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# Getting auctions for transportation capacity to roll*

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**Abstract**

An auction of transport capacity can only roll forward if competitive bidders show up at the start. To characterize bidding behavior, we develop a model with a single incumbent potentially in competition with a single challenger; should the challenger obtain slots, the two firms will engage post-auction in capacity constrained price competition. We show how the auction structure, that is, whether the slots are auctioned one at a time, and if not, how they are packaged affects the outcome. Our key finding is that the division of the available slots into tranches can significantly affect the outcome of the auction. Absent any set-asides, a single auction for all the slots will almost certainly be won by an incumbent. Set-asides can enable the challenger to win one or more packages of slots. Further, when the slots are split up, and auctioned one-at-a-time or in batches, a challenger's prospects improve significantly, and no longer rely only on set-asides. The implications of our analysis are (a) the outcome will depend crucially on auction design decisions, (b) set-asides for challengers can help and (c) an auction that results in successful entry by challengers may result in reduced auction revenues and industry profits.

Keywords: Rail transportation; Open access; Auctions; Regulation

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# 1 Introduction

Since the late 1990's, transport regulators in much of Europe and elsewhere have sought to introduce, or otherwise enhance, competition into the rail industry. However, competition in rail transport requires there being two or more carriers have access to the same or duplicative infrastructure, as well as significant amount of durable capital which may have little or no value outside of a national regional network. This means that an Infrastructure Manager ("IM") or national regulatory authority ("NRA") that hopes to promote competitive entry requires a mechanism to provide those entrants access to fixed infrastructure. This presents a challenging problem for regulators, and different approaches have been employed. Often this takes the form of direct regulation and the imposition of access charges. At other times, this takes the form of a procurement by means of a beauty contest or a competitive bid process. This procurement can in the form of either a railway franchise award (for-the-market competition) or an allocation slots to the bidders (in-the-market competition).

We study how auctions can be applied to share a capacity constrained route between an incumbent and a challenger, and how the division of that capacity will determine the level of competition in the downstream market for transportation services. We find that auction design decisions, both about format and how the available capacity is divided, can affect both the challenger's prospects and auction proceeds. Thus, the success of a liberalization policy intended to promote competition in-the-market for transportation services will often crucially depend on fine details of an auction design.

Our analysis begins with a model of downstream competition between firms with capacities determined by a competitive allocation process.

We first explain how, in the context of passenger rail service the Kreps and Scheinkman (1983) model[1] can be adapted to characterize competition between carriers. Key is that customer waiting cost is additive to the per customer operating costs. When customer waiting times are factored into costs and into pricing decisions, total cost of service will be a convex decreasing function of the capacity, or number of slots, available to a firm assuming other costs are also weakly convex.

We then proceed to provide an analysis of the conditions under which a liberalization program can succeed. Specifically, we solve for the outcome of different auctions. Throughout, we maintain the assumption that the incumbent can retain all the slots it has initially, and that the new slots will never be so much as to allow a challenger to *pass* the incumbent. Further, we also assume that the slots are arranged ex post to minimize expected waiting costs of all passengers. Thus, the entrants are not bidding to replace the incumbent. And, in practice, train operators will periodically adjust the departure times to maximize values, which in this case, means minimize expected waiting costs of their customers. The justification for these hypotheses is both practical and theoretical: on the theoretical: Allowing the new entrant to overtake the incumbent generates many mirror cases, and complicates the analysis. On the practical in most if not all previous experience the new entrant has not been able to undertake the investment and risk. to overtake the incumbent.

We start by showing that when there is a single auction for any number of new

[1] "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", Author(s): David M. Kreps and Jose A. Scheinkman. Source: The Bell Journal of Economics, Vol. 14, No. 2 (Autumn, 1983), pp. 326-337

slots in which an incumbent is competing against a de novo entrant, the incumbent will always have a higher willingness to pay; so any of the standard auctions will result in the incumbent winning, and entry is foreclosed. However, if the entrant has a "toe-hold" or there is a "set-aside" guarantying the entrant some capacity, we derive conditions under which it will win a one-shot auction. Moreover, and surprisingly, if the available capacity is split into two parts with separate auctions for each part, we find conditions under which the entrant will win both auctions. In particular, this will be the case if consumer willingness to pay is sufficiently high. Further, we find that the final outcome need not maximize sum of the firms' profits. The basic idea is that the incumbent may find the cost of blocking the challenger in repeated auctions as excessive, and therefore accommodate entry. We also find that the outcome with sequential auctions will result in higher consumer surplus, and possibly social welfare, as compared to the outcome of a single auction.

Lastly, we apply the model to simulate outcome of different auctions using data from the French railway on the Paris-Lyon route. Our simulations are drive home the point that auction design decisions can have a significant impact on the outcome and on whether liberalization does result in durable competition in the market.

## Literature review

A large part of the previous work on train "paths" allocations has been concerned with the analysis of different ways in which competition can be introduced for rail transport services. Luisa Affuso has written a series of papers, a number with David Newbery, exploring a number of issues with auctions for capacity on specific train paths (See Affuso et al. (2003), Affuso et al. (2002) and Newbery and Affuso (2000)). An important distinction is between auctions for an entire franchise including both the track and the equipment on the track versus auctions to serve specific schedules. A key issue, which is not the subject of this paper, is whether there should be vertical separation of the track ownership and maintenance from the operation of service. Another issue is whether any specific path or set of paths is best allocated to one operator, or is competition on a path both viable and welfare enhancing.[2]

A number of papers have explored some of the auction design options, such as one-shot, sealed-bids, ascending auctions and Vickrey auctions, cf. in particular Affuso et al. (2003), Borndörfer et al. (2006), Perennes (2014), Stojadinović et al. (2019), Montero-Pascual and Ramos Melero (2020). Some experiments and numerous simulation studies have been conducted to compare these alternatives - cf. Nilsson (1999), Nilsson (2003) and Cox et al. (2002). This literature has been unable to demonstrate that a split of the schedule on a single route or set of routes between carriers would even attract qualified entrants. Finger et al. (2016) provide a summary of recent experience, and all the experience they mention are about the effects of open access on regulated terms. The question of how viable competition can be in an auction even for a set of routes is probably still in unsettled cf. Perennes (2014)and Affuso and Newbery (2004). While the motivation here is on train slots, similar issues arise for airplane takeoff and landing slots.[3]

[2]See Lalive and Schmutzler (2008) for an analysis of the impact of for-the-market competition on the development of the level of service in German local passenger railway markets.

[3]See Rassenti et al. (1982) and Ball et al. (2018) for discussion. One main difference is that there is often a single IM for train slots and not for airplane slots.

Our paper relates to a fairly large literature on auctions for upstream inputs. Post-auction interaction generates externalities between the firms in the downstream market,[4] as well as among the bidders in the auction: each bidder's payoff depends not only on what it wins, but also on what its rivals win. A series of papers, most notably by Jehiel and Moldovanu, have explored single-object auctions with such externalities.[5]

A large branch of the literature on auctions with externalities is concerned with spectrum licenses.[6] Cramton et al. (2011) argue that, absent provisions to handicap large bidders, entrants and small participants are unlikely to win new spectrum; hence, regulators should concentrate their efforts on achieving an efficient allocation rather than revenue maximization. Janssen and Karamychev (2007), shows that auctions do not always select the most efficient firm.[7] Hoppe et al. (2006), show that limiting the number of licenses to be auctioned may foster entry, by exacerbating free-riding among incumbents' preemption strategies.[8]

Here we focus on the allocation of a limited capacity on a single path between an incumbent carrier and a challenger. We explain how the division of the available slots into two or more sequential auctions can affect the outcome, and also discuss the impact of other auction design decisions. The issue of how auctioning track capacity affects ex post market structure and performance addressed here is very much the same as is the case in auctions of spectrum bandwidth (Rey and Salant (2017)).

One of the striking results here is that allowing the auctioning authority the flexibility to divide the available slots into two or more auctions in any way it sees fit, can significantly affect an entrant's profits, auction revenues, consumer surplus and total welfare. This means that a non-competitive outcome of a train-path auction can be merely the result of unfortunate decisions about how the slots would be auctioned. Further, we find that when either the IM or the firms choose the timing of the slots, they will often both want to space slots to minimize average passenger waiting time; at times this will result in the firms choosing or being allocated nearly identical departure times.

There is a link with the literature on patent races. In that literature, an incumbent has a preemption incentive (Gilbert and Newbery (1982) and Reinganum (1983)) and the market heads to increasing dominance (Vickers, 1986) when there is Bertrand competition. Vickers (1986) and Budd et al. (1993) show that the evolution of market structure always moves in the direction of higher industry profits. This is also true in much of Rey

[4] Borenstein (1988) shows that the resulting discrepancy between private and social benefits can lead to inefficient outcomes.

[5] See Jehiel and Moldovanu (2001), Jehiel and Moldovanu (2000), and Jehiel et al. (1996). See Salant (2014) for a more extensive discussion

[6] Jehiel and Moldovanu (2003) consider several examples of auctions of fixed-size spectrum licenses, and discuss the likely market outcomes.

[7] Other papers include Janssen and Karamychev (2009), who study the impact of auctions on *ex post* prices, and Moldovanu and Sela (2001) in which a seller is conducting an all pay auction so as to maximize the sum of the bidder payments (or efforts). Eső et al. (2010) examines efficient capacity allocations when there is Cournot competition in the downstream market and Brocas (2013) examines optimal auction design of a single, indivisible object when there are externalities.

[8] Mayo and Sappington (2016) explore a Hotelling model in which a single block of spectrum is available. They show that an auction is unlikely to result in an optimal allocation and consider various corrective handicapping policies.

and Salant (2017). The intuition is clear. An incumbent will win a single auction if total industry profits are greater than would be the case if the new entrant wins. The same is true in our setting for a single auction. But this result no longer holds when there are multiple auctions.[9] A couple of papers consider multi-object auctions. Levin and Skrzypacz (2016) examine bidder incentives in a Combinatorial Clock Auction ("CCA"). They show that bidders may bid more aggressively on packages that they anticipate to lose, in order to increase the price paid by rivals. Kasberger (2017), examines auction designs that can achieve an optimal allocation, in a setting where Cournot competitors bid on the entire allocation across firms.

Finally Klemperer (2004) warns regulators against the temptation of taking measures to increase auction revenues at the cost of discouraging entry, and suggests instead the Anglo-Dutch hybrid auction as a way to balance the trade-off between revenues and post-auction concentration.

# 2 The Model

We assume that passengers decide on which train to ride after the train operators have determined their schedules, but in advance of their departures. More specifically, when booking, passengers do not necessarily know the most convenient time, and therefore will factor in expected cost of waiting into their decisions. We assume that the cost of the waiting is concave increasing in the waiting time and therefore expected waiting cost is a convex decreasing function of the number of slots possessed by the operator. As a consequence, the generalized price perceived by passengers for traveling with firm $j$ is the sum of the fare $f_j$ charged by the operator and the waiting cost, i.e.,

$$p_j = f_j + H(N_j)$$

where $N_j$ is the number of slots of firm $j$, and $H$ is a convex decreasing function. We assume that there are two operators, with identical constant marginal cost per passenger $\gamma$. This implies the incremental cost for firm $j$ of serving a customer is

$$\Gamma_j = \gamma + H(N_j).$$

Firms choose in advance capacity, or in other words the quantity of seats available at each departure time, and then vary price to clear demand. More precisely, we assume the following timing of the competition between two operators who must publish schedules and determine capacities before setting fares.

1. An incumbent starts with an initial allocation of $n_1$ slots. A new entrant starts with an initial allocation of $n_2 = 0$ slots;

2. $\Delta$ new slots are made available;

3. Each firm obtains (possibly via an auction) a share of the $\Delta$ new slots, and ends up with respectively $n_i' \geq n_i$ slots;

4. Each firm determines capacity, i.e., number of cars and seats available on each of their slots;

[9] This result does not conflict with Vickers (1986) in that he shows that the outcome maximizes the sum of the value functions for the continuation game after each patent is allocated.

5. Each firm's fare is determined so that demand equals capacity.

This is the framework of the Kreps and Scheinkman (1983) model. They show that the Cournot model applies assuming passengers are prioritized based on willingness to pay.[10] Letting $F$ denote the inverse demand function, the fare of firm $j$ is

$$f_j = F(q_1 + q_2) - H(N_j)$$

where $q_1$ and $q_2$ are the total number of seats provided by the two firms. Thus, in this formulation, more slots for a firm reduce expecting waiting time of its customers. This has the same effect on that firm as a reduction in its unit costs. In the following, we assume that the inverse demand function is linear

$$F(q) = a - bq$$

and that the new entrant won't have more slots than the incumbent even if it wins all the slots in the auction(s), that is

$$\Delta < n_1.$$

Before studying the allocation of slots, we first briefly describe the outcome for any allocation of the slots $(N_1, N_2)$ with $N_1 \geq n_1$ and $N_1 + N_2 \leq n_1 + \Delta$. As explained above, such an allocation implies that the incremental cost for firm $j$ of serving a customer is

$$\Gamma_j = \gamma + H(N_j).$$

Each firm maximizes its profit $\pi_i = Max\,(0, (a - b(q_1 + q_2) - \Gamma_i)q_i)$. Capacity choices are at the same as if the two firms were engaged in a Cournot competition. For a given firm $i$, we note thereafter $\neg i$ the other firm. One may check that at a Nash equilibrium:

- If $a > 2\Gamma_i - \Gamma_{\neg i}$ for $i = 1, 2$, both firms are active. Profits and consumer surplus are

$$\begin{aligned} \pi_i &= \frac{(a - 2\Gamma_i + \Gamma_{-i})^2}{9b} \\ CS &= \frac{(2a - \Gamma_1 - \Gamma_2)^2}{18b} \end{aligned}$$

- If $2\Gamma_{-i} - \Gamma_i > a > \Gamma_i$, then only firm $i$ is active. Profits and consumer surplus are

$$\begin{aligned} \pi_i &= \frac{(a - \Gamma_i)^2}{4b} \\ \pi_{\neg i} &= 0 \\ CS &= \frac{(a - \Gamma_i)^2}{8b} \end{aligned}$$

- If $a < \Gamma_i$ for both $i = 1, 2$, no firm is active, and profits and consumer surplus are 0.

In what follows, we will assume that

$$\begin{aligned} H(n_1) &< a - \gamma \\ 2H(\Delta) - H(n_1) &< a - \gamma \end{aligned}$$

which guarantees that the incumbent is always active, and that the new entrant is active if it gets all the new slots.

[10] See Kreps and Scheinkman (1983), Davidson and Deneckere (1986) and Friedman (1988).

# 3 Profit, Consumer Surplus, and Welfare Maximizing Slot Allocations

Before studying mechanisms for the allocation of new slots, it is useful to start with a brief discussion about allocations that maximize consumer surplus, total profit, and welfare.

### *3.1 Consumer Surplus Maximizing Slot Allocation*

First, we derive the allocation that maximizes the consumer surplus.

- If $a-\gamma > 2H(N_2)-H(N_1)$, both firms are active at equilibrium. Consumer surplus is
$$CS = \frac{\left(2\left(a-\gamma\right)-H(N_1)-H(N_2)\right)^2}{18b}$$
Recall that costs are a convex function of the number of slots $N_i$ allocated to each firms. Thus, minimizing $\Gamma_1+\Gamma_2$ implies that the new entrant should receive all the additional slots. The maximum CS if the new entrant is active is therefore
$$CS = \frac{2}{9b}\left(a-\gamma-\frac{H(n_1)+H(\Delta)}{2}\right)^2$$

- If $2H(N_2)-H(N_1) > a-\gamma$, then only firm 1 is active. Consumer surplus is maximized in that case if all the slots are given to the incumbent. The maximum CS if the new entrant is not active is therefore
$$CS = \frac{1}{8b}(a-\gamma-H(n_1+\Delta))^2$$

Summarizing, giving all slots to the entrant is optimal iff

$$\frac{a-\gamma-H(n_1+\Delta)}{a-\gamma-\frac{H(n_1)+H(\Delta)}{2}} < \frac{4}{3}$$

This result is stated in the next Proposition.

**Proposition 1.** *The allocation that maximizes the Consumer Surplus is either to give the new entrant all the slots, or to give the incumbent all the slots. It is the former iff*

$$\frac{a-\gamma-H(n_1+\Delta)}{a-\gamma-\frac{H(n_1)+H(\Delta)}{2}} < \frac{4}{3}$$

Proposition 1 illustrates the following trade-off. On the one hand, giving all the slots to the new entrant creates the most competitive setting for the downstream market. This results in lower prices, which is beneficial to the consumers. One the other hand, giving all the slots to the incumbent is the allocation that minimizes potential waiting cost for consumers. Depending on which effect dominates, consumer surplus is maximized at a corner solution: all the new slots should be given to one operator only. Note that when the demand parameter $a$, which corresponds to the highest passengers' willingness to pay, gets large enough, the former effect dominates, and consumer surplus is maximized when the new entrant gets all the slots.

### 3.2 Total Profit Maximizing Slot Allocation

**Proposition 2.** *The allocation that maximizes total profit is to give the incumbent all the slots.*

This result directly follows from the following remark:

**Remark**: *A monopoly generates more profit than a duopoly in which both operators are active and both operators' cost are greater or equal than monopoly's cost.*

**Proof:** Let $c$ denote the monopoly cost, and consider the set $\mathcal{I}$ of $\{c_1, c_2\}$ such that $c \leq c_i \leq a$ and $2c_i - c_{\neg i} \leq a$ for $i = 1, 2$. The interior of the subset spans cases where both firms are active. The function $F(c_1, c_2) = (a - 2c_1 + c_2)^2/9b + (a - 2c_2 + c_1)^2/9b$ gives then the total profit of both firms. The boundaries of $\mathcal{I}$ correspond partly to the cases where at least one firm remains inactive, and the function $F$ gives the total profit as well. Indeed, if $a = 2c_i - c_{\neg i}$, then $F(c_1, c_2) = (a - c_{\neg i})^2/4b$ which is the monopoly profit when only firm $\neg i$ stays active. If both $c_i$ are equal to $c$, then $F(c_1, c_2)$ is equal to $2(a-c)^2/9b$, which is also lower than the monopoly's profit $(a-c)^2/4b$. The result follows then from the fact that $F$ reaches its maximum on the boundaries of $\mathcal{I}$. This result from the convexity of $F$, i.e. its Hessian is positive semi-definite, which is the consequence of:

$$\frac{\partial F}{\partial c_i} = \frac{-2a + 10c_i - 8c_{\neg i}}{9b} \text{ so that } \frac{\partial^2 F}{\partial c_i^2} = \frac{10}{9b} \text{ and } \frac{\partial^2 F}{\partial c_i c_{\neg i}} = \frac{-8}{9b}$$

### 3.3 Welfare Maximizing Slot Allocation

Define welfare as

$$W = CS + \lambda\left(\pi_1 + \pi_2\right)$$

where $\lambda$ is the weight put on private profits.

It directly follows from the Propositions 1 and 2 that if

$$\frac{a - \gamma - H(n_1 + \Delta)}{a - \gamma - \frac{H(n_1)+H(\Delta)}{2}} > \frac{4}{3}$$

welfare is maximized by giving the incumbent all the new slots. In the opposite case, there is a conflict between maximizing consumer surplus, which requires giving the new entrant all the new slots and maximizing profit, which requires giving the incumbent all the slots. [11]

# 4 Single-Round Auctions

In this section, we assume that the IM will sell all $\Delta$ slots in a single one-shot, second price auction.

[11] An analysis by numerical simulations tends to show that the optimum is most often a corner solution, which amounts to giving all the slots to one or the other of the operators, but that, depending on the parameters, the optimum can also be an interior solution which consists in sharing the new slots between the two operators.

**Proposition 3.** *Consider a second-price sealed-bid single auction for a package of $\Delta$ slots. The incumbent wins the auction and gets all the slots.*

The proof follows from the observation that the outcome of such an auction is the one that maximizes total profit. Indeed, the willingness to pay of firm 1 is $\pi_1(n_1 + \Delta, 0) - \pi_1(n_1, \Delta)$ whereas that of firm 2 is $\pi_2(n_1, \Delta) - \pi_2(n_1 + \Delta, 0)$. We know from proposition 2 that among all the possible allocations, the one that maximizes total profit is to give the incumbent all the slots, so that in particular, total profit when the incumbent gets all the slots is larger than when the new entrant gets all the slots.

This is illustrated in the example depicted in figure 1. The incumbent and the new entrant start respectively with $n_1 = 5$ and $n_2 = 0$ slots. A total of $\Delta = 3$ additional slots is being auctioned. We assume that the demand parameter $b$ is equal to $1/9$ and consider two possible values for the demand parameter, $a$. The waiting cost function is assumed to be $H(N) = 40/N$.[12] At each node of the graph, a couple of numbers represents the net profit for respectively the incumbent and the new entrant. A sold line indicates the winner of the auction. The cases (a) and (b) correspond to two different values for the parameter $a$: the incumbent wins, even when the willingness to pay is very high. For instance, when $a = 80$, we see on the graph that the incumbent's profit would be about 12656 (resp. 5980) when winning (resp. losing) the auction. The incumbent is consequently ready to pay the difference, that is 6676, whereas the new entrant is ready to pay at most his profit when it wins, that is 3761. Thus, the incumbent wins, pay 3761 and ends up with a net profit about 8895 equal to the difference between his final gross profit 12656 and this payment (the figures presented here are rounded, which explains that a difference of the order of one unit can appear when one follows this simple arithmetic from the data provided on the graph).

(a) Single auction with $a = 80$ (b) Single auction with $a = 300$

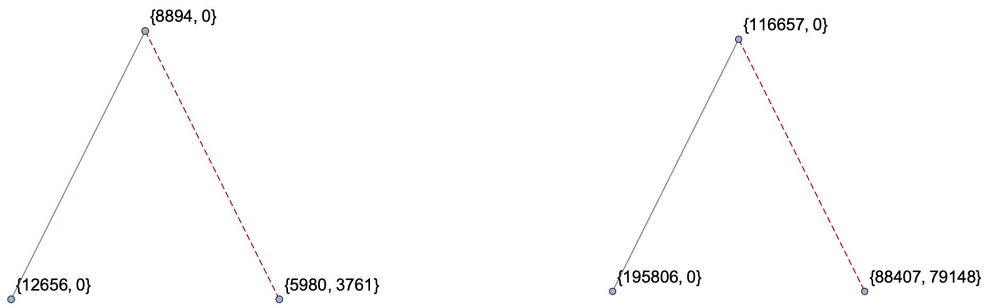


Figure 1: Decision tree for a single-round auction of 3 slots auctioned. The inverse demand function is $F = a - q/9$ where $a$ is either equal to 80 (left) or 300 (right).

Note that Proposition 2 has more general consequences. It implies that if the mechanism that allocates slots maximizes total profit, then the new entrant will not get any slots. This is the case if the slots are auctioned using a single Vickrey-Clarke-Groves (VCG) mechanism. A single VCG auction of $\Delta$ slots being sold to two bidders would have the following rules:

[12] The form of such function is discussed more generally in section 8.

- Each of the two firms submits a value function, that is, its value for $k$ slots, $k \leq \Delta$. Denote by $\widehat{v}_i(\delta_i)$ the value reported by firm $i$ for getting $\delta_i$ slots.
- The auctioneer selects the pair of offers that maximizes the sum of offers:

$$(\delta_1^*, \delta_2^*) \in ArgMax_{(\delta_1,\delta_2)} \left(\widehat{v}_1(\delta_1) + \widehat{v}_2(\delta_2)\right) \text{ s.t. } \delta_1 + \delta_2 \leq \Delta$$

- The price paid by bidder $i$ when the allocation is $(\delta_1^*, \delta_2^*)$ is then

$$p_i = \widehat{v}_j(\Delta) - \widehat{v}_j(\delta_j^*),\ j \neq i$$

Note that a bidder's report affects how many slots it wins, but not the price it pays (which only depends on the report of the other firm). Truthful reporting is a dominant strategy for each bidder in the VCG mechanism. Moreover, when each bidder reports truthfully, the outcome of the mechanism is one that maximizes total value (Theorem 1, Ch. 1, Ausubel et al. (2006)).[13]

In summary, this section reports results that have already been established and are familiar in the literature on rail auctions: with simultaneous auctions, the incumbent is ready to pay a large amount to foreclose entry. One-shot auctions without any restriction on how many slots the incumbent can win de facto prevent a de novo entrant of winning any slot.

# 5 Sequential Auctions

In this section, we consider the possibility that the new slots can be sold in two or more consecutive auctions. We first consider the case with two consecutive auctions. Unlike the case of a single auction, an entrant may win one or both sets of slots. Indeed, somewhat surprisingly, when the demand parameter $a$, which corresponds to the highest passengers' willingness to pay, gets large enough, the entrant will win both auctions.

**Proposition 4.** *Consider two consecutive second-price sealed-bid single auction for respectively packages $\Delta_1$ and $\Delta_2$ of slots, with $\Delta_1 + \Delta_2 = \Delta$. When the passenger's highest willingness to pay (parameter $a$) is sufficiently large, the entrant wins both auctions.*

**Proof:** See Section 10.2 in the Appendix.

Proposition 4 is reminiscent to the "action-reaction" result of Vickers (1986), where the firm with a higher cost can win an auction for a cost reducing patent, assuming Cournot competition. However, unlike Vickers, the outcome does not maximize the sum of firms' profits.

This result is illustrated by the decision trees depicted in figure 2, based on the same example as in figure 1. The cases (b) illustrates the result stated in proposition 4. The auction is split in two consecutive auctions for respectively $\Delta_1 = 2$ then $\Delta_2 = 1$ slots. When the willingness to pay is high enough (case (b) in figure 1), the new entrant wins at least one auction.

[13]This assumes bidders are trying to maximize their own profits, and are not concerned about how much rivals pay. See Salant (2014) and Levin and Skrzypacz (2016)).

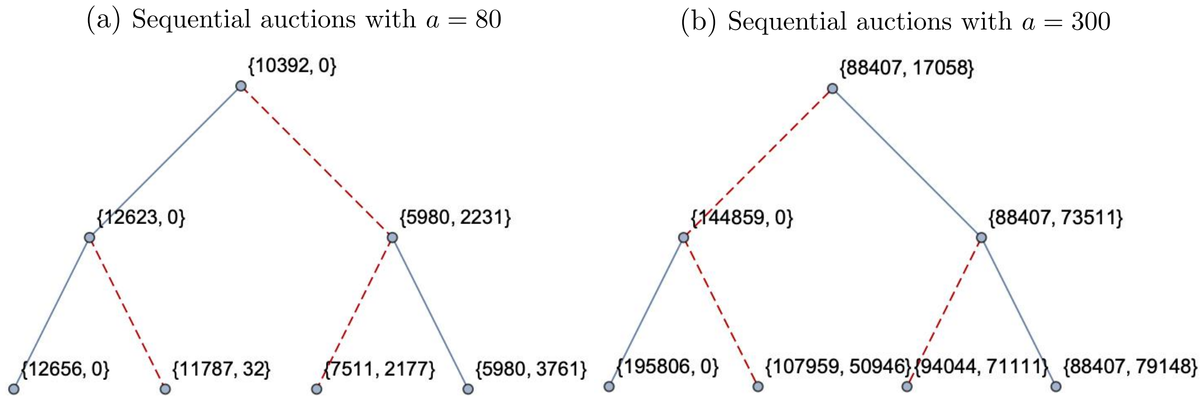


Figure 2: Decision tree for two sequential auction for 2 then 1 slots. The inverse demand function is $F = a - q/9$ where $a$ is either equal to 80 (left) or 300 (right).

The intuition for this results is as follows. If the parameter $a$ is large enough, one may check that whichever operator wins the first auction also wins the second. How much is the incumbent ready to pay at the first auction? It is the difference between the profit it gets from the final outcome and the amount that it will have to pay at the second auction. Similarly, how much is the new entrant ready to pay at the first auction? It is the difference between the profits it gets at the final outcome and the amount that it will have to pay at the second auction. It follows that the resulting allocation is the one that maximizes total profits minus the amount paid at the second auction. As seen above, total profit is maximized when the incumbent gets all the slots. But if the price to be paid at the second auction by the incumbent to win both slots is larger than the price that the new entrant has to pay to win both slots, the new entrant can win

More generally, when the auction is split, the incumbent will have to pay several times the full price to prevent entry into its market. One should therefore expect that entry will be easier as the auction is split into a multitude of consecutive auctions. This is illustrated in the following table, which shows the minimum value of $a$ that guarantees the new entrant to win at least one auction. We see that this threshold depends also on the ordering of auctions.

| sequence | max WTP |
|---|---|
| {1, 1, 1} | 159.03 |
| {1, 2} | 194.968 |
| {2, 1} | 213.83 |

Figure 3 shows what happens when $a = 180$. According to the previous table, the new entrant wins only when slots are auctioned one at a time. [14]

[14] We choose to present here polar examples where a single actor wins successive auctions, but it is important to note that this is not always the case, and that there are also intermediate situations, for example where the new entrant wins the first auction but loses the next one.

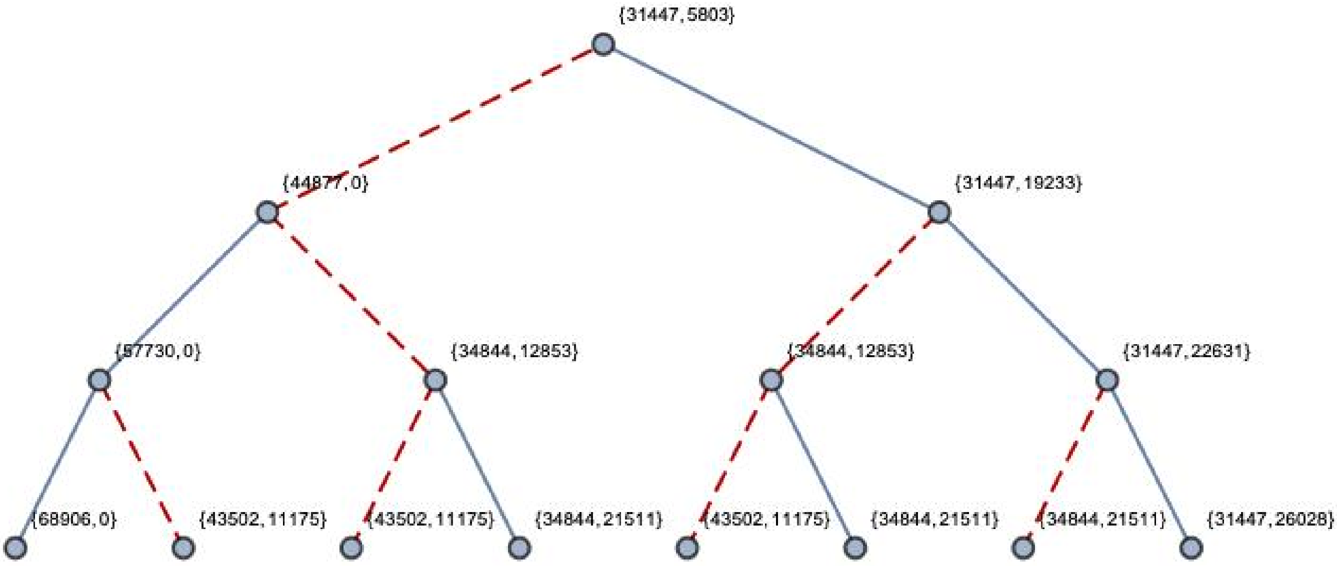


Figure 3: Decision tree for a one-per-one slot auction for a total of three slots. Incumbent has already 5 slots. New entrant has not slot. Same assumptions as in figure 2

# 6 Comparison Between Sequential and Simultaneous Auctions

We have seen that the the details of the auction format can make dramatic differences in slot allocation. Proposition 5 compares total welfare, consumer surplus and total revenue raised by the auctions across auction formats.

**Proposition 5.** *When the passenger's highest willingness to pay (parameter $a$) is large enough so that, according to Proposition 4, the new entrant wins both auctions in the case of sequential auctions, then*

- *Total revenue and total profit are higher under simultaneous auctions than under sequential auctions.*
- *Consumer surplus and welfare are higher under sequential auctions than under simultaneous auctions.*

**Proof:** See Section 10.3 in the Appendix.

Suppose there are $\Delta$, new slots, one incumbent, I, and one challenger, C. The case of Vickey auctions with no cap nor set-aside has been discussed in Section 4. For illustrative purposes, consider the example described in the two previous sections : the incumbent and the new entrant start respectively with $n_1 = 5$ and $n_2 = 0$ slots. A total of $\Delta = 3$ additional slots must be auctioned. We focus here on the example of Figure 3 where the demand parameter $a$ takes the value $a = 180$. Recall, that in a Vickrey auction each bidder submits an offer of a set of values $\widehat{v}_j(\delta_j)$ indicating his true value for $\delta_j$ slots, for each $\delta_j = 0, 1, 2, \ldots, \Delta$, and that truthful reporting is a dominant strategy for each bidder in the VCG mechanism (see Section 4).

Tables 1 and 2 illustrates sample values. More, precisely, it uses the data provided in Figure 3, which provides the profit made by each bidder based on the outcome of the

Table 1: Profits when getting x slots while the other competitor gets the remaining slots

| Bidder | 0 slot | 1 slot | 2 slots | 3 slots |
|---|---|---|---|---|
| Incumbent | 31447 | 34844 | 43502 | 68906 |
| Challenger | 0 | 11175 | 21511 | 26028 |

Table 2: Incremental value of getting x slots while the other competitor gets the remaining slots

| Bidder | 1 slot | 2 slots | 3 slots |
|---|---|---|---|
| Incumbent | 3437 | 12055 | 37459 |
| Challenger | 11175 | 21511 | 26028 |

auction.
Tables 1 and 2 gives the value for a bidder i winning $k$ slots and its rival, j, winning $\Delta - k$.. For instance, if the incumbent does not win an additional slot, it ends up with a profit equal to 31447. If it gets one slot while the challenger gets the two remaining slots, it ends up with a profit equal to 34844. Thus, his willingness to pay for one slot if 3437.

We can check in this example that:

- If I gets 0 slot and C 3 slots, total value is $0 + 26028 = 26028$;
- If I gets 1 slot and C 2 slots, total value is $3437 + 21511 = 24948$;
- If I gets 2 slots and C 1 slot, total value is $12055 + 11175 = 23230$;
- If I gets 3 slots and C 0 slot, total value is $37459 + 0 = 37459$.

Thus, the incumbent wins all the slots and pays 26028 in the case of a simultaneous Vickrey auction whereas, as shown in the previous section, the new entrant wins all the slot in the case of a sequential slot-by-slot auction.

# 7 Caps and Set-Asides

As seen in the previous sections, sequential auctions can be used as a way to facilitate entry. Obviously, more direct measures such as caps and set-asides can also be used. Provisions to promote entry or ensure competition after an auction for an upstream input are common in a few other regulated sectors. Perhaps, most notably, are spectrum auctions, which often have limits on how much of the input a bidder can win. Other examples include sports broadcasting rights and airport landing slots. In all these cases, regulators will limit the fraction of slots one firm can control. Here we examine the impact of various provisions a regulator can apply to auctions for train departure slots. More precisely, we consider Vickrey auctions combined with various restrictions (caps or set-asides). We illustrate the discussion based on the example described in detail in the previous section.

Consider first caps. A cap can be on the number of slots that a bidder can win in the auction, or on the total number of slots. Two cases must be distinguished:

- Assume first that the cap, $K$, is on the number of slots won in the auction, then the outcome will depend on whether $K < \Delta$. Obviously, if $K \geq \Delta$, the cap is not binding and it leaves the challenger with no slots. Assume that in the considered example, the cap, $K = 2$, where, recall, that 3 slots are available in the auction.

  In that case, the two allocations where either of the two bidders wins all the slots are no longer admissible. Among the remaning two allocations, the one where the incumbent gets 1 slot and the new entrant 2 slots generates the highest total value.

- Assume now that the cap $K$ is on the total number of slots that an operator may have following the auction. Obviously, if $K \geq n_1 + \Delta$, the cap is not binding and it leaves the challenger with no slots. Assume now that $n_1 \leq K < n_1 + \Delta$, preventing the incumbent from winning all the slots in the auctions. For illustrative purposes, consider the case $K = 7$ in the considered example. In that case, the allocation where the incumbent wins all the slots is not admissible, while all the others are. Among the remaning three allocations, the one where the incumbent gets no additional slot and the new entrant all the three auctioned slots generates the highest total value.

From this simple example, one can see that if a new entrant starts with a disadvantge (in our case, no slot at all), caps on the the total number of slots is more favorable to the entrant than caps on the number of slots that can be won in the auction.

Consider now set-asides. Assume a set-aside of $K$ slots. Then, absent competition from multiple challengers, the set-aside slots will sell at the reserve price, and the non-set-aside slots will sell at a premium. More specifically, the Vickrey price of the $K$ set-aside slots would be zero. Further, if the parameters of demand are such that the new entrant would win an auction for $\Delta - K$ slots in competition with the incumbent, when the challenger starts with $K$ slots, then it is possible with the set-aside that the challenger would win all the slots, but have to outbid the incumbent for the $\Delta - K$ slots that are not set-aside. In a Vickrey auction, the allocation of non-set-asides slots would maximize the sum of valuations given the challenger is awarded the set-aside slots. In the example discussed above in the case of caps, a set-aside of $K = 1$ slot would lead to the new entrant winning all the non-set-aside slots.

More generally, a corollary of proposition 4 is that the new entrant with some set-asides wins the auction when the parameter $a$ is large enough. Indeed, if $K$ slots are set-aside for the new entrant, and $\Delta - K$ slots are auctioned in a single round auction (or a Vickrey auction), this proposition tells us that the new entrant wins both auction when $a$ is large enough. It implies that it wins also the single auction for $\Delta - K$ slots once it has received a set-aside of $K$ slots. This is stated in the following proposition:

**Proposition 6.** *Consider a second-price sealed-bid single auction for $\Delta$ slots, with a set-aside of $K$ slots for the new entrant. When the passenger's highest willingness to pay (parameter $a$) is sufficiently large, the entrant wins all the non-set-aside slots.*

The following picture 4 illustrates this result, based on the same assumptions as in the previous examples. With 1 slot set-aside, the new entrant wins the remaining slots when $a = 300$ or $a = 180$ (see the discussion above), but not when when $a = 80$.

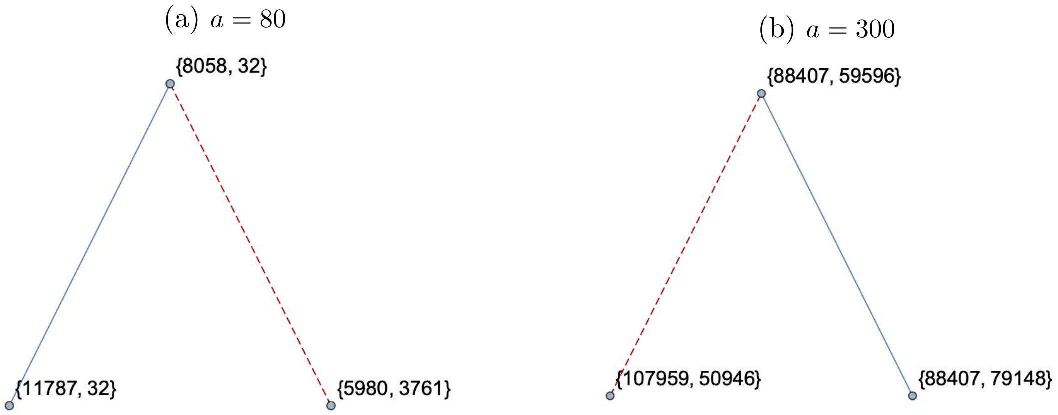


Figure 4: Decision tree for a single auction for two slots. Incumbent has already 5 slots. One slot is set-aside for the new entrant. Same assumptions as in figure 2

# 8 Application

To illustrate the previous results, we consider the case of the main high-speed-train route in France, connecting the cities of Paris and Lyon. With the 4th railway package (https://ec.europa.eu/transport/modes/rail/packages/2013_en), the European Union has set a legal framework that provides for an opening to competition of these high speed passenger services from 2020 onwards. Each Member State can choose the modalities on its own network, but this framework mandates open access competition. The State-owned incumbent, SNCF, currently has 9 daily slots on the Paris-Lyon path during the peak hours (with departure time between 7am and 10am, and between 4pm and 7pm). The Regulator is considering the possibility of opening up to in-the-market competition through an auction system. The following scenario is based on information Fench railway infrastructure manager:

- The incumbent starts with $n_1 = 9$ slots, and is initially a monopoly;
- A total of $\Delta = 8$ additional slots can be allocated during rush hours;
- Upon the opening competition, the incumbent will face one main serious challenger;
- Four distinct scenarios are considered;
  (1) No caps nor set asides, and a single second-price auction for the $\Delta$ new slots;
  (2) No caps nor set asides, and sequential auctions for the $\Delta$ new slots;
  (3) Set-asides $K = 1$ for the new entrant, and a single second-price auction for the remaining $\Delta - K$ slots;
  (4) Set-asides $K = 1$ for the new entrant, and sequential auctions for the remaining $\Delta - K$ slots.

The operational cost for the railway operator, not including tolls paid to the infrastructure manager, is about 39€ per customer. About 3.94 millions passengers per year travel on the Paris-Lyon railway link during peak hours at an average price of 90€ per passenger. Different assumptions can be made to define the passengers' waiting cost function $H(N)$, where $N$ is the operator's number of slots. In our example, the rush hour length is $L = 2 \times 3$ hours. We may assume that consumers are characterized by the maximal time at which they are ready to leave, which is assumed uniformly distributed on the rush hour period. Then, it would be optimal to equally space these slots. It is then easy to see that the average waiting time is given the formula $L/2N$. [15] Other assumptions can lead to slightly different formulas.[16] In the following numerical simulation, we choose to use these assumption, and calibrate this cost formula based on the time value for business passengers, according to a public French report on cost-benefit analysis (Quinet et al., 2014), equal to $w = 46.8$€ per hour. We then obtain that the cost of waiting is $H(N) = wL/2N = 140.4/N$.

As previously, we assume that the inverse demand function is linear. From the available estimates on the price elasticity of business passengers (Wardman, 2014), it is possible to roughly estimate a linear demand function. More precisely, we obtain $a = 278.7$ and $b = 43.94$. This estimate can only be considered as a rough approximation. In particular, as shown by Cherbonnier et al. (2017), the average train tickets price are too low compared to what can be predicted by a static model in which the rail incumbent (the first operator here) would only maximize its short-term profit. This can be the result of a downward pressure on prices exerted by the State, or as the outcome of a dynamic game integrating the reaction of other player. From this point of view, the calibrated parameter $a$ can be seen as a lower bound of the highest willingness to pay of French passengers.

Under these assumptions, we can now compare the outcomes of these scenarios :

- Under Scenario 1 where all the new slots are auctioned simultaneously, we find - in line with our theoretical results from Section 5, - that the new entrant wins all the slots and pays 122 millions euros**.**

- What would happen under scenario 2 is illustrated in figure 5 with two different auction designs : on the left, three consecutive auctions are implemented and the new entrant wins all of them. On the right, two consecutive auctions are implemented, and the incumbent wins both of them.

[15]For illustration purposes, let's assume that the incumbent has 4 slots. In the morning, the optimal allocation for the incumbent is to set a departure at 10h, and another one at 8h30 (assuming it is always better for the incumbent to satisfy the entire demand). Passengers' cost then depends on how long they actually wait before they catch their train, that is $45mn$ or $L/2N$.

[16]For instance, we may assume instead that what matters is the ideal departure's time of passengers, which is assumed uniformly distributed on the rush hour period. Then, it would be optimal to equally space these slots. On the morning rush hour time period, this means placing a departure at 7h45 and a second one at 9h15. The 'deshorage', that is the average value between their ideal departure time and the time at which they leave (assuming symmetry before and after this ideal departure time), is then $22'30''$, that is $L/4N$.

Three consecutive auctions 2-3-3 Two consecutive auctions 4-4

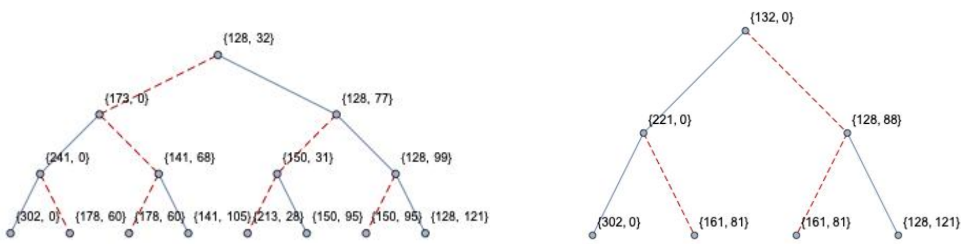


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Figure 5: Decision tree for a sequence of auctions for a total of 8 slots. Incumbent has already 9 slots. New entrant has no slots.

- Under scenario 3 (all the non-set-aside slots are auctioned simultaneously), we see from figure 5 that the incumbent would win all the non-set-aside slots.
- Under scenario 4 (sequential auctioning of the remaining 7 non-set-asides slots), one might expect that entry will be all the easier as the auction is split into a multitude of consecutive auctions. This is illustrated in the following table, that shows the minimum value of $a$ that guarantees that the new entrant wins at least one of these auctions, once it has received 1 slot as a set-aside. Recall that in our calibration, the actual value for $a$ is $a = 278.7$. We see that the new entrant would win at least one auction if the sequence is, for instance, 1-5-1, but would loose all auctions if the sequence were 5-1-1. The ordering of the sequence of auctions matters, but this ordering is not a simple reverse lexicographic order: for instance, the sequence 1-2-2-2 has a lower threshold than the sequence 1-1-1-2-2, which has a lower threshold than the sequence 2-2-1-1-1.

| sequence | max WTP | sequence | max WTP | sequence | max WTP | sequence | max WTP |
|---|---|---|---|---|---|---|---|
| {1, 2, 2, 2} | 199.264 | {1, 2, 1, 1, 2} | 207.399 | {1, 1, 1, 4} | 218.824 | {3, 3, 1} | 235.018 |
| {1, 1, 1, 2, 2} | 200.571 | {1, 2, 1, 2, 1} | 207.399 | {1, 3, 1, 1, 1} | 218.953 | {2, 1, 4} | 235.018 |
| {1, 1, 2, 1, 2} | 200.571 | {1, 1, 3, 1, 1} | 209.11 | {1, 1, 4, 1} | 219.847 | {3, 1, 3} | 237.935 |
| {1, 1, 2, 2, 1} | 200.571 | {1, 3, 2, 1} | 211.096 | {1, 2, 4} | 220.009 | {1, 1, 5} | 245.721 |
| {1, 1, 1, 1, 1, 2} | 201.673 | {1, 3, 1, 2} | 211.541 | {2, 2, 1, 1, 1} | 221.926 | {4, 2, 1} | 250.27 |
| {1, 1, 1, 1, 2, 1} | 201.673 | {2, 3, 2} | 211.947 | {2, 1, 1, 3} | 222.39 | {3, 4} | 250.762 |
| {1, 1, 1, 2, 1, 1} | 201.673 | {2, 2, 1, 2} | 212.157 | {2, 1, 3, 1} | 222.433 | {1, 5, 1} | 251.72 |
| {1, 1, 2, 1, 1, 1} | 201.673 | {1, 2, 1, 1, 1, 1} | 212.516 | {1, 4, 2} | 224.043 | {4, 1, 2} | 252.752 |
| {1, 1, 1, 1, 1, 1, 1} | 202.61 | {2, 2, 3} | 212.705 | {3, 2, 2} | 224.377 | {4, 1, 1, 1} | 254.662 |
| {1, 2, 2, 1, 1} | 203.653 | {2, 2, 2, 1} | 212.741 | {2, 3, 1, 1} | 224.383 | {4, 3} | 257.044 |
| {1, 1, 2, 3} | 205.472 | {2, 1, 2, 2} | 214.531 | {1, 4, 1, 1} | 228.249 | {2, 5} | 267.252 |
| {1, 1, 3, 2} | 205.472 | {2, 1, 1, 1, 2} | 214.987 | {3, 2, 1, 1} | 231.551 | {5, 1, 1} | 285.669 |
| {1, 2, 1, 3} | 205.472 | {2, 1, 1, 2, 1} | 214.987 | {3, 1, 1, 1, 1} | 232.06 | {5, 2} | 287.828 |
| {1, 2, 3, 1} | 205.472 | {2, 1, 2, 1, 1} | 214.987 | {2, 4, 1} | 232.649 | {1, 6} | 315.751 |
| {1, 1, 1, 1, 3} | 206.306 | {1, 3, 3} | 215.226 | {3, 1, 1, 2} | 232.991 | {6, 1} | 327.857 |
| {1, 1, 1, 3, 1} | 206.306 | {2, 1, 1, 1, 1, 1} | 215.337 | {3, 1, 2, 1} | 232.991 | {7} | 420.483 |

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Figure 6: Minimum level required for parameter $a$ (passenger's highest willingness to pay) for the new entrant to win at least one auction. Same assumptions as in figure 5 except that the new entrant has received 1 slot as a set-aside.

Our objective with this (admittedly very rough) simulation exercise is certainly not to predict what would actually happen under these different scenarios were the French IM to use them. A much more thorough econometric study would be needed for that.

Our aim with this example is simply to show that the fact that the details of the action design can have important consequences for the allocation of slots is not merely a theoretical curiosity, but is empirically relevant for realistic values of the demand and supply parameters.

# 9 Conclusion

Experience for train slot auctions suggests that auctions for train slots have had very limited success in attracting durable entry.[17] Our analysis shows how the prospects of a new entrant successfully challenging an incumbent for slots in an auction depends crucially on auction design. Our analysis suggests that it may be necessary to provide set-asides, or caps that preclude incumbents from total foreclosure. But set-asides are not sufficient in themselves. Indeed, our analysis shows that the choice of auction design to allocate the non-set-aside slots (whether auctions are simultaneous or sequential) can provide an additional hindrance to entry. In other sectors, too, such as telecommunications, set-asides in auctions for upstream capacity have not, since the introduction of the first set of 3G mobile operators, succeeded in attracting long-run entrants.[18] The experience of set-asides in spectrum auctions in the UK, Netherlands, Austria, Canada and elsewhere indicate set-asides can fail, unless these are large enough to ensure viability of the challenger for incremental capacity.

All through the analysis, we have assumed that the new slots are never so much as to allow the challenger to pass the incumbent. We believe it to be a reasonable assumption in practice, unless ALL slots are made available for auction so that the incumbent is not allowed to keep any of its existing slots on any related part of its network. Some of the insights derived in this paper will remain true. For example, when there is a single auction, the incumbent will always have a higher willingness to pay, and entry will be foreclosed. Results regarding sequential auctions will differ though, as a high consumer willingness to pay does not imply that the new entrant gets all the slots. We leave it to future research to completely characterize the outcomes under more general assumptions.

So far, we have considered one-shot Vickrey auctions and sequential auctions. Both auction formats have some drawbacks.[19] The Vickrey auction allows bidders to submit bids for any or all packages of slots, and will result in outcomes that maximize total profits. However, the Vickrey auction lacks transparency and “fairness” in that one bidder can win more slots and pay less than a rival. And sequential auctions can suffer what has been termed the “declining price anomaly” in that prices of later auctions tend to be lower than earlier auctions. Further, bidders need to guess the outcome of later auctions when bidding in earlier ones. For these reasons, other auction formats are often tried - and all have some limitations. We leave that for further study, but illustrate the

[17]See Cherbonnier et. al (2017) and Affuso (2003), both argue slot auctions do not typically result in durable competition.

[18]See https://www.ofcom.org.uk/spectrum/spectrum-management/spectrum-awards/awards-archive/800mhz-2.6ghz for a description of the UK 2013 4G auction which attracted three potential challengers to the four incumbents. But within 4 years, the market reverted to a four firms.

[19]See Salant (2014) and Levin and Skrzypacz (2016)) for discussion of the Vickrey auction and of sequential auctions.

case of the clock auction with an example provided in the appendix. The simultaneous multiple round auction is a variant of the clock auction that allow for pre-determined departure slots, which are heterogeneous.[20]. Another form of auction is one in which bidders submit demand schedules. In this case, the outcome is at the point at which total demand is equal to the number of slots available. The demand schedule auction may have no equilibrium, equilibrium only in mixed strategies or multiple equilbria depending precise circumstances. This type of auction tends to be used in regulated sectors, such as energy, where the regulator can potentially monitor bidding behavior.

While the recent Spanish auction and the experience in Italy indicate that open access can be viable, it may be too soon to judge. Given how sensitive the outcome of an auction for slots is to the design details, it is not too surprising that auctions for train slots have not generally resulted in durable post-auction competition.

[20]See Salant (2014) for a description.

# 10 Appendix

## 10.1 Clock auction example

A now common form of auction for selling multiple units of a single product is the clock auction.[21] In a clock auction, an auctioneer starts by an announcing a price per slot, and the bidders indicate the number of slots each would want to purchase at the announced price. This assumes that slots are re-arranged after the auction, so that the slots are generic. If there is excess demand, the auctioneer increases the price per slot, and bidder can maintain or reduce demand at the higher price. The auctioneer continues increasing the price until there is no longer any excess demand.

Table 3: Incremental Values

| Bidder | Value of the 1st slot | Value of the 2nd slot | Value of the 3rd slot |
|---|---|---|---|
| Incumbent | 3437 | 8618 | 25404 |
| Challenger | 11175 | 10336 | 4517 |

Table 4: Average slot values when getting x slots while the other competitors get the remaining auctioned slots

| Bidder | 1 slot | 2 slots | 3 slots |
|---|---|---|---|
| Incumbent | 3437 | 6027 | 12486 |
| Challenger | 11175 | 10755 | 8676 |

Clock auctions have the property that the outcome will be economically efficient assuming marginal values are decreasing and absent strategic withholding. Table 3 shows the incremental values for the example of section 6. For train departure slots marginal slot values are decreasing for the Challenger, but increasing for the Incumbent. When the marginal, or incremental slot, values are increasing over a range, bidders will face an "exposure problem." When this is the case a bidder may be willing to pay a higher per slot price for a larger number of slots and the outcome of such an auction may be indeterminant. In this example, when a cap prevents I from winning all the slots, the allocation will still be the same as is the case in the Vickrey auction: I wins one slot and C wins two slots. However the prices are not the same. I will pay 4517 for its one slot and C will pay 8638 for its two slots - assuming a three slot allocation to I is not permitted. Also, absent caps, the Vickrey and clock auctions will have the some allocations but not the same prices. In each, I will win all three slots. In the Vickrey auction I will pay 26028. However, in the clock auction, I will pay three times the value of one slot or 33525.[22]

[21] See Salant (2014) for a more complete description of a clock auction.

[22] At clock prices 3397 11175, I might choose drop out rather than risking winning only one slot for a price above its value. See Salant (2014) for a discussion.

### 10.2 Proof of proposition 4

The new slots are sold in two consecutive auctions, $(\Delta_1, \Delta_2)$ with $\Delta_1 > 0, \Delta_2 > 0$ and $\Delta_1 + \Delta_2 = \Delta$.

- If Firm 1 wins the first auction, we know from Proposition 2 that it also wins the second auction and pays

$$Max\left(0, \frac{1}{9b}\left(a - \gamma - 2H(\Delta_2) + H(n_1 + \Delta_1)\right)^2\right)$$

  for the second package of slots. Assuming that $a$ is large enough, it pays

$$\frac{1}{9b}\left[a - \gamma - 2H(\Delta_2 + H(n_1 + \Delta_1)\right]^2.$$

- If Firm 2 wins the first auction, it also wins the second auction if

$$\pi_1(n_1, \Delta) + \pi_2(n_1, \Delta) > \pi_1(n_1 + \Delta_2, \Delta_1) + \pi_2(n_1 + \Delta_2, \Delta_1)$$

  When $a$ is large enough, firm 2 is active whether it wins or loses the second auction, and condition above writes:

$$\begin{array}{rl} & (a - \gamma - 2H(n_1) + H(\Delta))^2 + (a - \gamma - 2H(\Delta) + H(n_1))^2 \\ > & (a - \gamma - 2H(n_1 + \Delta_2) + H(\Delta_1))^2 + (a - \gamma - 2H(\Delta_1) + H(n_1 + \Delta_2))^2 \end{array}$$

$$\Longleftrightarrow$$

$$\begin{array}{rl} & (a - \gamma)\left(-2H(n_1) + H(\Delta) - 2H(\Delta) + H(n_1)\right) \\ > & (a - \gamma)\left(-2H(n_1 + \Delta_2) + H(\Delta_1) - 2H(\Delta_1) + H(n_1 + \Delta_2)\right) + K \end{array}$$

$$\Longleftrightarrow$$

$$(a - \gamma)\left(H(n_1 + \Delta_2) - H(n_1) + H(\Delta_1) - H(\Delta_1 + \Delta_2)\right) > K$$

  where $K$ is a term that does not depend on $a$. Since $\Delta_1 < n_1$ and $H$ is convex, $H(\Delta_1) - H(\Delta_1 + \Delta_2) > H(n_1) - H(n_1 + \Delta_2)$. Therefore, when $a$ gets large enough, firm 2 wins the auction if it won the first. In that case, it pays for the second auction

$$\frac{1}{9b}[(a - \gamma - 2H(n_1 + \Delta_2) + H(\Delta_1)]^2 - \frac{1}{9b}[(a - \gamma - 2H(n_1) + H(\Delta_1 + \Delta_2)]^2$$

- Therefore, if Firm 1 wins the first auction, final payoffs for the two firms (net of the price paid for the second auction) are respectively:

$$\begin{array}{rcl} \pi_1' & = & \frac{1}{4b}[a - \gamma - H(n_1 + \Delta)]^2 - \frac{1}{9b}[a - \gamma - 2H(\Delta_2) + H(n_1 + \Delta_2)]^2 \\ \pi_2' & = & 0 \end{array}$$

whereas if firm 2 wins the first auction, payoffs for the two firms (net of the price paid for the second auction) are respectively:

$$\begin{aligned}
\pi_1' &= \frac{1}{9b}[a-\gamma-2H(n_1)+H(\Delta)]^2 \\
\pi_2' &= \frac{1}{9b}[a-\gamma-2H(\Delta)+H(n_1)]^2 \\
&\quad -\left[\begin{array}{c} \frac{1}{9b}[a-\gamma-2H(n_1+\Delta_2)+H(\Delta_1)]^2 \\ -\frac{1}{9b}[a-\gamma-2H(n_1)+H(\Delta)]^2 \end{array}\right]
\end{aligned}$$

Firm 2 wins the first auction iff

$$\begin{aligned}
& \frac{1}{9b}[a-\gamma-2H(\Delta)+H(n_1)]^2 - \frac{1}{9b}[a-\gamma-2H(n_1+\Delta_2)+H(\Delta_1)]^2 \\
& +\frac{1}{9b}[a-\gamma-2H(n_1)+H(\Delta)]^2 \\
> & \frac{1}{4b}[a-\gamma-H(n_1+\Delta)]^2 - \frac{1}{9b}[a-\gamma-2H(\Delta_2)+H(n_1+\Delta_2)]^2 \\
& -\frac{1}{9b}[a-\gamma-2H(n_1)+H(\Delta)]^2
\end{aligned}$$

which is satisfied when $a$ gets large enough.

### *10.3 Proof of Proposition 5*

- In the case of a simultaneous auction, Firm 1 wins the auction. Consumer surplus is
$$CS^{\text{Sim}} = \frac{1}{8b}\left[a-\gamma-H(n_1+\Delta)\right]^2,$$
total welfare is
$$W^{\text{Sim}} = \frac{1+2\lambda}{8b}\left[a-\gamma-H(n_1+\Delta)\right]^2,$$
and auction revenue is
$$R^{\text{Sim}} = \frac{1}{9b}\left[a-\gamma-2H(\Delta)+H(n_1)\right]^2.$$

- In the case of a sequential auction, Firm 2 wins both auctions. Consumer surplus is
$$CS^{\text{Seq}} = \frac{1}{9b}\left[a-\gamma-\frac{H(n_1)+H(\Delta)}{2}\right]^2$$
total welfare is
$$\begin{aligned}
W^{\text{Seq}} &= \frac{1}{9b}\left[a-\gamma-\frac{H(n_1)+H(\Delta)}{2}\right]^2 + \\
&\quad \frac{\lambda}{9b}\left[(a-\gamma)-2H(\Delta)+H(n_1)\right]^2 + \frac{\lambda}{9b}\left[(a-\gamma)-2H(n_1)+H(\Delta)\right]^2 +
\end{aligned}$$
What is the total revenue? From the proof of Proposition 4, we know that Firm 2 pays at the second auction
$$R_2^{\text{Seq}} = \frac{1}{9b}[a-\gamma-2H(n_1+\Delta_2)+H(\Delta_1)]^2 - \frac{1}{9b}[a-\gamma-2H(n_1)+H(\Delta_1+\Delta_2)]^2$$

and at the first auction

$$\begin{aligned} R_1^{\text{Seq}} &= \frac{1}{4b}[a-\gamma-H(n_1+\Delta)]^2 - \frac{1}{9b}[a-\gamma-2H(\Delta_2)+H(n_1+\Delta_2)]^2 \\ &\quad -\frac{1}{9b}[a-\gamma-2H(n_1)+H(\Delta)]^2 \end{aligned}$$

and thus total revenue is

$$R^{\text{Seq}} = R_1^{\text{Seq}} + R_2^{\text{Seq}}$$

- Conclusion: When $a$ gets large, consumer surplus and total welfare are higher under sequential auctions than simultaneous auctions, but total revenue is lower.

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