consumer surplus, and so maximize the device fee from consumers plus revenues from micro-payments (in-app purchases) from the marginal consumer. However, a higher price is better for a platform's app due to the infra-marginal consumers, since these consumers have higher willingness-to-pay for any given app quality x, so a higher fee enables the platform to extract more of their surplus. However, the platform does not want to go all the way to the monopoly price, $p^* = \lambda$. A slight drop in price has a second-order effect on revenues, but a first-order effect on increased consumer surplus. And it is the extra consumer surplus for the marginal consumer that parlays into a higher device fee for all of them. Therefore, holding all else constant, a platform can do strictly better by pricing $p_P^* \in (0,\lambda)$ than any price outside that interval.

Next consider the other performance dimensions, namely how the hybrid platform sets the marginal app and consumer types. We start with the consumer side. To guarantee a well-behaved problem of the platform we assume that

Assumption 2 *$B(\tilde{b})$ is strictly concave, equivalently, the elasticity of $f(\tilde{b})$ exceeds -1, or $f(\tilde{b}) + \tilde{b}f'(\tilde{b}) > 0$.*

Define $\alpha(\tilde{b}) \equiv \tilde{b}(1-F(\tilde{b})) + \frac{B(\tilde{b})}{2}$ and $\gamma(\tilde{b}) \equiv \tilde{b}(1-F(\tilde{b})) + \frac{p_P B(\tilde{b})}{\lambda+p_P}$ so that the equilibrium choice of the hybrid platform for the marginal consumer, $\tilde{b}^{h*}$, is the solution (for any given $\tilde{x}$) to

$$\frac{\partial \Pi^h}{\partial \tilde{b}}|_{\tilde{b}=\tilde{b}^{h*}} = \alpha'(\tilde{b}^{h*})\frac{Q^{3P}(\tilde{x})}{2\lambda}+\gamma'(\tilde{b}^{h*})\frac{Q^P}{\lambda+p_P} = 0. \qquad (23)$$

Recall that $\tilde{b}^*$ denotes the equilibrium level of $\tilde{b}$ in the absence of platform apps. We now show that $\tilde{b}^{h*} > \tilde{b}^*$ under Assumption 2. When $B(\tilde{b})$ is strictly concave, $\alpha'(\tilde{b})$ is strictly decreasing. Moreover, we have $\alpha'(\tilde{b}^*) = 0$ from (9) at $C = 0$ and

$$\gamma'(\tilde{b}^*) = \alpha'(\tilde{b}^*) - \frac{B'(\tilde{b}^*)}{2} + \frac{p_P B'(\tilde{b}^*)}{\lambda + p_P} = -B'(\tilde{b}^*)\left(\frac{1}{2} - \frac{p_P}{\lambda + p_P}\right) > 0,$$

since $p_P^* < \lambda$ by Lemma 5 and $B'(\tilde{b}^*) < 0$. Hence, we prove that

$$\frac{\partial \Pi^h}{\partial \tilde{b}}|_{\tilde{b}=\tilde{b}^*} > 0,$$

which implies $\tilde{b}^{h*} > \tilde{b}^*$ since $\Pi^h(\tilde{x}, \tilde{b}, p_P)$ is concave in $\tilde{b}$ under Assumption 2.[13] As a result, the hybrid platform induces a higher marginal consumer, and so *there are fewer consumers on-board in equilibrium when the platform owns some* of the apps.

In the Appendix (Lemma 8) we also show the companion property that $\tilde{x}^{h*} < \tilde{x}^*$ so that *there are more apps on-board in equilibrium when the platform owns some* of them. This property is more intricate to prove. There are two confounds. Referring to Figure 2, it is straighforward to show that the blue locus (the optimality condition with respect to $\tilde{x}$) shifts left for any given $\tilde{b}$. But, in conjunction with the result above that the black locus (the optimality condition with respect to $\tilde{b}$) shifts up for any given $\tilde{x}$, that alone is insufficient to show that the equilibrium outcome entails $\tilde{x}^{h*} < \tilde{x}^*$; to get there we show also that the shift in the blue locus is large enough. Second, the value of p_P^* changes with the extent of platform ownership and we need to factor that in to clinch the result.

These results enable us to draw strong conclusions for distributional and tarification effects of platform ownership. The result that the marginal consumer rises implies that all excluded consumers are worse off, as is the new marginal consumer. Consumers are better off to the extent that higher quality is available (both $\tilde{x}^{h*} < \tilde{x}^*$ and higher surplus on platform products) so in order for the marginal consumer to rise ($\tilde{b}^{h*} > \tilde{b}^*$) this can only be *more* than offset by a rise in S. As regards consumers with types $b > \tilde{b}^{h*}$, they are better off through the quality effects but worse off through the pricing effect, so what happens to them depends upon the details of the consumer distribution function. Higher-b consumers are more likely to be better off as they appreciate more the higher quality apps.

[13] Note first that $\alpha''(\tilde{b})$ has the sign of $-\left(5f(\tilde{b}) + \tilde{b}f'(\tilde{b})\right)$, which is negative under Assumption 2. To show that the left hand-side of (23) is strictly decreasing, it suffices to show that $\alpha'(\tilde{b}) - B'(\tilde{b})\left(\frac{1}{2} - \frac{p_P}{\lambda+p_P}\right)$ is strictly decreasing. Given that $B'(\tilde{b}) = -\tilde{b}f(\tilde{b}) < 0$, it suffices to show that $\left(\alpha'(\tilde{b}) + \frac{\tilde{b}f(\tilde{b})}{2}\right)' = \left(1 - F(\tilde{b}) - \frac{\tilde{b}f(\tilde{b})}{2}\right)' < 0$, which holds under the assumed concavity of $B'(\tilde{b})$.

The magnification effect upward on the device fee comes hand-in-hand with a magnification *down* in the commission rate τ. Marginal apps, and those now included, must be better off. But they serve fewer consumers, so it is only possible for there to be more entry if τ falls so much as to induce entry. Analogous to the consumer case, higher x apps lose more from consumer exit.

Proposition 8 *The hybrid platform induces more app entry and less consumer participation than if it had no own apps: $\tilde{x}^{h*} < \tilde{x}^*$ and $\tilde{b}^{h*} > \tilde{b}^*$. This allocation is supported by a lower app commission and a higher device fee: $\tau^{h*} < \tau^*$ and $S^{h*} > S^*$. At least some consumers are worse off while at least some apps earn more rents.*

To recapitulate, we started with an analysis of the effects of hybrid app platforms while assuming that in-house apps price like third-party ones. The upshot was that all consumers but the marginal one are strictly better off, with more app entry compensating higher device pricing. All apps are strictly better off, as is the platform itself. The only losers are the displaced apps. This rosy picture is largely overturned when we endogenize in-house app pricing. Surprisingly, the consequent lower price is extracted by the platform in a significantly higher subscription price, rendering at least some consumers worse off.

6 Different revenue models for apps

6.1 Ad-financed apps

We here specify an alternative version of the model which captures advertising finance as the monetization for apps. We then show the equivalence between the ad-financed apps model and the benchmark with in-app-purchase-financed apps, and we study the preferences of apps and the platform between these app revenue models.

Consider the benchmark model, but now suppose that consumers are subject to ads when using an app. Let γ denote consumer nuisance cost per ad and a denote the level of ads per minute on the platform. The dollar "price" to the consumer from spending q minutes on an app is the opportunity cost of time plus the ad nuisance, $(\lambda + \gamma a)\, q$. Notice that the in-app transaction price p is replaced by γa in consumer utility (1) and so the demand as a function of ad level a is $\frac{bx}{(\lambda+\gamma a)^2}$ (see (4)). The consumer indirect utility is then

$$V(b, \tilde{x}) \;=\; \frac{b}{\lambda + \gamma a} Q(\tilde{x}).$$

On the advertiser side, assume that advertisers are willing to pay r per ad per consumer. If the app permits an ad intensity a per minute then the app makes a revenue of ra per minute per consumer.

The profit of the app developer with quality x is then

$$\pi(a, x, \tilde{b}) = \int_{\tilde{b}}^{\bar{b}} \frac{bx}{(\lambda + \gamma a)^2} raf(b)db - K.$$

The app chooses an ad intensity a maximizing its profit and so the app's optimum solves $\gamma a^* = \lambda$. This can be seen by substituting the nuisance cost $\gamma = \frac{\hat{p}}{a}$ in the app profit: $\pi = \frac{bx}{(\lambda+\hat{p})^2}\hat{p}\frac{r}{\gamma} - K$, and so $\hat{p} = \lambda$ maximizes the latter.

We note several key points. First, when γ increases, the app developer decreases a^* proportionately so that the full price $\hat{p}$ paid by consumers remains the same – higher distaste for ads is exactly compensated by reducing a^* so that consumer nuisance cost stays the same per unit of app usage. Second if the platform collected the same commission, τ, from ad revenues as in-app purchases, app developers and the platform would prefer ad finance to in-app pricing if and only if $r > \gamma$. We next show that the platform actually achieves a better outcome (for all participants) by changing its equilibrium choices compared to the ones in the benchmark:

Proposition 9 *Let γ be the consumer nuisance cost per ad and r be the advertising revenue per ad per consumer. The platform induces more app participation (lower $\tilde{x}$) and more consumer participation (lower $\tilde{b}$) when all apps are ad-financed than when all apps are in-app-purchase-financed if and only if $r > \gamma$. Apps, consumers, and the platform prefer ad-finance if and only if $r > \gamma$.*

If we allow the platform to choose its commission for different app revenue models and $r > \gamma$, it can induce all apps to choose the ad-finance model by setting the same commission on in-app purchase revenues as for ad-finance revenues. The immediate implication of the previous proposition is then the following:[14]

Corollary 4 *Suppose the platform can set a different app commission for ad-financed apps than in-app-purchase-financed apps, and, after observing the platform's fees, apps choose their revenue model. In equilibrium, the platform chooses its app commissions to induce all apps to choose the ad-financed revenue model if and only if $r > \gamma$.*

To see why we cannot have a situation where some apps are ad-financed and some apps are in-app-purchase-financed, suppose this was the case. If $r > \gamma$, the platform induces all apps to choose ad-financed revenue model by choosing its commission for each revenue model (e.g., setting the same commission for both revenue models). The converse is true otherwise.

Our theory prediction that all apps choose the same revenue model might seem odd given that in practice some apps are ad-financed and some apps are financed by in-app purchases. We could

[14]The proof of Corollary 4 is in the Appendix within the proof of Proposition 9.

obtain different apps choosing different revenue models if we allow r/γ to differ across different app markets such that each app knows its own r/γ when it chooses its revenue model, but the platform knows only the distribution of r/γ when it chooses its device fee and app commission for each app revenue model. Suppose that r/γ is distributed over $[0, \infty)$ and this distribution is independent from the distribution of x and the distribution of b. In that case, given the platform's commission for each app revenue model, apps with sufficiently high draw of r/γ would choose the ad-financed model and other apps would choose the in-app-purchase model.

Now consider the comparative statics with respect to r. If all apps are financed by in-app purchases, increasing r above γ makes them switch to ad-finance (as the platform would then set commissions to induce this switch). As we show in Proposition 9, the platform then induces higher level of app entry and more consumer participation. If all apps are financed by ads, increasing r results in the platform allowing more app entry and consumer participation. This gives us the following:

Corollary 5 *Higher revenue per ad per consumer r implies higher app entry and more consumer participation in equilibrium.*

Access to ad revenue finance therefore expands participation on both sides when it is chosen by the platform. As advertising strength r increases, equilibrium participation on both sides expands beyond the levels with in-app purchases.

6.2 Apps charging only subscription fees

Suppose that there is no in-app purchase price for micro-payments ($p = 0$) and each app instead collects a lump-sum download or subscription price M from consumers if they want to use the app (in whatever amount they choose). Assume again that the platform collects τ^S commission over subscription revenues of its apps and for simplicity set $C = 0$. From the indirect utility function, consumer type b's usage of app type x is $q^{S*}(b, x) = \frac{bx}{\lambda^2}$ which leads to indirect utility $\frac{bx}{\lambda}$ for type b consumer from using app type x. To fix ideas, suppose that some large enough fraction of consumers, all those above some $\tilde{b}$ have joined the platform, and we will determine below the size of $\tilde{b}$. App type x chooses its subscription fee maximizing its profit subject to the marginal consumer, $\tilde{b}^S \geq \tilde{b}$, purchasing its subscription:

$$\pi^S(M, \tau^S, \tilde{b}^S) = (1 - \tau^S)M(1 - F(\tilde{b}^S)) - K \qquad \text{s.t.} \qquad \text{(i) } \frac{\tilde{b}^S x}{\lambda} \geq M, \qquad \text{(ii) } \tilde{b}^S \geq \tilde{b}.$$

Whatever marginal consumer type $\tilde{b}^S \geq \tilde{b}$, each app developer captures the entire surplus of the marginal consumer from transactions within its app, so:

$$M^*(x) = \frac{\tilde{b}^S x}{\lambda}.$$

Hence, in each app market the app developer extracts all transaction surplus of the marginal consumer once consumers already paid for their device fee. Anticipating this the marginal consumer is not willing to pay a positive device fee; this generates a hold-up problem for the platform and the platform has to set $S^{S*} = 0$ (there is no gains for the platform from setting a lower device price because already all consumers who will ever buy join). The profit of the platform comes only from commissions it collects from app subscriptions:

$$\Pi^S = \tau^S \frac{1}{\lambda} \tilde{b}^{S*}(1 - F(\tilde{b}^{S*}))Q(\tilde{x}^S).$$

We next show that this hold-up problem lowers consumer participation, but might increase app entry to the platform in equilibrium:

Proposition 10 *Suppose apps are subscription-based and the platform collects a percentage commission over app subscription revenues. Each app extracts the full surplus of the marginal consumer from its market and induces the same marginal consumer type. In equilibrium the platform cannot collect a positive device fee, $S^* = 0$. There are fewer consumers on the platform than the benchmark with in-app purchase pricing. There are more apps entering the platform if*

$$\epsilon_B \equiv -\frac{\tilde{b}^* B'(\tilde{b}^*)}{B(\tilde{b}^*)} \geq \frac{1}{3}, \text{ which holds for } b \sim U[0,1].$$

7 Conclusion

We provide a model of a software application platform capturing important features of this market. There is a continuum of app markets which differ in their quality and a continuum of consumers which differ in their willingness-to-pay for app quality. The platform sells devices to consumers and collects a percentage commission over in-app purchases. App developers enter if they incur a fixed cost and active apps choose their in-app purchase price. Consumers decide whether to buy a device and how much to purchase from each app. We characterize the equilibrium allocations of apps and consumers that are induced by the platform's pricing. Vertical differentiation of users on both sides enables us to find both the volume and quality of transactions on the platform, and illustrate distortions in both dimensions. We show that the platform sets excessive fees on both sides distorting down both consumer and app surpluses. Welfare maximizing fees are below cost and so subsidizes apps and consumers.

We next use our model to study several policy interventions that are discussed by regulators and policy makers in the EU and in the US. We show that a cap regulation on commissions increases equilibrium app entry and app surplus, but the cap lowers the equilibrium consumer participation and can harm consumers because the platform reacts to the cap by collecting a higher device fee from consumers. Whether total consumer surplus decreases from the cap depends on the distributional properties of consumer types and app qualities. The cap harms consumers when the losses of consumers from the higher device fee are higher than the gains of high valuation consumers from having more apps on the platform. This is the case for uniformly distributed consumer types and app qualities. We also study an alternative intervention, which has not been discussed by policy makers: a cap regulation on device fee. We show that the device fee cap increases consumer participation and lowers app participation since the platform raises its app commission as a reaction to the device fee cap. This decreases app surplus, but consumers can benefit from the cap (always for uniformly distributed types). Furthermore, following the EU's Digital Markets Act's imposed rule on gatekeeper app stores, we study the implications of allowing apps to use a third-party payment channel and show that this effectively mirrors a commission cap since apps can divert transactions off the platform by choosing different prices on and off the platform. A third-party payment channel lowers the equilibrium commission more when there is lower level of inconvenience that consumers face when their transaction is diverted off the platform. Like a cap regulation on commissions, this increases app surplus, but can lower consumer surplus.

We also study another concern of policy makers arising from the dominant hybrid platforms: does the platform ownership of some apps harm consumers and third-party apps? When we allow the platform to introduce its own apps by replacing some high-quality apps, we show that it sets a lower commission and a higher device fee resulting in lower consumer participation, more app entry and higher profits for the remaining apps, whereas replaced apps lose. The hybrid app platform lowers its commission to attract more apps and so more consumers since this generates revenues for its own apps (due to complementary app markets). The platform sets a lower in-app purchase price for its own apps than third-party app pricing since it takes into account the impact of app pricing on consumer participation, which third-party apps ignore. The platform does not lower its app price to zero, since a positive app price captures more value from higher value transactions. The platform raises its device fee to capture the increased consumer benefit from more apps. As a result, fewer consumers buy the device and more app markets are opened, but replaced apps lose and at least some consumers lose due to the hybrid mode.

Finally, we consider different app financing models (ad-financed and subscription-based). We show that if revenue per ad is higher than consumers' nuisance cost per ad, the platform induces more app participation and more consumer participation when apps are ad-financed than the benchmark of in-app-purchase-financed apps. In that case the ad-finance app model is better than in-app purchase app model for all market participants; apps, consumers, and the platform. The converse is true if revenue per ad is lower than consumers' nuisance cost per ad. Higher revenue per ad is a Pareto improvement by expanding the market to more consumers and more apps (true both when apps switch from being financed by in-app purchases to ad-financed model, and when all apps are ad-financed initially). We thereby argue that the public policies that lower ad revenue for apps harms consumers and apps, as well as the platform, by contracting the app market. This conclusion should be interpreted carefully as it does not take into account potential benefits of data protection policies, like privacy protection.

When apps are subscription financed (with zero in-app purchase price), the platform faces a hold-up problem since in each app market the developer sets a subscription fee extracting all consumer surplus from its app transactions. Anticipating this, consumers will not pay a positive device fee. This leads to lower consumer participation, but can increase app participation (the case for uniformly distributed consumer and app types).

Appendices

A Benchmark

Lemma 6 *Let $f(b)$ be a probability density function (PDF) over $[0, \bar{b}]$. If $f(b)$ satisfies the Monotone Hazard Rate Property (MHRP), $B(x) = \int_x^{\bar{b}} bf(b)\, db$ is log-concave in x.*[15]

Proof. We first define function $v(x)$:

$$v(x) \equiv \frac{B(x)}{1 - F(x)}. \tag{24}$$

Using integration by parts we rewrite $B(x)$ as

$$B(x) = \bar{b} - xF(x) - \int_x^{\bar{b}} F(b)db = x(1 - F(x)) + \int_x^{\bar{b}} (1 - F(b))db. \tag{25}$$

[15]Note that a stronger condition $f(b)$ log-concave (which implies $1 - F(b)$ log-concave) delivers a quick proof because the integrand is log-concave (it is the product of log-concave functions b and $f(b)$) then the integral is log-concave by the Prekopa-Borell theorem (Caplin and Nalebuff, 1991, p.36).

Substituting the latter into (24) we rewrite $v(x)$ as

$$v(x) = \frac{\int_x^{\bar{b}}(1-F(b))db}{1-F(x)} + x. \tag{26}$$

We next define function $Y(x)$:

$$Y(x) \equiv \int_x^{\bar{b}}(1-F(b))db. \tag{27}$$

We have $Y(\bar{b}) = 0$, $Y(x) > 0$ and $Y'(x) < 0$ for $x \in (0, \bar{b})$. Notice that the MHRP of $f(\cdot)$ is equivalent to $\frac{Y''(x)}{Y'(x)} = -\frac{f(x)}{1-F(x)}$ strictly decreasing. Lemma 1 of Bagnoli and Bergstrom (1989) shows that if $Y(\bar{b}) = 0$ and $Y'(x) < 0$, then the log-concavity of $Y'(x)$ ($\frac{Y''(x)}{Y'(x)}$ strictly decreasing) implies the log-concavity of $Y(x)$ ($\frac{Y'(x)}{Y(x)}$ strictly decreasing). Using this we next prove that $B(x)$ is log-concave.

Recall that $B(x) = (1-F(x))v(x)$ (24). We have

$$\begin{aligned} \ln(B(x)) &= \ln(1-F(x)) + \ln(v(x)), \\ (\ln(B(x)))' &= -\frac{f(x)}{1-F(x)} + \frac{v'(x)}{v(x)}. \end{aligned}$$

$B(x)$ is log-concave if and only if the right hand-side of the latter equation is strictly decreasing. From the MHRP we have $-\frac{f(x)}{1-F(x)}$ strictly decreasing. It is sufficient to show that $\frac{v'(x)}{v(x)}$ is decreasing. Using (26) and (27) we have $v(x) = -\frac{Y(x)}{Y'(x)} + x$, so $v(x) > 0$ (given that $Y(x) > 0$, $Y'(x) < 0$), $v'(x) = \frac{-(Y'(x))^2 + Y(x)Y''(x)}{(Y'(x))^2} < 0$ (given that $Y(x)$ is log-concave) and $v''(x) = \left(\frac{Y(x)}{Y'(x)}\frac{Y''(x)}{Y'(x)}\right)' < 0$ (given that $Y(x)$ and $Y'(x)$ are log-concave) for $x \in (0, \bar{b})$. Concavity of $v(x)$ implies that $v(x)$ is log-concave, $\left(\frac{v'(x)}{v(x)}\right)' < 0$, and so $B(x)$ is log-concave. ■

Proof of Lemma 1 The first-order condition of (8) with respect to the marginal consumer type is (9)

$$FOC_{\tilde{b}} : \left[1 - F(\tilde{b}) - \frac{3}{2}\tilde{b}f\left(\tilde{b}\right)\right]\frac{Q(\tilde{x})}{2\lambda} + Cf\left(\tilde{b}\right) = 0.$$

The second derivative of the platform's problem with respect to $\tilde{b}$ is:

$$\frac{\partial^2 \Pi}{\partial \tilde{b}^2} = -\frac{5}{4\lambda}f(\tilde{b})Q(\tilde{x}) - \left[\frac{3}{4\lambda}\tilde{b}Q(\tilde{x}) + C\right]f'(\tilde{b}),$$

which we rewrite by substituting $\frac{3}{4\lambda}\tilde{b}Q(\tilde{x})+C=\frac{Q(\tilde{x})}{2\lambda}\frac{1-F(\tilde{b})}{f(\tilde{b})}$ (from (9)) as

$$\frac{\partial^2\Pi}{\partial\tilde{b}^2}=-\frac{3Q(\tilde{x})}{4\lambda}f(\tilde{b})-\frac{Q(\tilde{x})}{2\lambda}\frac{(f(\tilde{b}))^2+\left[1-F(\tilde{b})\right]f'(\tilde{b})}{f(\tilde{b})}. \tag{28}$$

Hence, we prove that for $\tilde{b}$ satisfying (9) we have $\frac{\partial^2\Pi}{\partial\tilde{b}^2}<0$ since $(f(\tilde{b}))^2+\left[1-F(\tilde{b})\right]f'(\tilde{b})>0$ by the MHRP (Assumption 1(i)), $Q(\tilde{x})>0$, and $f(\tilde{b})>0$

The first-order condition for the equilibrium marginal app type is (10)

$$FOC_{\tilde{x}}:\frac{\partial\Pi}{\partial\tilde{x}}=\left[\frac{1}{2\lambda}\tilde{b}(1-F(\tilde{b}))+\frac{B(\tilde{b})}{4\lambda}-\frac{K}{\tilde{x}}\right]Q'(\tilde{x})+\left(\frac{K}{\tilde{x}^2}\right)Q(\tilde{x})=0.$$

We define $\varepsilon_Q(\tilde{x})=\frac{\tilde{x}Q'(\tilde{x})}{Q(\tilde{x})}<0$ as the elasticity of function $Q(\tilde{x})$ and rewrite the first-order condition with respect to $\tilde{x}$, (10), as

$$FOC_{\tilde{x}}:\frac{1}{2\lambda}\left[\tilde{b}(1-F(\tilde{b}))+\frac{1}{2}B(\tilde{b})\right]+\frac{K}{\tilde{x}}\left(\frac{1}{\varepsilon_Q(\tilde{x})}-1\right)=0,$$

where the first term (inside the brackets) is positive and the second one is negative. From (9) $\left[\tilde{b}(1-F(\tilde{b}))+\frac{1}{2}B(\tilde{b})\right]$ is increasing in $\tilde{x}$ (since $Q'(x)<0$). We have a unique solution to (10) if $\varepsilon_Q(\tilde{x})$ is decreasing.[16] Note that log-concavity of $Q(\tilde{x})$ (by Assumption 1(ii) and Lemma 6) implies that ε_Q is decreasing in $\tilde{x}$.[17] Hence, there is a unique $\tilde{x}>0$ that solves (10). This is the case for the uniform distribution. If x is uniformly distributed on $[0,1]$, we have $Q(\tilde{x})=\frac{1-\tilde{x}^2}{4\lambda}$, so $\varepsilon_Q=\frac{2}{1-\frac{1}{\tilde{x}^2}}<0$ and ε_Q is decreasing in $\tilde{x}$. Alternatively, it is straightforward to show that log-concavity of $Q(\tilde{x})$ implies $\frac{\partial^2\Pi}{\partial\tilde{x}^2}<0$ evaluated at $\tilde{x}$ satisfying (10). We have

$$\frac{\partial^2\Pi}{\partial\tilde{x}^2}=\left[\frac{1}{2\lambda}\tilde{b}(1-F(\tilde{b}))+\frac{B(\tilde{b})}{4\lambda}-\frac{K}{\tilde{x}}\right]Q''(\tilde{x})+\frac{K}{\tilde{x}^3}\left[\tilde{x}Q'(\tilde{x})-2Q(\tilde{x})\right]$$

[16]In this case the solution is at the intersection of a decreasing and positive curve (the left hand-side below) and increasing and positive curve (the right hand-side below) going through the origin:

$$-\left(\frac{1}{\varepsilon_Q(\tilde{x})}-1\right)=\tilde{x}\frac{\frac{1}{2\lambda}\left[\tilde{b}(1-F(\tilde{b}))+\frac{1}{2}B(\tilde{b})\right]}{K}$$

[17]Given that $\varepsilon_Q=\tilde{x}\frac{Q'(\tilde{x})}{Q(\tilde{x})}$, we have $\frac{d\varepsilon_Q}{d\tilde{x}}=\frac{\tilde{x}Q''(\tilde{x})Q(\tilde{x})+Q'(\tilde{x})Q(\tilde{x})-\tilde{x}\left(Q'(\tilde{x})\right)^2}{(Q(\tilde{x}))^2}$, and so $\frac{d\varepsilon_Q}{d\tilde{x}}<0$ if $(Q'(\tilde{x}))^2>Q''(\tilde{x})Q(\tilde{x})$, which is implied by the log-concavity of $Q(\tilde{x})$.

Using (10) we re-write the latter and show that it is negative:

$$\begin{aligned}\frac{\partial^2\Pi}{\partial\tilde{x}^2} &= -\frac{K}{\tilde{x}^{*2}}\frac{Q(\tilde{x})}{Q'(\tilde{x})}Q''(\tilde{x}) + \frac{K}{\tilde{x}^3}\left[\tilde{x}Q'(\tilde{x}) - 2Q(\tilde{x})\right] \\ &= \frac{K}{Q'(\tilde{x})\tilde{x}^2}\left[2(Q'(\tilde{x}))^2 - Q(\tilde{x})Q''(\tilde{x})\right] - \frac{K}{\tilde{x}^3}Q(\tilde{x}) < 0,\end{aligned}$$

since $(Q'(\tilde{x}))^2 - Q(\tilde{x})Q''(\tilde{x}) > 0$ by the log-concavity of $Q(\cdot)$, $Q'(\cdot) < 0$ and $Q(\cdot) > 0$.

Proof of Lemma 2 From (9) we show that the platform's profit is increasing in $\tilde{b}$ at $\tilde{b} = 0$ for all $\tilde{x} \in [0, \overline{x}]$ if $C > 0$ and for all $\tilde{x} \in [0, \overline{x})$ if $C = 0$:

$$\frac{\partial\Pi(0,\tilde{x})}{\partial\tilde{b}} = \frac{Q(\tilde{x})}{2\lambda} + Cf(0) > 0,$$

so $\tilde{b}^*(\tilde{x}) > 0$. The solution to (9) when $C = 0$, which we denote by $\tilde{b}_o$, is independent of $\tilde{x}$:

$$1 - F(\tilde{b}_o) - \frac{3}{2}\tilde{b}_o f\left(\tilde{b}^o\right) = 0.$$

If $C > 0$, condition (9) implies an increasing relationship between $\tilde{b}$ and $\tilde{x}$:

$$\frac{d\tilde{b}^*(\tilde{x})}{d\tilde{x}} = -\frac{\frac{\partial^2\Pi}{\partial\tilde{b}\partial\tilde{x}}}{\frac{\partial^2\Pi}{\partial\tilde{b}^2}} > 0, \tag{29}$$

given that $\frac{\partial^2\Pi(\tilde{x},\tilde{b}^*(\tilde{x}))}{\partial\tilde{b}^2} < 0$ (the platform profit is quasi-concave in $\tilde{b}$ by Assumption 1, as shown in the proof of Lemma 1) and

$$\frac{\partial^2\Pi(\tilde{x},\tilde{b}^*(\tilde{x}))}{\partial\tilde{b}\partial\tilde{x}} = \left[1 - F(\tilde{b}^*(\tilde{x})) - \frac{3}{2}\tilde{b}^*(\tilde{x})f(\tilde{b}^*(\tilde{x}))\right]\frac{Q'(\tilde{x})}{2\lambda} > 0,$$

since $\left[1 - F(\tilde{b}^*(\tilde{x})) - \frac{3}{2}\tilde{b}^*(\tilde{x})f(\tilde{b}^*(\tilde{x}))\right] = -\frac{2\lambda Cf\left(\tilde{b}^*(\tilde{x})\right)}{Q(\tilde{x})} < 0$ from (9) and $Q'(\tilde{x}) < 0$.

We next show that $\tilde{b}^*(\tilde{x})$ is a convex function of $\tilde{x}$ if $g(\tilde{x})$ is inelastic, which is the case for $g(\tilde{x})$ uniform. $\tilde{b}^*(\tilde{x})$ is a convex function of $\tilde{x}$ if and only if

$$\frac{d^2\tilde{b}^*(\tilde{x})}{d\tilde{x}^2} = -\frac{\frac{\partial^3\Pi}{\partial\tilde{b}\partial\tilde{x}^2}\frac{\partial^2\Pi}{\partial\tilde{b}^2} - \frac{\partial^2\Pi}{\partial\tilde{b}\partial\tilde{x}}\frac{\partial^3\Pi}{\partial\tilde{b}^2\tilde{x}}}{\left(\frac{\partial^2\Pi}{\partial\tilde{b}^2}\right)^2} > 0 \quad \text{or} \quad \frac{\partial^3\Pi}{\partial\tilde{b}\partial\tilde{x}^2}\frac{\partial^2\Pi}{\partial\tilde{b}^2} < \frac{\partial^2\Pi}{\partial\tilde{b}\partial\tilde{x}}\frac{\partial^3\Pi}{\partial\tilde{b}^2\tilde{x}},$$

We have

$$\frac{\partial^3\Pi}{\partial\tilde{b}\partial\tilde{x}^2} = \left[1 - F(\tilde{b}^*(\tilde{x})) - \frac{3}{2}\tilde{b}^*(\tilde{x})f(\tilde{b}^*(\tilde{x}))\right]\frac{Q''(\tilde{x})}{2\lambda} > 0$$

since $\left[1 - F(\tilde{b}^*(\tilde{x})) - \frac{3}{2}\tilde{b}^*(\tilde{x})f(\tilde{b}^*(\tilde{x}))\right] < 0$ as shown above and $Q''(\tilde{x}) < 0$ if $g(\tilde{x})$ is inelastic ($-(\tilde{x}\frac{g'(\tilde{x})}{g(\tilde{x})} < 1$). We also have $\frac{\partial^2\Pi}{\partial\tilde{b}^2} < 0$ from the quasi-concavity of the platform's profit and $\frac{\partial^2\Pi(\tilde{x},\tilde{b}^*(\tilde{x}))}{\partial\tilde{b}\partial\tilde{x}} > 0$, as proved earlier. Hence, a sufficient condition to have $\tilde{b}^*(\tilde{x})$ convex is that $\frac{\partial^3\Pi}{\partial\tilde{b}^2\tilde{x}} > 0$, which we show below by taking derivative of (28) with respect to $\tilde{x}$:

$$\frac{\partial^3\Pi}{\partial\tilde{b}^2\partial\tilde{x}} = -\frac{3Q'(\tilde{x})}{4\lambda}f(\tilde{b}) - \frac{Q'(\tilde{x})}{2\lambda}\frac{(f(\tilde{b}))^2 + \left[1 - F(\tilde{b})\right]f'(\tilde{b})}{f(\tilde{b})} > 0,$$

since $(f(\tilde{b}))^2 + \left[1 - F(\tilde{b})\right]f'(\tilde{b}) > 0$ by Assumption 1(i), $Q'(\tilde{x}) < 0$, and $f(\tilde{b}) > 0$.

Proof of Lemma 3 By taking the total derivative of (10) we express how $\tilde{x}$ varies with $\tilde{b}$:

$$\frac{d\tilde{x}^*(\tilde{b})}{d\tilde{b}} = -\frac{\frac{\partial^2\Pi}{\partial\tilde{b}\partial\tilde{x}}}{\frac{d^2\Pi}{d\tilde{x}^2}} = -\frac{\frac{1}{2\lambda}\left[1 - F(\tilde{b}) - \frac{3}{2}\tilde{b}f(\tilde{b})\right]Q'(\tilde{x}^*)}{\frac{d^2\Pi}{d\tilde{x}^2}}. \tag{30}$$

Note that the denominator is negative by the quasi-concavity of the platform's profit in $\tilde{x}$ (as shown in the proof of Lemma 1). Hence, $\frac{d\tilde{x}^*(\tilde{b})}{d\tilde{b}}$ has the sign of the numerator. When $\tilde{b}$ is close to $\bar{b}$, $\frac{d\tilde{x}^*(\tilde{b})}{d\tilde{b}} > 0$ since

$$\lim_{\tilde{b}\to\bar{b}}\left[1 - F(\tilde{b}) - \frac{3}{2}\tilde{b}f(\tilde{b})\right] = -\frac{3}{2}\bar{b}f(\bar{b}) < 0,$$

and $Q'(\tilde{x}) < 0$. When $\tilde{b}$ is close to 0, $\frac{d\tilde{x}^*(\tilde{b})}{d\tilde{b}} < 0$ since

$$\lim_{\tilde{b}\to 0}\left[1 - F(\tilde{b}) - \frac{3}{2}\tilde{b}f(\tilde{b})\right] = 1 > 0.$$

Moreover, $\left[1 - F(\tilde{b}) - \frac{3}{2}\tilde{b}f(\tilde{b})\right]$ is decreasing by the MHRP of $F(b)$, Assumption 1(i). At $\tilde{b}_o \in (0, \bar{b})$ satisfying $\left[1 - F(\tilde{b}_o) - \frac{3}{2}\tilde{b}_o f(\tilde{b}_o)\right] = 0$, we have $\frac{d\tilde{x}^*(\tilde{b}_o)}{d\tilde{b}} = 0$. For $\tilde{b} < \tilde{b}_o$, $\frac{d\tilde{x}^*(\tilde{b})}{d\tilde{b}} < 0$, and for $\tilde{b} > \tilde{b}_o$, $\frac{d\tilde{x}^*(\tilde{b})}{d\tilde{b}} > 0$.

Proof of Proposition 1 If $C = 0$, (9) characterizes the platform's optimal choice of $\tilde{b}$: $1 - F(\tilde{b}_o) - \frac{3}{2}\tilde{b}_o f(\tilde{b}_o) = 0$ (given that the platform's profit is quasi-concave in $\tilde{b}$ from Lemma 1). The platform's optimal choice of $\tilde{x}$ is then given by (10) evaluated at $\tilde{b}_o$. Lemma 3 shows that $\frac{d\tilde{x}^*(\tilde{b}_o)}{d\tilde{b}} = 0$. Moreover, from Lemma 2 we have $\tilde{b}^*(\tilde{x})$ is strictly increasing in C. Thus, for $C > 0$, $\tilde{b}^*(\tilde{x}) > \tilde{b}_o$. But then Lemma 3 implies that at the equilibrium if $C > 0$, we must have $\frac{d\tilde{x}^*(\tilde{b}^*)}{d\tilde{b}} > 0$. Finally, Lemma 2 implies that at the equilibrium if $C > 0$, $\frac{d\tilde{b}^*(\tilde{x}^*)}{d\tilde{x}} > 0$. The platform's profit is quasi-concave in $\tilde{b}$ and $\tilde{x}$ by Lemma 1. The remaining second-order condition holds, $\frac{d^2\Pi}{d\tilde{b}^2}\frac{d^2\Pi}{d\tilde{x}^2} > \left(\frac{d^2\Pi}{d\tilde{b}d\tilde{x}}\right)^2$, if and only if $\frac{d\tilde{b}^*(\tilde{x}^*)}{d\tilde{x}} < \frac{1}{\frac{d\tilde{x}^*(\tilde{b}^*)}{d\tilde{b}}}$, see (29) and (30). Thus, the smaller intersection of (9) and (10) characterizes the unique equilibrium allocations.

Proof of Lemma 4 The platform chooses $\tilde{b}_o$ satisfying (9): when $C = 0$, this is

$$1 - F(\tilde{b}_o) + \frac{3}{2}\tilde{b}_o f(\tilde{b}_o) = 0.$$

Thus, $\tilde{b}_o$ does not depend on $\tilde{x}$ or K. This means consumer participation stays the same, regardless of K. The platform chooses $\tilde{x}$ satisfying (10) at $\tilde{b} = \tilde{b}_o$:

$$FOC_{\tilde{x}} : \frac{\partial \Pi}{\partial \tilde{x}} = \left[\frac{1}{2\lambda}\tilde{b}_o(1 - F(\tilde{b}_o)) + \frac{B(\tilde{b}_o)}{4\lambda} - \frac{K}{\tilde{x}}\right] Q'(\tilde{x}) + \left(\frac{K}{\tilde{x}^2}\right) Q(\tilde{x}) = 0,$$

which we rewrite as

$$FOC_{\tilde{x}} : \left[\frac{1}{2\lambda}\tilde{b}_o(1 - F(\tilde{b}_o)) + \frac{B(\tilde{b}_o)}{4\lambda}\right] \frac{Q'(\tilde{x})}{\frac{Q(\tilde{x})}{\tilde{x}} - Q'(\tilde{x})} + \frac{K}{\tilde{x}} = 0.$$

Given that $\tilde{b}_o$ is independent of K, a lower K decreases the left-hand side of this condition. Given that the platform profit is quasi-concave in $\tilde{x}$, this results in the platform choosing a lower $\tilde{x}$ in equilibrium, and so more apps enter the platform when K is lower. The optimality condition for $\tilde{x}$ also implies that the fraction $\frac{K}{\tilde{x}}$ increases in K if and only if $\frac{Q'(\tilde{x})}{\frac{Q(\tilde{x})}{\tilde{x}} - Q'(\tilde{x})}$ decreases or

$$\begin{aligned} Q''(\tilde{x})\left(\frac{Q(\tilde{x})}{\tilde{x}} - Q'(\tilde{x})\right) - Q'(\tilde{x})\left(\frac{Q'(\tilde{x})}{\tilde{x}} - \frac{Q(\tilde{x})}{(\tilde{x})^2} - Q''(\tilde{x})\right) &< 0, \text{ or} \\ Q''(\tilde{x})Q(\tilde{x}) - (Q'(\tilde{x}))^2 + Q'(\tilde{x})\frac{Q(\tilde{x})}{\tilde{x}} &< 0, \end{aligned}$$

which is the case since $Q(\tilde{x})$ is log-concave by Assumption 1(ii) and Lemma 6. Thus, we conclude that a lower K decreases $\frac{K}{\tilde{x}}$. Using (7), $\tau = 1 - \frac{4\lambda K}{B(\tilde{b})\tilde{x}}$, this implies that the platform sets a higher commission when K decreases. From (6), we have $S = \frac{\tilde{b}_o}{2\lambda}Q(\tilde{x})$. We show above that $\tilde{b}_o$ is constant in K and equilibrium $\tilde{x}$ decreases when K decreases. This in turn implies that the platform sets a higher device fee S when K is lower ($Q'(\tilde{x}) < 0$).

Proof of Proposition 3 Let $\tilde{x}^*, \tilde{b}^*$ denote the unconstrained equilibrium levels of the marginal app and the marginal consumer, respectively. Similarly, $\tilde{x}^{C*}, \tilde{b}^{C*}$ denote the equilibrium levels of the marginal app and the marginal consumer with the binding cap on τ: $\overline{\tau} < \tau^*$. Let $\tilde{x}^{\overline{\tau}}(\tilde{b})$ denote $\tilde{x}$ that satisfies the constraint at $\tilde{b}$. A binding τ constraint means that the platform would induce a higher marginal app type (lower app participation) at $\tilde{b} = \tilde{b}^*$ if it was not constrained: $\tilde{x}^* > \tilde{x}^{\overline{\tau}}(\tilde{b}^*)$. Using the unconstrained equilibrium equation (10) and the concavity of the platform's profit with respect to $\tilde{x}$ at the unconstrained choice $\tilde{b}^*$, this implies that

$$\left.\frac{\partial \Pi}{\partial \tilde{x}}\right|_{\tilde{b}=\tilde{b}^*} = \left[\frac{1}{2\lambda}\tilde{b}^*(1-F(\tilde{b}^*)) + B(\tilde{b}^*)\frac{1}{4\lambda} - \frac{K}{\tilde{x}^{\overline{\tau}}(\tilde{b}^*)}\right] Q'(\tilde{x}^{\overline{\tau}}(\tilde{b}^*)) + \frac{K}{\left(\tilde{x}^{\overline{\tau}}(\tilde{b}^*)\right)^2} Q(\tilde{x}^{\overline{\tau}}(\tilde{b}^*)) > 0.$$

From the cap constraint we have $\frac{d\tilde{x}^{\overline{\tau}}}{d\tilde{b}} = -\frac{B'(\tilde{b})\tilde{x}}{B(\tilde{b})} > 0$. Hence, at the constrained equilibrium (with the binding cap on τ), using the constrained equilibrium condition (16), we obtain

$$\left.\frac{d\Pi}{d\tilde{b}}\right|_{\tilde{b}=\tilde{b}^*,\, \tilde{x}=\tilde{x}^{\overline{\tau}}(\tilde{b}^*)} > 0,$$

given $(1-F(\tilde{b}^*)) - \frac{3}{2}\tilde{b}^* f\left(\tilde{b}^*\right) = 0$ from the unconstrained equilibrium equation (9) at $C = 0$. The latter inequality implies a higher marginal consumer type at the new equilibrium: $\tilde{b}^* < \tilde{b}^{C*}$, and so lower consumer participation. Above we have shown that $\tilde{x}^* > \tilde{x}^{\overline{\tau}}(\tilde{b}^*)$ and that at the constrained equilibrium $\frac{d\tilde{x}^{\overline{\tau}}}{d\tilde{b}} > 0$. These together with $\tilde{b}^* < \tilde{b}^{C*}$ imply that $\tilde{x}^{\overline{\tau}}(\tilde{b}^*) < \tilde{x}(\tilde{b}^{C*}) = \tilde{x}^{C*}$. Moreover, we have $(1-F(\tilde{b}^{C*})) - \frac{3}{2}\tilde{b}^{C*} f(\tilde{b}^{C*}) < 0$ since $\tilde{b}^* < \tilde{b}^{C*}$ (and using the concavity of platform profit at $\tilde{b}^*$, which is proved in Proposition 1), so

$$\left.\frac{\partial \Pi}{\partial \tilde{x}}\right|_{\tilde{b}=\tilde{b}^{C*}, \tilde{x}=\tilde{x}^{C*}} = \left[\frac{1}{2\lambda}\tilde{b}^{C*}(1-F(\tilde{b}^{C*})) + B(\tilde{b}^{C*})\frac{1}{4\lambda} - \frac{K}{\tilde{x}^{C*}}\right] Q'(\tilde{x}^{C*}) + \frac{K}{(\tilde{x}^{C*})^2} Q(\tilde{x}^{C*}) > 0.$$

Thus, $\tilde{x}^* > \tilde{x}^{C*}$.

Proof of Proposition 4 Let $\tilde{x}^*, \tilde{b}^*$ denote the unconstrained equilibrium levels of the marginal app and the marginal consumer, respectively. Similarly, $\tilde{x}^{C*}, \tilde{b}^{C*}$ denote the equilibrium levels of the marginal app and the marginal consumer with the binding cap on S: $\overline{S} < S^* = \frac{1}{2\lambda}\tilde{b}^* Q(\tilde{x}^*)$. Let $\tilde{x}^{\overline{S}}(\tilde{b})$ denote $\tilde{x}$ that satisfies the constraint at $\tilde{b}$. Given that $\overline{S} = \frac{1}{2\lambda}\tilde{b}^* Q(\tilde{x}^{\overline{S}}(\tilde{b}^*)) < S^*$ and $Q'(\cdot) < 0$, the platform would induce a lower marginal app type (higher app participation) at $\tilde{b} = \tilde{b}^*$ if it was not constrained: $\tilde{x}^{\overline{S}}(\tilde{b}^*) > \tilde{x}^*$. Using the unconstrained equilibrium equation (10) and the concavity of the platform's profit with respect to $\tilde{x}$ at the unconstrained choice $\tilde{b}^*$, this implies that

$$\left.\frac{\partial \Pi}{\partial \tilde{x}}\right|_{\tilde{b}=\tilde{b}^*} = \left[\frac{1}{2\lambda}\tilde{b}^*(1-F(\tilde{b}^*)) + B(\tilde{b}^*)\frac{1}{4\lambda} - \frac{K}{\tilde{x}^{\overline{S}}(\tilde{b}^*)}\right] Q'(\tilde{x}^{\overline{S}}(\tilde{b}^*)) + \frac{K}{\left(\tilde{x}^{\overline{S}}(\tilde{b}^*)\right)^2} Q(\tilde{x}^{\overline{S}}(\tilde{b}^*)) < 0.$$

From the cap constraint we have $\frac{d\tilde{x}^{\overline{S}}}{d\tilde{b}} = -\frac{Q(\tilde{x})}{\tilde{b}Q'(\tilde{x})} > 0$. Hence, at the constrained equilibrium (with the binding cap on τ), using the constrained equilibrium condition (17), we obtain

$$\left.\frac{d\Pi}{d\tilde{b}}\right|_{\tilde{b}=\tilde{b}^*,\, \tilde{x}=\tilde{x}^{\overline{S}}(\tilde{b}^*)} < 0,$$

given $(1-F(\tilde{b}^*)) - \frac{3}{2}\tilde{b}^* f(\tilde{b}^*) = 0$ from the unconstrained equilibrium equation (9) at $C = 0$. The latter inequality implies a lower marginal consumer type at the new equilibrium: $\tilde{b}^* > \tilde{b}^{C*}$, and so greater consumer participation. Above we have shown that $\tilde{x}^* < \tilde{x}^{\overline{S}}(\tilde{b}^*)$ and that at the constrained equilibrium $\frac{d\tilde{x}^{\overline{S}}}{d\tilde{b}} > 0$. These together with $\tilde{b}^* > \tilde{b}^{C*}$ imply that $\tilde{x}^{\overline{S}}(\tilde{b}^*) > \tilde{x}^{\overline{S}}(\tilde{b}^{C*}) = \tilde{x}^{C*}$. Moreover, we have $(1-F(\tilde{b}^{C*})) - \frac{3}{2}\tilde{b}^{C*} f(\tilde{b}^{C*}) > 0$ since $\tilde{b}^* > \tilde{b}^{C*}$ (and using the concavity of platform profit at $\tilde{b}^*$, which is proved in Proposition 1), so

$$\left.\frac{\partial\Pi}{\partial\tilde{x}}\right|_{\tilde{b}=\tilde{b}^{C*},\tilde{x}=\tilde{x}^{C*}} = \left[\frac{1}{2\lambda}\tilde{b}^{C*}(1-F(\tilde{b}^{C*})) + B(\tilde{b}^{C*})\frac{1}{4\lambda} - \frac{K}{\tilde{x}^{C*}}\right] Q'(\tilde{x}^{C*}) + \frac{K}{(\tilde{x}^{C*})^2} Q(\tilde{x}^{C*}) < 0.$$

which implies that (given $\tilde{b}^* > \tilde{b}^{C*}$ and $\tilde{b}^*$ is the maximizer of $\frac{1}{2\lambda}\tilde{b}(1-F(\tilde{b})) + B(\tilde{b})\frac{1}{4\lambda}$)

$$\left.\frac{\partial\Pi}{\partial\tilde{x}}\right|_{\tilde{b}=\tilde{b}^*,\tilde{x}=\tilde{x}^{C*}} = \left[\frac{1}{2\lambda}\tilde{b}^*(1-F(\tilde{b}^*)) + B(\tilde{b}^*)\frac{1}{4\lambda} - \frac{K}{\tilde{x}^{C*}}\right] Q'(\tilde{x}^{C*}) + \frac{K}{(\tilde{x}^{C*})^2} Q(\tilde{x}^{C*}) < 0.$$

Thus, $\tilde{x}^* < \tilde{x}^{C*}$.

B Hybrid platform

Proof of Lemma 5 The platform's equilibrium own-app price solves

$$\begin{aligned}\frac{\partial\Pi^h}{\partial p_P} &= -\left(\tilde{b}(1-F(\tilde{b})) + \frac{p_P B(\tilde{b})}{\lambda+p_P}\right)\frac{Q^P}{(\lambda+p_P)^2} + \left(\frac{B(\tilde{b})(\lambda+p_P) - p_P B(\tilde{b})}{(\lambda+p_P)^2}\right)\frac{Q^P}{\lambda+p_P} = 0,\\ &= \frac{Q^P}{(\lambda+p_P)^2}\left(\frac{(\lambda-p_P)B(\tilde{b})}{\lambda+p_P} - \tilde{b}(1-F(\tilde{b}))\right) = 0\end{aligned}$$

The expression in parentheses on the last line is strictly decreasing in p_P over the relevant range of prices for positive demand ($p_P > -\lambda$) implying quasi-concavity in p_P of profit $\Pi^h(\tilde{x}, \tilde{b}, p_P)$. This profit has a unique maximizer $p_P^* \in (0, \lambda)$ because $\frac{\partial\Pi^h}{\partial p_P}|_{p_P=0} = \frac{Q^P}{\lambda^2}\left(B(\tilde{b}) - \tilde{b}(1-F(\tilde{b}))\right) > 0$ (as $B(\tilde{b}) = \int_{\tilde{b}}^{\overline{b}} bf(b)db > \tilde{b}(1-F(\tilde{b}))$) and $\frac{\partial\Pi^h}{\partial p_P}|_{p_P=\lambda} = -\frac{Q^P}{4\lambda^2}\tilde{b}(1-F(\tilde{b})) < 0$.

Lemma 7 *The hybrid platform induces a lower level of consumer participation than the platform which has no own apps in its app store:* $\tilde{b}^h > \tilde{b}^*$. *Consumer participation increases when the* *platform's own app amount decreases,* $\frac{d\tilde{b}^h}{dQ^P} < 0$*, and* $lim_{Q^P \to 0}\tilde{b}^h = \tilde{b}^*$.

Proof.

We show that $\tilde{b}^h$ is decreasing in the amount of third-party apps, $Q^{3P}(\tilde{x})$, or increasing in $\tilde{x}$. Given $\tilde{b}^h > \tilde{b}^*$ and $\alpha'(\tilde{b})$ decreasing, we have $\alpha'(\tilde{b}^h) < 0$. Thus, we have

$$\frac{d\tilde{b}^h}{d\tilde{x}} = -\frac{\alpha'(\tilde{b}^h)\frac{Q^{3P\prime}(\tilde{x})}{2\lambda}}{\frac{\partial^2 \Pi^h}{\partial \tilde{b}^2}} > 0.$$

since $Q^{3P\prime}(\tilde{x}) < 0$ and $\frac{\partial^2 \Pi^h}{\partial \tilde{b}^2} < 0$ by the second-order condition (assumption).

Similarly, it is straightforward to show that $\tilde{b}^h$ increases in the amount of platform-owned apps, Q^P.

$$\frac{d\tilde{b}^h}{dQ^P} = -\frac{\gamma'(\tilde{b}^h)\frac{1}{\lambda + p_P}}{\frac{\partial^2 \Pi^h}{\partial \tilde{b}^2}} > 0.$$

since $\gamma'(\tilde{b}^h) > 0$ given that $\alpha'(\tilde{b}^h) < 0$ and (9).

When the platform's app amount is close to zero, Q^P goes to zero, $Q^{3P}(\tilde{x})$ goes to $Q(\tilde{x})$, the platform's optimal $\tilde{b}^h$ goes to $\tilde{b}^*$, since the platform's objective becomes close to its objective without own apps, see (21). When the platform's app amount increases, $Q^{3P}(\tilde{x})$ decreases (or $\tilde{x}$ increases), this increases $\tilde{b}^h$, since the first(negative)-term in (23) decreases in magnitude and the second(positive)-term increases in magnitude. ■

Lemma 8 *The hybrid platform induces a lower level of marginal app type (more entry of apps at the margin) than the platform which has no own apps in its app store:* $\tilde{x}^h < \tilde{x}^*$. *App entry increases in the platform's own app amount,* $\frac{d\tilde{x}^h}{dQ^P} > 0$, $lim_{Q^P \to 0}\tilde{x}^h = \tilde{x}^*$*, and* $lim_{Q^{3P} \to 0}\tilde{x}^h = 0$

Proof. Consider the first order condition of (22) with respect to $\tilde{x}$ (and evaluating at $\tilde{b}^h$):

$$\frac{\partial \Pi(\tilde{x}, \tilde{b}^h, p_P)}{\partial \tilde{x}} = \frac{K}{\tilde{x}^2}Q^{3P}(\tilde{x}) + \left(\tilde{b}^h(1 - F(\tilde{b}^h)) + \frac{B(\tilde{b}^h)}{2} - \frac{2\lambda K}{\tilde{x}}\right)\frac{1}{2\lambda}\frac{dQ^{3P}(\tilde{x})}{d\tilde{x}} = 0.$$

Note that the platform's own app amount Q^P does not depend on the marginal app type since the platform's apps are infra-marginal ones (by assumption). When there were no platform apps we had (10):

$$\frac{K}{\tilde{x}^{*2}}Q(\tilde{x}^*) + \left(\tilde{b}(1 - F(\tilde{b}^*)) + \frac{B(\tilde{b})}{2} - \frac{2\lambda K}{\tilde{x}^*}\right)\frac{1}{2\lambda}Q'(\tilde{x}^*) = 0.$$

Let $x^*(\tilde{b})$ denote the solution to the latter condition. Let $x^{h*}(\tilde{b})$ denote the solution to the hybrid platform's optimality condition:

$$\frac{\partial \Pi(\tilde{x}, \tilde{b}, p_P)}{\partial \tilde{x}} = \frac{K}{\tilde{x}^{h*2}} Q^{3P}(\tilde{x}^{h*}) + \left(\tilde{b}(1 - F(\tilde{b})) + \frac{B(\tilde{b})}{2} - \frac{2\lambda K}{\tilde{x}^{h*}} \right) \frac{1}{2\lambda} \frac{dQ^{3P}(\tilde{x}^{h*})}{d\tilde{x}} = 0. \quad (31)$$

Note that we have $Q'(\tilde{x}) = \frac{dQ^{3P}(\tilde{x})}{d\tilde{x}}$ (marginal type is always a third-party seller) and $Q^{3P}(\tilde{x}) < Q(\tilde{x})$ (some third-party apps are replaced). It is then straightforward to show that, for a given $\tilde{b}$, $x^{h*}(\tilde{b}) < x^*(\tilde{b})$ since $\frac{\partial \Pi^h}{\partial \tilde{x}}|_{\tilde{x}=x^*(\tilde{b})} < 0$ as $Q^{3P}(\tilde{x}) < Q(\tilde{x})$ and Π^h is concave in $\tilde{x}$ (by assumption). We therefore have $x^{h*}(\tilde{b}^*) < x^*(\tilde{b}^*)$. Taking the total derivative of (31) gives

$$\frac{dx^{h*}}{d\tilde{b}} = -\frac{\alpha'(\tilde{b}) \frac{1}{2\lambda} \frac{dQ^{3P}(\tilde{x}^{h*})}{d\tilde{x}}}{\frac{\partial^2 \Pi}{\partial \tilde{x}^2}}$$

Thus, we show that $\frac{dx^{h*}}{d\tilde{b}} = 0$ at $\tilde{b}^*$, x^{h*} is increasing for $\tilde{b} > \tilde{b}^*$ and x^{h*} is decreasing for $\tilde{b} < \tilde{b}^*$. Given that $\tilde{b}^h < \tilde{b}^*$ (as we show in Lemma 7), $x^{h*}(\tilde{b}^*) < x^{h*}(\tilde{b}^h)$.

Note also that $\alpha(\tilde{b}^h) < \alpha(\tilde{b}^*)$ since $\tilde{b}^*$ is the maximizer of $\alpha(\tilde{b})$ and $\tilde{b}^h > \tilde{b}^*$. Drawing these points together, we have $\tilde{x}^h(\tilde{b}^h) < x^*(\tilde{b}^*)$ if and only if $\frac{\partial \Pi^h}{\partial \tilde{x}}|_{\tilde{x}=\tilde{x}^*} < 0$ or

$$\frac{K}{\tilde{x}^{*2}} Q^{3P}(\tilde{x}^*) + \left(\alpha(\tilde{b}^h) - \frac{2\lambda K}{\tilde{x}^*} \right) \frac{1}{2\lambda} \frac{dQ^{3P}(\tilde{x}^*)}{d\tilde{x}^*} < \frac{K}{\tilde{x}^{*2}} Q(\tilde{x}^*) + \left(\alpha(\tilde{b}^*) - \frac{2\lambda K}{\tilde{x}^*} \right) \frac{1}{2\lambda} \frac{dQ^{3P}(\tilde{x}^*)}{d\tilde{x}} = 0$$

If $Q^{3P}(\tilde{x})$ is close to 0 (the platform has taken nearly all products) then the second, negative, term should dominate, implying that $\tilde{x}^{h*}$ is close to 0. In that case, he platform does not earn revenue via τ, since most of apps is its own, so lowers τ close to zero to encourage entry of marginal apps as this facilitates raising S.

If $Q^{3P}(\tilde{x})$ is close to $Q(\tilde{x})$, $\tilde{b}^{h*}$ goes to $\tilde{b}^*$ (as shown in Lemma 7), so $\frac{\partial \Pi^h}{\partial \tilde{x}}|_{\tilde{x}=\tilde{x}^*}$ goes to zero, that is, $\tilde{x}^h$ goes to $\tilde{x}^*$. By the continutity of the hybrid platform's profit in $Q^{3P}(\tilde{x})$ and in $\tilde{b}^h$, and using Lemma 7), we show the lemma. ■

The hybrid problem internalizes entire revenue from its own app transactions, and therefore puts less weight on extracting third-party app revenue via τ, but more weight on extracting surplus from consumers via S. As a result, it lowers its app commission, which induces more entry of apps at the margin, and raises its device fee. This induces lower consumer participation when the platform prices its own apps, since then it lowers in-app purchase below the monopoly level to extract more rent from infra-marginal consumers via raising S further.

C Ad-financed app model

Proof of Proposition 9 Suppose all apps are ad-financed. As we show in Section 6.1, each app sets $a^* = \frac{\lambda}{r}$ and app type x earns:

$$\pi = (1-\tau)\int_{\tilde{b}}^{\bar{b}} \frac{bxr}{4\lambda\gamma} f(b)db - K = (1-\tau)\frac{B(\tilde{b})}{4\lambda} x \frac{r}{\gamma} - K.$$

The platform's profit is

$$\Pi = (S-C)(1-F(\tilde{b})) + \tau \frac{1}{4\lambda}\frac{r}{\gamma} B(\tilde{b})Q(\tilde{x}),$$

which we rewrite by substituting $S = \frac{\tilde{b}Q(\tilde{x})}{2\lambda}$ and $\tau = 1 - \frac{4\lambda K}{B(\tilde{b})\tilde{x}}\frac{\gamma}{r}$:

$$\Pi = \left(\frac{1}{2\lambda}\tilde{b}(1-F(\tilde{b})) + \frac{r}{\gamma}\frac{B(\tilde{b})}{4\lambda} - \frac{K}{\tilde{x}}\right) Q(\tilde{x}) - C(1-F(\tilde{b})).$$

The first-order condition with respect to the marginal consumer type is

$$\frac{\partial \Pi}{\partial \tilde{b}} = \left(1 - F(\tilde{b}^A) - \left(1 + \frac{r}{\gamma}\right)\tilde{b}^A f(\tilde{b}^A)\right)\frac{Q(\tilde{x})}{2\lambda} + Cf(\tilde{b}^A) = 0,$$

which determines the equilibrium marginal consumer type as a function of the marginal app type $\tilde{b}^{A*}(\tilde{x})$. Recall that $\tilde{b}^*(\tilde{x})$ denotes the benchmark equilibrium choice of the platform, satisfying (9). Comparing these, we conclude that for a given marginal app type, the platform sets a lower marginal consumer in the ad-financed model, $\tilde{b}^{A*}(\tilde{x}) < \tilde{b}^*(\tilde{x})$, if and only if $\frac{r}{\gamma} > 1$. Moreover, $\tilde{b}^{A*}(\tilde{x})$ is decreasing in $\frac{r}{\gamma}$.

The first-order condition with respect to the marginal app type is

$$\frac{\partial \Pi}{\partial \tilde{x}} = \left(\frac{1}{2\lambda}\tilde{b}(1-F(\tilde{b})) + \frac{r}{\gamma}\frac{B(\tilde{b})}{4\lambda} - \frac{K}{\tilde{x}^A}\right) Q'(\tilde{x}^A) + \frac{K}{\tilde{x}^A}Q(\tilde{x}^A) = 0,$$

which determines the equilibrium marginal app type as a function of the marginal consumer type $\tilde{x}^{A*}(\tilde{b})$. Comparing this with the optimal marginal app in the benchmark (10) we conclude that for a given marginal consumer type, the platform sets a lower marginal app in the ad-financed model, $\tilde{x}^{A*}(\tilde{b}) < \tilde{x}^*(\tilde{b})$, if and only if $\frac{r}{\gamma} > 1$. Furthermore, $\tilde{x}^{A*}(\tilde{b})$ is decreasing in $\frac{r}{\gamma}$.

Hence, both optimality conditions involve more participation for any participation on the other side, compared to the benchmark, that is, $\tilde{b}^{A*}(\tilde{x}) < \tilde{b}^*(\tilde{x})$ and $\tilde{x}^{A*}(\tilde{b}) < \tilde{x}^*(\tilde{b})$, if and only if $\frac{r}{\gamma} > 1$. Recall from Proposition 1 that the benchmark equilibrium conditions have positive slopes,

$\frac{d\tilde{b}^*(\tilde{x}^*)}{d\tilde{x}} > \frac{1}{\frac{d\tilde{x}^*(\tilde{b}^*)}{d\tilde{b}}} > 0$. We therefore conclude that the platform induces more apps and more consumers when apps are ad-financed than when they are in-app-purchase-financed if and only if $\frac{r}{\gamma} > 1$.

In addition, participation on both sides, platform profits, and all user surpluses increase with $\frac{r}{\gamma}$. If the platform set the same consumer fee and the same app commission in the two models, all apps would prefer the ad-financed revenue model to in-app-purchases if and only if $\frac{r}{\gamma} > 1$. In that case we previously showed that the platform's pricing induces more app participation and more consumer participation under ad-finance, and so all apps, all consumers, and the platform prefer ad-finance to in-app-purchases.

D Subscription-based app model

Proof of Proposition 10 Given $\tilde{b}^S \geq \tilde{b}$, app type x chooses the marginal consumer type $\tilde{b}^S$ to maximize its profit

$$\pi^S(\tau^S, \tilde{b}^S) = (1-\tau^S)\frac{\tilde{b}^S x}{\lambda}(1-F(\tilde{b}^S)) - K.$$

The first-order condition determines the app's equilibrium choice:

$$\frac{\partial \pi^S}{\partial \tilde{b}^S} = 1 - F(\tilde{b}^{S*}) - \tilde{b}^{S*} f(\tilde{b}^{S*}) = 0.$$

Thus, every app chooses the same marginal consumer type, $\tilde{b}^{S*}$ regardless of its quality. App type x sets $M^*(x) = \frac{\tilde{b}^{S*}x}{\lambda}$ to implement $\tilde{b}^{S*}$. For $C = 0$, this is higher than the equilibrium marginal consumer of our benchmark model, $\tilde{b}^{S*} > \tilde{b}^*$, given that $\tilde{b}^*$ is the solution to (9):[18]

$$1 - F(\tilde{b}^*) - \frac{3}{2}\tilde{b}^* f(\tilde{b}^*) = 0, \tag{32}$$

where $1 - F(b) - \frac{3}{2}bf(b)$ is a decreasing function (by Assumption 1). Anticipating that in each app market the marginal consumer pays its entire surplus, the marginal consumer is not willing to pay a positive device fee. Thus, we have $S^{S*} = 0$ (there is no gains for the platform from setting a lower device price because already all consumers who will ever buy join). The platform sets τ^S to maximize its revenue from app subscriptions under the condition that $S^{S*} = 0$ and the marginal consumer is $\tilde{b}^{S*}$:

$$\Pi^S = \tau^S \frac{1}{\lambda}\tilde{b}^{S*}(1 - F(\tilde{b}^{S*}))Q(\tilde{x}^S).$$

[18]This delivers $\tilde{b}^{S*} = 0.5$ for a uniformly distributed types b over $[0, 1]$, as opposed to $\tilde{b}^* = 0.4$ of the benchmark.

After replacing $\tau^S = 1 - \frac{\lambda K}{(1-F(\tilde{b}^{S*}))\tilde{b}^{S*}\tilde{x}^S}$ (setting the profit of marginal app at zero) we re-write the platform's problem as choosing $\tilde{x}^S$ to maximize

$$\Pi^S = \left(\frac{\tilde{b}^{S*}(1-F(\tilde{b}^{S*}))}{\lambda} - \frac{K}{\tilde{x}^{S*}} \right) Q(\tilde{x}^S),$$

which gives the first-order condition:

$$\frac{d\Pi^S}{d\tilde{x}} = \left(\frac{\tilde{b}^{S*}(1-F(\tilde{b}^{S*}))}{\lambda} - \frac{K}{\tilde{x}^{S*}} \right) Q'(\tilde{x}^{S*}) + Q(\tilde{x}^{S*}) \frac{K}{(\tilde{x}^{S*})^2} = 0,$$

Given that $\tilde{b}^{S*}$ is the maximizer of $\tilde{b}(1-F(\tilde{b}))$ (from the problem of the app developer, see (32)), $\tilde{b}^{S*} > \tilde{b}^*$, and $Q'(\cdot) < 0$, we obtain

$$\frac{d\Pi^S}{d\tilde{x}}|_{\tilde{b}=\tilde{b}^{S*},\tilde{x}=\tilde{x}^{S*}} = 0 < \frac{d\Pi^S}{d\tilde{x}}|_{\tilde{b}=\tilde{b}^*,\tilde{x}=\tilde{x}^{S*}} = \left(\frac{\tilde{b}^*(1-F(\tilde{b}^*))}{\lambda} - \frac{K}{\tilde{x}^{S*}} \right) Q'(\tilde{x}^{S*}) + Q(\tilde{x}^{S*}) \frac{K}{(\tilde{x}^{S*})^2}.$$

If the average consumer type at the initial equilibrium is smaller than twice the marginal consumer type, that is, if

$$\frac{B(\tilde{b}^*)}{1-F(\tilde{b}^*)} \leq 2\tilde{b}^*, \tag{33}$$

then we have

$$\frac{d\Pi^S}{d\tilde{x}}|_{\tilde{b}=\tilde{b}^*,\tilde{x}=\tilde{x}^{S*}} \leq \frac{d\Pi}{d\tilde{x}}|_{\tilde{b}=\tilde{b}^*,\tilde{x}=\tilde{x}^{S*}} = \left(\frac{\tilde{b}^*(1-F(\tilde{b}^*))}{2\lambda} + \frac{B(\tilde{b}^*)}{4\lambda} - \frac{K}{\tilde{x}^{S*}} \right) Q'(\tilde{x}^{S*}) + Q(\tilde{x}^{S*}) \frac{K}{(\tilde{x}^{S*})^2}$$

where $\frac{d\Pi}{d\tilde{x}}$ is given by (10) and we have $\frac{d\Pi}{d\tilde{x}}|_{\tilde{b}=\tilde{b}^*,\tilde{x}=\tilde{x}^*} = 0$. Using the concavity of Π in $\tilde{x}$, $\frac{d\Pi}{d\tilde{x}}|_{\tilde{b}=\tilde{b}^*,\tilde{x}=\tilde{x}^{S*}} > 0$ implies that $\tilde{x}^{S*} < \tilde{x}^*$. Note that condition (33) holds strictly for $b \sim U[0,1]$ since then $\tilde{b}^* = 0.4$ in the benchmark, so $\frac{B(\tilde{b}^*)}{1-F(\tilde{b}^*)} = \frac{\int_{0.4}^1 b db}{1-0.4} = 0.7 < 0.8$. Moreover, using (9) we can rewrite (33) as $\epsilon_B(\tilde{b}^*) \equiv -\frac{\tilde{b}^* B'(\tilde{b}^*)}{B(\tilde{b}^*)} \geq \frac{1}{3}$.

The profit of an app type x is

$$\pi^S(\tau^S, \tilde{b}^S) = (1-\tau^S)\frac{\tilde{b}^S x}{\lambda}(1-F(\tilde{b}^S)) - K.$$

E Online Appendix

E.1 Alternative Demand Specification

This section aims to show that the paper's main results go through under an alternative demand specification in which different app types charge different prices for in-app purchases. In particular, we assume that in stage 4 of the game, consumer utility maximization results in transaction demand given by

$$q^*(b, x, p) = b(1 - \frac{p}{x}) \tag{34}$$

This alternative demand specification implies that apps with different types x will offer a different price p for in-app purchases. Specifically, for a type x app, properties of linear demand imply that the price for transactions given by

$$p^* = \frac{x}{2} \tag{35}$$

for which the app profit is given by

$$\pi(x, \tilde{b}, \tau) = (1 - \tau)\frac{xB(\tilde{b})}{4} \tag{36}$$

The marginal app $\tilde{x}$ has zero net profit from joining so we obtain the app participation constraint

$$K = (1 - \tau)\frac{\tilde{x}B(\tilde{b})}{4} \tag{37}$$

The indirect utility from participating for a type b consumer is given by

$$V(b, \tilde{x}) = \frac{bQ(\tilde{x})}{8} \tag{38}$$

and the marginal consumer type $\tilde{b}$ has zero surplus from participating so we obtain the consumer participation constraint

$$S = \frac{\tilde{b}Q(\tilde{x})}{8} \tag{39}$$

We can now write the platform's profit as

$$\Pi(S, \tau, \tilde{x}, \tilde{b}) = (S - C)(1 - F(\tilde{b})) + \tau \int_{\tilde{x}}^{\bar{x}} \int_{\tilde{b}}^{\bar{b}} p^* q^* dbdx$$

Substituting S and τ from the consumer and app participation constraints respectively, we obtain

$$\Pi(\tilde{x},\tilde{b}) = \left[\frac{\tilde{b}}{8}\left(1-F(\tilde{b})\right) + \frac{B(\tilde{b})}{4} - \frac{K}{\tilde{x}}\right] Q(\tilde{x}) - C\left(1-F(\tilde{b})\right) \tag{40}$$

Thus, the platform solves

$$\max_{\tilde{b},\tilde{x}} \Pi$$

Using the fact that $B'(\tilde{b}) = -\tilde{b}f(\tilde{b})$ we have the FOC with respect to $\tilde{b}$:

$$\frac{1}{8}[1-F(\tilde{b}) - 3\tilde{b}f(\tilde{b})]Q(\tilde{x}) + Cf(\tilde{b}) = 0 \tag{41}$$

The FOC with respect to $\tilde{x}$ is

$$\left[\frac{\tilde{b}}{8}\left(1-F(\tilde{b})\right) + \frac{B(\tilde{b})}{4} - \frac{K}{\tilde{x}}\right] Q'(\tilde{x}) + \frac{K}{\tilde{x}^2}Q(\tilde{x}) = 0 \tag{42}$$

By inspecting the expressions for consumer participation, app participation, profit, and the first-order conditions, we can see that only scaling constants change between the the new and the original demand cases. Thus, using the exact same arguments, we can obtain direct analogs of Lemmas 1,2 and 3, Proposition 1 and Corollary 1.

E.1.1 Consumer Surplus, App Surplus and Welfare

The consumer surplus from the app store is the sum of transaction surplus of consumers minus device fee aggregated over those consumer types who join the platform:

$$CS(S,\tilde{x},\tilde{b}) = \int_{\tilde{b}}^{\bar{b}} \left(\frac{bQ(\tilde{x})}{8} - S\right) f(b)db \tag{43}$$

Substituting S for consumers' participation constraint yields

$$CS(\tilde{x},\tilde{b}) = \frac{1}{8}\left[B(\tilde{b}) - \tilde{b}(1-F(\tilde{b}))\right] Q(\tilde{x}) \tag{44}$$

For given marginal types $(\tilde{x},\tilde{b})$ and the app commission τ, app surplus is given by:

$$AS((\tau,\tilde{x},\tilde{b}) = \int_{\tilde{x}}^{\bar{x}} \left((1-\tau)\frac{x}{4}B(\tilde{b}) - K\right) g(x)dx = (1-\tau)\frac{B(\tilde{b})Q(\tilde{x})}{4} - K(1-G(\tilde{x}))$$

Substituting the commission that induces $\tilde{x}$ given $\tilde{b}$ we obtain

$$AS(\tilde{x}) = K\left(\frac{Q(\tilde{x})}{\tilde{x}} - (1 - G(\tilde{x})\right) \tag{45}$$

which is exactly the same expression as in the main model. Corollary 2 continues to hold, as once more, only the scaling constant in the expression for consumer surplus has changed.

Total welfare is the sum of consumer and app surplus and platform profits. That is, summing equations 40, 44 and 45 we obtain

$$W(\tilde{x}, \tilde{b}) = \frac{3}{8}B(\tilde{b})Q(\tilde{x}) - C(1 - F(\tilde{b})) - K(1 - G(\tilde{x})) \tag{46}$$

The analog of Proposition 2 is now the following:

Proposition 2': Taking as given the equilibrium app pricing, $p^* = \frac{x}{2}$, the welfare-maximizing marginal consumer type and the marginal app type are respectively

$$\tilde{b}^W = \frac{8C}{3B(\tilde{x}^W)}, \qquad \tilde{x}^W = \frac{8C}{3B(\tilde{b}^W)}.$$

which are supported by fees below cost: $S^W = \frac{C}{3} < C$ and $\tau^W = -\frac{1}{2}$, and the platform makes an operating loss.

E.1.2 Cap on Commission

Suppose there is a cap $\bar{\tau}$ on commission. The new platform optimization problem is given by

$$\max_{\tilde{b},\tilde{x}} \left[\frac{\tilde{b}}{8}\left(1 - F(\tilde{b})\right) + \frac{B(\tilde{b})}{4} - \frac{K}{\tilde{x}}\right] Q(\tilde{x}) - C\left(1 - F(\tilde{b})\right)$$

subject to $\bar{\tau} = 1 - \frac{4K}{\tilde{x}B(\tilde{b})}$ or equivalently $\tilde{x} = \frac{4K}{(1-\bar{\tau})B(\tilde{b})}$

The constrained first order condition is given by

$$\frac{1}{8}[1 - F(\tilde{b}) - 3\tilde{b}f(\tilde{b})]Q(\tilde{x}) + Cf(\tilde{b}) + \left[\left[\frac{\tilde{b}}{8}\left(1 - F(\tilde{b})\right) + \frac{B(\tilde{b})}{4} - \frac{K}{\tilde{x}}\right] Q'(\tilde{x}) + \frac{K}{\tilde{x}^2}Q(\tilde{x})\right]\frac{d\tilde{x}}{d\tilde{b}} = 0 \tag{47}$$

It is straightforward to verify that since only scaling constants change relative to the main model, the proof of Proposition 3 goes through essentially verbatim. Here, we illustrate the argument for completeness: Let $\tilde{x}^*, \tilde{b}^*$ denote the unconstrained equilibrium levels of the marginal app and the marginal consumer, respectively. Similarly, $\tilde{x}^{C*}, \tilde{b}^{C*}$ denote the equilibrium levels of the marginal app and the marginal consumer with the binding cap on τ: $\bar{\tau} < \tau^*$. Let $\tilde{x}^{\bar{\tau}}(\tilde{b})$ denote $\tilde{x}$ that satisfies the constraint at $\tilde{b}$. A binding τ constraint means that the platform would induce a higher marginal app type (lower app participation) at $\tilde{b} = \tilde{b}^*$ if it was not constrained: $\tilde{x}^* > \tilde{x}^{\overline{\tau}}(\tilde{b}^*)$. Using the unconstrained equilibrium equation (42) and the concavity of the platform's profit with respect to $\tilde{x}$ at the unconstrained choice $\tilde{b}^*$, this implies that

$$\left.\frac{\partial \Pi}{\partial \tilde{x}}\right|_{\tilde{b}=\tilde{b}^*} = \left[\frac{1}{8}\tilde{b}^*(1-F(\tilde{b}^*)) + \frac{B(\tilde{b}^*)}{4} - \frac{K}{\tilde{x}^{\overline{\tau}}(\tilde{b}^*)}\right] Q'(\tilde{x}^{\overline{\tau}}(\tilde{b}^*)) + \frac{K}{\left(\tilde{x}^{\overline{\tau}}(\tilde{b}^*)\right)^2} Q(\tilde{x}^{\overline{\tau}}(\tilde{b}^*)) > 0.$$

From the cap constraint we have $\frac{d\tilde{x}^{\overline{\tau}}}{d\tilde{b}} = -\frac{B'(\tilde{b})\tilde{x}}{B(\tilde{b})} > 0$. Hence, at the constrained equilibrium (with the binding cap on τ), using the constrained equilibrium condition (47), we obtain

$$\left.\frac{d\Pi}{d\tilde{b}}\right|_{\tilde{b}=\tilde{b}^*,\, \tilde{x}=\tilde{x}^{\overline{\tau}}(\tilde{b}^*)} > 0,$$

given $\frac{1}{8}\left((1-F(\tilde{b}^*)) - 3\tilde{b}^* f\left(\tilde{b}^*\right)\right) = 0$ from the unconstrained equilibrium equation (41) at $C = 0$. The latter inequality implies a higher marginal consumer type at the new equilibrium: $\tilde{b}^* < \tilde{b}^{C*}$, and so lower consumer participation. Above we have shown that $\tilde{x}^* > \tilde{x}^{\overline{\tau}}(\tilde{b}^*)$ and that at the constrained equilibrium $\frac{d\tilde{x}^{\overline{\tau}}}{d\tilde{b}} > 0$. These together with $\tilde{b}^* < \tilde{b}^{C*}$ imply that $\tilde{x}^{\overline{\tau}}(\tilde{b}^*) < \tilde{x}(\tilde{b}^{C*}) = \tilde{x}^{C*}$. Moreover, we have $\frac{1}{8}\left(1 - F(\tilde{b}^{C*}) - 3\tilde{b}^{C*} f(\tilde{b}^{C*})\right) < 0$ since $\tilde{b}^* < \tilde{b}^{C*}$ (and using the concavity of platform profit at $\tilde{b}^*$, which is proved in Proposition 1), so

$$\left.\frac{\partial \Pi}{\partial \tilde{x}}\right|_{\tilde{b}=\tilde{b}^{C*}, \tilde{x}=\tilde{x}^{C*}} = \left[\frac{1}{8}\tilde{b}^{C*}(1-F(\tilde{b}^{C*})) + \frac{B(\tilde{b}^{C*})}{4} - \frac{K}{\tilde{x}^{C*}}\right] Q'(\tilde{x}^{C*}) + \frac{K}{(\tilde{x}^{C*})^2} Q(\tilde{x}^{C*}) > 0.$$

Thus, $\tilde{x}^* > \tilde{x}^{C*}$. We can see that we have increased app participation and lower consumer participation in the constrained equilibrium. Since $\tilde{x}^* > \tilde{x}^{C*}$ and $AS'(\tilde{x}) < 0$, it follows that app surplus increases.

The main difference in the analysis with this new demand specification is the following result:

Corollary 6 *Suppose consumer and app types (b, x) are uniformly distributed over $[0, 1]$. A binding cap regulation on third-party app commission may increase or lower aggregate consumer surplus.*

We illustrate this corollary in Figures 13, 14 and 15. In particular, Figure 13 shows the effect of the commission cap when the cost of app development is high. In this case, the platform optimally sets a low commission (approximately 2%) to ensure app participation. A cap in this commission that induces more participation results in unambiguously making consumers better off. The opposite happens when the app development cost is minimal, as illustrated in Figure 15. The platform does not worry about low app participation and charges a high commission (approximately 40%). A cap, in this case, unambiguously decreases consumer surplus as it induces too high app participation. Finally, for cases in the middle, the effect of the cap on commission depends on the cap size as illustrated in Figure 14. In this case, if the cap is close to the unconstrained optimal, consumers are better off, while if the cap is very low, consumers are worse off. This happens because a cap that is close to the platform's optimal increases app participation but not by too much relative to the case where the cap is lower.

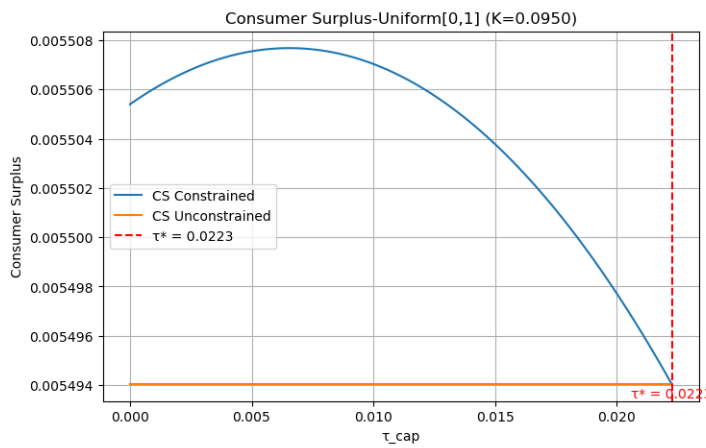

Figure 13: Commission Cap increases CS

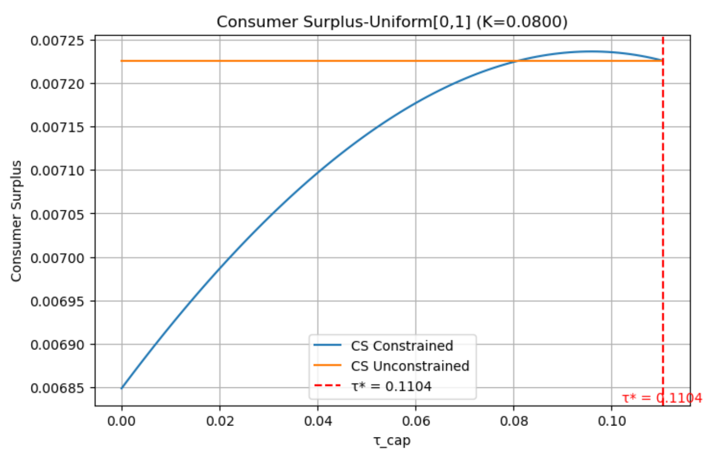

Figure 14: Ambiguous Effect of Commission Cap on CS

E.2 Vertically separated app platform

Suppose the platform provides the operating system and an independent manufacturer sells devices on which consumers run the operating system and make transactions (in-app purchases) on its app

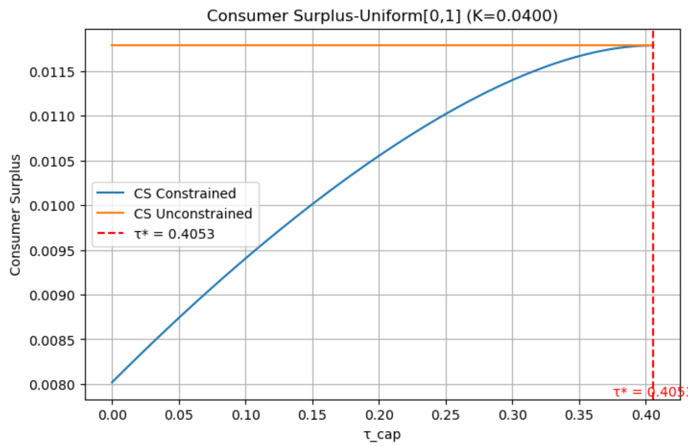

Figure 15: Commission Cap decreases CS

store. This structure represents Google's Android system and app store Google Play since independent manufacturers, like Samsung, sell devices that operate with the Android system. We describe below the main changes in the benchmark model to capture the vertically separated structure.

There is one platform A running the app store and one manufacturer D producing devices. A licenses its operating system to D in exchange for a licensing fee L per consumer paid by D.[19] We modify only the first stage of the benchmark game by adding two stages:

1. The platform sets a licensing fee L per consumer and a percentage commission τ
2. The manufacturer sets its device price S.

The last three stages of the game are the same as the benchmark and the equilibrium of the subgame starting at stage 3 is the same as before. All apps set $p^* = \lambda$. In app market of quality x consumer type b consumes $q^*(b, x) = \frac{bx}{4\lambda^2}$ (4) and gets indirect utility of $v(b, x) = \frac{bx}{2\lambda}$ (5).

E.2.1 Homogenous apps and heterogenous consumers

To simplify the solution of the model we will consider first the case where all apps have the same type $x = \overline{x} > 0$ and keep consumer heterogeneity as in the benchmark. Given τ all apps join the platform if and only if they make non-negative profits, so the participation condition for apps is

$$PC_A : (1 - \tau)\frac{B(\tilde{b})\overline{x}}{\lambda} \geq K, \tag{48}$$

[19] We allow L to be negative in which case it is a payment from A to D. We show that $L > 0$ in equilibrium.

and the marginal consumer type $\tilde{b}$ has zero surplus:

$$PC_C : \frac{\tilde{b}\overline{x}}{2\lambda} = S. \tag{49}$$

Consumer demand for devices is $1 - F(\tilde{b})$. The manufacturer D's profit is its net revenues from device sales after paying the licensing fee L to the platform and the cost of the device C:

$$\Pi_D(S) = (S - C - L)\left(1 - F(\tilde{b})\right).$$

In Stage 2, given τ and L, D sets S by maximizing its profit $\Pi_D(S)$ subject to (48) and (49). We write D's problem by substituting S from constraint (49), and so D chooses the marginal consumer type subject to the app participation condition:

$$\max_{\tilde{b}} \left(\frac{\tilde{b}\overline{x}}{2\lambda} - C - L\right)\left(1 - F(\tilde{b})\right) \qquad \text{s.t.} \quad (1-\tau)\frac{B(\tilde{b})\overline{x}}{4\lambda} \geq K, \tag{50}$$

The best-response of D, $\tilde{b}^{BR}(\tau, L)$, is the unconstrained solution:[20]

$$\frac{\overline{x}}{2\lambda}\left[1 - F(\tilde{b}^D) - \tilde{b}^D f(\tilde{b}^D)\right] + (C+L)f(\tilde{b}^D) = 0 \quad \text{or} \quad \tilde{b}^D = (C+L)\frac{2\lambda}{\overline{x}} + \frac{1 - F(\tilde{b}^D)}{f(\tilde{b}^D)}, \tag{51}$$

if this solution satisfies the app participation constraint, $(1-\tau)\frac{B(\tilde{b}^D)\overline{x}}{4\lambda} \geq K$. In that case $\tilde{b}^{BR}$ increases in licensing fee L. Otherwise, the constraint determines the equilibrium marginal app type: $(1-\tau)\frac{B(\tilde{b}^C)\overline{x}}{4\lambda} = K$, where $\tilde{b}^C < \tilde{b}^D$ since $B'(\cdot) < 0$:

$$\tilde{b}^{BR}(\tau, L) = \begin{cases} \tilde{b}^D & \text{if} \quad (1-\tau)\frac{B(\tilde{b}^D)\overline{x}}{4\lambda} \geq K, \\ \tilde{b}^C & \text{otherwise} \end{cases} \tag{52}$$

In Stage 1 the platform A sets L anticipating the best-response of D. A's profit is the sum of licensing fee revenues plus commission revenues:

$$\Pi_A = L(1 - F(\tilde{b}^{BR})) + \tau\frac{B(\tilde{b}^{BR})\overline{x}}{4\lambda}$$

Define the vertically integrated profits of the platform and device manufacturer after replacing (49):

$$\Pi_{VI}(\tilde{b}, \tau) = \left(\frac{\tilde{b}\overline{x}}{2\lambda} - C\right)\left(1 - F(\tilde{b})\right) + \tau\frac{B(\tilde{b})\overline{x}}{4\lambda}.$$

[20] Note that the profit of D is strictly quasi-concave in $\tilde{b}$ by the MHRP of $F(b)$ (Assumption 1).

and consider $\tilde{b}^{VI}, \tau^{VI}$ that maximize $\Pi_{VI}(\tilde{b}, \tau)$ subject to (48). The integrated profits increase in τ and so are maximized by setting $\tau^{VI}(\tilde{b}) = 1 - \frac{4\lambda K}{B(\tilde{b})\overline{x}}$ which captures all profits of apps and $\tilde{b}^{VI}$ maximizing total profit:

$$\tilde{b}^{VI} = \arg\max_{\{\tilde{b}\}} \left[\left(\frac{\tilde{b}\overline{x}}{2\lambda} - C \right) \left(1 - F(\tilde{b}) \right) + \frac{B(\tilde{b})\overline{x}}{4\lambda} - K \right], \tag{53}$$

The following shows that A implements the integrated outcome in equilibrium:

Proposition 11 *In equilibrium the vertically separated platform implements the vertically integrated outcome by setting $L^* = \frac{\tilde{b}^{VI}\overline{x}}{2\lambda} - C$ and $\tau^* = 1 - \frac{4\lambda K}{B(\tilde{b}^{VI})\overline{x}}$, where $\tilde{b}^{VI}$ is given by (53).*

The platform A internalizes the entire industry profit since it captures the device manufacturer's entire margin via a licensing fee and it captures all app profit via a commission. Thus, A wants to implement the vertically integrated outcome. When the platform sets $L^* = \frac{\tilde{b}^{VI}\overline{x}}{2\lambda} - C$ and $\tau^* = 1 - \frac{4\lambda K}{B(\tilde{b}^{VI})\overline{x}}$, it induces D to choose $\tilde{b}^{VI}$. To see that consider D's problem of choosing $\tilde{b}$ subject to the app participation (48) given A sets L^* and τ^*:

$$\max_{\tilde{b}} \Pi_D(\tilde{b}) = (\tilde{b} - \tilde{b}^{VI}) \frac{\overline{x}}{2\lambda} \left(1 - F(\tilde{b}) \right) \quad \text{st.} \quad \frac{B(\tilde{b})}{B(\tilde{b}^{VI})} \geq 1.$$

D wants to set $\tilde{b} \geq \tilde{b}^{VI}$, since otherwise it would obtain negative profits. The constraint is binding ($B'(\tilde{b}) < 0$), so D's best-response to L^* and τ^* is to set $\tilde{b}^{VI}$ and implement the integrated outcome. This result is striking since the platform does not suffer from the fact that the manufacturer sets the device price and D ignores the app revenues. The platform could perfectly control D's choice via setting τ^* which also captures entire app revenue when $\tilde{b} = \tilde{b}^{VI}$. The platform then uses the licensing fee to capture D's profits. Hence, the platform does not have a profitable deviation from (L^*, τ^*) since this gives the platform the highest industry profit. Note that the platform has first mover-advantage when it chooses τ before D chooses $\tilde{b}$ (S): A induces D to choose the right level of $\tilde{b}^{VI}$ by setting τ^* at the same time as L^*. If we considered an alternative timing where A first chooses L in stage 1, then D chooses S and A chooses τ simultaneously in stage 2, the equilibrium characterized above would still be an equilibrium, but there could be other equilibria due to simultaneous choices of A and D.

We next illustrate that the platform could not implement the vertically integrated outcome if there is also downward sloping demand (unobserved heterogeneity) on the app side.

E.2.2 Heterogenous apps and heterogenous consumers- TBC

We now assume apps are heterogenous like in the benchmark model. The participation conditions of apps and consumers are given by (7) and (6). To simplify the analysis we consider simultaneous timing: A sets τ and D sets S simultaneously in Stage 2. This changes the problem of A when choosing τ, since now A does not account for the best-reply of D to τ. This change makes it easier to show that the separated platform cannot implement the integrated outcome.We then argue that this result would be valid also in the sequential timing.

Given τ and L, D sets S by maximizing its profit subject to (7) and (6). By substituting S from (6) we write D's problem as choosing the marginal consumer type subject to (7):

$$\max_{\tilde{b}} \left(\frac{\tilde{b}Q(\tilde{x})}{2\lambda} - C - L \right) \left(1 - F(\tilde{b}) \right) \qquad \text{s.t.} \quad (1-\tau)\frac{B(\tilde{b})\tilde{x}}{4\lambda} = K, \tag{54}$$

Note that, given τ and L, when D changes $\tilde{b}$, this implements a different level of $\tilde{x}$. Hence, the platform cannot perfectly control D's choice via τ (which it was able to do in the case of homogenous apps in the previous subsection). Besides, D ignores the positive effect of selling more devices on the revenues from apps, which the platform internalizes. By substituting (7) implicitly we rewrite D's problem as

$$\max_{\tilde{b}} \left(\frac{\tilde{b}Q(\tilde{x}^{PC}(\tilde{b},\tau))}{2\lambda} - C - L \right) \left(1 - F(\tilde{b}) \right), \tag{55}$$

where $\tilde{x}^{PC}(\tilde{b}) = \frac{4\lambda K}{B(\tilde{b})(1-\tau)}$ is the solution to (7). Observe that $\tilde{x}^{PC}(\tilde{b})$ is the relation illustrated in Figure 1 by the increasing and convex orange curve. Note also that increasing τ shifts the curve right, resulting in a higher $\tilde{x}$ for a given $\tilde{b}$, formally $\frac{\partial^2 \tilde{x}^{PC}}{\partial \tilde{b} \partial \tau} > 0$. D's equilibrium choice of the marginal consumer type is the solution to the first-order condition (note that the second-order condition is satisfied under (Assumption 1)):

$$\frac{Q(\tilde{x}^{PC})}{2\lambda}\left[1 - F(\tilde{b}) - \tilde{b}f(\tilde{b})\right] + (C+L)f(\tilde{b}) + \tilde{b}\frac{Q'(\tilde{x}^{PC})}{2\lambda}\frac{\partial \tilde{x}^{PC}}{\partial \tilde{b}}\left(1 - F(\tilde{b})\right) = 0, \tag{56}$$

which determines the best-response of D to (L,τ): $\tilde{b}^{BR}(\tau, L)$.

A's profit is the sum of licensing fee revenues plus commission revenues:

$$\Pi_A(\tilde{b},\tilde{x}) = L(1 - F(\tilde{b})) + \tau\frac{B(\tilde{b})Q(\tilde{x})}{4\lambda}.$$

In Stage 2, given S and L, A chooses τ to maximize $\Pi_A(\tilde{b},\tilde{x})$ subject to (7). By substituting (7) we rewrite A's second stage problem as

$$\max_{\tilde{x}} \left(L(1-F(\tilde{b})) + \frac{B(\tilde{b})Q(\tilde{x})}{4\lambda} - \frac{KQ(\tilde{x})}{\tilde{x}} \right).$$

In Stage 1 the platform A sets L and τ anticipating $\tilde{b}^{BR}(\tau, L)$. A's profit is the sum of licensing fee revenues plus commission revenues:

$$\Pi_A = L(1-F(\tilde{b}^{BR})) + \tau \frac{B(\tilde{b}^{BR})Q(\tilde{x}^{PC}(\tilde{b}^{BR}, \tau))}{4\lambda}$$

Observe that the vertically integrated profits of the platform and device manufacturer after replacing (7) correspond to the platform profit in the benchmark (8), so the vertically integrated outcome is implemented by setting $(\tilde{b}^*, \tilde{x}^*)$.

Proposition 12 *In equilibrium the vertically separated platform implements a lower level of participation on both sides of the market than the vertically integrated equilibrium of the benchmark.*

The proposition shows that the vertically separated platform is less efficient than the integrated platform of the benchmark. This inefficiency is due to three factors: 1) The device manufacturer does not internalize the positive effect of selling more devices on revenues from apps, 2) The platform does not internalize the positive effect of lowering τ on device sale revenues, 3) a positive licensing fee increases the cost of the device manufacturer and so increases the device fee.

We prove the proposition first considering the case where $L = 0$. We write the vertically integrated profit as a function of $\tilde{b}$ and τ:

$$\Pi_{VI} = \left(\frac{\tilde{b}Q(\tilde{x}^{PC}(\tilde{b}))}{2\lambda} - C \right) \left(1 - F(\tilde{b})\right) + \tau \frac{B(\tilde{b})Q(\tilde{x}^{PC})}{4\lambda}$$

where $\tilde{x}^{PC}(\tilde{b}) = \frac{4\lambda K}{B(\tilde{b})(1-\tau)}$.

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