Platform Competition and Interoperability: The Net Fee Model*

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Abstract

Is more competition the key to mitigating dominance by large tech platforms? Could regulation of such markets be a better alternative? We study the effects of competition and interoperability regulation in platform markets. To do so, we propose an approach of competition in *net fees*, which is well-suited to situations where users pay additional charges, after joining, for on-platform interactions. Compared to existing approaches, the net fee model expands the tractable scope to allow variable total demand, platform asymmetry and merger analysis. Regarding competition, we find that adding more platforms to the market may lead to the emergence of a dominant firm. In contrast, we find that interoperability can play a key role in reducing market dominance and lowering prices. Broadly speaking, our results favor policy interventions that assure the formidability of the competition that dominant platforms face.

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1 Introduction

Large Internet platforms (e.g., Alibaba, Alphabet, Amazon, Apple, Meta, Tencent, X, etc.) are at the center of many of today's most important public policy debates. Platforms invite much criticism, including claims that they are too dominant and should, in some form, have their power reigned in.[1] Some argue that platforms should face more competition, while others focus on regulation. Behind this debate lies a set of basic economic questions. Can more competition in platform markets mitigate dominance? If not, what other remedies might work? One prominent proposal is regulation that would make competing platforms "interoperable" and thus less proprietary. Does this show promise?

This paper offers a modeling approach that sheds light on these two issues: the effects of increasing competition and requiring platform interoperability. Our approach, which we call the *net fee model*, brings about a high level of flexibility to the technically challenging topic of platform competition. The basic idea is that platforms compete by setting a kind of price that we call their "net fee." In most existing models of platform competition, the platforms charge each user either only a "membership" fee (e.g., a subscription) or only "interaction" fees (e.g., charges incurred when transacting with other users). Our approach is a form of hybrid between these two polar cases, in which platforms charge any given user both types of fees.

Surprisingly, this hybrid fee structure makes analyzing platform competition more straightforward than it is under either of the two aforementioned approaches, not more complicated. In a nutshell, as in a pure membership fee model, our approach assumes platforms compete with one another by announcing membership fees. The key difference is as follows: in a pure membership fee model, once these membership fees are announced and users make joining decisions, no additional money ever passes between users and platforms. Under our approach, platforms and users anticipate that, after users have joined a platform, the latter will levy further interaction fees (or subsidies), and they both incorporate this into their decision-making. We thus label the membership fees that

[1] For instance, see a trio of recent, high-profile policy reports addressing these issues (Crémer, de Montjoye and Schweitzer, 2019, Furman et al., 2019, Scott Morton et al., 2019).

platforms set in our model as *net fees* – prices that are net of interaction fees – and we contrast them with the *total prices* that platforms charge in pure membership fee models.

Crucially, by adopting this approach, we can solve a general discrete-choice model with network effects that accommodates asymmetries across platforms and variable total market participation. We provide three main sets of results, pertaining to (i) pricing and characterization of equilibrium, (ii) the effects of competition, and (iii) the consequences of interoperability. With regard to the latter two points we particularly focus on how these forces affect market dominance and total platform demand.

In the first category, we show that net fee competition leads to a straightforward pricing formula. We further show that this can be compared with numerous familiar pricing benchmarks, including from standard oligopoly competition and from earlier work on platform pricing. In line with pricing formulas in the existing literature on platforms, ours features the same three components – marginal cost, market power and network discount – while having the benefit of being more broadly applicable.

The second set of results deals with the effects of competition on the level of dominance enjoyed by one platform in a market. We show that adding more competitors may lead one platform to become dominant or may enhance the position of an already dominant platform. Moreover, a potential merger between two small platforms can reduce the dominance of a large one, and the scope for this to occur grows as network effects become stronger. That is, in a potential merger, network effects can serve as a substitute for cost synergies. To see the underlying mechanism behind these results, consider a setting with one dominant platform and one or more niche platforms. The larger a platform's user base, the stronger its incentive is to discount its net fee. As competition increases, niche platforms' user bases get divided up, and the discounts they offer shrink. Thus, adding a new platform can lead to splintering among the niche players, enabling the dominant platform to capture a larger market share.

The third set of results studies the impact of allowing platforms to be at least partially *interoperable*. By this, we mean that, when two platforms are interoperable with one another, a user who joins either platform can enjoy the network externalities that come from the user bases of both platforms. Think, for instance, of the way a subscriber to one phone company can have conversations with subscribers to other companies. In our model, rather than being a discrete variable, interoperability can take on any value between 0 and 1.

Here, our findings tilt strongly in favor of interoperability. We first analyze the effect that interoperability has on a market that is dominated by one platform. We show that increasing the level of interoperability in the market leads to a reduction in this platform's dominance. This occurs because a higher degree of interoperability reduces the disparity between the network discounts, mentioned above, offered by the dominant platform and its smaller rival(s), and this disparity is the driving force behind market dominance. Second, in a symmetric but otherwise quite general setting, we show that greater interoperability typically leads to lower prices, greater user participation and higher consumer surplus.

In the broad public policy debate regarding platforms, two topics that have received particular interest are, (i) whether to promote greater competition in markets led by a dominant platform, and (ii) what the impact could be of requiring interoperability. Our result that entry may increase an incumbent's dominance urges caution with respect to the first. In contrast, our analysis of interoperability offers an encouraging view of such regulation, suggesting that such proposals should be explored in more detail.

1.1 Related Literature

This paper contributes to the broad literature that has come to be known as "platform economics." The earliest works in this area, which formalize the study of network effects in "one-sided" settings, include Rohlfs (1974), Katz and Shapiro (1985), Farrell and Saloner (1985). A significant step forward occurred when the concept of "multi-sidedness" was introduced, incorporating multiple groups of agents with interdependent demand. Pioneering works on multi-sided platforms include Caillaud and Jullien (2003), Evans (2003), Rochet and Tirole (2003, 2006), Rysman (2004), Anderson and Coate (2005), Parker and Van Alstyne (2005), Armstrong (2006), Hagiu (2006) and Armstrong and Wright (2007). In the monopoly context, Weyl (2010) provides a general synthesis of the incentives influencing a platform's pricing.[2] Jullien, Pavan and Rysman (2021) offers an excellent, recent

[2]Also see Veiga, Weyl and White (2017), which further generalizes the Weyl (2010) model to allow for selection effects as well as network effects.

survey of the platform literature.

In the existing work on platform competition, the most conventional approach is to assume what is sometimes called "pure membership" conduct, which, throughout the paper, we call "total pricing." As mentioned above, the total pricing and net fee approaches share the common feature that platforms compete by setting membership fees. Unlike with net fees, under the total pricing approach, the membership fee is an all-encompassing measure of the revenue that a platform receives from any given user. The best known example of total pricing is the Armstrong (2006) "two-sided single-homing" model, a Hotelling setup which has served as a workhorse in much subsequent literature.

Recently, Tan and Zhou (2021) provides an important generalization of the total pricing approach, incorporating general demand, an arbitrary number of competing platforms, and an arbitrary number of sides of the market. At the same time, that work sheds light on the technical hurdles that are inherent to the total pricing approach. Despite the expansive nature of Tan and Zhou's generalization, their analysis is, nevertheless, constrained to environments with fixed total demand and symmetric platforms playing symmetric equilibrium. A crucial contribution of our work, therefore, is to provide a framework that can be readily used to study situations with variable total demand, and, especially, asymmetries across platforms, which can be both *ex ante* and/or *ex post* in nature. This feature is essential for studying questions of platform dominance.

In the settings where Tan and Zhou (2021) show total pricing is a feasible approach – i.e., platform symmetry with fixed total demand, we compare the results generated by that approach with those arising under net fee competition, and we find a high degree of qualitative similarity. A first such comparison is between pricing formulas, detailed in Section 2.1. Second, a main result of Tan and Zhou (2021) is the so-called "perverse pattern," whereby adding competition may lead the equilibrium price to increase. Like total pricing, net fee conduct also generates this perverse pattern. Due to the similarity in this aspect across the two approaches, we omit the analysis of it from the main text and, instead, include it in Appendix D. Nevertheless, it is worth noting that, since the net fee approach incorporates variable total demand, the perverse pattern encompasses not only price increases but also a reduction of consumption. Third, in Section 6.2, when analyzing the effect of increasing interoperability on equilibrium prices, we compare the predictions of the net fee and total pricing approaches. There, we find more contrast, with the latter offering less optimism. In that section, we explain the logic behind these opposing predictions.

Another important recent contribution using the total pricing approach is Peitz and Sato (2024). It develops a technique to analyze asymmetric platforms, making use of the Logit demand form and logarithmic network effects. Like our paper, it addresses questions about the effects of competition and interoperability in settings with potentially dominant platforms. Apart from the difference in its conduct assumption compared to ours, another point of differentiation is that it focuses on settings with a unique equilibrium in the pricing stage of the platforms' oligopoly game. In contrast, our paper pays particular attention to settings where multiple pricing equilibria may arise and where their presence is the key force leading to asymmetric outcomes.

In addition to pure membership, the literature has considered several other assumptions on platforms' conduct. As mentioned, some models, most notably Rochet and Tirole (2003), take a pure interaction fee approach, while the Armstrong (2006) "competitive bottlenecks" model (see Section 7) includes analysis in which platforms charge only interaction fees on one side of the market and only membership fees on the other side. In addition, Armstrong (2006) considers the possibility of competition in two-part tariffs with platforms simultaneously setting both membership fees and interaction fees. In that context, the paper offers a rather discouraging result whereby any demand profile yielding nonnegative profits can be supported as an equilibrium. Reisinger (2014) offers an approach to restore equilibrium uniqueness under two-part tariff competition, drawing on Klemperer and Meyer (1989)'s introduction of demand uncertainty to restore uniqueness in models of competition in supply functions. Correia-da Silva et al. (2019) offer a Cournot model of platform competition.

Our approach is closely related to one developed in an earlier, now inactive working paper by one of the current authors, White and Weyl (2016), called "insulated equilibrium." Both embed, in an oligopoly framework, Dybvig and Spatt's (1983) insight regarding monopoly, further developed by Becker (1991) and Weyl (2010), that appropriately designed prices can alleviate potential coordination problems for users. Insulated equilibrium differs from our approach in that it applies a refinement to select from among multiple equilibria arising in a higher-dimensional strategy space. Whereas that paper focuses on the effects of different forms of user heterogeneity, this paper derives novel implications on the effects of competition and regulation in platform markets. One separate but related line of research focuses on dynamic platform competition (Cabral, 2011) and explores the link between dynamic competition and static models of conduct (Cabral, 2019). In considering potential interoperability between competing platforms, our paper builds on a stream of literature that includes Crémer, Rey and Tirole (2000) and Malueg and Schwartz (2006), which we discuss further in Section 6. Another related line of research focuses on multi-homing. The simplest model of multi-homing is the competitive bottlenecks model (Anderson and Coate, 2005, Armstrong, 2006, Armstrong and Wright, 2007, Anderson and Peitz, 2020) to which the net fee approach can be readily applied. An active topic has been the study of more fine-grained models of user multi-homing, such as by Athey, Calvano and Gans (2018), Bakos and Halaburda (2020) and Teh et al. (2023).[3]

On a technical level, our proof of equilibrium existence extends a result of Caplin and Nalebuff (1991). In order to derive results on uniqueness, we use aggregative game (Selten, 1970) techniques from Anderson, Erkal and Piccinin (2020). Anderson and Peitz (2020) and Peitz and Sato (2024) also use aggregative game techniques to aid in the analysis of platforms.

The rest of the paper is organized as follows. Section 2 presents the model and motivates the net fee approach. Section 3 derives pricing formulas. Section 4 proves existence of equilibrium and provides a result on uniqueness. Sections 5 and 6 analyze policy questions. The former focuses on the effects of increasing competition and the latter on interoperability regulation. These two sections make their key points in a simplified environment, which Appendices B and C generalize in numerous ways. Section 7 extends the net fee approach to the competitive bottlenecks setting. Appendix D replicates and extends the Tan and Zhou perverse pattern under net fee competition. Appendix E collects all the proofs.

[3] White (2022) discusses some similarities between the economic forces governing multi-homing in Teh et al. (2023) and interoperability in the current paper.

2 The Model

Users can each choose to join at most one platform. Their options are indexed by $j \in \mathcal{J} \cup \{0\} = \{0, 1, ..., J\}$, where $J \geq 1$ is the number of platforms and 0 denotes the outside option. "Sides of the market" are indexed by $s \in \mathcal{S} = \{1, ..., S\}$, where $S \geq 1$. Each side of the market has a unit mass of users, each user belongs to exactly one side, and each platform serves all sides.

Users of a side s are identified by a type $\theta_s = (\theta_s^0, \theta_s^1, ..., \theta_s^J) \in \mathbb{R}^{J+1}$ which captures their *membership value* (standalone taste) for each platform as well as for the outside option. Types are distributed according to cumulative distribution function (CDF) F_s. We assume that F_s admits a density f_s which is continuously differentiable and strictly positive on $\mathbb{R}^{J+1}$.

Payoffs from joining platform j may also depend on how many other users join j. Denote by n_s^j the fraction of side-s users that join platform j, and denote by p_s^j platform j's total side-s price. Users have quasilinear preferences with respect to money, and the payoff to user θ_s from joining platform j is

$$u_s^j := \theta_s^j + \sum_{\hat{s} \in \mathcal{S}} \gamma_{s\hat{s}}^j n_{\hat{s}}^j - p_s^j, \tag{1}$$

where $\gamma_{s\hat{s}}^j$ denotes the *interaction value* with side-$\hat{s}$ users on the same platform.[4] That is, it measures the marginal externality that a user on side $\hat{s}$ of platform j contributes to users on side s of platform j. The payoff from choosing the outside option is $u_s^0 := \theta_s^0$.

Platforms compete by posting membership fees, which we call *net fees*, $t^j = (t_1^j, ..., t_S^j)$. We assume that once a user joins a platform by paying the net fee, all of the utility she derives on the platform from interacting with other users will be extracted by the platform. As a result, a net fee t_s^j guarantees a user with type θ_s a payoff from joining platform j of

$$u_s^j = \theta_s^j - t_s^j. \tag{2}$$

This payoff does not depend on the joining decisions of other users. A net fee t^j implies a

[4] For simplicity, we assume interaction utility to be linear, but this is not essential for our approach. Note that these are allowed to be negative.

user who joins platform j pays a total transaction price of

$$p_s^j := t_s^j + \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j n_{\hat{s}}^j, \tag{3}$$

where the first term is the net fee and the second term is the interaction utility generated by the platform. Appendix A provides a microfoundation in which this interaction utility is extracted by interaction fees that platforms set explicitly.

Given a profile of net fees $t_s = (t_s^1, ..., t_s^J)$ charged by platforms, a user on side s with type θ_s chooses the $j \in \mathcal{J} \cup \{0\}$ yielding the maximal u_s^j. The demand for platform j on side s is then

$$n_s^j(t_s) = \int \mathbf{1}_{\{u_s^j \geq u_s^k, \forall k \in \mathcal{J}\cup\{0\}\}} f_s(\theta_s) \mathrm{d}\theta_s. \tag{4}$$

Let $t \in \mathbb{R}^{J\times S}$ denote the vector of all net fees charged by all platforms on all sides, and let c_s^j denote platform j's marginal cost on side S, which we assume to be constant. Platform j earns, from side-s users, profits of

$$\pi_s^j(t) = \left(p_s^j - c_s^j\right) n_s^j(t_s) \tag{5}$$

$$= \left(t_s^j + \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j n_{\hat{s}}^j(t_{\hat{s}}) - c_s^j\right) n_s^j(t_s), \tag{6}$$

which can be summed to give total profits of

$$\pi^j(t) = \sum_{s\in\mathcal{S}} \pi_s^j(t) \tag{7}$$

$$= \sum_{s\in\mathcal{S}} (t_s^j - c_s^j) n_s^j(t_s) + \sum_{s,\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j n_s^j(t_s) n_{\hat{s}}^j(t_{\hat{s}}). \tag{8}$$

We write consumer surplus for users on side s as

$$V_s(t_s) := \int \max_{j\in\mathcal{J}\cup\{0\}} \{\theta_s^j - t_s^j \cdot \mathbf{1}_{j\in\mathcal{J}}\} f_s(\theta_s) \mathrm{d}\theta_s, \tag{9}$$

noting that $\frac{\partial V_s}{\partial t_s^j} = -n_s^j$. Define total surplus by $W(t) := \sum_{j \in \mathcal{J}} \pi^j(t) + \sum_{s \in \mathcal{S}} V_s(t_s)$.

In the game, platforms simultaneously announce net fees, which determine the demand and the profits. We focus on pure-strategy Nash equilibria, $t = (t^1, ..., t^J)$, where each platform $j \in \mathcal{J}$ chooses a vector of net fees, t^j, that maximizes $\pi^j(\cdot, t^{-j})$, where t^{-j} denotes the vector of net fees announced by platforms other than j.

2.1 Discussion: Competition in Net Fees

In our model, the strategic variable is the net fee, which is a membership fee (or subsidy). This fee does not encompass all of the money that a platform earns per user. In addition, users that join a given platform interact with other users, generating interaction utility, all of which the platform extracts. The microfoundation in Appendix A represents a scenario in which this occurs endogenously in an extensive-form game.

The key ingredient in this microfoundation is that, after making a decision to join a platform, users become captive. The platform is then able to set interaction fees at levels that are just low enough to induce users to be fully active on the platform while still capturing all of the surplus generated from interaction among users. As an illustrative example, consider the choice a user makes when deciding which video game console to purchase. At the time of the joining decision, users have the opportunity to shop around among different gaming platforms. They do so knowing that, after having joined, they will be at the mercy of the platform's fee structure for games. The gaming platforms also know this, and they compete to get users to sign up. In setting console prices (i.e., membership fees), platforms take into account their anticipated future stream of revenue from game purchases, which, in practice, they earn from levying royalties (i.e., interaction fees) on per-copy-sold basis.

Of course, this is a stylized modeling approach, and in some circumstances it will be a more useful tool than in others. Broadly speaking, it seems best-suited for analyzing markets where

- it is feasible for platforms to charge both membership fees and interaction fees;
- platforms can effectively construct a "moat" around users' activity once they join.

Such moat construction has been recognized, in the literature, as both an important objective and challenge for platforms.[5] In addition to gaming, other types of platforms that display these characteristics include software operating systems (and associated app stores), online food and grocery delivery services, and both traditional and new media platforms that involve standalone subscription fees, advertisement, and up-charges for access to popular content creators. Industries in which it is feasible to charge only a membership fee or only a per-transaction fee would seem to more closely match the total pricing approach (Armstrong, 2006, Tan and Zhou, 2021) or the pure-interaction fee approach (Rochet and Tirole, 2003), respectively. An example of the former could be dating platforms, on which users might object to per-interaction fees for non-economic reasons. An example of the latter could be certain payment platforms (e.g., Venmo), where, in practice, joining is typically free for all users. In reality, platforms' pricing mechanism tend to be both innovative and complex, and so any of these models involves significant abstraction, but we believe the net fee approach has a realistic flavor. Moreover, for proponents of the total pricing approach, it is reassuring that the net fee approach delivers qualitatively similar results while being usable in a much wider range of environments.

Remarks. We briefly highlight the following properties of the model.

1. Adopting net fee conduct contributes two features to the model that expand the scope of possible analysis.

   A. Net fees lead demand to be fully determined, so there is no problem of equilibrium multiplicity among users. Thus, for the sake of tying down demand, we invoke no constraints on the strength of network effects nor must we apply an equilibrium selection to the continuation game played by users.

   B. Under the net fee approach, the standard oligopoly demand system, as determined by the distribution of membership values, and network effects enter into platform j's profit function in a separable way. (See eq. (6).) In other words, given a demand

[5] See, for instance, Gu and Zhu (2021), analyzing platforms' efforts to avoid "disintermediation" by users who join platforms and then seek alternative ways to interact with one another in order to avoid paying interaction fees.

system, our game with nonzero γ's and net fees has an analog involving the same demand system $n^j(t)$, no network effects, differentiated Bertrand competition, and variable marginal cost $c^j - \sum_{\hat{s}\in\mathcal{S}} \gamma^j_{s\hat{s}} n^j_{\hat{s}}$ for each platform. This makes first-order conditions straightforward to express in a general environment. Also, it means that underlying properties of a given demand system are preserved.[6]

2. Independently of the conduct assumed in the game, the vector of net fees is the relevant argument in the demand system we study.[7] That is, holding fixed an arbitrary profile of platform strategies (which might be net fees, total prices, or other, as long as there is no within-side price discrimination), the demand profile on side S depends precisely on the values of u^j_s that users receive. In this more general case, a net fee t^j_s can be defined as the difference between membership value θ^j_s and utility u^j_s, which is the same for all side-s users. Thus, the net fee is the relevant measure for demand and consumer surplus, as noted in our analysis of symmetric competition in Section 6.2 and Appendix D.

3 Pricing

3.1 Pricing under Net Fee Competition

First, we analyze the net fees that platform j chooses as a best response when the competing platforms choose t^{-j}. Consider the impact of a marginal effect on j's profits resulting from a change in t^j_s, holding fixed t^{-j}. This is given by

$$\frac{\partial \pi^j(t)}{\partial t^j_s} = \left(p^j_s - c^j_s\right)\frac{\partial n^j_s(t_s)}{\partial t^j_s} + n^j_s(t_s)\left(1 + \gamma^j_{ss}\frac{\partial n^j_s(t_s)}{\partial t^j_s}\right) + \frac{\partial\left(\sum_{\hat{s}\in\mathcal{S}\setminus\{s\}} \pi^j_{\hat{s}}(t)\right)}{\partial t^j_s}. \tag{10}$$

The first two terms capture $\frac{\partial \pi^j_s(t)}{\partial t^j_s}$, i.e., the effect of the fee increase on j's profits arising directly from side s, by taking the derivative of eq. (6). These contain the usual effects

[6] For example, net fee conduct can preserve the aggregative property of demand, which Anderson, Erkal and Piccinin (2020) shows to be useful in analyzing oligopoly.

[7] This point extends to settings where users have nonlinear interaction values, as in Tan and Zhou (2021).

that appear under differentiated Bertrand competition without network effects as well as an additional factor, $\gamma^j_{ss}\frac{\partial n^j_s(t_s)}{\partial t^j_s}$, representing the within-side externality that j's side-s users exude on one another. The last term captures the impact that changing t^j_s has on j's profits from the other sides of the market. Plugging in $\frac{\partial \pi^j_{\hat{s}}(t)}{\partial t^j_s} = \frac{\partial n^j_s(t_s)}{\partial t^j_s}\gamma^j_{\hat{s}s}n^j_{\hat{s}}(t_{\hat{s}})$, the right-hand side of eq. (10) simplifies to

$$\left(p^j_s - c^j_s + \frac{n^j_s(t_s)}{\frac{\partial n^j_s(t_s)}{\partial t^j_s}} + \sum_{\hat{s}\in\mathcal{S}}\gamma^j_{\hat{s}s}n^j_{\hat{s}}(t_{\hat{s}})\right)\frac{\partial n^j_s(t_s)}{\partial t^j_s}. \tag{11}$$

The last term, $\frac{\partial n^j_s(t_s)}{\partial t^j_s}$, is strictly negative, because the density of types is strictly positive everywhere. Thus, the first-order condition that must hold in any best response implies that the bracketed term in eq. (11) must equal zero. Hence, we can immediately obtain the pricing formula of Proposition 1.

Proposition 1. *At any equilibrium, the net fee that platform j charges to users on side s satisfies*

$$t^j_s = c^j_s + \frac{n^j_s(t_s)}{-\frac{\partial n^j_s(t_s)}{\partial t^j_s}} - \sum_{\hat{s}\in\mathcal{S}}(\gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s})n^j_{\hat{s}}(t_{\hat{s}}). \tag{12}$$

This proposition says that a platform's net fee is equal to the sum of (i) its marginal cost of serving a user, (ii) its standard "one-sided" market power, $\frac{n^j_s(t_s)}{-\frac{\partial n^j_s(t_s)}{\partial t^j_s}}$, and (iii) a term we refer to as the *network discount*. The last term captures the total interaction value that is generated when an additional side-s user joins platform j. It can be decomposed as follows. The first component, $\sum_{\hat{s}}\gamma^j_{s\hat{s}}n^j_{\hat{s}}$, equals the additional money that a side-s user pays the platform, beyond the net fee. The second component, $\sum_{\hat{s}}\gamma^j_{\hat{s}s}n^j_{\hat{s}}$, measures the marginal interaction value that an additional side-s user creates for other users across all sides, which the platform extracts from them. All proofs are collected in Appendix E.

Example: Logit Demand. A particularly convenient functional form, which we use in Sections 5 and 6 on policy analysis, involves demand that takes on the Logit form, i.e.,

$$n_s^j(t_s) = \frac{e^{-t_s^j}}{e^{z_s} + \sum_{k\in\mathcal{J}} e^{-t_s^k}}, \tag{13}$$

where z_s parameterizes the outside option for side-s users.[8] Note that this gives $\frac{\partial n_s^j(t_s)}{\partial t_s^j} = -n_s^j(1-n_s^j)$, and so the net fee in Proposition 1 becomes

$$t_s^j = c_s^j + \frac{1}{1-n_s^j(t_s)} - \sum_{\hat{s}\in\mathcal{S}}(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j)n_{\hat{s}}^j(t_{\hat{s}}). \tag{14}$$

3.2 Relationship to Benchmarks

The pricing formula in Proposition 1 relates as follows to these notable benchmarks.

1. Compared to the net fee that maximizes total surplus, $W(t)$,

$$t_s^j = c_s^j - \sum_{\hat{s}\in\mathcal{S}}(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j)n_{\hat{s}}^j(t_{\hat{s}}), \tag{15}$$

   it has the additional one-sided market power term.[9] The final terms in eqs. (12) and (15) coincide, because each platform fully internalizes the network effects that are created by adding a marginal user.[10]

2. It generalizes the one from standard differentiated Bertrand competition, in discrete choice models without network effects, where all γ's are equal to zero.

3. In the special case of two-sided monopoly ($J = 1$, $S = 2$), it coincides with the "pure-membership" pricing formulas of Rochet and Tirole (2006) and Armstrong (2006).[11]

[8] Demand as in eq. (13) arises when the membership values of side-s users are drawn independently, with $\theta_s^1, \ldots, \theta_s^J \sim \text{Gumbel}(0,1)$ and $\theta_s^0 \sim \text{Gumbel}(z_s, 1)$.

[9] Equation (15) can be obtained by noting that the first-order condition for maximization of total surplus, $\frac{\partial \pi^j(t)}{\partial t_s^j} + \sum_{k\in\mathcal{J}\setminus\{j\}} \frac{\partial \pi^k(t)}{\partial t_s^j} + \frac{\partial V_s}{\partial t_s^j} = 0$, implies that $t_s^j = c_s^j - \sum_{\hat{s}\in\mathcal{S}}(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j)n_{\hat{s}}^j$.

[10] Note that, although these final terms coincide in the two expressions, they take on different values because the n_j^s's are endogenous. See Tan and Wright (2018) for discussion of this point in the case of monopoly.

[11] See Rochet and Tirole's Proposition 1(iii), which encompasses Armstrong's Section 3 monopoly pricing

Under S-sided monopoly, it coincides with the formula of Weyl (2010), and under two-sided oligopoly, it coincides with the formula of White and Weyl (2016), both specialized to affine, homogeneous-within-side interaction values.

4. Compared to Tan and Zhou's (2021) symmetric-equilibrium oligopoly pricing formula, our expression relates in the following way. Their paper makes a significant generalization of Armstrong (2006)'s classic "two-sided single-homing" Hotelling model.[12] In order to derive their formula, they assume that the distribution of membership values is symmetric across the platforms, marginal costs are identical across platforms and equal to c_s, interaction values are identical across platforms and equal to $\gamma_{s\hat{s}}$.[13] Furthermore, users have no outside option. For each $s \in \mathcal{S}$, let H_s and h_s denote the cumulative distribution function (CDF) and probability density function (PDF) of $\theta_s^1 - \max\{\theta_s^2, ..., \theta_s^J\}$.[14]

   At the symmetric equilibrium they study, the pricing formula is

$$p_s = c_s + \frac{1 - H_s(0)}{h_s(0)} - \frac{1}{J-1} \sum_{\hat{s} \in \mathcal{S}} \gamma_{\hat{s}s}. \tag{16}$$

   In our model, under these assumptions, expressing eq. (12) as a total price gives

$$p_s = c_s + \frac{1 - H_s(0)}{h_s(0)} - \frac{1}{J} \sum_{\hat{s} \in \mathcal{S}} \gamma_{\hat{s}s}, \tag{17}$$

   whose only difference from eq. (16) is in the denominator of the final term.[15]

formula. In RT's notation, their expression $p^i = \frac{p^i}{\eta^i} - b^j$ can be rewritten by substituting $p^i = \frac{A^i - C^i}{N^j}$, $\frac{p^i}{\eta^i} = \frac{N^i}{-\frac{\partial N^i}{\partial p^i}} = \frac{1}{N^j}\frac{N^i}{-\frac{\partial N^i}{\partial A^i}}$ and then translated into ours by noticing their generic A^i, C^i correspond to generic p^j, c^j in our notation.

[12]See Section 4 of Armstrong (2006).

[13]They also provide further generalization of allowing network effects to be nonlinear in demand.

[14]Notice that due to the symmetry of the distribution of membership values, H_s is independent of the platform.

[15]In terms of net fees, Tan and Zhou's formula becomes

$$t_s = c_s + \frac{1 - H_s(0)}{h_s(0)} - \frac{1}{J-1} \sum_{\hat{s} \in \mathcal{S}} \gamma_{\hat{s}s} - \frac{1}{J} \sum_{\hat{s} \in \mathcal{S}} \gamma_{s\hat{s}},$$

4 Equilibrium Existence and Uniqueness

In order to guarantee the existence of equilibrium, we make two assumptions that ensure both the sufficiency of the first-order conditions for profit maximization and that the profit-maximizing fees are bounded. We start with the following definition, followed by the two assumptions. In the following, we denote the marginal density of θ_s^j by $f_{s,j}\left(\theta_s^j\right)$, the conditional density of θ_s^k (conditioned on θ_s^j) by $f_{s,k|j}(\theta_s^k|\theta_s^j)$, and the conditional CDF of θ_s^0 (conditioned on θ_s^j) by $F_{s,0|j}\left(\theta_s^0|\theta_s^j\right)$.

Definition 1. For each side s, define $\overline{g}_s$ to be the supremum of the conditional density function of the membership values of any pair of alternatives, i.e.,

$$\overline{g}_s := \sup_{j\in\mathcal{J},k\in\{0\}\cup\mathcal{J}\setminus\{j\}} \sup_{\theta_j,\theta_k} f_{s,k|j}\left(\theta_s^k|\theta_s^j\right). \tag{18}$$

Assumption A1. $\forall s \in \mathcal{S}$, there exists $\rho_s \geq -\frac{1}{J+2}$, such that

(a) the joint distribution of side-s users' membership values, $f_s(\theta_s)$, is ρ_s-concave;

(b) $\left(\gamma_{ss}^j + \sum_{\hat{s}\neq s}\left|\frac{\gamma_{s\hat{s}}^j+\gamma_{\hat{s}s}^j}{2}\right|\right)\cdot \overline{g}_s \leq \frac{1}{2J}\left[1+\frac{\rho_s}{1+(J+1)\rho_s}\right], \forall j \in \mathcal{J}$.

Assumption A2. $\forall s \in \mathcal{S},\ j \in \mathcal{J}$, we have $\lim_{t_s^j\to\infty} t_s^j \cdot \int_{-\infty}^{\infty} F_{s,0|j}\left(\theta_s^j - t_s^j|\theta_s^j\right) f_{s,j}\left(\theta_s^j\right)\mathrm{d}\theta_s^j = 0$.

Assumption A1 pertains to both the membership value distribution and the magnitudes of interaction values. It ensures that each platform's profit function, $\pi^j(t^j, t^{-j})$, is quasiconcave in t^j, for every t^{-j}. Roughly speaking, part (b) says that, as network effects grow larger, the degree of concavity that must be imposed in part (a) on the distribution of membership values becomes more stringent. In the special case without network effects, i.e., when $\gamma_{s\hat{s}}^j = 0$ for all $s, \hat{s} \in \mathcal{S}$, this assumption reduces to Assumption A2 in Caplin and Nalebuff's (1991) seminal work on the existence of equilibrium in oligopoly.[16]

and our eq. (12) specializes to

$$t_s = c_s + \frac{1-H_s(0)}{h_s(0)} - \frac{1}{J}\sum_{\hat{s}\in\mathcal{S}}\gamma_{\hat{s}s} - \frac{1}{J}\sum_{\hat{s}\in\mathcal{S}}\gamma_{s\hat{s}}.$$

Also note, in particular, that eqs. (16) and (17) share the same limit behavior as the number of platforms grows large.

[16] A slight difference in accounting is that Caplin and Nalebuff label the dimensionality of users' types as n, whereas we label it as $J+1$.

Assumption A2 ensures that the measure of users who prefer platform j to the outside option goes to zero sufficiently fast as j's net fee, t_s^j, grows large. It implies that platforms do not charge arbitrarily high fees, and thus it provides a bound for the set of best responses. Using standard techniques, including the Brouwer's fixed point theorem, we obtain the following result. Note that this assumption requires the existence of an outside option.

Proposition 2. *Under Assumptions A1 and A2, there exists a pure-strategy Nash equilibrium.*

The following result, dealing with Logit demand, provides an existence condition that applies even when users do not have an outside option.

Proposition 2′. *Assume demand takes the Logit form as specified in eq. (13), and either $J \geq 2$ or users have an outside option. If $\gamma_{ss}^j + \sum_{\hat{s}\neq s} \left|\frac{\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j}{2}\right| \leq 3.375, \forall j \in \mathcal{J}, s \in \mathcal{S}$, then there exists a pure-strategy Nash equilibrium.*

4.1 Uniqueness under Logit

Proposition 3 contains a result on uniqueness of equilibrium in the Logit case.

Proposition 3. *Assume demand takes on the Logit form, either $J \geq 2$ or users have an outside option, and the market is one-sided ($S = 1$). If $\gamma^j < 2.610, \forall j \in \mathcal{J}$, there exists a unique pure-strategy Nash equilibrium.*

In order to obtain this result, we make use of two features. The first is that, in a single market without network effects, when demand is of the Logit form, standard differentiated Bertrand competition gives rise to profits for each firm that have the *aggregative* property. That is, to calculate a given firm's profits, it is sufficient to know a sum that depends on all competitors' prices; it is not necessary to know each of their prices individually. The second feature we make use of is the one discussed in remark 1B of Section 2. This is the fact that, in markets with nonzero γ's, when platforms compete in net fees, the demand system and the network effects enter into each platform's profits in a separable manner. Consequently, in a one-sided platform context, the aforementioned aggregative property is preserved. We can thus make use of the technique provided by Anderson, Erkal and Piccinin (2020) to establish our bound for equilibrium uniqueness.[17]

Together, Propositions 2′ and 3 establish that the following straightforward configuration holds in one-sided markets with Logit demand. When network effects are not too strong ($\gamma^j \leq 3.375$, $\forall j$), equilibrium exists; if they also satisfy a tighter upper bound ($\gamma^j < 2.610$, $\forall j$), then it is unique. We now turn to the effects of competition, studying both settings with potential equilibrium multiplicity and with guaranteed uniqueness.

5 Effects of Competition

A much-discussed concern regarding platform industries is the dominance of one firm in a given market (e.g., Alphabet, Amazon, Meta, etc.). In the context of such discussion, it is sometimes proposed that such dominance could be alleviated by the entry of more players into the given market. In this section, we use our model to address two issues related to such discussion. We first show that, when strong network effects lead to multiple equilibria, more competition could in fact help tip the market in favor of one dominant platform. Second, we analyze a potential merger between two small platforms that compete against a dominant one. We show that, when network effects are weak enough to guarantee equilibrium uniqueness, these network effects play a substitutable role to that of standard costs synergies that are typically required to justify such a merger. Both results suggest that, in platform industries, the best "pro-competitive" policies are not those that merely increase the number of competitors. To the contrary, they may be those that pit dominant platforms up against appropriately formidable challengers.

Throughout this section we make several assumptions that ease the exposition of our results. First, we assume that demand takes on the Logit form, as specified in eq. (13), and that there is no outside option. We are thus in an environment covered by Propositions 2′

[17] An idea that at first blush seems tempting is to apply the multi-product technique of Nocke and Schutz (2018) to establish uniqueness in the case of $S \geq 2$. Unfortunately, such an intuition is misguided, because multi-sidedness in platform competition corresponds to an oligopoly setting with multiple markets, not a setting, like the one they study, where a given firm can sell multiple products within one market. See Peitz and Sato (2024) for a recent advancement in the use of aggregative game analysis to study two-sided platform competition under Logit demand.

and 3. We further assume that $S = 1$. Finally, we assume that network effect strength is common across all platforms, i.e., $\gamma^j = \gamma$, $\forall j$.

In two appendices, we consider more general environments and provide a set of results that extend those that are presented below in the main text. In particular, Appendix B deals with "multi-sidedness." It shows that any one-sided version of our model is isomorphic to a class of multi-sided markets. Although the isomorphic class of multi-sided models features a form of symmetry across the different sides, this form of symmetry is surprisingly weak, especially in the case of two-sided markets ($S = 2$), where the isomorphic class includes models with "indirect network effects" that differ in strength across the two sides. Appendix C deals with the case of general demand, S sides and J platforms. As usual in oligopoly models, crisp results are hard to come by without imposing functional forms. However, this generalization makes the case that the results in the main text do not flow in some peculiar way from the logit assumption.[18]

5.1 Competition and Market Dominance

A frequent concern regarding markets with network effects is the idea that they are prone to "tipping" towards dominance by one, or perhaps a small number, of platforms. Here, we state this section's first result on the potential unintended consequences of competition.

Proposition 4 (Competition may increase dominance)**.** *Assume platforms are* ex ante *identical and* $\gamma \in (2.71, 3.375]$. *There exists an equilibrium under triopoly in which a dominant platform's market share is greater than the market share of any platform in any duopoly equilibrium.*

Note that this result pertains to a region of γ in which equilibrium existence is guaranteed but, for an arbitrary number of platforms, equilibrium uniqueness is not. In the lower region of the assumed interval, duopoly has a unique equilibrium which is symmetric,

[18] Another form of generalization that we do not address in the aforementioned appendices is heterogeneity in the interaction values (i.e., the γ's) across users within a given side of the market. The only approach we are aware of that can accommodate this in general oligopoly is "insulated equilibrium" due to White and Weyl (2016). That approach assumes platforms' strategies are contingent functions and is thus more complex. It is most useful when the questions at hand relate to the quality distortion most closely identified with Spence (1975) and further explored in the context of platforms by Chan (2024). The insulated equilibrium approach is complementary to the net fee approach, as indicated, for example, by the comparison of pricing formulas given in Section 3.2.

where each platform has a market share of 1/2. Under triopoly, there is an asymmetric equilibrium in which the dominant platform's market share is greater than 1/2. When γ is in the upper region of the assumed interval, duopoly has an asymmetric equilibrium. Throughout this interval of γ, starting from duopoly and adding a new platform to the market can lead the dominant platform to become even more dominant. Figure 1 shows the largest possible equilibrium market share of any platform under duopoly and triopoly, with different strengths of network externality γ.

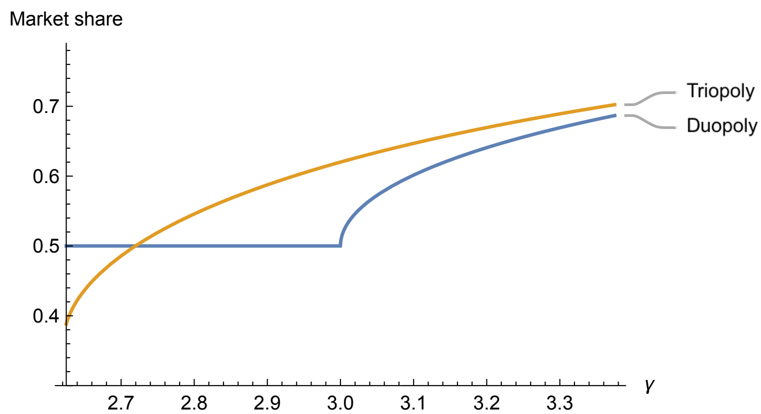

Figure 1: Largest possible equilibrium market share of any platform

The underlying mechanism that allows competition to increase dominance can be understood, somewhat heuristically, in the following way. Observe that, in the pricing formula of a generic platform j,

$$t^j = \frac{1}{1-n^j} - 2\gamma n^j,$$

the network discount, $2\gamma n^j$, is increasing in j's market share n^j. Under duopoly, denote by $n \leq 1/2$ the initial market share of the (weakly) smaller "incumbent" platform, and note that, because there is no outside option, the dominant incumbent platform serves the $N = 1 - n$ remaining users.

Now consider the arrival of an "entrant" into the market, and imagine that the entrant and the smaller incumbent compete most directly with one another. Specifically, consider a candidate triopoly equilibrium at which the smaller incumbent and the entrant adopt the same strategy as one another and each serve $n/2$ users, while the dominant incumbent continues to serve N users. At this candidate equilibrium, the dominant platform does not face any direct incentive to change its net fee; however, the smaller incumbent and the entrant each have an incentive to lower their network discount and thus raise their net fee. As the proof of Proposition 4 shows formally, this incentive for the two smaller firms to raise their net fee, starting from this candidate equilibrium, implies that an actual equilibrium exists in which they each serve fewer than $n/2$ users, and the dominant platform's demand grows to a level larger than N.

5.2 Merger Analysis

We now study the effects of a possible merger between two smaller platforms in a triopoly market that includes a dominant platform. In this analysis, we restrict attention to an environment in which equilibrium is unique (see Proposition 3), but we allow for the platforms to be *ex ante* asymmetric. In particular, we take into account cost synergies (i.e., reductions in marginal costs) brought about by the potential merger, which play a central role in standard merger analysis. We address the question of how the strength of network effects in a given market influences the amount of cost synergy that is needed in order for a merger between two smaller platforms to help reduce the large platform's dominance.

The environment is as follows. In the pre-merger setting, the dominant platform has some market share of at least 1/2, and the remaining users, who have no outside option, are equally split between the two non-dominant platforms. The dominant platform's marginal cost is assumed to be zero, whereas the smaller platforms have some positive marginal cost, $c > 0$. The particular demand profile in question can be supported by some combination of cost difference, c, and network effect strength, γ.[19] In the event of a merger, the two smaller platforms become one entity, which enjoys both combined network effects and cost synergies, given by $\Delta c \in (0, c)$.[20] We now state Proposition 5.

[19] The condition for the large platform to have market share of at least 1/2 is $c + \gamma/2 > 1.36$.

[20] There are different possible ways to model a merger. We take the approach of assuming one of the merged platforms shuts down, allowing the two user bases to be combined. An alternative assumption, which is also compatible with the net fee modeling approach, allows the two merged platforms to continue to operate as separate entities but with a single agent setting both of their prices.

Proposition 5. *Assume $\gamma < 2.610$. In a merger between the two non-dominant platforms, the minimum cost synergy needed to reduce the market share of the dominant platform decreases with the strength of network effects.*

To interpret this proposition, first consider a traditional oligopoly setting without network effects. There, following a merger, if there were no cost synergies, the merged firm would have an incentive to raise its price compared to the pre-merger level. This decreases its market share and thus increases the dominance of the non-merging firm. Hence, a significant cost synergy would be necessary in order for a merger not to cause the large firm to become more dominant. In a market with network effects, however, since the merged entity benefits from a larger user base, post-merger it incorporates a larger network discount into its pricing. This larger network discount plays a role that can substitute for the one played by cost synergies. Thus, the stronger the network externality, the smaller the required cost synergy to prevent the dominant firm from growing.

To conclude this section, we note that Propositions 4 and 5 both depend on essentially the same underlying mechanism. From a technical perspective, Proposition 4 relies on network effects being strong enough to generate multiple equilibria in platforms' oligopoly game and does not contemplate *ex ante* exogenous differences across platform. On the other hand, Proposition 5 relies on exogenous cost differences among platforms without requiring network effects that are strong enough to generate multiplicity. Both results highlight the point that network effects can undermine the effectiveness of policies whose intent is to be pro-competitive. This theme of potential ineffectiveness of competition policy is also reflected in the symmetric case with variable total demand, which we cover in Appendix D. There, we show that an increase in the number of platforms can cause net fees to rise substantially enough to induce market contraction, even taking into account that entry mechanically provides users with better expected idiosyncratic matches with their preferred platform. The key force underlying each of these results is that the arrival of more platforms can interfere with existing competitive pressures by dividing up user bases.

6 Interoperability

In policy debates on platform governance, it is sometimes argued that regulation, not more competition, is a better approach to tempering the dominance of large platforms. In this vein, a particular policy that is sometimes proposed is a requirement that competing platforms be (at least partially) compatible or "interoperable" with one another.[21] The basic idea is that a user who joins one platform could be able to interact with not just other users of the same platform but also with users of its competitors.[22] This section explores the effects of such a requirement.

It is already understood in the literature that a tradeoff may arise as platforms' interoperability increases. As Farrell and Klemperer (2007) point out,[23] on the one hand, when network effects are fully proprietary, users can be inefficiently divided up across platforms; on the other hand, more proprietary network effects may drive platforms to compete more intensely. In this section, we use an extended version of the net fee model to study two competitive configurations in which this tradeoff arises and where interesting mechanics governing this tradeoff emerge.

First, we consider an environment that is completely analogous to the one studied in Section 5.1. This allows for an apt comparison between the possible impacts on market dominance of increasing interoperability and increasing competition under asymmetric equilibrium in the platforms' oligopoly game. We show that a possible effect of increased interoperability can indeed be to eliminate such asymmetric equilibria. As in Section 5.1, the result in the main text considers a logit model with no outside option and $S = 1$, and we generalize things in Appendix C.2.

Second, we study a more general demand setup, with J symmetric platforms and an outside option. There, we reveal the crucial role played by the diversion ratio in governing the aforementioned efficiency-competition tradeoff. We show that, in settings with an outside option or more than two platforms, interoperability reduces net fees and improves

[21] For consistency, we stick to "interoperability," but we view this term as interchangeable with "compatibility," as it is often used in the literature.

[22] To fix ideas, contrast the case of Facebook, on which users can be friends only with other Facebook users, with phone service, where subscribers can call one another, regardless of their respective networks.

[23] See, especially, Section 3.8.1.

consumer surplus. In both environments, we find reason for optimism about the effects of increasing interoperability.

6.1 Interoperability and Market Dominance

The key ingredient we add to the model is the parameter, $\lambda \in [0, 1]$, denoting the *degree of interoperability*. For simplicity, we assume that this single parameter captures the level of interoperability between any two platforms in the market, although a more complicated configuration would be consistent with our framework. As in Section 5.1, we consider a duopoly in which the two platforms are *ex ante* identical, demand takes the Logit form with no outside option, and $S = 1$.

When there are J platforms, the expression (updated from eq. (1)) for the gross utility derived by a user who joins platform j is

$$u^j := \theta^j + \gamma n^j + \lambda \sum_{k \in \mathcal{J} \setminus \{j\}} \gamma n^k - p^j. \tag{19}$$

The notion of the net fee extends naturally to cover all externalities that the user receives from joining the platform, i.e.,

$$p^j := t^j + \gamma n^j + \lambda \sum_{k \in \mathcal{J} \setminus \{j\}} \gamma^j n^k.$$

In the assumed environment, the equilibrium net fee is then

$$t^j = c + \frac{1}{1 - n^j} - 2\gamma n^j - \gamma\lambda(1 - n^j) + \gamma\lambda n^j. \tag{20}$$

We now state our result on the effect of interoperability in a duopoly that, under the *status quo*, features a dominant platform.

Proposition 6 (Interoperability may mitigate dominance)**.** *Assume $\gamma > 0$. Consider any two levels of interoperability $\underline{\lambda} < \overline{\lambda}$. For any duopoly equilibrium under $\overline{\lambda}$ in which the dominant platform has market share $\overline{n}^1 > 1/2$, when $\lambda = \underline{\lambda}$, there is an equilibrium with $\underline{n}^1 > \overline{n}^1$.*

The logic behind this result is that an increase in interoperability, λ, changes the equilibrium in a way similar to a decrease in the network externality, γ. To see this, rewrite the right-hand side of the net fee in eq. (20) as

$$(c - \gamma\lambda) + \frac{1}{1 - n^j} - 2\gamma(1 - \lambda)n^j.$$

A platform market with marginal cost c, network externality γ, and interoperability λ is equivalent to a market with marginal cost $\tilde{c} := c - \gamma\lambda$, network externality $\tilde{\gamma} := \gamma(1 - \lambda)$, and no interoperability. Hence, increasing λ is equivalent to decreasing γ, leading to equilibrium market shares that are closer to one another.

In this current context, we can revisit Figure 1, whose blue line plots the largest market share of any platform under duopoly. As network externality γ increases, the larger platform grows even larger. At a given level of externality γ, increasing interoperability λ lowers the effective externality $\tilde{\gamma}$, which brings the two market shares closer to each other. Indeed, once λ goes beyond a certain threshold, there no longer exists any asymmetric equilibrium. Figure 2 illustrates this, plotting the market share of the largest platform under duopoly under different levels of interoperability λ, holding fixed $\gamma = 3.375$.

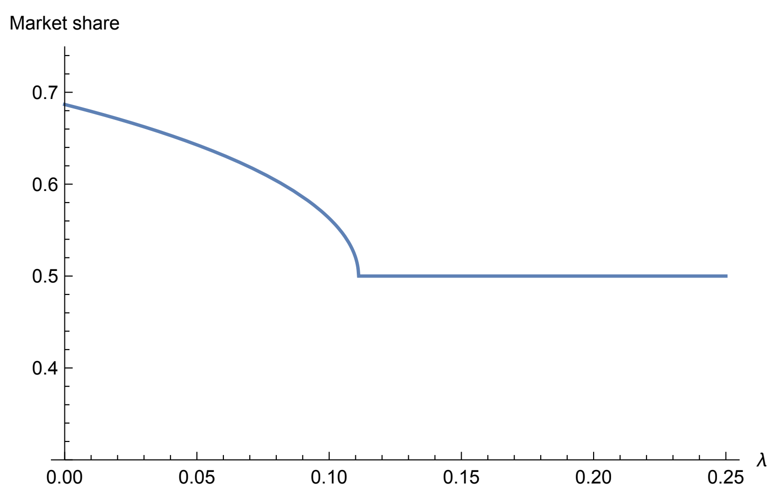

Figure 2: Largest possible duopoly equilibrium market share, as a function of interoperability

The effect that Proposition 6 highlights is closely related to what Crémer, Rey and Tirole (2000) refer to as the (diminishing) "quality differentiation effect" that arises from increased interoperability. A key differentiating feature of our model, however, is that market asymmetries may arise endogenously. This endogenous asymmetry is allowed by the fact that our approach tractably accommodates strong network effects, which gives rise to straightforward analysis of cases where the market tips towards one of two *ex ante* symmetric platforms. In contrast, that paper bounds network effects in order to rule out tipping and assumes that one platform exogenously has a larger installed base than its rival.[24]

6.2 Interoperability in a Broader Environment Under Symmetry

We continue to focus on the case of one-sided platforms and now allow for J competitors with general demand and an outside option. The question we now address is what the effect is on net fees of an increase in λ. If net fees were exogenously fixed, it is clear that, as long as network effects are positive, then users would benefit from an increase in λ. This is because such a change would simply allow users to enjoy network effects from other platforms as well as the one they choose to join. However, net fees are endogenous. Moreover, as we noted when discussing the intuition behind Proposition 4, large, fully proprietary network effects tend to push net fees down. Thus, *a priori*, from users' perspective, an increase in interoperability brings about a potential tradeoff between expanding the reach of network effects across platforms and discouraging them from competing intensely.

Proposition 7 gives the expression for equilibrium net fees and provides a result on this tradeoff. It makes use of notation $\varphi^j(t) := \frac{\sum_{k \in \mathcal{J} \setminus \{j\}} \frac{\partial n^k(t)}{\partial t^j}}{-\frac{\partial n^j(t)}{\partial t^j}} \in [0, 1]$, denoting platform j's *diversion ratio*. This captures the share of new users that platform j would attract from other platforms, rather than from the outside option, if it were to decrease its net fee by a small amount.

[24] A related point is made by Malueg and Schwartz (2006), which notes that, in a setting with one dominant platform and multiple smaller platforms, the smaller ones may mutually choose to become interoperable with one another, even if the dominant platform does not participate in such an arrangement.

Proposition 7. *(a) When $\lambda \in [0,1]$, at symmetric equilibrium, platforms' net fee satisfies*

$$t = c + \frac{n^j}{-\frac{\partial n^j}{\partial t^j}} - \left(2 + \lambda(J - 1 - \varphi^j)\right)\gamma n^j. \tag{21}$$

(b) Stronger interoperability gives rise to a lower net fee, except under no-outside-option duopoly, where it has no effect. Formally, consider any two levels of interoperability, $\underline{\lambda}$ and $\overline{\lambda}$, such that $\underline{\lambda} < \overline{\lambda}$. Under a duopoly with outside option or when $J \geq 3$, let $\underline{t}$ denote a net fee that arises at symmetric equilibrium when $\lambda = \underline{\lambda}$. When $\lambda = \overline{\lambda}$, there is a symmetric equilibrium with $\bar{t} < \underline{t}$. Under duopoly with no outside option, the same statement is true except that $\bar{t} = \underline{t}$.

Regarding the net fee in part (a), compared to the no-interoperability case, the factor $\lambda(J - 1 - \varphi^j)$ is new, and it reflects the following tradeoff. When adding a small mass of additional users, platform j extracts the "off network" interaction utility that they will derive. Letting $\widehat{N} := \sum_{j \in \mathcal{J}} n^j$, the new users enjoy a per-interaction benefit of $\lambda\gamma$ with each of the $\frac{J-1}{J}\widehat{N}$ users that join other platforms. This is included in the platform's marginal gain from adding an additional user, but it must be excluded when calculating the platform's net fee. On the other hand, when platform j adds this small mass of new users, a fraction of these, measured by φ^j, switch to j from other platforms, rather than from the outside option. This flow of φ^j users from other platforms to j eats away at the revenue from "off network" interaction that j can extract from its existing n^j users, at a rate of γ per interaction.

Part (b) regards the relative magnitudes of these two effects. It says that, following a shift from some $\underline{\lambda}$ to a greater $\overline{\lambda}$, the former effect dominates, in terms of its effect on the equilibrium net fee, except in the case of duopoly with no outside option, when these two effects balance each other out.

Consider, first, the special case of duopoly with no outside option. Here, there are two key driving features. First, due to the lack of an outside option, platform j's diversion ratio is always equal to one (as in Section 6.1), since any new user must come from the other platform. Second, in a symmetric duopoly with no outside option, the mass of users on the other platform, which influences the size of the former effect, is the same as the mass of users on platform j, which influences the size of the latter effect.

In order to depart from this special case, it suffices to consider duopoly with an outside option, so that $\varphi < 1$. Now, when platform j adds a small mass of users, each of these interact "off-network" with all users on the other platform, measured by $\widehat{N}/2$. However, of these new users arriving at platform j, only $\varphi^j\widehat{N}/2$ arrived from the competing platform.

Overall, Proposition 7 paints an optimistic picture regarding the effects on users of interoperability in symmetric settings. Further note that, in this setting, as net fee t decreases with an increase in interoperability λ, it is straightforward to show that consumer surplus increases. Under positive network effects, one would typically expect such a change to also lead to an increase in total surplus, although we do not know this to be generally true, given the need to compare the magnitude of the last two terms in Equation (21) at different demand levels.[25]

It is interesting to briefly compare our findings here to the ones that arise when platforms compete in total prices. Under that conduct assumption, the most tractable case in which to analyze the effect of interoperability is one with two platforms and Hotelling demand.[26] Let τ denote the standard transportation cost parameter, assume full market coverage, and note that this is a special case of duopoly with no outside option. As Proposition 7 implies, under net fee competition, equilibrium t does not depend on λ, and it is given by $t = c + \tau - \gamma$. By contrast, under competition in total prices, the equilibrium total price, $\hat{p}$, which we derive in Appendix E.2, satisfies $\hat{p} = c + \tau - (1-\lambda)\gamma$, which implies a net fee, $\hat{t}$, of $\hat{t} = c + \tau - \frac{1}{2}(3-\lambda)\gamma$, which is strictly increasing in λ. Therefore, at least in some settings, total price competition paints a more negative picture of the effects of interoperability.

The intuition for this difference is as follows. Under total price competition, when network effects are proprietary, platforms have a particularly strong incentive to intensively compete with one another to gain users. As interoperability increases and users become more of a common resource across platforms, this incentive decreases quickly. Under net fee competition, adding interoperability does less to undermine the level of competition that prevails in the baseline setting with $\lambda = 0$.

[25] See Tan and Wright (2021) for analysis of this general issue.

[26] This exercise adds a λ parameter to a one-sided version of Armstrong's (2006) single-homing model.

7 Competitive Bottlenecks

Our model, as described most generally in Section 2, assumes that all users join at most one platform. This section briefly considers "multi-homing." In particular, we modify our assumptions on users' joining patterns to match the canonical two-sided "competitive bottlenecks" configuration with indirect network effects studied by Anderson and Coate (2005), Armstrong (2006), Armstrong and Wright (2007), Anderson and Peitz (2020), among others.

Assume there are two platforms, 1 and 2, and an outside option, and there are two sides of the market, A and B. For notational convenience, suppose the platforms are symmetric to one another, but this is not essential to what follows. Let γ_A denote side-A users' interaction value with side-B users and γ_B denote side-B users' interaction value with side-A users. There are no within-side externalities.

Side A is the multi-homing side. Each side-A user makes two separate joining decisions: join platform 1 if and only if $\theta_A^1 - t_A^1 = \theta_A^1 + \gamma_A n_B^1 - p_A^1 \geq \theta_A^0$, and the analogous choice regarding platform 2. Note that, under this configuration, platform j's demand on side A depends on its own net fee, t_A^j, but not on t_A^k, $k \neq j$. Denote this side-A demand for platform j by $\widetilde{n}_A^j(t_A^J)$. Side B is the single-homing side. As in the main model, users on this side join only their most preferred platform, if any. These users choose the maximal option from the set $\{\theta_B^0, \theta_B^1 - t_B^1, \theta_B^2 - t_B^2\}$, so each platform's demand continues to depend on the vector t_B.[27] Proposition 8 states equilibrium pricing in the net fee model.

Proposition 8. *In the competitive bottlenecks model, platform $j = 1, 2$ charges net fees given by*

$$t_A^j = c_A + \frac{\widetilde{n}_A^j(t_A^j)}{-\frac{\partial \widetilde{n}_A^j(t_A^j)}{\partial t_A^j}} - (\gamma_A + \gamma_B) n_B^j(t_B), \tag{22}$$

$$t_B^j = c_B + \frac{n_B^j(t_B)}{-\frac{\partial n_B^j(t_B)}{\partial t_B^j}} - (\gamma_A + \gamma_B) \widetilde{n}_A^j(t_A^j). \tag{23}$$

[27]For concreteness, consider the example in which Side A is comprised of app developers who choose independently whether to participate on each of two competing software platforms, and side B features end-users who stick to one platform or the other.

The following two points stand out about this result. First, note that the side-A net fee in Equation (22) features a monopoly markup term, whereas the side-B net fee in Equation (23) features the same differentiated Bertrand markup term as appears in Proposition 1. Thus, this proposition captures, in a transparent way, the pattern often noted in competitive bottlenecks literature whereby platforms compete more intensively on the single-homing side than they do on the multi-homing side.[28]

Second, in the competitive bottlenecks setting, net fee conduct is, in fact, equivalent to the conduct assumed in Section 5 of Anderson and Coate (2005). In that section, on the "advertiser" side, the competing platforms each set the number of advertisers to serve, and on the "viewer" side, they each set a total price, referred to as a "subscription." To see why, in a competitive bottlenecks setting, this conduct is equivalent to net fee conduct, consider the incentives facing platform j, assuming that the choices of the competing platform, k, that are being held fixed are k's side-A demand, $\widetilde{n}_A^k$, and its side-B total price, p_B^k. First, since there is a one-to-one mapping between $\widetilde{n}_A^k$ and platform k's side-A net fee, t_A^k, the latter must also remain fixed. Second, since k's side-B total price satisfies $p_B^k = t_B^k + \gamma_B \widetilde{n}_A^k$, platform k's side B net fee, t_B^k must also remain fixed. Therefore, these two representations present platform j with the same set of feasible choices.

Beyond the competitive bottlenecks model, multi-homing is an important topic in platform economics. In particular, a number of works, such as Athey, Calvano and Gans (2018), Bakos and Halaburda (2020), and Teh et al. (2023) make significant contributions to our understanding of situations in which some of the same users may interact across different platforms, potentially multiple times and potentially with the ability to choose, in a fine-grained way, on which platform to meet. We leave the study of this issue in the context of net fees to future research.

[28] Here, however, this tendency is less stark than, for instance, Proposition 3 of Armstrong and Wright (2007), in which there is no smooth distribution of membership values for side-A users (side-S in their notation), which leads those users to be left with zero surplus.

8 Conclusion

In the era of big tech, understanding the way platform markets operate is of great importance to managerial decision-makers and policymakers. For managers, it is crucial to have a modeling approach that can help clarify the range of plausible market outcomes without relying on restrictive assumptions (e.g., on function forms, fixed market demand, firm symmetry). This paper delivers such a tool. For policymakers, an issue that is widely perceived to need further clarification is the relative merit of competition-based versus regulation-based interventions. This paper uncovers possible effects on market concentration of two different policy interventions, which could feature prominently in policy evaluation.

The distinguishing feature of our approach is that we assume platforms compete by setting *net fees*. Under this approach, as is often assumed, platforms compete by setting membership fees. However, in contrast to the standard approach in which membership fees represent platforms' only revenue stream, in net fee competition, platforms have an additional revenue stream. This additional revenue comes from the fees they charge users who interact with one another after joining the platform. Taking this approach brings about a great degree of analytical tractability to the study of platform competition. In particular, it opens the door to studying asymmetry among platforms and variable total demand, which are essential features for analyzing questions surrounding dominant platforms.

Using our modeling approach we address a set of policy questions related to platform dominance that attract significant debate. We show that increasing competition may have the unintended consequences of tipping the market towards a dominant platform. We also show that, in the context of mergers, strong network effects can act as a substitute for cost synergies. Moreover, we study the effects of interoperability regulation. There, we show that interoperability tends to mitigate market dominance and lower prices. Within the policy analysis, our focus has been on identifying the key mechanisms driving these results. However, we believe that the net fee approach can be useful in addressing further theoretical and empirical questions in a wide range of platform settings.

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A Microfoundation

Here we provide an example of a microfoundation underpinning the net fee model that we study throughout the main text. The key idea is that users first make the decision of which platform to join. Once they have joined any given platform, they make interaction decisions which generate utility that platforms can extract. Except as noted below, all notation remains unchanged. In the context of this microfoundation, we assume all interaction values to be strictly positive. The game has the following stages.

1. Each platform j sets its vector of membership fees, $t^j = (t_1^j, ..., t_S^j)$.

2. Each user chooses which platform to join or select the outside option. The mass of users that joins platform j on each side of the market is denoted by $n^j = (n_1^j, ..., n_S^j)$.

3. Platforms simultaneously set a vector of per-interaction fees facing users on each side of the market. Denote such a vector facing j's side s users by $w_s^j = (w_{s1}^j, ..., w_{sS}^j)$. $w_{s\hat{s}}^j$ is the amount of money a side-s user must pay to interact with each side-$\hat{s}$ user.

4. Each user who has joined platform j on side s chooses how many (mass of) users to interact with on each side of platform j. For a generic side-s user of platform j, denote this choice by $q_s^j = (q_{s1}^j, ..., q_{sS}^j)$, with each $q_{s\hat{s}}^j \in [0, n_{\hat{s}}^j]$.

A side-s user's payoff from joining platform j is given by

$$u_s^j := \theta_s^j + \sum_{\hat{s} \in \mathcal{S}} \left(\gamma_{s\hat{s}}^j - w_{s\hat{s}}^j\right) q_{s\hat{s}}^j - t_s^j,$$

or $u_s^0 := \theta_s^0$ from the outside option. A side-s user who joins platform j and chooses to interact with $q_{s\hat{s}}^j$ users on side $\hat{s}$ generates profits for the platform of

$$t_s^j - c_s^j + \sum_{\hat{s} \in \mathcal{S}} w_{s\hat{s}}^j q_{s\hat{s}}^j.$$

Consider stages 3 and 4, taking as given arbitrary vectors of membership fees and joining decisions, where we asssume $n^j > 0, \forall j \in \mathcal{J}$. In the unique subgame perfect equilibrium of the game defined by these vectors, each platform j sets interaction fee $w_{s\hat{s}}^j = \gamma_{s\hat{s}}^j, \forall s, \hat{s} \in \mathcal{S}$, and each side-$s$ user who has joined platform j chooses interaction level $q_{s\hat{s}}^j = n_{\hat{s}}^j, \forall \hat{s} \in \mathcal{S}$.

To justify this claim, first consider why this a subgame perfect equilibrium. Given $w_{s\hat{s}}^j = \gamma_{s\hat{s}}^j, \forall s, \hat{s} \in \mathcal{S}$, each user is indifferent among all feasible interaction levels. Thus, our claim regarding users' choices in stage 4 can be supported. In stage 3, if some platform

j chose some $w^j_{s\hat{s}} < \gamma^j_{s\hat{s}}$, the same profile of user actions could still be supported in the continuation game, but this platform's profits would strictly decrease. If, on the other hand, some platform j chose some $w^j_{s\hat{s}} > \gamma^j_{s\hat{s}}$, then interaction level $q^j_{s\hat{s}} = n^j_{\hat{s}}$ could not be supported, as side s users on platform j would find it optimal to choose an interaction level of 0, which would yield strictly lower profits for platform j because $\gamma^j_{s\hat{s}} n^j_{\hat{s}} > 0$.

Similar reasoning can be used to argue that no other outcome can be part of a subgame perfect equilibrium in this game. To see this, observe that once the users have made their platform choices, they are locked in to the platform, and the platform can guarantee a profit that is arbitrarily close to $\gamma^j_{s\hat{s}} n^j_{\hat{s}}$ by charging $w^j_{s\hat{s}}$ arbitrarily close to $\gamma^j_{s\hat{s}}$ from a side s user's interaction with side $\hat{s}$ users. Moreover, any price $w^j_{s\hat{s}} > \gamma^j_{s\hat{s}}$ would lead to the consumer choosing no interaction with side $\hat{s}$ users, and resulting in no profit from these interactions. Therefore, the subgame perfect equilibrium we found is unique.

B Multiple Sides Isomorphism

This section provides a result showing that there is an isomorphism between the one-sided case ($S = 1$), which Sections 5 and 6 and Appendix D focus on, and a comparable multi-sided case where sides are symmetric.

In order to establish the result, we clarify the relevant primitives. A game with S sides and J platforms takes as primitives (i) the CDF $F_s(\cdot)$ of distribution of membership value θ_s on each side s, (ii) the marginal cost c^j_s for each platform j on each side s, and (iii) the interaction value $\gamma^j_{s\hat{s}}$ for each platform j for each ordered pair of interaction $(s, \hat{s})$.

Proposition 9. *In a market with one side, characterized by membership value distribution $F(\cdot)$, cost $\{c^j\}_{j\in\mathcal{J}}$, and externalities $\{\gamma^j\}_{j\in\mathcal{J}}$, which satisfy Assumptions A1 and A2, suppose that there exists an equilibrium $\{t^j\}_{j\in\mathcal{J}}$. Then in a market with S sides, characterized by membership value CDFs $\{F_s(\cdot)\}_{s\in\mathcal{S}}$, cost $\{c^j_s\}_{j\in\mathcal{J},s\in\mathcal{S}}$, and externalities $\{\gamma^j_{s\hat{s}}\}_{j\in\mathcal{J},s,\hat{s}\in\mathcal{S}}$, which satisfy Assumptions A1 and A2, there exists an equilibrium with*

$$t^j_s = t^j, \;\; n^j_s = n^j, \;\; \forall j \in \mathcal{J}, s \in \mathcal{S}, \tag{24}$$

if the primitives of the S-sided market are comparable *to the one-sided market as follows*

$$F_s(\cdot) = F(\cdot), \;\; \forall s \in \mathcal{S}, \tag{25}$$

$$c^j_s = c^j, \;\; \forall j \in \mathcal{J}, s \in \mathcal{S}, \tag{26}$$

$$\gamma_{ss}^j + \sum_{\hat{s}\neq s} \frac{\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j}{2} = \gamma^j, \ \ \forall j \in \mathcal{J}, s \in \mathcal{S}. \tag{27}$$

We note that this isomorphism result requires that sides are symmetric as defined by (25-27), but allows for (both ex ante and ex post) asymmetry across firms. An immediate implication is that, among symmetric firms, if there exists an asymmetric equilibrium in a one-sided market, there also exists an asymmetric equilibrium in a comparable multi-sided market. Therefore, analogous results to those presented in Sections 5 and 6 with $S = 1$ regarding competition, interoperability and dominance hold in a class of multi-sided markets with an arbitrary number of sides.

We point out that, while the isomorphism implies that for a given firm j, its net fees $t_s^j = t^j$ and market shares $n_s^j = n^j$ are the same across all sides s, its total prices p_s^j can vary across sides as $p_s^j = t^j + \sum_{\hat{s}} \gamma_{s\hat{s}}^j n^j$ in which the added term is not symmetric. Once again, looking at the net fees instead of the total prices can be more informative in this analysis.

Last, we observe that (27) simplifies to, in the case of two-sided market ($S = 2$),

$$\gamma_{11}^j = \gamma_{22}^j = \gamma^j - \frac{\gamma_{12}^j + \gamma_{21}^j}{2}, \ \ \forall j \in \mathcal{J}. \tag{28}$$

Assume the demand and marginal cost are symmetric across two sides ($F_1(\cdot) = F_2(\cdot), c_1^j = c_2^j, \forall j$). Such a two-sided market features a set of equilibria that exists in a comparable one-sided market, as long as the own-side externalities ($\gamma_{11}^j, \gamma_{22}^j$) are the same across two sides, even when the cross-side externalities ($\gamma_{12}^j, \gamma_{21}^j$), sometimes called "indirect network effects," differ from one another.

C Competition and Interoperability in Multi-sided Markets under General Demand

C.1 Competition and Dominance

Now we give conditions under which there exists an asymmetric equilibrium with one firm larger than the others, among J number of ex ante symmetric firms under general demand functional form.

For side s, denote the CDF of $\theta_s^1 - \max\{\theta_s^2 - \tau, \theta_s^3, \ldots, \theta_s^J\}$ as $H_s(\cdot; \tau, J)$ and its PDF as $h_s(\cdot; \tau, J)$. We denote the inverse of $H_s(\cdot; 0, J)$ as $H_s^{-1}(\cdot; J)$, i.e., $H_s(H_s^{-1}(n; J); 0, J) = n, \forall n \in (0, 1)$.

Proposition 10. *Under Assumption A1, among J ex ante symmetric firms in a market with S sides and no outside option, for any $n^* = (n_s^*)_{s\in\mathcal{S}} \in (0, \frac{J-1}{J})^S$, there exists an equilibrium with a dominant firm which occupies a market share of at least $1 - n_s^*$ on each side s and other firms equally splitting the rest of the market, if $(\gamma_{s\hat{s}})_{s,\hat{s}\in\mathcal{S}}$ are large enough so that the following inequalities hold,*

$$\gamma_{s\hat{s}} + \gamma_{\hat{s}s} \geq 0, \quad \forall s, \hat{s} \in \mathcal{S}, s \neq \hat{s}, \tag{29}$$

$$\sum_{\hat{s}\in\mathcal{S}} \left(1 - \frac{J}{J-1} n_s^*\right)(\gamma_{s\hat{s}} + \gamma_{\hat{s}s}) \geq H_s^{-1}(n_s; J) + \frac{1 - n_s^*}{h_s(H_s^{-1}(n_s^*); 0, J)} - \frac{n_s^*}{(J-1)h_s(0; -H_s^{-1}(n_s^*; J), J)}, \quad \forall s \in \mathcal{S}. \tag{30}$$

This result illustrates how the mechanism in Proposition 4 carries through to an environment with general demand. Equilibrium can feature one platform with an arbitrary level of dominance, so long as the network externality is strong enough.

C.2 Interoperability

Interoperability in a General Environment. In a S-sided market, the utility of a side-s user who joins platform j is,

$$u_s^j := \theta^j + \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j \left(n_{\hat{s}}^j + \lambda_{s\hat{s}} \sum_{k\in\mathcal{J}\setminus\{j\}} n_{\hat{s}}^k \right) - p_s^j, \tag{31}$$

which generalizes eq. (19) to allow for a set of interoperability parameters $\{\lambda_{s\hat{s}}\}$. The net fee accommodates all externalities the user receives, i.e.,

$$p_s^j := t_s^j + \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j \left(n_{\hat{s}}^j + \lambda_{s\hat{s}} \sum_{k\in\mathcal{J}\setminus\{j\}} n_{\hat{s}}^k \right).$$

We extend the pricing formula of Proposition 1 to show the equilibrium net fees under interoperability. It makes use of notation $\varphi_s^j(t) := \frac{\sum_{k\in\mathcal{J}\setminus\{j\}} \frac{\partial n_s^k(t)}{\partial t_s^j}}{-\frac{\partial n_s^j(t)}{\partial t_s^j}} \in [0,1]$, denoting platform j's *diversion ratio* on side s, under general demand form. This captures the share of new users that platform j would attract from other platforms, rather than from the outside option, if it were to decrease its net fee by a small amount.

Under Logit demand, the side-s diversion ratio φ^j equals

$$\varphi_s^j = \frac{\sum_{k\in\mathcal{J}\setminus\{j\}} n_s^k}{1-n_s^j} = \frac{\sum_{k\in\mathcal{J}\setminus\{j\}} \mathrm{e}^{-t_s^k}}{\mathrm{e}^z + \sum_{k\in\mathcal{J}\setminus\{j\}} \mathrm{e}^{-t_s^k}}, \tag{32}$$

which is independent of t_s^j.

With interoperability, under general demand there exists a pure-strategy Nash equilibrium under Assumptions A1 and A2, as long as $\frac{\partial \varphi_s^j}{\partial t_s^j} \leq 0$ holds globally, extending our baseline existence result (Proposition 2). Under Logit demand, this condition $\frac{\partial \varphi_s^j}{\partial t_s^j} \leq 0$ is trivially met as φ_s^j is independent of t_s^j, and we can dispense with Assumption A2 as before.

Proposition 11 (Pricing formula under interoperability)**.** *At any equilibrium, the net fee that platform j charges to users on side s satisfies*

$$t_s^j = c_s^j + \frac{n_s^j}{-\frac{\partial n_s^j}{\partial t_s^j}} - \sum_{\hat{s}\in\mathcal{S}} \left[\gamma_{\hat{s}s}^j(1-\lambda_{\hat{s}s}\varphi_{\hat{s}}^j) + \gamma_{s\hat{s}}^j\left(1+\lambda_{s\hat{s}}\frac{\sum_{k\in\mathcal{J}\setminus\{j\}} n_{\hat{s}}^k}{n_{\hat{s}}^j}\right)\right] n_{\hat{s}}^j. \tag{33}$$

The equilibrium net fee (20) in Section 6 under Logit demand in a one-side market with no outside option is a special case of (33), with $-\frac{\partial n_s^j}{\partial t_s^j} = n_s^j(1-n_s^j)$ and $\varphi^j = 1$.

Interoperability and Dominance.

Proposition 12. *Assume no outside option. Consider any two levels of interoperability $\underline{\lambda}, \overline{\lambda}$ such that $\gamma_{s\hat{s}}\underline{\lambda}_{s\hat{s}} + \gamma_{\hat{s}s}\underline{\lambda}_{\hat{s}s} \in [0, \gamma_{s\hat{s}}\overline{\lambda}_{s\hat{s}} + \gamma_{\hat{s}s}\overline{\lambda}_{\hat{s}s}), \forall s, \hat{s} \in \mathcal{S}$. For any equilibrium under $\overline{\lambda}$ among J ex ante symmetric firms in a market with S sides in which a dominant firm (referred to as firm 1) has market share $\overline{n}^1 = (\overline{n}_s^1)_{s\in\mathcal{S}} \in (\frac{1}{J}, 1)^S$ and the other firms equally split the rest of the market, when $\lambda = \underline{\lambda}$, there is an equilibrium with $\underline{n}^1 = (\underline{n}_s^1)_{s\in\mathcal{S}}$ such that $\underline{n}_s^1 > \overline{n}_s^1, \forall s \in \mathcal{S}$.*

This proposition extends to a general environment the insight from Proposition 6 that interoperability may mitigate dominance.

D Symmetric Competition and Market Contraction

Here we assume that platforms are *ex ante* identical. Thus, without loss of generality, we normalize platforms' common marginal cost c to zero, since the sum of cost and outside option, $c + z$, is what matters for equilibrium market shares. Proposition 13 shows that, under strong network effects, increasing the number of competing platforms can lead to *market contraction*, i.e., lower total participation by users.

Proposition 13 (Market contraction)**.** *Assume* $\gamma \in (2.71, 3.375]$. *There exists an interval of outside option* z *such that total demand is lower at the symmetric equilibrium of the duopoly model than it is under monopoly.*

This proposition resembles and builds on the "perverse pattern" pricing result of Tan and Zhou (2021). That model assumes fixed total demand, and that result focuses on possible price increases as the number of competitors goes up. On the one hand, this result mirrors that one, suggesting that the current paper's net fee framework gives rise to results that are qualitatively similar to those of the more standard total pricing conduct. At the same time, this result extends the Tan and Zhou (2021) perverse pattern result, in that it incorporates, into the perverse pattern, rationing of consumption by some users, an aspect which is precluded when one assumes fixed total demand. While we present the market contraction result in the special case of monopoly to duopoly under Logit demand, the mechanism applies more broadly, if one analyzes an increase from J platforms to J' platforms under general demand form.

E Proofs

E.1 Proofs of Equilibrium Existence

We first establish that the pricing formula in Proposition 1 is the best response of each platform, by showing that each platform's profit is quasiconcave in t^j. Then we prove that there exists a fixed point to the set of pricing formulas. Thus there exists an equilibrium. Towards the end, we briefly discuss how the existence condition extends to the case with interoperability.

E.1.1 General Demand

Lemma 1. *Under Assumption A1, given any* t_{-j}, *we have* $-\frac{\partial n_s^j}{\partial t_s^j} \in \left(0, J \cdot \overline{g}_s\right)$, *for any* $s \in S$.

Proof of Lemma 1. The demand n_s^j is the mass under probability measure $f_s(\theta_s)$ of set

$$\begin{aligned} A &= \{\theta_s | \theta_s^j - t_s^j \geq \max\{\theta_s^0, \max_{k \neq j} \theta_s^k - t_s^k\}\} \\ &= \cap_{k \neq j}\{\theta_s | \theta_s^j - t_s^j \geq \theta_s^k - t_s^k\} \cap \{\theta_s | \theta_s^j - t_s^j \geq \theta_s^0\}. \end{aligned}$$

We have $-\frac{\partial n_s^j}{\partial t_s^j} > 0$ since f_s has full support. The shrinkage of this set A resulting from a marginal increase in t_s^j satisfies

$$\frac{\partial A}{\partial t_s^j} \subset \bar{A} = \cup_{k \neq j}\{\theta_s | \theta_s^j - t_s^j = \theta_s^k - t_s^k\} \cup \{\theta_s | \theta_s^j - t_s^j = \theta_s^0\},$$

and thus the slope of demand satisfies

$$\begin{aligned}
-\frac{\partial n_s^j}{\partial t_s^j} &\leq \int_{\bar{A}} f_s(\theta_s) \mathrm{d}\theta_s \\
&= \sum_{k \neq j} \int f_{s,k|j}(\theta_s^j - t_s^j + t_s^k | \theta_s^j) f_{s,j}(\theta_s^j) \mathrm{d}\theta_s^j + \int f_{s,0|j}(\theta_s^j - t_s^j | \theta_s^j) f_{s,j}(\theta_s^j) \mathrm{d}\theta_s^j \\
&\leq \sum_{k \neq j} \int \overline{g}_s f_{s,j}(\theta_s^j) \mathrm{d}\theta_s^j + \int \overline{g}_s f_{s,j}(\theta_s^j) \mathrm{d}\theta_s^j \\
&\leq J \cdot \overline{g}_s.
\end{aligned}$$

□

Lemma 2. *Under Assumption A1, $\pi^j(n^j, t^{-j})$ is concave in n^j, given any $t^{-j} := \left(t^k\right)_{k \in \mathcal{J} \setminus \{j\}}$.*

Proof of Lemma 2. Suppress t^{-j} for brevity, acknowledging that we are holding t^{-j} fixed. First, we show the mapping $n^j(t^j)$ is globally univalent, and thus we can think of platform j's optimization problem as choosing n^j. Second, we show π^j is concave in n^j.

First, the Jacobian of $n^j(t^j)$ is a $S \times S$ diagonal matrix with negative diagonals $\frac{\partial n_s^j}{\partial t_s^j} < 0$ from Lemma 1. Thus the Jacobian is negative definite and thus globally univalent. (Gale and Nikaido, 1965).

Second, we have

$$\frac{\partial \pi^j}{\partial n_s^j} = \frac{\frac{\partial \pi^j}{\partial t_s^j}}{\frac{\partial n_s^j}{\partial t_s^j}} = \frac{n_s^j}{\frac{\partial n_s^j}{\partial t_s^j}} + \sum_{\hat{s}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right) n_{\hat{s}}^j + t_s^j - c_s^j,$$

and

$$\frac{\partial^2 \pi^j}{\partial n_s^j \partial n_{\hat{s}}^j} = \frac{\frac{\partial \frac{\partial \pi^j}{\partial n_s^j}}{\partial t_{\hat{s}}^j}}{\frac{\partial n_{\hat{s}}^j}{\partial t_{\hat{s}}^j}} = \left(2 - \frac{n_s^j \frac{\partial^2 n_s^j}{\partial (t_s^j)^2}}{\left(\frac{\partial n_s^j}{\partial t_s^j}\right)^2}\right) \frac{1}{\frac{\partial n_s^j}{\partial t_s^j}} \cdot 1_{s=\hat{s}} + \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right).$$

For the Hessian $\left(\frac{\partial^2 \pi^j}{\partial n_s^j \partial n_{\hat{s}}^j}\right)_{s,\hat{s}\in S}$ to be globally negative semi-definite, it suffices for it to be a diagonally dominant matrix with non-positive diagonals, i.e.

$$\frac{\partial^2 \pi^j}{\partial (n_s^j)^2} + \sum_{\hat{s}\neq s} \left| \frac{\partial^2 \pi^j}{\partial n_s^j \partial n_{\hat{s}}^j} \right| \leq 0, \ \forall s \in S. \tag{34}$$

The LHS of (34) equals,

$$\left(2 - \frac{n_s^j \frac{\partial^2 n_s^j}{\partial (t_s^j)^2}}{\left(\frac{\partial n_s^j}{\partial t_s^j}\right)^2}\right) \frac{1}{\frac{\partial n_s^j}{\partial t_s^j}} + 2\gamma_{ss}^j + \sum_{\hat{s}\neq s} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right|.$$

Thus the inequality (34) simplifies to

$$2\gamma_{ss}^j + \sum_{\hat{s}\neq s} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| \leq \left(2 - \frac{n_s^j \frac{\partial^2 n_s^j}{\partial (t_s^j)^2}}{\left(\frac{\partial n_s^j}{\partial t_s^j}\right)^2}\right) \frac{1}{-\frac{\partial n_s^j}{\partial t_s^j}}. \tag{35}$$

By Theorem 1 in the Appendix of Caplin and Nalebuff (1991), Assumption A1 implies $n_s^j(t_s^j)$ is $\left(\frac{\rho_s}{1+(J+1)\rho_s}\right)$-concave, further implying

$$-\frac{n_s^j \frac{\partial^2 n_s^j}{\partial (t_s^j)^2}}{\left(\frac{\partial n_s^j}{\partial t_s^j}\right)^2} \geq \frac{\rho_s}{1+(J+1)\rho_s} - 1,$$

and Lemma 1 shows

$$-\frac{\partial n_s^j}{\partial t_s^j} \in \left(0, J \cdot \overline{g}_s\right).$$

Thus the inequality (35) holds if

$$\left(\gamma_{ss}^j + \sum_{\hat{s}\neq s} \left|\frac{\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j}{2}\right|\right) \cdot \overline{g}_s \leq \frac{1}{2J}\left(1 + \frac{\rho_s}{1+(J+1)\rho_s}\right).$$

Under Logit demand, the inequality (35) takes the form of

$$2\gamma^j_{ss} + \sum_{\hat{s}\neq s} \left|\gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s}\right| \leq \frac{1}{n^j_s\left(1-n^j_s\right)^2}, \tag{36}$$

the RHS of which is minimized at $n^j_s = \frac{1}{3}$ to obtain 6.75. This is a weaker bound, as we make use of the specific demand functional form. □

Proof of Proposition 2. Lemma 2 implies that given any t^{-j}, $\pi^j(t^j, t^{-j})$ is maximized at at most one t^j under Assumption A1, though it could be monotonic in t^j_s for some $s \in S$. The full set of pure-strategy equilibria is thus the set of solutions to the system of pricing formulas for all J platforms. Now we claim that, with Assumption A2 in addition, there exists a solution. It suffices to show that it is without loss of generality to restrict best responses to $[L, U]^{JS}$, so that we can apply Brouwer's fixed point theorem.

We can write

$$\begin{aligned}\pi^j(t^j, t^{-j}) &= \sum_s \left(t^j_s + \sum_{\hat{s}} \gamma^j_{s\hat{s}} n^j_{\hat{s}} - c^j_s\right) n^j_s \\ &= (t^j_s - c^j_s) n^j_s + \sum_{\hat{s}} \left(\gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s}\right) n^j_s n^j_{\hat{s}} + g(t^j_{-s}, t^{-j}),\end{aligned}$$

in which $g(t^j_{-s}, t^{-j})$ is independent of t^j_s. Define

$$h(t^j_s, t^j_{-s}, t^{-j}) := \pi^j(t^j, t^{-j}) - g(t^j_{-s}, t^{-j}) = \left[t^j_s - c^j_s + \sum_{\hat{s}} \left(\gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s}\right) n^j_{\hat{s}}(t^j_{\hat{s}}, t^{-j}_{\hat{s}})\right] n^j_s(t^j_s, t^{-j}_s).$$

For the lower bound, let $L := \min_{j,s}\left(c^j_s - \sum_{\hat{s}}\left|\gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s}\right|\right) - 1$. It follows that $h(t^j_s, t^j_{-s}, t^{-j}) < 0$ as long as $t^j_s < L$. However, if the platform j sets $t^j_s = \check{t}^j_s := c^j_s + \sum_{\hat{s}}\left|\gamma^j_{s\hat{s}} + \gamma^j_{\hat{s}s}\right| + 1$, it ensures $h(t^j_s, t^j_{-s}, t^{-j}) > 0$. Formally, we have, for any t^j_{-s}, t^{-j}, if $t^j_s < L$, then

$$\pi^j(t^j_s, t^j_{-s}, t^{-j}) < \pi^j(\check{t}^j_s, t^j_{-s}, t^{-j}),$$

Thus j would never set $t^j_s < L$. This also implies that it is without loss of generality to restrict to t^j_s such that $h(t^j_s, t^j_{-s}, t^{-j}) > 0$.

For the upper bound, we notice that when $h(t^j_s, t^j_{-s}, t^{-j})$ is positive, since n^j_s is increasing in each competitor's side-s net fee, it is bounded between

$$h(t_s^j, t_{-s}^j, t^{-j}) \geq h^L(t_s^j, t_{-s}^j, t^{-j}) := \left[t_s^j - c_s^j + \sum_{\hat{s}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j \right) n_{\hat{s}}^j(t_{\hat{s}}^j, t_{\hat{s}}^{-j}) \right] n_s^j(t_s^j, L),$$

and

$$h(t_s^j, t_{-s}^j, t^{-j}) \leq h^\infty(t_s^j, t_{-s}^j, t^{-j}) := \left[t_s^j - c_s^j + \sum_{\hat{s}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j \right) n_{\hat{s}}^j(t_{\hat{s}}^j, t_{\hat{s}}^{-j}) \right] n_s^j(t_s^j, \infty).$$

It is guaranteed that at $t_s^j = \check{t}_s^j$, the lower bound $h^L(\check{t}_s^j, t_{-s}^j, t^{-j}) \geq \underline{h} := n_s^j(\check{t}_s^j, L) > 0$. Meanwhile, the upper bound at any t_s^j satisfies $h^\infty(t_s^j, t_{-s}^j, t^{-j}) \leq \overline{h}(t_s^j) := \left(t_s^j - c_s^j + \sum_{\hat{s}} \left| \gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j \right| \right) n_s^j(t_s^j, \infty)$. Under Assumption A2, $\lim_{t_s^j \to \infty} \overline{h}(t_s^j) = 0$, and hence there exists U such that, for any $t_s^j > U$, we have $\overline{h}(t_s^j) < \underline{h}$, implying $h(t_s^j, t_{-s}^j, t^{-j}) < h(\check{t}_s^j, t_{-s}^j, t^{-j})$. Consequently, we have, for any t_{-s}^j, t^{-j}, if $t_s^j > U$, then

$$\pi^j(t_s^j, t_{-s}^j, t^{-j}) < \pi^j(\check{t}_s^j, t_{-s}^j, t^{-j}).$$

Therefore, j would never set $t_s^j > U$, completing our proof. □

E.1.2 Logit Demand

Proof of Proposition 2′. The extension we accommodate here is that there is no outside option but multiple platforms. We have already shown that the first-order condition is the best-response function, when inequality (36) holds. Now we show that there is a fixed point to the system of best response functions, even when there is no outside option ($e^z = 0$).

The system of best response functions is

$$t_s^j = T_s^j(t) := c_s^j + \frac{1}{1 - n_s^j} - \sum_{\hat{s} \in \mathcal{S}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j \right) n_{\hat{s}}^j, \ \forall j \in \mathcal{J}, s \in \mathcal{S},$$

with

$$n_s^j(t) = \frac{e^{-t_s^j}}{e^z + \sum_{k \in \mathcal{J}} e^{-t_s^k}}.$$

Use Schaefer's fixed point theorem recited as follows. Assume that X is a Banach space and that $T : X \to X$ is a continuous compact mapping. Moreover assume that the set

$$\cup_{0\leq\mu\leq 1}\{x \in X : x = \mu T(x)\}$$

is bounded. Then T has a fixed point.

Our $T(t)$ is continuous. And in Euclidean space, a continuous mapping is a compact mapping. Thus it suffices to show that for our $T(t)$, $M := \cup_{0\leq\mu\leq 1}\{t \in \mathbb{R}^{JS} : t = \mu T(t)\}$ is bounded. Claim that there exists $L \leq 0, U \geq 0$ such that $M \subset [L, U]^{JS}$, which would imply our existence result.

For the lower bound, as

$$\begin{aligned} T_s^j(t) &= \frac{1}{1-n_s^j} - \sum_{\hat{s}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right) n_{\hat{s}}^j + c_s^j \\ &\geq -\sum_{\hat{s}} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| + c_s^j, \end{aligned}$$

letting $L := \min_{j,s}\{0, -\sum_{\hat{s}} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| + c_s^j\}$, we have $\mu T_s^j(t) \geq L, \forall \mu \in [0,1], t \in \mathbb{R}^{JS}$. Thus any t with $t_s^j < L$ will not be in M.

For the upper bound, we study a candidate t that satisfies $t = \mu T(t)$ for some $\mu \in [0,1]$, and we show that there exists a constant U such that $t_s^j \leq U, \forall j, s$. From $t = \mu T(t)$, we have

$$\begin{aligned} t_s^j &= \mu \frac{1}{1-n_s^j} + \mu \left[-\sum_{\hat{s}} \left(\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right) n_{\hat{s}}^j + c_s^j\right] \\ &\leq \frac{1}{1-n_s^j} + c_s^j + \sum_{\hat{s}} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| \\ &= \frac{n^j}{1-n_s^j} + 1 + c_s^j + \sum_{\hat{s}} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| \\ &= \frac{\mathrm{e}^{-t_s^j}}{\mathrm{e}^z + \sum_{l\neq j} \mathrm{e}^{-t_s^l}} + 1 + c_s^j + \sum_{\hat{s}} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right| \\ &= \frac{\mathrm{e}^{-t_s^j}}{\mathrm{e}^z + \sum_{l\neq j} \mathrm{e}^{-t_s^l}} + d_s^j, \end{aligned} \tag{37}$$

with $d_s^j := 1 + c_s^j + \sum_{\hat{s}} \left|\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j\right|$ as a constant. For any side s, pick two generic platforms indexed by j, k, there are three possible possibilities: 1. Two inequalities $t_s^j > d_s^j, t_s^k > d_s^k$ both hold; 2. only one of them holds; 3. neither holds. We claim that all three cases lead to some upper bound for t.

In the first case where $t_s^j > d_s^j, t_s^k > d_s^k$, from inequality (37) we have

$$\begin{aligned} t_s^j &\leq \mathrm{e}^{-t_s^j}\mathrm{e}^{t_s^k} + d_s^j, \\ t_s^k &\leq \mathrm{e}^{-t_s^k}\mathrm{e}^{t_s^j} + d_s^k. \end{aligned} \tag{38}$$

Move the d terms to the LHS and multiply two inequalities. We get

$$(t_s^j - d_s^j)(t_s^k - d_s^k) \leq 1,$$
$$t_s^k \leq \frac{1}{t_s^j - d_s^j} + d_s^k.$$

Plugging this back into inequality (38) gives a new inequality solely dependent on t_s^j, d_s^j, d_s^k,

$$t_s^j \leq \mathrm{e}^{-t_s^j} \exp\left(\frac{1}{t_s^j - d_s^j} + d_s^k\right) + d_s^j.$$

The LHS increases to ∞ and the RHS decreases to d_s^j as $t_s^j \to \infty$. Thus there exists a threshold $u_{j,k,s}^1$ such that $t_s^j \leq u_{j,k,s}^1$. We set $U^1 = \max_{j,k,s} u_{j,k,s}^1$.

In the second case, we let $t_s^j > d_s^j$ but $t^k \leq d_s^k$. Then from inequality (37) we have

$$\begin{aligned} t_s^j &\leq \mathrm{e}^{-t_s^j}\mathrm{e}^{t_s^k} + d_s^j \\ &\leq \mathrm{e}^{-t_s^j}\mathrm{e}^{d_s^k} + d_s^j. \end{aligned}$$

Once again, we observe that the LHS is increasing to ∞ and the RHS is decreasing to d_s^j as $t_s^j \to \infty$, and hence there exists $u_{j,k,s}^2$ such that $t_s^j \leq u_{j,k,s}^2$. We set $U^2 = \max_{j,k,s} u_{j,k,s}^2$.

In the third case where $t_s^j \leq d_s^j, t_s^k \leq d_s^k$, we simply set $U^3 = \max_{j,s} d_s^j$.

Taking stock, we let $U := \max\{U^1, U^2, U^3\}$, which would guarantee that any candidate t that satisfies $t = \mu T(t)$ would have $t_s^j \leq U, \forall j, s$, completing our proof. □

E.1.3 Interoperability

Here we sketch how the existence condition extends to the case with interoperability, covered in Section 6 and Appendix C.2. We analyze the case with $S = 1$ under general demand; the case with an arbitrary S is similar. As before, first, each platform's best response is characterized by its first-order condition, under the additional assumption that $\frac{\partial \varphi^j}{\partial t^j} \leq 0$ holds globally. Second, there is a fixed point.

In one-sided markets ($S = 1$), we have $\pi^j = (t^j + \gamma^j n^j + \gamma^j \mu n^{-j} - c^j) n^j$, with $n^{-j} := \sum_{k \in \mathcal{J} \setminus \{j\}} n^k$.

The marginal profit is

$$\frac{\partial \pi^j}{\partial n^j} = \frac{\frac{\partial \pi^j}{\partial t^j}}{\frac{\partial n^j}{\partial t^j}} = \frac{n^j}{\frac{\partial n^j}{\partial t^j}} + 2\gamma^j n^j + \mu\gamma^j n^{-j} - \mu\gamma^j\varphi^j n^j + t^j - c^j,$$

and

$$\frac{\partial^2 \pi^j}{\partial (n^j)^2} = \frac{\frac{\partial \frac{\partial \pi^j}{\partial n^j}}{\partial t^j}}{\frac{\partial n^j}{\partial t^j}} = \left(2 - \frac{n^j \frac{\partial^2 n^j}{\partial (t^j)^2}}{\left(\frac{\partial n^j}{\partial t^j}\right)^2}\right)\frac{1}{\frac{\partial n^j}{\partial t^j}} - \mu\gamma^j n^j \frac{\frac{\partial \varphi^j}{\partial t^j}}{\frac{\partial n^j}{\partial t^j}} + 2\gamma^j(1-\mu\varphi^j).$$

It is non-positive, if $\frac{\partial \varphi^j}{\partial t^j} \leq 0$ holds globally, in addition to Assumptions A1 and A2. (See the proof of Lemma 2 for bounds on terms unrelated to μ.) Under Logit demand, the diversion ratio φ^j is independent of t^j as shown in eq. (32), and thus this condition is met.

In terms of a fixed point, the pricing formula under interoperability is, reciting eq. (33),

$$\begin{aligned} t^j &= c^j + \frac{n^j}{-\frac{\partial n^j}{\partial t^j}} - \left(2 + \mu\left(\frac{\sum_{k\in\mathcal{J}\setminus\{j\}} n^k}{n^j} - \varphi^j\right)\right)\gamma n^j, \\ &= c^j + \frac{n^j}{-\frac{\partial n^j}{\partial t^j}} - \left(2 - \mu(1+\varphi^j)\right)\gamma n^j - \mu\gamma(1-n^0). \end{aligned}$$

Compared to the no-interoperability case, γn^j is now multiplied by $2 - \mu(1+\varphi^j)$ instead of simply 2. However, since μ, φ^j are bounded between 0 and 1, it is straightforward to repeat the proofs of Propositions 2 and 2′ to establish a fixed point.

E.2 Proofs of Other Results in the Main Text

Proof of Proposition 1. It is obtained by setting (11) to zero and plugging in (3). □

Proof of Proposition 3. We use the aggregative games approach from Anderson, Erkal and Piccinin (2020) to establish uniqueness. We recap their assumptions; A1-A3 ensure existence and A4 ensures uniqueness. Each platform plays $a^j \geq 0$ and the aggregate is $A = \sum_j a^j$, including the outside option if any. And let $A^{-j} = A - a^j$. In the Logit specification, we have $a^j = \mathrm{e}^{-t^j}$. The *best response* (br) function is defined as $r^j(A^{-j})$. As $(r^j)' > -1$ and $A^{-j} + r^j(A^{-j})$ strictly increases in A^{-j} implied by A3 shown below, define the *inclusive best response* (ibr) as $\tilde{r}^j(A)$.

- A1 (competitiveness): $\pi^j(A^{-j} + a^j, a^j)$ strictly decreases in A^{-j} for $a^j > 0$.

- A2 (payoffs): (a) $\pi^j(A^{-j}+a^j, a^j)$ is twice differentiable, and strictly quasi-concave in a^j, with a strictly negative second derivative with respect to a^j at an interior maximum.

  (b) $\pi^j(A, a^j)$ is twice differentiable, and strictly quasi-concave in a^j, with a strictly negative second derivative with respect to a^j at an interior maximum.

- A3 (reaction function slope): $\frac{\mathrm{d}^2\pi^j}{\mathrm{d}(a^j)^2} < \frac{\mathrm{d}^2\pi^j}{\mathrm{d}a^j \mathrm{d}A^{-j}}$.

- A4 (slope condition): $(\tilde{r}^j)'(A) < \frac{\tilde{r}^j(A)}{A}$.

Using br $r^j(A^{-j})$ instead of ibr $\tilde{r}^j(A)$, A4 is equivalently expressed as[29]

- A4′ (slope condition): $(r^j)'(A^{-j}) < \frac{r^j(A^{-j})}{A^{-j}}$.

We now proceed to characterize the threshold on γ^j that satisfies A3 and A4′, so as to show equilibrium uniqueness.

Our FOC is

$$\begin{aligned} t^j - c^j &= \frac{1}{1-n^j} - 2\gamma n^j, \\ -\ln r^j(A^{-j}) - c^j &= \frac{r^j(A^{-j}) + A^{-j}}{A^{-j}} - 2\gamma^j \frac{r^j(A^{-j})}{r^j(A^{-j}) + A^{-j}}, \end{aligned}$$

which implies

$$(r^j)'(A^{-j}) = \frac{\left[\frac{1}{(A^{-j})^2} - \frac{2\gamma^j}{\left(r^j(A^{-j})+A^{-j}\right)^2}\right] r^j(A^{-j})}{\left[\frac{1}{(A^{-j})^2} - \frac{2\gamma^j}{\left(r^j(A^{-j})+A^{-j}\right)^2}\right] r^j(A^{-j}) + \frac{1}{A^{-j}}} \frac{r^j(A^{-j})}{A^{-j}}.$$

Simplify the denominator of the first term,

$$\begin{aligned} \left[\frac{1}{(A^{-j})^2} - \frac{2\gamma^j}{\left(r^j(A^{-j}) + A^{-j}\right)^2}\right] r^j(A^{-j}) + \frac{1}{A^{-j}} &= \frac{r^j}{\left(r^j + A^{-j}\right)^2}\left[\frac{(r^j + A^{-j})^3}{r^j(A^{-j})^2} - 2\gamma^j\right], \\ &= \frac{r^j}{\left(r^j + A^{-j}\right)^2}\left[\frac{1}{n^j(1-n^j)^2} - 2\gamma^j\right]. \end{aligned}$$

As $\frac{1}{n^j(1-n^j)^2}$ obtains its minimum of 6.75 when $n^j = 1/3$, $\gamma^j \leq 3.375$ ensures the denominator is positive. Since the numerator of the first term is smaller than the denominator, it is ensured the first term is smaller than 1 and thus A4′ holds.

[29]Rewrite $(\tilde{r}^j)'(A) = \frac{(r^j)'}{1+(r^j)'}$ and $\frac{\tilde{r}^j}{A} = \frac{r^j}{A^{-j}+r^j}$. We have $\frac{(r^j)'}{1+(r^j)'} < \frac{r^j}{A^{-j}+r^j}$ if and only if $(r^j)' < \frac{r^j}{A^{-j}}$. the equivalence of A4 and A4′ holds only when there is a well-defined ibr, which is true when $(r^j)'(A^{-j}) > -1$.

Our Logit game with generic γ^j admits an ibr if $(r^j)'(A^{-j}) > -1$, which is the Lemma 1 in Anderson, Erkal and Piccinin (2020) implied by their A3. When $\gamma^j \le 3.375$, the denominator is positive, and thus $(r^i)'(A^{-j}) \ge -1$ is simplified to

$$\left[\frac{1}{(A^{-j})^2} - \frac{2\gamma^j}{(r^j + A^{-j})^2}\right](r^j + A^{-j}) + \frac{1}{r^j} \ge 0$$
$$\frac{1}{n^j} + \frac{1}{(1-n^j)^2} \ge 2\gamma^j,$$

the LHS of which obtains its minimum of ≈ 5.219 when $n^j \approx 0.361$. Thus when $\gamma^j \lesssim 2.610, \forall j$, our Logit game admits an ibr, which combined with A4′ yields equilibrium uniqueness. □

Proof of Proposition 4. Here we study an asymmetric equilibrium in which there is one dominant platform, and $(J-1)$ symmetric smaller platforms. We use superscript 1 to denote the dominant platform and 2 for a generic smaller platform. We define $n := (J-1)n^2$ as the total demand of all smaller platforms, with $n^1 = 1-n, n^2 = n/(J-1)$.

The demand of the dominant platform relative to a smaller platform satisfies,

$$\frac{\mathrm{e}^{-t^1}}{\mathrm{e}^{-t^2}} = \frac{n^1}{n^2} = \frac{1-n}{\frac{n}{J-1}},$$

and the pricing formulas are,

$$t^1 = c + \frac{1}{1-n^1} - 2\gamma n^1 = c + \frac{1}{n} - 2\gamma(1-n)$$
$$t^2 = c + \frac{1}{1-n^2} - 2\gamma n^2 = c + \frac{1}{1-\frac{n}{J-1}} - 2\gamma\frac{n}{J-1}.$$

We can combine these three and arrive at a characterization function $g(n;J)$ whose zeros are equilibria,

$$g(n;J) = \ln n - \ln(J-1) - \ln(1-n) + \frac{1}{1-\frac{n}{J-1}} - 2\gamma\frac{n}{J-1} - \left(\frac{1}{n} - 2\gamma(1-n)\right).$$

We notice $\lim_{n\to 0} g(n;J) = -\infty$.

As before, suppose the duopoly has an equilibrium featuring a demand of the smaller platform n, which solves $g(n;2) = 0$. With 3 platforms, if $g(n;3) > 0$, then by the intermediate value theorem, there must exists $n' < n$ that satisfies $g(n';3) = 0$. As the dominant platform's market share is $1-n'$, that means, the dominant platform is more dominant under triopoly than under duopoly. Since $g(n;2) = 0$, an equivalent condition to $g(n;3) > 0$ is $g(n;3) -$

$g(n;2) > 0$, and we have

$$g(n;3) - g(n;2) = -\ln 2 + \frac{1}{1-n/2} - \frac{1}{1-n} + \gamma n.$$

This shows that, at a given n, the network externality γ has to be relatively strong for the difference to be larger than zero, very similar to the previous market contraction result.

However, as we restrict attention to *ex ante* identical platforms, the asymmetric equilibrium market outcome is also solely driven by γ. We rewrite $g(n;J) = 0$ as

$$\gamma = f(n, J) := \frac{1}{2} \frac{\ln \frac{n}{1-n} - \ln(J-1) + \frac{J-1}{J-1-n} - \frac{1}{n}}{\frac{n}{J-1} - (1-n)},$$

which takes on a U-shape in n when $J = 2, 3$. In the relevant parameter range $\gamma \in (2.71, 3.375]$ that we are interested in, we can verify that at any n that solves $f(n,2) = \gamma$, we have $f(n,3) < f(n,2)$, suggesting that there exists $n' < n$ that solves $f(n',3) = \gamma$. We conclude that there exists an equilibrium under triopoly in which a dominant platform's market share is greater than the market share of any platform in any duopoly equilibrium. □

Proof of Proposition 5. Let 1 denote one of the two symmetric firms with marginal cost c to be merged, and 2 denote the other firm with zero marginal cost. Before the merger, with a market share of $n/2$, firm 1's FOC is

$$t^1 = c + \frac{1}{1-\frac{n}{2}} - 2\gamma\frac{n}{2},$$

and firm 2's FOC is

$$t^2 = \frac{1}{n} - 2\gamma(1-n).$$

Their relative demand satisfies

$$\frac{e^{-t^1}}{e^{-t^2}} = \frac{n/2}{1-n}.$$

Combining these 3 equations to cancel t^1, t^2, we arrive at a characterization of the equilibrium n in terms of c, γ,

$$f(n) = c + \frac{1}{1-\frac{n}{2}} - \gamma n - \frac{1}{n} + 2\gamma(1-n) + \ln\frac{n}{2(1-n)} = 0. \tag{39}$$

Notice that $\lim_{n\to 0} f(n) = -\infty, f(\frac{2}{3}) = c > 0, \lim_{n\to 1} f(n) = \infty$, Further, when $\gamma < 2.62$ which we assume, $f(n)$ is increasing in n. Thus there is a unique solution $n \in \left(0, \frac{2}{3}\right)$, which is decreasing in c. When $n \in \left(0, \frac{2}{3}\right)$, $f(n)$ is also increasing in γ, suggesting n is decreasing in γ. As long as $c + \frac{\gamma}{2} > \frac{2}{3} + \ln 2 \approx 1.36$ so that $f\left(\frac{1}{2}\right) > 0$, it is guaranteed that $n < \frac{1}{2}$. That is, the most efficient firm (with zero marginal cost) has a market share that is larger than one half.

After the merger, if the merged identity has a marginal cost of $c' > 0$, its equilibrium market share n' is characterized by

$$g(n') = c' + \frac{1}{1-n'} - 2\gamma n' - \frac{1}{n'} + 2\gamma(1-n') + \ln\frac{n'}{1-n'} = 0.$$

Similarly, we observe that $g(0) = -\infty, g(\frac{1}{2}) = c' > 0, g(1) = \infty$. Further, when $\gamma \le 3$, $g(n')$ is increasing in n'. There is a unique solution $n' \in \left(0, \frac{1}{2}\right)$ that is decreasing in both c' and γ.

Suppose the pre-merger equilibrium features n, i.e. $f(n) = 0$. We have

$$g(n) = g(n) - f(n) = -\gamma n + \frac{n}{(1-n)(2-n)} + \ln 2 + c' - c.$$

The post-merger equilibrium entails $n' > n$ if and only if $g(n) < 0$, i.e.

$$\Delta c := c - c' > \frac{n}{(1-n)(2-n)} - \gamma n + \ln 2.$$

Here we see that, given n, a larger γ leads to a smaller threshold of Δc. □

Proof of Proposition 6. From eq. (20) we have

$$\begin{aligned} t^j &= c + \frac{1}{1-n^j} - 2\gamma n^j - \gamma\lambda(1-n^j) + \gamma\lambda n^j \\ &= (c - \gamma\lambda) + \frac{1}{1-n^j} - 2\gamma(1-\lambda)n^j. \end{aligned}$$

We use superscript 1 for the larger platform and 2 for the smaller one. The demand function gives that

$$\frac{e^{-t^1}}{e^{-t^2}} = \frac{n^1}{n^2}.$$

Combining these to cancel t^1, t^2 and plugging in $n^2 = 1 - n^1$, we get

$$\zeta(n^1;\lambda) := \ln\frac{n^1}{1-n^1} + \frac{1}{1-n^1} - \frac{1}{n^1} - 2\gamma(1-\lambda)(2n^1 - 1) = 0$$

Observe that $\lim_{n^1 \to 1} \zeta(n^1; \lambda) = \infty, \forall \lambda$. Consider two levels of interoperability $\underline{\lambda}, \overline{\lambda}$, and suppose the dominant platform has a market share $\overline{n}^1 > 1/2$ under $\overline{\lambda}$, i.e. $\zeta(\overline{n}^1; \overline{\lambda}) = 0$. At this $\overline{n}^1$ under a lower level of interoperability $\underline{\lambda}$, we have

$$\begin{aligned} \zeta(\overline{n}^1; \underline{\lambda}) &= \zeta(\overline{n}^1; \underline{\lambda}) - \zeta(\overline{n}^1; \overline{\lambda}) \\ &= 4(\underline{\lambda} - \overline{\lambda})\gamma \overline{n}^1 < 0. \end{aligned}$$

As $\lim_{n^1 \to 1} \zeta(n^1; \underline{\lambda}) = \infty$, by the intermediate value theorem, there exists $\underline{n}^1 > \overline{n}^1$ that solves $\zeta(\underline{n}^1; \underline{\lambda}) = 0$. That is, when the level of interoperability is lower, there exists an equilibrium in which the dominant platform has an even larger market share. □

Proof of Proposition 7. Part (a) follows from setting $S = 1$ and imposing symmetry among platforms in Equation (33), which is derived in the proof of Proposition 11.

For part (b), note that, in a symmetric equilibrium among symmetric platforms, we have $n^{-1} = (J-1)n^1$ and all platforms charge the same t^1. We define

$$\xi(t; \lambda) = -t + c + \frac{1}{1 - n^j} - 2\gamma n^j - \gamma \lambda (J - 1 - \varphi^j) n^j,$$

with all platforms charging the same t. Any solution to $\xi(t; \lambda) = 0$ is a symmetric equilibrium, and conversely any symmetric equilibrium would satisfy $\xi = 0$.

Suppose there is a symmetric equilibrium with interoperability $\underline{\lambda}$ featuring $t^j = \underline{t}$, and we are to find a new symmetric equilibrium with higher interoperability $\overline{\lambda}$ featuring $t^j = \overline{t}$. We can write

$$\xi(t; \overline{\lambda}) = \xi(t; \underline{\lambda}) - \gamma(\overline{\lambda} - \underline{\lambda})(J - 1 - \varphi^j) n^j,$$

the latter term of which enters negatively unless $J = 2$ and $\varphi^j = 1$, in which case $\xi(t; \overline{\lambda}) = \xi(t; \underline{\lambda}), \forall t$ and thus there exists a new equilibrium with $\overline{t} = \underline{t}$.

If, however, $J > 2$ or $\varphi^j < 1$, then given any $\overline{\lambda} > \underline{\lambda}$, we have $\xi(t; \overline{\lambda}) < \xi(t; \underline{\lambda}), \forall t$. In this case, $\xi(\underline{t}; \overline{\lambda}) < 0$, since $\xi(\underline{t}; \underline{\lambda}) = 0$. To show that there exists $\overline{t} < \underline{t}$ satisfying $\xi(\overline{t}; \overline{\lambda}) = 0$, it suffices to show that there exists t_- such that $\xi(t; \overline{\lambda}) > 0, \forall t < t_-$ and then the intermediate value theorem establishes the existence of such a $\overline{t} \in (t_-, \underline{t})$. We can choose any t_- such that

$$t_- < c - 2\gamma - \gamma \overline{\lambda}(J - 1),$$

which implies, $\forall t < t_-$,

$$\begin{aligned}\xi(t;\overline{\lambda}) &= -t + c + \frac{1}{1-n^j} - 2\gamma n^j - \gamma\overline{\lambda}(J-1-\varphi^j)n^j \\ &> \frac{1}{1-n^j} + 2\gamma(1-n^j) + \gamma\overline{\lambda}\varphi^j n^j \\ &> 0,\end{aligned}$$

completing our proof.

Since we are studying symmetric equilibria with a fixed number of platforms, total market participation $\widehat{N} = Jn^j$ is inversely related to t, unless there is no outside option, in which case $\widehat{N}$ is always equal to 1.

□

Derivation of $\hat{p}$ in Section 6.2. Assume $S = 1$, $\gamma > 0$, and demand takes on the standard Hotelling form, with two symmetric platforms competing in a fully "covered" market on the unit interval, with transport cost parameter τ. Competing in total prices, platform 1's demand is

$$\begin{aligned}n^1 &= \frac{1}{2} + \frac{\gamma(1-\lambda)(n^1-n^2) - (p^1-p^2)}{2\tau} \\ &= \frac{1}{2} - \frac{p^1-p^2}{2\tau - 2\gamma(1-\lambda)},\end{aligned}$$

and its profit is $\pi^1 = (p^1 - c)n^1$.

Solving for symmetric Nash equilibrium in total prices gives

$$\hat{p} = c + \tau - \gamma(1-\lambda),$$

which implies that, at the equilibrium in total prices, the net fee equivalent, $\hat{t}$, is

$$\begin{aligned}\hat{t} &= \hat{p} - \gamma(n^1 + \lambda n^2) \\ &= c + \tau + \frac{1}{2}(3-\lambda)\gamma.\end{aligned}$$

□

Proof of Proposition 8. To obtain this result, note that platform j's profits are equal to $(t_A^j + \gamma_A n_B^j(t_B^j, t_B^k) - c_A)\widetilde{n}_A^j(t_A^j) + (t_B^j + \gamma_B \widetilde{n}_A^j(t_A^j) - c_B)n_B^j(t_B^j, t_B^k)$, $k \neq j$, and maximize with respect to t_A^j and t_B^j. □

E.3 Proofs of Results in the Appendix

Proof of Proposition 9. When Assumptions A1 and A2 hold for any market, a solution to the FOCs is an equilibrium in the market and vice versa. Thus an equilibrium in a multi-sided market satisfies, based on Proposition 1,

$$t_s^j = c_s^j + \frac{n_s^j(t_s)}{-\frac{\partial n_s^j(t_s)}{\partial t_s^j}} - \sum_{\hat{s}\in\mathcal{S}} (\gamma_{s\hat{s}}^j + \gamma_{\hat{s}s}^j) n_{\hat{s}}^j(t_{\hat{s}}),$$

whereas the demand follows

$$n_s^j(t_s) = \int \mathbf{1}_{\{u_s^j \geq u_s^k, \forall k\in\mathcal{J}\cup\{0\}\}} f_s(\theta_s) \mathrm{d}\theta_s = \int \mathbf{1}_{\{\theta_s^j - t_s^j \geq \theta_s^k - t_s^k, \forall k\in\mathcal{J}\cup\{0\}\}} f_s(\theta_s) \mathrm{d}\theta_s.$$

Under (25-27), a side-symmetric equilibrium in the S-sided market is characterized by

$$t^j = c^j + \frac{n^j(t)}{-\frac{\partial n^j(t)}{\partial t^j}} - 2\gamma^j n^j(t),$$

and

$$n^j(t) = \int \mathbf{1}_{\{\theta^j - t^j \geq \theta^k - t^k, \forall k\in\mathcal{J}\cup\{0\}\}} f(\theta) \mathrm{d}\theta.$$

These are exactly the expressions of pricing formulas and demand function in a comparable one-sided market. Thus any equilibrium in the one-sided market is a solution to this system, which is in turn a side-symmetric equilibrium in the S-sided market. It is straight forward to see that the isomorphism result extends to the case with interoperability too. □

Proof of Proposition 10. We observe the following limits $H_s(\infty;\tau, J) = 1, \forall\tau$, $H_s(-\infty;\tau, J) = 0, \forall\tau$, and $H_s(x;-\infty, J) = 0, \forall x$, $H_s(x;\infty, J) = H_s(x;0, J-1) \in (0,1], \forall x$. The first two are properties of H_s as a CDF. The third follows from that if one competitor charges a price that is infinitely lower, then the demand of other firms is zero. The last follows from that one competitor charging an infinitely high price amounts to it dropping out of the market.

For the inverse H_s^{-1}, since $H_s(0;0, J) = \frac{J-1}{J}$ holds for a symmetric market share configuration, we have $H_s^{-1}(\frac{J-1}{J}; J) = 0$.

Now we characterize the equilibrium with J firms in which firm 1 has a unique market share whereas the other $(J-1)$ firms equally split the rest of the market. Firm 2 signifies a generic firm among the other firms.

$$t_s^1 = \frac{1 - H_s(\tau_s; 0, J)}{h_s(\tau_s; 0, J)} - \beta_{ss}(1 - H_s(\tau_s; 0, J)) - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}(1 - H_{\hat{s}}(\tau_{\hat{s}}; 0, J)),$$

$$\begin{aligned} t_s^2 &= \frac{1 - H_s(0; -\tau_s, J)}{h_s(0; -\tau_s, J)} - \beta_{ss}(1 - H_s(0; -\tau_s, J)) - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}(1 - H_{\hat{s}}(0; -\tau_{\hat{s}}, J)) \\ &= \frac{H_s(\tau; 0, J)}{(J-1)h_s(0; -\tau_s, J)} - \beta_{ss}\frac{H_s(\tau; 0, J)}{J-1} - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}\frac{H_{\hat{s}}(\tau; 0, J)}{J-1}, \end{aligned}$$

where $\beta_{s\hat{s}} = \gamma_{s\hat{s}} + \gamma_{\hat{s}s}$ and we use $1 - H_s(\tau; 0, J) + (J-1)(1 - H_s(0; -\tau, J)) = 1$ in the last simplification. We use the inverse function H_s^{-1} and $t_s^1 - t_s^2 = -\tau_s$ to cancel t_s^1, t_s^2, τ_s,

$$\begin{aligned} g_s(n_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; J) =\ & H_s^{-1}(n_s; J) + \frac{1 - n_s}{h_s(H_s^{-1}(n_s; J); 0, J)} - \beta_{ss}(1 - n_s) - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}(1 - n_{\hat{s}}) \\ & - \left(\frac{n_s}{(J-1)h_s(0; -H_s^{-1}(n_s; J), J)} - \beta_{ss}\frac{n_s}{J-1} - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}\frac{n_{\hat{s}}}{J-1} \right) \\ =\ & -\sum_{\hat{s}} \left(1 - \frac{J}{J-1} n_{\hat{s}}\right) \beta_{s\hat{s}} + H_s^{-1}(n_s; J) + \frac{1 - n_s}{h_s(H_s^{-1}(n_s; J); 0, J)} - \frac{n_s}{(J-1)h_s(0; -H_s^{-1}(n_s; J), J)}. \end{aligned}$$

A solution $\{n_s\}$ to $\{g_s(n; J) = 0\}$ is an equilibrium in which one firm has a market share of $1 - n_s$ while the others equally split n_s. We note that $n_s = \frac{J-1}{J}$ is always a solution, characterizing the symmetric equilibrium.

We make two crucial observations on g_s:

(i) $\lim_{n_s \to 0^+} g_s(n_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; J) = \infty$. This is because $\lim_{n_s \to 0^+} H_s^{-1}(n_s; J) = \infty$ (since the membership value distribution has full support), $\frac{1-n_s}{h_s(H_s^{-1}(n_s;J);0,J)}$ is always positive, β-related terms are all finite, and $\lim_{n_s \to 0^+} \frac{n_s}{(J-1)h_s(0;-H_s^{-1}(n_s;J),J)} = \lim_{n_s \to 0^+} \frac{1-H_s(0;-H_s^{-1}(n_s;J))}{h_s(0;-H_s^{-1}(n_s;J),J)} = \frac{1-H_s(0;0,J-1)}{h_s(0;0,J-1)}$ is finite too.

(ii) $g_s(n_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; J)$ increases in $n_{\hat{s}}$ if $\beta_{s\hat{s}} > 0$. As a result, if $g_s(n_s^*, \{n_{\hat{s}}^*\}; J) < 0$, we have $g_s(n_s^*, \{n_{\hat{s}}\}_{\hat{s} \neq s}; J) < 0$ if $n_{\hat{s}} < n_{\hat{s}}^*$ for all $\hat{s} \neq s$. By inspecting the formula, it is evident that the same conclusion holds if $\beta_{s\hat{s}} = 0$.

Thus, under inequalities (29, 30), for any side s, we have that $\lim_{n_s \to 0^+} g_s(n_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; J) = \infty$, $g_s(n_s^*, \{n_{\hat{s}}\}; J) < 0$ if $n_{\hat{s}} < n_{\hat{s}}^*$ for all $\hat{s} \neq s$. The Poincaré–Miranda theorem, which is a generalization of the intermediate value theorem from the unidimensional case to multidimensional case, implies that there exists a solution n to $g = 0$ that satisfies $n_s < n_s^*$ for any s. □

Proof of Proposition 11. Denote $\pi^j(t^j, t^{-j})$ as platform j's profit . We have

$$\pi^j(t^j, t^{-j}) = \sum_{s\in\mathcal{S}} \left[t_s^j + \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j \left(n_{\hat{s}}^j + \lambda_{s\hat{s}} \sum_{k\in\mathcal{J}\setminus\{j\}} n_{\hat{s}}^k \right) - c_s^j \right] n_s^j,$$

and thus

$$\frac{\partial \pi^j(t^j, t^{-j})}{\partial t_s^j} = (p_s^j - c_s^j)\frac{\partial n_s^j}{\partial t_s^j} + n_s^j \left(1 + \gamma_{ss}^j (1 - \lambda_{ss}\varphi_s^j) \frac{\partial n_s^j}{\partial t_s^j} \right) + \sum_{s\in\mathcal{S}\setminus\{s\}} n_{\hat{s}}^j \gamma_{\hat{s}s}^j (1 - \lambda_{\hat{s}s}\varphi_{\hat{s}}^j) \frac{\partial n_s^j}{\partial t_s^j}$$

in which $\varphi_s^j(t) := \frac{\sum_{k\in\mathcal{J}\setminus\{j\}} \frac{\partial n_s^k(t)}{\partial t_s^j}}{-\frac{\partial n_s^j(t)}{\partial t_s^j}} \in [0,1]$, denoting platform j's *diversion ratio* on side s. Setting $\frac{\partial \pi^j(t^j, t^{-j})}{\partial t_s^j} = 0$ leads to

$$\begin{aligned}
t_s^j &= p_s^j - \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j \left(n_{\hat{s}}^j + \lambda_{s\hat{s}} \sum_{k\in\mathcal{J}\setminus\{j\}} n_{\hat{s}}^k \right) \\
&= c_s^j + \frac{n_s^j}{-\frac{\partial n_s^j}{\partial t_s^j}} - \sum_{\hat{s}\in\mathcal{S}} \gamma_{\hat{s}s}^j (1 - \lambda_{\hat{s}s}\varphi_{\hat{s}}^j) n_{\hat{s}}^j - \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j \left(1 + \lambda_{s\hat{s}} \frac{\sum_{k\in\mathcal{J}\setminus\{j\}} n_{\hat{s}}^k}{n_{\hat{s}}^j} \right) n_{\hat{s}}^j \\
&= c_s^j + \frac{n_s^j}{-\frac{\partial n_s^j}{\partial t_s^j}} - \sum_{\hat{s}\in\mathcal{S}} \left[\gamma_{\hat{s}s}^j (1 - \lambda_{\hat{s}s}\varphi_{\hat{s}}^j) + \gamma_{s\hat{s}}^j \left(1 + \lambda_{s\hat{s}} \frac{\sum_{k\in\mathcal{J}\setminus\{j\}} n_{\hat{s}}^k}{n_{\hat{s}}^j} \right) \right] n_{\hat{s}}^j.
\end{aligned}$$

□

Proof of Proposition 12. When there is no outside option, (33) simplifies to

$$\begin{aligned}
t_s^j &= c_s^j + \frac{n_s^j}{-\frac{\partial n_s^j}{\partial t_s^j}} - \sum_{\hat{s}\in\mathcal{S}} \left[\gamma_{\hat{s}s}^j (1 - \lambda_{\hat{s}s}) + \gamma_{s\hat{s}}^j \left(1 + \lambda_{s\hat{s}} \left(\frac{1}{n_s^j} - 1 \right) \right) \right] n_{\hat{s}}^j \\
&= \left(c_s^j - \sum_{\hat{s}\in\mathcal{S}} \gamma_{s\hat{s}}^j \lambda_{s\hat{s}} \right) + \frac{n_s^j}{-\frac{\partial n_s^j}{\partial t_s^j}} - \sum_{\hat{s}\in\mathcal{S}} \left[\gamma_{\hat{s}s}^j (1 - \lambda_{\hat{s}s}) + \gamma_{s\hat{s}}^j (1 - \lambda_{s\hat{s}}) \right] n_{\hat{s}}^j,
\end{aligned}$$

as $\varphi_s^j = 1$ and $\sum_{k\in\mathcal{J}\setminus\{j\}} n_{\hat{s}}^k = 1 - n_s^j$.

Now we characterize the equilibrium with J ex ante symmetric firms in which firm 1 has a unique market share whereas the other $(J-1)$ firms equally split the rest of the market.

Firm 2 signifies a generic firm among the other firms.

$$t_s^1 = c_s + \frac{1 - H_s(\tau_s; 0, J)}{h_s(\tau_s; 0, J)} - \beta_{ss}(\lambda)(1 - H_s(\tau_s; 0, J)) - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}(\lambda)(1 - H_{\hat{s}}(\tau_{\hat{s}}; 0, J)),$$
$$t_s^2 = c_s + \frac{1 - H_s(0; -\tau_s, J)}{h_s(0; -\tau_s, J)} - \beta_{ss}(\lambda)(1 - H_s(0; -\tau_s, J)) - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}(\lambda)(1 - H_{\hat{s}}(0; -\tau_{\hat{s}}, J))$$
$$= \frac{H_s(\tau; 0, J)}{(J-1)h_s(0; -\tau_s, J)} - \beta_{ss}(\lambda)\frac{H_s(\tau; 0, J)}{J-1} - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}(\lambda)\frac{H_{\hat{s}}(\tau; 0, J)}{J-1},$$

where $\beta_{s\hat{s}}(\lambda) = \gamma_{s\hat{s}}(1 - \lambda_{s\hat{s}}) + \gamma_{\hat{s}s}(1 - \lambda_{\hat{s}s})$ and we use $1 - H_s(\tau; 0, J) + (J-1)(1 - H_s(0; -\tau, J)) = 1$ in the last simplification. We use the inverse function H_s^{-1} and $t_s^1 - t_s^2 = -\tau_s$ to cancel t_s^1, t_s^2, τ_s,

$$\begin{aligned} g_s(n_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; \lambda, J) = \; & H_s^{-1}(n_s; J) + \frac{1 - n_s}{h_s(H_s^{-1}(n_s; J); 0, J)} - \beta_{ss}(\lambda)(1 - n_s) - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}(\lambda)(1 - n_{\hat{s}}) \\ & - \left(\frac{n_s}{(J-1)h_s(0; -H_s^{-1}(n_s; J), J)} - \beta_{ss}(\lambda)\frac{n_s}{J-1} - \sum_{\hat{s} \neq s} \beta_{s\hat{s}}(\lambda)\frac{n_{\hat{s}}}{J-1} \right) \\ = \; & -\sum_{\hat{s}} \left(1 - \frac{J}{J-1} n_{\hat{s}}\right) \beta_{s\hat{s}}(\lambda) + H_s^{-1}(n_s; J) + \frac{1 - n_s}{h_s(H_s^{-1}(n_s; J); 0, J)} - \frac{n_s}{(J-1)h_s(0; -H_s^{-1}(n_s; J), J)}. \end{aligned}$$

A solution $\{n_s\}$ to $\{g_s(n; \lambda, J) = 0\}$ is an equilibrium in which one firm has a market share of $1 - n_s$ while the others equally split n_s under interoperability λ.

Suppose that there exists an equilibrium with $\overline{n}$ under $\overline{\lambda}$, such that $\overline{n}_s < \frac{J-1}{J}, \forall s$, i.e. $g_s(\overline{n}; \overline{\lambda}, J) = 0$. Then we have,

$$\begin{aligned} g_s(\overline{n}_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; \underline{\lambda}, J) &= g_s(\overline{n}_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; \underline{\lambda}, J) - g_s(\overline{n}; \underline{\lambda}, J) + g_s(\overline{n}; \underline{\lambda}, J) - g_s(\overline{n}; \overline{\lambda}, J) \\ &= -\sum_{\hat{s} \neq s} \frac{J}{J-1} (\overline{n}_{\hat{s}} - n_{\hat{s}}) \beta_{s\hat{s}}(\underline{\lambda}) - \sum_{\hat{s}} \left(1 - \frac{J}{J-1}\overline{n}_{\hat{s}}\right) \left(\beta_{s\hat{s}}(\underline{\lambda}) - \beta_{s\hat{s}}(\overline{\lambda})\right) \\ &= -\sum_{\hat{s} \neq s} \frac{J}{J-1} (\overline{n}_{\hat{s}} - n_{\hat{s}}) \beta_{s\hat{s}}(\underline{\lambda}) - \sum_{\hat{s}} \left(1 - \frac{J}{J-1}\overline{n}_{\hat{s}}\right) \left(\gamma_{s\hat{s}}(\overline{\lambda}_{s\hat{s}} - \underline{\lambda}_{s\hat{s}}) + \gamma_{\hat{s}s}(\overline{\lambda}_{\hat{s}s} - \underline{\lambda}_{\hat{s}s})\right) \end{aligned}$$

This is negative if the following conditions hold: (i) $\gamma_{s\hat{s}}(\overline{\lambda}_{s\hat{s}} - \underline{\lambda}_{s\hat{s}}) + \gamma_{\hat{s}s}(\overline{\lambda}_{\hat{s}s} - \underline{\lambda}_{\hat{s}s}) > 0, \forall (s, \hat{s})$, (ii) $n_{\hat{s}} \leq \overline{n}_{\hat{s}}, \forall \hat{s} \neq s$, and (iii) $\beta_{s\hat{s}}(\underline{\lambda}) \geq 0, \forall (s, \hat{s})$.

We observe that, as in the proof of Proposition 10: (i) $\lim_{n_s \to 0^+} g_s(n_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; \lambda, J) = \infty$, and (ii) $g_s(n_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; \underline{\lambda}, J)$ increases in $n_{\hat{s}}$ if $\beta_{s\hat{s}}(\underline{\lambda}) \geq 0$.

Thus, for any side s, we have that $\lim_{n_s \to 0^+} g_s(n_s, \{n_{\hat{s}}\}_{\hat{s} \neq s}; \underline{\lambda}, J) = \infty, g_s(\overline{n}_s, \{n_{\hat{s}}\}; \underline{\lambda}, J) < 0$ if $n_{\hat{s}} < \overline{n}_{\hat{s}}$ for all $\hat{s} \neq s$. The Poincaré–Miranda theorem implies that there exists a solution $\underline{n}$ to

$g = 0$ that satisfies $\underline{n}_s < \overline{n}_s$ for any s. That is, there exists an equilibrium in which firm 1 is even more dominant under $\underline{\lambda}$ than under $\overline{\lambda}$. □

Proof of Proposition 13. With J platforms, a symmetric equilibrium with total demand $\widehat{N}$ is characterized by the demand function

$$\frac{e^{-t}}{e^{z}} = \frac{n^j}{n^0} = \frac{\widehat{N}/J}{1-\widehat{N}},$$

together with the pricing formula,

$$t = c + \frac{1}{1-n^j} - 2\gamma n^j = c + \frac{1}{1-\widehat{N}/J} - 2\gamma\frac{\widehat{N}}{J}.$$

Combining these gives a characterization function $g(\widehat{N};J)$ whose zeros are symmetric equilibria,

$$g(\widehat{N};J) = z + c + \ln\widehat{N} - \ln J - \ln(1-\widehat{N}) + \frac{1}{1-\widehat{N}/J} - 2\gamma\frac{\widehat{N}}{J}.$$

We notice $\lim_{\widehat{N}\to 0} g(\widehat{N};J) = -\infty$.

Suppose the monopoly has a market share of $\widehat{N}$, which solves $g(\widehat{N};1) = 0$. With 2 platforms, if $g(\widehat{N};2) > 0$, then by the intermediate value theorem, there must exist $\widehat{N}' < \widehat{N}$ that satisfies $g(\widehat{N}';2) = 0$. That is, the total demand under duopoly is lower than under monopoly. Since $g(\widehat{N};1) = 0$, an equivalent condition to $g(\widehat{N};2) > 0$ is $g(\widehat{N};2) - g(\widehat{N};1) > 0$, and we have

$$g(\widehat{N};2) - g(\widehat{N};1) = -\ln 2 + \frac{1}{1-\widehat{N}/2} - \frac{1}{1-\widehat{N}} + \gamma\widehat{N},$$

which is positive if and only if γ is larger than a threshold,

$$\gamma \geq \frac{\ln 2}{\widehat{N}} + \frac{1}{(2-\widehat{N})(1-\widehat{N})}.$$

The RHS is convex in $\widehat{N}$. Numerically, we can find that the RHS obtains its minimum of 2.708 when $\widehat{N} \approx 0.470$. Thus, for any $\gamma \gtrsim 2.708$, an interval of $\widehat{N}$ that satisfies this inequality exists. For any $\widehat{N}$ in this interval, the $z + c$ that supports it as a monopoly equilibrium can be found from $g(\widehat{N},1) = 0$. Therefore, there exists an interval of $z + c$ such that total demand is lower at the symmetric equilibrium of the duopoly model than it is under monopoly. □