The Agency and Wholesale Models When a Platform Can Charge Entry Fees^{*}

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Abstract

We study the agency and wholesale models of intermediation in a bilateral monopoly setting where a platform can charge entry fees. In the benchmark case where the platform can use full-profit-extracting entry fees, the agency model is (weakly) superior for all agents: consumers face lower final prices, the platform makes higher profits and the seller makes no profits in either intermediation model. We next study how this insight generalizes in settings where the platform can only extract a fraction of the profit of the seller via entry fees. For arbitrary demand functions, the agency model leads to lower prices provided the fraction of the seller's profit extracted by the platform is sufficiently large. In the special case where demand satisfies Marshall's second law of demand, the platform continues to prefer the agency model no matter how much surplus it can extract from the seller (including nothing at all), and so do consumers; however, the seller prefers the wholesale model provided that rent-extraction via entry fees is sufficiently low. We extend the analysis to settings where the platform faces uncertainty about the seller's willingness to pay for entry. In both intermediation models, the platform does not always charge an entry fee. When it does, and provided Marshall's second law of demand holds, the final consumer price is lower under agency than under wholesale. Consumer surplus depends not only on the final price, but also on the seller's entry probability. We provide conditions under which both the platform and consumers prefer a switch to the agency model, to the detriment of the seller.

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1 Introduction

Entry fees are common in many industries where platforms act as intermediaries for consumers. These fees serve different purposes and can take different forms, depending on the industry and the business model. For example, Amazon charges professional sellers a monthly fee of \$39.99 for placing their products on the platform, along with a commission that varies by product category and ranges from 6% to 45% of the total transaction value, including shipping costs.¹ The magnitude of entry fees varies significantly across platforms. Some platforms impose substantial entry fees on sellers; for example, Alibaba charges up to \$1,833 per month to maintain an online store. In contrast, other platforms, such as Facebook Marketplace, do not charge any entry fee at all. Entry fees are also common in traditional retail. For example, supermarket chains impose various fees on manufacturers, such as slotting fees and pay-to-stay fees, for access to their shelf space. A survey conducted by the Federal Trade Commission (FTC, 2003) reports that retailers impose slotting allowances for 50% to 90% of new grocery product introductions (see also Klein and Wright, 2007; Bonnet et al., 2013; Bonnet and Dubois, 2015). While these fees vary from country to country, they represent a significant portion of the retailers' profits,² leading some experts to argue that "[s]upermarkets today are as much about selling shelves to food companies as they are about selling food to customers"(Rivlin, 2016).

In this paper, we study and compare two intermediation models widely used in online markets, namely the agency and the wholesale models, in terms of final consumer prices, entry probability and agent profits. In the wholesale model, the platform first sets the entry fee, after which the seller decides whether to enter and then sets its wholesale price. Finally, the platform acts as the retailer and sets the final consumer price. The wholesale model has been the core business model of Amazon for many years and continues to be so for many product categories. Other examples of platforms using the wholesale model include Walmart, Newegg, and Wayfair. In the agency model, the platform first sets an entry fee and a commission rate, the seller decides to enter, and then sets the final consumer price. Many platforms, including Booking, Uber and the Apple App Store, employ the agency model.

Using a bilateral monopoly setting where the seller must commercialize its product using the intermediation services of a platform, we compare the wholesale and the agency models in three distinct settings. As a benchmark, we first consider the case where the platform can charge full-profit-extracting entry fees. In such a case, we show that the agency model results in a final

¹Similarly, app stores charge developers a fee to list their mobile applications, and commissions on app and in-app purchases. LinkedIn charges recruiters \$1,680 annually for its 'Recruiter Lite' service. Doctolib, an intermediary connecting patients and doctors in France, charges general practitioners a monthly fee between ϵ 135 and $\in 274$, plus a 1% commission on teleconsultations. OpenTable charges restaurants a monthly fee ranging from \$149 to \$499 depending on the contract, plus a 2% commission. Artsy, an online marketplace, charges galleries a monthly fee of \$425 and a 10% commission.

 2 In the U.S. grocery yogurt market, Hristakeva (2022) estimates that these payments made by the suppliers to the retailers represent on average 19.9% of retailers profits.

consumer price equal to the price that maximizes industry profit, thereby eliminating the doublemarginalization problem altogether. The wholesale model, by contrast, fails to eliminate double marginalization and hence results in prices that are excessive from the viewpoint of industry profits maximization. With full-profit-extracting entry fees, the seller makes no profit in either intermediation model. As a result, both the platform and the consumers prefer the agency model to the wholesale model, irrespective of the shape of demand, while the seller is indifferent. This result is in stark contrast with Johnson (2017), who compares the two intermediation models in a bilateral monopoly setting without entry fees.

After studying the performance of the two intermediation models when the platform can charge full-profit-extracting entry fees, in line with Calzolari et al. (2020), we turn to a more realistic setting in which the seller can only afford to pay a part of its profit to enter. For example, this may be because the seller is required to provide a minimum return to its investors or need to retain a share of its profits to invest in advertising or R&D. This variant of the model collapses to that in Johnson (2017) in the limit case where the seller's willingness to pay for entry is zero. We show that, no matter the demand function, the agency model leads to lower prices provided that the platform extracts a sufficiently large fraction of the seller's profit. In the special case where demand satisfies Marshall's second law of demand, the agency model leads to lower prices irrespective of how much surplus it can extract from the seller (including nothing at all, like in Johnson (2017).³ The platform prefers the agency model and so do consumers. However, the seller prefers the wholesale model when the amount of rents extracted by the platform via entry fees is sufficiently low.

We finally turn to an extension of the model where the platform is uncertain about the seller's willingness to pay for entry. While in the case without uncertainty the platform always chooses its entry fee so as to leave the seller indifferent between entering and not entering, in this setting the platform picks its entry fee so as to resolve the following trade-off: a higher entry fee increases the probability that the seller refuses to enter, while it increases the platform's payoff in case of entry. A priori, thus, it not clear that a platform will employ entry fees at all. We show that the platform's choice of entry fee can conveniently be reformulated as a choice of the fraction of the seller's profit it wants to extract.

For each of the intermediation models, we establish a condition under which this trade-off makes it optimal for the platform to charge no entry fee. This condition is quite intuitive and implies that the platform will refrain from using entry fees when the probability mass at zero willingness to pay for entry is sufficiently large. This result then adds to the literature by providing a possible rationale for why some platforms do use entry fees while others do not, or why some platforms use entry fees for some –but not all– product categories.

³This implies that one can dispense with assumptions on demand stronger than just Marshall's second law of demand (in contrast to Johnson, 2017; Proposition 3). The reason for this is that our proof is somewhat different and this allows us to derive a weaker sufficient condition.

Interestingly, in the wholesale model, the condition under which the platform does not employ entry fees relates to the pass-through rate of the wholesale price to the final consumer price, which is equal to the platform's margin relative to the seller's margin (Bresnahan and Reiss, 1985). In particular, the higher the pass-through rate, the more likely it is that the platform abstains from charging an entry fee. Moreover, when the platform charges a strictly positive entry fee, the fraction of the seller's profit captured by the platform via the entry fee is larger the lower the pass-through. Because the pass-through of higher marginal costs is larger for the platform than for the seller if and only if the demand curvature is increasing (Spengler, 1950), an increase in wholesale or retail costs results in the platform capturing a smaller fraction of the seller's profit.

In the agency model, the lower the seller's marginal cost, the more likely it is that the platform does not charge entry fees, in which case the bulk of the platform profits is derived from commissions. The reason for this is that entry fees and commissions are negatively related. As the seller's marginal cost goes up, commissions cause a greater double marginalization problem and the platform relies more heavily on entry fees to raise profits. When the platform charges a positive entry fee, exactly like in the wholesale model, the fraction of the seller's profit captured by the platform is larger the higher the platform's margin relative to the seller's margin.

A comparison of the two intermediation models when the platform finds it optimal to charge an entry fee yields a necessary and sufficient condition for the final price to be lower in the agency model than in the wholesale model. This condition requires that the platform extracts a sufficiently large fraction of the seller's profit in the agency model and holds for any demand function that satisfies Marshall's second law of demand. This result significantly generalizes the finding in Johnson (2017).

To rank consumer surplus levels across intermediation models, it is no longer sufficient to compare final consumer prices because the seller's entry probability is also relevant. We show that when the demand satisfies Marshall's second law of demand and the margin of the platform is sufficiently large, the seller's entry probability under the agency model is higher and the price lower than under the wholesale model. As a result, consumer surplus is higher under agency and platform's profits too. Finally, we observe that the condition ensuring that the entry probability is higher under the agency model implies that the seller prefers the wholesale model.

Related literature

We contribute to the literature that compares the wholesale and agency models. This literature, to the best of our knowledge, has so far not allowed for entry fees. Johnson (2017) compares these two intermediation models in a bilateral monopoly setting. As is known from the taxation literature, he reports that the retail price is lower in the agency model than in the wholesale model if Marshall's second law of demand holds (Bishop, 1968). Gaudin and White (2014a)

reports a similar result. Because the agency model with per-unit commissions is similar to the wholesale model, Gaudin and White (2014b), Gaudin and White (2021) and Llobet and Padilla (2016) contain distinct versions of this result.⁴ De Los Santos et al. (2024) show that this price ranking may be reversed if the platform and the seller are allowed to bargain over the terms of trade and the bargaining power of the platform is sufficiently large.

Our paper generalizes these results to settings where platforms can charge sellers for entry. Specifically, we allow for full-profit-extracting and partial-profit-extracting entry fees.⁵ Our contribution is to show that the final consumer price under the agency model is always lower than under the wholesale model provided that the platform can extract sufficient surplus form the seller via entry fees. We also show that when the platform can charge full-profit-extracting entry fees, the agency model is efficient, while the wholesale model is not. We extend our results to a setting where there is uncertainty about the seller's willingness to pay for entry and show that the platform will not always charge entry fees.

Johnson (2017) also shows conditions on demand under which a shift from the wholesale model to the agency model may result in a conflict of interest, with the platform benefiting at the expense of the seller. We show that the platform prefers the agency model under weaker conditions than those in Johnson (2017), even if entry fees are not charged, and establish that a conflict of interest arises when rent-extraction via entry fees is not too large.

Hagiu and Wright (2015) study the choice of a monopoly intermediary to adopt the agency model or the wholesale model. They show that this choice depends on whether the intermediary or the sellers have better information to conduct marketing activities. Loertscher and Niedermayer (2020) show that a monopoly intermediary may adopt the agency model to prevent the emergence of a competing platform. Foros et al. (2017) study the endogenous choice of the wholesale or agency model under platform competition. They show that agency is never adopted in equilibrium when it is pro-competitive. In contrast, Abhishek et al. (2016) show that competing platforms adopt the agency model when their online channel diverts sales from their traditional (brick-and-mortar) channels.

Finally, Johnson (2020) studies the agency and wholesale intermediation models in a model where two retail platforms compete over two periods in the presence of consumer lock-in. He shows that, relative to the wholesale model, the agency model can lead to higher prices in the first period but lower prices in the second period.

The rest of the paper is organized as follows. In Section 2, we set up the model of bilateral

⁴This result has been generalized in various competitive settings. In Section 2 of Johnson (2017), the author extends the result to a setting in which there is imperfect competition at both layers of the supply chain using a conduct-parameter approach. Gu and Huang (2024) generalizes this result to a setting in which there is a platform and a bunch of sellers selling homogeneous products and competing in quantities. They also have a section with two competing platforms in which sellers multi-home; however, they assume that sellers' quantities on different platforms are independent of each other.

⁵In a similar spirit, Calzolari et al. (2020) develop a model of exclusive dealing where the upstream firms cannot efficiently extract the full profit of the downstream firms through fixed fees.

monopoly. In Section 3, we compare the wholesale and agency models for the case in which the platform can charge full-profit-extracting entry fees. In Section 4, we consider the case where the platform can only extract a fraction of the seller's profit. In Section 5, we consider the case where the platform faces uncertainty about the seller's willingness to pay for entry. We conclude in Section 6.

2 The model

We study a bilateral monopoly market where a seller (S) produces a good and sells it to final consumers via a platform (P). We assume that the platform has a marginal cost equal to $c_p > 0$; the marginal cost of the seller is $c_s > 0.6$ Let the demand for the seller's product be denoted by $Q(p)$, where p is the price of the good and $Q'(p) < 0$. As in Johnson (2017), it is useful to define the function:

$$
\lambda(p) \equiv -\frac{Q(p)}{Q'(p)} > 0,
$$

where $\lambda(p)$ is a measure of the price sensitivity of demand.⁷ We also assume that $\lambda(p)$ and $\lambda(p)(2 - \lambda'(p))$ have a slope less than 1, which is necessary to have second-order conditions (SOCs) satisfied. The notion of curvature of demand, $\sigma(p) \equiv \frac{Q(p)Q''(p)}{(Q'(p))^2}$ $\frac{\mathcal{U}(p)Q^{\prime\prime}(p)}{(Q'(p))^2}$, is related to $\lambda(p)$ and it is easy to check that $\sigma(p) = \lambda'(p) + 1$. Finally, we assume that the intercept of demand is high enough so that trade is profitable: $Q(c_p + c_s) > 0$.

Differently from Johnson (2017), we allow the platform to charge the seller a positive entry fee, which we denote F. However, following Calzolari et al. (2020) , we assume that extracting rents through entry fees potentially creates dead-weight losses, which implies that the amount that the platform can charge for entry may be limited by the seller's willingness to pay.⁸ To keep the model as realistic as possible, we assume that a seller is not able to enter the market if the entry fee is larger than a share β of its expected profits.⁹ We assume that β is distributed according to the cdf G with support $[0, \bar{\beta}]$, with $0 < \bar{\beta} \leq 1$. Without loss of generality, we

 6 If $c_s = 0$, the final price set by the seller in the agency model is not affected by the commission chosen by the platform. See Kind and Koethenbuerger (2018) for a discussion in the context of taxation of printed and digital books.

Tote that $\lambda(p) = \frac{p}{\varepsilon(p)}$, where $\varepsilon(p) \equiv \frac{-pQ'(p)}{Q(p)}$ $\frac{pQ(p)}{Q(p)}$ is the price elasticity of demand. $\lambda(p)$ can also be interpreted as the negative of the inverse of the slope of log-demand. Marshall's second law of demand, which refers to demands for which the price elasticity increases in price, is here equivalent to $\lambda(p) - p\lambda'(p) \geq 0$. When demand stems from a unit mass of consumers with random valuations following the distribution $F(p)$ and density $f(p)$, $\lambda(p)$ is equal to the inverse hazard rate of the random valuations (*i.e.*, the Mills ratio): $\lambda(p) = \frac{1-F(p)}{f(p)}$. Hence, $\lambda'(p) < 0$ is equivalent to log-concavity of demand, $\lambda'(p) > 0$ to log-convexity, and $\lambda'(p) = 0$ to log-linearity.

⁸ In a similar vein, Gaudin and White (2021) assume a platform with limited ability to charge consumers for access.

⁹For example, in line with Calzolari et al. (2020), if the platform charges an entry fee F, its costs $(1 + \kappa)F$ to the seller, where $\kappa \geq 0$ represents a dead-weight loss. Thus, if the seller's profit is π^S , the seller enters if and only if $\beta \pi^S \geq F$, with $\beta \equiv 1/(1+\kappa) \leq 1$. Other interpretations of β are possible. For example, $1-\beta$ may be the share of the seller's profits necessary to be able to operate (e.g., to pay corporate taxes or remunerate investors). See also the discussion in Section 4.

normalize $\bar{\beta} = 1$. Let g be the density of G and define the inverse hazard rate by $\mu(\beta) \equiv \frac{1-G(\beta)}{G(\beta)}$ $\frac{-G(\beta)}{g(\beta)}$. We assume that the density g is finite and, for the SOCs to hold, increasing. This implies that $\mu(\beta)$ is decreasing.¹⁰

The case where the platform cannot charge an entry fee is studied in Johnson (2017) and it arises here when G is degenerated at $\beta = 0$. Our formulation is general enough to capture the following alternative cases, which we analyze in turn in the rest of the paper:

- full-profit-extracting entry fees. The benchmark case in which the platform can charge entry fees that extract the full profit of the seller arises when G is degenerated at $\beta = 1$. We present the analysis with full-profit-extracting entry fees in Section 3.
- partial-profit-extracting entry fees. Because full-profit-extracting entry fees may be impractical (see above), we next examine in Section 4 the case where G is degenerated at $\beta \in [0, 1)$, which implies that the platform can only extract a fraction β of the seller's profit.
- partial-profit-extracting entry fees with seller heterogeneity. To model the idea that the platform may be uncertain about the seller's willingness to pay for entry, we let G to be a proper distribution. Interestingly, in this case we shall show that there are conditions, which depend on the primitives of the model, under which the platform abstains from charging entry fees. We place this analysis in Section 5.

We study (and compare in terms of final consumer prices, entry probability and profits) two business models that are widely used in practice by platforms. First, we consider the wholesale model, in which the platform first sets the entry fee to be paid by the seller, who then sets a wholesale price before the platform chooses the price at which it sells the good to final consumers. We then analyze the *agency model*, in which the platform first sets the entry fee and the commission it charges to the seller, and then the seller picks the price at which it sells the good to final consumers. These two modes of intermediation have been compared by Johnson (2017) for the case in which entry fees are not allowed.

Before moving to the analysis of the two intermediation models in the different settings, it is useful for later use to consider the benchmark case in which the platform and the seller are vertically integrated and choose the final price to maximize the joint industry profit. Suppose the market is controlled by a monopolist bearing the costs c_p and c_s per unit of output. In such a setting, the monopolist would maximize its profit $\pi^m(p) = (p - c_p - c_s)Q(p)$. Because the SOC $1 - \lambda'(p) > 0$ holds,¹¹ the price p^m that maximizes the joint profit of the industry is the solution

 10 We ignore the possibility that the platform pays the seller for participating, because negative entry fees are strictly dominated: an entry fee equal to zero is sufficient to induce the entry of the seller.

¹¹The SOC is $2Q'(p) + Q''(p)(p - c_p - c_s) < 0$, which simplifies to $2Q'(p) - Q''(p)Q(p)/Q'(p) < 0$ after using the FOC (1). Dividing by $Q'(p)$ and noting that $1 + \lambda'(p) = Q''(p)Q(p)/(Q'(p))^2$ gives the condition $1 - \lambda'(p) > 0$.

to the first-order condition (FOC):

$$
p - c_p - c_s - \lambda(p) = 0. \tag{1}
$$

At this price, the monopolist makes a profit equal to $\pi^m(p^m) = \lambda(p^m)Q(p^m)$.

3 Full-profit-extracting entry fees

In this section, we study the case in which the platform can charge a full-profit-extracting entry fee to the seller. This case, which arises when the distribution of G is degenerated at 1, can be regarded as the benchmark case of entry fees.

3.1 The wholesale model

We start by analyzing the wholesale model. In the wholesale model, the platform first chooses its entry fee, $F \geq 0$, and then the seller decides whether to enter or not. If the seller enters, in the second stage it chooses the wholesale price, w . In the last stage, the platform chooses the final consumer price, p.

To solve for an equilibrium, we proceed backwards. Because the entry fee is sunk when the seller and the platform decide on the wholesale and final prices, the latter are not affected by the level of the entry fee. Hence, the determination of the seller's and the platform's prices follows from Johnson (2017), which we reproduce here for completeness. Consider the platform's choice of final consumer price p . This choice must maximize the platform's profits:

$$
\pi^P(p) = (p - w - c_p)Q(p).
$$

The FOC is $Q(p) + (p - w - c_p)Q'(p) = 0$, which can be rewritten as:

$$
\lambda(p) - (p - w - c_p) = 0. \tag{2}
$$

Because the SOC $1 - \lambda'(p) > 0$ holds, the payoff $\pi^P(p)$ has a unique stationary point, $p(w)$, which is a maximum and increases in $w¹²$. In equilibrium, the platform makes a positive profit since its margin $p - c_p - w$ equals $\lambda(p)$ and $\lambda(p) > 0$.

We now move backwards to the choice of wholesale price w by the seller. Because the entry fee is sunk once the seller has entered, the seller, anticipating the final price $p(w)$ chosen by the platform, sets his wholesale price to maximize the expression:

$$
\pi^{S}(w) = (w - c_s)Q(p(w)).
$$

¹²The SOC is $2Q'(p) + Q''(p)(p-w-c_p) < 0$, which simplifies to $2Q'(p) - Q''(p)Q(p)/Q'(p) < 0$ after using the FOC (2). Dividing by $Q'(p)$ and noting that $1 + \lambda'(p) = Q''(p)Q(p)/(Q'(p))^2$ gives the condition $1 - \lambda'(p) > 0$. It follows that $p'(w) = (1 - \lambda'(p(w)))^{-1} > 0$.

As $p(w)$ is monotone, this problem is equivalent to optimizing the seller's profits in the final consumer price p that it wishes to induce via its choice of wholesale price w :

$$
\pi^{S}(p) = (p - c_p - c_s - \lambda(p))Q(p),
$$

where we have used the FOC of the platform (2) to rewrite the problem. The FOC is

$$
(1 - \lambda'(p))Q(p) + (p - c_p - c_s - \lambda(p))Q'(p) = 0.
$$
\n(3)

Dividing by $Q'(p)$, this condition can be rewritten as:

$$
p - c_p - c_s = \lambda(p)(2 - \lambda'(p)),\tag{4}
$$

which implicitly characterizes the equilibrium final price, p^* , and the total industry margin. Because $\lambda(p)(2 - \lambda'(p))$ has a slope less than 1, the payoff $\pi^{S}(p)$ has a unique stationary point that is a maximum.¹³ Note that the equilibrium price p^* is higher than the price set by the vertically-integrated monopolist, given by (1), due to double marginalization.

In equilibrium, the sharing of profits between the platform and the seller depends only on the shape of demand. From (2), the platform's margin is $\lambda(p)$ and, using (4), the seller's margin can be written as:

$$
w^* - c_s = \lambda(p^*)(1 - \lambda'(p^*)).
$$

As a result, the ratio of the platform's margin to the seller's margin is $\frac{1}{1-\lambda'(p^*)}$. Hence, the seller's margin is equal to the platform's margin for log-linear demands, higher for log-concave demands, and lower for log-convex demands.

Finally, in the first stage, the platform sets an entry fee $F \geq 0$ to maximize its profit. Because the platform profit, $(p^* - w^* - c_p)Q(p^*) + F$, is monotone increasing in F, it is clear that F should extract the full profit of the seller. Hence:

$$
F^* = \lambda(p^*)(1 - \lambda'(p^*))Q(p^*).
$$

The next lemma summarizes our findings:

Lemma 1 In the wholesale model, when G is degenerated at 1 and hence the platform can extract the full profit of the seller, the final price p^* charged to consumers solves the equation $p - c_p - c_s = \lambda(p)(2 - \lambda'(p))$, while the wholesale price is $w^* = p^* - c_p - \lambda(p^*)$. The entry fee equals $F^* = \lambda(p^*)(1 - \lambda'(p^*))Q(p^*)$. The seller obtains zero profits, i.e., $\pi_W^S = 0$, while the platform makes profits equal to $\pi_W^P = \lambda(p^*)(2 - \lambda'(p^*))Q(p^*)$.

¹³The SOC is $-\lambda''(p)Q(p) + 2(1 - \lambda'(p))Q'(p) + (p - c_p - c_s - \lambda(p))Q''(p) < 0$. After using the FOC (2), this can be rewritten as $-\lambda''(p)Q(p) + 2(1 - \lambda'(p))Q'(p) - \frac{(1-\lambda'(p))Q(p)}{Q'(p)}Q''(p) < 0$. Dividing by $-Q'(p)$ and noting that $1 + \lambda'(p) = Q''(p)Q(p)/(Q'(p))^2$ gives the condition $-\lambda(p)\lambda''(p) - (1 - \lambda'(p))^2 < 0$. This is the same as requiring $\lambda(p)(2 - \lambda'(p))$ to have a slope strictly less than 1.

3.2 The agency model

We now move to the agency model with revenue-sharing contracts and full-profit-extracting entry fees. In the agency model, the platform first chooses the entry fee $F \geq 0$ and the commission $t \in [0, 1)$ that the seller must pay to the platform per unit sold. Then, if the seller enters, it chooses the final consumer price p.

To solve the game, we proceed backwards again. Because F is sunk at the moment the seller decides on the final consumer price, its choice of price is the same as in Johnson (2017). That is, the seller will charge a price p to maximize:

$$
\pi^{S}(p) = (p(1-t) - c_s)Q(p).
$$

The FOC is

$$
(1-t)Q(p) + (p(1-t) - c_s) Q'(p) = 0.
$$

After dividing by $Q'(p)$, this FOC can be rewritten as:

$$
p(1-t) - c_s = (1-t)\lambda(p).
$$
 (5)

Because $1 - \lambda'(p) > 0$, the payoff $\pi^{S}(p)$ has a unique stationary point, which is a maximum.¹⁴ Let us denote by $p(t)$ the seller's profit-maximizing price and note that $p(t)$ increases in t:

$$
p'(t) = \frac{p(t) - \lambda(p(t))}{(1 - t)(1 - \lambda'(p(t)))} \ge 0.
$$

Inspection of (5) further reveals that $p(t)$ satisfies $p(t) > c_s + \lambda(p(t))$ for all $t \in (0,1)$ and $p(0) = c_s + \lambda(p(0)).$

We now move backwards to the choice of commission $t \in [0, 1)$ and entry fee $F \geq 0$ by the platform. Anticipating how the seller will price, the problem of the platform is to solve:

$$
\max_{F,t} (tp(t) - c_p) Q(p(t)) + F
$$

subject to

$$
F \le (p(t)(1-t) - c_s) Q(p(t)),
$$

where $p(t)$ is the solution to (5). Since the platform can raise the entry fee until the constraint binds, for a given t, the profit-maximizing choice of entry fee is $F^a(t) = (p(t)(1-t) - c_s) Q(p(t))$. Plugging the constraint into the objective function and simplifying, the platform's problem becomes

$$
\max_t (p(t) - c_p - c_s) Q(p(t)),
$$

¹⁴The SOC requires $2Q'(p) + (p - \frac{c_s}{1-t})Q''(p) < 0$, which can be rewritten as $-2 + (p - \frac{c_s}{1-t})\frac{1+\lambda'(p)}{\lambda(p)}$ $-1 + \lambda'(p) < 0$, where we have used the FOC (5) in the last equality.

or equivalently,

$$
\max_{p} (p - c_p - c_s) Q(p).
$$

This expression is equal to the profit a vertically integrated firm would make. Denoting by p^a the final consumer price, we thus have $p^a = p^m$. Using (5), it follows that the entry fee is $F^a = (1 - t^a) \lambda(p^a) Q(p^a)$ and the commission is $t^a = \frac{c_p}{c + 1}$ $\frac{c_p}{c_p+c_s}$, with $t^a \in (0,1)$ as $c_p, c_s > 0$.

In conclusion, when the platform is certain that the dead-weight loss of entry fees is zero, it can extract the full profit of the seller and implement an efficient outcome from the industry point of view by setting a commission equal to $t^a = \frac{c_p}{c_+}$ $\frac{c_p}{c_p+c_s}$. With this commission, the effective marginal cost of the seller becomes $c_p + c_s$, so the seller ends up producing the industry profit-maximizing output.

Lemma 2 In the agency model, when G is degenerated at 1 and hence the platform can extract the full profit of the seller, the final consumer price p^a equals the price p^m that maximizes industry profits, which solves the equation $p - c_s - c_p - \lambda(p) = 0$. The platform's optimal entry fee equals $F^a = (1-t^a)\lambda(p^a)Q(p^a)$, while its optimal commission is $t^a = \frac{c_p}{c+1}$ $\frac{c_p}{c_p+c_s}$. The seller obtains zero profits, i.e., $\pi_A^S = 0$, while the platform gets a profit equal to the monopoly profit $\pi_A^P = \pi^m(p^m)$.

A comparison between the two modes of intermediation gives the following sharp result in favor of the agency model:

Proposition 1 Assume G is degenerated at 1 so that the platform can use a full-profit-extracting entry fee. Then, $p^* > p^a = p^m$, the platform prefers the agency model to the wholesale model, while the seller is indifferent. Consumer surplus is also higher in the agency model. As a consequence, a shift from the wholesale model to the agency model results in a Pareto improvement.

Proof. It remains to be shown that $p^* > p^m$. To do this, we evaluate the first-order derivative of the seller's payoff under wholesale at the monopoly price p^m . Using (3), we have:

$$
\frac{d\pi^{S}(p)}{dp}\bigg|_{p=p^{m}} = Q(p^{m})(1 - \lambda'(p^{m})) + Q'(p^{m})(p^{m} - c_{p} - c_{s} - \lambda(p^{m})) = Q(p^{m})(1 - \lambda'(p^{m}) > 0,
$$

where we have used (1) to simplify. Thus, $p^* > p^m$, because the profit function is concave.

Proposition 1 stands in sharp contrast to the result presented in, e.g., Bishop (1968), Gaudin and White (2014a), Johnson (2017) and Llobet and Padilla (2016), where entry fees are excluded. While the ability to extract the full profit of the seller eliminates the double-marginalization problem in the agency model, this is not the case in the wholesale model because the entry fee is sunk when the firms make their pricing decisions and the platform cannot commit to a marginal cost markup policy.¹⁵ This makes the agency model better for the platform and

¹⁵If the platform could credibly commit to a marginal cost markup policy $p = w + c_p$, then the seller would choose a wholesale price that maximizes the joint profit, and the two business models would be equivalent.

consumers, regardless of the nature of demand. When entry fees are excluded, by contrast, a double-marginalization problem exists in both models and the price ranking depends on the convexity of demand. In the presence of this double-marginalization problem, Johnson (2017) shows that the platform prefers the agency model to the wholesale model when demand is logconcave, log-linear or constant elasticity (see his Proposition 3). With full-profit extraction via entry fees, the platform always prefers the agency model.¹⁶

Proposition 1 shows that with full-profit-extracting entry fees, the agency model is efficient while the wholesale model is not. This result is fundamentally linked to the timing of pricing decisions and does not depend on the nature of the wholesale price and the commission. In the agency model, the platform simultaneously sets the entry fee and the commission rate in the first stage and uses this two-part tariff to induce the seller to produce the industry profit-maximizing output and capture the full industry profit. By contrast, because the entry fee is sunk when the agents make their pricing decisions, the wholesale model gives rise to double marginalization.

To see that the efficiency of the agency model does not depend on the ad valorem nature of the commission, suppose that in the agency model the platform used a per-unit instead of an ad valorem commission. Then, in the second stage, the seller would maximize its profit $(p-t-c_s)Q(p)$ by setting a price p satisfying the FOC $p-t-c_s-\lambda(p) = 0$. In the first stage, for a given t, the profit-maximizing entry fee set by the platform would be $F(t) = (p(t) - t - c_s)Q(p(t))$. As a result, the platform would choose its per-unit commission to maximize the profit π^P = $(t-c_p)Q(p(t)) + F = (p(t)-c_s-c_p)Q(p(t))$. This would again result in the efficient equilibrium price p^m . This equivalence between ad valorem and per-unit commissions contrasts with earlier results in the literature stating that ad valorem commissions are better than per-unit (see e.g. Shy and Wang, 2011 and Gaudin and White, 2014b).

Likewise, to see that the inefficiency of the wholesale model does not depend on the perunit nature of the wholesale price, suppose that in the wholesale model the seller used an ad valorem commission τ instead of a per-unit price w. Then, in stage 3, the platform would maximize its profit $\pi^P = (p(1 - \tau) - c_p)Q(p)$ by setting a final price $p(\tau)$ satisfying the FOC $p - \frac{c_p}{1-\tau} - \lambda(p) = 0$. In stage 2, the seller would set its commission τ to maximize its profit $\pi^{S} = (\tau p(\tau) - c_s) Q(p(\tau))$. It is easy to see that maximizing this profit in τ is equivalent to maximizing $\pi^S = \left(p - c_p - c_s - \frac{\lambda c_p}{n} \right)$ $\left(\frac{\lambda c_p}{p-\lambda}\right) Q(p)$ in p. Because this profit function clearly differs from the joint profit $(p - c_p - c_s) Q(p)$, it is obvious that the final price would not be equal to p^m . Once again, because the entry fee is sunk when the agents set the terms of trade, double marginalization arises in the wholesale model and the equilibrium final price does not maximize industry profits.

The key distinction across models is hence the timing of pricing decisions. In the wholesale

 $16A$ byproduct of the analysis in the next section on partial-profit extraction is to show that demand satisfying Marshall's second law of demand is a weaker sufficient condition than than in Johnson (2017) for the platform to prefer the agency model over the wholesale model when entry fees are restricted to be zero.

model, full-profit-extracting entry fees affect only the division of profits without impacting final prices or consumer surplus. In contrast, in the agency model, such entry fees influence both prices and the profit distribution.

4 Partial-profit-extracting entry fees

In the previous section, we have studied the case in which the platform can use a full-profitextracting entry fee. In reality, however, sellers may only be able to allocate a fraction of their profits to cover entry costs. There are multiple reasons for this. For example, sellers often need to distribute a fraction of their profits among investors to be able to finance their operations. Sometimes, profits are also shared with employees as end-of-year bonuses. Moreover, corporate taxation naturally reduces the amount of profits sellers can afford to pay for entry. Finally, sellers may also need to retain a share of their profits to invest in advertising or R&D. Accordingly, in this section we assume that G is degenerated at some $\beta \in [0,1)$, which represents the share of profit a seller can afford to pay for entry. Naturally, the case where G is degenerated at $\beta = 0$ is equivalent to Johnson $(2017).^{17}$

4.1 The wholesale model

We start by analyzing the wholesale model. The wholesale and final prices are the same as in the previous section where the platform uses full-profit-extracting entry fees (and as in Johnson, 2017) because the entry fee becomes sunk once the seller has entered. Moving directly to the first stage and noting that the payoff of the platform is monotone increasing in F , the platform will set an entry fee exactly equal to the seller's willingness to pay for entry, i.e.,

$$
F^* = \beta \lambda(p^*)(1 - \lambda'(p^*))Q(p^*).
$$

As a result:

Lemma 3 In the wholesale model, when G is degenerated at $\beta \in [0, 1)$ and so the platform can only extract part of the seller's profit, the final price p^* charged to consumers solves the equation $p-c_p-c_s = \lambda(p)(2-\lambda'(p))$, while the wholesale price is $w^* = p^* - c_p - \lambda(p^*)$. The entry fee equals $F^* = \beta \lambda(p^*)(1 - \lambda'(p^*)Q(p^*)$. The seller obtains a profit equal to $\pi_W^S = (1 - \beta)\lambda(p^*)(1 - \lambda'(p^*)Q(p^*))$. while the platform gets a profit equal to $\pi_W^P = \lambda(p^*)Q(p^*)[1 + \beta(1 - \lambda'(p^*)]$.

¹⁷Corporate taxation, dividends paid to shareholders, and end-of-year bonuses are all profit-sharing schemes. As such, they reduce the seller's willingness to pay for entry by an amount that depends on the equilibrium price. By contrast, fixed costs such as those incur to produce reduce the seller's willingness to pay for entry by an amount that is independent of price. This distinction turns outo to be crucial. We show later that the results in Section 3 generalize to the case in which the seller has to retain some profits to pay a fixed cost of production. Therefore, in the remaining of the paper, we focus on situations where the platform has to "compensate" the entry of the seller by an amount that depends on the equilibrium price.

4.2 The agency model

In the agency model, the final consumer price that the seller charges is chosen to maximize the same profit as in the previous section, so the FOC (5) continues to give the final consumer price as a function of the commission t .

Moving backwards, the platform chooses its entry fee $F \geq 0$ and commission $t \in [0,1)$ to solve the problem:

> $\max_{F,t} (tp(t) - c_p) Q(p(t)) + F,$ subject to $\beta(1-t)\lambda(p(t))Q(p(t)) > F.$

Because the objective function of the platform increases in F , the constraint will bind and the platform's program thus boils down to

$$
\max_{t}[tp(t) - c_p + \beta(1-t)\lambda(p(t))]Q(p(t)).
$$
\n(6)

As argued above, $p(t)$ is monotone increasing. Therefore, as in Johnson (2017), we can reformulate the problem of the platform as choosing p instead of t . Once p is determined, the commission is pinned down from (5): $t = 1 - \frac{c_s}{n-1}$ $\frac{c_s}{p-\lambda(p)}$. Hence, replacing t by $1-c_s/(p-\lambda(p))$ in problem (6), we can rewrite the platform's problem as:

$$
\pi^{P}(p) = \left[\left(1 - \frac{c_s}{p - \lambda(p)} \right) p - c_p + \beta \frac{c_s}{p - \lambda(p)} \lambda(p) \right] Q(p),
$$

or, adding and subtracting c_s , as:

$$
\pi^{P}(p) = \left(p - c_p - c_s - c_s(1 - \beta)\frac{\lambda(p)}{p - \lambda(p)}\right)Q(p). \tag{7}
$$

Note that except when $\beta = 1$, the platform does not internalize the entire industry profit.

The price p^a that maximizes the platform profit must then satisfy the FOC for profit maximization:¹⁸

$$
p - c_p - c_s - \lambda(p) - c_s(1 - \beta) \frac{p\lambda(p)(1 - \lambda'(p))}{(p - \lambda(p))^2} = 0.
$$
 (8)

As a result:

¹⁸The SOC is:

$$
\frac{d^2\pi^P(p)}{dp^2} = \left(p - c_p - c_s - \frac{c_s(1-\beta)\lambda(p)}{p-\lambda(p)}\right)Q''(p) + 2\left(1 - c_s(1-\beta)\frac{p\lambda'(p) - \lambda(p)}{(p-\lambda(p))^2}\right)Q'(p) \n- c_s(1-\beta)\frac{p(p-\lambda(p))\lambda''(p) - 2(1-\lambda'(p))(p\lambda'(p) - \lambda(p))}{(p-\lambda(p))^3}Q(p) \le 0.
$$

Using the facts that $Q'' = -Q' \frac{1+\lambda'}{\lambda}$ $\frac{+\lambda'}{\lambda}$, $Q = -\lambda Q'$ and the FOC (8), we have:

$$
p - c_p - c_s - \frac{c_s (1 - \beta) \lambda(p)}{p - \lambda(p)} = \frac{-Q(p)}{Q'(p)} \left(1 - c_s (1 - \beta) \frac{p \lambda'(p) - \lambda(p)}{(p - \lambda(p))^2} \right).
$$

Lemma 4 In the agency model, when G is degenerated at $\beta \in [0,1)$ and hence the platform can only extract part of the profit of the seller, the final consumer price p^a solves the equation

$$
p - c_p - c_s - \lambda(p) - c_s(1 - \beta) \frac{p\lambda(p)(1 - \lambda'(p))}{(p - \lambda(p))^2} = 0.
$$

The platform's optimal entry fee equals $F^a = \beta(1-t^a)\lambda(p^a)Q(p^a)$, while its optimal commission is $t^a = 1 - \frac{c_s}{r^a - \lambda}$ $\frac{c_s}{p^a - \lambda(p^a)}$. The seller gets a profit $\pi_A^S = (1 - \beta) \frac{c_s}{p^a - \lambda}$ $\frac{c_s}{p^a - \lambda(p^a)} \lambda(p^a) Q(p^a)$, while the platform *receives profits equal to* $\pi_A^P = \left(1 + \frac{(1-\beta)c_s[\lambda(p^a)-p^a\lambda'(p^a)]}{(p^a-\lambda(p^a))^2}\right)$ $\frac{c_s[\lambda(p^a)-p^a\lambda'(p^a)]}{(p^a-\lambda(p^a))^2}\bigg)\,\lambda(p^a)Q(p^a).$

In the agency model, when G is degenerated at $\beta \in [0, 1)$, the platform can no longer capture the whole industry profit through the entry fee. Consequently, it increases its commission at the detriment of the joint profit, which gives rise to the usual double marginalization inefficiency. This inefficiency worsens as the share of the profit captured by the platform becomes lower; as a result, the equilibrium price decreases in β ¹⁹

The comparison between the two business models yields the following result.

Proposition 2 Assume G is degenerated at $\beta \in [0, 1)$ so that the platform can only extract part of the profit of the seller. Then:

- 1. Both the final consumer price under the wholesale model p^* and under the agency model p^a are higher than p^m , the price that maximizes the joint profit.
- 2. The final price under the agency model is lower than under the wholesale model, i.e., $p^a \lt p^*$, if and only if $\beta > \hat{\beta} \equiv \max\left\{0, 1 - \frac{(p^* - \lambda(p^*))^2}{c n^*}\right\}$ $\frac{(-\lambda(p^*))^2}{c_s p^*}$. As a result, $p^a < p^*$:
	- for any demand if β is sufficiently large
	- for any demand that satisfies Marshall's second law of demand, regardless of β .
- 3. For any demand that satisfies Marshall's second law of demand, the platform prefers the agency model. Moreover, if $\beta - \frac{\lambda(p^a) - p^a \lambda'(p^a)}{p^a (1 - \lambda'(p^a))}$ $\frac{(p^{\alpha})-p^{\alpha} \lambda^{r}(p^{\alpha})}{p^{\alpha}(1-\lambda^{r}(p^{\alpha}))} < 0$, then the seller prefers the wholesale model.

Proof.

It then follows that:

$$
\frac{d^2\pi^P(p)}{dp^2} = \left(1 - c_s(1-\beta)\frac{p\lambda'(p) - \lambda(p)}{(p-\lambda(p))^2}\right)(1-\lambda'(p))Q'(p) \n+ c_s(1-\beta)\lambda(p)\left(\frac{p(p-\lambda(p))\lambda''(p) + 2(1-\lambda'(p))(\lambda(p) - p\lambda'(p))}{(p-\lambda(p))^3}\right)Q'(p).
$$

Hence, a sufficient condition for the SOC to hold is that $\lambda(p) - p\lambda'(p) \ge 0$ (i.e., demand satisfies Marshall's second law of demand) and $\lambda''(p) \geq 0$.

¹⁹Assuming the SOC holds, applying the implicit function theorem in the FOC (8) immediately shows that $\partial p^a/\partial \beta < 0.$

1. We already know that $p^* > p^m$ (see Proposition 1). It remains to show that $p^a > p^m$. Using (8) , we have:

$$
\frac{d\pi^{P}(p)}{dp}\Big|_{p=p^{m}} = -Q'(p^{m}) \left[c_{s}(1-\beta) \frac{p^{m} \lambda(p^{m})(1-\lambda'(p^{m}))}{(p^{m}-\lambda(p^{m}))^{2}} - (p^{m} - c_{p} - c_{s} - \lambda(p^{m})) \right]
$$

$$
= Q(p^{m})c_{s}(1-\beta) \frac{p^{m}(1-\lambda'(p^{m}))}{(p^{m}-\lambda(p^{m}))^{2}} > 0,
$$

which implies that $p^a > p^m$, assuming that the profit function is concave (see footnote 18).

2. To show this result, we evaluate the first-order derivative of the payoff in the agency model at the optimal price in the wholesale model, p^* . This gives:

$$
\frac{d\pi^{P}(p)}{dp}\Big|_{p=p^{*}} = \left(1 - c_{s}(1-\beta)\frac{p^{*}\lambda'(p^{*}) - \lambda(p^{*})}{(p^{*} - \lambda(p^{*}))^{2}}\right)Q(p^{*}) + \left(p - c_{p} - c_{s} - c_{s}(1-\beta)\frac{\lambda(p^{*})}{p^{*} - \lambda(p^{*})}\right)Q'(p^{*})
$$
\n
$$
= -Q'(p^{*})\left[\left(1 - c_{s}(1-\beta)\frac{p^{*}\lambda'(p^{*}) - \lambda(p^{*})}{(p^{*} - \lambda(p^{*}))^{2}}\right)\lambda(p^{*}) - \left(\lambda(p^{*})(2-\lambda'(p^{*})) - c_{s}(1-\beta)\frac{\lambda(p^{*})}{p^{*} - \lambda(p^{*})}\right)\right]
$$
\n
$$
= -Q'(p^{*})\lambda(p^{*})(1 - \lambda'(p^{*}))\left(\frac{c_{s}(1-\beta)p^{*}}{(p^{*} - \lambda(p^{*}))^{2}} - 1\right),
$$

The previous expression is negative if and only if

$$
\beta > \hat{\beta} \equiv \max \left\{ 0, 1 - \frac{(p^* - \lambda(p^*))^2}{c_s p^*} \right\},\,
$$

in which case the payoff under the agency model is decreasing at p^* , so $p^a < p^*$. Note that $\hat{\beta}$ is strictly less than 1, so for any demand function there always exists a sufficiently large β for which $p^a < p^*$.

Moreover, for demands for which the Mills ratio is less than 1, $\hat{\beta} = 0$, so for those demands $p^a < p^*$ no matter β . For this, we need to show that $1 - \frac{(p^* - \lambda(p^*))^2}{c}$ $\frac{-\lambda(p^*)^2}{c_s p^*} < 0$, which is equivalent to $c_s p^* < (p^* - \lambda(p^*))^2$. This is always true for demands that satisfy Marshall's Second Law because:

$$
c_s p^* - (p^* - \lambda(p^*))^2 = [p^* - c_p - \lambda(p^*)(2 - \lambda'(p^*))^2
$$

= $-p^*c_p - p^*\lambda(p^*)(1 - \lambda'(p^*)) + \lambda(p^*)(p^* - \lambda(p^*))$
 $< -p^*\lambda(p^*)(1 - \lambda'(p^*)) + \lambda(p^*)(p^* - \lambda(p^*))$
= $-\lambda(p^*) [\lambda(p^*) - p^*\lambda'(p^*))] < 0$,

where we have used (4) to replace c_s , the first inequality follows from dropping $-p^*c_p$, and the second inequality follows from Marshall's second law of demand.

3. We first show that the platform prefers the agency model for any demand that satisfies Marshall's second law of demand. To do this, let us suppose that in the agency model,

rather than choosing its commission optimally, the platform chooses (sub-optimally) a commission \tilde{t} to induce a final price equal to the price p^* prevailing under the wholesale model:

$$
\tilde{t} = \frac{c_p + \lambda(p^*)(1 - \lambda'(p^*))}{p^* - \lambda(p^*))},
$$

To obtain \tilde{t} , we use expression (4) that gives the final price in the wholesale model p^* , which implies that $p^* - c_s - \lambda(p^*) = \lambda(p^*)(1 - \lambda'(p^*)) + c_p$, and equation (5) that defines the final price in the agency model, which can be rewritten as $p - c_s - \lambda(p) = t(p - \lambda(p)).$ To implement p^* , the platform then sets \tilde{t} so that $\lambda(p^*)(1 - \lambda'(p^*)) + c_p = \tilde{t}(p^* - \lambda(p^*))$. Note also that we have $\tilde{t} < 1$ as $p^* - \lambda(p^*) = c_p + c_s + \lambda(p^*)(1 - \lambda'(p^*) > c_p + \lambda(p^*)(1 - \lambda'(p^*))$ so that p^* can be implemented by an interior commission.

Given this choice of commission, the entry fee that the platform would set equals \tilde{F}^a = $\beta(1-\tilde{t})\lambda(p^*)Q(p^*)$. As a result, the platform would obtain a profit equal to:

$$
\pi_A^P(\tilde{t}, \tilde{F}) = (\tilde{t}p^* - c_p)Q(p^*) + \beta(1 - \tilde{t})\lambda(p^*)Q(p^*)
$$

=
$$
[\tilde{t}p^* - c_p + \beta(1 - \tilde{t})\lambda(p^*)]Q(p^*)
$$

=
$$
\frac{\lambda(p^*)Q(p^*)}{p^* - \lambda(p^*)}[c_p(1 - \beta) + p^*(1 - \lambda'(p^*)) + \beta(p^* - \lambda(p^*)(2 - \lambda'(p^*))].
$$

We now show that this sub-optimal profit level is greater than the maximal profit the platform would make under the wholesale model, which is equal to $\pi_W^P(p^*,F^*) = \lambda(p^*)Q(p^*)[1+$ $\beta(1-\lambda'(p^*))$] (see Lemma 3). In fact, $\pi_A^P(\tilde{t}, \tilde{F}) > \pi_W^P(p^*, F^*)$ if and only if

$$
\frac{1}{p^* - \lambda(p^*)} [c_p(1 - \beta) + p^*(1 - \lambda'(p^*)) + \beta(p^* - \lambda(p^*)(2 - \lambda'(p^*))] > 1 + \beta(1 - \lambda'(p^*))
$$

This inequality holds if:

$$
c_p(1-\beta) + p^*(1-\lambda'(p^*)) + \beta(p^*-\lambda(p^*)(2-\lambda'(p^*))] > (p^*-\lambda(p^*))[1+\beta(1-\lambda'(p^*))],
$$

which can be simplified to

$$
(1 - \beta) [c_p + \lambda(p^*) - p^* \lambda'(p^*)] > 0.
$$

This inequality is always true provided that demand satisfies Marshall's second law of demand. As a result, the platform always prefers the agency model. Note that the inequality can also be satisfied for demands that do not satisfy Marshall's second law of demand if c_p is sufficiently high, i.e., $c_p > p^* \lambda'(p^*) - \lambda(p^*)$.

We now show that the seller prefers the wholesale model provided that β is sufficiently small. Suppose that in the wholesale model the seller does not choose its wholesale price optimally, but chooses a wholesale price \tilde{w} to induce the platform to charge a final price equal to p^a , the price prevailing in the agency model:

$$
\tilde{w} = c_s + c_s (1 - \beta) \frac{p^a \lambda(p^a)(1 - \lambda'(p^a))}{(p^a - \lambda(p^a))^2}.
$$

To obtain \tilde{w} , we use equation (2), which defines the final price in the wholesale model, and implies that $\tilde{w} = p^a - c_p - \lambda(p^a)$, and equation (8) that defines the final price in the agency model. With this choice of wholesale price, the seller's payoff in the wholesale model would be equal to

$$
\pi_W^S(p^a) = (1 - \beta)(p^a - c_s - c_p - \lambda(p^a))Q(p^a).
$$

While in the agency model, the seller would make a profit equal to

$$
\pi_A^S(p^a) = (1 - \beta) \frac{c_s \lambda(p^a)}{p^a - \lambda(p^a)} Q(p^a).
$$

Comparing these two profits gives:

$$
\pi_W^S(p^a) - \pi_A^S(p^a) = (1 - \beta) \left(p^a - c_s - c_p - \lambda(p^a) - \frac{c_s \lambda(p^a)}{p^a - \lambda(p^a)} \right) Q(p^a).
$$

Hence, $\pi_W^S(p^a) > \pi_A^S(p^a)$ if and only if $p^a - c_s - c_p - \lambda(p^a) - \frac{c_s\lambda(p^a)}{p^a - \lambda(p^a)}$ $\frac{c_s \lambda(p^a)}{p^a - \lambda(p^a)} > 0$. Using the FOC (8), this requires

$$
c_s(1-\beta) \frac{p^a \lambda(p^a)(1 - \lambda'(p^a))}{(p^a - \lambda(p^a))^2} - \frac{c_s \lambda(p^a)}{p^a - \lambda(p^a)} > 0.
$$

After simplifying, this condition can be written as:

$$
\beta < \frac{\lambda(p^a) - p^a \lambda'(p^a)}{p^a (1 - \lambda'(p^a))},
$$

which is the condition given in the proposition. Note that this condition may only hold for demands satisfying the Marshall's second law of demand. If the condition holds, then the seller would prefer the wholesale model to the agency model if p^a was the final consumer price. Because p^a is sub-optimal from the point of view of the seller in the wholesale model, it follows that $\pi_W^S(p^*) > \pi_W^S(p^a) > \pi_A^S(p^a)$. So, the seller prefers the wholesale model.

Finally, because $p^a \leq p^*$ for any demand that satisfies Marshall's second law of demand, the joint profit under the agency model is greater than under the wholesale model. The fact that $\pi_W^S(p^*) > \pi_A^S(p^a)$ immediately implies that $\pi_A^P(p^a) > \pi_W^P(p^*)$.

Proposition 2 lends strong support to the idea that platforms prefer the agency model over the wholesale model. The same applies to consumers because the final consumer price is typically lower under the agency model than under the wholesale model. In contrast, a shift from the wholesale model to the agency model will be detrimental to the seller if its willingness to pay for entry is low enough, in which case a conflict of interest arises. It is worth emphasizing that our result in Proposition 5 covers the case of $\beta = 0$, which gives Johnson's (2017) results. This means that one can dispense with stronger assumptions on demand than just Marshall's second law of demand (in contrast to Johnson's (2017) Proposition 3) to show the platform's preference for the agency model. The reason for this difference in results is that our distinct proof allows us to derive a weaker sufficient condition. In the next section we significantly generalize this result by showing that the platform continues to prefer the agency model when it faces uncertainty about the seller's willingness to pay for entry and hence the amount of rent extraction is the outcome of an interior solution (rather than chosen to leave the seller indifferent between entering and not entering the market).

We illustrate Proposition 2 with Figures 1 and 2. To construct these figures, we set $c_p =$ $c_s = 1/4$. In Figure 1, we plot the equilibrium prices and profits for the linear demand function $Q(p) = 1 - p$, which satisfies Marshall's second law of demand. Panel (a) shows the equilibrium prices under the wholesale and agency models, along with the price that maximizes joint profits. As Proposition 2 states, the price under agency is always lower than under wholesale. Moreover, it decreases in β and converges to the monopoly price when rent extraction is complete $(\beta = 1)$. In panel (b), we plot the equilibrium profits. As the graph shows, there is a conflict of interest because the platform prefers the agency model, while the seller prefers the wholesale model, irrespective of the extent to which the platform can extract the seller's surplus.

Figure 1: Linear demand $Q(p) = 1 - p$.

In Figure 2, we perform the same exercise for the demand function $Q(p) = \frac{1}{p} + \frac{1}{p^2}$ $\frac{1}{p^2}$, which does not satisfy Marshall's second law of demand. Panel (a) shows the equilibrium prices under the wholesale and agency models, along with the price that maximizes joint profits. Consistent with Proposition 2, the price is lower under agency than under wholesale if β is sufficiently high; otherwise, the wholesale model is better for consumers. This example thus illustrates a case where, without entry fees, the final consumer price is lower under the wholesale model; however, with entry fees, the price ranking may reverse. Panel (b) shows the equilibrium profits. There is still a conflict of interest, but for this non-typical demand its nature is quite different. The seller prefers the agency model, while the platform prefers the wholesale model, unless rent extraction is very high because in that case the final price in the agency model converges to the monopoly

price and the double marginalization problem vanishes.

Figure 2: Non-Marshall demand: $Q(p) = \frac{1}{p} + \frac{1}{p^2}$ $\frac{1}{p^2}.$

5 Seller heterogeneity

In the most general version of our model, G is a proper distribution function. One interpretation is that the platform is uncertain about the seller's willingness to pay for entry. An alternative interpretation is that the platform interacts with many independent sellers with different willingness to pay for entry but similar demand. While in the case without uncertainty the platform always chooses its entry fee so as to leave the seller indifferent between entering and not entering the market and hence entry always occurs in equilibrium, this is no longer true in the presence of uncertainty. Instead, the platform picks its entry fee to solve the following trade-off: a higher entry fee increases the probability that the seller refuses to enter, while it increases the platform's payoff in case of entry. As a result, the amount of rent extraction via entry fees becomes interior, and hence the probability of entry need not be equal to 1 in equilibrium. As we will see, this more general model delivers a rationale for why platforms sometimes do not charge entry fees at all.

5.1 The wholesale model

We start with the wholesale model. As already mentioned, in the wholesale model the continuation equilibrium prices and profits are independent of the entry fee F , which is sunk. As a result, the wholesale price w^* and the final consumer price p^* continue to be the solution to the FOCs (2) and (4), respectively.

Moving to the first stage, the platform sets an entry fee $F \geq 0$ to maximize the probability that the seller enters times the payoff in case of entry:

$$
\max_{F} \left\{ \Pr[\beta \lambda(p^*)(1 - \lambda'(p^*)) Q(p^*) \ge F] (\lambda(p^*) Q(p^*) + F) \right\},\
$$

which is equivalent to:

$$
\max_{F} \left[1 - G\left(\frac{F}{\lambda(p^*)(1 - \lambda'(p^*))Q(p^*)}\right)\right] \left(\lambda(p^*)Q(p^*) + F\right).
$$

It is useful to reformulate the problem of the platform as maximizing over the share of the seller's profit it wants to extract, rather than maximizing with respect to F . For this purpose, let us define:

$$
z \equiv \frac{F}{\lambda(p^*)(1 - \lambda'(p^*))Q(p^*)},
$$

which represents the share of the seller's profit extracted through the fixed fee, i.e., $F =$ $z[\lambda(p^*)(1 - \lambda'(p^*))Q(p^*)].$

The platform's problem can then be reformulated as:

$$
\max_{z} (1 - G(z)) \left[\lambda(p^*) Q(p^*) + z(1 - \lambda'(p^*)) \lambda(p^*) Q(p^*) \right]. \tag{9}
$$

The FOC wrt z is: 20

$$
-g(z) \left[\lambda(p^*) Q(p^*) + z(1 - \lambda'(p^*)) \lambda(p^*) Q(p^*) \right] + \left[1 - G(z) \right] (1 - \lambda'(p^*)) \lambda(p^*) Q(p^*) = 0,
$$

Rearranging, we obtain that the equilibrium share z^* of the seller's profit extracted by the platform via its entry fee is the solution in z to the equation:

$$
\mu(z) = z + \frac{1}{1 - \lambda'(p^*)} \tag{10}
$$

if it exists. Otherwise, $z^* = 0$. Hence, we have the following result:

Proposition 3 In the wholesale model with seller heterogeneity, the final price p^* charged to consumers solves the equation $p - c_p - c_s = \lambda(p)(2 - \lambda'(p))$, while the wholesale price is $w^* = p^* - c_p - \lambda(p^*)$. The platform charges an entry fee $F^* > 0$ if and only if $g(0) < 1 - \lambda'(p^*)$, in which case F^* is equal to a share z^* of the seller's profits $\pi^S(p^*)$, where z^* solves the equation $\mu(z) = z + (1 - \lambda'(p^*))^{-1}$. Otherwise, $F^* = 0$.

Proof. The optimal prices follow from the above analysis. Since the density g is increasing –this is is necessary for the second-order conditions to hold– μ is a decreasing function, hence equation (10) has a solution whenever $\mu(0) > (1 - \lambda'(p^*))^{-1}$ or $g(0) < 1 - \lambda'(p^*)$.

Proposition 3 shows that the platform will charge an entry fee if and only if $g(0) < 1 - \lambda'(p^*)$. This condition depends on all the model primitives, but in particular on the properties of G. Since the equilibrium price does not depend on G , this condition is more likely to be satisfied when the mass of sellers who cannot afford to pay anything to enter the platform's market is

²⁰The SOC is satisfied if $-2g(z)(1 - \lambda'(p^*)) - g'(z) [1 + z(1 - \lambda'(p^*))] \leq 0$, which holds because $g' \geq 0$.

sufficiently small. Interestingly, for any distribution function with no mass at 0 (i.e, $g(0) = 0$),²¹ the platform will charge a strictly positive entry fee. Conversely, if G has positive mass at zero (e.g. when G is uniform), the platform risks deterring the seller from entering the market if it imposes a positive entry fee.

Proposition 1 also shows that the fraction z^* of the seller's profit captured by the platform as entry fee is lower the higher the pass-through from the wholesale price to the final price in equilibrium: $p^{*'}(w) = (1 - \lambda'(p^{*}(w))^{-1}$. This means that, holding G fixed, measuring passthrough is sufficient to learn about rent extraction: in markets where demand and cost conditions lead to a greater pass-through, the share of profits extracted by the platform will be smaller.

Finally, we observe that the pass-through in equilibrium equals the ratio of the platform's margin to the seller's margin, as demonstrated by Bresnahan and Reiss (1985). Hence, at the equilibrium price, equation (10) can be rewritten as:

$$
\mu(z) = z + RM^*(p^*),
$$

where

$$
RM^*(p) \equiv \frac{\lambda(p)}{p - c_p - c_s - \lambda(p)}
$$

is the ratio of profit margins of the platform and the seller. 22

Proposition 4 In the wholesale model with seller heterogeneity, when $g(0) < 1 - \lambda'(p^*)$ so that the platform finds it optimal to charge a positive entry fee, the fraction z^* of the seller's profit captured by the platform through the entry fee is larger for a demand and cost system that yields a lower pass-through rate or, equivalently, a lower ratio of platform margin over seller margin.

Compared to Sections 3 and 4, when there is seller heterogeneity, the extent of rent extraction is interior. Therefore, it is valuable to examine how rent extraction varies with the parameters of the model. Our next results presents the comparative statics of increasing the platform's and seller's marginal costs.

Proposition 5 In the wholesale model with seller heterogeneity, the final price p^* increases in the platform's marginal cost c_p and in the seller's marginal cost c_s . Moreover, an increase in either c_p or c_s leads to a lower fraction z^* of the seller's profit captured by the platform if and only if the curvature of demand is increasing in p (i.e. $\sigma'(p^*) > 0$).

Proof. Using (4), we find that

$$
\frac{dp^*}{dc_p} = \frac{1}{(1 - \lambda'(p^*))^2 + \lambda(p^*)\lambda''(p^*)} > 0,
$$

²¹This is the case, for example, if G is the power distribution, i.e, $G(\beta) = \beta^{\alpha}$, with $\alpha > 1$.

 22 Note that in equilibrium, the margin of the platform is larger than the margin of the seller if and only if $\lambda'(p^*) < 0$: this is necessarily true for log-concave demands, whereas the reverse is true for log-convex demands.

where the denominator is positive because the SOC should hold (see footnote 13). Therefore, a lower marginal cost c_p leads to a lower final price p^* . The derivative with respect to c_s is identical. Besides, the fraction z^* of the seller's profit captured by the platform is the solution to $\mu(z) = z + r(p^*)$, with $r(p) \equiv (1 - \lambda'(p))^{-1}$. So, we have:

$$
\frac{r(p^*)}{dc_p} = \frac{\partial r}{\partial p}\Big|_{p=p^*} \frac{dp^*}{dc_p}
$$

=
$$
\frac{\lambda''(p^*)}{(1-\lambda'(p^*))^2 \left[(1-\lambda'(p^*))^2 + \lambda(p^*)\lambda''(p^*) \right]},
$$

which is positive if and only if $\lambda''(p^*) > 0$. The derivative with respect to c_s is the same. So, a higher c_p or a higher c_s leads to a higher fraction z^* of the seller's profit captured by the platform. Since $\sigma'(p) = \lambda''(p)$, the result follows.

Proposition 5 on how the extent of rent extraction depends on costs is related to the literature on cost pass-through in vertical relations (Spengler, 1950). It is well known that when the demand curvature is increasing in price, the pass-through rate is lower for the upstream firm than for the downstream firm. As a result, an increase in the marginal cost of either the seller or the platform results in an increase in the ratio of the platform's margin to the seller's margin, which decreases the share of the seller's profits that the platform extracts in equilibrium.

We finish this section with a relatively simple example illustrating that the platform sometimes chooses not to charge an entry fee, depending on G and the shape of the demand function.

Example: the uniform distribution

Consider the case where G is the uniform distribution. Because G has strictly positive density at zero, the platform will only use an entry fee if the demand function is sufficiently concave. To see this, note that $\mu(z) = 1 - z$. Using (10), it is straightforward to derive that the equilibrium entry fee extracts a fraction of the seller's profits equal to $z^* = -\lambda'(p^*)/2(1 - \lambda'(p^*))$. As a result:

$$
F^* = \begin{cases} -\frac{\lambda(p^*)\lambda'(p^*)Q(p^*)}{2}, & \text{if } \lambda'(p^*) < 0\\ 0, & \text{otherwise.} \end{cases}
$$
(11)

Hence, with G uniform, the platform will choose $F^* = 0$ for all log-linear and log-convex demands and $F^* > 0$ for all log-concave demands.

5.2 The agency model

In the agency model, the final consumer price that the seller charges for a given commission continues to be the solution to equation (5). Moving back to stage 1, the platform chooses its commission $t \in [0, 1)$ and entry fee $F \geq 0$ to solve:

$$
\max_{t,F} \left\{ \Pr[\beta(p(t)(1-t) - c_s)Q(p(t)) \ge F] \left[(tp(t) - c_p)Q(p(t)) + F \right] \right\}
$$
\n
$$
= \max_{t,F} \left\{ \left[1 - G\left(\frac{F}{(p(t)(1-t) - c_s)Q(p(t))} \right) \right] \left[(tp(t) - c_p)Q(p(t)) + F \right] \right\}. \tag{12}
$$

In this expression, the first factor represents the likelihood of seller entry. The second factor is the payoff the platform receives conditional on the seller entering the market.

As we did before, because $p(t)$ is monotone, we can reformulate the problem of the platform as choosing p instead of t . Once p is determined, the commission is given by the relationship $t = 1 - \frac{c_s}{n-1}$ $\frac{c_s}{p-\lambda(p)}$. Substituting t by $1-c_s/(p-\lambda(p))$ in problem (28), we can rewrite the platform's objective function as

$$
\max_{p,F} \left\{ \left[1 - G\left(\frac{(p - \lambda(p))F}{c_S \lambda(p) Q(p)} \right) \right] \left[\left(p - c_p - \frac{pc_S}{p - \lambda(p)} \right) Q(p) + F \right] \right\}.
$$
 (13)

In regard to the platform's choice of entry fee, as we have done in the analysis of the wholesale model, it is also useful here to reformulate the problem of the platform as maximizing over the share of the seller's profit it wants to extract. Thus, let us define:

$$
y \equiv \frac{F}{\frac{c_s \lambda(p)Q(p)}{p - \lambda(p)}},
$$

which represents the share of the seller's profit extracted through the fixed fee, i.e.,

$$
F = y(p(1-t) - c_s)Q(p) = y(1-t)\lambda(p)Q(p) = y\frac{c_s\lambda(p)Q(p)}{p - \lambda(p)}
$$

The platform's problem (23) is then reformulated as:

$$
\max_{p,y} \left\{ (1-G(y)) \left[\left(p - c_p - \frac{pc_S}{p - \lambda(p)} \right) Q(p) + y \frac{c_s \lambda(p) Q(p)}{p - \lambda(p)} \right] \right\},\,
$$

which we can rewrite as:

$$
\max_{p,y} \left\{ (1 - G(y)) \left(p - c_p - c_s - c_s (1 - y) \frac{\lambda(p)}{p - \lambda(p)} \right) Q(p) \right\}.
$$
 (14)

.

Notice that, conditional on the entry of the seller, the platform's payoff is exactly the same as the payoff in (14) but with y replacing β . Hence, the FOC with respect to price is the same as that in (8) but with y replacing β :

$$
p - c_p - c_s - \lambda(p) - c_s(1 - y) \frac{p\lambda(p)(1 - \lambda'(p))}{(p - \lambda(p))^2} = 0.
$$
 (15)

Taking now the FOC of (14) with respect to y gives:

$$
-g(y)\left(p - c_p - c_s - c_s(1 - y)\frac{\lambda(p)}{p - \lambda(p)}\right) + (1 - G(y))\frac{c_s\lambda(p)}{p - \lambda(p)} = 0,
$$

which can be rewritten as:

$$
\mu(y) = y + \frac{p\left(p - c_p - c_s - \lambda(p)\right)}{c_s \lambda(p)} + \frac{c_p}{c_s}.\tag{16}
$$

If an interior equilibrium exists, then it is given by the solution to the FOCs (15) and (16) in p and y, respectively. We then have the following result:

Proposition 6 In the agency model:

(A) If $g(0) < \frac{c_s \lambda(p^a)}{p^a (p^a - c - c - \lambda)p^b}$ $\frac{c_s\lambda(p^a)}{p^a(p^a-c_s-c_p-\lambda(p^a))+c_p\lambda(p^a)},$ the platform charges a commission $t^a=1-\frac{c_s}{p^a-\lambda}$ $\frac{c_s}{p^a-\lambda(p^a)}$ and an entry fee equal to:

$$
F^{a} = Q(p^{a}) \left(\frac{c_{s} \lambda(p^{a})}{p^{a} - \lambda(p^{a})} - \frac{(p^{a} - \lambda(p^{a}))(p^{a} - c_{p} - c_{s} - \lambda(p^{a}))}{p^{a} (1 - \lambda'(p^{a}))} \right),
$$
(17)

where the final consumer price p^a is the solution to:

$$
\mu \left(1 - \frac{(p - \lambda(p))^2 (p - c_p - c_s - \lambda(p))}{c_s p \lambda(p) (1 - \lambda'(p))} \right) - \frac{(p - \lambda(p)) \left[(p - c_p - c_s) (\lambda(p) - p \lambda'(p)) + \lambda(p)(p - \lambda(p)) \right]}{c_s p \lambda(p) (1 - \lambda'(p))} = 0. \tag{18}
$$

(B) Otherwise, $F^a = 0$ and the final price p^a , as in Johnson (2017), is given by the solution to

$$
p - c_p - c_s = \lambda(p) \left[1 + \frac{c_s p (1 - \lambda'(p))}{(p - \lambda(p))^2} \right].
$$
\n(19)

Proof. In an interior equilibrium, the FOCs (15) and (16) have to hold simultaneously. Inspection of (15) immediately reveals that it must be the case that $p^a > c_s + \lambda(p^a)$ for otherwise (15) would not hold. This ensures that $t^a \in (0,1)$. Solving the FOC (15) for y, we find that:

$$
y = 1 - \frac{(p - \lambda(p))^2 [p - c_p - c_s - \lambda(p)]}{c_s p \lambda(p) (1 - \lambda'(p))}.
$$
 (20)

and plugging this expression for y into (16) yields equation (18) , whose solution is the equilibrium price p^a in an interior equilibrium. Having obtained p^a , we can now obtain y^a using (20). Because $y^a = F^a \frac{p - \lambda(p)}{c - \lambda(p)Q(a)}$ $\frac{p-\lambda(p)}{c_s\lambda(p)Q(p)}$, solving for F^a gives the expression for the equilibrium fixed fee as a function of price given in (17).

To obtain the condition under which $F^a > 0$, we use the FOC (16) and note that, because μ is a decreasing function, $F^a > 0$ if and only if

$$
\mu(0) > \frac{p^a(p^a - c_s - c_p - \lambda(p^a)) + c_p \lambda(p^a))}{c_s \lambda(p^a)},
$$

which is equivalent to:

$$
g(0) < \frac{c_s \lambda(p^a)}{p^a (p^a - c_s - c_p - \lambda(p^a)) + c_p \lambda(p^a))}.\tag{21}
$$

Hence, if (21) holds at the price p^a that solves (18), then $F^a > 0$. However, if (21) does not hold, then the equilibrium is not interior and we have Case (B). In such a case, $F^a = 0$ and the final price follows from setting $y = 0$ in the FOC (15) and solving it for p. This gives equation (19) , which characterizes the agency price in Johnson (2017) .

Proposition 6 demonstrates that, also in the agency model, there are conditions under which the platform will refrain from charging a positive entry fee. In fact, the platform will charge positive entry fees only when the inequality $g(0) < \frac{c_s \lambda(p^a)}{n^a (p^a - c_s - c_s - \lambda(p^a))}$ $\frac{c_s \lambda(p^{\alpha})}{p^a (p^a - c_s - c_p - \lambda(p^a)) + c_p \lambda(p^a)}$ holds. This condition is different from the one in the wholesale model, but it is also more likely to hold if the mass of sellers who cannot afford to pay for entry is sufficiently small. The intuition is the same.

Unlike in the wholesale model, in the agency model the fraction y^a of the seller's profit captured by the platform as an entry fee is not directly related to the pass-through of commissions to final consumer prices. However, as in the wholesale model, the ratio of margins of the platform and the seller is sufficient to tell whether there is more or less rent extraction. This is because equation (16) can be written as:

$$
\mu(y) = y + RM^a(p),
$$

where

$$
RM^{a}(p) \equiv \frac{p - c_{p} - \frac{pc_{s}}{p - \lambda(p)}}{\frac{c_{s}\lambda(p)}{p - \lambda(p)}}
$$

.

Because $RM^a(p)$ is increasing in p and p^a is decreasing in y, we can state that:

Proposition 7 In the agency model with seller heterogeneity, when $g(0) < \frac{c_s\lambda(p^a)}{n^a(n^a-c-c-1)n^c}$ $\frac{c_s\lambda(p^{\alpha})}{p^a(p^a{-}c_s{-}c_p{-}\lambda(p^a)){+}c_p\lambda(p^a)},$ the fraction y^a of the seller's profit captured by the platform as entry fee is larger for a demand and cost system that yields a lower ratio of platform margin over seller margin.

Example (continued)

Proposition 6 characterizes the equilibrium in the agency model. As in the wholesale model, whether or not the platform charges an entry fee depends on the distribution G and the nature of demand. However, the conditions under which the platform chooses to charge a positive entry fee differ across intermediation models.

For example, we saw that in the wholesale model, if G is uniform and demand is log-concave, the platform will charge a positive entry fee. We now show that in the agency model, the platform will refrain from charging any entry fee if the demand satisfies Marshall's second law of demand. Because log-concave demands are in the class of demands that satisfy Marshall's second law of demand, this means that no entry fees will be charged in the agency model, while they will be in the wholesale model. As a consequence, the seller will always enter in the agency model while not always in the wholesale model.

We show this by contradiction. Suppose a positive entry fee is charged in the agency model, in which case, using (17), it must hold that

$$
\frac{c_s \lambda(p^a)}{p^a - \lambda(p^a)} > \frac{(p^a - \lambda(p^a))(p^a - c_p - c_s - \lambda(p^a))}{p^a (1 - \lambda'(p^a))}.
$$
\n(22)

Since the final price p^a must satisfy (18), using the fact that $\mu(x) = 1 - x$ for the uniform distribution and rearranging implies that the equilibrium price p^a must satisfy:

$$
p^{a} - c_{s} - c_{p} = \frac{2\lambda(p^{a})(p^{a} - \lambda(p^{a}))}{p^{a} - 2\lambda(p^{a}) + p^{a}\lambda'(p^{a})},
$$
\n(23)

which, naturally, implies that $p^a - 2\lambda(p^a) + p^a \lambda'(p^a) > 0$. Using (23) in (22) and simplifying gives:

$$
\frac{c_s\lambda(p^a)}{p^a - \lambda(p^a)} > \frac{\lambda(p^a)(p^a - \lambda(p^a))}{p^a - 2\lambda(p^a) + p^a\lambda'(p^a)}.
$$

This inequality holds whenever:

$$
c_s (p^a - \lambda(p^a) - (\lambda(p^a) - p^a \lambda'(p^a))) > (p^a - \lambda(p^a))^2.
$$

Because $c_s < p^a - \lambda(p^a)$, this inequality cannot hold for any demand function that satisfies Marshall's second law of demand.

The uniform G example with log-concave demand thus constitutes an instance in which the entry probability will be higher under the agency model than under the wholesale model. If the final price under the agency model happens to be lower than under the wholesale model, then the agency model will definitely yield a higher level of social welfare. In the next section, we compare the two modes of intermediation more generally.

5.3 Comparison of final prices, entry probabilities, firm profits and consumer surplus

We have already shown in Proposition 1 that the final consumer price under the wholesale model is higher than the monopoly price, i.e., $p^* > p^m$. We have also shown in Proposition 2 that the final consumer price under the agency model is higher than the monopoly price for any exogenous level of rent-extraction $\beta \in [0, 1)$, i.e., $p^a > p^m$. This implies that no matter the endogenous level of rent-extraction y^a , it is also the case that $p^a > p^m$.

We now proceed to compare the equilibrium prices under the wholesale model and the agency model when the level of rent-extraction is endogenous. When demand satisfies Marshall's second law of demand, we can directly apply Proposition 2 and conclude that $p^a \lt p^*$. This is because Proposition 2 demonstrates that $p^a < p^*$ for any β , and hence for any y^a . However, when demand does not satisfy Marshall's law, we do not know whether the equilibrium level of rent extraction y^a is below or above the threshold $\hat{\beta}$ defined in Proposition 2. The following result provides the necessary and sufficient condition under which $p^a \leq p^*$ in this general case.

Corollary of Proposition 2 In the model with seller heterogeneity, assume that entry fees are strictly positive in both intermediation settings, for which $g(0) = 0$ suffices. Then, the final price under the agency model p^a is lower than under the wholesale model p^* if and only if

$$
y^{a} > 1 - \frac{(p^* - \lambda(p^*))^2}{c_s p^*}.
$$

As a result, $p^a < p^*$ for any demand function that satisfies Marshall's second law of demand.

To rank consumer surplus levels across intermediation models, this corollary is no longer sufficient because, in addition to how prices compare across models, the seller's entry probability is also relevant. Our final result compares the probability of entry across intermediation models when entry fees and commissions are positive.

Proposition 8 In the model with seller heterogeneity, assume that entry fees are strictly positive in both models of intermediation, for which $g(0) = 0$ suffices. Assume also that the demand function satisfies Marshall's second law of demand so that $p^a \leq p^*$. Then, the seller's entry probability under the agency model is higher than under the wholesale model if:

$$
p^{a} - c_{p} - \frac{p^{a}c_{s}}{p - \lambda(p^{a})} > \lambda(p^{a}),
$$
\n(24)

or equivalently,

$$
y^a < \frac{\lambda(p^a) - p^a \lambda'(p^a)}{p^a (1 - \lambda'(p^a))}.\tag{25}
$$

As a result, if these conditions hold, the agency model results in a higher consumer surplus, higher platform profits but lower seller's profits. On aggregate, welfare under the agency model is higher than under the wholesale model.

Proof. The seller's entry probability under the wholesale model is given by $1 - G(z^*)$, where z^* is the solution to equation (10). Likewise, the seller's entry probability under the agency model is given by $1 - G(y^a)$, where y^a is given by the solution to equation (16). Hence, to compare the entry probabilities, it suffices to compare z^* and y^a .

We first note that the RHS of (16) can be rewritten as

$$
z + \frac{p(p - c_s - c_p - \lambda(p)) + c_p \lambda(p)}{c_s \lambda(p)}
$$

and, as noted in Proposition 7, this expression increases in p.

Likewise, the RHS of (10) can be rewritten as

$$
z+\frac{\lambda(p)}{p-c_s-c_p-\lambda(p)},
$$

which decreases in p if demand satisfies Marshall's second law of demand.

Hence, $y^a < z^*$ (and so the seller's entry probability is higher under agency than under wholesale) if and only if:

$$
\frac{p^{a}(p^{a} - c_{s} - c_{p} - \lambda(p^{a})) + c_{p}\lambda(p^{a})}{c_{s}\lambda(p^{a})} > \frac{\lambda(p^{*})}{p^{*} - c_{s} - c_{p} - \lambda(p^{*})}
$$
(26)

Because $\frac{\lambda(p^a)}{p^a-c-c}$ $\frac{\lambda(p^a)}{p^a-c_s-c_p-\lambda(p^a)} > \frac{\lambda(p^*)}{p^*-c_s-c_p-\lambda(p^*)}$ $\frac{\lambda(p^*)}{p^*-c_s-c_p-\lambda(p^*)}$, for (26) to hold, it suffices that:

$$
\frac{p^a(p^a - c_s - c_p - \lambda(p^a)) + c_p \lambda(p^a)}{c_s \lambda(p^a)} > \frac{\lambda(p^a)}{p^a - c_s - c_p - \lambda(p^a)}.
$$

Or:

$$
p^{a}(p^{a}-c_{s}-c_{p}-\lambda(p^{a}))^{2}+c_{p}\lambda(p^{a})(p^{a}-c_{s}-c_{p}-\lambda(p^{a})) -c_{s}\lambda(p^{a})^{2} > 0,
$$

which can be rewritten as:

$$
(p^{a} - c_{s} - c_{p}) [(p^{a} - \lambda(p^{a}))^{2} - c_{p}(p^{a} - \lambda(p^{a})) - c_{s}p^{a}] > 0.
$$

Hence, the condition we need is simply:

$$
(p^{a} - \lambda(p^{a}))^{2} - c_{p}(p^{a} - \lambda(p^{a})) - c_{s}p^{a} > 0,
$$

which can be rewritten as

$$
p^{a} - c_{p} - \frac{p^{a}c_{s}}{p - \lambda(p^{a})} > \lambda(p^{a}),
$$
\n(27)

which is the first condition is the proposition. The FOC (15) implies that:

$$
p^{a} - c_{p} - \lambda(p^{a}) = c_{s} + c_{s}(1-y) \frac{p^{a} \lambda(p^{a})(1 - \lambda'(p^{a}))}{(p^{a} - \lambda(p^{a}))^{2}}.
$$

Using this to rewrite condition (27) gives:

$$
c_s - \frac{p^a c_s}{p - \lambda(p^a)} + c_s(1 - y) \frac{p^a \lambda(p^a)(1 - \lambda'(p^a))}{(p^a - \lambda(p^a))^2} = -\frac{\lambda(p^a)c_s}{p^a - \lambda(p^a)} + c_s(1 - y) \frac{p^a \lambda(p^a)(1 - \lambda'(p^a))}{(p^a - \lambda(p^a))^2} > 0.
$$

Multiplying by $(p^a - \lambda(p^a))^2$ and dividing by $\lambda(p^a)c_s$, condition (27) becomes

$$
-(p^{a} - \lambda(p^{a})) + (1 - y^{a})p^{a}(1 - \lambda'(p^{a})) > 0,
$$

which is equivalent to

$$
y^{a} < \frac{\lambda(p^{a}) - p^{a} \lambda'(p^{a})}{p^{a} (1 - \lambda'(p^{a}))},
$$

which is the second condition in the proposition.

Because $p^a < p^*$ and $y^a < z^*$, consumer surplus is higher under the agency model. Moreover, from Proposition 2 it follows that, conditional on seller's entry, platform's profits are higher and seller's profits lower. On aggregate, because $p^a \leq p^*$, welfare is higher conditional on seller's entry. The proof is now complete. \blacksquare

Proposition 8 generalizes Proposition 2 to settings where the platform is uncertain about the seller's willingness to pay for entry. In such a case, the seller's entry probability is no longer equal to one. When the demand function satisfies Marshall's second law of demand, and when the condition in Proposition 8 that the platform's margin is sufficiently large is met, the seller's entry probability in the agency model is higher than in the wholesale model. Because we know from Proposition 2 that the equilibrium price is lower in the agency model than in the wholesale model when the demand function satisfies Marshall's second law of demand, consumer surplus in the agency model is also higher than in the wholesale model. Moreover, as the total industry profit $\pi^m(p)$ is concave and maximized at p^m with $p^m < p^a < p^*$, industry profit when entry occurs is higher in the agency model as well. Hence, whenever entry fees are strictly positive in both models of intermediation (for which $g(0) = 0$ suffices) and (25) holds, total welfare is larger in the agency model than in the wholesale model.

6 Conclusions

In this paper, we have studied the agency and wholesale models of intermediation. Two empirical observations have inspired our model. First, many intermediaries charge entry fees, but, perhaps surprisingly, not all of them do. For example, supermarkets often use slotting fees and some platforms such as Amazon and eBay charge entry fees in addition to commissions. However, other platforms such as Facebook Marketplace, Airbnb or Expedia do not use entry fees in their pricing policies. Second, Amazon's entry fees at levels such as \$39.99 per month for some product categories seem relatively low and hence do not probably extract the full profit of the sellers.

Using a bilateral monopoly market, we have compared the agency and wholesale models of intermediation when platforms can charge entry fees. In the benchmark case where the platform can employ full-profit-extracting entry fees, the agency model results in no double marginalization and hence the final consumer price maximizes industry profit. By contrast, because in the wholesale model entry fees have no bearing on the the terms of trade, double marginalization is not eliminated. As a result, the wholesale model is (weakly) inferior for all agents: consumers face higher final prices, the platform makes lower profits and the seller makes no profits in either of the intermediation models. These results hold no matter the shape of demand.

As mentioned above, the assumption that the platform can extract the full profit of the seller via entry fees is unrealistic for several reasons. Sellers often have to distribute part of their profits to shareholders or investors, pay bonuses to workers, and settle corporate taxes. Sometimes, sellers also have to keep part of their profits to invest in advertising and R&D. This has motivated us to study the agency and wholesale models in settings where the platform cannot extract the full profit of the seller, but just a fraction.

When studying such settings, we have followed Calzolari et al. (2020) and assumed that the

seller's profit share that can be extracted via entry fees depends on the final price.²³ We have shown that, for arbitrary demand functions, the agency model leads to lower prices provided the fraction of the seller's profit extracted by the platform is sufficiently large. In the special case where demand satisfies Marshall's second law of demand, the platform continues to prefer the agency model no matter how much surplus it can extract from the seller (including nothing at all, as in Johnson (2017)), and so do consumers. However, the seller prefers the wholesale model provided that rent-extraction via entry fees is sufficiently low.

Finally, we have extended our model to a setting where the platform faces uncertainty about the seller's willingness to pay for entry. An important result has been the characterization of the conditions under which the platform charges an entry fee in the two intermediation models. Charging an entry fee may cause the seller to refuse entry, thereby fully destroying the intermediation surplus. For both models, we have shown that entry fees are not always used. They are only charged provided that the frequency with which sellers are not willing to pay for entry is sufficiency low. Moreover, we have seen that for the two intermediation models the fraction of the seller's profit captured by the platform via the entry fee is larger the greater the platform's margin relative to the seller's margin. We have also compared the two intermediation models on consumer welfare grounds. For this, a ranking of the final consumer prices is not sufficient because the seller's entry probability is also relevant. Regarding the ranking of prices across intermediation models, we have provided a necessary and sufficient condition for the final price to be higher in the wholesale model than in the agency model and we have shown that the condition holds for any demand function that satisfies Marshall's second law of demand. Hence,

$$
\max_{t,F} \left\{ \Pr[E \le (p(t)(1-t) - c_s)Q(p(t)) - F] \left[(tp(t) - c_p)Q(p(t)) + F \right] \right\}
$$
\n
$$
= \max_{t,F} G \left[(p(t)(1-t) - c_s)Q(p(t)) - F \right] \left[(tp(t) - c_p)Q(p(t)) + F \right].
$$
\n(28)

or

$$
\max_{p,F} G\left[\frac{c_s \lambda(p)}{p - \lambda(p)} Q(p) - F\right] \left[\left(p - c_p - c_s - \frac{\lambda(p)c_s}{p - \lambda(p)}\right) Q(p) + F\right].
$$

Define now the amount of surplus left to the seller by the platform as $\phi \equiv \frac{c_s \lambda(p)}{p-\lambda(p)}Q(p) - F$. Then, the problem of the platform can be reformulated as:

$$
\max_{p,\phi} G(\phi) [(p - c_p - c_s)Q(p) - \phi].
$$

It is now obvious that p^m is the price that maximizes this payoff.

²³An alternative, and less interesting, situation arises when the seller's needs to keep a fixed amount of profit to pay a fixed cost (cf. footnote 17). In that case, the results obtainprofit ed do not differ from the situation where the platform can extract the full profit of the seller via entry fees. The reason for this is simply that when the amount of surplus the platform has to leave to the seller is independent from price, the platform's choice of commission becomes totally independent from the platform's choice of entry fee. In that case, commissions are set at the industry profit-maximizing level. To see this, consider a modified version of our most general model where the platform is uncertain about the fixed cost E the seller has to incur to be able to operate. Let G be the distribution of E . Then, in the agency model, the final consumer price that the seller charges for a given commission continues to be the solution to equation (5). In stage 1, the platform would choose its commission $t \in [0, 1)$ and entry fee $F \geq 0$ to solve:

for the special class of Marshall demands, on the price account, the agency model performs better for consumers than the wholesale model. Regarding the seller's entry probability, a sufficient condition stating that the platform's margin is sufficiently large ensures that entry probability in the agency model is larger than in the wholesale model if the demand function satisfies Marshall's second law. In such cases, on the entry probability account, the agency model also performs better for consumers than the wholesale model. Because a lower price leads to greater industry profits, we conclude that the agency model is better for welfare under the conditions that demand satisfies Marshall's second law and the platform margin is large enough.

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