Supplementary Material for 'Entry-Proofness and Discriminatory Pricing under Adverse Selection' (For Online Publication)

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January 5, 2021

Abstract

This online appendix presents supplementary material for the main text. Appendix A provides the proofs of technical lemmas. Appendix B extends the analysis of entry-proofness in inactive markets to arbitrary distributions of types. Appendix C illustrates the range of possible applications of our model. Appendix D shows the relevance of Condition EP for markets on which the weak adverse-selection condition is not satisfied. Appendix E studies the tightness of our assumptions. Appendix F investigates to which extent the assumption that the market tariff be convex can be relaxed.

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Appendix A: Proofs of Technical Lemmas

Proof of Lemma 1. For the sake of clarity, the index i is hereafter omitted. The proof consists of three steps.

Step 1 We begin by analyzing the statement $p < \tau^T(q', t')$. By definition, $\tau^T(q', t')$ is the supremum of the set of prices p such that

$$\begin{aligned}\max\{u(q+q', T(q)+t') : q\} &= u^T(q', t') \\ &< \max\{u^T(q'+q'', t'+pq'') : q''\} \\ &= \max\{u(q+q'+q'', T(q)+t'+pq'') : q, q''\} \\ &= \max\{u(q+q', T \,\square\, T_p(q)+t') : q\},\end{aligned}$$

where T_p is the linear tariff with slope p and $T \,\square\, T_p(q) \equiv \min\{T(q')+p(q-q') : q' \in [0, q]\}$ is the infimal convolution of T and T_p (Rockafellar (1970, Theorem 5.4)). Thus the statement $p < \tau^T(q', t')$ is equivalent to

$$\max\{u(q+q', T(q)+t') : q\} < \max\{u(q+q', T \,\square\, T_p(q)+t') : q\}. \tag{A.1}$$

Notice that $T \,\square\, T_p \leq T$, and that both tariffs coincide up to some quantity q_p, beyond which the inequality is strict. Two cases can arise. Either the maximization problem on the right-hand side of (A.1) admits a solution at most equal to q_p. Then (A.1) is an equality. Or all the solutions to this problem are strictly higher than q_p. Then (A.1) cannot be an equality because, if it were, then there would exist a solution to the maximization problem on the left-hand side of (A.1) at most equal to q_p, and thus this solution would also be a solution to the maximization problem on the right-hand side of (A.1), a contradiction; therefore, (A.1) must hold because in any case $T \,\square\, T_p \leq T$. Overall, we have shown that the statement $p < \tau^T(q', t')$ is equivalent to the statement that all the solutions to the maximization problem on the right-hand side of (A.1) are strictly higher than q_p.

Step 2 Next, fix t' and, for any quantities q'_0 and q'_1 such that $q'_0 < q'_1$, define the following quasiconcave functions:

$$v_0(q, t) \equiv u(q+q'_0, t+t'), \quad v_1(q, t) \equiv u(q+q'_1, t+t').$$

Assumption 2 expresses that the indifference curves for v_0 are everywhere steeper than the indifference curves for v_1. Therefore, if two buyers with utilities v_0 and v_1 face the same tariff $t = T(q)$, then the lowest optimal quantity choice for the buyer with utility v_0 is at least as large as the lowest optimal quantity choice for the buyer with utility v_1.

Step 3 Now, suppose that $p < \tau^T(q'_1, t')$. From Step 1, we first obtain that all the solutions to the maximization problem on the right-hand side of (A.1) (with q' replaced by q'_1) are strictly higher than q_p. From Step 2, we next obtain that all the solutions to the maximization problem on the right-hand side of (A.1) (with q' replaced by q'_0) are strictly higher than q_p. From Step 1 again, we finally obtain $p < \tau^T(q'_0, t')$. Because p is arbitrary, this shows that $\tau^T(q'_1, t') \leq \tau^T(q'_0, t')$ for all t' and $q'_0 < q'_1$, and, therefore, that the property expressed by Assumption 2 is inherited by $\tau^T(q', t')$ from $\tau(q, t)$; a fortiori, Assumption 1 holds for $\tau^T(q', 0)$. The result follows. ■

Proof of Lemma 2. By assumption, there exists a type i such that $D_i(\bar{c}_i) > 0$. Thus, as D_i is continuous, there exists n_0 such that $D_i(\bar{c}_i + \Delta_{n_0}) > 0$; define $p \equiv \bar{c}_i + \Delta_{n_0}$. Because $\bar{c}_i$ belongs to the price grid with tick size Δ and the price grids for tick sizes $\Delta_n = \Delta/2^n$, $n \in \mathbb{N}$, are nested, p belongs to the price grid with tick size Δ_n for all $n \geq n_0$.

Fix some $n \geq n_0$, and an equilibrium of Γ_n. Suppose first that the aggregate quantity q purchased by type I in equilibrium satisfies $q < D_i(p)$. Then $q < D_I(p)$ by single-crossing. As type I overall purchases q, the aggregate supply at prices lower than or equal to p must be such that $S(p) \leq q$. In equilibrium, aggregate revenues are constrained by individual rationality, because no type would accept to pay more than $U_I(q) - U_I(0)$. By ignoring costs, we obtain that the aggregate expected profits are at most $U_I(q) - U_I(0)$.

Now, any seller can deviate when price $p > \bar{c}_i$ is quoted by supplying $D_i(p) - q > 0$ at price p and nothing afterwards. The aggregate supply at prices $p' < p$ is unchanged, and is at most $S(p) \leq q$. Thus, as revenues are nonnegative, the deviating seller's expected profit at prices $p' < p$ is at worst $-\bar{c}_I q$. Second, trading with any type $j < i$ at price p is always profitable as $c_j \leq \bar{c}_i < p$. Thus, from the deviating seller's perspective, these types will at worst choose not to trade with him at price p. Third, the aggregate supply at prices $p' \leq p$ following the deviation is at most $S(p) + D_i(p) - q \leq D_i(p)$. Thus type i has a unique best response at price p that involves purchasing $D_i(p) - q$ from the deviating seller. By strict single-crossing, this a fortiori holds for types $j > i$. Finally, the deviating seller earns zero profits at prices $p' > p$. In equilibrium, this deviation cannot be profitable, so that, a fortiori,

$$-\bar{c}_I q + \left(\sum_{j \geq i} m_j\right)(p - \bar{c}_i)[D_i(p) - q] \leq U_I(q) - U_I(0).$$

Because $D_i(p) > 0$ and $p > \bar{c}_i$, this inequality is violated at $q = 0$. This shows that the aggregate quantity q purchased by type I is bounded away from zero in all equilibria in which $q < D_i(p)$. As $q \geq D_i(p) > 0$ in all other equilibria, it follows that there exists $\underline{q} > 0$

such that type I purchases at least $\underline{q}$ in any equilibrium. In particular, because $D_i(\overline{c}_i) > 0$ by assumption, we can select $\underline{q} < D_i(\overline{c}_i)$; and we can select $\underline{q}$ independently of the equilibrium of Γ_n, and independently of $n \geq n_0$.

Finally, because type I purchases at least $\underline{q}$, she is not willing to trade at prices $p > \overline{p} \equiv U_I'(\underline{q})$. Moreover, as expected profits are nonnegative in equilibrium, she must purchase her aggregate quantity q at a price at least equal to $\overline{c}_1$. This implies that she purchases at most $\overline{q} \equiv D_I(\overline{c}_1)$, which is finite and strictly higher than $\underline{q}$. The result follows. ■

Proof of Lemma 3. Suppose first that (21) holds and that the tariff T implements a budget-feasible allocation $(q_i, T(q_i))_{i=1}^I$. Then, for all i and p, $p > \overline{c}_i$ implies $D_i(p) \leq S(p^-)$; otherwise, $B(p,s)$ would be strictly positive for s small enough, a contradiction. Thus no type i is willing to trade at prices $p > \overline{c}_i$ along T; that is, for each i, $U_i'(q_i) \leq \overline{c}_i$ and $T(q_i) - T(q_{i-1}) \leq \overline{c}_i(q_i - q_{i-1})$. By a now standard argument, budget-feasibility implies that these last inequalities hold as equalities. If $q_{i-1} = q_i$, then we obtain $U_i'(q_{i-1}) \leq \overline{c}_i$, and property (ii) in Theorem 2 is satisfied. If $q_{i-1} < q_i$, then, because $\partial^- T(q_i) \leq U_i'(q_i) \leq \overline{c}_i$ and T is convex with $T(q_i) - T(q_{i-1}) = \overline{c}_i(q_i - q_{i-1})$, it must be that T is affine with slope $\overline{c}_i$ over the interval $[q_{i-1}, q_i]$, as required by property (ii) in Theorem 2; hence $\partial^- T(q_i) = U_i'(q_i) = \overline{c}_i$, and property (ii) in Theorem 2 is again satisfied. Thus $(q_i, T(q_i))_{i=1}^I$ is the JHG allocation, and T is, up to inessential modifications beyond q_I, the JHG tariff. Conversely, consider the JHG tariff T and the JHG allocation $(q_i^*, t_i^*)_{i=1}^I$ it implements, which is budget-balanced by construction. By Theorem 2(ii), $U_i'(q_i^*) \leq \overline{c}_i$ and $q_i^* \leq S(\overline{c}_i)$ for all i. Consider any price p. If $p \leq \overline{c}_1$, then (21) clearly holds. If $p > \overline{c}_i$ for some i, then $p > U_i'(q_i^*)$ and, hence, $D_i(p) \leq q_i^* \leq S(\overline{c}_i) \leq S(p^-)$, so that (21) again holds. The result follows. ■

Proof of Lemma 4. We prove each statement in turn.

(i) By Lemma 2, we have $D_I(p) \leq S_n(p^-)$ for all $p > \overline{p}$ and n large enough. Taking limits, this implies that $D_I(p) \leq S_\infty(p^-)$ at any continuity point $p > \overline{p}$ of S_∞, so that $D_i(p) \leq D_I(p) \leq S_\infty(p^-)$ for all i and $p > \overline{p}$ by continuity and single-crossing. Therefore, $B_\infty^*(p) = 0$ for all $p > \overline{p}$ by definition of B_∞, and, hence, $\hat{p}_\infty \leq \overline{p}$ by definition of $\hat{p}_\infty$.

(ii) Summing by parts the expression (21) for $B_\infty(p,s)$ yields

$$\sum_i \left(\sum_{j \geq i} m_j \right) (p - \overline{c}_i)(\min\{[D_i(p) - S_\infty(p^-)]^+, s\} - \min\{[D_{i-1}(p) - S_\infty(p^-)]^+, s\}),$$

which is at most zero if and only if $p > \overline{c}_i$ implies $D_i(p) \leq S_\infty(p^-)$ for all i.

(iii) The set $\mathcal{I}_\infty \equiv \{i : \hat{p}_\infty > \overline{c}_i \text{ and } D_i(\hat{p}_\infty) \geq S_\infty(\hat{p}_\infty^-)\}$ must be nonempty; otherwise, for all i and $p < \hat{p}_\infty$ close enough to $\hat{p}_\infty$, $p > \overline{c}_i$ would imply $D_i(p) < S_\infty(p^-)$, contradicting

(ii) and the definition of $\hat{p}_\infty$. Define $\hat{\imath}_\infty \equiv \max \mathcal{I}_\infty$. Then $\overline{c}_{\hat{\imath}_\infty} < \hat{p}_\infty \leq \overline{c}_{\hat{\imath}_\infty+1}$ and $D_{\hat{\imath}_\infty}(\hat{p}_\infty) \geq S_\infty(\hat{p}_\infty^-)$, so that, for each $p < \hat{p}_\infty$ close enough to $\hat{p}_\infty$, $\overline{c}_{\hat{\imath}_\infty} < p \leq \overline{c}_{\hat{\imath}_\infty+1}$ and $D_{\hat{\imath}_\infty}(p) > S_\infty(p^-)$. Hence, for any such p, $\hat{\imath}_\infty$ is the highest i satisfying the property in (ii). This holds for p in an open left-neighborhood of $\hat{p}_\infty$, because, if this holds for some p, then $\overline{c}_{\hat{\imath}_\infty} < p' < p \leq \overline{c}_{\hat{\imath}_\infty+1}$ and $D_{\hat{\imath}_\infty}(p') - S_\infty(p'^-) > D_{\hat{\imath}_\infty}(p) - S_\infty(p^-) > 0$ for all p' close enough to p. Finally, (23) is a direct consequence of (21) and of the definition of $\hat{\imath}_\infty$. The result follows. ■

Proof of Lemma 5. Observe first that, by Lemma 2, $D_I(p) \leq S_n(p)$ for all $p > \overline{p}$ and n large enough. As in the proof of Lemma 4(i), this implies that $D_i(\overline{p}) \leq S_n(\overline{p})$ and $D_i(\overline{p}) \leq S_\infty(\overline{p})$ for all i by continuity and single-crossing. Therefore, $\overline{p}_{i,\infty}$ is well-defined. That condition (27) determines a unique measure over the Borel sets of $[0, \overline{p}]$ is standard (Billingsley (1995, Theorem 12.4)). We only need to show that $(\boldsymbol{\sigma}_{i,n}([0,p]))_{n\in\mathbb{N}}$ converges to $\boldsymbol{\sigma}_{i,\infty}([0,p])$ at any continuity point p of $p \mapsto \boldsymbol{\sigma}_{i,\infty}([0,p])$. By (27), the set of such points is included in the set of continuity points of S_∞. Moreover, using the definition (26) of $\boldsymbol{\sigma}_{i,n}$, we can check that

$$\text{For each } p \in [0, \overline{p}],\ \boldsymbol{\sigma}_{i,n}([0,p]) = \min\{S_n(p), D_i(\overline{p}_{i,n})\},$$

where, according to our preliminary observation, $\overline{p}_{i,n} \equiv \inf\{p \in [0, \overline{p}] : S_n(p) \geq D_i(p)\}$ is well-defined for n large enough. As D_i is continuous and $\lim_{n\to\infty} S_n(p) = S_\infty(p)$ at any continuity point of S_∞, we thus only need to prove that $\lim_{n\to\infty} \overline{p}_{i,n} = \overline{p}_{i,\infty}$. To this end, consider a subsequence of $(\overline{p}_{i,n})_{n\in\mathbb{N}}$ whose elements all satisfy $\overline{p}_{i,n} < \overline{p}_{i,\infty}$, and suppose, by way of contradiction, that it does not converge to $\overline{p}_{i,\infty}$. Then there exists $\varepsilon > 0$ and a subsubsequence whose elements all satisfy $\overline{p}_{i,n} < \overline{p}_{i,\infty} - \varepsilon$. Using the definition of $\overline{p}_{i,n}$ and $\overline{p}_{i,\infty}$ and the monotonicity of supply and demand functions, we finally obtain that, for each $p \in (\overline{p}_{i,\infty} - \varepsilon, \overline{p}_{i,\infty})$, we have $S_n(p) \geq D_i(p) > S_\infty(p)$ for any such n, a contradiction as $\lim_{n\to\infty} S_n(p) = S_\infty(p)$ if p is a continuity point of S_∞. A symmetric argument applies to a subsequence of $(\overline{p}_{i,n})_{n\in\mathbb{N}}$ whose elements all satisfy $\overline{p}_{i,n} > \overline{p}_{i,\infty}$. The result follows. ■

Proof of Lemma 6. Because $\overline{p}_{j,\infty}$ is nondecreasing in j, we can partition the integration interval into (i) successive intervals $(\overline{p}_{j-1,\infty}, \overline{p}_{j,\infty})$, on which $\boldsymbol{\sigma}_{i,\infty}$ puts a mass if and only if $i \geq j$ by (27), and (ii) possible mass points at each bound, once more using (27) to compute the mass. To avoid double-counting in (ii), we let A be the set of $\overline{p}_{j,\infty}$, and for $p \in A$ we let $j(p)$ be the lowest type such that $\overline{p}_{j,\infty} = p$. We obtain

$$\begin{aligned}
&\sum_i \int_{(p_1, \overline{p}]} m_i(p - c_i)\, \boldsymbol{\sigma}_{i,\infty}(\mathrm{d}p) \\
&= \sum_j \int_{(\max\{\overline{p}_{j-1,\infty}, p_1\}, \max\{\overline{p}_{j,\infty}, p_1\})} \left[\sum_{i\geq j} m_i(p - c_i)\right] \mathrm{d}S_\infty(p)
\end{aligned}$$

$$+\sum_{p\in A, p>p_1}\sum_{j\geq j(p)} m_j(p-c_j)[\min\{D_j(p), S_\infty(p)\} - S_\infty(p^-)]^+,$$

with $\int_\emptyset \equiv 0$. For each integral on the right-hand side, when $p < \overline{p}_{j,\infty}$ we have $D_j(p) > S_\infty(p) \geq S_\infty(p^-)$; but $p > p_1 > \hat{p}_\infty$ implies $B^*_\infty(p) = 0$, so that $D_j(p) > S_\infty(p^-)$ implies $p \leq \overline{c}_j$ by Lemma 4(ii). Each of these integrals is thus at most zero. For the second term on the right-hand side, fix $p \in A$, $p > p_1$. Then we have $p = \overline{p}_{j(p),\infty}$, and for $j < j(p)$ we have $p > \overline{p}_{j,\infty}$ by definition of $j(p)$. Therefore, $\boldsymbol{\sigma}_{j,\infty}$ puts no mass on p if $j < j(p)$, so that we can extend the sum $\sum_{j\geq j(p)}$ to all types. Hence this sum is equal to

$$\sum_j m_j(p-c_j)\min\{[D_j(p) - S_\infty(p^-)]^+, [S_\infty(p) - S_\infty(p^-)]^+\} = B_\infty(p, S_\infty(p) - S_\infty(p^-)),$$

which is at most zero as $p_1 > \hat{p}_\infty$. The result follows. ■

Appendix B: Arbitrary Distributions

In this appendix, we extend Theorem 1 to arbitrary distributions of types with bounded support $\mathcal{I}$ over the real line. Denote by i the buyer's type, and by $\boldsymbol{m}$ the corresponding distribution; $\boldsymbol{m}$ may be continuous, discrete, or mixed. It will sometimes be convenient to think of any point in $\mathcal{I}_0 \equiv [\min \mathcal{I}, \max \mathcal{I}]$ as a type, even if it does not belong to $\mathcal{I}$. We impose the same conditions on the utility functions u_i and on the upper-tail conditional expectations of unit costs $\overline{c}_i^{\boldsymbol{m}} \equiv \mathbf{E}^{\boldsymbol{m}}[c_j \,|\, j \geq i]$ as in Section 2, and we moreover assume that $u_i(q,t)$ is jointly continuous in (i,q,t) and that c_i is continuous in i.

The proof that Condition EP is necessary for entry-proofness is exactly the same as in Section 3. There only remains to show that Condition EP is sufficient for entry-proofness. According to the taxation principle, there is no loss of generality in letting the entrant offer a tariff specifying a transfer $T(q)$ to be paid as a function of the quantity q demanded by the buyer, with $T(0) \equiv 0$. We assume that the domain of T is a compact set with lower bound 0 and that T is bounded from below and lower semicontinuous. These minimal regularity conditions ensure that any type i's maximization problem

$$\max\{u_i(q, T(q)) : q \geq 0\} \tag{B.1}$$

has a solution. The following result then holds.

Lemma B.1 *There exists for each i a solution q_i to (B.1) such that*

(i) The mapping $i \mapsto q_i$ is nondecreasing.

(ii) The mapping $i \mapsto T(q_i) - c_i q_i$ is bounded from below and lower semicontinuous.

Proof. As in Step 1 of the proof of Theorem 1, the weak single-crossing condition ensures that we can select the buyer's best response in such a way that the mapping $i \mapsto q_i$ is nondecreasing. This implies (i). As for (ii), observe first that, because T has a compact domain and is bounded from below, the mapping $i \mapsto T(q_i) - c_i q_i$ is bounded from below no matter the buyer's best response. To show that the buyer's best response can be chosen in such a way that this mapping is lower semicontinuous, it is useful to fix a best response $i \mapsto q_i$ and some type $i_0 \in \mathcal{I}_0$, and then to distinguish two cases.

Case 1 Suppose first that $i \mapsto q_i$ is continuous at i_0. Then, as T is lower semicontinuous and c_i is continuous in i, we have $\liminf_{i \to i_0} T(q_i) - c_i q_i \geq T(q_{i_0}) - c_{i_0} q_{i_0}$.

Case 2 Suppose next that $i \mapsto q_i$ is discontinuous and left-continuous at i_0. (The other types of jump discontinuities can be treated in a similar way.) Because the domain of T is a compact set, it must include $q^+_{i_0} \equiv \lim_{i \downarrow i_0} q_i$; moreover, T must be right-continuous at $q^+_{i_0}$; otherwise, some type $i > i_0$ would be strictly better off purchasing $q^+_{i_0}$ instead of q_i, a contradiction. Now, type i_0 must be indifferent between the trades $(q_{i_0}, T(q_{i_0}))$ and $(q^+_{i_0}, T(q^+_{i_0}))$. Indeed, we clearly have $u_{i_0}(q_{i_0}, T(q_{i_0})) \geq u_{i_0}(q^+_{i_0}, T(q^+_{i_0}))$ and, if we had $u_{i_0}(q_{i_0}, T(q_{i_0})) > u_{i_0}(q^+_{i_0}, T(q^+_{i_0}))$, then, by continuity of u_i in i, some type $i > i_0$ would be strictly better off purchasing q_{i_0} instead of q_i, a contradiction. We can thus select the trade of type i_0 so that $\liminf_{i \to i_0} T(q_i) - c_i q_i \geq T(q_{i_0}) - c_{i_0} q_{i_0}$. The result follows. ■

The next step of the analysis consists in checking that any distribution that satisfies Condition EP can be approximated by a sequence of discrete distributions that satisfy Condition EP. Specifically, the following result holds.

Lemma B.2 *If $\boldsymbol{m}$ satisfies Condition EP, then it is the weak* limit of a sequence of discrete distributions $(\boldsymbol{m}_n)_{n \in \mathbb{N}}$ such that*

$$\text{For all } n \text{ and } i,\ \overline{c}_i^{\boldsymbol{m}_n} \geq \overline{c}_i^{\boldsymbol{m}}.$$

Proof. The proof is a simple adaptation of Hendren (2013, Supplementary Material, Lemma A.7), using the fact that c_i is continuous in i and that, as $\overline{c}_i^{\boldsymbol{m}}$ is nondecreasing in i, $c_{\max \mathcal{I}} \geq c_i$ for all i. Hendren's (2013) proof establishes that the sequence of cumulative distribution functions associated to the sequence $(\boldsymbol{m}_n)_{n \in \mathbb{N}}$ can be chosen so as to uniformly converge to the cumulative distribution function associated to $\boldsymbol{m}$. The result follows. ■

We are now ready to complete the proof of Theorem 1 for arbitrary distributions. Let

$\boldsymbol{m}$ be a distribution that satisfies Condition EP. Fix a tariff T as above and, for each i, a solution q_i to (B.1) such that properties (i)–(ii) in Lemma B.1 hold. Lemma B.2 implies that there exists a sequence of discrete distributions $(\boldsymbol{m}_n)_{n\in\mathbb{N}}$ with weak* limit $\boldsymbol{m}$ and such that each $\boldsymbol{m}_n$ satisfies Condition EP. Taking advantage of the fact that the mapping $i \mapsto q_i$ is nondecreasing, we can apply the version of Theorem 1 for discrete distributions provided in the main text to obtain

$$\text{For each } n, \int [T(q_i) - c_i q_i]\, \boldsymbol{m}_n(\mathrm{d}i) \leq 0.$$

Because the mapping $i \mapsto T(q_i) - c_i q_i$ is bounded from below and lowersemicontinuous, the weak* convergence of the sequence $(\boldsymbol{m}_n)_{n\in\mathbb{N}}$ to $\boldsymbol{m}$ then yields

$$\int [T(q_i) - c_i q_i]\, \boldsymbol{m}(\mathrm{d}i) \leq \liminf_{n\to\infty} \int [T(q_i) - c_i q_i]\, \boldsymbol{m}_n(\mathrm{d}i) \leq 0$$

according to a corollary of the portmanteau theorem (Aliprantis and Border (2006, Theorem 15.5)). Hence, if the distribution $\boldsymbol{m}$ satisfies Condition EP, no tariff can guarantee the entrant a strictly positive expected profit, which is the desired result.

Appendix C: Examples

In this appendix, we illustrate the range of possible applications of our model. As in Section 4, we focus on situations where types are ordered according to the strict single-crossing condition and the more demanding Assumption 2 holds.

C.1 Trade with Quasilinear Utility

We may first suppose, as in the models of trade on financial markets studied by Glosten (1989, 1994), Biais, Martimort, and Rochet (2000), Mailath and Nöldeke (2008), and Back and Baruch (2013), that every type i's preferences are quasilinear,

$$u_i(q,t) \equiv U_i(q) - t,$$

for some concave utility function U_i. The strict single-crossing condition is satisfied if $\partial^+ U_i(q)$ is strictly increasing in i for all q, and the concavity of U_i ensures that Assumption 2 holds.

C.2 Insurance Economies

We now consider variations on the standard Rothschild and Stiglitz (1976) insurance economy, in which a consumer has initial wealth w_0 and faces the risk of a nonnegative loss $\tilde{L}$

distributed according to a cumulative distribution function F_i with strictly positive density f_i over the support of F_i relative to a fixed measure $\boldsymbol{l}$. Here q is the fraction of the loss that is insured and t is the premium paid in return, so that the consumer bears a fraction $1-q$ of the loss. Under nonexclusivity, the intended interpretation is that (q,t) results from the aggregation of coinsurance contracts offered by different insurers. This captures markets where multiple policies pay for the same loss, such as life insurance. This also applies to cases in which the loss $\tilde{L}$ can be divided into units—such as drugs, care, and various indemnities for pain or loss of income—and consumers can cover different units with different insurers, the assumption being that all these units are fungible.

C.2.1 Expected Utility (Rothschild and Stiglitz (1976))

Suppose first that every type i's preferences over coverage-premium pairs (q,t) have an expected-utility representation

$$u_i(q,t) \equiv \int v(w_0 - (1-q)l - t) f_i(l)\, \boldsymbol{l}(\mathrm{d}l),$$

where v is a differentiable, strictly increasing, and strictly concave utility index. Then type i's marginal rate of substitution of coverage for premium can be written as

$$\tau_i(q,t) = \int l g_i(l \,|\, q,t)\, \boldsymbol{l}(\mathrm{d}l),$$

where $g_i(\cdot \,|\, q,t)$ is the risk-neutral density

$$g_i(l \,|\, q,t) \equiv \frac{v'(w_0 - (1-q)l - t) f_i(l)}{\int v'(w_0 - (1-q)\ell - t) f_i(\ell)\, \boldsymbol{l}(\mathrm{d}\ell)}.$$

We assume that $j > i$ implies that f_j dominates f_i in the monotone-likelihood-ratio order; then $g_j(\cdot \,|\, q,t)$ dominates $g_i(\cdot \,|\, q,t)$ in the monotone-likelihood-ratio order as well, and thus

$$\tau_j(q,t) > \tau_i(q,t),$$

which implies strict single-crossing. As for costs, we have

$$c_j \equiv \int l f_j(l)\, \boldsymbol{l}(\mathrm{d}l) > \int l f_i(l)\, \boldsymbol{l}(\mathrm{d}l) \equiv c_i,$$

so that the weak adverse-selection condition (1) is a fortiori satisfied.

Example 1 In the Rothschild and Stiglitz (1976) original specification, $\tilde{L} \in \{0, L\}$ for some $L > 0$, $\boldsymbol{l}$ is the counting measure on $\{0, L\}$, and $f_i(L)$ is strictly increasing in i.

There remains to find conditions under which Assumption 2 holds. Denoting by $\alpha \equiv$

$-v''/v'$ the buyer's absolute risk-aversion index, we have

$$\frac{\partial \tau_i}{\partial q} = \int l \, \frac{(v''l \int v' f_i \, \mathrm{d}\boldsymbol{l} - v' \int v'' l f_i \, \mathrm{d}\boldsymbol{l}) f_i}{(\int v' f_i \, \mathrm{d}\boldsymbol{l})^2} \, \mathrm{d}\boldsymbol{l} = -\int l \left(\alpha l - \int \alpha l g_i \, \mathrm{d}\boldsymbol{l} \right) g_i \, \mathrm{d}\boldsymbol{l},$$

where we have omitted the arguments of the functions for the sake of clarity. We thus obtain the following covariance formula:

$$\frac{\partial \tau_i}{\partial q}(q,t) = -\mathbf{Cov}_{g_i(\cdot|q,t)}\left[\tilde{L}, \alpha(w_0 - (1-q)\tilde{L} - t)\tilde{L}\right].$$

This implies, in particular, that $\tau_i(q,t)$ is strictly decreasing in q—and thus that Assumption 2 holds—if $\tilde{L} \in \{0, L\}$ for some $L > 0$, or, for more than two loss levels, if v has constant absolute risk-aversion or if $q \in [0,1]$ and v has decreasing absolute risk-aversion.

C.2.2 Rank-Dependent Expected Utility (Quiggin (1982))

Suppose now that every type i's preferences over coverage-premium pairs (q,t) have a rank-dependent expected-utility representation

$$u_i(q,t) \equiv \int v(w) \, \mathrm{d}(-T(1 - W_i(w))),$$

where v is a differentiable, strictly increasing, and strictly concave utility index v, and $T : [0,1] \to [0,1]$ is a differentiable and strictly increasing probability weighting function, with $T(0) \equiv 0$ and $T(1) \equiv 1$, which acts on the true decumulative distribution function $1 - W_i$ of type i's final wealth $w_0 - (1-q)\tilde{L} - t$. Let W_i^{-1} be the right-continuous inverse of W_i and F_i^{-1} be the right-continuous inverse of F_i; then

$$u_i(q,t) = \int_0^1 v(W_i^{-1}(z))T'(1-z) \, \mathrm{d}z = \begin{cases} \int_0^1 v(w_0 - (1-q)F_i^{-1}(z) - t)T'(z) \, \mathrm{d}z & \text{if } q < 1, \\ \int_0^1 v(w_0 - (1-q)F_i^{-1}(z) - t)T'(1-z) \, \mathrm{d}z & \text{if } q \geq 1. \end{cases}$$

We now consider two examples in turn.

Example 2 Suppose first that $\tilde{L} \in \{0, L\}$ for some $L > 0$. Then

$$u_i(q,t) = \begin{cases} [1 - T(1 - f_i(L))]v(w_0 - (1-q)L - t) + T(1 - f_i(L))v(w_0 - t) & \text{if } q < 1, \\ T(f_i(L))v(w_0 - (1-q)L - t) + [1 - T(f_i(L))]v(w_0 - t) & \text{if } q \geq 1. \end{cases}$$

As v is concave, type i's preferences are convex if and only if $\tau_i(1^-, t) \geq \tau_i(1^+, t)$ for all t, for which a sufficient condition is that $T(z) + T(1-z) \leq 1$ for all $z \in [0,1]$. This property is satisfied by any convex weighting function T, and also by some S-shaped weighting functions. We assume that $f_i(L)$ is strictly increasing in i, which implies strict single-crossing as T is strictly increasing, as well as (1). The concavity of v ensures that Assumption 2 holds.

Example 3 Suppose next that there is a continuum of loss levels, that $\boldsymbol{l}$ is Lebesgue measure, that F_i is continuously differentiable, and that the support of F_i is a closed interval of $\mathbb{R}_+$ over the interior of which f_i is strictly positive. Then type i's marginal rate of substitution of coverage for premium can be written as

$$\tau_i(q,t) = \int l g_i^T(l \,|\, q,t)\,\mathrm{d}l,$$

where $g_i^T(\cdot \,|\, q,t)$ is the risk-neutral density

$$g_i^T(l \,|\, q,t) \equiv \begin{cases} \dfrac{v'(w_0-(1-q)l-t)T'(F_i(l))f_i(l)}{\int v'(w_0-(1-q)\ell-t)T'(F_i(\ell))f_i(\ell)\,\mathrm{d}\ell} & \text{if } q<1, \\[2ex] \dfrac{v'(w_0-(1-q)l-t)T'(1-F_i(l))f_i(l)}{\int v'(w_0-(1-q)\ell-t)T'(1-F_i(\ell))f_i(\ell)\,\mathrm{d}\ell} & \text{if } q \geq 1. \end{cases}$$

As v is concave, type i's preferences are convex if and only if $\tau_i(1^-,t) \geq \tau_i(1^+,t)$ for all t, that is, if and only if

$$\int l\,\frac{T'(F_i(l))f_i(l)}{\int T'(F_i(\ell))f_i(\ell)\,\mathrm{d}\ell}\,\mathrm{d}l \geq \int l\,\frac{T'(1-F_i(l))f_i(l)}{\int T'(1-F_i(\ell))f_i(\ell)\,\mathrm{d}\ell}\,\mathrm{d}l.$$

This property is satisfied by any convex weighting function T, and also if T' is symmetric around 1/2. As for single-crossing, we can first directly assume that $j>i$ implies that $g_j^T(\cdot \,|\, q,t)$ dominates $g_i^T(\cdot \,|\, q,t)$ in the monotone-likelihood-ratio order; then

$$\tau_j(q,t) > \tau_i(q,t),$$

which implies strict single-crossing. As for costs, we assume that $j>i$ implies that f_j dominates f_i in the monotone-likelihood-ratio order, so that (1) holds. These properties are satisfied, for instance, when type i's losses are distributed according to an exponential distribution with a parameter λ_i that is strictly decreasing in i, and the probability weighting function is $T(z) \equiv (\mathrm{e}^z-1)/(\mathrm{e}-1)$.

Alternatively, one can avoid making joint assumptions on the functions f_i and T by restricting q to belong to $[0,1]$ and by observing that, if $j>i$ implies that f_j dominates f_i in the monotone-likelihood-ratio order, then

$$\begin{aligned}\tau_i(q,t) &= \int_0^1 F_j^{-1}(z)\,\frac{v'(w_0-(1-q)F_j^{-1}(z)-t)T'(z)}{\int v'(w_0-(1-q)F_j^{-1}(\zeta)-t)T'(\zeta)\,\mathrm{d}\zeta}\,\mathrm{d}z \\ &> \int_0^1 F_i^{-1}(z)\,\frac{v'(w_0-(1-q)F_j^{-1}(z)-t)T'(z)}{\int v'(w_0-(1-q)F_j^{-1}(\zeta)-t)T'(\zeta)\,\mathrm{d}\zeta}\,\mathrm{d}z,\end{aligned}$$

for $q \in [0,1)$. Hence $\tau_j(q,t) > \tau_i(q,t)$ for any such q if the mapping

$$z \mapsto \frac{v'(w_0 - (1-q)F_j^{-1}(z) - t)}{v'(w_0 - (1-q)F_i^{-1}(z) - t)}$$

is nondecreasing. Because $q < 1$, a sufficient condition is that v have nonincreasing risk-aversion and that $f_i \circ F_i^{-1}$ be nonincreasing in i, that is, higher types have more dispersed loss distributions (Shaked and Shanthikumar (2007, Chapter 3.B.1)).

There remains to find conditions under which Assumption 2 holds. Proceeding as in Section C.2.1, we obtain the following covariance formula:

$$\frac{\partial \tau_i}{\partial q}(q,t) = -\mathbf{Cov}_{g_i^T(\cdot|q,t)}\Big[\tilde{L}, \alpha(w_0 - (1-q)\tilde{L} - t)\tilde{L}\Big].$$

This implies, in particular, that $\tau_i(q,t)$ is strictly decreasing in q—and thus that Assumption 2 holds—if v has constant absolute risk-aversion or if $q \in [0,1]$ and v has decreasing absolute risk-aversion.

C.2.3 Robust Control (Hansen and Sargent (2001))

Suppose next that every type i's preferences over coverage-premium pairs (q,t) have a robust-control representation

$$u_i(q,t) \equiv \min\left\{\int v(w_0 - (1-q)l - t)\tilde{f}_i(l)\,\boldsymbol{l}(\mathrm{d}l) + \rho R(\tilde{f}_i \| f_i) : \tilde{f}_i\right\},$$

where v is a differentiable, strictly increasing, and strictly concave utility index, ρ is a strictly positive constant, and $R(\tilde{f}_i \| f_i)$ is the relative entropy of $\tilde{f}_i$ with respect to f_i,

$$R(\tilde{f}_i \| f_i) \equiv \int \tilde{f}_i(l) \log_2\left(\frac{\tilde{f}_i(l)}{f_i(l)}\right)\boldsymbol{l}(\mathrm{d}l),$$

which can be interpreted as a measure of "distance" between f_i and $\tilde{f}_i$. As u_i is a minimum of concave functions, it is itself concave. Solving the above minimization problem subject to the constraint that $\tilde{f}_i$ be a density,

$$\int \tilde{f}_i(l)\,\boldsymbol{l}(\mathrm{d}l) = 1,$$

we obtain that the solution $f_i^R(\cdot\,|\,q,t)$ satisfies

$$\text{For each } (l,l'),\ \frac{f_i^R(l'\,|\,q,t)}{f_i^R(l\,|\,q,t)} = \frac{f_i(l')}{f_i(l)}\,\mathrm{e}^{\frac{\ln 2}{\rho}[v(w_0-(1-q)l-t)-v(w_0-(1-q)l'-t)]}.$$

Then, by the envelope theorem, type i's marginal rate of substitution of coverage for premium can be written as

$$\tau_i(q,t) = \int l g_i^R(l\,|\,q,t)\,\boldsymbol{l}(\mathrm{d}l),$$

where $g_i^R(\cdot\,|\,q,t)$ is the risk-neutral density

$$g_i^R(l\,|\,q,t) \equiv \frac{v'(w_0-(1-q)l-t) f_i^R(l\,|\,q,t)}{\int v'(w_0-(1-q)\ell-t) f_i^R(\ell\,|\,q,t)\,\boldsymbol{l}(\mathrm{d}\ell)}.$$

We assume that $j > i$ implies that f_j dominates f_i in the monotone-likelihood-ratio order, so that (1) holds; then $f_j^R(\cdot \mid q,t)$ dominates $f_i^R(\cdot \mid q,t)$ and likewise $g_j^R(\cdot \mid q,t)$ dominates $g_i^R(\cdot\,|\,q,t)$ in the monotone-likelihood-ratio order, and thus

$$\tau_j(q,t) > \tau_i(q,t),$$

which implies strict single-crossing.

There remains to find conditions under which Assumption 2 holds. Denoting by f_{iq}^R the partial derivative of f_i^R with respect to q, we have

$$\begin{aligned}
\frac{\partial \tau_i}{\partial q} &= \int l \, \frac{\left[\left(v''l + v'\,\frac{f_{iq}^R}{f_i^R}\right)\int v' f_i^R \,\mathrm{d}\boldsymbol{l} - v' \int \left(v''l + v'\,\frac{f_{iq}^R}{f_i^R}\right) f_i^R \,\mathrm{d}\boldsymbol{l}\right] f_i^R}{(\int v' f_i^R \,\mathrm{d}\boldsymbol{l})^2} \,\mathrm{d}\boldsymbol{l} \\
&= -\int l \left[\alpha l - \frac{f_{iq}^R}{f_i^R} - \int \left(\alpha l - \frac{f_{iq}^R}{f_i^R}\right) g_i^R \,\mathrm{d}\boldsymbol{l}\right] g_i^R \,\mathrm{d}\boldsymbol{l},
\end{aligned}$$

where we have omitted the arguments of the functions for the sake of clarity. We thus obtain the following covariance formula:

$$\frac{\partial \tau_i}{\partial q}(q,t) = -\mathbf{Cov}_{g_i^R(\cdot|q,t)}\left[\tilde{L}, \alpha(w_0-(1-q)\tilde{L}-t)\tilde{L} - \frac{f_{iq}^R}{f_i^R}(\tilde{L}\,|\,q,t)\right].$$

Denoting by $\lambda(q,t)$ the Lagrange multiplier of the constraint that f_i^R be a density in the minimization constraint that yields $f_i^R(\cdot\,|\,q,t)$, we have

$$f_i^R(l\,|\,q,t) = f_i(l)\,\mathrm{e}^{\frac{\ln 2}{\rho}[\lambda(q,t)-v(w_0-(1-q)l-t)]-1}$$

and, hence,

$$\frac{f_{iq}^R(l\,|\,q,t)}{f_i^R(l\,|\,q,t)} = \frac{\ln 2}{\rho}\left[\frac{\partial \lambda}{\partial q}(q,t) - v'(w_0-(1-q)l-t)l\right].$$

Therefore, we can rewrite the covariance formula as

$$\frac{\partial \tau_i}{\partial q}(q,t) = -\mathbf{Cov}_{g_i^R(\cdot|q,t)}\left[\tilde{L}, \left[\alpha(w_0-(1-q)\tilde{L}-t) + \frac{\ln 2}{\rho}\,v'(w_0-(1-q)\tilde{L}-t)\right]\tilde{L}\right].$$

This implies, in particular, that $\tau_i(q,t)$ is strictly decreasing in q—and thus that Assumption 2 holds—if $\tilde{L} \in \{0, L\}$ for some $L > 0$, or, for more than two loss levels, if $q \in [0,1]$ and v has nonincreasing absolute risk-aversion.

C.2.4 Smooth Ambiguity Aversion (Klibanoff, Marinacci, and Mukerji (2005))

Suppose finally that every type i's preferences over coverage-premium pairs (q,t) have a smooth ambiguity-aversion representation

$$u_i(q,t) \equiv \int \phi\Big(\int v(w_0-(1-q)l-t) f_i(l\,|\,\theta)\,\boldsymbol{l}(\mathrm{d}l)\Big) h_i(\theta)\,\mathrm{d}\theta,$$

where v is a differentiable, strictly increasing, and strictly concave utility index, ϕ is a differentiable, strictly increasing, and strictly concave ambiguity index, and $\{f_i(\cdot\,|\,\theta)\}_{\theta\in\Theta}$ is a family of loss densities indexed by a parameter θ whose distribution has a strictly positive density h_i over a closed interval $\Theta \subset \mathbb{R}$ with respect to Lebesgue measure. As v and ϕ are concave, so is u_i. Then type i's marginal rate of substitution of coverage for premium can be written as

$$\tau_i(q,t) = \int l g_i^{\phi}(l,\theta\,|\,q,t)\,\boldsymbol{l}\otimes\boldsymbol{\lambda}(\mathrm{d}l,\mathrm{d}\theta) = \iint l g_i^{\phi}(l\,|\,\theta,q,t)\,\boldsymbol{l}(\mathrm{d}l)\,h_i(\theta)\,\mathrm{d}\theta$$

by Fubini's theorem, where $\boldsymbol{\lambda}$ is Lebesgue measure, $g_i^{\phi}(\cdot,\cdot\,|\,q,t)$ is the risk-neutral density

$$g_i^{\phi}(l,\theta\,|\,q,t) \\ \equiv \frac{\phi'(\int v(w_0-(1-q)l-t)f_i(l\,|\,\theta)\,\boldsymbol{l}(\mathrm{d}l))v'(w_0-(1-q)l-t)f_i(l\,|\,\theta)h_i(\theta)}{\iint \phi'(\int v(w_0-(1-q)\ell-t)f_i(\ell\,|\,\vartheta)\,\boldsymbol{l}(\mathrm{d}\ell))v'(w_0-(1-q)\ell-t)f_i(\ell\,|\,\vartheta)\,\boldsymbol{l}(\mathrm{d}\ell)\,h_i(\vartheta)\,\mathrm{d}\vartheta},$$

and $g_i^{\phi}(\cdot\,|\,\theta,q,t)$ is the risk-neutral density of losses conditional on θ,

$$g_i^{\phi}(l\,|\,\theta,q,t) \equiv \frac{g_i^{\phi}(l,\theta\,|\,q,t)}{\int g_i^{\phi}(\ell,\theta\,|\,q,t)\,\boldsymbol{l}(\mathrm{d}\ell)}.$$

We assume that, for each θ, $j>i$ implies that $f_j(\cdot\mid\theta)$ weakly dominates $f_i(\cdot\mid\theta)$ in the monotone-likelihood order; then $g_j^{\phi}(\cdot\,|\,\theta,q,t)$ weakly dominates $g_i^{\phi}(\cdot\,|\,\theta,q,t)$ in the monotone-likelihood-ratio order as well, and thus

$$\int l g_j^{\phi}(l\,|\,\theta,q,t)\,\boldsymbol{l}(\mathrm{d}l) \geq \int l g_i^{\phi}(l\,|\,\theta,q,t)\,\boldsymbol{l}(\mathrm{d}l).$$

We in addition assume that, for each i, $\theta'>\theta$ implies that $f_i(\cdot\,|\,\theta')$ weakly dominates $f_i(\cdot\,|\,\theta)$ in the monotone-likelihood order; then $g_i^{\phi}(\cdot\mid\theta',q,t)$ weakly dominates $g_i^{\phi}(\cdot\mid\theta,q,t)$ in the monotone-likelihood-ratio order as well, and thus the mapping

$$\theta \mapsto \int l g_i^{\phi}(l\,|\,\theta,q,t)\,\boldsymbol{l}(\mathrm{d}l)$$

is nondecreasing. We finally assume that $j>i$ implies that h_j weakly dominates h_i in the monotone-likelihood order, and thus

$$\iint l g_i^{\phi}(l\,|\,\theta,q,t)\,\boldsymbol{l}(\mathrm{d}l)\,h_j(\theta)\,\mathrm{d}\theta \geq \iint l g_i^{\phi}(l\,|\,\theta,q,t)\,\boldsymbol{l}(\mathrm{d}l)\,h_i(\theta)\,\mathrm{d}\theta.$$

Requiring that the first or the third of these orderings be strict yields

$$\tau_j(q,t) > \tau_i(q,t),$$

which implies strict single-crossing. As for costs, we assume that every type i is well-calibrated in the sense that

$$\text{For each } l,\ f_i(l) = \int f_i(l\,|\,\theta)h_i(\theta)\,\mathrm{d}\theta.$$

Our assumptions on h_i and $f_i(\cdot\,|\,\cdot)$ then ensure that $j > i$ implies that f_j dominates f_i in the monotone-likelihood order, so that (1) holds.

There remains to find conditions under which Assumption 2 holds. Denoting by $\beta \equiv -\phi''/\phi'$ the buyer's absolute ambiguity-aversion index, we have

$$\begin{aligned}\frac{\partial\tau_i}{\partial q} &= \int l\left[\frac{\phi''\int v'lf_i\,\mathrm{d}\boldsymbol{l}\,v' + \phi'v''l}{\int\phi'v'f_ih_i\,\mathrm{d}\boldsymbol{l}\otimes\boldsymbol{\lambda}} - \frac{\phi'v'\int\left(\phi''\int v'lf_i\,\mathrm{d}\boldsymbol{l}\,v' + \phi'v''l\right)f_ih_i\,\mathrm{d}\boldsymbol{l}\otimes\boldsymbol{\lambda}}{(\int\phi'v'f_ih_i\,\mathrm{d}\boldsymbol{l}\otimes\boldsymbol{\lambda})^2}\right]f_ih_i\,\mathrm{d}\boldsymbol{l}\otimes\boldsymbol{\lambda}\\ &= -\int l\left[\alpha l + \beta\int v'lf_i\,\mathrm{d}\boldsymbol{l} - \int\left(\alpha l + \beta\int v'lf_i\,\mathrm{d}\boldsymbol{l}\right)g_i^\phi\,\mathrm{d}\boldsymbol{l}\otimes\boldsymbol{\lambda}\right]g_i^\phi\,\mathrm{d}\boldsymbol{l}\otimes\boldsymbol{\lambda},\end{aligned}$$

where we have omitted the arguments of the functions for the sake of clarity. We thus obtain the following covariance formula:

$$\frac{\partial\tau_i}{\partial q}(q,t) = -\mathbf{Cov}_{g_i^\phi(\cdot,\cdot|q,t)}\left[\tilde{L}, \alpha(w_0-(1-q)\tilde{L}-t)\tilde{L} + X(\tilde{\theta})\right],$$

where

$$X(\theta) \equiv \beta\Big(\mathbf{E}_{f_i(\cdot|\theta)}\left[v(w_0-(1-q)\tilde{L}-t)\right]\Big)\mathbf{E}_{f_i(\cdot|\theta)}\left[v'(w_0-(1-q)\tilde{L}-t)\tilde{L}\right].$$

As $\theta' > \theta$ implies that $f_i(\cdot\,|\,\theta')$ weakly dominates $f_i(\cdot\,|\,\theta)$ in the monotone-likelihood order,

$$\mathbf{Cov}_{g_i^\phi(\cdot,\cdot|q,t)}\left[\tilde{L}, X(\tilde{\theta})\right] = \mathbf{Cov}_{g_i^\phi(\cdot,\cdot|q,t)}\left[\mathbf{E}_{f_i(\cdot|\tilde{\theta})}[\tilde{L}], X(\tilde{\theta})\right] \leq 0$$

if X is nondecreasing. This implies, in particular, that $\tau_i(q,t)$ is strictly decreasing in q—and thus that Assumption 2 holds—if $\tilde{L} \in \{0, L\}$ for some $L > 0$ and ϕ has constant absolute ambiguity aversion, or, if $q \in [0,1]$ and ϕ has nonincreasing absolute ambiguity aversion, if $\tilde{L} \in \{0, L\}$ for some $L > 0$ or v has nonincreasing absolute risk-aversion.

Appendix D: Equity Issuance under Background Risk

In this appendix, we use a variant of Leland and Pyle's (1977) model of equity issuance to illustrate how Condition EP can be relevant even if (1) does not hold. In this model, an entrepreneur can *sell* a fraction $q \in [0,1]$ of a project with cash-flow $\tilde{X}_1$ against a transfer t. In addition, she earns a nontradeable income $\tilde{X}_2$, which thus represents a background risk. We assume that $\tilde{X}_1$ and $\tilde{X}_2$ are jointly normally distributed,

$$\begin{pmatrix}\tilde{X}_1\\ \tilde{X}_2\end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}, \begin{pmatrix}\sigma_1^2 & \rho\sigma_1\sigma_2\\ \rho\sigma_1\sigma_2 & \sigma_2^2\end{pmatrix}\right),$$

where $\mu_1 \equiv \mathbf{E}[\tilde{X}_1]$ and $\rho \equiv \mathbf{Corr}[\tilde{X}_1, \tilde{X}_2]$ are the entrepreneur's private information, with joint distribution $\boldsymbol{d}$ from the perspective of an investor *purchasing* a fraction of $\tilde{X}_1$.

We suppose that the entrepreneur's preferences over quantity-transfer pairs (q,t) have an expected-utility representation with constant absolute risk-aversion α,

$$-\mathbf{E}\left[\mathrm{e}^{-\alpha[(1-q)\tilde{X}_1+\tilde{X}_2+t]}\right],$$

for which an alternative representation is

$$t + \mu_1(1-q) - \frac{\alpha}{2}\,[\sigma_1^2(1-q)^2 + 2\rho\sigma_1\sigma_2(1-q)].$$

The corresponding marginal rate of substitution is

$$\tau_i(q,t) \equiv -\alpha\sigma_1^2(1-q) + i,$$

where

$$i \equiv \mu_1 - \alpha\rho\sigma_1\sigma_2$$

is the entrepreneur' type. Types are clearly ordered according to the strict single-crossing condition; in this setting, where the roles of the buyer and of the seller are switched compared to main text, this expresses the fact that higher types are *less* willing to increase their sales as lower types are; that is, they are more willing to retain a high equity stake.

Unlike in the original one-dimensional framework of Leland and Pyle (1977), however, a higher type i need not stem from a higher expected value μ_1, but rather from a lower—even negative—correlation ρ: intuitively, this reflects that, for a fixed expected value of the project, a lower correlation between the cash-flow of the project and the nontradeable income makes the entrepreneur less willing to sell equity. Formally, this means that the expected value of the project for an investor who observes the entrepreneur's type i but cannot disentangle μ_1 from ρ,

$$v_i \equiv \mathbf{E}[\mu_1 \,|\, \mu_1 - \alpha\rho\sigma_1\sigma_2 = i],$$

is not necessarily increasing in i. Indeed, it is easy—though tedious—to construct examples in which v_i is strictly decreasing in i, which correspond to situations of positive selection. Now, if an investor cannot observe the entrepreneur's type, the relevant expected value for him is the *lower*-tail conditional expectation

$$\underline{v}_i \equiv \mathbf{E}[v_j \,|\, j \leq i].$$

Again, $\underline{v}_i$ is not necessarily increasing in i, so that the analogue of (1) need not hold.

To conclude, consider Condition EP, whose analogue in this setting is

$$\text{For each } i,\ \underline{v}_i \leq \tau_i(0,0).$$

Given the above observations, it should now be obvious that it is possible that this condition be satisfied, implying market breakdown by Corollary 1, despite $\underline{v}_i$ being strictly decreasing over some range. All that matters is that, for every low enough type i, the conditional distribution $\boldsymbol{d}(\cdot \mid \mu_1 - \alpha\rho\sigma_1\sigma_2 = i)$ puts enough weight on characteristics (μ_1, ρ) such that there are no gains from trade at the fair, complete-information price μ_1, that is,

$$\mu_1 < \tau_{\mu_1 - \alpha\rho\sigma_1\sigma_2}(0,0) = -\alpha\sigma_1^2 + \mu_1 - \alpha\rho\sigma_1\sigma_2,$$

or, equivalently,

$$\rho < -\frac{\sigma_1}{\sigma_2},$$

which obviously requires $\sigma_2 > \sigma_1$. In particular, Condition EP is satisfied only if there is some positive correlation between μ_1 and ρ; it is always violated, for instance, if the support of $\boldsymbol{d}$ is a rectangle $[\underline{\mu}_1, \overline{\mu}_1] \times [-1, 1]$, for an entrepreneur with characteristics $(\underline{\mu}_1, 1)$ is always ready to sell equity at price $\underline{\mu}_1$.

Appendix E: Counterexamples

In this appendix, we study the tightness of our assumptions for the results of Section 3. The following example shows if Assumption 1 does not hold, entry with a menu of contracts can be profitable even though Condition EP is satisfied.

Example 1 Consider a two-type economy in which type i has preferences represented by

$$u_i(q,t) \equiv (q+1)(\theta_i q - t),$$

where $\theta_2 > \theta_1 > 0$. These preferences are convex over the relevant range $\{(q,t) : u_i(q,t) \geq u_i(0,0)\} = \{(q,t) : \theta_i q \geq t\}$, with

$$\tau_i(q,t) = \theta_i\left(1 + \frac{q}{q+1}\right) - \frac{t}{q+1},$$

so that the strict single-crossing condition is satisfied. However, $\tau_i(q, 0)$ is strictly increasing in q, so that Assumption 1 does not hold. Now, fix quantities $q_2 > q_1 > 0$ and, for some small $\eta > 0$, consider an entrant offering a menu $\{(q_1, t_1), (q_2, t_2)\}$ such that

$$t_1 \equiv \theta_1 q_1 - \eta,$$

so that type 1 has a slight preference for (q_1, t_1) over $(0, 0)$, and

$$t_2 \equiv \theta_2 q_2 - \frac{q_1 + 1}{q_2 + 1}(\theta_2 q_1 - t_1) - \eta,$$

so that type 2 has a slight preference for (q_2, t_2) over (q_1, t_1). Hence each type has a unique best response, and the entrant's expected profit is $m_1(t_1 - c_1 q_1) + m_2(t_2 - c_2 q_2)$. To compute this expected profit, set up costs so that $\bar{c}_1 \equiv \theta_1 + \varepsilon$ and $\bar{c}_2 \equiv \theta_2 + \varepsilon$ for some small $\varepsilon > 0$. Because $\tau_i(0, 0) = \theta_i$, Condition EP is satisfied. As in (2), the entrant's expected profit can be rewritten as $t_1 - \bar{c}_1 q_1 + m_2[t_2 - t_1 - \bar{c}_2(q_2 - q_1)]$; this in turn simplifies into

$$m_2(\bar{c}_2 - \bar{c}_1)(q_2 - q_1)\frac{q_1}{q_2 + 1} - \varepsilon(m_1 q_1 + m_2 q_2) - \eta\left(1 + m_2 \frac{q_1 + 1}{q_2 + 1}\right),$$

which is strictly positive for arbitrary quantities $q_2 > q_1 > 0$ if ε and η are small enough. Notice that the entrant makes a profit when trading with type 1 and a loss when trading with type 2; but he only incurs a small expected loss on the first quantity layer q_1, which he more than recoups on the second quantity layer $q_2 - q_1$.

Concerning market breakdown, the first difficulty is that there may exist menus of contracts for which the buyer has multiple best responses, some of which may be more favorable to the entrant than others. This difficulty can be overcome by requiring that types be ordered according to the strict single-crossing condition. This assumption is tight: the following example shows that, when types are only ordered according to the weak single-crossing condition, zero-expected-profit entry can take place even though Condition EP is satisfied.

Example 2 Consider a two-type economy in which both types have the same strictly convex preferences represented by

$$u(q, t) \equiv q - q^2 - t,$$

so that $\tau(0, 0) = 1$, but different costs such that $c_1 < 1 < \bar{c}_1 < c_2$, so that Condition EP is satisfied. Both types are indifferent between not trading and purchasing the quantity $1 - c_1$ at unit price c_1. An entrant offering the contract $(1 - c_1, c_1(1 - c_1))$ thus earns zero expected profit if type 1 accepts, and type 2 chooses not to trade with him.

Even under strict single-crossing, it is possible that the entrant's expected profit be exactly zero on every strictly positive quantity layer $q_i - q_{i-1}$. A simple and natural way to rule out this knife-edged situation is to assume that the buyer's preferences are strictly convex. Indeed, under this additional assumption, the inequalities (3) directly imply that the expected profit from any strictly positive quantity layer $q_i - q_{i-1}$ is strictly negative. Again, this assumption is tight: the following example shows that, when the buyer's preferences are only weakly convex, zero-expected-profit entry can take place even though types are ordered according to the strict single-crossing condition and Condition EP is satisfied.

Example 3 Consider, in line with Samuelson (1984), Myerson (1985), and Attar, Mariotti, and Salanié (2011), an economy in which a divisible good is traded subject to a capacity constraint $q \in [0, 1]$. Every type i has linear preferences represented by

$$u_i(q, t) \equiv \overline{c}_i q - t,$$

where $\overline{c}_i$ is strictly increasing in i. Under this highly nongeneric assumption, strict single-crossing is satisfied and Condition EP is satisfied with equality for each type. Suppose now that the entrant offers a menu of contracts $\{(q_1, t_1), \ldots, (q_I, t_I)\}$ with strictly positive quantities q_i that are strictly increasing in i and transfers t_i such that $t_i - t_{i-1} = \overline{c}_i(q_i - q_{i-1})$. Any such allocation yields zero expected profit to the entrant and features strict gains from trade for types $i > 1$. The intuition is that Condition EP rules out gains from trade for any type i on the quantity layer $q_i - q_{i-1}$ but not necessarily, for $i > 1$, on the inframarginal quantity layers $q_j - q_{j-1}$, $j < i$. Hence, whereas strictly profitable entry is ruled out by Theorem 1, zero-expected-profit entry is possible, in many different ways, if every type i accepts to trade (q_i, t_i), even though she could as well choose to trade (q_{i-1}, t_{i-1}).

Appendix F: Beyond Convexity

In Section 4, we have highlighted the key role single-crossing plays in our analysis. This property itself resulted from the combination of two assumptions: that types be ordered according to the strict single-crossing condition, and that the market tariff be convex. In this appendix, we examine to which extent this second assumption can be relaxed.

To this end, we propose a more direct characterization of the budget-feasible allocations $(q_i, T(q_i))_{i=1}^I$ that are implemented by an *arbitrary* entry-proof market tariff T. Because types are ordered according to the strict single-crossing condition, the optimal quantities q_i remain nondecreasing in i. A careful reading of the proof of Theorem 2 then reveals that, supposing (7) to hold, we can conclude that $(q_i, T(q_i))_{i=1}^I$ must be the JHG allocation; indeed, from (7) on, the convexity of T is not required to derive the desired equalities (10)–(11). Thus, what is needed is to directly establish (7) in a parsimonious way.

The most intuitive path is to proceed by contradiction, as follows. Suppose that (7) does not hold for some i, and consider a solution (q, q') to the maximization problem in (7). Then an entrant can offer the contract (q', t') with $t' \equiv \bar{c}_i q' + \varepsilon$ for some small $\varepsilon > 0$. Let J be the set of types that are attracted by this contract; by construction, J contains type i. To reach a contradiction, we must find conditions ensuring that the contract (q', t') is profitable. We explore two avenues in turn.

A Condition on the Distribution of Costs The first avenue is as follows. The worst case for the entrant is when J maximizes the expected cost $\mathbf{E}[c_j \,|\, j \in J]$ under the constraint $i \in J$. If the distribution of costs is such that the worst case occurs when the contract (q', t') attracts all types $j \geq i$, then entry is profitable as $t' > \bar{c}_i q'$. This implies the following result.

Lemma F.1 *If $c_i \leq \bar{c}_i \leq c_{i+1}$ for all i, then the only budget-feasible allocation implemented by an entry-proof market tariff is the JHG allocation.*

The assumption of Lemma F.1 is twofold: the first inequalities are equivalent to (1), while the second inequalities ensure that the worst case occurs when all types $j \geq i$ are attracted. In the two-type case, both inequalities hold as soon as $c_1 \leq c_2$. This shows that the convexity requirement is not needed in this simple case.[1]

Corollary F.1 *In the two-type case, the only budget-feasible allocation implemented by an entry-proof market tariff is the JHG allocation.*

By contrast, the assumption of Lemma F.1 becomes quite restrictive when the number of types grows large: indeed, in the limit, the corresponding set of cost distributions reduces to the private-value case in which c_i does not depend on i.

A Condition on the Market Tariff The second avenue is as follows. Once the contract (q', t') is offered, every type j purchases an aggregate quantity Q'_j and, as types are ordered according to the strict single-crossing condition, the quantities Q'_j are nondecreasing in j. Now, consider two types in J, say, to fix ideas, types 1 and 3. Then $Q'_1 \geq q'$, $Q'_3 \geq q'$, and

$$T(Q'_1) > T(Q'_1 - q') + t' \quad \text{and} \quad T(Q'_3) > T(Q'_3 - q') + t'. \tag{F.1}$$

At this point, let us assume that $T(q) - T(q - q')$ is quasiconcave in $q \geq q'$. Then, because

[1]This result also appears in Attar, Mariotti and Salanié (2020), with a different proof.

the intermediate type 2 purchases an aggregate quantity $Q'_2 \in [Q'_1, Q'_3]$, (F.1) implies

$$T(Q'_2) > T(Q'_2 - q') + t',$$

and thus type 2 is also attracted by the contract (q', t'). This shows that J is connected. Under (1), the worst connected set is $\{j : j \geq i\}$, with expected cost $\overline{c}_i$, and we once more obtain that entry is profitable as $t' > \overline{c}_i q'$. There only remains to find a condition on T ensuring that $T(q) - T(q - q')$ is quasiconcave in $q \geq q'$ for all q'. The following result provides such a condition, which allows for tariffs exhibiting quantity discounts.

Corollary F.2 *The only budget-feasible allocation implemented by an entry-proof market tariff that is first convex and then concave is the JHG allocation.*

Overall, we have used the convexity of the market tariff only to ensure that adverse selection is sufficiently severe. Corollaries F.1–F.2 show that we can significantly relax this assumption without threatening the special status of the JHG allocation. To go further, one would have to envision tariffs such that an entrant can attract a nonconnected set of types with an associated expected cost exceeding the upper-tail conditional expectation of unit costs, so that entry would be deterred even though (7) does not hold. In light of the above, this seems implausible, but we must acknowledge that the general problem remains open.

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