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Fair Gatekeeping in Digital Ecosystems

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Abstract. Do users receive their fair contribution to digital ecosystems? The frequent accusations of excessive platform fees and self-preferencing leveled at dominant gatekeepers raise the issue of the standard gatekeepers should be held to. The paper provides a framework to explain business strategies and assess regulation. It stresses the key role played by the zero lower bounds on core and app prices in the setting of privately and socially optimal platform fees. Finally, it derives a simple rule for regulating access conditions and analyses its implementation.

Keyworks. Platforms, ecosystems, fair access, price and non-price foreclosure, zero lower bounds.

JEL numbers. L12, L4.

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1 Introduction

Dominant gatekeepers – the platforms controlling "core services" such as search, e-commerce, app stores, or social networking – are often suspected of charging excessive platform fees to business users (merchants, apps, advertisers) and/or practicing self-preferencing (favoring their own offerings when hybrid,[1] i.e., when competing in the markets they operate). This has led to numerous recent investigations and lawsuits concerning platforms such as Amazon, Apple, Booking, and Google;[2] similar questions will probably surface as AI-based platforms in tech and in health come to the fore.

Detecting "excessive fees" and "self-preferencing", the two prongs of the regulators' equity concern (e.g., in the EU Digital Market Act, DMA), is notoriously difficult or costly; and so regulators must pick their fights, which requires looking for smoking gun evidence that business users may overpay or not receive their fair contribution to digital ecosystems. Unfortunately, current regulations contain only broadly scripted prescriptions such as the DMA's requirement that access conditions be fair, reasonable and nondiscriminatory (FRAND). This leaves open the question of what "fair and reasonable" conceptually means, even putting aside the measurement issue. Our paper attempts to fill this void.

Why are policymakers preoccupied with business users' welfare in particular? After all, over twenty years of research have taught us that the "see-saw effect" in two-sided markets (that a price increase on one side increases the profitability of attracting users on the other side and induces a concomitant price decrease there) implies that antitrust analysis should consider the entire market and not just its business side. Similarly, the hypothesis of self-preferencing runs counter the Chicago School argument that a rich ecosystem brings product variety and lower prices, which can be monetized on the consumer side.

The paper builds a framework capable of accounting for existing business strategies and assessing regulation over a rich array of digital platform environments. It explains why there is a good reason to be preoccupied with equity for business users in the context of digital platforms: The important role played by two zero lower bounds (ZLBs) on core and app services (whose prices cannot be negative because of arbitrage and so are most often equal to 0)[3] in the setting of

[1]Pure platform players (like Airbnb or Booking, which operate markets, but do not compete in them) cannot engage in self-preferencing, although they might enter into "sweet deals" with selected business users to the same effect.

[2]Examples of self-preferencing include the 2017 EU Google Shopping decision, the 2021 Google case in Italy (Android Auto did not accept an Enel's app that competed with Google Maps), and investigations into Amazon's prominent display of Amazon-branded goods and favoring its own logistics services (FBA).

Regarding excessive fees, several antitrust cases (Epic Games v. Apple; Spotify v. Apple; 2024 EU investigation of Apple and Google's non-compliance with the Digital Market Act) concern 3rd party apps trying to circumvent the 30% app store fee they deem unfairly high. The clampdown on most-favored-nation clauses similarly aims at capping access fees paid by merchants. Regulators may also directly set caps on access fees. Many local governments in the US introduced caps on the fees that food delivery platforms charge restaurants during the COVID-19 pandemic, and several of them then made their fees caps permanent. The major platforms (Uber Eats, Grubhub and DoorDash) typically charge a 30% fee, and most governments capped these fees to 15%.

[3]As of December 2024, 97% and 95% of apps in Google Play and Apple's App Store were freely available. These include some of the most common 3rd party apps (e.g., PayPal, Dropbox), as well as the competing in-house apps by Apple and Google (e.g., Apple Pay and Google Pay, iCloud and Google Drive, respectively). On the

privately and socially optimal platform fees. Despite their importance for strategic behavior and policy and their endogeneity, the multi-sided-platform literature has mostly ignored ZLBs, or else posited free consumer access to the platform.

From a bird's eye view, platforms, whether search engines, app stores, e-commerce, OTAs, or social networks, all provide business users (merchants, apps, advertisers) with access to the consumers. Business users may thereby sell their goods or services and, importantly, receive ancillary benefits from attracting a consumer: Advertising revenues (content providers), data collection (most apps), sales of premium services (e.g., Dropbox, Spotify, Zoom), or else the future profits attached to repeat purchases and upgrades; we capture these per-consumer ancillary benefits in a given app market by a variable $b > 0$. Ancillary benefits imply that the marginal cost is negative for digital goods, making the app ZLB particularly relevant (in contrast, $b < 0$ for most physical goods, whose marginal production cost must be subtracted when computing the ancillary benefit).

For digital goods, an incentive for self-preferencing arises when the platform is vertically integrated and makes more money by supplying, even for free, the good or app itself than by being paid for giving access, i.e., when $b > a$, where a is the access fee paid by the app or merchant to the platform. A second cost of low or nil access fees is that, even if there is no such foreclosure, they invite entry by me-too apps, that add little value to the ecosystem but extract a nonnegligible share of it, because competition in the app market is hindered by the app ZLB.

While regulation may keep access fees low or nil, laissez-faire in contrast generates extractive access fees that squeeze business users; furthermore, the core ZLB, when relevant, blocks the seesaw effect and prevents consumers from indirectly benefitting from the squeeze. Such extractive fees both induce a suboptimal usage of apps and discourage their creation in the first place.

The paper derives a simple rule for the optimal regulation of access conditions and analyses its implementation. A "Pigouvian rule" ($\hat{a} = b$) allows the 3$^{\text{rd}}$ party apps to capture their contribution to the ecosystem, promoting the right level of innovation; it does so by pricing the unpriced positive externality (ancillary benefit) enjoyed by an app that receives access to the consumer. It also minimizes double marginalization in the set of access fees that do not induce self-preferencing.

We demonstrate that neither the promotion of platform competition nor that of app store competition, two potentially useful interventions aimed at creating multiple paths from business users to consumers, solve the equity concern. Platform competition transfers value from the platform to the consumer but, provided platforms still control access to their consumers, does nothing to solve the equity concern for business users. App store competition (triggered by regulations forcing Apple and Google to host app stores that compete with theirs, so as to promote consumer multi-homing) is not effective if the dominant platform downlists multihoming apps; but even if it is, the absence of access fee makes the platform too app friendly;

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core side, most digital platforms, such as the major app stores, e-commerce platforms, search engines and social networks, grant free access to consumers.

put differently, app store competition requires levying the optimal access fee b, this time from the alternative 3^{rd} party app stores rather than from now-disintermediated 3^{rd} party business users.

The paper then discusses the costs and benefits of various approaches to implementing the Pigouvian rule in the real-world context of heterogenous app sub-markets: regulatory measurement of b (perhaps triggered by an appeal), a constraint on the distribution of access fees set by the platform, and finally elicitation from business users (as is already the case, as we show, for sponsored search and display ads).

Finally, the conclusion lists some take-home points and, pointing at the richness of our framework for analyzing digital platforms by illustrating the four possibilities (according to whether app and core ZLBs are binding or not), shows how ZLBs provide guidance to find smoking gun evidence educating policymakers.

The paper is organized as follows. Section 2 develops the basic framework (making a number of assumptions that we later relax), derives equilibria with and without the possibility of self-preferencing, points at the wedge between profit- and welfare-maximizing access fees, and shows that through appropriate relabeling the analysis accommodates physical devices to access the platform, physical goods rather than digital apps, and asymmetric ancillary benefits. Section 3 extends the framework to endogenous innovation in the app market, and downward-sloping demands for either the platform or for app quality. It derives a simple optimal, Pigouvian regulation in which the optimal access fee is equal to the ancillary benefit ($\hat{a} = b$). Section 4 analyzes two interventions aimed at creating multiple paths from business users to consumers: platform competition and app store competition. Section 5 studies implementation under asymmetric information. Section 6 reviews the relevant literature, and Section 7 concludes. Omitted proofs and additional material can be found in the Online Appendices, A and B, respectively.

2 Impact of the access fee

2.1 Basic framework

Consider a two-sided digital platform (e.g., an app store) that connects sellers of digital goods (hereafter, apps) with a mass 1 of consumers.[4] Digital goods entail negligible marginal costs of production and distribution. Rather, their usage by consumers brings ancillary benefits for the app providers, such as advertising revenues, fees collected from merchants selling their products through the app, consumers' data that can be monetized, or repeat purchases. Hence, their opportunity cost is negative. App developers face a zero lower bound constraint because negative prices are subject to arbitrage: Bots and uninterested consumers may take advantage of the payment for usage, and yet bring no profit for merchants and advertisers and provide

[4]Throughout the paper, we posit that the platform exists. The viability of dominant platforms is not the current concern of antitrust authorities in light of the large profits and substantial market power they gather. Online Appendix B extends the model to entry in the platform market.

valueless data.[5]

Multiple (a mass 1 of) app markets coexist on the platform. We will analyze a representative app market for expositional simplicity; the multiplicity of app markets serves to better motivate the "platform pivotality" assumption below, but is otherwise inessential. In each app market, two apps[6] compete for the platform's customers, a superior (high quality) one and an inferior (low quality) one. Without loss of insight for the equity question, the superior one is owned by an independent app provider: were it owned by the platform, the self-preferencing and squeeze issues would not arise. The platform in contrast may either own the inferior app (the platform is then hybrid), or be a pure platform (the inferior app is also independently owned).

The platform: (i) is a gatekeeping platform and charges its consumers a fixed access price; and (ii) adopts an agency business model (app providers pay access fees for distributing their apps and set their prices). Figure 1 depicts the case in which the platform operates an hybrid marketplace.

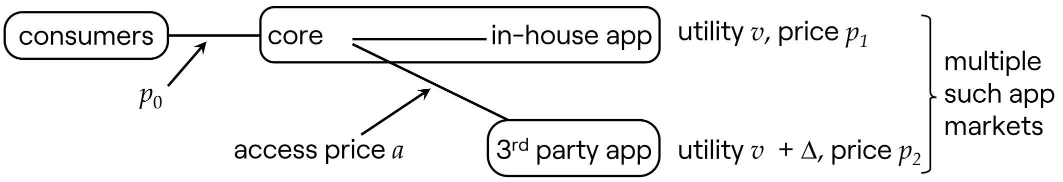

Figure 1: Two-sided market (hybrid case).

Consumers have a unit demand each, and utility $v \geq 0$ (resp. $v + \Delta$, with $\Delta > 0$) when using the inferior (resp. the superior) app. To avoid the standard "openess problem", we assume that consumers select the superior app when indifferent between the two apps ($p_2 - p_1 = \Delta$); and that they buy an app when indifferent between doing so and not buying at all. We assume for the moment that the platform brings no per-se value to consumers (independently of app consumption).

In a representative app market, let $x_2 = 1$ if consumers buy the superior app and $x_1 = 1$ if they buy the inferior one. In the following, $b \geq 0$ denotes the per-consumer ancillary benefit accruing to the app provider, p_0 the consumers' access price to the platform, and p_1 (resp. p_2) the price of the inferior (resp. the superior) app. The platform levies a unit access fee $a \geq 0$ on apps distributed by a 3$^{\text{rd}}$ party provider (Online Appendix B shows that, provided $\Delta > b$, our insights are unchanged considering instead ad-valorem access fees)[7]. The profit of app provider

[5]Alternatively, non-negative price constraints arise because of technical difficulties in operationalizing negative prices – as in the following quote from the Stigler Report (p. 30) *"It is possible that a digital market has an equilibrium price that is negative; in other words, because of the value of target advertising, the consumer's data is so valuable that the platform would pay for it. But the difficulty of making micropayments might lead a platform to mark up this negative competitive price to zero."*

[6]There can be more than two apps, as when there is a fringe of previous generation (i.e., inferior) apps. The case where there is only one app is captured in our notation by $v = 0$.

[7]Under ad-valorem fees, without foreclosure, the platform can capture Δ, which the superior app passes through to consumers, but cannot capture b. Therefore, if $b > \Delta$, self-preferencing is always optimal for an hybrid platform.

$i = 1, 2$ (or division if owned by the platform) is

$$\pi_i \equiv x_i(p_i + b - a).$$

A pure-player platform's profit is

$$\pi_0 = p_0 + a(x_1 + x_2),$$

whilst an hybrid platform's profit can be written as the profit π_0 it would make as a pure platform, plus the extra profit, π_1, its app division makes if it captures the app market – i.e., it is given by $\pi_0 + \pi_1$.

Left unmonitored, the platform can, if it wants, make apps less attractive (e.g., through downlisting). It then chooses $\{\delta_1 \leq 0, \delta_2 \leq \Delta\}$ so that the value of the app i for the consumer becomes $v + \delta_i$. Strict inequalities correspond to non-price foreclosure. In the case of an hybrid platform, self-preferencing corresponds to policy $\{\delta_1 = 0,\ \delta_2 < \Delta\}$. We will employ "non-price foreclosure" and "self-preferencing" indifferently in the hybrid context. The concept of "non-price foreclosure" is broader – to the extent that the hybrid platform won't handicap its own app – as it applies also to the pure-platform case. In contrast, regulatory monitoring of equal access forces the platform to select $\{\delta_1 = 0,\ \delta_2 = \Delta\}$, which eliminates stage (2) below.

Throughout the analysis in this section, we take the access fee a as given (whether set by regulation or by the platform) and consider simultaneous pricing choices.[8] The timing is given in Figure 2.

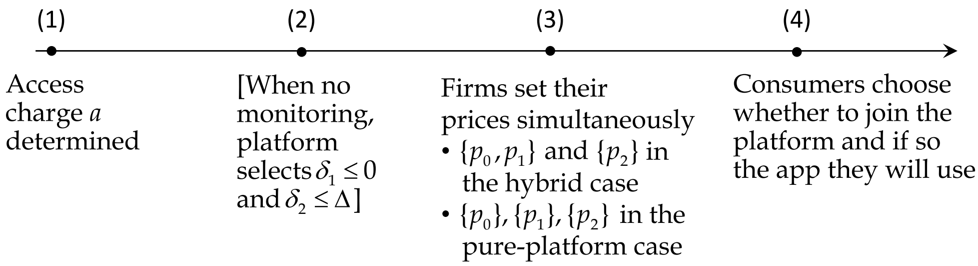

Figure 2: Timing.

Equilibrium concept. The simultaneity of price choices gives rise to a multiplicity of pure-strategy equilibria, as is familiar in Nash demand games – e.g., in the literature on tying (Choi and Stefanadis, 2001, Carlton and Waldman, 2002). We will make the reasonable assumption that the platform is "pivotal" for consumers' participation:

Definition (platform pivotality). An equilibrium of the pricing subgame (stage (3)) exhibits platform pivotality if an independent app i maximizes its profit taking the mass of consumers present on the platform as given; that is, independent app providers do not perceive themselves

[8] That a is set first is natural if this access fee is regulated. Under laissez-faire, the platform needs to commit to a for some time to attract sellers (in reality, access fees charged by major digital platforms are stable over time). Put differently, the timing allows the platform to squeeze the superior 3[rd] party app, but not to hold it up.

as pivotal for the consumers' decision of whether to join the platform.[9]

The platform pivotality assumption is innocuous when the platform faces a downward-sloping demand and there are many apps (see Section 3.2.1), as apps are then too small to have much effect on consumers' participation decisions. Unless otherwise stated, we will focus on the following equilibria (insights are not much affected by this focus; we will also characterize all equilibria):

Definition (equilibrium). An equilibrium of the pricing subgame is a set of pure strategies that (i) are undominated and (ii) satisfy platform pivotality.

Let us introduce some further definitions:

Definition (competitive neutrality). The access fee a is competitively neutral in a range $[\underline{a}, \bar{a}]$ if, in this range, (i) the platform has no incentive to use non-price instruments to foreclose (even if it can), and (ii) the equilibrium profits and the allocation $\{x_i\}_{i=1,2}$ are independent of a over the range.

Definition (fairness and squeeze). The superior app receives its fair share of its contribution to the ecosystem if $\pi_2^*(a) = \Delta$. The superior app is squeezed if (i) the platform does not foreclose it ($\delta_2 = \Delta$), but (ii) $\pi_2^*(a) < \Delta$.

Definition (zero lower bounds). The app zero lower bound (ZLB) binds if $p_1^* = 0$. The core ZLB binds if $p_0^* = 0$.[10]

2.2 Equilibrium in the absence of non-price foreclosure

We first assume that non-price foreclosure is prevented through regulatory monitoring. Note that, in both cases of a pure and an hybrid platform, both apps have the same opportunity cost of selling $(a - b)$. Under platform pivotality, the unique Bertrand equilibrium outcome in undominated strategies corresponds to[11]

$$p_1^* = \max(0, a - b) \text{ and } p_2^* = \min(p_1^* + \Delta, v + \Delta).$$

[9] Letting $x_i(\mathbf{p}^k, \delta^k) \in \{0, 1\}$ denote the demand for app i in market k as function of $\mathbf{p}^k \equiv (p_1^k, p_2^k)$ and $\delta^k \equiv (\delta_1^k, \delta_2^k)$, independent app i maximizes $[p_i^k - (a - b)]x_i(p_i^k, p_j^k)$ over p_i^k.

[10] $p_2 > 0$ always holds in the basic model (the "superior app ZLB" is never binding). This ZLB binds, for low access fees levels, if the demand for the superior app is downward sloping (Section 3.2.2) or it adopts a freemium business model (see Online Appendix B).

[11] Consider a pure or hybrid platform. Prices p_1 strictly below $a - b$ are dominated by price $a - b$: $p_1 < a - b$ would make app 1 regret having won the consumer if app 2 charges an unexpectedly high price. Conversely, prices p_1 above $a - b$ cannot be equilibrium prices. (a) Either the superior app is not constrained by users' willingness to pay ($p_2 < v + \Delta$) and then $p_2 = p_1 + \Delta$ by platform pivotality. The consumers are indifferent between the two apps and so app 1 could gain $a - b > 0$ by lowering its price by ε. (b) Or, if the superior app is constrained by users' willingness to pay ($p_2 = v + \Delta$), then $p_1 \geq v$ for app 2 to win the market and so app 1 is out of the market. This is the case if and only if $a - b \geq v$: Otherwise app 1 could charge $p_1 = a - b(+\varepsilon)$, take the market and gain relative to the presumed equilibrium behavior. Note that in this region of parameters, the exact value of p_1 is irrelevant as app 1 will not be considered by consumers; so there is no loss of generality in positing $p_1 = a - b$.

That is, the app providers do not charge below the opportunity cost $a - b$ in an equilibrium in undominated strategies. The unique equilibrium takes four configurations as a increases:

Muted app competition. When the opportunity cost is negative ($a - b < 0$), app 1 cannot charge an app price p_1 below 0 due to the app ZLB and therefore sets $p_1^* = 0$, while app 2 is priced at $p_2^* = \Delta$. The superior app does not feel the full competitive pressure from app 1, and so makes supranormal profit ($\pi_2^*(a) = \Delta + (b - a) > \Delta$). The consumers obtain surplus $v > 0$ and so $p_0^* = v$.

Access fee neutrality. If the opportunity cost is non-negative ($a - b \geq 0$), the standard Bertrand equilibrium in undominated strategies has $p_1^* = a - b$ (the app ZLB does not bind) and $p_2^* = (a-b)+\Delta$, so that $\pi_2^*(a) = \Delta$ (fair reward): A change in the access fee increases one-for-one both apps' opportunity cost. Access fee pass-through is feasible in our model as long as consumers keep purchasing the app; using the fact that consumers are in equilibrium indifferent between the two apps, $p_0^* = v - (a-b)$, and so it must be that $a - b \leq v$. By making charging the consumer or app 2 for access perfect substitutes, pass-through causes the access fee to be neutral in this region. The neutrality region exhibits the familiar "see-saw property" of two-sided-market theory, in which an increase in the merchant fee translates (in our case one-for-one) into a decrease in the consumer fee. The validity of Chicago School's "rich ecosystem argument"[12] for the hybrid case thus requires an access fee that exceeds the ancillary benefit of attracting consumers on the app.

Squeeze. When $a > b + v$, app 2 can no longer apply mark-up Δ over app 1: $p_2 = p_1 + \Delta = (a - b) + \Delta > v + \Delta$. So, to sell to consumers on the platform it must lower its mark-up and sell at $p_2^* = v + \Delta$. Because the consumers do not benefit from apps, the core ZLB is binding ($p_0^* = 0$) and the access fee is no longer neutral, as the app 2 developer must absorb its increase to keep customers. The app 2 developer's margin is squeezed ($\pi_2^*(a) < \Delta$, with $\pi_2^*(a)$ strictly decreasing in a and $\pi_2^*(b + v + \Delta) = 0$), and the platform appropriates, at least in part, app 2's contribution to the ecosystem.

Superior app's exit. When $a > b + v + \Delta$, app 2 would have to sell at a price below the fee paid to the platform minus the ancillary benefit. It is then excluded from the app market. Such price foreclosure benefits neither the gatekeeper nor app 2's developer.

Proposition 1 (retail prices in the absence of self-preferencing)**.** *Suppose that the platform cannot use non-price instruments to foreclose. Whether the platform is pure or hybrid, the equilibrium is unique. Because consumers are homogeneous, their surplus is extracted ($p_0^* + p_2^* = v + \Delta$), and $\pi_0^*(a) + \pi_2^*(a) = b + v + \Delta$ (and $\pi_1^*(a) = 0$) for all $a \leq b + v + \Delta$. Furthermore,*

[12] The old Chicago School critique of foreclosure theory can be stated for the platform context in the following way: "*Aside from efficiency motives, an hybrid platform (the monopoly segment) has no incentive to foreclose a 3rd party app (an independent player in the competitive market): A rich ecosystem benefits consumers in two ways, product variety and enhanced competition, and allows the platform to raise its consumer price to extract the associated increase in consumer surplus.*"
This argument is akin to several others recommending a focus on price levels, but non on price structures: in public utility regulation (delegation of individual prices to utilities under the umbrella of a price cap solely aimed at reducing the overall price level), in the antitrust of two-sided markets (the see-saw argument and the concomitant recommendation of looking at a single market), and in authorities' agnostic stance with regards to (second- and third-degree) price discrimination.

- *when $a < b$:*

$$\begin{cases} \textit{App ZLB: } p_1^* = 0 \quad \textit{and} \quad p_2^* = \Delta, \\ \textit{Supranormal app profit: } \quad \pi_2^*(a) = \Delta + (b - a) > \Delta, \end{cases}$$

- *when $b \leq a < b + v$:*

$$\begin{cases} \textit{Passthrough: } p_1^* = a - b \quad \textit{and} \quad p_2^* = p_1^* + \Delta, \\ \textit{Fair reward: } \quad \pi_2^*(a) = \Delta, \end{cases}$$

- *when $b + v \leq a \leq b + v + \Delta$:*

$$\begin{cases} \textit{Core ZLB } (p_0^* = 0)\textit{: } p_1^* = a - b \quad \textit{and} \quad p_2^* = v + \Delta < p_1^* + \Delta, \\ \textit{Squeeze: } \quad \pi_2^*(a) = b + v + \Delta - a < \Delta, \end{cases}$$

- *when $a > b + v + \Delta$:*

$$\begin{cases} \textit{App 2's exit: } p_0^* + p_1^* = b + v \textit{ in hybrid platform case; } \pi_i^*(a) = 0 \textit{ for all } i \textit{ in pure platform case}, \\ \textit{Price foreclosure: } \quad \pi_2^*(a) = 0. \end{cases}$$

Proof of Proposition 1. The proof of uniqueness can be found in Online Appendix A. There, we allow for differentiated app markets rather than a representative one – an app market k is then characterized by a triple $\{a^k, b^k, \Delta^k\}$ – and an elastic demand for the platform. ■

Remark (other Nash equilibria). We have focused on the unique equilibrium satisfying undominated strategies and platform pivotality. Are there other Nash equilibria? When $a > b$, there are other Nash equilibria satisfying platform pivotality, in which app 1 charges a price p_1 below its opportunity cost $a - b$ – i.e., $p_1 \in [0, a - b)$ and, as above, $p_2^* = \min(p_1 + \Delta, v + \Delta)$ and $p_0^* = v + \Delta - p_2^*$, provided that app 2 makes a non-negative profit. These equilibria with below-cost pricing involve a squeeze of the superior app (i.e. $\pi_2^*(a) < \Delta$ for all $a > b$). At prices below $a - b$, app 1 would lose money if app 2 were to raise its price and surrender the market to app 1; therefore, such equilibria are ruled out by the requirement (i) of the equilibrium definition that dominated strategies be eliminated. Finally, for all values of the access fee, there are also Nash equilibria in which p_2 is again low, but for another reason: App 2 could internalize the constraint $p_0 + p_2 \leq v + \Delta$, i.e. perceive itself as pivotal (for example it would charge nothing if $p_0 = v + \Delta$); hence the need for condition (ii) of the equilibrium definition as well.

Remark (the link between ECPR and the ZLBs). The equilibrium characterization unveils a simple connection between the ZLBs and Baumol and Willig's ECPR rule for a vertically integrated firm providing access to a rival:

Definition (ECPR level). The access fee is below (equal to, above) the Baumol-Willig efficient component pricing rule level if a is smaller than (equal to, higher than) the unit profit, $p_1 + b$, lost by the hybrid platform when the 3$^{\text{rd}}$ party app attracts a consumer.

Corollary 1 (ECPR). *In equilibrium, the access fee is*

- *below the ECPR level ($a < p_1^* - (-b)$) if and only if the app ZLB binds ($a < b$);*
- *at the ECPR level ($a = p_1^* - (-b)$) if and only if no ZLB binds ($b \leq a \leq b + v$);*

- *above the ECPR level ($a > p_1^* - (-b)$) if and only if the core ZLB binds ($a > b + v$).*

2.3 Self-preferencing and vertical integration

Let us augment the platform's strategy space by letting it engage in non-price foreclosure (the platform is left unmonitored).

Proposition 2 (self-preferencing and vertical integration)**.**

(i) Left unmonitored, an hybrid platform engages in self-preferencing if and only if $a < b$. In contrast, a pure platform never benefits from using non-price foreclosure.

(ii) When $a \geq b$ or when a regulator prevents non-price foreclosure, the ownership of the inferior app is irrelevant. In contrast, if $a < b$ and non-price foreclosure cannot be prevented, the platform (but not society) benefits from vertical integration.

Proof of Proposition 2.

(i) First, we show that the platform has no incentive to engage in non-price foreclosure ($\delta_2 < \Delta$ and/or $\delta_1 < 0$) when both apps are independent. Suppose that $\delta_2 > \delta_1$; if app 2 is not constrained by consumers' willingness to pay, a reduction in δ_2 has no impact on consumer surplus from apps as it decreases the superior app's markup and its value to the consumer by the same amount. In contrast, a decrease in δ_1 reduces app competition and hurts the consumers, so the platform could raise δ_1 as well as the price p_0 (keeping consumer membership constant), thereby increasing its profit.[13] When app 2 is constrained by consumers' willingness to pay, $p_0 = 0$ and so the platform's profit equals $\pi_0 = a$ for all (δ_1, δ_2); then, setting δ_2 below Δ would reduce app 2's profit ($\pi_2 = \delta_2 + b - a$), which the platform can appropriate under laissez faire, and so is never optimal for the platform.

Second, consider an hybrid platform. Without loss of generality, we can assume that $\delta_1 = 0$ and either $\delta_2 = \Delta$ (no foreclosure) or $\delta_2 = -v$ (full foreclosure). Intuitively, the platform's choice determines which among the in-house and 3rd party apps the consumers will select. In the former case, making the 3rd party app worthless involves no loss of generality. In the latter case, picking $\delta_2 < \Delta$ does not benefit the platform. When foreclosing, only the total price $p_0 + p_1$ matters; and so we can assume w.l.o.g. that $p_1 = 0$, and so $p_0 = v$. The platform can achieve profit $v + b$, i.e., the value it creates on a stand-alone basis. When not foreclosing, the platform makes profit $\pi_0^*(a) = \min(v + a, v + b)$ if there is no squeeze (and more when there is a squeeze). So, the platform forecloses if and only if $a < b$.

(ii) As seen in Proposition 1, absent foreclosure, prices are the same irrespective of the inferior app's ownership. Thus, vertical integration into the app segment is profitable if and only if the

[13] More precisely, fix an arbitrary access fee a and let $u(a)$ denote consumer net surplus when buying the best quality-price proposition in the app market. Suppose, without loss of generality, that $\delta_2 \geq \delta_1$; then either the consumer buys no app at all, and so $u(a) = 0$ and the platform further receives no access fee or membership fee, or the consumer buys app 2. In the latter case, the undominated-strategies assumption yields $p_1 = \max(a - b, 0)$ and $p_2 = p_1 + \delta_2 - \delta_1 \geq p_1 + \delta_2$, and so $u(a) \leq (v + \delta_2) - [\max(a - b, 0) + \delta_2] \leq (v + \Delta) - [\max(a - b, 0) + \Delta]$: the consumer's surplus from the app is smaller than in the absence of non-price foreclosure. So foreclosure does not raise access fee revenue and does not allow the platform to raise p_0 either.

hybrid platform has incentives to engage in self-preferencing – i.e., for all $a < b$. Intuitively, for low access fees, vertical integration enables the platform to reap the benefit $b - a$ from foreclosing the superior app; this benefit would go to a 3rd party app provider otherwise. ■

In a nutshell, for $a < b$ (i.e., when the app ZLB binds), a vertically integrated platform does not have enough skin in the game to want to give access to its rival. Existing (GDPR, DMA) and forthcoming regulations aim at restricting the use of data and thereby reduce the ancillary benefit b. Such a decrease in b reduces the incentive for self-preferencing, keeping the access fee constant.

Figure 3 depicts how the platform's and app 2's profits vary with the access fee, with and without self-preferencing, and with and without vertical integration.

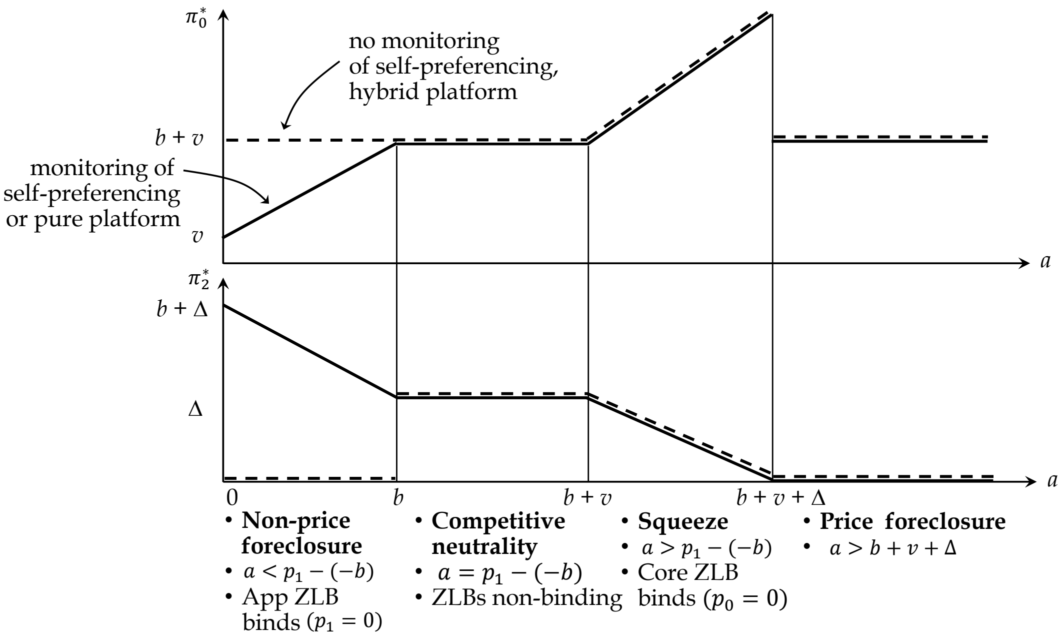

Figure 3: Profits. The dashed lines represent the profits when self-preferencing is feasible and the full lines the profits when it is not. They differ only when $a < b$.

2.4 Platform-optimal, welfare-optimal, and fair access pricing

We define (ex-post) social welfare W as the sum of consumer net surplus S and the firms' profit: $W = S + \sum_{i=0,1,2} \pi_i$.

Proposition 3 (optimal access fees)**.** *When the apps pre-exist the platform's policy:*

(i) Welfare-optimal fees. *Any access fee such that app 2 is not foreclosed maximizes ex-post social welfare. This means any* $a \in [0, b + v + \Delta]$ *in the absence of self-preferencing, and any* $a \in [b, b + v + \Delta]$ *when the platform is hybrid and can engage in self-preferencing.*

(ii) Profit-maximizing access fee. *The platform's profit is maximized at the extractive access fee* $a^* = b + v + \Delta$*, yielding* $\pi_2^*(a^*) = 0$.

(iii) Fair access fees. *The independent developer receives a fair reward for its contribution to the ecosystem if and only if* $a \in [b, b+v]$.

Proof of Proposition 3. Consumer surplus is always extracted by the platform through the access price, and $W^* = b + v + x_2\Delta$ is maximized whenever there is no price ($a > b + v + \Delta$) or non-price foreclosure ($a < b$ if self-preferencing cannot be monitored), so that $x_2 = 1$, from which (i) follows. For $a = a^*$: $\pi_0^*(a^*) = W^*$, which establishes (ii).[14] Finally, the result in (iii) follows from the equilibrium profit $\pi_2^*(a)$ given in Proposition 1. ∎

Remark (other Nash equilibria). We noted that there are other, lower-app-price equilibria when $a > b$. Does the equilibrium selection matter for the fair levels of the access fee?[15] For all $a > b$, all equilibria with $p_1 < a - b$ yield $\pi_2^*(a) < \Delta$. In contrast, our later Pigouvian recommendation $\hat{a} = b$ yields a fair reward and is not subject to the multiplicity issue provided that independent apps do not perceive themselves as pivotal (part (ii) of our equilibrium definition).

2.5 Extensions

The basic model presumes that goods are digital. The value chain may be more complex and embody non-negligible production costs incurred when supplying the devices needed to connect to the platform or in the production of physical goods. In this section, we will let $b^\dagger$ denote the ancillary benefit and show how we can define an adjusted benefit, b, such that the analysis carries over to such environments with this adjusted benefit. Finally, we show that under another simple relabellisation the model applies to asymmetric ancillary benefits.

2.5.1 Physical goods

Our analysis is applicable to platforms hosting sellers of physical goods (e.g., e-commerce platforms such as Amazon and eBay) or services (e.g., OTAs or ride-hailing platforms such as Booking.com and Uber) that entail positive marginal costs.

Intuitively, the cost of physical goods makes the "app" ZLB less likely to bind and reduces concerns about self-preferencing. Let $c_a > 0$ denote this unit cost. With ancillary benefit $b^\dagger > 0$, the adjusted ancillary benefit b is then

$$b \equiv b^\dagger - c_a.$$

If this adjusted ancillary benefit is non-negative, the self-preferencing region in the hybrid case is still $a < b$; otherwise it is empty.

[14] This full-squeeze result rests on the assumptions that the superior app faces no entry cost and has no alternative ways to reach the platform's consumers. Else, the unregulated platform would set a lower access fee, allowing the superior 3rd party developer to recover entry costs and/or to match its option to reach consumers through other, less efficient, distribution channels – e.g., own website rather than app (see `https://vwo.com/blog/10-reasons-mobile-apps-are-better`).

[15] The welfare-optimal and profit-maximizing access fees are as in Proposition 3 (i)-(ii) in all Nash equilibria satisfying platform pivotality.

Proposition 4 (physical goods)**.** *Letting* $b \equiv b^\dagger - c_a$ *(the difference between the ancillary benefit and the cost of the final good), the self-preferencing region is empty whenever the adjusted ancillary benefit is negative* ($b < 0$)*. The platform-optimal level,* $a^* = v + b + \Delta$*, strictly exceeds the fair levels of the access fee* $a \in [\max\{b, 0\}, v + b]$*.*

2.5.2 Physical devices

Consumers can access an app store (Apple App Store, Google Play, Microsoft Store...) only upon purchase of a costly physical device (smartphone, laptop, or game console). To what extent must Propositions 1 through 3 be amended to account for this extra layer between the consumer and the gatekeeper? Suppose that the device brings stand-alone value v_d, the same for all consumers. For example the smartphone can be used for "non-gatekeeping purposes" such as taking pictures and making calls; likewise desktops have other usages than supporting services intermediated by a gatekeeper. Let c_d denote the device's production cost (the ancillary benefit is again denoted $b^\dagger$). As in the basic model, all prices are set simultaneously and consumers then take their consumption decision.

If the device is "cheap" relative to its stand-alone value, that is if $v_d \geq c_d$, consumers own the device regardless of the app store policy and of whether there is a competitive original equipment manufacturers (OEM) sector or a monopoly, vertically integrated platform, and so Propositions 1 through 3 are literally unchanged. The platform squeezes the superior app by setting $a^* = b + v + \Delta$, with $b \equiv b^\dagger$.

Next consider a "costly" device, i.e. $v_d < c_d$.

(a) Suppose first that the platform is *not vertically integrated into device manufacturing* (and cannot subsidize device manufacturers); instead, the device (say, an Android-powered smartphone) is manufactured by a competitive OEM industry. The device is sold at cost, i.e. at price c_d.

The consumer's surplus from the apps must exceed $c_d - v_d > 0$. For that reason, the equilibrium cannot be in the squeeze region, as this would imply that app 2 is constrained by, and charges the consumer's willingness to pay for the app. This means that app 2 must receive Δ and that the gross surplus to be divided between the consumer and the platform is $v + b^\dagger$. Letting

$$b \equiv b^\dagger - (c_d - v_d),$$

the platform must set $a \leq b + v$.[16] The market exists if and only if $v + v_d - c_d = v + b - b^\dagger \geq 0$, because the core ZLB prevents the ancillary benefit $b^\dagger$ from being passed through to consumers absent vertical integration into the device segment (as would be the case of $v + \Delta$ if app 2 were squeezed).

(b) *When the device is produced by a vertically integrated platform* (as is the case for, e.g., videogame platforms) or the platform can subsidize device manufacturers, the core ZLB may

[16] Any $\{a \leq b + v,\, p_0 = b + v - a\}$ gives the same profit $v + b$ to the platform.

be circumvented by subsidizing the device. The platform can then (i) squeeze the superior app by setting $a^* = b^\dagger + v + \Delta$, and (ii) charge price v_d for the device (implying a loss $c_d - v_d$ per device) and $p_0 = 0$ for the app store. Concretely, the platform can bundle device and app store and sell the bundle at v_d (in contrast, lowering a to keep apps cheap may attract consumers, but is suboptimal for the platform, which no longer squeezes the superior app). The market exists provided that the total surplus is positive: $v + b + \Delta \geq 0$.

Proposition 5 (devices)**.** *Suppose that the device is produced by a competitive sector. Let $b^\dagger$ denote the ancillary benefit and $b \equiv b^\dagger - \max\{0, c_d - v_d\}$ denote the adjusted ancillary benefit.*

(i) When the device is cheap ($b = b^\dagger$), Propositions 1 through 3 are literally unchanged as the consumers own it irrespective of the platform's policy.

(ii) When the device is costly ($b = b^\dagger - (c_d - v_d)$):

- *When the platform is not vertically integrated into device manufacturing, the market exists if and only if $v + b - b^\dagger \geq 0$. The platform optimally sets $a^* = b + v$ and $p_0 = 0$. It does not squeeze the superior app.*
- *When the platform is vertically integrated into device manufacturing, the market exists if and only if $v + b + \Delta \geq 0$. The platform optimally sets $a^* = b^\dagger + v + \Delta$ and $p_0 = 0$, and provides subsidy $(c_d - v_d)$ for the purchase of the device (equivalently it bundles device and app store at price v_d). It squeezes the superior app, which provides the platform with an incentive to vertically integrate into device manufacturing.*
- *Preventing self-preferencing and guaranteeing a fair reward requires $b^\dagger \leq a \leq b + v$; but if $v + b - b^\dagger < 0 < v + b + \Delta$, allowing vertical integration and (at least "some") squeeze is necessary for the ecosystem's viability.*

An interesting result here is that vertical integration into devices can enable ecosystem viability. The intuition relates to the familiar Tinbergen rule requiring at least as many independent instruments as there are targets: the access fee alone cannot achieve two contradictory goals. To make the platform more attractive to consumers, a must make apps cheap and therefore be low. But that may not suffice to induce the consumers to purchase the costly device. The second instrument is a platform subsidy to such purchases. In turn, the vertically integrated platform may not want to pay this subsidy unless the apps themselves are put to contribution through a high access fee. This high fee can be offset on the user side through the subsidy instrument.[17]

Remark (valuable core services). The analysis of Proposition 5 (i) applies beyond physical devices. Indeed, consumers attach per se value to Google's search engine and Facebook's social network, which are core products. This case is equivalent to the above analysis with a platform manufacturing the device and $v_d > c_d = 0$, where v_d is now the consumption value of the core (rather than the device).

[17] Note also that the possibility that some squeeze is needed to ensure viability applies not only to the case of costly physical devices but also when entry costs in the core segment are very large, and so the viability of the platform is not a foregone conclusion.

2.5.3 Asymmetric ancillary benefits

So far, we assumed that all apps reap the same benefit from app distribution. This need not be the case. First, the platform may obtain a share of benefits when a 3^{rd} party app is sold, as Google does when sharing data with independent apps in its ecosystem. Second, the benefits from app distribution may depend on provider-specific features. We therefore allow the ancillary benefit to take the form $b^{\dagger}_{ix_i}$ for app i, where $x_i = 1$ if the consumer chooses app i and $x_i = 0$ otherwise. Letting $b_1 \equiv b^{\dagger}_{11} - b^{\dagger}_{10} > 0$ (with $b^{\dagger}_{10} > 0$ in the hybrid platform case and $b^{\dagger}_{10} = 0$ in the pure platform case) and $b_2 \equiv b^{\dagger}_{21} - b^{\dagger}_{20} = b^{\dagger}_{21} > 0$, and assuming, without loss of generality,[18] that $\Delta > b_1 - b_2$, the analysis carries through with appropriate modifications:

Proposition 6 (asymmetric ancillary benefits)**.** *With asymmetric ancillary benefits $b^{\dagger}_{ix_i}$, denoting $b_1 \equiv b^{\dagger}_{11} - b^{\dagger}_{10}$ and $b_2 \equiv b^{\dagger}_{21} - b^{\dagger}_{20}$:*

(i) An hybrid platform has an incentive to engage in self-preferencing if and only if $a < b_1$.

(ii) Any $a \in [b_1, b_1 + v]$ yields a fair reward to the superior app; these levels are strictly lower than the platform's profit-maximizing fee $a^* = v + b_2 + \Delta$.

Proof of Proposition 6. Irrespective of the ownership of the inferior app, equilibrium app prices are $p_1^* = \max(a - b_1, 0)$ and $p_2^* = \min(p_1^* + \Delta, v + \Delta)$. The 3^{rd} party app's profit is then $\pi_2^* = \Delta + (b_2 - b_1)$ in the neutrality region $[b_1, b_1 + v]$, and $\pi_2^* = b_2 + (v + \Delta) - a$ in the squeeze region. So $a^* = v + b_2 + \Delta$. The superior 3^{rd} party app's contribution to the ecosystem is now $\Delta + (b_2 - b_1)$. Therefore, picking access fees in the neutrality region creates neither a squeeze nor an incentive for self-preferencing, and yields the fair reward. ■

Remark (application). The end of current arrangements under which the platform shares with its apps their data would thus increase the incentive for self-preferencing (by reducing $b^{\dagger}_{10}$ and so increasing b_1); put differently, such a move would have to be accompanied with increased regulatory monitoring and/or reduced regulatory pressure on access-fee setting.

Remark (freemium model). The freemium model can be studied within our framework. The apps come in a basic version, that the consumers can try. If satisfied by the app and if they need/want, they can buy an upgraded version, a paying premium version. The expected profit made by the app on the premium version can be modeled as an ancillary benefit associated with the consumption of the basic version of the app, which is split between the app and the platform (as the platform levies a fee on the sale of the premium version). The app ZLB can then bind for both apps on their basic version, vindicating the "freemium" terminology (see Online Appendix B for a simple example).

[18] $\Delta + b_2 < b_1$ would mean that the "superior" app creates less total value than the "inferior" app, a case that we noted is uninteresting.

3 Pigouvian regulation

In the simple model of Section 2, all outcomes in which there is no (price or non-price) foreclosure are equivalent from a total welfare standpoint, since consumer surplus is entirely captured by the platform directly via the consumer price p_0 or indirectly through the impact of the access fee a on the price of apps, and superior 3rd party developers' margin squeeze has only redistributional effects. Hence, even the fully extractive unregulated outcome ($a = a^*$) is socially efficient, so that one might argue that there is little scope for regulation.

In this section, we first show that this indifference is unwarranted if the introduction of a superior 3rd party app in the marketplace is endogenous and depends on the independent developer's incentives to invest in product innovation (Section 3.1). We then extend the model by allowing for heterogeneous consumer valuations (Section 3.2), deriving a Pigouvian principle that underlies optimal access fee regulation in more general environments. Concretely, we show that the regulated access fee should coincide with the ancillary benefit associated with app distribution:

$$\hat{a} = b.$$

The reason why this can be interpreted as the Pigouvian level of the access fee is that b represents an unpriced externality that is internalized when app suppliers are charged $\hat{a} = b$.

3.1 Endogenous innovation

Suppose that, absent innovation, both apps bring value v to consumers. In this case, perfect Bertrand competition in the app market implies that app 2 makes zero profit, whereas the pure or hybrid platform obtains $b + v$. Upon observing the access conditions, app 2 decides whether to sink a cost $\gamma > 0$ to introduce a superior version of the app, which brings an extra-value Δ to consumers. Suppose that the development cost γ is distributed according to a smooth cdf $G(\gamma)$, with density $g(\gamma)$ and monotone hazard rate on $\mathbb{R}^+$, and its realization is privately observed by the developer.

Proposition 7 (fairness and innovation)**.** *With privately observed app developer's cost distributed according to cdf $G(\cdot)$ with density $g(\cdot)$ and monotone hazard rate, welfare is maximized at $\hat{a} \in [b, b + v]$. These levels are strictly lower than the platform's profit-maximizing level $a^* \in (b + v, b + v + \Delta)$, given by*

$$a^* = b + v + \frac{G(v + \Delta + b - a^*)}{g(v + \Delta + b - a^*)}.$$

Proof of Proposition 7. For $a < b$, either the anticipation of being foreclosed gives the 3rd party developer no incentive to innovate, or the supranormal profit obtained if the platform is a pure player or foreclosure is monitored implies that some socially inefficient innovations – namely, those with development cost $\gamma \in (\Delta, \Delta + b - a]$ – are undertaken. For $a \in [b, b + v]$, the innovation takes place if and only if $\gamma \leq \Delta$, i.e. whenever it is socially optimal. For any $a > b + v$, the 3rd party developer innovates if and only if $\gamma \leq v + \Delta + b - a$, so that the platform's expected profit is $\pi_0^*(a) = v + b + [a - (b + v)]G(v + \Delta + b - a)$, which is maximized at $a^* > b + v$ characterized above. This implies that, under laissez faire, some socially optimal innovations – namely, those with development cost $\gamma \in (v + \Delta + b - a^*, \Delta]$ – are not undertaken. ■

Squeezed 3rd party sellers have a suboptimal incentive to develop their apps. The impact of the access fee on the richness of the ecosystem is accounted for by the platform, but incompletely so. As a result, an inefficiently low amount of innovation takes place under laissez-faire. Capping, by regulation, the access fee to any level in the competitive neutrality region (i.e., $a \in [b, b + v]$) is needed to maximize social welfare. As noted in Section 2.4, $\hat{a} = b$ is the only access fee that maximizes welfare for any equilibrium satisfying platform pivotality. Considering independent developers' innovation incentives unveils a natural link between fair access pricing and welfare maximization.

Remark (when the consumer standard and welfare maximization coincide). The call for 3rd-party developer reward Δ was made from the point of view of efficiency/welfare maximization. A consumer standard might seem to lead to a social demand for some "taxation" of innovation in the form of a squeeze on app profits, provided that the increase in access fee is passed through to consumers via a reduction in the core price. But there is here no trade-off between consumer and innovator surplus in this intermediated environment because there is no pass through (the squeeze region coincides with the core ZLB one); and so the consumer standard and welfare maximization lead to the same conclusion on the innovation front. This conclusion is robust to platform or app store competition (Section 4), because the core ZLB again binds and so there is zero pass-through to consumers.

Remark (how plausible is "excessive innovation"?). Our second remark relates to the excessive innovation that arises when the access fee lies below the ancillary benefit ($a < b$) and vertical integration is not an option or self-preferencing can be monitored. One may be suspicious of worries about excessive innovation. Yet, this possibility is natural in the digital economy. A me-too innovation in the app segment, bringing along a small improvement $\Delta = \varepsilon$ in app quality, allows the innovator to corner the app market, engendering profit $(b - a)$. Similarly, assuming that entry can occur at value proposition v (no innovation relative to the current generation), me-too entry will occur as long as the investment cost is smaller than b/n, where n is the number of active apps, when there is no access fee for example: The ancillary benefit is thus dissipated.

3.2 Consumer heterogeneity

This section allows consumers to differ in the overall demand for the platform (Section 3.2.1) or in their demand for the superior apps relative to the in-house ones (Section 3.2.2). These extensions, as well as the study of platform and app store competition in Section 4, allow us to test the robustness of our insights; they also generate a positive consumer surplus and thereby introduce a meaningful distinction between the welfare standard and the consumer standard.

3.2.1 Elastic platform demand

Assume that consumers directly derive utility from the core service, independently of apps. Their willingness to pay for the core service, v_c, is heterogeneous, has wide support (in $\mathbb{R}$), and is distributed according to a smooth cdf $F(v_c)$ with density $f(v_c)$, and monotone (inverse) hazard rate $\rho(v_c) \equiv [1 - F(v_c)]/f(v_c)$. A negative value of v_c corresponds to a learning or an opportunity cost. To more easily identify the value created by apps from that created by the core, let $v_a \equiv v$ denote the gross value created by the inferior app (and $v_a + \Delta$ that created by the superior one).

In the absence of vertical integration, the two independent apps take platform consumer membership as given (part (ii) of our equilibrium definition), and play the undominated Bertrand equilibrium (part (i) of our equilibrium definition), which is unique and equal to $\{p_1^* = \max(0, a - b), p_2^* = \min(p_1^* + \Delta, v + \Delta)\}$.

Things are more complex in the vertical integration case, as reductions in p_1 and in p_0 are alternative ways of attracting more consumers to the platform. An hybrid platform may want to set app prices p_1^k that would be "too low" (dominated) from the point of view of the narrowly-construed app-market-k profit π_1^k: a low app price attracts more consumers to the platform, and thereby generates additional consumer fees and, unless $a = 0$, access fee revenue in other app markets as well. Thus, the concept of undominated strategy must be interpreted at the multi-product level $\{p_0, \{p_1^k\}\}$ for the hybrid platform.

In contrast, even if platform pivotality is not invoked, when the number of app markets is high enough, it is still the case that the 3$^{\text{rd}}$ party apps do not feel responsible for attracting consumers to the platform and so maximize their profit, taking platform membership as given. Put differently, the platform pivotality assumption is always satisfied with a large number of (independent) app markets, because the presence and price of an individual (superior) 3$^{\text{rd}}$ party app in one app market has a negligible impact on consumers' overall utility from access to the platform.[19]

If there are no superior apps (or if the superior apps are foreclosed) and the platform is hybrid, both the core and app ZLBs bind ($p_0 = p_1 = 0$) whenever

$$\arg \max_{\{p_0 + p_1\}} (p_0 + p_1 + b)[1 - F(p_0 + p_1 - v_a)] \leq 0 \iff b \geq \rho(-v_a),$$

given that the cutoff is given by $v_c^* + v_a = p_0 + p_1$. As this is a novel feature compared with the foregoing analysis (where at most one ZLB binds), we restrict attention to this region of parameters:

[19] A heuristic proof goes as follows: Letting v_c^* denote the cutoff and A the total app revenue made by the platform, the FOC with respect to p_0 yields $1 - F(v_c^*) = f(v_c^*)(p_0 + A)$. Suppose there are K identical app markets (instead of a continuum), each with weight $1/K$. Suppose that the superior app in market k reduces its price by ε. The increase in profit for the app is equal to $-\varepsilon[1 - F(v_c^*)] + f(v_c^*)(\varepsilon/K) \propto (1/K) - (p_0 + A)$; it is easy to show that the per-consumer platform revenue $p_0 + A$ is bounded away from 0. And so, for K large enough, the superior app has no incentive to lower its price below the maximal level that allows it to capture the market.

Proposition 8 (elastic platform demand). *Augment the basic model by adding a consumer utility from core services, v_c, distributed according to a smooth cdf $F(v_c)$ with density $f(v_c)$ in $\mathbb{R}$, and monotone (inverse) hazard rate $\rho(v_c) \equiv [1 - F(v_c)]/f(v_c)$. Suppose that $b \geq \rho(-v_a)$ where v_a is the value of the inferior apps, so that the core and app ZLB both bind ($p_0 = p_1 = 0$) when the superior apps do not exist or are foreclosed.*

(i) If left unmonitored, an hybrid platform engages in self-preferencing if and only if $a < b$. If $a < b$, and the platform is a pure player or cannot engage in self-preferencing, the two ZLBs bind if and only if $a \geq \rho(-v_a)$.

(ii) In the class of access fees that do not trigger self-preferencing (and in the entire class of access fees when self-preferencing cannot be prevented), welfare is maximized for $\hat{a} = b$. In contrast, under laissez-faire, $a^* \geq \hat{a}$.

Remark (equilibrium multiplicity). When the platform is hybrid, a continuum of equilibria that satisfy conditions (i)-(ii) of our equilibrium definition exist when $a > b$. In particular, there exists a strictly increasing function $\bar{p}(a)$ for $a \geq b$, with $\bar{p}(b) = 0$ and $\bar{p}(a) < a - b$ for $a > b$, such that for any $p_1 \in [0, \bar{p}(a)]$, the triple $\{p_0 = 0, p_1, p_2 = p_1 + \Delta\}$ is an equilibrium. As noted above, such below-cost pricing would not emerge in equilibrium if the inferior app were independently owned. This implies that the platform has incentives to vertically integrate even for $a > b$. In these circumstances, vertical integration, by intensifying price competition in the marketplace, is socially beneficial.

Yet, similar to the basic model, only setting $\hat{a} = b$ ensures that consumers can find the superior app at the lowest possible price ($p_2 = \Delta$) in any Nash equilibrium satisfying platform pivotality; the (pure or hybrid) platform may instead optimally set a higher access fee to profitably squeeze the superior app providers.

Remark (lower bound and the rich ecosystem argument). When $b > \rho(-v_a)$, the core ZLB necessarily binds, with the implication that the platform no longer passes through the increase in ecosystem quality to the consumers. The platform's incentive to provide a rich ecosystem rather than extract the business users' surpluses through squeezes and self-preferencing depends on whether, at the margin, the platform can monetize the ecosystem on the consumer side. The core ZLB is a simple and robust reason why such monetization may be infeasible. But it is not the only reason; suppose, e.g., that, in contrast with our specification, the inframarginal consumers value the app store more that the marginal ones. Online Appendix B develops a simple such environment. Consumers get utility from two services: a non-platform one (say, pictures and calls for an iPhone) and, for a subset of them only, the app store. Then, a marginal improvement in app store quality does not induce the platform to lower its price on the device, as this improvement is valued only by inframarginal users. There is de facto a lower bound, with similar implications as a core ZLB, but it is not 0. This second reason for the absence of pass-through is reminiscent of Spence's (1975) observation that a monopolist's incentive to (over or under) supply quality depends on the relationship between the marginal and the average consumer's willingnesses to pay for quality, where "quality" in our context can be understood as "low app prices".

3.2.2 Heterogeneous preference for the superior app

This section introduces heterogeneity with respect to the perceived extra quality of the superior app: One might think of the superior app as adding a functionality relative to the rival one; consumers value this extra functionality diversely. We assume that for a given consumer Δ is the same across app markets.

Proposition 9 (efficient choice of app)**.** *Suppose that consumers have heterogeneous valuations Δ for the extra value brought about by the superior apps (distributed according to a smooth cdf $H(\Delta)$ with support $\mathbb{R}^+$ and a monotone hazard rate). Left unmonitored, an hybrid platform engages in self-preferencing if and only if $a < b$. Consumer surplus and social welfare are then uniquely maximized at $\hat{a} = b < a^*$.*

By encouraging an excessive consumption of the in-house apps, platform's below-opportunity-cost pricing in the competitive segment, which emerges in the unique Nash equilibrium for all $a > b$,[20] harms both consumers and the superior 3$^{\text{rd}}$ party app providers. Thus, in the absence of monitoring of self-preferencing, even neglecting fairness considerations, optimal access fee regulation must follow the Pigouvian principle.

While an in-house inferior app is priced below-cost, a 3$^{\text{rd}}$ party seller would price its inferior app above cost. This implies that the platform always has an incentive to vertically integrate to exert competitive pressure on the superior app providers – i.e., Proposition 9 applies if platform business model is an endogenous choice.

If, on the contrary, vertical integration is not an option, or the platform is hybrid but non-price foreclosure can be monitored, then consumer surplus and social welfare would be maximized by granting free access to the marketplace (i.e., for $a = 0$), though this could raise viability concerns for the platform and excessive innovation on the app side.

3.3 Summing up the normative analysis

Welfare in the digital ecosystem depends on the availability and consumption of creative apps. This in turn focuses attention on

- *The absence of self-preferencing*, which always amounts to the access fee exceeding the ancillary benefit: $a \geq b$.
- *A fair compensation for the 3rd party app developers.* In the basic model, this is ensured by $a = b$.

[20] The mechanism is similar to the one at play in Chen and Rey (2012), who provide a rationale for loss leading in the retailing industry. By pricing the competitive good below cost, and raising the price for the monopolized good (that is, consumers' access price) accordingly, the platform: (i) maintains the total price charged to consumers with low (extra-) willingness to pay for the 3$^{\text{rd}}$ party app (corresponding to one-stop shoppers in Chen-Rey), who buy the in-house app; (ii) increases the margin earned on those with higher willingness to pay, who buy the 3$^{\text{rd}}$ party app (Chen-Rey's multi-stop shoppers) in the monopolized segment; and (iii) induces the 3$^{\text{rd}}$ party app to reduce its price (hence, squeezes its margin).

- *The minimization of double marginalization.* Double marginalization in the value chain is linked with a high access fee, which jeopardizes the consumption of superior apps, directly and indirectly. By raising the price of the superior app, a higher access fee triggers an excessive substitution toward the inferior app. A high access fee also reduces the usage of the platform (and therefore that of the superior app) if it cannot be compensated by a reduction in the core price, i.e., if the core ZLB binds. Both distortions (insufficient consumption of the superior app and of the platform) call for as low an access fee as possible.

Combining the three desiderata, we see that

(i) If it is hard to monitor self-preferencing, the optimal access fee obeys the Pigouvian rule:

$$\hat{a} = b.$$

(ii) A regulator opting for the monitoring of self-preferencing can reduce double marginalization relative to the Pigouvian rule, but then occurs two costs: that of monitoring and the incentivization of me-too app development.

4 Contested bottlenecks

Does either platform or app store competition eliminate the scope for access fee regulation? To answer this question, a prior query is "do platform and app store competition promote multi-homing?"; for, it is well known that consumer single-homing on an intermediary (platform or app store) provides this intermediary with the monopoly of access to the consumer, regardless of whether it had to compete with other intermediaries to enlist the consumer. Accordingly, the intermediary is a "gatekeeper" or a "bottleneck" pursuant to acquiring the consumer, and can sell access to this consumer at a monopoly price (a^* in the basic model), with the negative consequences that we described earlier. On the other hand, whether a platform captures single-homing consumers could depend on the fee a, high fees inducing high app prices; so we need to look into the mechanics of competition to become the bottleneck.

Even in the presence of competition among several intermediaries, consumer multi-homing may not emerge for at least two reasons: (a) the intermediary is associated with a costly device (few people have both an iPhone and a Samsung), and (b) habit formation and familiarity imply that consumers may multi-home in membership and single-home in usage (most consumers use Google search even though also Bing is available on any browser, or systematically consult Booking even though they have as easy an access to Expedia).

In this section, which returns to the framework of Section 2, independent apps multi-home (indeed most popular 3$^{\text{rd}}$ party apps are available on both Apple's and Google's app stores: Bresnahan et al., 2015). By convention and to illustrate the two polar cases, we talk about "competing platforms" when consumers single-home (maybe, but not necessarily because of a costly device) and "competing app stores" when consumers multi-home on rival intermediaries. Needless to say, this is a highly stylized representation; applying the theory to a specific environment requires making assumptions about how much multi-homing the introduction of competition induces.

4.1 Platform competition

Consider $N \geq 2$ (symmetric) competing platforms, indexed by i (Figure 4). Each platform is hybrid (the case of pure-player platforms is discussed below) and owns an inferior app (valued v by consumers) in the representative app market; the independent superior app (valued $v+\Delta$) multi-homes on all platforms. Let $U^i \equiv u^i - p_0^i$ denote consumers' net value from access to platform i's ecosystem, where $u^i \equiv \max\{v - p_1^i,\ v+\Delta - p_2^i,\ 0\}$, and $\{p_0^i, p_1^i, p_2^i\}$ are consumers' access price, in-house and 3rd party app prices on platform i, respectively. To analyse platform competition in the starkest way, we consider perfect competition. That is, we suppose that all consumers patronize only the platform offering the highest net value U^i. As a tie-breaking condition, we assume that platforms offering the same utility split equally the demand, though this does not affect our results. The timing is the same as with a single platform: (1) The access fees $\{a^i\}$ are selected either by the platform or through regulation; (2) The platforms select the 3rd party app's realized quality advantage $\{\delta_2^i\}$;[21] (3) The platforms and the representative apps select their prices $\{p_0^i, p_1^i\}$ and p_2^i; (4) Consumers choose their platform, and their app on that platform. We can skip the self-preferencing decision (the choice of δ_2^i) because under perfect platform competition, a platform has no incentive to degrade its ecosystem even if $a^i < b$.

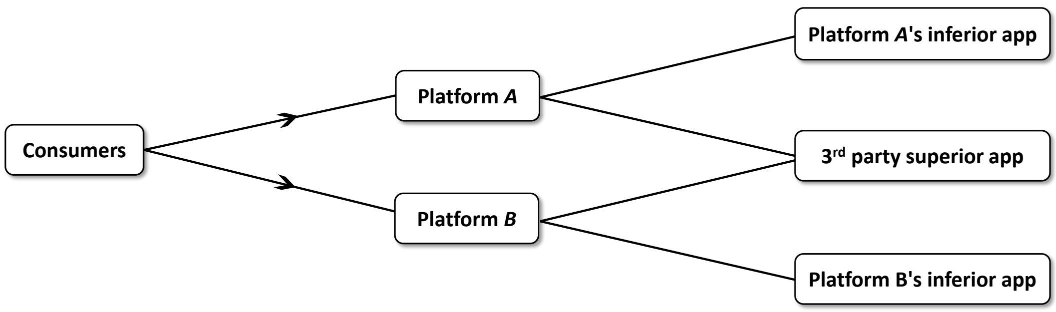

Figure 4: Competing (hybrid) platforms ($i = A, B$) under consumer single-homing.

In equilibrium, all platforms offer the same net utility $U^* = v$ to consumers and the core ZLB binds. The presence of platform competition forces hybrid platforms to price their in-house app at zero even if $a^i > b$. Indeed, because in equilibrium consumers are indifferent between the in-house and the 3rd party app, any $p_1^i > 0$ would give room for platform i to undercut its rivals. By the same reasoning, the core price p_0^i must be equal to 0. The analysis is similar to that with a monopoly platform in which the core ZLB binds; indeed, it is optimal for each platform to fully squeeze the 3rd party app: $a^* = b + \Delta$. The only difference with the monopoly platform case is a transfer of value v from the platform to the consumers.

[21] It is straightforward to check that, in this simple model, whether decisions in stages (1)-(2) are observed by rival platforms is immaterial to the results.

Proposition 10 (platform competition)**.** *Consider $N \geq 2$ identical competing hybrid platforms, indexed by i.*

(i) Laissez-faire. *In the laissez-faire equilibrium, both ZLBs are binding ($p_0^i = p_1^i = 0$) and $p_2^i = \Delta$. All platforms select access fee $a^* = b + \Delta$ and make profit $(b + \Delta)/N$ each. The core ZLB prevents total platform profit $b + \Delta$ from being competed away. Consumers receive net surplus v each, and the 3rd party app is fully squeezed.*

(ii) Access fee regulation. *A regulator concerned with fairness optimally sets $\hat{a} = b < a^*$, yielding per-platform profit b/N and 3rd party app profit Δ. Consumers still receive net surplus v each.*

The laissez-faire result aligns with the conventional wisdom in platform economics[22] that the multi-homing side does not benefit from platform competition, while the single-homing one (the competitive bottleneck) does, because the platform is the gatekeeper for users on the single-homing side: Platform competition allows consumers to get positive net surplus v. The novel feature of our framework is that perfectly competing platforms collectively earn a high profit $(b + \Delta)$ under laissez faire. The first component of this unit profit is the ancillary benefit from app distribution; the second component is the value brought about by superior app developers, which is extracted through the access fee squeeze. Both revenues are not competed away by price competition because of the core ZLB.

Thus, enforcing the Pigouvian rule through a cap on the access fees is still needed to guarantee independent app developers' proper incentives to invest, even in the presence of fierce platform competition.

Remark (vertical integration into device manufacturing). Let us add a physical device (as in Section 2.5.2). When platforms are vertically integrated into device manufacturing, competing platforms can pass through to consumers, via a below-cost price of their devices, the profits earned by squeezing 3rd party apps through an access fee $a^* = b + \Delta$: As long as the core ZLB does not bind, access fees above the Pigouvian level in this case benefit consumers. However, under the welfare-oriented criterion with endogenous app entry, the optimal access fee is $\hat{a} = b$.[23] Platforms that do not manufacture their own device may similarly circumvent the core ZLB constraint on the app store by subsidizing external device manufacturers provided that the latter's devices is incompatible with rival platforms.

[22] See Caillaud and Jullien (2003), Armstrong (2006), Armstrong and Wright (2007) and, more recently, Teh et al. (2023). Armstrong and Wright (2007) explore the implications of a ZLB constraint on the access price charged to the single-homing side, which competing platforms would like to subsidize.

[23] That is, as each app store earns a per consumer (provided the 3rd party app is viable: $a \leq b + \Delta$), devices including access to the app stores would be sold at $c_d - a$ in equilibrium, as long as the core ZLB does not bind (or, more generally, the device price is not so low as to attract users that are not interested in the apps). In these cases, consumers would reap the benefits from the margin squeeze of the superior apps. Note that we have assumed that these apps already exists. If not, the prospect of being fully squeezed will discourage them from entering, even for a small entry cost. To remedy this, platforms may voluntarily cap their business users' access fees. Assuming that such a commitment is feasible, it still would not bring about a fair access fee. As long as app developers have negligible multi-homing costs, 3rd party app entry is a public good from the point of view of platforms, and free riding would be expected (this would necessarily be the case if Δ were random): see Jeon and Rey (2024).

Remark (pure-player platforms). The results of Proposition 10 hinge on the assumption that platforms are vertically integrated into the app segment: If the low-value apps were offered by (single- or multi-homing) 3$^{\text{rd}}$ party providers, then $a^* = b$ would prevail in the laissez-faire equilibrium, which would eliminate the scope for regulation. The reason is that, as non-pivotal 3$^{\text{rd}}$ party apps set their prices as in the basic model, the superior app is priced at $p_2^i = \Delta$ for all $a^i \leq b$, whilst any larger access fee implies $p_2^i > \Delta$ and so $U^i < v$ and no customer for platform i. These results imply that, in the presence of platform competition, under laissez-faire each platform has incentives to vertically integrate into the app segment to be able to squeeze superior apps – i.e., if platforms' business model is a strategic choice, the results are as in Proposition 10.

4.2 App store competition on a platform

The DMA and the proposed Open App Markets Act require Apple and Google to guarantee 3$^{\text{rd}}$ party app stores' access to their respective devices. These alternative paths from business users to consumers are meant to discipline the currently monopolistic app stores and bring higher quality to consumers and lower fees to business users.[24] As the regulatory texts are silent as to the access conditions, we look at a benchmark in which 3$^{\text{rd}}$ party app stores must be given free access to the platform. Does the availability of competing app stores on a single device eliminate the scope for access fee regulation?

We now have a sequence of "platforms", so we must clarify the terminology. In the following, "platform" will keep designating the gatekeeper to the consumer, "app stores" will be the entities interacting with business users: see Figure 5. Consider a monopoly platform, hereafter denoted by A, vertically integrated into device manufacturing. As in Section 2.5.2, its device brings value v_d to consumers and is produced at marginal cost $c_d > 0$. Let p_0 denote its price. On its app store, whose access is priced at p_0^A,[25] consumers can find, in a representative app market, an inferior, in-house or 3$^{\text{rd}}$ party, app valued v_a and a superior 3$^{\text{rd}}$ party app valued $v_a + \Delta$, at prices p_1^A and p_2^A respectively. A's in-house app store faces competition from a 3$^{\text{rd}}$ party app store B priced at p_0^B, where consumers can find the respective inferior (in-house or 3$^{\text{rd}}$ party) app, bringing value v_a, at price p_1^B, and the same, multi-homing 3$^{\text{rd}}$ party app available on A's store at price p_2^B.

Suppose consumers multi-home across app stores, which they can access for free (this is always the case in equilibrium).[26] Then the superior 3$^{\text{rd}}$ party app would serve all consumers on the

[24] Scott Morton et al. (2024) argue that the Apple's App Store offers poor-quality discovery and curation, and that rival app stores could innovate in the two dimensions and further offer lower fees to app developers.

[25] When multiple app stores compete for consumers on the same device, its vertically integrated manufacturer is forced to unbundle its two core products (the device and the app store), charging two different prices. In what follows, we refer to the app stores as the core products.

[26] If instead consumers always single-home (because of, e.g., habit formation, or else each downloads at most one app store), then, as all app stores are equally constrained by the core ZLB ($p_0^A \geq 0$ and $p_0^B \geq 0$) the analysis is as in Section 4.1 (with the only difference that the monopoly manufacturer appropriates consumer surplus charging $p_0 = v_a + v_d$ for the device; this value would instead be appropriated by consumers in the presence also of platform competition). Pigouvian regulation is thus still needed to fairly reward the superior 3$^{\text{rd}}$ party app provider.

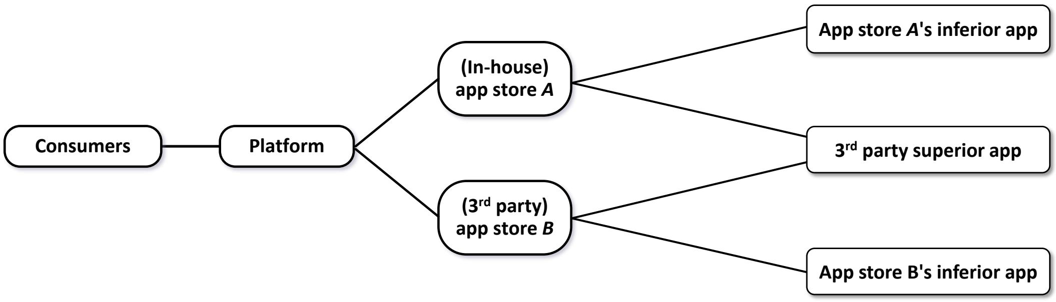

Figure 5: Competing (hybrid) app stores.

least expensive platform: App stores de facto engage in Bertrand competition for the superior app, which dissipates their profits – i.e., $a^* = 0$ in equilibrium.

Proposition 11 (app store competition)**.** *Suppose that the regulator mandates app store competition on devices, with app stores enjoying free access to the device, and that consumers multi-home on app stores on their device. Because the superior app steers the consumer to the lowest access fee app store, Bertrand competition among pure-player or hybrid app stores induces them to charge nothing for consumer access and to levy no fee on 3rd party apps. The superior app then makes supranormal profit $\Delta + b$. The Pigouvian access fee ($\hat{a} = b$, where now b is a floor rather than a cap) is needed to ensure fairness and avoid over-entry in the app market.*

In this simple model, the fair outcome can be alternatively achieved by allowing the platform to levy on 3rd party app stores a unit access fee α for each app sold through their stores. As Bertrand competition among app stores with opportunity cost α implies that they will in turn charge $a^i = \alpha$ to the 3rd party apps, setting by regulation $\hat{\alpha} = b$ ensures fairness. Thus, whether for apps or for app stores, the proper concept of FRAND access pricing boils down to the Pigouvian principle.

This conclusion would be supported also by a consumer surplus standard in a model where consumers have heterogeneous valuations v_d for the device, because A could react to the reduced profitability of the app store (due to competition) by increasing the device price p_0.

Remark (disintermediation). Closely related to interventions mandating app store competition are those forcing the platform to accept that apps publicize on their websites opportunities for bypassing the platform ("disintermediation")[27]. A separate issue with disintermediation as well as with app store competition is its enforceability. An analogy is the ban on most-favored-nation clauses (MFNs), which has had so far little effect because the threat of downlisting is powerful and substitutes for a formal MFN (Ma et al., 2024). Platforms may similarly downlist apps that take advantage of the disintermediation possibility; if so, the promotion of disintermediation by the regulator is ineffective. A case in point is the refusal by the Google Play Store to participate

[27] "*The gatekeeper shall allow business users, free of charge, to communicate and promote offers, including under different conditions, to end users acquired via its core platform service or through other channels, and to conclude contracts with those end users, regardless of whether, for that purpose, they use the core platform services of the gatekeeper.*" (DMA Articles 5(4) and 5(5)). Note that, while the DMA does not specify a fee at which competing app stores should be given access, it here clearly specifies that the fee should be 0.

in the promotion of the One Store app store in Korea, a behavior for which Google was fined.[28]

5 Implementation

Our analysis, which calls for Pigouvian regulation ($\hat{a} = b$), posits a representative app market. The theory trivially generalizes to heterogeneous app markets, indexed by $k \in [0, 1]$. Letting the ancillary benefit, the in-house app value and the competitive advantage of the 3$^{\text{rd}}$ party app in app market k be denoted b^k, v^k, and Δ^k, respectively, the platform's and socially optimal access fees are

$$a^{k*} = v^k + b^k + \Delta^k > \hat{a}^k = b^k.$$

Even under laissez-faire, the platform may not finely tailor the access fee to the specific app market (as we will see, an exception to this rule is search advertising). This is for at least two reasons. The first is the complexity cost: The platform would have to define individual app markets and estimate the profit-maximizing fee in each of them. The second is related to commitment: a very-fine-grained policy may discourage innovation in the app market (or equivalently the porting of apps to the particular platform). In such circumstances, the platform may prefer a uniform policy (such as the app stores' familiar 30% cut) to a fine-grained one. But, in the class of uniform fees, it is still the case that the platform's optimal fee exceeds the socially optimal one.

As is the case for optimal taxes in public finance, the theoretical benchmark – here the ancillary benefit obtained when the app acquires a customer – must be supplemented with an empirical methodology to measure the relevant data. A weak spot of the DMA is its limited guidance regarding both the theoretical benchmark and its implementation. It contains broadly scripted conditions[29] and alludes to FRAND (Fair, Reasonable and Non-Discriminatory) access fees. To go beyond such general statements, the regulator may (1) engage in information collection, or (2) elicit this information from the parties.

In the first approach, the regulator estimates the ancillary benefit (b) or the existence of an "unfair downlisting" (which requires measuring Δ). Measuring the ancillary benefit is the path taken in the EU for capping the merchant fees for card payments. The investigation of unfair downlisting (self-preferencing) has been undertaken in a few recent academic papers on Amazon's vertical integration.[30] Note that the detection of unfair downlisting speaks to the self-preferencing question, but does not address the excessive-fees one.

The heterogeneity of app markets however hinders either endeavor. For instance, app categories differ substantially in terms of the ancillary benefit their distribution generates. There are data-poor and data-rich markets - e.g., social media and food delivery apps sell much more

[28]We are grateful to Jay-Pil Choi for this example.

[29]E.g., *"The gatekeeper shall not engage in any behaviour that undermines effective compliance with the obligations of Articles 5, 6 and 7"* (Article 13(4)).

[30]See Farronato et al. (2023), Lee-Musolff (2024), and Waldfogel (2024) for recent studies, and Etro (2024) for a survey of some earlier studies.

personal data to 3rd party advertisers than videoconferencing apps.[31] The industry has private information about these values that is hardly available to the regulator. The alternative is to elicit the value of the benefit b from the industry, or possibly combine both approaches. The remainder of this section explores these approaches.

5.1 Eliciting the information from the platform: An impossibility result

We first consider an elicitation of ancillary benefits from the platform. To examine how the regulator's limited knowledge of market-specific ancillary benefits affects access fee regulation in the simplest possible model, we consider the best-case scenario in which the regulator knows their cumulative distribution $K(b)$ in the population of app markets $k \in [0,1]$. A necessary condition for the Pigouvian access fee to be implemented in all markets is that the distribution of (observed) access fees be equal to the distribution of benefits.

Proposition 12 (impossibility of elicitation from the platform)**.** *Suppose the regulator knows the distribution $K(b)$ of ancillary benefits and lets the platform choose $(a^k)_{k\in[0,1]}$ subject to the constraint that the distribution of access fees mimics that of benefits (i.e., follows $K(a)$). Then, if self-preferencing cannot be monitored, setting $a^k = b^k$ for all $k \in [0,1]$ is not incentive-compatible for an hybrid platform.*

If self-preferencing cannot be monitored, the Pigouvian rule is not implementable in all markets even if the regulator knows the distribution of b, and so can require that a and b have the same distribution $K(\cdot)$.[32] The reason is that, rather than charging $a^k = b^k$ in all markets, the hybrid platform can profitably charge higher fees in markets where b is lower, so as to squeeze 3rd party developers' margins in these markets, and foreclose developers in markets where b is higher, where it is constrained to set lower fees, which allows it not to lose profit in these markets. Thus, under no monitoring of self-preferencing, market-specific fees cannot be enforced under asymmetric information, and the regulator faces a trade-off between preventing foreclosure of developers in high-b markets and allowing margin squeeze (hence, dampening innovation incentives) in low-b ones.

5.2 Eliciting the information from business users

Section 5.1's impossibility result hinged on the assumption that fee setting is delegated to the platform. We now reverse the roles in access fee setting.

5.2.1 An illustration: Ad-supported platforms

When analyzing ad-supported media and services, one should think of ads as giving merchants access to the consumer. In this respect, Facebook, TikTok or Google Search are not that different

[31] See `https://www.pcloud.com/it/invasive-apps`.

[32] Whether the regulator observes (v^k, Δ^k) or not is immaterial for Proposition 12. Note also that the impossibility result holds a fortiori if the regulator sets a global access fee cap, which would be a less stringent regulation. If vertical integration is not an option, or under monitoring of foreclosure, and provided the regulator knows the distribution $K(b)$ of benefits, the Pigouvian rule can instead be implemented by delegating fee setting to the platform under the constraint that the distributions of a and b be the same.

from Amazon or Booking. Our goal here is not to contribute to the theory of digital advertising markets,[33] but rather to demonstrate the analogy between an access fee and a bid in an ad-auction. An interesting feature is that the access fee, a, is here elicited from business users rather than set by a platform or a regulator.

Consider search advertising and one advertising slot auctioned off by the platform following on a consumer's search for an item (market k). Assume that the platform cannot see the actual sales triggered by the ad.[34] Because there is no uncertainty about which brand the consumer will prefer, we can here focus on the auction of a single slot. Then, consumers observe only the ad of the winner of the auction, who can therefore charge the monopoly price (v^k for the inferior seller, $v^k + \Delta^k$ for the superior one). To the extent that merchants, while benefitting from obtaining the access to the consumer, incur a cost of selling their final products or services to the consumer, the merchants most often satisfy $b^k < 0$. Thus, our "app providers" can also be 3[rd] party sellers advertising their products on the platform – e.g., display advertising on social media platforms (Facebook, TikTok) or search advertising on search engines (Google, Microsoft Bing).[35] Rather than setting an access fee, however, these platforms award any ad-space by running an auction: a thus corresponds to the bid paid by the winner of the auction.

Proposition 13 (ad-auctions)**.** *The superior app receives the fair share for its contribution to the ecosystem ($\pi_2^{k*} = \Delta^k$) if a slot is auctioned off without reserve price. This result does not hinge on the platform observing the existence of a sale (through an app store, say).*

Proof of Proposition 13. In app market k, the inferior app, whether in-house or 3[rd] party, bids $v^k + b^k$, its profit if it wins the auction. And so, assuming a second-price auction, the superior app pays $a^k = v^k + b^k$, yielding $\pi_2^{k*} = \Delta^k$; and $p_0 = 0$ for consumers to access the platform. The access fee thus reflects the ancillary benefit, and the 3[rd] party app receives its fair reward. In contrast, if allowed to set a reserve price, the platform can squeeze the superior app by setting a high reserve price (requiring bids $a^k \geq v^k + b^k + \Delta^k$). ■

Note that while the Pigouvian outcome is achieved, the resulting access fee is now $a^k = v^k + b^k$ rather than b^k. This is because app competition is now *for* the market rather than *in* the market.

[33]For an interesting recent contribution to this, see Bergemann et al. (2024) on managed ad-campaigns. They assume that the platform is privately informed about its consumers' valuation for all products and makes a take-it-or-leave-it offer to each seller, specifying (1) a steering policy, assigning consumers to firms, (2) a pricing policy, i.e., the price of the seller's product for each consumer to which it is shown, and (3) a fixed fee. In our setting, under these assumptions, the platform can fully squeeze the superior sellers. See also Ichihashi and Smolin (2024) for a rationing of ad-slots that is contingent on prices offered by sellers, and Janssen et al. (2024) for the assignment of products to sponsored positions and the obfuscation of the organic positions' informational content.

[34]In practice, it can avail itself of a proxy, namely the number of clicks. However, this proxy is a noisy measure of the advertiser's willingness to pay for the slot; to recover the latter, the platform would need to estimate (a) the net conversion rate (how many sales are triggered by a click that would not happen otherwise) and (b) the per-sale markup of the seller over the opportunity cost (which include b). For expositional simplicity we ignore the proxy, even though it is relevant in practice.

[35]Note that, in the case of organic search, where de facto $a^k = 0$, the search engine has incentive to engage in self-preferencing in markets where it is present as a competitor to 3[rd] parties, as, e.g., in the Google Shopping and Google Flight cases.

Remark (more general auctions). In this simple example, and in the absence of a reserve price (which we argued would allow the platform to set the access fee), it is optimal for the platform to restrict the number of advertising slots so as to squeeze the apps, as argued by Prat and Valletti (2022). Needless to say, when horizontal or vertical differentiation affects the consumer's optimal choice, more slots are needed to match buyer and seller; but the general idea that limited space benefits the platform remains.

5.2.2 Alternative elicitation schemes

We now consider the elicitation of information from app developers without restricting the number of slots. The challenge for such elicitation is that app developers want the lowest possible access fee; we saw that for digital goods the absence of access fee encourages the development of me-too apps, whose main purpose is to steal value from existing apps and which create little value for the consumer. To prevent the access fee from providing such incentives, the regulator may refrain from monitoring foreclosure.

In the case of vertical integration (the platform owns the inferior app), we elicit information only from the 3$^{\text{rd}}$ party app. For a pure platform (the inferior app is also a 3$^{\text{rd}}$ party app): (1) the inferior and superior 3$^{\text{rd}}$ party apps in market k propose access fees a_1^k and a_2^k; (2) the platform selects $\{\delta_i^k\}_{i\in\{1,2\},\, k\in[0,1]}$ – i.e., it decides to give access to no, one or the two app providers; then (3) the platform and the (non foreclosed) app providers set their prices $\{p_0, p_1^k, p_2^k\}$; and (4) consumers make their consumption choice.

Proposition 14 (alternative information-light implementation of Pigouvian rule)**.** *The fair reward ($\pi_2^{k*} = \Delta^k$) can be guaranteed in all markets by letting 3rd party app developers pick their access fee subject to the threat of foreclosure. When the platform is hybrid, this can be accomplished by just eliciting the access fee from the 3rd party app in each app market k. When it is a pure platform, this requires that the inferior app pays its proposed access fee only in case of a sale (as is feasible in an app store).*

Proof of Proposition 14.

(i) *Hybrid platform*: If foreclosure is not monitored, then choosing $a^k \in \left[b^k, b^k + v^k\right]$ is optimal for the 3$^{\text{rd}}$ party app in app market k, as it is foreclosed for $a^k < b^k$ and squeezed for $a^k > b + v^k$.

(ii) *Pure platform*: Let the inferior and superior 3$^{\text{rd}}$ party apps in market k propose access fees a_1^k and a_2^k. We claim that $a_1^k = a_2^k = v^k + b^k$: The inferior app knows that it can win consumers if and only if it is the more rewarding app from the platform's standpoint; it cannot afford paying more than $v^k + b^k$, though. The superior app must bid $v^k + b^k$ as well, as otherwise it would be foreclosed by the platform, which would bring it a higher access fee. The platform lets both apps operate, and app prices are $p_1^k = v^k$ and $p_2^k = v^k + \Delta^k$. Hence, consumers patronize solely the superior app, the platform sets $p_0 = 0$ and receives $v^k + b^k$ in market k, and the superior 3$^{\text{rd}}$ party app obtains its fair contribution to the ecosystem ($\pi_2^{k*} = \Delta^k$). ■

By contrast, under monitoring of foreclosure independent app developers would choose $a^k = 0$;

this would overincentivize me-too 3rd party apps to enter.

Remark (a caveat). Although this section's results are encouraging, they rely on the platform maximizing its profit in each app submarket. Yet, the fact that platforms are engaged in a variety of B2B relationships across apps and across time, gives them the possibility to build a reputation for toughness, or, put differently, to extract higher access fees through predation (by adopting behaviors that do not maximize their short-term profit). This may be the case if the superior app determines the access fee. The platform may downlist this app when the latter offers a socially optimal access fee but refuses to "self-squeeze" (offer above $v^k + b^k$ in our model). Such downlisting would "teach a lesson" to the app developer or, more to the point, its colleagues.

Future research should investigate how to prevent such predation. A preliminary idea that comes to mind is to make sure that the access fee is not determined by the superior app, so the latter cannot be pressured to self-squeeze. Take the case in which a superior app competes with suppliers of an inferior app, the platform can observe the existence of a sale on its app store (see Proposition 14), and the access fee is determined by the second bid a_i^k. The equilibrium access fee is then $v^k + b^k$, and the platform cannot extort a higher fee from the superior app, which then receives Δ^k. Another idea is to introduce appeals, in a way similar to the study in the next sub-section (there, appeals will relate to high access fees set by the platform rather than to the platform downlisting apps otherwise paying proper access fees; but the logic is the same).

5.3 Off-path measurement and appeals

Measuring b systematically would imply considerable costs and delays. At best can one, when the access fee is determined by the platform, allow appeals that hopefully will not be frequent if the incentive scheme is designed properly. Let us focus on the more interesting case of an hybrid platform (as an inferior 3rd party app is never foreclosed anyway). Suppose the regulator is equipped with a noisy measure of the ancillary benefit if called upon by a party. More precisely, the *platform* chooses the access conditions $\{a^k, \delta_2^k\}$ (of which the regulator observes only a), then prices are set, and finally the superior 3rd party app chooses whether to appeal "against a high access fee". In this appeal procedure, the authority observes a noisy, but unbiased, version $\tilde{b}^k$ of the ancillary benefit, with cdf $R(\tilde{b}^k)$ such that $\int_{\mathbb{R}} \tilde{b}^k \mathrm{d}R(\tilde{b}^k) = b^k$. If $a^k > \tilde{b}^k$, then the access fee is assessed to be unfair, and the defendant (the platform) must pay a fine $\tau(a^k - \tilde{b}^k)$ (with $\tau > 0$) to the plaintiff (the 3rd party app); and vice versa if $a^k \leq \tilde{b}^k$.

The outcome of the appeal procedure interferes neither with the platform's choice of whether to foreclose the 3rd party app, nor with access and app prices chosen by the firms before the appeal. This implies that the 3rd party app will appeal whenever $\int_{\mathbb{R}} \tau(a^k - \tilde{b}^k)\mathrm{d}R(\tilde{b}^k) > 0 \Leftrightarrow a^k > b^k$. So if $\tau \geq 1$, the platform does not gain from inflating the access fee beyond b^k.

This analysis however understates the platform's ability to extort high access fees. In the spirit of the caveat above, let us therefore "empower" the platform by allowing it to foreclose the 3rd party app *after* the latter has appealed (silence means assent, so the absence of appeal means that the proposed a applies). Suppose that the platform faces a sequential entry of superior 3rd party apps in distinct but identical app submarkets and discounts future profits at a rate β. The platform can therefore build a reputation for preying on apps that dare to appeal. The following proposition is proved in Online Appendix A:

Proposition 15. *Give the 3rd party app a right to appeal against a high access fee chosen by the platform. If the regulator can produce a noisy measure $\tilde{b}^k$ of the ancillary benefit, and impose sufficiently large fines to the platform if it loses the appeal (namely, $\tau(a^k - \tilde{b}^k)$ if $a^k > \tilde{b}^k$, with $\tau \geq \max\{1, \beta/(1-\beta)\}$), then the Pigouvian rule can be implemented even when the platform can build a reputation for engaging in foreclosure after being challenged by an app.*

6 Relevant literature

(a) Older regulation literature. There is a large literature on foreclosure practices and the essential facility doctrine (e.g., Hart and Tirole, 1990; Rey and Tirole, 2007), and on access pricing for one-sided markets (e.g., Laffont and Tirole, 1994) and for telecom and payment card markets (e.g., Armstrong, 1998; Laffont et al., 1998; Rochet and Tirole, 2002, 2011). The literature on access to regulated bottlenecks showed that the monitoring of access is needed, as a vertically integrated incumbent has little incentive to provide access at a capped price to competitors. In this literature, what constrains the core price upwards is regulation rather that a demand for subsidization at the core ZLB. A celebrated rule, the ECPR (or Baumol-Willig) rule states that the access fee should be no greater than the vertically integrated monopolist's lost margin in the competitive retail segment. Its properties are analysed in Laffont and Tirole (1994); obviously it just connects two prices and says little about their absolute level. Another classic implication of the theoretical analysis is that an access markup does not necessarily mean that competitors are disadvantaged, as the markup increases the opportunity cost of the vertically integrated firm and its rivals alike.[36]

(b) App ZLB and incentive for self-preferencing: are they equivalent? Given that many recent antitrust investigations against Google and the other large platforms focus on self-preferencing and exclusion, it is worth noting that our framework sheds light on when the incentive and the feasibility of self-preferencing concur. The basic model made the same prediction as the literature on the regulation of a bottleneck (power grid, railroad tracks, local loop...): A low a deprives the bottleneck owner from the possibility of making money on the potentially competitive segment; this gives it an incentive to engage in non-price foreclosure (for a public utility) or self-preferencing (platforms, provided that $a < b$).

But if a low a creates an incentive for self-preferencing, will the bottleneck owner actually engage in the practice? The answer is "not necessarily" for two reasons. The first, also developed in Section 2.3, likewise shares with the public-utility literature the idea that the bottleneck must

[36]This is important because marginal-cost pricing of access need not be the right welfare benchmark. It jeopardizes the recovery of fixed costs for the essential infrastructure owner and it further incentivizes foreclosure ("self-preferencing" in modern parlance), requiring heavy investment in regulatory capacity: The vertically integrated firm cannot make money by selling access and therefore must make its money on the competitive segment.

be vertically integrated into the competitive segment in order to take advantage of the rival's degraded access. The other reason why a low a may not trigger self-preferencing is novel to a platform environment.[37] App store competition (Section 4.2) discourages non-price foreclosure; to be certain, an app store ceteris paribus benefits from self-preferencing if $a < b$; but the degraded service makes consumers move to the substitute app store: the 3rd party app can slightly lower its price, at $\Delta - \epsilon$, on the rival app store and make sales there, so that the in-house app of the deviating platform is not sold anyway.

(c) Platform is not a gatekeeper. The literature has studied the regulation of platform fees when the consumer and the merchant can transact through multiple channels: the platform and another channel (direct purchases, other platforms, other payment methods in the case of a payment platform). Because the consumer chooses the channel, the welfare analysis is naturally grounded in the externalities associated with this choice.

Some contributions suppose that the merchant offers the same price regardless of the channel (there is a most-favored-nation, MFN, clause); the merchant's revenue from a sale is then channel-independent, which does not mean that its markup is. The merchant may enjoy a convenience benefit from the platform channel, as in Rochet and Tirole (2011): A card payment may dominate cash and cheque in terms of expediency, fraud prevention, accounting, or absence of hold up. The socially optimal access fee corrects for externalities of consumer channel choice upon merchants, and the socially optimal access fee (which in payment networks is at least partially passed through by issuers to consumers) is equal to the merchant benefit from a card usage; this internalization principle is the so-called *tourist test*. In Gomes and Mantovani (2024), the platform creates an informational and a convenience benefits for consumers; in particular, the platform offers products that they were unaware of. This improved-opportunities benefit of the platform is internalized by consumers. But, consumers' access to the platform being assumed free, they do not directly reward the platform for it, which is a problem if the platform is created only if sufficiently profitable. The platform however can charge consumers indirectly through the competing merchants' access fee, then passed through to consumers. Gomes and Mantovani show that, provided the presence of the platform does not increase aggregate sales, the welfare-maximizing access fee equals the sum of these two benefits. In both papers, $a^* > \hat{a}$.

Alternatively, there may be no MFN. Prices are lower on the platform if it displays tougher merchant competition than the direct sale channel. The consumers may then choose to transact through the platform not because they prefer this channel, but because the latter lowers merchants' markups, at least in part a redistributive effect (Wang and Wright, 2024). The privately optimal fee may now fall short of the socially efficient one, which equals the platform's marginal cost of implementing the transaction plus the amount by which the platform, by intensifying seller competition, decreases the merchants' margins. Again, the merchants' pass-through of the access fee is key to restoring proper consumer incentives.

In contrast with these papers, which hinge on consumers' choice of channel to interact with

[37] The public-utility literature has mostly assumed that the bottleneck segment is unassailable.

merchants, we assume that consumers single-home, whether there is platform competition or not: the platform is a "gatekeeper". The set of potential externalities under consideration is then rather different: (a) a vertically integrated gatekeeping platform may use non-price instruments to prevent consumers from accessing the best product; (b) the platform may jeopardize the existence of superior 3^{rd} party apps by squeezing them through a high access fee; (c) the 3^{rd} party app enjoys supranormal profit when the app ZLB binds. The welfare-maximizing access fee is then equal to the opportunity cost for the platform of letting 3^{rd} party sellers serve consumers, rather than to the benefits it brings to one or both sides of the market.

(d) Platform presence in app markets. A number of recent papers examine platforms' incentive to vertically integrate, and the welfare effect of this vertical integration, in the presence of foreclosure and/or imitation concerns: see Anderson and Bedre-Defolie (2024a,b), Etro (2021, 2023), Gutiérrez (2021), Hagiu et al. (2022) and Zennyo (2022). Yet, these works, as the ones on platform fees' regulation, assume non-negative opportunity costs (i.e., rule out an app ZLB) and do not consider access pricing on the consumer side.[38] To be certain, one may argue that the widespread assumption that platforms grant free access to consumers in these papers reflects a core ZLB.[39] However, they do not connect the validity of the underlying assumption with the level of seller access fees.

Another closely related contribution to our paper is Choi and Jeon (2021). They show that tying may help a firm circumvent a non-negative price constraint in the tied (complementary) product market that prevents it from squeezing superior sellers in that market. Zero lower bounds do not usually emerge in standard models (e.g., Choi and Stefanadis, 2001, Carlton and Waldman, 2002), which assume that the tied market involves a positive marginal cost.[40] Unlike in this literature on tying, which does not consider access pricing, in our paper margin squeeze of superior 3^{rd} party sellers by the platform does not necessarily occur via below-cost pricing in the tied (competitive) good market, but primarily via fees: In this case, it is the core ZLB, rather than the ZLB in the tied market (the app ZLB in our terminology), that binds.

7 Conclusion

Brief summary. Gatekeeping platforms control businesses' access to us. Policymakers dealing with platform access have met with the difficulty that welfare analyses in two-sided markets are generally ambiguous. The see-saw effect, and its distant parent, the Chicago school rich ecosystem argument, hold that self-preferencing and high access fees, by degrading the ecosystem and making it unattractive to the consumer side, do not benefit the platform. Relatedly, capping

[38]By considering access pricing both on consumer and seller side, our work relates to the literature on optimal pricing by two-sided platforms pioneered by Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2003, 2006). This literature however is not concerned with hybrid platforms and mostly ignores ZLB constraints.

[39]In other papers on hybrid platforms, including Etro (2023) and Padilla et al. (2022), app stores are bundled with physical devices, so that consumers are always charged a positive price.

[40]For earlier work on the effects of tying in two-sided markets where ZLB constraints may bind, see Amelio and Jullien (2012). They show that, in situations where a platform would like to set negative prices on one side of the market, tying serves as a mechanism to introduce implicit subsidies on that side. As a result, it can raise participation on both sides and benefit consumers.

access fees for business users leads to higher prices on the consumer side.[41] This paper argues that this logic does not apply to the zero-lower-bounds environment of digital markets.

The *core ZLB* (the impossibility for digital platforms to charge negative access prices to consumers) creates incentives for harmful behaviors:

1. The see-saw effect no longer operates. Benefits from a better ecosystem are not passed through to consumers as the platform is reluctant to raise prices. This gives the platform incentives to maximally extract the surplus of business users through high access fees. Extractive access fees create a double marginalization and induce a suboptimal usage of apps. They furthermore discourage the creation of apps.
2. The core ZLB is more likely to bind if there is platform competition, or, in its absence, a high elasticity of consumer demand for the platform. It is less likely to bind if a costly device is part of the bottleneck.

The *app ZLB* (the infeasibility of negative app prices) limits competition in the app markets and generates two inefficiencies:

3. The greater profit made in-house relative to providing access (which arises when $a < b$) creates incentives for self-preferencing, all the more so, the larger the ancillary benefit b (e.g., goods are digital rather than physical) relative to the access fee a. Unfortunately, antitrust watchdogs find it notoriously difficult to discern and demonstrate self-preferencing.
4. Low or zero access fees dissipate value by inviting business stealing by me-too apps, that add little value to the ecosystem but extract a non-negligible share of it.

Overall, the argument for capping access fees and more generally enforcing equitable access to gatekeeping platforms is definitely stronger in the presence of ZLBs. In this respect, the paper provides guidance for policy-making:

5. The Pigouvian rule ($\hat{a} = b$) allows the 3rd party apps to capture their contribution to the ecosystem. Furthermore, this access fee minimizes double marginalization in the set of access fees that do not induce self-preferencing.

The diversity of digital environments. A benefit from our framework is that, despite its simplicity, it accounts for the rich diversity of digital environments. Figure 6 provides illustrations of the various situations. Because detecting "self-preferencing" and "excessive fees", the two prongs of the regulators' equity concern, is notoriously difficult or costly, regulators must pick their fights, which requires looking for "smoking guns". Figure 6 shows that ZLBs provide guidance for finding smoking gun evidence.

(a) Regarding the first concern, which obviously arises only when the platform is hybrid (or can enter sweat deals with favored 3rd party apps), our analysis points out that self-preferencing benefits the platform more (or is less costly for a platform engaging in predatory behavior) if the access fee is small in relation to the ancillary benefit (app ZLB). Conversely, it is less of a

[41] See Anderson and Bedre-Defolie (2024b).

concern if the consumers can easily find the app outside the platform.

(b) App store competition (although not platform competition) may help bring down excessive fees paid by business users. We however pointed out that the threat of downlisting may be an effective deterrent to attempts to bypass the in-house app store, and thereby enforce an implicit exclusivity. Why are we concerned about high access fees? Recall the two-sided-market-theory finding, echoed in the neutrality region of Proposition 1 (i), that there may be no presumption that a platform price structure be excessively tilted in the direction of consumers or business users. However high access fees do not translate into better terms for consumers through a see-saw effect if the core ZLB is binding. A binding core ZLB might therefore be a smoking gun that high access fees are detrimental.

The regulatory challenge. The paper stressed that laissez-faire – in the sense of a lack of interference with the platforms' preferred access policies – breeds unfair access conditions for these business users. Furthermore, we should not expect competition to solve the gatekeeping problem in the digital world of ZLBs. Indeed, the core ZLB constraint prevents platform competition from disciplining access policies. We also showed that platform competition and app store competition work very differently. While platform competition is too business-user unfriendly, app store competition (if effective) is too business-user friendly.

The overall picture is therefore a need for overseeing the terms and conditions offered by platforms to business users. In this we concur with the spirit of recent regulatory developments. The latter however remain nebulous when it comes to specific recommendations, and the occasional invocation of the need for "fair, reasonable and non-discriminatory" terms is not helpful. The paper's main insight concerning regulation relates to the social benefits of setting the access fee at the ancillary benefit associated with acquiring a customer. This level discourages self-preferencing and thereby spares intrusive assessment of whether access conditions are actually fair; it also provides app developers with a fair return and therefore a proper incentive to innovate; finally, it minimizes double marginalization conditional on intrusive regulation being infeasible or too costly.

Despite these clear theoretical messages, meeting the empirical challenge of regulating platforms' access policies remains as difficult as it is essential. The task of answering whether a 10% or 30% merchant fee is appropriate is marred with asymmetric information. We made real progress on the question of how to implement the theoretical benchmark; but we feel that more work is necessary to properly tame the gatekeeping platforms while not preventing them from offering innovative services to consumers and businesses alike. As new AI-based platforms are entering the e-commerce, search, and health markets, this question should remain a priority.

	App ZLB binding $(a < b)$	App ZLB non-binding $(a > b)$
Core ZLB binding	• (Effective) app store competition [1] • Hybrid apps (easy bypass of app store) [2] • App stores, free apps [3] SP? • Regulatory cap on a [4] SP	• App stores, in-app purchases [6] EF • E-commerce $(b<0)$ [7] EF ○ Amazon (hybrid player) ○ Booking, Uber (pure players) • Ad-supported media/services $(b<0)$ [7] EF? ○ Facebook, TikTok (display ads) ○ Google (sponsored search ads)
Core ZLB non-binding	• iPhone + Apple AppStore (and free apps, or capped fee) [5] SP?	• Videogame platforms [5], [8] EF?

Figure 6: (taxonomy of digital platforms)
Concerns about self-preferencing (SP) or excessive fee (EF)

(1) Effective competition in the app store market (efficient competitors installing without charge their app store on Apple or Android devices) implies that $p_0 = a = 0$: Section 4.2. The caveat "effective" refers to the fact that a dominant app store may deprive an entrant app store of apps by downlisting apps that multi-home.

(2) Hybrid apps are known, highly visible apps that consumers can access through the browser for free (connecting to one's bank, Deliveroo, Amazon, Uber). Because they are free web apps that are also available for download on the App Store, $a = 0$. The app store then has a limited gatekeeping ability. To be certain, native apps may use the app store's functionalities to offer better service than on the web. Thus, the case considered here may approximate reality.
Apps such as Google Maps, Gmail, and Google Flights are both free apps and hybrid apps. They bring ancillary benefits for Google ($b > 0$): data for Google Maps and Gmail, merchant fees for Google Flights for instance. They can be obtained through Google's Play-Store, another app store, or as they are identifiable through the browser.

(3) Apps may be free because the app designer wants to create a service for the benefit of the community (alternatively, they may want to maximize downloads for signaling or ego purposes). These range from giant Wikipedia to the many small apps that are created by individuals and groups without commercial purpose. Still, the platform may occasionally want to promote its own version of the app that it may now or later monetize/use for obtaining ancillary benefits such as data.

(4) A regulatory intervention or threat thereof (cap on merchant or interchange fees in payment card systems, or DMA's requirement of giving access to alternative payment methods or alternative app stores at zero or FRAND access fee) imposes de facto a cap on a.

(5) Devices may be bundled with a "in-house platform". Together they constitute the "core". The core ZLB is then unlikely to bind, as the physical device has a positive manufacturing cost: Section 2.5.2. An example of the lower-left corner may be a smartphone and associated app store. An example of the lower-right corner is videogames, which are captured by our framework even when they face no substitutes (although they always do to some extent, if only because games compete for gamers' time). $a > b_1$ in that case (in the notation of Section 2.5.3). To be certain, $b > 0$ as games may benefit from repeat purchases (upgrades) and data collection facilitating the promotion of look-alike games. But the surplus is large ($v + \Delta$ if there is no substitute), and there is scope for the videogame platform to charge access fees per game that exceed the ancillary benefit.

(6) When the platform is a gatekeeper and is not regulated: Section 2.

(7) The app-ZLB rarely binds for physical products (for which usually $b < 0$): e-commerce of physical products (Amazon) and services (Uber), products and services advertised through search engines (sponsored search) or social media/publishers (display advertising): Section 2.5.1.

(8) EF is still a concern (squeeze of apps), but less so because of the see-saw effect.

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Online Appendix

Appendix A: Proofs

Proof of equilibrium uniqueness in Proposition 1

Encompassing both the hybrid- and pure-platform cases, and decomposing the representative app market into individual app markets $k \in [0,1]$, pure strategies are

(a) in the hybrid case, choices $\{p_0, \{p_1^k\}\}$ by the platform, and a choice $\{p_2^k\}$ by each independent app $k \in [0,1]$;

(b) in the pure-platform case, a choice p_0 by the platform, and choices $\{p_1^k\}$ and $\{p_2^k\}$ by independent app providers in each app market k.

We allow for more generality the access fee to be app-market contingent (a^k). Because the uniqueness proofs are very similar, we include the case of an elastic demand for the platform: as in Section 3.2.1, we assume that the consumers also derive utility from the platform v_c independently of its apps, where $v_c \gtreqless 0$ (a negative value is a learning or opportunity cost) follows a smooth cdf $F(v_c)$ with density $f(v_c)$ on $\mathbb{R}$.

Assume, without loss of generality that $\delta_2^k \geq \delta_1^k$. Let us first show that we can assume, also without loss of generality, that $p_1^k \leq \max(a^k - b^k, 0)$. Either $v + \delta_1^k < p_1^k$ and app 1 is out of app market k; then charging $\max(a^k - b^k, 0)$ cannot do worse for the platform than charging p_1^k, regardless of whether app 1 is owned by the platform or an independent app developer (and in the case of vertical integration, may increase consumers' surplus, which benefits the platform, which can raise p_0). Or $v + \delta_1^k \geq p_1^k$; because app 2 wins the market, if app 2 is owned by an independent app developer, $p_2^k = p_1^k + \delta_2^k - \delta_1^k$. If $p_1^k > a^k - b^k > 0$, app 1 can charge $p_2^k - (\delta_2^k - \delta_1^k) - \varepsilon < p_1^k$, win the market and make a strictly higher profit; furthermore, if app 1 is owned by the platform, the consumers' utility from app market k slightly increases.

Next suppose that $p_1^k < a^k - b^k$ (which requires $a^k > b^k$). Let us show that price p_1^k is strictly dominated for an independent owner of app 1 by price $a^k - b^k$. If $\tilde{p}_2^k \equiv p_2^k - (\delta_2^k - \delta_1^k)$ lies below p_1^k, whether app 1 is priced at p_1^k or $a^k - b^k$ makes no difference; but if it lies above p_1^k, charging p_1^k rather than $a^k - b^k$ implies a loss of $a^k - b^k - p_1^k > 0$ if app 1 is owned by an independent developer; in this case, the only undominated behavior involves $p_1^k = a^k - b^k$.

Now suppose that the platform owns app 1. Suppose that demand is elastic (the distribution $F(v_c)$ is smooth). If $p_0 > 0$ (the case where the core ZLB binds is covered in Proposition 8), then letting A denote the per-consumer access revenue collected by the platform, the FOC is: $(p_0 + A) f(v_c^*) = 1 - F(v_c^*)$. This implies that purely redistributional shifts between platform and consumers (say, an increase in app price cashed by the platform, as happens when $\tilde{p}_2^k > a^k - b^k$ and the app price is raised from p_1^k to $a^k - b^k$) do not affect platform profit. In contrast, if $\tilde{p}_2^k$ lies between p_1^k and $a^k - b^k$, the consumers lose $p_2^k - (\delta_2^k - \delta_1^k) - p_1^k > 0$ when app 1 increases its price from p_1^k to $a^k - b^k$ while the platform gains in that market $a^k - (p_1^k + b^k) > \tilde{p}_2^k - p_1^k =$

$p_2^k - (\delta_2^k - \delta_1^k) - p_1^k$. The FOC then implies that the platform gains. So again $p_1^k < a^k - b^k$ is dominated when app 1 is owned by the platform. ■

Proof of Proposition 8

$\boldsymbol{a < b}$. When $a < b$ (and more generally $a \leq b$), whether the platform is vertically integrated or not does not matter; intuitively, the inferior app is available at price 0 and the platform cannot boost consumer membership by reducing its in-house app's price. We can therefore focus on the vertically integrated case. As we saw in the text, $b \geq \rho(-v_a)$ implies that under self-preferencing $p_0 = p_1 = 0$ and the platform's profit is $b[1 - F(-v_a)]$. Can the platform do better by granting access to the 3rd party app? Note that $p_2 \geq \Delta$ (any price below or at Δ allows it to corner the app market). Because $p_1 \geq 0$, the utility of the consumers from apps is at most v_a. And hence the platform's profit if the 3rd party app serves the app market is at most $(p_0 + a)[1 - F(p_0 - v_a)] < (p_0 + b)[1 - F(p_0 - v_a)] \leq b[1 - F(p_0 - v_a)]$. Finally, as long as $p_1 \leq v_a$ (so the inferior app is at least considered by consumers), $p_1 + \Delta \leq v_a + \Delta$, the superior app corners the market at $p_1 + \Delta$, profitably so: $p_1 + \Delta + b > a$.

Next, suppose that self-preferencing is monitored. The unique price equilibrium when $a \leq b$ is $p_1 = 0$ and $p_2 = \Delta$. For, if $p_2 > \Delta$, the platform could charge $p_1 = (p_2 - \Delta) - \varepsilon$ and obtain $p_1 + b > a$. The core price is given by $\max_{p_0} \left\{(p_0 + a)[1 - F(p_0 - v_a)]\right\}$, and so $p_0 = 0$ if and only if $a \geq \rho(-v_a)$.

$\boldsymbol{a \geq b}$. We consider the two cases:

(i) Pure platform. When the platform is not vertically integrated, the undominated equilibrium when apps are unconcerned with platform consumer membership is $\{p_1^* = a - b, p_2^* = \min(p_1^* + \Delta, v + \Delta)\}$. The consumers' utility from the apps is then $u(a) = v_a - (a - b)$ if $a \leq v_a + b$ and $u(a) = 0$ otherwise. The platform's profit from the apps is $a[1 - F(a - b - v_a)]$ in the range $a \in [b, v_a + b]$ in which increases in a are passed through one-for-one; our assumption on ρ implies that $a = b$ is optimal in this range, yielding overall profit $\max_{p_0}[p_0 + b][1 - F(-v_a + p_0)\} = b[1 - F(-v_a)]$. In the range $a \in [v_a + b, v_a + b + \Delta]$, increases in the access fee are no longer passed through, and so the profit-maximizing access fee in that range is $v_a + b + \Delta$, yielding profit $\max_{p_0}[p_o + v_a + b + \Delta][1 - F(p_0)] = [v_a + b + \Delta][1 - F(0)]$. Thus the optimal access fee is either $a = b$ or $a = v_a + b + \Delta$ depending on $b[1 - F(-v_a)] \gtrless [v_a + b + \Delta][1 - F(0)]$.

(ii) Hybrid platform. Suppose that the ZLBs bind under foreclosure – i.e., $b > \rho(-v_a)$. Consider $a \geq b$ and the following candidate equilibrium:

$$\{p_0 = 0, p_1, p_2 = p_1 + \Delta\},$$

with $p_1 \in [0, v_a]$. This constitutes an equilibrium for all p_1 such that

$$[1 - F(p_1 - v_a)]a \geq [1 - F(-v_a)]b,$$

which defines an interval $[0, \overline{p}(a)]$, where $\overline{p}(a) \in (0, a-b)$ is increasing in a.

To see this, note that given $p_1 \in [0, v_a]$, the non-pivotal 3$^{\text{rd}}$ party app optimally charges $p_2 = p_1 + \Delta$. Then,

$$\arg\max_{p_0}[1 - F(p_0 + p_1 + v_a)](p_0 + a) = 0 \iff a \geq \rho(p_1 - v_a),$$

which is implied by $a > b$ and (the hazard rate being decreasing) $p_1 \geq 0$; so, $p_0 = 0$. Since increasing p_1 has no effect on the equilibrium profits (consumers still buy the 3$^{\text{rd}}$ party app), the only possible deviations to consider are to $\tilde{p}_1 < p_1$. Any such deviation implies that the in-house app is sold. Then, the optimal deviation is to $\tilde{p}_1 = 0$, yielding the foreclosure profit $[1 - F(-v_a)]b$. It follows that p_1 is an equilibrium price for the inferior app if and only if $[1 - F(p_1 - v_a)]a \geq [1 - F(-v_a)]b$. Hence, $p_1 = 0$ is always an equilibrium, but it is not unique: The larger a, the larger the upper bound on p_1, denoted by $\bar{p}(a)$, that can be sustained as an equilibrium. Still, $b > \rho(-v_a)$ implies that $\bar{p}(a) < a - b$.

Note that here the concept of equilibrium in undominated strategies does not help selecting an equilibrium. Compare an equilibrium price p_1 and consider an alternative price $\hat{p}_1$. A price $\hat{p}_1 < p_1$ (if any) increases platform profit if price p_2 (not necessarily the equilibrium price) is such that app 1 has the market regardless of $\hat{p}_1$ or p_1; it decreases platform profit if price p_2 is such that app 1 is selected under $\hat{p}_1$, but not under p_1, as $\hat{p}_1 + b < a$. Similarly, a price $\hat{p}_1 > p_1$ decreases platform profit if price p_2 (not necessarily the equilibrium price) is such that app 1 has the market regardless of $\hat{p}_1$ or p_1; it increases platform profit if price p_2 is such that app 1 is selected under p_1, but not under $\hat{p}_1$, as $p_1 + b < a$.

Therefore, the hybrid platform gains at least the foreclosure profit for all $a \geq b$, and it can gain strictly more for some $a^* > b$, depending on which equilibrium is played – e.g., in the equilibrium with $p_0 = p_1 = 0$ and $p_2 = \Delta$, the platform's profit is maximized at $a^* = b + \Delta$, at which the superior app is fully squeezed.

Welfare-optimal access fee. Finally, we show that $\hat{a} = b$ is a welfare optimal access fee. Given that $u \leq (v_a + \Delta) - p_2 \leq v_a$,

$$W = \int_{-u}^{+\infty} (v_c + v_a + \Delta) dF(v_c)$$

is maximized at $u = v_a$, which requires $p_2 = \Delta \iff p_1 = 0$. But when $a = b$, $p_1 = 0$ and $p_2 = \Delta$ is the unique equilibrium as noted earlier. ■

Proof of Proposition 9

Consider an hybrid platform. For $p_0 + p_1 \leq v$, all consumers buy one app. Since a consumer with type Δ prefers the 3$^{\text{rd}}$ party app if and only if $\Delta \geq p_2 - p_1$, letting $H(\cdot)$ denote the cdf of consumers' type Δ, firms' profits in the representative market are

$$\pi_0^* + \pi_1^* = p_0 + a + H(p_2 - p_1)(p_1 + b - a),$$

and

$$\pi_2^* = [1 - H(p_2 - p_1)](p_2 + b - a).$$

Since its profit is increasing in p_0, the platform optimally sets $p_0 = v - p_1 \in [0, v]$, so that all consumers buy one app, and those buying the in-house app are left with no surplus. By doing so, it achieves a higher profit compared with the one attainable setting prices so that $p_0 + p_1 > v$.[42] For any given a, the equilibrium app prices and firms' profits are as follows:

Lemma. *Non-price foreclosure is optimal for the platform if and only if $a < b$. There are two thresholds $(\underline{a}, \overline{a})$, with $b < \underline{a} < b + v < \overline{a}$, such that:*

- *For $a \leq \underline{a}$, the app ZLB binds: $p_1^* = 0 \leq p_2^*$, with p_2^* being strictly positive and increasing in a for all $a \in [b, \underline{a}]$, and $p_0^* = v$.*
- *For $a \in (\underline{a}, \overline{a})$: $0 < p_1^* < a - b < p_2^*$, and $p_0^* = v - p_1^* > 0$, with $(p_2^* - p_1^*)$ and firms' profits being constant when a varies.*
- *For $a \geq \overline{a}$, the core ZLB binds ($p_0^* = 0$) and $p_1^* = v < p_2^*$, with p_2^* being strictly increasing in a. The platform's profit is maximized at $a^* > \overline{a}$.*

Proof of Lemma. Denoting by $h(\cdot)$ the pdf of consumers' type on $\mathbb{R}^+$ and by $\rho_\Delta(\tilde{\Delta}) \equiv [1 - H(\tilde{\Delta})]/h(\tilde{\Delta})$ the inverse hazard rate, which we assumed is decreasing, we have

$$\frac{\partial[\pi_0^* + \pi_1^*]}{\partial p_1} = -h(p_2 - p_1)(p_1 + b - a) - 1 + H(p_2 - p_1) = 0 \iff a - b - p_1 = \rho_\Delta(p_2 - p_1), \quad (1)$$

and

$$\frac{\partial \pi_2^*}{\partial p_2} = -h(p_2 - p_1)(p_2 + b - a) + 1 - H(p_2 - p_1) = 0 \iff p_2 - (a - b) = \rho_\Delta(p_2 - p_1). \quad (2)$$

As Δ is distributed on $\mathbb{R}^+$, $p_2^* \geq p_1^*$ in any equilibrium, with strict inequality whenever the app ZLB does not bind. First, consider an equilibrium where $p_1^* = 0 \leq p_2^*$. By (1), this is the case if

[42] For $p_0 + p_1 > v$, only the 3rd party app is bought in equilibrium, and firms' profits are

$$\pi_0 + \pi_1 = \pi_1 = [1 - H(p_0 + p_2 - v)](p_0 + a),$$

and

$$\pi_2 = [1 - H(p_0 + p_2 - v)](p_2 + b - a).$$

For the deviation $p_0 = v - p_1$ (so p_2 remains the same): $p_0 + p_2 - v = p_2 - p_1$, and so

$$\pi_0^* + \pi_1^* = [1 - H(p_0 + p_2 - v)](p_0 + a) + H(p_0 + p_2 - v)(b + v) > \pi_1.$$

Therefore, $p_0^* = v - p_1$ is set so that all consumers access the platform in equilibrium.

and only if

$$\left.\frac{\partial[\pi_0^* + \pi_1^*]}{\partial p_1}\right|_{p_1=0} \leq 0 \iff a - b \leq \rho_\Delta(p_2) \leq p_2 - (a-b) \iff p_2 \geq 2(a-b), \tag{3}$$

where the second inequality uses (2), which holds with equality as long as $p_2 > 0$. Hence, in equilibrium $p_1^*(a) = p_2^*(a) = 0$ if $\frac{\partial[\pi_0^*+\pi_1^*]}{\partial p_1}|_{p_1=p_2=0} \leq 0$ and $\frac{\partial \pi_2^*}{\partial p_2}|_{p_1=p_2=0} \leq 0$, which gives $a < b - \rho_\Delta(0)$. In turn, from (2),

$$p_2 \geq 2(a-b) \iff a - b \leq \rho_\Delta(2(a-b)), \tag{4}$$

which, as the LHS (resp. RHS) is increasing (resp. decreasing) in a, is satisfied if and only if $a \leq \underline{a}$, with $\underline{a} > b$. The platform's profit is

$$\pi_0^*(a) + \pi_1^*(a) = v + a + H(p_2^*)(b-a).$$

For $a \in [0, b - \rho_\Delta(0)]$, as $p_2^* = p_2^* = 0$ and $H(0) = 1$ (i.e., all consumers buy the 3rd party app), $\pi_0^*(a) + \pi_1^*(a) = v + a < v + b \equiv \pi^F$, with π^F denoting the platform's profit under foreclosure. For $a \in (b - \rho_\Delta(0), \underline{a}]$, $p_2^* > 0$, and, by the implicit function theorem,

$$\frac{\partial[\pi_0^* + \pi_1^*]}{\partial a} = h(p_2^*)\frac{\partial p_2^*}{\partial a}(b-a) - H(p_2^*) + 1 > 0 \iff \frac{\partial p_2^*}{\partial a}(a-b) < \rho_\Delta(p_2^*) = p_2^* - (a-b),$$

which is satisfied for all $a < b$, as $\frac{\partial p_2^*}{\partial a} > 0$ (since $p_2^* > 0 > a - b$), and for $a \in [b, \underline{a}]$ as well, by (3), as $\frac{\partial p_2^*}{\partial a} < 1$ (the monotone hazard rate assumption implies $\frac{\partial p_2^*}{\partial a} \in (0,1)$). Therefore, we can conclude that $\frac{\partial[\pi_0^*+\pi_1^*]}{\partial a} > 0$ for all $a \in [0, \underline{a}]$. Given that $\pi_0^*(b) + \pi_1^*(b) = \pi^F$, it then follows that non-price foreclosure is optimal for the platform if and only if $a < b$.

Next, consider an equilibrium where $p_2^* > p_1^* \in (0, v)$. In this equilibrium, (1)-(2) imply

$$a - b - p_1 = \rho_\Delta(p_2 - p_1) = p_2 - (a-b) \iff p_1 + p_2 = 2(a-b). \tag{5}$$

As $p_2^* > p_1^*$, it must be $p_1^* < a - b < p_2^*$. Using (5), (1) rewrites as

$$a - p_1 - b = \rho_\Delta(2(a - p_1 - b)). \tag{6}$$

As the LHS (resp. RHS) is decreasing (resp. increasing) in p_1, this equilibrium exists if and only if

$$p_1^* > 0 \iff a - b > \rho_\Delta(2(a-b)) \iff a > \underline{a},$$

and, using (2),

$$p_1^* < v \iff a - b - v < \rho_\Delta(2(a-b-v)) \iff a < \overline{a},$$

where, comparing the two above inequalities, it follows that $\overline{a} > \underline{a}$. From (6) it follows that $p_1^* - a$ is constant varying a. Since $p_1 + p_2 = 2(a-b)$ is equivalent to $p_1 - a = a - p_2 - 2b$, this implies that $a - p_2^*$ is constant in a as well, and so also $p_2^* - p_1^*$ does not vary with a. This shows a neutrality result: $\pi_0^*(a) + \pi_1^*(a) = [1 - H(p_2^* - p_1^*)](v - p_1^* - a) + H(p_2^* - p_1^*)(b + v)$ is independent of a in this range. However, $\pi_0^*(a) + \pi_1^*(a) > \pi^F$ since $p_1^* < a - b$.

Finally, we consider an equilibrium where $p_1^* = v < p_2^*$ (and so $p_0^* = 0$). By (1) and (2), this is the case if and only if

$$\left.\frac{\partial[\pi_0 + \pi_1]}{\partial p_1}\right|_{p_1 = v} \geq 0 \iff a - b - v \geq \rho_\Delta(p_2 - v) = p_2 - (a - b) \iff a - b - v \geq \rho_\Delta(2(a - b - v)),$$

which holds if and only if $a \geq \overline{a}$, with $\overline{a} > b + v$ implying $p_1^* = v < a - b$. The platform's profit is

$$\pi_0^*(a) + \pi_1^*(a) = H(p_2^* - v)(b + v - a) + a.$$

We then have:
$$\frac{\partial[\pi_0^* + \pi_1^*]}{\partial a} = h(p_2^* - v)\frac{\partial p_2^*}{\partial a}(b + v - a) - H(p_2^* - v) + 1,$$

where, by the monotone hazard rate assumption, $\frac{\partial p_2^*}{\partial a} \in (0, 1)$ is characterized using the implicit function theorem. We then obtain:

$$\frac{\partial[\pi_0^* + \pi_1^*]}{\partial a} > 0 \iff a - b - v < 2(p_2^* + b - a) + \frac{h'(p_2^* - v)}{h(p_2^* - v)}(p_2^* + b - a)^2. \tag{7}$$

At $a = \overline{a}$, $p_2^* = 2(a - b) - v$. Substituting into (7) and simplifying gives

$$\frac{h'(2(a - b - v))}{h(2(a - b - v))}(a - b - v) = \frac{h'(2(a - b - v))}{h(2(a - b - v))}\rho_\Delta(2(a - b - v)) > -1,$$

where the equality follows from the definition of $\overline{a}$. This inequality is always satisfied as it is equivalent to the assumption of decreasing inverse hazard rate. Therefore, the platform's equilibrium profit is still increasing at $a = \overline{a}$, and so $a^* > \overline{a}$. ■

As $p_0^* + p_1^* = v$, consumers purchasing the platform's in-house app have zero surplus, and consumer surplus writes as

$$S^* = \int_{\Delta \geq p_2^* - p_1^*} [\Delta - (p_2^* - p_1^*)] dH(\Delta) > 0,$$

if there is no foreclosure, and $S^F = 0$ with foreclosure. Social welfare is given by

$$W^* = b + v + \int_{\Delta \geq p_2^* - p_1^*} \Delta dH(\Delta),$$

if there is no foreclosure, and $W^F = b + v < W^*$ with foreclosure.

Hence, both consumer surplus and social welfare are lower under foreclosure: If non-price foreclosure cannot be monitored, it must be that $a \geq b$. Moreover, both welfare objectives are decreasing in the relative price $p_2^* - p_1^*$. The access fee thus affects S^* and W^* only through its impact on the equilibrium prices. Then:

- for $a \in [0, \underline{a}]$, as p_2^* is increasing in a (strictly so for $a > b - \rho_\Delta(0)$) and $p_1^* = 0$, S^* and W^* are decreasing in a (strictly so for $a > b - \rho_\Delta(0)$);
- for $a \in (\underline{a}, \overline{a})$, $p_2^* - p_1^*$, and hence S^* and W^*, are constant when a varies;
- for $a \geq \overline{a}$, as p_2^* is strictly increasing in a and $p_1^* = v$, S^* and W^* are strictly decreasing in a.

Hence, if monitoring non-price foreclosure is not feasible, the optimal access fee, both from a consumer-surplus and a total-welfare standpoint, is $\hat{a} = b$. If, on the contrary, non-price foreclosure could be monitored, then any $a \in [0, b - \rho_\Delta(0)]$ would maximize both welfare objectives. ■

Proof of Proposition 10

As $u^i \equiv \max\{v - p_1^i,\ v + \Delta - p_2^i,\ 0\}$, and the superior app always charges $p_2^{i*} \geq \Delta$, we have $u^i \in [0, v]$ for all i. As $p_0 \geq 0$ and consumers' outside option is zero, also $U^i \in [0, v]$. Next, take two platforms i' and i'' and suppose they offer different utility levels to consumers $v \geq U^{i'} > U^{i''} \geq 0$, with $U^{i'} = \max_j\{U^j\}$ so that platform i' has strictly positive market share. Then, platform i'' would face no demand and make zero profit. By foreclosing the 3rd party app and setting prices $p_0^i + p_1^i \leq v - U^{i'}$, it would offer utility $U^{i''} \geq U^{i'}$ and make a positive profit.

As a result, all platforms must offer the same utility U^* in equilibrium. Hence, their profit is $\frac{1}{N}(p_0^i + p_1^i + b)$ with foreclosure, with $p_0^i + p_1^i = v - U^*$ and $\frac{1}{N}(p_0^i + a^i)$ without foreclosure, with $p_0^i = v + \Delta - p_2^* - U^*$. If $p_0^i > 0$ for some i, then, no matter whether it forecloses or not the 3rd party app, platform i would find it optimally to deviate, charging a slightly lower access price to consumers to serve all demand. Therefore, $p_0^{1*} = \ldots = p_0^{N*} = 0$ in equilibrium.

Whenever its rivals are expected to provide $U^* = v$ in equilibrium, any platform i has no profitable deviation to $U^i \neq U^*$: offering $U^i < v$ drives its profit to zero, and, as shown above, it is never possible to provide $U^i > v$. As $U^* = v$ can always be provided by foreclosing the 3rd party app and setting $p_0^i = p_1^i = 0$, it follows that an equilibrium where $U^* = v$ always exists. We next characterize the corresponding subgame perfect equilibrium prices for any given $(a^i, \delta^i = \Delta)_{i=1,\ldots,N}$ (no foreclosure). Suppose that in equilibrium $p_1^i > 0$. Then, as $p_2^{i*} = \min\{p_1^i + \Delta, v + \Delta\}$, $u^i = \max\{v - p_1^i, 0\} < v$. Given that rival platforms offer higher value $U^* = v$, the considered platform makes no profit. It has therefore a strictly profitable deviation: It can set $p_1^i = 0$ and thus, by selling its in-house app, offer value $U^i = v$ to consumers, so as to attract some of them and make positive profits (given that the marginal cost is negative). Hence, the app ZLB binds: $p_1^{i*} = 0$ for all i and $(a^i, \delta^i = \Delta)_{i=1,\ldots,N}$. Anticipating this, the (non-foreclosed) 3rd party seller must set $p_2^{i*} = \Delta$ to sell its app. It optimally does so whenever selling its app yields positive profit (i.e., as long as $\Delta + b - a^i \geq 0$).

As each platform's profit $\frac{a^i}{N}$ is increasing in the access fee, unregulated platforms set the highest a subject to the 3rd party app's participation constraint: $a^* = b + \Delta$. Hence, $\pi_0^* = \frac{b+\Delta}{N}$ exceeds the foreclosure profit $\frac{b}{N}$ (given that a platform foreclosing the superior app can provide utility

$U^* = v$ to consumers and serve a share $1/N$ of them only by setting $p_0 = p_1 = 0$). By contrast, as it makes $\Delta + b - a^i$ per-consumer on any platform i, for the 3rd party app to receive its fair reward, the access fee must be capped by regulation at the Pigouvian level ($\hat{a} = b$), at which $\pi_0 = b/N$ coincides with the foreclosure payoff. ■

Proof of Proposition 11

As in Proposition 10, app store competition implies $p_0^{j*} = p_1^{j*} = 0$, for all a^j and app stores j, including the in-house app store. As all app stores can be accessed for free, consumers multi-home. An equilibrium where the 3rd party app is foreclosed by all app stores cannot exist: Given that consumers would prefer to buy the superior app at any price $p_2^j \leq \Delta$, any app store would deviate by granting access to the superior app at the extractive access fee $a^j = b + \Delta$.

Therefore, the 3rd party provider can sell its app to all consumers on any app store j at any price $p_2^j \leq \Delta$. Therefore, it will optimally sell at a price (slightly below) Δ on the app store charging the lowest access fee. Whenever $a^j \geq a^{j'} > 0$ and app store j' attracts some 3rd party app sales, app store j can profitably undercut its rival (i.e., set $a^j = a^{j'} - \epsilon$) so as to induce the 3rd party provider to rather serve all consumers through its app store. It then follows that in equilibrium $a^* = 0$ for all app stores, and so $\pi_2^* = \Delta + b$, while app stores make zero profits.

As the equilibrium apps' and app stores' prices are the same for all values of the access fee (provided the 3rd party app is viable), $\pi_2^* = \Delta$ if and only if $a = b$ (so that app stores collectively make profit b).

Finally, both under laissez faire and with regulated access fees, consumers get net value v_a from the app stores and v_d from the device, and so the monopoly platform optimally sets $p_0 = v_a + v_d$. ■

Proof of Proposition 12

With multiple heterogeneous app markets, consumers' utility from accessing the app store is

$$U \equiv \int_{k \in [0,1]} u^k \, dk - p_0,$$

with p_0 again denoting consumers' access price, and u^k being the utility obtained from app market $k \in [0, 1]$:

$$u^k \equiv \max\{v^k - p_1^k,\ v^k + \Delta^k - p_2^k,\ 0\},$$

where p_1^k and p_2^k denote the prices for in-house app and 3rd party app, respectively, in the considered market k.

As seen in the basic model, irrespective of the ownership of the inferior app in each market k, equilibrium prices are

$$p_1^{k*} = \max\{a^k - b^k, 0\}, \quad p_2^{k*} = \min\{p_1^{k*} + \Delta^k, v^k + \Delta^k\},$$

whenever $a^k \leq b^k + v^k + \Delta^k$ (any larger access fee implies access-price foreclosure of the 3rd party app).

Then, p_0 is set so as to satisfy consumers' participation constraint with equality ($U = 0$):

$$p_0^* = \int_{k\in[0,1]} u^{k*} \, dk.$$

Hence, denoting $x^k = 0$ (resp. $x^k = 1$) if the inferior app (resp. superior app) is sold in market k, platform's profit writes

$$p_0^* + \int_{\{k:\, x^k=0\}} (p_1^{k*} + b^k) \, dk + \int_{\{k:\, x^k=1\}} a^k \, dk = \int_{\{k:\, x^k=0\}} \pi^k(x^k = 0) \, dk + \int_{\{k:\, x^k=1\}} \pi^k(x^k = 1) \, dk,$$

where

$$\pi^k(x^k = 0) \equiv v^k + b^k, \qquad \pi^k(x^k = 1) \equiv v^k + \Delta^k - p_2^{k*} + a^k,$$

are the per-market profits with and without foreclosure, respectively (inclusive of the revenues from optimally setting consumers' access price).

Note that (i) there is no scope for acquiring the high-value app given that the sum of platform's and high-value app provider's profit is always (excluding the uninteresting Pareto-dominated price-foreclosure region) $b^k + v^k + \Delta^k$, which coincides with the vertically integrated platform's profit; (ii) as the inferior app always makes zero profit in equilibrium, the platform can vertically integrate by acquiring it at a negligible cost, and can gain from vertical integration only by foreclosing the superior app.

If not foreclosed, the superior 3rd party seller in market k makes

$$\pi_2^{k*} = p_2^{k*} + b^k - a^k.$$

In any market k where $a^k < b^k$, absent foreclosure, consumers purchase the superior app at $p_2^{k*} = \Delta^k$ and obtain utility $u^{k*} = v^k$. As this is the same utility that they would obtain under foreclosure and $p_1^{k*} = 0$, it follows that by foreclosing superior rivals in any such market the platform can charge the same access price p_0^* to consumers, but obtains higher unit revenues $b^k > a^k$. Therefore, vertical integration with the inferior app and non-price foreclosure occur for all $b^k > a^k$. In any market k where $a^k \in [b^k, b^k + v^k]$, absent foreclosure, consumers purchase the superior app at $p_2^{k*} = a^k - b^k + \Delta^k$ and obtain utility $u^{k*} = v^k - (a^k - b^k) > 0$. From any such market, the platform obtains profit $\pi^k(x^k = 1) = v^k + b^k = \pi^k(x^k = 0)$, and so is indifferent between foreclosing or not. The 3rd party seller gains $\pi_2^{k*} = \Delta^k$. Finally, in any market k where $a^k \in (b^k + v^k, b^k + v^k + \Delta^k]$, absent foreclosure, consumers purchase the superior app at $p_2^{k*} = v^k + \Delta^k < a^k - b^k + \Delta^k$ and obtain utility $u^{k*} = 0$. From any such market, the platform obtains profit $\pi^k(x^k = 1) = a^k > v^k + b^k = \pi^k(x^k = 0)$, and so is strictly better off than under foreclosure. The superior app is squeezed: $\pi_2^{k*} = v^k + \Delta^k + b^k - a^k < \Delta^k$. Clearly, the profit-maximizing fee in market k is $a^* = v^k + \Delta^k + b^k$.

To show that setting $a^k = b^k$ is not incentive-compatible for the platform, take two markets k' and k'' such that $v^{k'} \leq b^{k''} - b^{k'} \leq v^{k'} + \Delta^{k'}$. By the above analysis, if the platform sets $a^k = b^k$ for $k \in \{k', k''\}$, it obtains profit $\pi^k = v^k + b^k$ from each of these markets. By setting instead $a^{k'} = b^{k''}$ and $a^{k''} = b^{k'}$, and foreclosing the 3rd party app in market k'', it still obtains profit $\pi^{k''} = v^{k''} + b^{k''}$ in the higher-b market, but now makes a larger profit $\pi^{k'} = a^{k'} = b^{k''} > b^{k'} + v^{k'}$ from the lower-b market.[43] ■

Proof of Proposition 15

The arguments outlined in the text imply that the 3rd party app in market k will appeal whenever $\int_{\mathbb{R}} \tau(a^k - \tilde{b}^k)\mathrm{d}R(\tilde{b}^k) > 0 \Leftrightarrow a^k > b^k$.

Moving backwards to the pricing stage, $p_1^k = 0$ and $p_2^k = \Delta^k$ is the worst-case scenario for the 3rd party app in any platform-pivotality equilibrium. Then, for all $a^k \leq b^k + \Delta^k$ (no access-price foreclosure), the 3rd party app's profit, absent non-price foreclosure, is at least $\pi_2^k = \Delta^k + b^k - a^k$. Because total profit is at most $v^k + b^k + \Delta^k$, the platform's maximal expected profit from setting any $a^k > b^k$ and therefore being challenged is

$$v^k + a^k - \int_{\mathbb{R}} \tau(a^k - \tilde{b}^k)\mathrm{d}R(\tilde{b}^k),$$

which is decreasing in a^k provided $\tau > 1$. In contrast, the platform makes profit $v^k + b^k$ either by setting $a^k < b^k$ and foreclosing the superior app, or by choosing $a^k = b^k$, while any other (a^k, δ_2^k)-choice yields strictly lower profit. Therefore, the Pigouvian principle can always be implemented by giving the platform a tiny advantage in the appeal procedure – e.g., the appeal benefits the 3rd party app if and only if $a^k > \tilde{b}^k + \epsilon$ for a small positive ϵ, so that a small squeeze is tolerated and the platform strictly prefers not to foreclose.

The observation that the 3rd party app appeals for any $a^k > b^k(+\epsilon)$ crucially hinges on the fact that appealing has no impact on its market profit. This would not be the case if the platform had the possibility (and the incentive) to foreclose it *post* appeal (as discussed in the text). When such a post-appeal foreclosure threat is credible, the 3rd party app does not appeal whenever $\Delta^k + b^k - a^k \geq \int_{\mathbb{R}} \tau(a^k - \tilde{b}^k)\mathrm{d}R(\tilde{b}^k)$, or equivalently $a^k \leq a^\dagger \equiv b^k + \frac{\Delta^k}{\tau+1}$. If τ is large enough relative to the platform's discount factor β, however, such reputation building strategy can be prevented. To see this, suppose for simplicity that, by foreclosing after $a = a^\dagger$ is appealed in the first market, the platform is able to secure profit $v^k + a^\dagger$ forever after, which implies a discounted extra profit $\frac{\beta\Delta^k}{(1-\beta)(1+\tau)}$ from future markets relative to the profit $v^k + b^k$ it obtains by proposing $a^k = b^k$,[44] at an expected loss $\int_{\mathbb{R}} \tau(a^\dagger - \tilde{b}^k)\mathrm{d}R(\tilde{b}^k) = \tau\frac{\Delta^k}{1+\tau}$ from the appeal. Therefore, setting

[43] Note that this deviation is not profitable if the platform cannot foreclose the 3rd party app in the high-b market k''. Indeed, the deviation profit equals now $\pi^{k''} = v^{k''} + b^{k'}$ in market k'', and so $\pi^{k'} + \pi^{k''} = b^{k'} + b^{k''} + v^{k''}$ is lower than the equilibrium profit $(b^{k'} + b^{k''} + v^{k'} + v^{k''})$. This suffices to prove that, if foreclosure can be monitored, then, under the constraint that the distribution of a mimic that of b, setting $a^k = b^k$ for all $k \in [0,1]$ is incentive-compatible for the platform.

[44] For $\tau > 1$, this profit in turn exceeds the profit from proposing $a^\dagger$ in the first market and not foreclosing after being challenged, thereby failing to build a reputation for foreclosing future apps.

$\tau \geq \frac{\beta}{1-\beta}$ prevents such reputation building strategy. ■

Appendix B: Simple extensions

Lower bound and the rich ecosystem argument

Suppose a device with production cost c_d brings non-platform benefits $v_d > c_d$. This device is manufactured by the platform, and we will let p_0 denote the price of the bundle device cum app store. There are two categories of consumers: A fraction κ do not use the app store and thus receive net surplus $v_d - p_0$ from buying the device. A fraction $1-\kappa$ further use the app store and obtain utility $v_d - p_0 + u(a)$ where $u(a)$ is their net surplus obtained from the apps. Let $\pi(a)$ denote the platform's profit associated with apps.

Finally, suppose a continuum of app markets. In each app market, there is a single app and this app is a 3[rd] party app. A fraction α of consumers using the app store value this app at Δ while the complementary fraction value it at $\zeta \in (0, \Delta)$. The draws are independent across app markets and so the payoffs of consumers are deterministic. Assuming $\zeta > \alpha\Delta$, let $\bar{a}$ denote the access fee such that an app owner is indifferent between screening high WTP consumers and selling to all types:

$$\zeta - \bar{a} = \alpha(\Delta - \bar{a})$$

So for $a \leq \bar{a}$, $u(a) = \alpha(\Delta - \zeta)$ and $\pi(a) = a$, while for $a > \bar{a}$, $u(a) = 0$ and $\pi(a) = \alpha a$. The platform w.l.o.g. either chooses to be extractive ($a = a^* = \Delta$) or goes for a lower access fee ($a = \bar{a}$). If the platform attracts all consumers, including those who do not use the app store, then $a = a^*$and $u(a) = 0$ if and only if $\alpha\Delta > \bar{a}$. If the platform attracts only consumers who use the app store, it maximizes $u(a) + \pi(a)$ and so chooses $a = a^*$ if and only if $\alpha\Delta > \bar{a} + \alpha(\Delta - \zeta)$ (or $\alpha\zeta > \bar{a}$). We conclude that:

Proposition. *The access fee is weakly higher when the inframarginal consumer does not benefit from app store quality (where quality is defined as the surplus obtained by the consumer on the app store).*

This analysis carries a clear intuition, but may be criticized for not allowing unbundling. Given the positive correlation between overall WTP and WTP for the apps among consumers, the platform could do better by selling the device and access to the app store separately. We leave this puzzle for future research.

Platform viability and entry

Assume that the social welfare function is $U + \omega\Pi$, where Π is total profit (platforms and apps) and $\omega \in (0, 1)$ is the weight on industry profits relative to consumer surplus.[45] Suppose that

[45] Under a social welfare standard ($\omega = 1$), platform competition just entails socially wasteful duplicative entry costs: welfare maximization dictates $N = 1$. On the contrary, under a consumer surplus standard ($\omega = 0$), as a monopolist brings zero net value to consumers, entry by any number $N \geq 2$ of platforms would be optimal (i.e., there is never excessive entry in equilibrium from consumers' standpoint).

there is (sequential) free entry into the platform segment, with entry cost J. Suppose further that self-preferencing cannot be monitored, and so the access fee must be no lower than b.

The socially optimal number of platforms is at most two, because extra platforms beyond $N = 2$ do not alter the consumer surplus and variable profit, but add entry costs and so are necessarily suboptimal if $N > 2$. Given that, with $N = 3$, each entrant makes $\frac{b}{3}$ under the Pigouvian rule, and higher profits under laissez-faire, for all $J \leq \frac{b}{3}$ there is always too much entry into the platform segment, which, without monitoring of non-price foreclosure (or a ban on the hybrid platform model), cannot be prevented by access fee regulation.

Under a monopoly platform, the consumers obtain no surplus ($U = 0$). A second entrant increases consumer surplus by v, at the expense of platform total profit, but also entails a socially wasteful entry cost J. Formally, $U + \omega\Pi = v + \omega(b + \Delta - 2J)$ under duopoly and $U + \omega\Pi = \omega(b + v + \Delta - J)$ under monopoly. Hence, a duopoly is preferred to monopoly if and only if $(1 - \omega)v \geq \omega J$, or $J \leq \frac{1-\omega}{\omega}v$.

Thus, assuming that the access fee is set, by regulation, at the Pigouvian level, for $J \in [\max\{\frac{1-\omega}{\omega}v, \frac{b}{3}\}, \frac{b}{2}]$ there is again too much entry, as two platforms enter but it would be optimal to have one. The region of parameters where excessive entry prevails of course expands when platforms are free to set access fees, as the absence of regulation increases their profits. If, on the contrary, $J \in (\frac{b}{2}, \min\{\frac{1-\omega}{\omega}v, b + v\}]$, then spurring the welfare-maximizing second entry requires setting the access fee above the Pigouvian level. Similarly, if $J \in (b + v, b + v + \Delta]$, there is a potential trade-off between the first platform's viability, which requires a squeeze in the app's profit, and app viability, which calls for staying away from the squeeze region to obtain the proper level of innovation.

In sum, we have:

Proposition. *Because the core ZLB prevents platform profits from being competed away, socially excessive entry prevails when the entry cost is low and foreclosure cannot be monitored. By contrast, for high entry costs, setting access fees above the Pigouvian level is desirable to spur platform entry, if no other instrument is available (as we saw, $a > b$ introduces distortions).*

These results suggest that, while access fee regulation is an effective instrument to achieve fairness, thereby promoting efficient entry and investment decisions in the app segment, it may not be a jack of all trades, able to take on extra tasks such as ensuring contestability of the core segment.

Ad-valorem access fees

Throughout the paper we considered for simplicity linear (per-unit) access fees. Here we show that our results are robust when considering instead ad-valorem fees (which are more often employed in reality): For each app sold by the 3$^{\text{rd}}$ party seller at price p_2, the platform gets tp_2 and the seller $(1 - t)p_2$, with $t \in [0, 1]$. Let us first consider the hybrid platform case.

Lemma. *For any ad-valorem access fee $t \in [0, 1]$, the equilibrium has the following features:*

1. *If $b \geq \Delta$, the platform is better off foreclosing the 3^rd^ party app for all t; if foreclosure can be monitored, $\pi_2^*(t) = (1-t)\Delta + b \geq b$.*

2. *If $b < \Delta$:*

   - *For $t \in [0, \frac{b}{\Delta})$: $p_1^* = 0$, $p_2^* = \Delta$, and $p_0^* = v$; hence, $\pi_0^*(t) + \pi_1^*(t) = v + t\Delta < v + b \equiv \pi^F$: the platform is better off foreclosing the 3rd party app; if foreclosure can be monitored, $\pi_2^*(t) = (1-t)\Delta + b > \Delta$.*
   - *For $t \in \left[\frac{b}{\Delta}, \frac{b+v}{v+\Delta}\right]$, $p_1^* = \frac{t\Delta - b}{1-t}$, $p_2^* = \frac{\Delta - b}{1-t}$, and $p_0^* = v - \frac{t\Delta - b}{1-t}$; for any such t, $\pi_0^*(t) + \pi_1^*(t) = \pi^F$ and $\pi_2^*(t) = \Delta$ (neutrality).*
   - *For $t \in (\frac{b+v}{v+\Delta}, 1]$: $p_1^* = t(v+\Delta) - b$, $p_2^* = v + \Delta$, and $p_0^* = 0$; hence, $\pi_0^*(t) + \pi_1^*(t) = t(v+\Delta) > \pi^F$ and $\pi_2^*(t) = (1-t)(v+\Delta) + b < \Delta$ (squeeze).*

Proof of Lemma. In this setting, the platform prefers selling its own app as long as $tp_2 < b + p_1$. Hence, given the ZLB constraints, equilibrium app prices are

$$p_1^* = \max\{0, tp_2^* - b\} \quad \text{and} \quad p_2^* = \min\{p_1^* + \Delta, v + \Delta\}.$$

As long as app providers are unconstrained by consumers' willingness to pay, the equilibrium app prices are

$$\begin{cases} p_1^* = \frac{t\Delta - b}{1-t}, p_2^* = \frac{\Delta - b}{1-t} & \text{if } t \geq \frac{b}{\Delta} \\ p_1^* = 0, p_2^* = \Delta & \text{if } t < \frac{b}{\Delta} \end{cases}$$

Hence, for $t < \frac{b}{\Delta} \in (0,1)$, $p_0^* = v$, and foreclosure is optimal (this result holds for all $t \in [0,1]$ when $b > \Delta$): $\pi_0^*(t) + \pi_1^*(t) = v + t\Delta < v + b \iff t < \frac{b}{\Delta}$. For $t \geq \frac{b}{\Delta}$, $p_0^* = v + \Delta - p_2^* = v + \frac{b - t\Delta}{1-t}$, and so $\pi_0^*(t) + \pi_1^*(t) = v + b = \pi^F$ and $\pi_2^*(t) = v$. This is the equilibrium outcome as long as $p_2^* < v + \Delta$, which requires $t < \frac{b+v}{v+\Delta}$. For $t \geq \frac{b+v}{v+\Delta}$, $p_2^* = v + \Delta$, and so $p_1^* = t(v+\Delta) - b \in (v, p_2^* - \Delta)$ and $p_0^* = 0$. In this case, $\pi_0^*(t) + \pi_1^*(t) = t(v+\Delta) > \pi^F$ and $\pi_2^*(t) = (1-t)(v+\Delta) + b < \Delta$. ■

Under ad-valorem fees, the platform can capture Δ, which is charged by the superior app, but cannot capture b. As a result, if t is not regulated foreclosure is always optimal if $b > \Delta$.

If instead $b < \Delta$, the equilibrium characterization mirrors the one under unit fees: In these cases, for low (resp. high) values of the access fee, the app (resp. core) ZLB binds, and the platform is strictly better off foreclosing (resp. not foreclosing) the 3rd party app. For intermediate values of t, no ZLB binds, and the neutrality result holds. Accordingly, it is easy to derive the following results:

Proposition (optimal access fees). *Suppose $b < \Delta$. Then:*

(i) Welfare-optimal access fees. *Any access fee such that the 3rd party app is not foreclosed maximizes ex-post social welfare: $t \in [\frac{b}{\Delta}, 1]$ if non-price foreclosure cannot be monitored, $t \in [0,1]$ under monitoring of self-preferencing;*

(ii) Profit-maximizing access fee. *Platform's profit is maximized at $t^* = 1$;*

(iii) Fair access fees. *The independent developer receives a fair reward for its contribution to the ecosystem if and only if* $t \in \left[\frac{b}{\Delta}, \frac{b+v}{v+\Delta}\right]$.

Proof of Proposition. As consumer surplus is always extracted by the platform through the access price, social welfare is simply $W^* = b + v + \Delta x$, and so is maximized whenever there is no price or non-price foreclosure, so that $x = 1$, from which (i) follows. Platform's profit is continuous, non-decreasing in t for $t \leq \frac{b+v}{v+\Delta}$, and strictly increasing for larger values of t, hence it is maximized at $t^* = 1$, which establishes (ii). Finally, the result in (iii) follows from the equilibrium profit $\pi_2^*(t)$ given in Lemma 7. ■

Note that the lowest assess charge such that the platform has no incentives to practice self-preferencing, $\hat{t} = \frac{b}{\Delta}$, is such that $p_2^*(\hat{t}) = \Delta$, so that the platform obtains $\hat{t}p_2^*(\hat{t}) = b$ from distributing the 3rd party app. Hence, optimal access fee regulation still follows a Pigouvian principle: The superior seller must internalize that, for each app it sells, it "steals" b from the platform.

Remark (Pure-player platform). If the inferior app is also provided by a 3rd seller, it finds it optimal to sell if and only if $(1-t)p_1 + b \geq 0$, and so $p_1 = \max\{0, -\frac{b}{1-t}\} = 0$. As then $p_2 = \min\{p_1 + \Delta, v + \Delta\}$, we obtain that, for all t, $p_1^* = 0$ and $p_2^* = \Delta$: Unlike under linear fees, here an inferior 3rd party developer is a tougher competitor to the superior seller relative to the platform.

The superior app makes profit $\pi_2^*(t) = (1-t)\Delta + b$, whereas the pure-player platform obtains $\pi_0^*(t) = v + t\Delta$. Therefore:

- For all $t < \frac{b}{\Delta}$ (again, always if $b > \Delta$), the superior app makes a supranormal profit. The platform has incentives to vertically integrate, by acquiring the inferior app at a negligible price, and foreclose the superior app;
- For $t = \frac{b}{\Delta}$, the superior app obtains its fair reward, and the platform has no strict incentives to vertically integrate;
- For all $t > \frac{b}{\Delta}$, the superior app is squeezed, and the platform is strictly better off by operating as a pure-player platform.

Hence, fair compensation obtains if and only if $b < \Delta$ and the access fee is set at the "Pigouvian level": Any lower level of t either gives the platform incentives to vertically integrate and foreclose the superior seller or generates inefficient me-too entry in the app segment, whereas any larger access fee results in margin squeeze.

Freemium apps

While richer versions are available, let us give a simple example, in the context of an ad-valorem access fee equal to $t \in [0, 1)$. The basic version of either app brings utility v to the consumer, who has time to try only one of the apps. The premium app brings extra utility $V_i - v$ for $i \in \{1, 2\}$. Assume the absence of commitment to the premium price. So, using the notation introduced in Section 2.5.3, $b^{\dagger}_{i1} \equiv (1-t)(V_i - v)$ and $b^{\dagger}_{i0} \equiv 0$ in the case of a pure platform. Assume that $b^{\dagger}_{21} \geq b^{\dagger}_{11}$ and that consumers select app 2 when indifferent (this assumption can be relaxed in a more general version with heterogeneous valuations for the premium app). Then the prices for the basic versions of the apps are $p_1 = p_2 = 0$. This result would also hold for an hybrid platform owning app 1 provided that t is not too large – e.g., it is set by regulation at the fair level.

The assumption of non commitment to the premium-version price can also be relaxed. For example, the apps may charge $p_i = 0$ even if they can commit to the premium-version price, as making the basic version free may serve as an introductory price, i.e., be interpreted as a signal of a high probability that the consumer will like the app.