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Free entry in a Cournot market with overlapping ownership

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Abstract

We examine the effects of overlapping ownership among existing firms deciding whether to enter a product market. We show that in most cases—and especially when overlapping ownership is already widespread, an increase in the extent of overlapping ownership will harm welfare by softening product market competition, reducing entry, thereby (in contrast to standard results) inducing insufficient entry, and magnifying the negative impact of an increase of entry costs on entry. Overlapping ownership can mostly be beneficial only under substantial increasing returns to scale, in which case industry consolidation (induced by overlapping ownership) leads to sizable cost efficiencies. (JEL D43, L11, L13, L21, L41)

Keywords: common ownership, cross-ownership, institutional ownership, minority shareholdings, oligopoly, entry, competition policy, antitrust

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1 Introduction

Overlapping ownership, be it in the form of common or cross-ownership, has generated concern for its potential anti-competitive impact (Elhauge, 2016; Posner, Morton and Weyl, 2017), especially due to the rising shares of large investment funds in multiple competitors in several industries; for example airlines (Azar, Schmalz and Tecu, 2018), banks, and supermarkets (Schmalz, 2018). Antitrust authorities take seriously, indeed, the impact of overlapping ownership (e.g., in the 2023 Merger Guidelines). Azar and Vives (2019, 2021) and Backus, Conlon and Sinkinson (2021*b*) document the dramatic rise in common ownership among publicly listed U.S. companies in the last decades.[1] Cross-ownership is also far from rare. Although antitrust authorities scrutinize horizontal mergers, non-controlling investments in rival firms have gone largely unregulated and are common in multiple industries and countries (e.g., see Huse, Ribeiro and Verboven, 2023; Shelegia and Spiegel, 2023).

At the same time, firm entry patterns have been argued to pose a significant impact on the aggregate economy. Gutiérrez and Philippon (2019) document a decline in entry of firms in the U.S. economy and estimate the elasticity of entry with respect to Tobin's Q to have dropped to zero since the late 1990s, up to which point it was positive and significant. Gutiérrez, Jones and Philippon (2021) argue that increases in entry costs have had a considerable impact on the U.S. economy over the past 20 years, leading to higher concentration, as well as lower entry, investment, and labor income.[2] Figure 1 shows the increase in regulatory restrictions that has accompanied the decrease in entry.

The literature above documents the decline in firm entry rates (accompanied by a milder decrease in firm exit rates) and a concurrent increase in overlapping ownership over close to 40 years in the U.S. economy (see Figure 1). There are several explanations for the decreased entry dynamism, an increase in entry costs (for technological or regulatory reasons) being a prominent one. It is possible also that apart from softening competition in pricing, overlapping ownership also contributes to diminishing entry dynamism. Some recent empirical work points in this direction (Ruiz-Pérez, 2019; Xie and Gerakos, 2020; Xie, 2021; Newham, Seldeslachts and Banal-Estañol, 2022).

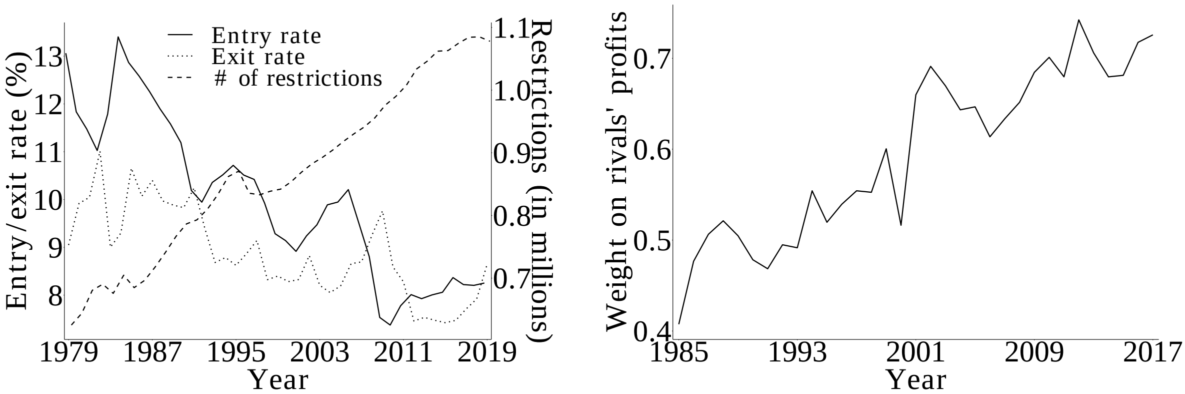
However, these empirical results do *not* imply that entry decreases with overlapping ownership in any market or under any conditions. More importantly, they do not speak

[1]Ownership links also exist among private firms, for example through the stakes of private equity firms (e.g., see Eldar and Grennan, 2023), which has generated antitrust concerns (Wilkinson and White, 2006). Particularly, Li, Liu and Taylor (2023) find that venture capital firms (VCs) that fund multiple pharmaceutical startups shut down the startups' lagging R&D projects encouraging them to turn to alternative projects. They provide evidence that VCs do so not only to limit the duplication of R&D costs but also to create market power for successful startups.

[2]Gourio, Messer and Siemer (2016) also argue for the positive effects of entry on the macroeconomy. Apart from a generalized decline in entry, Decker et al. (2016) document a particular decline in high-growth young firms in the U.S. since 2000, when such firms could have had a major contribution to job creation.

Figure 1: Firm entry, regulatory restrictions, and overlapping ownership trends in the U.S.

(a) Firm entry/exit rates and regulatory restrictions **(b)** Average weight on competing firms' profits



Note: Firm count and death data are from U.S. Census Bureau Business Dynamics Statistics. The firm entry (resp. exit) rate in year t is calculated as the count of age-zero firms (resp. firm deaths) in year t divided by the average count of firms in year t and $t-1$. The total number of regulatory restrictions data are from McLaughlin et al. (2021). Panel (b) shows the average intra-sector Edgeworth sympathy coefficient for the largest 1500 firms by market capitalization (*i.e.*, the average weight placed by a firm on the profits of another firm in the same sector relative to a weight of 1 placed on its own profit), as calculated in Azar and Vives (2021) based on Thomson-Reuters 13F filings data on institutional ownership.

to the welfare effects of overlapping ownership (through its effect on both product market competition and entry). A theoretical analysis will be useful to (i) examine the relationship between entry and overlapping ownership and (ii) gauge the welfare effects of overlapping ownership in markets without barriers to entry.

In this paper, we provide a framework to study the effects of overlapping ownership in a Cournot oligopoly with free entry. We study an industry which established firms with existing ownership ties consider whether to enter; that is, there is pre-entry overlapping ownership. This fits today's markets with extensive common ownership links among public firms.[3] In Appendix B.10 we consider the case of post-entry overlapping ownership.

The main takeaway of our analysis is that in most relevant cases—and especially when overlapping ownership is already high, an increase in the extent of overlapping ownership will harm welfare not only by (i) softening product market competition but also by (ii) reducing entry, thereby (in contrast to standard results) inducing or exacerbating insufficient entry, and (iii) magnifying the negative impact of an increase in entry costs on entry. Overlapping ownership can mostly be beneficial only under increasing returns to scale (IRS), in which case industry consolidation (induced by overlapping ownership) leads to significant cost efficiencies. For example, overlapping ownership among pharmaceuticals that focus on drug discovery—which, due to large R&D costs, is characterized by strong IRS—can be socially beneficial, unlike overlapping ownership involving generic drug

[3]For example, chain stores decide whether to pay a fixed cost to open a new branch, pharmaceuticals whether to incur R&D and regulatory costs to enter a new drug market, tech firms whether to perform R&D to enter new product markets, and airlines which routes to serve.

manufacturers, which is more likely to suppress generic entry without generating cost efficiencies, thereby damaging welfare.

Overlapping ownership differs from collusion in terms of both the mechanism through which it affects competitive outcomes and the actual competitive effects. Both pre-entry overlapping ownership and collusion induce firms to internalize the effects of their actions on other firms' profits, but we show that the former gives rise to trade-offs and forces that are not present under collusion.[4]

The prospect of entry acts as a constraint on incumbents and limits the attainable collusive outcomes since colluding incumbents increase prices enhancing the incentives for entry. Similarly, post-entry overlapping ownership (*i.e.*, when ownership links develop after entry) tends to spur entry by softening pricing competition (thereby increasing profits), since a firm decides whether to enter only seeking to maximize its own profit.

Pre-entry overlapping ownership induces a novel trade-off (absent in the context of collusion or post-entry overlapping ownership) in terms of its effect on entry. We distinguish the three channels (not specific to our assumption of Cournot competition) through which an increase in the level of pre-entry overlapping ownership affects entry. Overlapping ownership tends to limit entry by increasing the degree of internalization of the negative externality of entry on other firms' profits but also tends to increase equilibrium profits in the product market competition stage, which tends to increase entry. Overlapping ownership also changes the magnitude of the entry externality, and this channel has an ambiguous effect on entry.

The effect of overlapping ownership on entry will depend on the magnitude of the different channels and the direction of the ambiguous channel's effect. We find that an increase in the degree of overlapping ownership can limit but also spur entry. For markets with many active firms and low levels of overlapping ownership, the rise in own profit due to an increase in overlapping ownership can dominate. However, in markets with only a few firms or already high levels of overlapping ownership, competition in the product market is already soft enough, so further increases in overlapping ownership suppress entry. Common ownership among U.S.-listed firms is indeed already widespread and increasing (see Figure 1) and several U.S. markets are dominated by a few large public companies. Thus, further increases in common ownership are likely to limit entry by public firms into product markets where other public firms already operate (or also consider operating).

However, it is not immediately obvious that this suppression of entry is welfare-damaging. Particularly, if entry is excessive (compared to the total surplus-maximizing level of entry)—as is generally the case in homogeneous product markets without overlapping ownership (see Mankiw and Whinston, 1986; Amir, Castro and Koutsougeras, 2014), then the suppression of entry caused by overlapping ownership will tend to improve

[4]The effect of overlapping ownership on incentives could in turn have coordinated effects facilitating or impeding collusion (e.g., see Malueg, 1992; Gilo, Moshe and Spiegel, 2006).

welfare. This tendency toward excessive entry is due to business-stealing competition (*i.e.*, the fact that the individual quantity decreases with the number of firms). Namely, if by entering a firm causes incumbent firms to reduce output, entry is more attractive to the entrant than it is socially desirable.

But—given that ownership links cause firms to partly internalize the effect that their entry would have on other firms' profits—does this general excessive entry result apply to markets *with* overlapping ownership? We show that it does not. In the standard case of business-stealing competition, we find that under decreasing returns to scale (DRS) and high levels of overlapping ownership, entry is insufficient.[5] Then, any decrease in entry (e.g., due to an expansion of overlapping ownership) will harm welfare. On the other hand, entry is (weakly) excessive under substantial increasing returns to scale (IRS). In that case, the socially optimal level of entry under both a total surplus and a consumer surplus standard is a monopoly, which can be achieved through extreme (high) levels of overlapping ownership.

Also, we show that under common assumptions, overlapping ownership exacerbates the negative impact of an increase in entry costs on entry. Therefore, overlapping ownership could magnify the negative macroeconomic implications documented in Gutiérrez, Jones and Philippon (2021).

The plan of the paper is as follows. After a discussion of related literature, section 2 presents the model and studies the quantity-setting stage. Section 3 studies the entry stage, existence, and uniqueness of equilibrium in the complete game with entry. Section 4 studies the effects of overlapping ownership under free entry. Section 5 discusses the robustness of our results and considers post-entry overlapping ownership and the entry of maverick firms. Last, section 6 concludes. Proofs are gathered in Appendix A. Supplementary material (including the case of post-entry overlapping ownership) and proofs thereof are in Appendix B.

Related literature. Research attention to the possible anti-competitive effects of overlapping ownership dates back to at least Rubinstein and Yaari (1983) and Rotemberg (1984). Recently, interest in the topic has revived given the rising shares of large diversified funds.[6] Multiple empirical studies have been conducted and there is a debate on whether and how common ownership affects corporate conduct and softens competition.[7]

[5]That is because the planner takes advantage of cost savings due to entry to a greater extent than firms do. Although—with overlapping ownership—a firm also prefers lower average costs both for itself and for the other firms, the internalization of the price effect (that its entry would have) suppresses entry relative to the planner's solution. Under business-enhancing competition (*i.e.*, when the individual quantity increases with the number of firms in the Cournot game), entry is again insufficient.

[6]Banal-Estañol, Seldeslachts and Vives (2020) show that the profit loads firms place on competing firms increase if the holdings of more diversified investors increase relative to those of less diversified investors.

[7]While He and Huang (2017), Azar, Schmalz and Tecu (2018), Banal-Estañol, Seldeslachts and Vives (2020), and Antón et al. (2023) find evidence in favor of this hypothesis, others have found little to no

The received literature treats the number of firms in the industry as exogenous.[8] Sato and Matsumura (SM; 2020) provide a circular-market model with constant marginal cost of production and free entry under pre-entry symmetric common ownership. In their setting, they show that entry always decreases with common ownership.[9] Welfare has an inverted-U shaped relationship with the degree of common ownership: for low levels of common ownership entry is excessive while for high it is insufficient. Our model differs from theirs: (i) We consider quantity instead of price competition and discuss the main forces behind our results, which do not depend on the mode of competition. (ii) We consider general demand (in SM demand is inelastic) and cost functions. (iii) In our setting, total surplus depends on equilibrium objects not only through the number of firms. This allows us to delineate three channels through which pre-entry overlapping ownership affects entry (and the effect of the entry cost on entry), examine how returns to scale mediate the welfare effects of overlapping ownership and test the robustness of the results obtained in SM.[10]

Our work can be seen as an extension of the literature on free entry in homogeneous product markets. Mankiw and Whinston (1986) show that in a symmetric homogeneous product market with free entry and non-decreasing marginal cost (MC) where in the quantity-setting stage (i) the total quantity increases with the number of firms and (ii) the business-stealing effect is present, entry is never insufficient by more than one firm. Amir, Castro and Koutsougeras (2014) extend these results to the case of mildly decreasing MC, showing that under business-stealing competition, entry is never insufficient by more than one firm. We extend those results to the case of competition under overlapping ownership, showing that under business-enhancing competition, entry is always insufficient. However, we show that under business-stealing competition, overlapping ownership can lead to insufficient entry (by more than one firm) when returns to scale are decreasing.

2 The Cournot-Edgeworth λ-oligopoly model with free entry

There is a (large enough) finite set $\mathcal{F} \coloneqq \{1,2,\ldots,N\}$ of N symmetric firms that can potentially enter a market. The game has two stages, the entry stage and the quantity-setting stage. In the first stage, each firm chooses whether to enter by paying a fixed

effect (e.g., see Backus, Conlon and Sinkinson, 2021*a*; Koch, Panayides and Thomas, 2021; Lewellen and Lowry, 2021).

[8] Li, Ma and Zeng (2015) show that in a Cournot duopoly, the incumbent firm can strategically develop cross-ownership to deter the other firm from entering.

[9] In their model, the welfare effects of common ownership are directly implied by its effects on entry. Welfare only depends on the number of firms, the cost of transportation, and the entry cost.

[10] For example, SM find that entry always decreases with overlapping ownership, while in our case it can spur entry. In addition, in our model equilibrium total surplus can behave in multiple different ways as the extent of overlapping ownership changes—contrary to the inverted-U relationship found in SM.

cost $f > 0$.[11] In the quantity-setting stage, entrants compete à la Cournot. Namely, each firm i chooses its production quantity, $q_i \in \mathbb{R}_+$, simultaneously with the other firms. We denote by $s_i \coloneqq q_i/Q$ firm i's share of the total quantity $Q \coloneqq \sum_{i=1}^n q_i$. We also write $\boldsymbol{q}$ and $\boldsymbol{q}_{-i}$ to denote the production profile of all firms and all firms expect i, respectively; also, $Q_{-i} \coloneqq \sum_{j\neq i} q_j$.

2.1 The quantity-setting stage

Each firm i's production cost is given by the function $C : \mathbb{R}_+ \to \mathbb{R}_+$ with $C'(q) > 0$ for every q.[12] Denote by $E_C(q) \coloneqq C'(q)q/C(q)$ the elasticity of the cost function. We say that there is decreasing/constant/increasing marginal cost (MC) when C' is (globally) strictly decreasing/constant/strictly increasing, respectively. [13] When $C(q) = cq^\kappa/\kappa$ for some $c,\kappa > 0$, firms have constant-elasticity costs and $E_C(q) \equiv \kappa$. (i) For $\kappa = 1$, we have constant MC, (ii) for $\kappa \in (0,1)$, we have decreasing MC, (iii) for $\kappa > 1$, increasing MC. $AC(q) \coloneqq C(q)/q$ is the average variable cost.

The inverse demand function $P : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies $P'(Q) < 0$ for every $Q \in [0,\overline{Q})$, where $\overline{Q} \in (0, +\infty]$ is such that $P(Q) > 0$ if and only if $Q \in [0,\overline{Q})$. We assume that there exists $\overline{q} > 0$ such that $P(q) < AC(q)$ for every $q > \overline{q}$, and that P and C are twice differentiable.[14] For $Q < \overline{Q}$ we denote by $\eta(Q) \coloneqq -P(Q)/(QP'(Q))$ the elasticity of demand, and by $E_{P'}(Q) \coloneqq -P''(Q)Q/P'(Q)$ the elasticity of the slope of inverse demand. An inverse demand function with constant elasticity of slope (CESL), $E_{P'}(Q) \equiv E$, allows for log-concave and log-convex demand encompassing linear and constant elasticity specifications.[15] When we refer to linear demand, we mean $P(Q) = \max\{a - bQ, 0\}$. Every result applies to generic cost and inverse demand functions unless otherwise stated. We assume that the optimal (gross) monopoly profit is higher than the entry cost; that is, $\max_{Q\geq 0}\{P(Q)Q - C(Q)\} > f$.

Suppose n firms enter. A quantity profile $\boldsymbol{q}^*$ is an equilibrium of the quantity-setting stage if for each firm $i \in \{1,\ldots,n\}$, $q_i^* \in \arg\max_{q_i\geq 0}\{\pi_i(q_i,\boldsymbol{q}_{-i}^*) + \lambda\sum_{j\neq i}\pi_j(q_i,\boldsymbol{q}_{-i}^*)\}$, where $\pi_i(\boldsymbol{q}) \coloneqq P(Q)q_i - C(q_i)$ and $\lambda \in [0,1]$ is the (exogenous) Edgeworth coefficient of effective sympathy among firms.[16] This coefficient can for example arise from a symmetric

[11] We study pure-strategy equilibria. If firms decide whether to enter sequentially, this is without loss of generality. However, if they decide simultaneously, then there can also be equilibria where firms mix in their entry decisions (e.g., see Cabral, 2004).

[12] This production cost is in addition to the entry cost.

[13] Notice that under decreasing or constant MC, returns to scale (which take into account the fixed entry cost f) are increasing, while increasing MC tends to make returns to scale decreasing. Because of the fixed entry cost, returns to scale will not be globally decreasing. That is, the average total cost of production $(f + C(q))/q$, will not be globally increasing in the firm's output.

[14] P is required to be differentiable for $Q \in (0,\overline{Q})$. $P(Q)$ and its derivatives may be undefined for $Q = 0$ (e.g., with $\lim_{Q\downarrow 0} P(Q) = +\infty$ and $\lim_{Q\downarrow 0} P'(Q) = -\infty$).

[15] Appendix B.2 presents the functional form of CESL demand. E measures the curvature (*i.e.,* relative degree of concavity) of the inverse demand function. Demand is concave (resp. convex) for $E \leq 0$ (resp. $E \geq 0$). It is log-concave (resp. log-convex) for $E \leq 1$ (resp. $E \geq 1$).

[16] Appendix B.1 presents models that give rise to this objective function. λ may depend on N (*i.e.,*

overlapping ownership structure (be it common or cross-ownership) as in López and Vives (2019) or Azar and Vives (2021).

Given a quantity profile $\boldsymbol{q}$ where the number of firms that have entered is $n \equiv \dim(\mathbf{q})$, total surplus is given by $\mathrm{TS}(\boldsymbol{q}) \coloneqq \int_0^Q P(X)dX - \sum_{i=1}^n C(q_i) - nf$, while the Herfindahl–Hirschman index (HHI) and modified HHI (MHHI) are given by $\mathrm{HHI}(\boldsymbol{q}) \coloneqq \sum_{i=1}^n s_i^2$ and $\mathrm{MHHI}(\boldsymbol{q}) \equiv (1-\lambda)\,\mathrm{HHI}(\boldsymbol{q}) + \lambda$. We denote the MHHI at a symmetric equilibrium by $H_n \coloneqq (1+\lambda(n-1))/n$.

The setting of symmetric firms with a symmetric overlapping ownership structure that we consider preserves the symmetry of the Cournot game, which allows for extensions of existing oligopoly results (e.g., see Vives, 1999) to the case of competition under overlapping ownership.[17]

2.2 Equilibrium in the quantity-setting stage

2.2.1 Existence and uniqueness of a quantity-setting stage equilibrium

Having described the environment we first derive conditions for equilibrium existence and uniqueness in the quantity-setting stage using lattice-theoretic methods as in Amir and Lambson (AL; 2000). Let $\Delta(Q,Q_{-i}) \coloneqq 1 - \lambda - C''(Q - Q_{-i})/P'(Q)$ be defined on the lattice $L \coloneqq \left\{(Q,Q_{-i}) \in \mathbb{R}^2_{++} : \overline{Q} > Q \geq Q_{-i}\right\}$. $\Delta > 0$ allows for increasing, constant, and mildly decreasing MC, while $\Delta < 0$ allows for substantially decreasing MC.[18] In more detail, $\Delta > 0$ under non-decreasing MC. Under decreasing MC, $\Delta > 0$ (resp. $\Delta < 0$) when the price decreases fast (resp. slowly) with total output, MC decreases slowly (resp. fast) with output, and λ is low (resp. high).

Proposition 1. The following statements hold:

(i) Assume $\Delta(Q,Q_{-i}) > 0$ on L. Then, in the quantity-setting stage

  (a) there exists a symmetric equilibrium and no asymmetric equilibria,

  (b) if also $E_{P'}(Q) < (1 + \lambda + \Delta(Q,Q_{-i})/n)/H_n$ on L, then there exists a unique and symmetric equilibrium.

the number of potential entrants) depending on its microfoundation (see also López and Vives, 2019). In the case of common ownership studied in section B.10.1, λ is independent of N, while in the case of cross-ownership studied in section B.1.2, $\lambda = \alpha/[1-(N-2)\alpha]$, where α is the share that each firm holds of each other firm.

[17] We extend the results of Amir and Lambson (2000), who use lattice-theoretic methods to study equilibrium existence and comparative statics with respect to the (exogenous) number of firms in a symmetric Cournot market.

[18] Studies have indeed found evidence of declining marginal costs ($C'' < 0$), as required by $\Delta < 0$, across several industries (e.g., see Ramey, 1991; Betancourt and Malanoski, 1999; Diewert and Fox, 2008). $\Delta < 0$ can also arise in an endogenous sunk cost model where firms choose both output and cost-reducing R&D, in which case C is a reduced form (which obtains given that the optimal cost-reducing R&D is a function of only the firm's output).

(ii) Assume that $\Delta(Q,Q_{-i}) < 0$ and $E_{P'}(Q) < \frac{1+\lambda+\Delta(Q,Q_{-i})}{1-(1-\lambda)(1-s_i)}$ on L. Then, in the quantity-setting stage

(a) for every $m \in \{1,2\ldots,n\}$ there exists a unique quantity q_m such that any quantity profile where each of m firms produces quantity q_m and the remaining $n-m$ firms produce 0 is an equilibrium,

(b) no other equilibria exist.

Corollary 1.1 studies the existence and uniqueness of the quantity-setting stage equilibrium under linear demand and linear-quadratic cost. The linear-quadratic cost function is of the form $C(q) = c_1 q + c_2 q^2/2$, where $c_1 \geq 0$, for (i) $q \in [0,+\infty)$ if $c_2 \geq 0$, (ii) $q \in [0,-c_1/c_2]$ if $c_2 < 0$.[19]

Corollary 1.1. Let demand be linear, $P(Q) = \max\{a - bQ, 0\}$, and cost be linear-quadratic with $a > c_1 \geq 0$ and $c_2 > -2bc_1/a$. Then,

(i) if $c_2 > -b(1-\lambda)$, then $\Delta > 0$ on L, and a unique and symmetric equilibrium exists,

(ii) if $c_2 < -b(1-\lambda)$, then $\Delta < 0$ on L, and a unique (in the class of symmetric equilibria) symmetric equilibrium exists.

In light of Proposition 1 we maintain from now on the following assumption unless otherwise stated in a specific result. The assumption should be understood to hold at the relevant values of (n,λ) for each result.[20]

Maintained Assumption. The conditions in part (i-a,b) or part (ii) of Proposition 1 hold.

Remark 2.1. When in a result we assume $\Delta > 0$ (resp. $\Delta < 0$) it is thus understood that the additional assumption of part (i) (resp. part (ii)) of Proposition 1 also holds.

Remark 2.2. Necessary and sufficient conditions related to 1(i) and 1(ii) in Proposition 1 are: (1) $\Delta(Q,Q_{-i}) > 0$ on L for every $\lambda \in [0,1)$ if and only if $C''(q) \geq 0$ for every $q < \overline{Q}$ and (2) $E_{P'}(Q) < (1+\lambda)/H_n$ for every $n \in [2,+\infty)$ (resp. $n \in [1,2]$) and every $\lambda \in [0,1]$ if and only if $E_{P'}(Q) < 2$ (resp. $E_{P'}(Q) < 1$).

The maintained assumption guarantees that firms will play a symmetric equilibrium in the quantity-setting stage subgame of any subgame-perfect equilibrium.[21] Given that

[19] Cost is indeed increasing over $q \leq -c_1/c_2$ when $c_2 < 0$. The value of $C(q)$ for higher q will not matter in applications, as parameter values will be such that firms do not produce more than $-c_1/c_2$.

[20] For example, for global comparative statics of the Cournot game as λ changes, the assumption is assumed to hold for fixed n and every $\lambda \in [0,1]$. For the existence of a free-entry equilibrium for a fixed λ, it is sufficient that the assumption hold for every $n \in \mathbb{R}_{++}$ and that fixed λ.

[21] An analysis of the quantity-setting equilibrium stability is available upon request.

monopoly profit is positive, that equilibrium will be interior. When $\Delta < 0$, the quantity-setting subgame also has asymmetric equilibria; however, these cannot be played on the equilibrium path of an SPE of the complete game, since the entering firms that do not produce would prefer to avoid the entry cost by not entering.

We denote by $\boldsymbol{q}_n$ the symmetric Cournot equilibrium when n firms are in the market (which is unique under our maintained assumption) and with some abuse of notation by q_n the quantity each firm produces in that profile, where the subscript n now does *not* refer to the identity of the n-th firm; we also write $Q_n \coloneqq nq_n$, $\mathrm{TS}_n \coloneqq \mathrm{TS}(\boldsymbol{q}_n)$.[22] For any $n > 0$ we denote by $\Pi(n,\lambda) \coloneqq P(Q_n)\, q_n - C(q_n)$ the individual (gross) profit in the symmetric equilibrium of the Cournot game with n firms and Edgeworth coefficient λ, and refer to $\Pi(n,\lambda) - f$ as net profit. When we ignore the integer constraint on n, we allow all equilibrium objects, such as $\Pi(n,\lambda)$, to be defined for $n \in \mathbb{R}_{++}$. The Cournot equilibrium pricing formula is

$$\frac{P(Q_n) - C'(q_n)}{P(Q_n)} = \frac{H_n}{\eta(Q_n)}. \tag{1}$$

2.2.2 Comparative statics of the quantity-setting stage equilibrium

As a first step in examining the effects of overlapping ownership, Proposition 2 describes some comparative statics for the quantity-setting stage (*i.e.*, with a fixed number of firms). The total effect of overlapping ownership on consumer and total surplus will then be the sum of two effects: (a) the direct effect (*i.e.*, with the number of firms fixed) studied in part (i) of the proposition and (b) the indirect effect through its effect on entry, which is studied in part (iv) of the proposition and in section 4.

Proposition 2. The following statements hold:

(i) the total and individual quantity, and total surplus (resp. individual profit) are decreasing (resp. increasing) in λ,

(ii) the individual profit is decreasing in n,

(iii) if $E_{P'}(Q) < 2$ (resp. $E_{P'}(Q) > (1+\lambda)/\lambda$) for every $Q < \overline{Q}$, then the individual quantity is decreasing (resp. increasing) in n over $n \geq 2$,[23]

(iv) if $\Delta > 0$ (resp. $\Delta < 0$), then the total quantity is increasing (resp. decreasing) in n.

[22] To simplify notation, we suppress the dependence of these objects on λ.

[23] The condition $E_{P'}(Q) > (1+\lambda)/\lambda$ is very strong, especially given the assumption $E_{P'}(Q) < (1+\lambda+\Delta/n)/H_n$ on L. Also, it pushes against profit concavity in its own quantity, which can even make the monopolist's problem ill-behaved. For example, with CESL demand, when $E > 2$, $\lim_{Q\downarrow 0}(P(Q)Q - C(Q)) = +\infty$.

Part (iii) says that competition is business-stealing (*i.e.*, q_n is decreasing in n) under standard assumptions. Part (iv) says that, as in AL, for $\Delta > 0$ the Cournot market is quasi-competitive (*i.e.*, Q_n is increasing in n) while for $\Delta < 0$ it is quasi-anticompetitive (*i.e.*, Q_n is decreasing in n). This result can be interpreted as follows. Under non-decreasing MC (in which case $\Delta > 0$), an increase in the number of firms tends to increase total output both because competition increases and because production becomes (weakly) more efficient as the number of firms increases. On the other hand, under decreasing MC, an increase in the number of firms tends to make production less efficient. In that case, an increase in the number of firms tends to increase (resp. decrease) total output when the competition effect dominates (resp. is dominated by) the efficiency effect. The competition (resp. efficiency) effect dominates if $\Delta > 0$ (resp. $\Delta < 0$), which is the case when (i) the price decreases fast (resp. slowly) with total output—which makes externalities and competition among firms strong (resp. weak), (ii) MC is mildly (resp. substantially) decreasing—which makes the efficiency effect weak (resp. strong), and (iii) λ is low (resp. high)—which means that competition intensifies slowly (resp. fast) as the number of firms increases.

Part (i) shows the direct effect of an increase in overlapping ownership on consumer surplus to be negative. Combined with part (iv), this implies that if $\Delta > 0$, then an increase in overlapping ownership that suppresses entry will harm consumer welfare, as both the direct and the indirect effect push towards this direction. If $\Delta < 0$, then an increase in overlapping ownership that suppresses entry will indirectly boost consumer welfare. Then, the combined effect on consumer surplus will depend on whether the direct or the indirect effect dominates. The sign of the indirect effect is positive because when $\Delta < 0$, increasing returns to scale are strong enough to make the cost-savings (induced by a reduction in the number of firms) that are passed on to the consumer more than compensate for the harm due to increased market power (that also comes with industry consolidation).

Part (i) shows the direct effect of an increase in overlapping ownership on total surplus to also be negative. Since total surplus is generally not monotone in the level of entry, the sign of the indirect effect will depend on whether the equilibrium level of entry is excessive or insufficient (from a welfare standpoint). Provided that total surplus is single-peaked in n, if entry is insufficient (which section 4.2 shows to be the case under increasing MC and λ high), then an increase in overlapping ownership that suppresses entry will harm total welfare, as both the direct and the indirect effect push towards this direction. On the other hand, if it is excessive (which section 4.2 shows to be the case under decreasing MC or λ low), then the combined effect on total surplus of an entry-suppressing increase in overlapping ownership will depend on whether the direct or the indirect effect dominates.

3 The entry stage

To study the indirect effects, we first need to describe the entry stage. Assume that potential entrants have overlapping ownership with a coefficient of effective sympathy $\lambda \in [0,1]$. Given that $n-1$ firms enter, it is optimal for an n-th firm to enter if and only if $(1+\lambda(n-1))\left(\Pi(n,\lambda)-f\right) \geq \lambda(n-1)\left(\Pi(n-1,\lambda)-f\right)$. This can equivalently be written as

$$(2) \qquad \Psi(n,\lambda) \coloneqq \overbrace{\Pi(n,\lambda)}^{\substack{\text{own profit} \\ \text{from entry}}} - \lambda \overbrace{(n-1)\left(\Pi(n-1,\lambda)-\Pi(n,\lambda)\right)}^{=:\Xi(n,\lambda)>0,\text{ entry externality on other firms}} \geq \overbrace{f}^{\substack{\text{cost of} \\ \text{entry}}},$$

where $\Xi(n,\lambda)$ denotes the externality that the entry of the n-th firm poses on the other firms (*i.e.*, the absolute value of the reduction in the aggregate profits of all other firms caused by the entry of the n-th firm). $\Psi(n,\lambda)$ is a firm's "internalized profit" from entry (own profit from entry minus the part of the entry externality that is internalized by the firm, *i.e.*, the entry externality multiplied by λ). We assume that when indifferent, firms enter. Then, $\boldsymbol{q}_n$ is a free-entry equilibrium if and only if[24]

$$(3) \qquad \Psi(n,\lambda) \geq f > \Psi(n+1,\lambda),$$

which for $\lambda = 0$ reduces to the standard free entry condition $\Pi(n,0) \geq f > \Pi(n+1,0)$. For $\lambda = 1$, it reduces to $n\Pi(n,1) - (n-1)\Pi(n-1,1) \geq f > (n+1)\Pi(n,1) - n\Pi(n,1)$; firms enter as long as entry increases aggregate gross profits by enough to cover entry costs. We assume that $\Psi(N,\lambda) < f$ for every λ.

In deciding whether to enter a firm compares the profit it will make to the cost of entry and the negative externality its entry will pose to the other firms.[25] Overlapping ownership directly alters the incentives of firms to enter in a way additional to its effect on individual profit in the Cournot game.

The planner's problem. We will consider the problem of a total surplus-maximizing planner who takes λ as given and chooses the number of firms that will compete à la Cournot. Denote by $n^o(\lambda) \coloneqq \arg\max_{n\in\mathbb{N}} \mathrm{TS}_n$ the number of firms that given λ maximizes total surplus.[26] If the planner could control both n and λ, she would set $\lambda = 0$, since total

[24]When $\Delta < 0$, there are also asymmetric quantity-setting stage equilibria as described in Proposition 1, which are not played on an SPE path. However, these asymmetric quantity-setting stage equilibria can be played on out-of-equilibrium paths, which leads to SPE multiplicity. We focus on SPE where only symmetric quantity-setting stage action profiles are played on out-of-equilibrium paths. Provided that $\Psi(n,\lambda)$ is decreasing in n, this restricts attention to the SPE with the maximum number of entrants.

[25]If we compare this with the post-entry overlapping ownership case (see section 5.5), where—modulo the integer constraint—net profit is zero, we see that investors would prefer to become common owners before rather than after entry.

[26]Since monopoly net profit is positive, it follows that $n^o(\lambda) \geq 1$. Also, the planner can give subsidies in case the net profit in the symmetric Cournot equilibrium is negative.

surplus is decreasing in λ (this is also true under a consumer surplus standard). Define also $\widehat{n}^o(\lambda) \coloneqq \arg\max_{n\in\mathbb{R}_+} \mathrm{TS}_n$, the number of firms that given λ maximizes total surplus if we ignore the integer constraint on n.

Existence and uniqueness of equilibrium. Define $\Delta\Pi(n,\lambda) \coloneqq \Pi(n,\lambda) - \Pi(n-1,\lambda) < 0$, the decrease in individual profit caused by the entry of an extra firm. Proposition 3 identifies a condition under which a unique equilibrium exists. We treat n as a continuous variable and differentiate with respect to it.

Define $E_{\Delta\Pi,n}(n,\lambda) \coloneqq -\frac{\frac{\partial \Delta\Pi(n,\lambda)}{\partial n}}{\Delta\Pi(n,\lambda)}(n-1)$, a measure of the elasticity with respect to n of the slope of individual profit with respect to n, and $\varepsilon(n,\lambda) \coloneqq \partial\Pi(\nu,\lambda)/\partial\nu|_{\nu=n-1} / \Delta\Pi(n,\lambda) - 1$. $\varepsilon(n,\lambda)$ is close to 0, since by the mean value theorem $\Delta\Pi(n,\lambda) = \partial\Pi(\nu,\lambda)/\partial\nu|_{\nu=\nu^*}$ for some real number $\nu^* \in [n-1, n]$.

Proposition 3. Assume that for every $n \in [1, +\infty)$

$$E_{\Delta\Pi,n}(n,\lambda) < \frac{(n-1)(1+\lambda+\varepsilon(n,\lambda))}{1+\lambda(n-1)}.$$

Then, $\Psi(n,\lambda)$ is decreasing in n, so a unique equilibrium with free entry exists.

Remark 3.1. It can be checked that for $\lambda < 1$, $\Psi(n,\lambda)$ is indeed decreasing in n under linear demand and linear-quadratic cost with $a > c_1 \geq 0$ and $c_2 \geq 0$ (and also for $n \geq 5/2$ if $c_2 > -b(1-\lambda)$).[27]

The condition in Proposition 3 requires that equilibrium profit in the quantity-setting stage be not too convex in n; that is, the rate at which individual profit decreases with n should not decrease (in absolute value) too fast with n. Concerning internalized profit $\Psi(n,\lambda)$, see (2), an increase in n (i) decreases the first term $\Pi(n,\lambda)$, (ii) tends to increase the entry externality $\Xi(n,\lambda)$ through the increase in $(n-1)$ (as entry affects the profits of more firms), which tends to decrease $\Psi(n,\lambda)$, and (iii) affects $\Xi(n,\lambda)$ through its effect on the magnitude of the entry externality $\Pi(n-1,\lambda) - \Pi(n,\lambda)$ on a single firm. As long as the per-firm entry externality does not decrease with n too fast, $\Psi(n,\lambda)$ decreases with n.[28] We maintain the assumption that $\Psi(n,\lambda)$ is decreasing in n. Then, for a given λ, the number $\widehat{n}^*(\lambda)$ of firms that enter in equilibrium if we ignore the integer constraint on n is pinned down by $\Psi(\widehat{n}^*(\lambda),\lambda) = f$, and $n^*(\lambda) = \lfloor \widehat{n}^*(\lambda) \rfloor$ is the number of firms that enter if we respect the integer constraint.

[27]However, $\Psi(n,\lambda)$ is not always (globally) decreasing in n when $c_2 < 0$ and $\lambda > 0$ (Proposition 2 guarantees that $\Psi(n,\lambda)$ is decreasing in n when $c_2 < 0$ and $\lambda = 0$). See Appendix B.4 for a discussion.

[28]If $\Pi(n,\lambda)$ is concave in n, then the condition is satisfied given $\varepsilon(n,\lambda) \approx 0$. For $\lambda = 0$ the condition reduces to $\Pi(n,\lambda)$ being decreasing in n, which has been shown in Proposition 2. For $\lambda = 1$, provided that entry does not lead to significant savings in variable costs (*i.e.*, MC is not too increasing), only one firm enters.

4 The effects of overlapping ownership under free entry

This section studies the indirect effects of overlapping ownership (through its impact on entry). Particularly, it addresses the following concerns about the anti-competitive effects that overlapping ownership can have: suppress entry by inducing firms to internalize the effect that their entry would have on other firms' profits (subsection 4.1), induce or exacerbate socially sub-optimal levels of entry (subsection 4.2), and magnify the impact of entry costs on entry (subsection 4.4). Subsection 4.3 summarizes our main findings (except for those of subsection 4.4) using the case of linear demand and linear-quadratic cost.

4.1 Overlapping ownership effects on entry

The effect of changes in λ on entry will be determined by the sign of the (partial) derivative of $\Psi(n,\lambda)$ with respect to λ. If $\partial\Psi(n,\lambda)/\partial\lambda$ is positive (resp. negative), then increases in λ should be met with increases (resp. decreases) in n for (3) to continue to hold. Proposition 4 studies the effects of overlapping ownership on entry. This exercise amounts to changing the level of overlapping ownership *before* firms make their entry decisions. Therefore, it can be thought of as a counterfactual or a comparison of otherwise similar markets that have different levels of overlapping ownership (before firms enter).

Proposition 4. Equilibrium entry (locally) changes with λ in direction given by

$$\operatorname{sgn}\left\{\frac{d\widehat{n}^*(\lambda)}{d\lambda}\right\} = \operatorname{sgn}\left\{\overbrace{\frac{1}{\lambda}\frac{E_{\Pi,\lambda}(\widehat{n}^*(\lambda),\lambda)}{\widehat{E}_{\Pi,n}(\widehat{n}^*(\lambda),\lambda)}}^{\substack{\text{increase in own}\\ \text{profit from entry}}} - \overbrace{E_{\Xi,\lambda}\left(\widehat{n}^*(\lambda),\lambda\right)}^{\substack{\text{change in magnitude}\\ \text{of entry externality}}} - \overbrace{1}^{\substack{\text{increase in}\\ \text{internalization of}\\ \text{entry externality}}}\right\},$$

where $\widehat{E}_{\Pi,n}(n,\lambda) \coloneqq -\left(\Pi\left(n,\lambda\right) - \Pi\left(n-1,\lambda\right)\right)(n-1)/\Pi(n,\lambda) > 0$ is a measure of the elasticity of individual profit with respect to n, $E_{\Pi,\lambda}(n,\lambda) \coloneqq \lambda\partial\Pi(n,\lambda)/\partial\lambda/\Pi(n,\lambda) > 0$ is the elasticity of individual profit with respect to λ, and $E_{\Xi,\lambda}(n,\lambda) \coloneqq \lambda\partial\Xi(n,\lambda)/\partial\lambda/\Xi(n,\lambda)$ is the elasticity of the entry externality with respect to λ.[29]

An increase in overlapping ownership affects entry through three separate channels. First, it increases the degree of internalization of the negative externality of entry on other firms' profits; this increased internalization tends to limit entry. Second, it tends to increase equilibrium profits in the Cournot game, increasing entry.[30] Last, there is a

[29] $E_{\Xi,\lambda}$ is also a measure of the elasticity with respect to λ of the slope of individual profit with respect to n. With the integer constraint $n^*(\lambda)$ does not change with an infinitesimal change $d\lambda$ in λ unless we are in the knife-edge case where $\Psi(n^*(\lambda),\lambda) = f$. Thus, as λ increases, everything will behave as in the case with a fixed number of firms until λ reaches knife-edge cases causing a jump in $n^*(\lambda)$ as implied by Proposition 4.

[30] Stenbacka and Van Moer (2022) displays two similar forces with two firms choosing how much to invest in product innovation where they can only produce if they innovate.

channel with an ambiguous effect on entry: overlapping ownership changes the magnitude of the entry externality; that is, it affects how strongly equilibrium profits in the quantity-setting stage decrease with the number of firms. A high (and positive) elasticity $E_{\Xi,\lambda}$ of the entry externality Ξ with respect to λ tends to make entry decreasing in λ, while a negative $E_{\Xi,\lambda}$ tends to make entry increasing in λ. The magnitude of the entry externality $\Xi(n^*(\lambda),\lambda)$ can increase or decrease with λ.[31]

Remark 4.1. Evaluating the expressions in Propositions 3 and 4 requires evaluation of profits and derivatives thereof in different equilibria of the quantity-setting stage (with n and $n-1$ firms). This is possible under parametric assumptions while the problem remains intractable in general.[32] In what follows, we present numerical results.

Numerical Result 1. Under CESL demand, constant MC, $\lambda < 1$ and $\widehat{n}^*(\lambda) \geq 2$, it holds that

(i) entry is decreasing in λ if (a) $E \in (1,2)$ and $\lambda \geq 1/2$, or (b) $E < 1$ and $\lambda \geq 2/5$, or (c) $E \in (1,2)$, $\lambda \leq 3/10$, and $\widehat{n}^*(\lambda) \leq 3$, or (d) $E < 1$ and $\widehat{n}^*(\lambda) \leq 3$,

(ii) entry is increasing in λ if $\widehat{n}^*(\lambda) \geq 7$ and (a) $E \in (1,2)$ and $\lambda \leq 1/4$, or (b) $E \in [0,1)$ and $\lambda \leq 1/5$,

(iii) the total quantity is decreasing in λ.

These results can be loosely interpreted as follows. For λ low and entry high, competition is intense, so there is ample room for an increase in λ to soften it and increase individual profit in the Cournot game. For λ high and/or entry low, competition is already soft enough, so the increase in the internalization of the entry externality (due to an increase in λ) dominates and entry decreases with λ.[33]

The case of already high λ seems most relevant. In the U.S., common ownership levels among publicly listed firms have been "high enough" during at least the last decade (see Figure 1). The average λ (across pairs of firms in the same sector) among publicly traded firms has recently surpassed 0.7. If private firms are treated as a competitive fringe that only affects the residual demand in the oligopolies of public firms, then this will be the average λ among the oligopolists. Further increases in common ownership among the latter are likely to reduce entry by public firms in product markets where other public

[31] Under the parametrization of Figure 3b, Ξ is decreasing in λ. However, Ξ is increasing in λ under the parametrization of Figure 3a.

[32] Appendix B.11 presents a more tractable, differential version of the model.

[33] Let $\Pi(n,\lambda)$, $n \geq 2$, be (strictly) concave in λ (which is the case, for example, with linear demand and linear-quadratic cost under the conditions $a > c_1 \geq 0$ and $c_2 > -2bc_1/a$ of Corollary 1.1). Then, when λ is already high, the individual profit in the Cournot game increases slowly with λ, and the internalization of the entry externality (due to an increase in λ) dominates and entry decreases with λ.

firms are already present (see section 5.4).[34]

Under $\Delta > 0$, when entry is decreasing in λ, the price is increasing in λ, since both the increase in λ and the resulting decrease in entry tend to increase the price. On the other hand, for low levels of overlapping ownership and high levels of entry, overlapping ownership spurs entry (up to the point where λ is too high and then entry decreases with it). However, even in that case, Numerical Result 1 asserts that with constant MC, the direct effect of λ on the total quantity dominates, so that the price (resp. consumer surplus) always increases (resp. decreases) with λ.

4.2 Equilibrium versus socially optimal levels of entry

In section 4.1 we saw that an increase in the level of overlapping ownership decreases entry unless the extent of overlapping ownership is low to start with and many firms are active. In the latter case, an increase in the level of overlapping ownership will increase entry. Yet, neither of these two effects (especially the decrease in entry) will a priori necessarily reduce welfare. In this section, we study whether equilibrium entry is excessive or insufficient assuming that TS_n is single-peaked.[35] We show that under increasing MC and high overlapping ownership, entry is insufficient. Therefore, the suppression of entry induced by a further increase in the extent of overlapping ownership will be detrimental to welfare in that case. On the other hand, under substantially decreasing MC, entry is excessive, so any entry suppression caused by overlapping ownership is beneficial.

We have $d\,\mathrm{TS}_n\,/dn|_{n=\widehat{n}^*(\lambda)} = \Pi(\widehat{n}^*(\lambda),\lambda) - f + n(P(Q_n) - C'(q_n))\partial q_n/\partial n|_{n=\widehat{n}^*(\lambda)}$, which gives

$$(4) \qquad \left.\frac{d\,\mathrm{TS}_n}{dn}\right|_{n=\widehat{n}^*(\lambda)} \propto \underbrace{\lambda\frac{\Xi\left(\widehat{n}^*(\lambda),\lambda\right)}{\Pi\left(\widehat{n}^*(\lambda),\lambda\right)}}_{\substack{\in[0,1)\\ \text{internalized}\\ \text{normalized}\\ \text{entry externality}}} + \underbrace{\left(1 - \frac{E_C\left(q_{\widehat{n}^*(\lambda)}\right) - 1}{\dfrac{P\left(Q_{\widehat{n}^*(\lambda)}\right)}{AC\left(q_{\widehat{n}^*(\lambda)}\right)} - 1}\right)}_{\substack{>0\\ \text{If increasing MC, then} < 1.\\ \text{If constant MC, then} = 1.\\ \text{If decreasing MC, then} > 1.}} \underbrace{\left.\frac{\partial q_n}{\partial n}\frac{n}{q_n}\right|_{n=\widehat{n}^*(\lambda)}}_{\substack{<0 \text{ if } E_{P'}\left(Q_{\widehat{n}^*(\lambda)}\right)<2\\ \text{and } \widehat{n}^*(\lambda)\geq 2.\\ \text{If negative, the higher}\\ \text{in absolute value,}\\ \text{the stronger the}\\ \text{business-stealing effect.}}},$$

[34]Using data on advanced and emerging economies, Díez, Fan and Villegas-Sánchez (2021) find that, indeed, in the period 2000-2015, markups of listed firms have been higher and increased faster than markups of private firms, suggesting that private firms have lower market power than listed ones. If, instead, private firms are also treated as oligopolists but have weaker overlapping ownership links, then the average λ will be lower. For example, assuming that private firms have no overlapping ownership links, Azar and Vives (2021) calculate the average λ in the US economy to have increased from 0.07 in 1985 to 0.124 in 2017. In particular, between U.S. airlines, Azar and Vives (2022) calculate λ to be on average equal to 0.34.

[35]Lemma 1 in Appendix B provides sufficient conditions for it to be concave. For example, it is concave in n under linear demand and linear-quadratic costs with $c_2 \geq 0$.

where we have used the $\Psi(\widehat{n}^*(\lambda),\lambda) = f$ entry condition. The first term is equal to 1 minus the share of gross profit spent on the entry cost (*i.e.*, $1 - f/\Pi(\widehat{n}^*(\lambda),\lambda)$). Let us have a closer look at the two terms in the above expression. $\Xi(n,\lambda)/\Pi(n,\lambda) \equiv (n-1)(\Pi(n-1,\lambda) - \Pi(n,\lambda))/\Pi(n,\lambda)$ is the normalized entry externality.[36]

$$1 - \frac{E_C(q) - 1}{\frac{P(nq)}{AC(q)} - 1} = \frac{P(nq) - C'(q)}{P(nq) - AC(q)} > 0$$

is a (coarse) measure of the elasticity of the cost function, and thus of the extent to which marginal cost is decreasing or increasing. For example, $(E_C(q) - 1)/(P(nq)/(AC(q)) - 1)$ is higher than (resp. lower than/equal to) 0 if and only if the marginal cost is increasing (resp. decreasing/constant).

Whether entry is excessive or insufficient will depend on (i) the level of overlapping ownership λ, (ii) the magnitude of the normalized entry externality, (iii) whether marginal cost is decreasing or increasing and to what extent, and (iv) whether competition is business-stealing or business-enhancing, and to what degree $\left|\frac{\partial q_n}{\partial n}\frac{n}{q_n}\right|$. Factors (i) and (ii) can also be measured by the share of gross (*i.e.*, net of variable costs) profit that is spent on entry costs.

Under business-stealing competition and all else fixed, we distinguish the following forces. First, increases in the level of overlapping ownership or the magnitude of the entry externality, tend to make entry insufficient; these forces are complements in inducing insufficient entry. Equivalently, entry tends to be excessive when (due to a low level of overlapping ownership) a large share of revenue goes to the fixed entry cost. Second, increasing MC—which pushes towards DRS—tends to make entry insufficient, since the planner takes advantage of cost savings due to entry to a greater extent than firms.[37] Conversely, decreasing MC—which pushes towards IRS—tends to make entry excessive. Third, increases in the magnitude of the business-stealing effect make entry excessive. Under business-enhancing competition, entry is always insufficient.

The above analysis is valid *all else fixed.* It is informative when we want to compare two otherwise similar markets (e.g., with similarly strong business-stealing effects) that differ in their type and magnitude of returns to scale. We know then that in the market with increasing MC and high overlapping ownership, entry tends to be insufficient, so a suppression of entry caused by an expansion of overlapping ownership will likely harm welfare. However, in the market with decreasing MC or low overlapping ownership (and thus excessive entry), the indirect effect (*i.e.*, through entry) of an entry-suppressing increase in overlapping ownership on welfare will be positive.

[36] It is equal to $(n-1)$ times the percentage increase in the profit of each of the $n-1$ other firms when the n-th firm decides *not* to enter compared to the case where it did enter.

[37] Firms neither fully internalize the variable cost-savings of entry (except with complete indexation) nor internalize the increase in consumer surplus due to higher entry.

If we want to gauge the effects of changes in the extent of overlapping ownership *within* a certain market, then we need to take into account how these changes affect all equilibrium objects. To this end, we now study how overlapping ownership affects the relationship between the equilibrium and socially optimal levels of entry without holding all else fixed. First, define

$$\phi(n,\lambda) \coloneqq \frac{(n-1)\left(\Pi(n,\lambda) - \Pi(n-1,\lambda)\right)}{n\partial\Pi(n,\lambda)/\partial n} \approx 1,$$

which is positive and close to 1.[38]

Proposition 5. Assume that TS_n is single-peaked in n. Then $\widehat{n}^*(\lambda) \overset{(\text{resp. } <)}{>} \widehat{n}^o(\lambda)$ if and only if, evaluated at $n = \widehat{n}^*(\lambda)$,

$$\lambda\Delta(Q_n,(n-1)q_n) \overset{(\text{resp. } >)}{<} \frac{1-\lambda\phi(n,\lambda)}{\phi(n,\lambda)}\left[1+\lambda(n-1)\right]\left(1+\lambda - H_n E_{P'}(Q_n)\right).$$

Substituting $\lambda = 0$ we recover the standard excessive entry result: entry is excessive if and only if $E_{P'}\left(Q_{\widehat{n}^*(\lambda)}\right) < \widehat{n}^*(\lambda)$, which is indeed satisfied under standard assumptions on demand.[39] Proposition 5 also asserts that (as already discussed), all else (*i.e.*, $\lambda, E_{P'}, P', \widehat{n}^*(\lambda), \phi$) fixed, entry is excessive (resp. insufficient) for C'', and thus also Δ, low (resp. high).[40]

4.2.1 Markets with significantly decreasing MC ($\Delta < 0$)

Taking into account the integer constraint on the number of firms, the following remark shows formally that under substantially decreasing MC ($\Delta < 0$, and thus substantial IRS), entry is weakly excessive. Yet, the socially optimal level of entry, one firm, arises in equilibrium when the industry is fully indexed (*i.e.*, $\lambda = 1$).

Remark 4.2. If $\Delta < 0$, then (i) entry is weakly excessive, $n^*(\lambda) \geq n^o(\lambda)$. Particularly, (ii) $n^o(\lambda) = 1$, as $n = 1$ maximizes both Q_n and $n\Pi(n,\lambda)$, and (iii) for $\lambda = 1$, $n^*(1) = n^o(1) = 1$.[41]

[38] It is positive, because the numerator and denominator are negative. See Appendix B for an argument of why it is close to 1.

[39] Given $\widehat{n}^*(\lambda) \geq 2$ and $\Delta > 0$, Proposition 2 asserts that the total quantity in the quantity-setting stage is increasing in n and competition is business-stealing, which are the conditions under which Mankiw and Whinston (1986) show that entry is excessive. However, we see that $\Delta > 0$ is not necessary, consistent with Amir, Castro and Koutsougeras (2014), who show that entry is excessive under business-stealing competition and $\Delta > 0$ or $\Delta < 0$.

[40] Put differently, all else fixed, an increase in the elasticity E_C of the cost function (which means that returns to scale become more decreasing) tends to make entry insufficient.

[41] Notice also that if $1 - C''(Q - Q_{-i})/P'(Q) < 0$ on L, then $\Delta < 0$ on L for every $\lambda \in [0,1]$. In that case, then, $n^o(\lambda) = 1$ for every $\lambda \in [0,1]$, so a planner that controls either overlapping ownership or entry (but not both) will still achieve what a planner that can control both would (*i.e.*, a monopoly). Remember, however, that under $\Delta < 0$, $n^*(\lambda)$ is not the unique equilibrium level of entry, unless $n^*(\lambda) = 1$ (see footnote 24). Particularly, the monopoly solution is always a free-entry equilibrium. Therefore, what $\lambda = 1$ achieves is to break all socially sub-optimal equilibria with higher levels of entry, making a monopoly the unique free-entry equilibrium.

4.2.2 Markets with increasing, constant, or mildly decreasing MC ($\Delta > 0$)

While a result as general and clean as Remark 4.2 cannot be derived for the case of $\Delta > 0$ (partly because cases of both excessive and insufficient entry by more than one firm are possible under $\Delta > 0$), we have already seen that, all else fixed, a high λ and increasing MC tend to make entry insufficient. Numerical Result 2 in section 4.3 indeed shows that this holds without the "all else fixed" qualifier under linear demand and linear-quadratic cost. Namely, under non-decreasing MC, entry is excessive (resp. insufficient) for λ low (resp. high).

Under constant MC, if we let for simplicity $\phi = 1$, then Proposition 5 says that entry is excessive if $\lambda < [1 + \lambda(\widehat{n}^*(\lambda) - 1)] \left(1 + \lambda - H_{\widehat{n}^*(\lambda)} E_{P'} \left(Q_{\widehat{n}^*(\lambda)}\right)\right)$, which always holds if $E_{P'} < 1$ (and thus, also holds under linear demand). Therefore, under constant MC, entry is excessive for every λ (unless $E_{P'}$ is high). Then, an increase in λ that decreases entry tends to indirectly increase total surplus. This indirect effect is particularly strong when the entry cost is high, since in that case, a decrease in entry leads to substantial savings in entry costs. Thus, a high entry cost (which pushes towards IRS) tends to make increases in overlapping ownership more socially desirable.

4.2.3 Results under a consumer surplus standard

Remark 4.3 shows that if instead of a total surplus, the planner follows a consumer surplus standard, then entry is insufficient (resp. excessive) under increasing, constant, or mildly decreasing MC (resp. significantly decreasing MC).

Remark 4.3. Under a consumer surplus standard,

(i) if $\Delta > 0$, then $n^o(\lambda) = \infty$ (since Q_n is increasing in n), so $n^*(\lambda) < n^o(\lambda)$,

(ii) if $\Delta < 0$, then $n^o(\lambda) = 1$ (since Q_n is decreasing in n), so $n^*(\lambda) \geq n^o(\lambda)$.

4.3 Overlapping ownership effects: the linear-quadratic model

In this section, we summarize our main findings so far using the case of linear demand, $P(Q) = \max\{a - bQ, 0\}$, and linear-quadratic cost, $C(q) = c_1 q + c_2 q^2/2$.[42]

First, Figure 2 verifies the results of section 4.2 on the comparison between the equilibrium and socially optimal levels of entry. For increasing MC (*i.e.*, c_2 high), entry is insufficient under high levels of overlapping ownership. Particularly, Numerical Result 2 shows that under non-decreasing MC (and modulo the integer constraint), there exists a threshold $\overline{\lambda}$ such that entry is excessive (resp. insufficient) for λ lower (resp. higher)

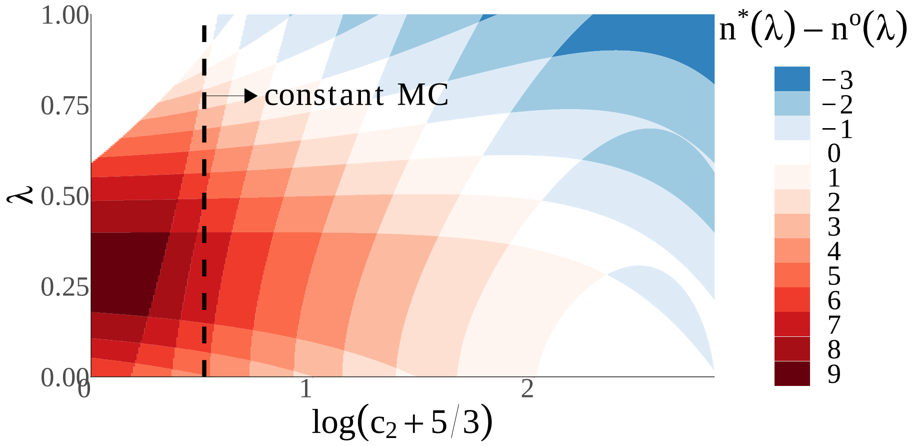
[42] Claim 2 in Appendix B.8 provides additional results in line with our findings.

than $\overline{\lambda}$.[43] Of course, when MC is not substantially increasing (*i.e.*, c_2 low), entry may be excessive for every λ and the result holds for $\overline{\lambda} > 1$.

Numerical Result 2. Let demand be linear and cost be linear-quadratic with $c_2 \geq 0$ (so that $\Delta \geq 0$ for every λ) and assume that $\widehat{n}^*(0) \geq 2$. Then, there exists $\overline{\lambda}$ (which depends on parameters) such that $\widehat{n}^*(\lambda) \overset{(\text{resp. } <)}{>} \widehat{n}^o(\lambda)$ if and only if $\lambda \overset{(\text{resp. } >)}{<} \overline{\lambda}$.

Entry is excessive under decreasing MC and/or low λ. Particularly, notice in Figure 2 the area with substantially decreasing MC (*i.e.*, $\Delta < 0$) and high λ in the upper-left corner, where a socially optimal monopoly arises in equilibrium (*i.e.*, $n^*(\lambda) = n^o(\lambda) = 1$), as shown in Remark 4.2.

Figure 2: Difference between equilibrium and socially optimal level of entry $n^*(\lambda) - n^o(\lambda)$ (with linear demand and linear-quadratic costs) as a function of the level of λ and the level of c_2 (indicating decreasing, constant, or increasing MC)



Note: $a = 3$, $b = c_1 = 1$, $c_2 \in [-2/3, 15]$, $f = 0.05$. For better readability, an increasing transformation is applied on the x-axis (*i.e.*, c_2). On the dashed line, there is constant MC (*i.e.*, $c_2 = 0$). On the left (resp. right) of the line, marginal cost is decreasing (resp. increasing).

Second, in section 2 we saw that the direct effect (*i.e.*, if we hold the number of firms fixed) of overlapping ownership on welfare is negative. To gauge its total effect on welfare, we next look at how overlapping ownership affects entry. Observe in Figure 3a that for λ low and entry high, the rise in own profit due to increases in λ dominates the other two channels, and thus, entry increases with λ. However, for high λ or low levels of entry, competition in the product market is already soft enough, and thus further increases in λ suppress entry.

Last, we look at the (total) welfare effects of overlapping ownership. With increasing MC (Figure 3c and Figure 6 in Appendix A.1), overlapping ownership tends to harm consumer and total surplus. With substantially decreasing MC (Figure 3d and Figure 6 in Appendix A.1), a high level of overlapping ownership is optimal both for consumer and

[43] We ignore the integer constraint on n. This is not important, since cases of both excessive and insufficient entry by more than one firm are possible under $\Delta > 0$ (see Figure 2).

total surplus. Particularly, a high value of λ (e.g., $\lambda = 1$) induces entry by only one firm, which is socially optimal. The planner also takes into account the impact of overlapping ownership on total entry costs through its effect on entry. For low entry cost (and thus weak IRS; Figure 3a and Figure 6 in Appendix A.1), a λ-controlling planner would choose no overlapping ownership, while for higher entry cost (which makes IRS stronger), higher values of λ are optimal (Figure 3b and Figure 6 in Appendix A.1). Overall, DRS (*i.e.*, low entry cost f and increasing MC, c_2 high) tend to make overlapping ownership welfare-damaging. Particularly, $\lambda = 0$ or very low tends to maximize total surplus. IRS (*i.e.*, high entry cost f and/or decreasing MC, c_2 low) tend to make high levels of overlapping ownership welfare-optimal. That is because (under IRS) high overlapping ownership leads to entry by only one firm, which implies substantial efficiency gains (compared to the case of low overlapping ownership, where more firms would enter).[44]

4.4 Entry cost effect on entry

We have seen so far that overlapping ownership is beneficial under substantial IRS. On the other hand, an increase in the extent of overlapping ownership tends to harm welfare under DRS—especially when overlapping ownership is high to start with. At the same time, the rising entry costs documented by Gutiérrez, Jones and Philippon (2021) tend to reduce entry. But how do the two forces—increasing overlapping ownership and entry costs—interact? Is their combined impact on competition worse than the sum of the two effects, or does overlapping ownership mitigate the suppression of entry caused by rising entry costs?

Proposition 6 studies the effect of the entry cost on entry, as well as how this effect depends on the extent of overlapping ownership (with the level of entry held fixed). Note that λ affects the slope $d\widehat{n}^*(\lambda)/df$ directly but also through its effect on $n^*(\lambda)$. We are interested in the direct effect so we keep $n^*(\lambda)$ fixed as we vary λ.

Proposition 6. Ignore the integer constraint on n (so that entry is given by $\widehat{n}^*(\lambda)$). Then

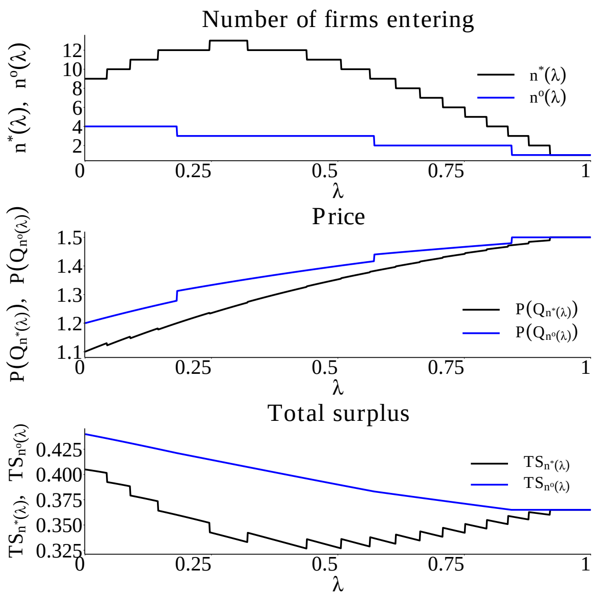
(i) entry is decreasing in the entry cost,

(ii) if λ increases and other parameters x (e.g., demand, cost) change infinitesimally with $\widehat{n}^*(\lambda)$ staying fixed and $\partial^2\Psi(n,\lambda)/(\partial x \partial n) = 0$ (e.g., (f,λ) changes in direction $\mathbf{v} := (-(d\widehat{n}^*(\lambda)/d\lambda)/(d\widehat{n}^*(\lambda)/df), 1))$, then $|d\widehat{n}^*(\lambda)/df|$ changes in direction given by $\operatorname{sgn}\{\partial^2\Psi(n,\lambda)/(\partial\lambda\partial n)|_{n=\widehat{n}^*(\lambda)}\}$.

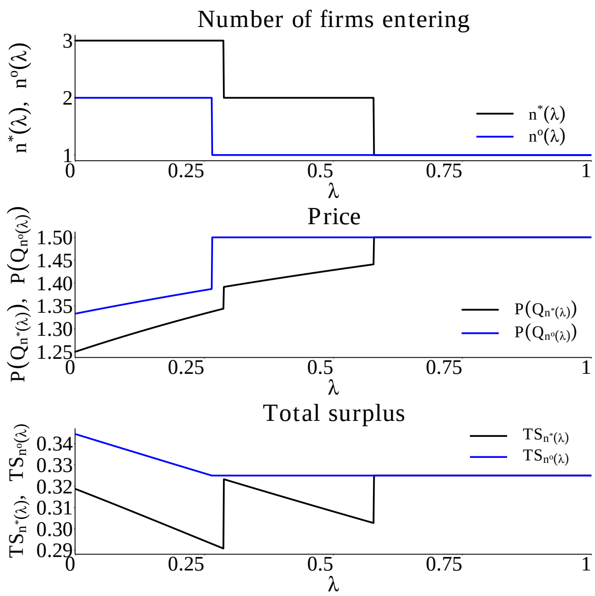
[44] When the market is so small (*i.e.*, production costs are large relative to demand) that it can accommodate entry by only one firm absent overlapping ownership, then a high level of overlapping ownership is not necessary for the efficiency gains from IRS to materialize. However, overlapping ownership does not harm welfare either, since only one firm enters with or without overlapping ownership (and when only one firm enters, there is no room for overlapping ownership to soften product market competition).

Figure 3: Equilibrium and socially optimal outcomes for varying λ

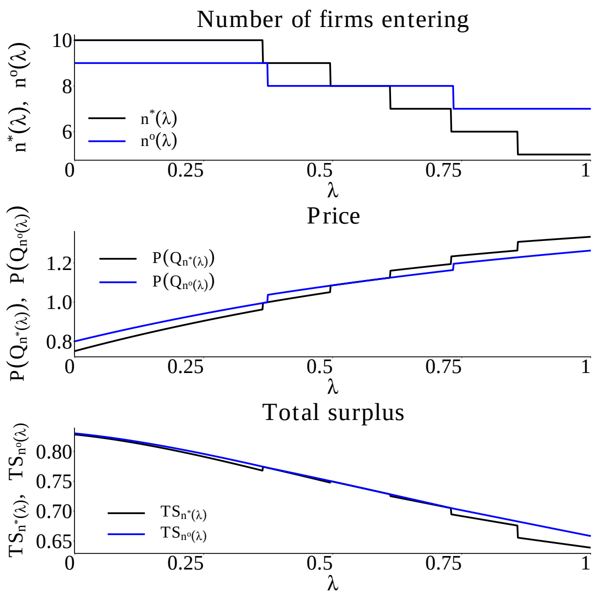
(a) linear demand, constant MC: $a = 2$, $b = c = 1$, $f = 0.01$



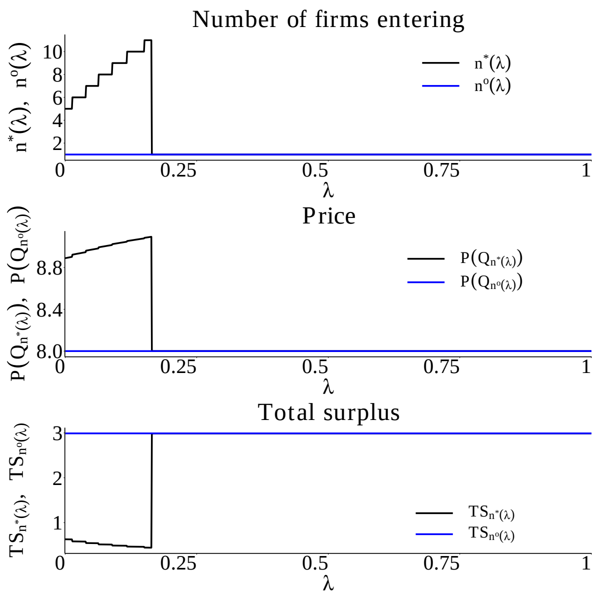
(b) linear demand, constant MC: $a = 2$, $b = c = 1$, $f = 0.05$



(c) linear demand, linear-quadratic costs (increasing MC): $a = 2$, $b = c_1 = 1$, $c_2 = 5$, $f = 0.05$



(d) linear demand, linear-quadratic costs (decreasing MC): $a = 10$, $b = 1$, $c_1 = 9$, $c_2 = -1.5$, $f = 0.01$

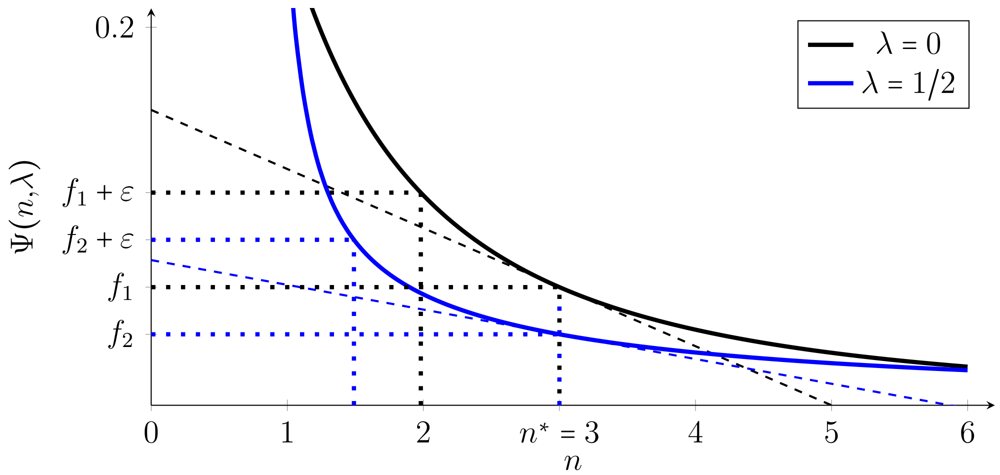


Note: Black lines represent values in equilibrium; blue represent values in the (entry-controlling) planner's solution.

As long as $\Psi(n,\lambda)$ is decreasing in n, the results of Proposition 6 are not specific to Cournot competition. Part (ii) says that if an increase in λ makes the internalized profit in the quantity-setting stage equilibrium more (resp. less) strongly decreasing in the number of firms, then an increase in the entry cost needs to be met with a smaller (resp. larger) increase (resp. decrease) in the number of firms for the condition $\Psi(\widehat{n}^*(\lambda),\lambda) = f$ to continue to hold.

Figure 4 explains the reasoning behind this. There are initially $n^* = 3$ firms, which can be a result of $\lambda = 0$ and $f = f_1$, or $\lambda = 1/2$ and $f = f_2$. Also, an increase in λ from 0 to 1/2 makes the internalized profit less strongly decreasing in n (*i.e.*, $\partial^2\Psi(n,\lambda)/(\partial\lambda\partial n) > 0$).[45] Thus, an increase in the entry cost by ε will decrease entry by more when $\lambda = 1/2$ (and initially $f = f_2$) compared to when $\lambda = 0$ (and initially $f = f_1$).

Figure 4: Entry cost effect on entry mediated by λ under linear demand and constant MC



Note: $a = 2$, $b = 1$, $c = 1$. The black and blue solid lines represent $\Psi(n,0)$ and $\Psi(n,1/2)$, respectively. The black and blue dashed lines are tangent to the corresponding solid lines at $n = n^*$.

Numerical result 3 provides conditions under which the cross derivative of $\Psi(n,\lambda)$ is positive, which by Proposition 6 implies that overlapping ownership exacerbates the negative effect of the entry cost on entry.

Numerical Result 3. Under CESL demand and constant MC, $\partial^2\Psi(n,\lambda)/(\partial\lambda\partial n) > 0$ if (i) $E \in (1,1.7]$ and $n \in [2,7]$, or (ii) $E < 1$ and $n \in [2,8]$.[46]

Under CESL demand and constant MC, markets with low entry are particularly susceptible to further decreases in entry when there is overlapping ownership. In such markets, apart from the direct effect it has on entry, overlapping ownership also makes entry more strongly decreasing in the entry cost. This means that overlapping ownership could exacerbate the negative macroeconomic implications of rising entry costs in moderately or severely concentrated markets.[47]

[45] For example, at $n = 3$, the slope of $\Psi(n,1/2)$ (see the tangent blue dashed line) is smaller in absolute value than the slope of $\Psi(n,0)$ (see the tangent black dashed line). $\Psi(n,\lambda)$ decreases less strongly with n when $\lambda = 1/2$ than when $\lambda = 0$.

[46] Empirical estimates for various markets place E in the range specified in Numerical Result 3. See for example Duso and Szücs (2017), Mrázová and Neary (2017), and Bergquist and Dinerstein (2020).

[47] Remember that by Proposition 2, when $\Delta > 0$, a decrease in the number of firms causes consumer surplus to fall. Also, in section 4.2 we saw that increasing MC and high overlapping ownership tend to make entry insufficient, in which case a decrease in the number of firms will also reduce total surplus.

5 Extensions and robustness

In this section, we present extensions and discuss the robustness of our results.

5.1 Asymmetric overlapping ownership or costs

Our model with symmetric overlapping ownership is rich enough to capture the main forces and allows us to study several issues.[48] Although the assumption of a unique Edgeworth coefficient of effective sympathy λ across all firm pairs is a simplification, an increase in λ in comparative statics exercises captures a particularly relevant phenomenon. It can for instance represent the expansion of an investment fund's holdings across all firms in an industry, as has recently been the trend that has spurred the antitrust interest in overlapping ownership. This is also the reason why proposed policies have emphasized the industry-wide holdings of each investor rather than only individual stock trades. For instance, Posner, Morton and Weyl (2017) and Posner (2021) propose that an investor holding shares of more than one firm in an oligopoly be not allowed to own more than 1% of the market shares unless they commit to being purely passive.

While a model with asymmetric overlapping ownership would allow for the analysis of additional issues (e.g., on the effects of a change in the ownership links between only a subset of firms), we expect our main insights on the effects of a *uniform* increase in overlapping ownership (*i.e.*, an increase in λ's across all firm pairs) to hold.[49] Particularly, similar forces will be at play. A uniform increase in λ's will tend to (i) directly increase the price and profits (*i.e.*, with the number of firms held fixed), (ii) indirectly affect welfare through its impact on entry, which will depend on the balance of the three forces identified in section 4.1.

Yet, in a model with asymmetric cost functions, there will be an additional force, which can make overlapping ownership more attractive from a welfare standpoint. Although with symmetric costs a uniform increase in λ affects production efficiency (through its effect on entry) based on the type of returns to scale, with asymmetric costs an increase in overlapping ownership can also induce cost efficiencies by causing production to shift towards low-cost firms.[50]

5.2 Endogenous overlapping ownership

Although standard in the overlapping ownership literature, the assumption that the ownership structure is exogenous may be questioned. However, even if λ is affected by parameters of the model (e.g., if common ownership tends to be higher in markets with

[48] The assumption of a symmetric overlapping ownership structure greatly facilitates the analysis.

[49] Our main insights also hold in an extension of the model (see section 5.4) where there is also a competitive fringe of maverick firms (without overlapping ownership).

[50] This effect is highlighted in Azar and Vives (2021).

larger demand), the analysis is still valid if there are factors that affect λ but are unrelated to the demand and production costs in the specific market that firms consider entering. For example, it is not unrealistic to think that an index fund can grow in size while a specific market's demand, as well as entry and production costs remain fixed. An increase in the level of overlapping ownership may be due to (i) an increase in stock market participation (by small investors, who invest mostly through diversified investment funds) or (ii) a shift in investor behavior from investing on individual stocks to investing through index funds. Stock market participation or such a shift in investor behavior is arguably affected mostly by factors other than the demand and production costs in a specific market. These factors can be increased financial literacy, the rise of fintech, and lower fund management fees.

5.3 Product differentiation and alternative modes of competition

We now discuss how our main insights still apply when the product market competition stage is not a homogeneous product Cournot market. In doing so, we emphasize the fundamental forces behind our results, which are present in most forms of product market competition.

Consumer surplus. If (i) consumers benefit from increased competition and enhanced product variety induced by higher entry, and (ii) overlapping ownership (iia) tends to harm consumer surplus by undermining product market competition (*i.e.*, ignoring its effects on entry), and (iib) suppresses entry (*i.e.*, the internalization of the entry externality channel dominates), overlapping ownership will still tend to harm consumer surplus.

Part (i) should hold in many markets but may fail if there are substantial IRS, so our finding that IRS tend to make overlapping ownership enhance welfare through entry generalizes. Part (iia) is also expected to hold with differentiated (substitute) goods. Whether part (iib) holds will depend on the balance of the same three channels identified in section 4.1 (which are not specific to homogeneous product Cournot competition). However, we expect the magnitude of the effects of overlapping ownership to be diminished, since with differentiated goods, there are smaller (pricing, production, and entry) externalities among firms to be internalized due to overlapping ownership.

Total surplus. The effects on total surplus will depend on (i) the direct effect (*i.e.*, ignoring effects on entry) of overlapping ownership on total surplus (see section 5.1 for a discussion on when the direct effect may be positive), (ii) its effect on entry, as discussed above, and (iii) whether entry is excessive or insufficient.

With regard to part (iii), the forces identified in equation (4) apply regardless of the mode of competition (e.g., they apply under both Cournot and Bertrand competition). However, with differentiated products, an additional force arises. Entry benefits consumers not only by leading to lower prices and higher output, but also by expanding product variety (see section 4 in Mankiw and Whinston, 1986). The benefit of entry to consumers due to their preference for variety is not internalized by the marginal entrant, which pushes towards insufficient entry.

5.4 Entry under the presence of maverick firms

We have examined the effects of overlapping ownership under a symmetric overlapping ownership structure. In that context, overlapping ownership can suppress entry by inducing firms to internalize the negative externality that their entry would have on other firms. However, if there are also potential entrants without ownership ties—which we call maverick firms, then limited entry by commonly-owned firms may spur entry by maverick ones.[51] This could enhance the incentives of a commonly-owned firm to enter.

In Appendix B.9 we model the maverick firms as a competitive fringe that in the first stage (where oligopolists enter) submit an aggregate supply schedule. We show that the (prospect of) entry by maverick firms essentially changes the demand faced by the commonly-owned firms by depressing it and making it more elastic. With demand adjusted accordingly, the results of the previous sections on the effects of overlapping ownership on entry and the price continue to hold (with the number of firms n not counting maverick firm entry), as does the comparison between the equilibrium and socially optimal levels of entry. Since demand is depressed, we expect lower levels of entry by commonly-owned firms. Also, given that higher (resp. lower) entry by commonly-owned firms leads to lower (resp. higher) entry by maverick firms, we expect entry to be less sensitive to overlapping ownership due to the presence of maverick firms.[52]

5.5 Post-entry overlapping ownership

Post-entry overlapping ownership applies to the case of a new industry that is to be mostly populated by start-ups without overlapping ownership that will develop ownership links after entry.[53] In this case, firms do not internalize the negative externality their entry has on other firms, as in the standard Cournot model with free entry. Thus, modulo the integer constraint on the number of firms, firms enter until the individual gross profit is equal to the entry cost, so in equilibrium the net profit is zero. Nevertheless, when

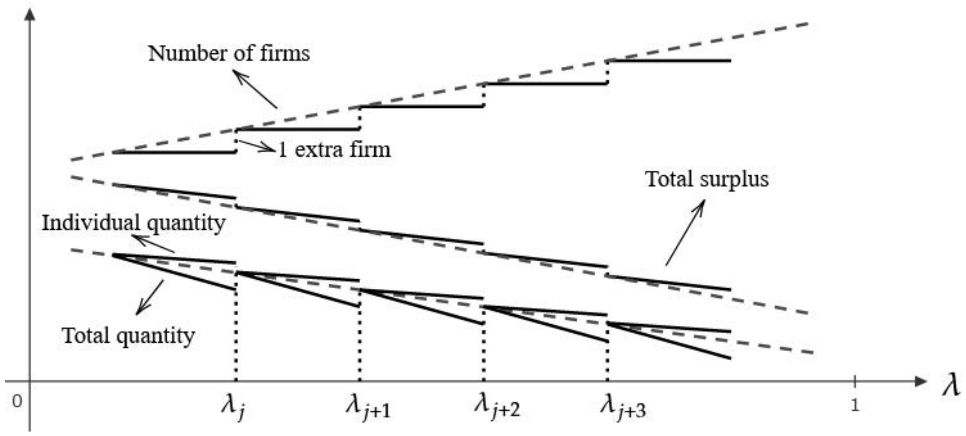
[51]Indeed, Eldar and Grennan (2021) argue that by softening competition, common ownership of public firms gives start-ups the opportunity to enter. However, they also provide evidence that—acknowledging this opportunity—VCs concentrate their activities on markets with extensive common ownership. This causes VC-induced common ownership to increase with public-firm common ownership.

[52]That is because all channels through which λ affects entry will diminish in magnitude when there are maverick firms. First, quantity-setting stage profit will not increase as strongly with λ, because maverick firms will produce more when oligopolists reduce production as λ increases (*i.e.*, when maverick firms are present, the demand that the oligopolists face is more elastic, so the externality that one oligopolist imposes on the others by producing is lower). Second, the entry externality is also smaller, since by entering an oligopolist limits maverick entry.

[53]The post-entry overlapping ownership case can be interpreted to address pre-entry overlapping ownership when firms do notinternalize their entry externality.

deciding whether to enter, they take into account how overlapping ownership will affect product market outcomes. Naturally, an increase in the degree of post-entry overlapping ownership spurs entry, since it tends to increase profits by softening pricing competition (see Proposition 2). Appendix B.10 (which studies the model with *post-entry* overlapping ownership) shows that the anti-competitive effect of overlapping ownership prevails causing the price to increase and consumer, as well as total surplus to fall. The results on the effects of post-entry overlapping ownership are schematically summarized in Figure 5.

Figure 5: Equilibrium with post-entry overlapping ownership for varying λ



Note: The solid lines represent equilibrium values under the integer constraint; from bottom to top they represent the behavior of the total quantity, individual quantity, total surplus, and number of firms. The dashed lines represent equilibrium values when we ignore the integer constraint; from bottom to top the first line represents the behavior of both the total and the individual quantity, the second of total surplus, and the third of the number of firms. The solid total quantity line is drawn for the case $\Delta > 0$. The solid individual quantity is drawn above the dashed one given that q_n is decreasing in n (see Proposition 2). To draw the solid total surplus line above the dashed one, we assume that the total surplus is single-peaked in n, and that $n^*(\lambda) \geq n^o(\lambda)$ (see Proposition 11 in Appendix B.10). Only the signs of the slopes of the lines and the directions of the jumps are part of the result; the lines are drawn linearly and the spacing of the jumps has been chosen for simplicity in depiction.

The comparison between the equilibrium and the optimal level of entry also changes: the result that entry tends to be excessive under business-stealing competition generalizes. Entry is never insufficient by more than one firm as in the standard Cournot model with free entry (see Mankiw and Whinston, 1986; Amir, Castro and Koutsougeras, 2014).

6 Conclusion

In the central case of a Cournot oligopoly market with free entry, potential entrants are established firms with overlapping ownership and decide whether to enter a new industry or product market.

Pre-entry overlapping ownership provides an additional channel for competitive effects on top of the collusive-like effect of raising prices and—thereby—possibly inducing entry. We find that in most relevant cases—and particularly when overlapping ownership is already widespread, an expansion of overlapping ownership harms welfare. That is because as the level of overlapping ownership increases, not only (i) does competition among active firms soften, but also (ii) fewer firms are active in the market with entry being lower than socially optimal. At the same time, (iii) entry becomes more sensitive to the entry cost (*i.e.*, it decreases faster with the entry cost). Overlapping ownership can mostly be beneficial only under strong increasing returns to scale (IRS), in which case industry monopolization (induced by overlapping ownership) generates considerable cost-savings.

These results suggest that regulators should be most concerned about overlapping ownership among firms operating (or considering operating) in industries with decreasing returns to scale (DRS) or weak IRS. Particularly, markets with DRS and already high and increasing overlapping ownership warrant the most regulatory scrutiny. On the other hand, overlapping ownership among firms operating in markets with substantial IRS seems less problematic—if not beneficial. For example, overlapping ownership among generic drug manufacturers or between a generic and a brand name should be more of a concern compared to overlapping ownership among pharmaceuticals that focus on drug discovery. That is because drug discovery—in contrast to generic drug manufacturing—requires sizable R&D costs (and is thus characterized by strong IRS), the duplication of which can be avoided when there is extensive overlapping ownership.[54] Overlapping ownership involving generic manufacturers is more likely to soften competition and suppress generic entry without generating important cost efficiencies, thereby harming welfare. Newham, Seldeslachts and Banal-Estañol (2022), Xie and Gerakos (2020), and Xie (2021) find that common ownership between a brand firm and potential generic entrants suppresses generic entry.

We derive the following testable implications for markets that existing firms with overlapping ownership consider entering. First, for low levels of overlapping ownership, an increase in overlapping ownership will (i) increase (resp. decrease) entry if there is low (resp. high) market concentration. Second, for high levels of overlapping ownership, further increases in it will suppress entry. Thus, entry will either depend negatively on overlapping ownership or have an inverted-U relationship with it. Third, unless there are IRS, an increase in the extent of overlapping ownership will increase prices. Fourth, increases in the entry cost can suppress entry more in industries with higher levels of overlapping ownership.

Finally, given that the extent to which ownership ties affect firm conduct is an open empirical question, our results suggest a test of the common ownership hypothesis. If the

[54]Particularly, apart from direct R&D costs, the cost of entry can also be understood to include the opportunity cost associated with foregoing R&D of drugs for other conditions. Then, overlapping ownership will induce each pharmaceutical to focus on drug discovery for a different condition rather than compete with other pharmaceuticals in the development of drugs for the same condition. Li, Liu and Taylor (2023) provide evidence pointing in this direction. Namely, they find that venture capital firms that fund multiple pharmaceutical startups hold back projects, withhold funding, and redirect innovation at lagging startups—partly to prevent R&D cost duplication.

common ownership hypothesis fails completely (*i.e.*, common ownership does not affect firm behavior and investors understand this), then entry (and other market outcomes) should be independent of common ownership. If the common ownership hypothesis is only *partially* correct in the sense that common ownership influences pricing behavior but does *not* cause the entry externality to be internalized, then entry is expected to increase with the level of common ownership. If, on the other hand, common ownership induces firms to internalize the entry externality without affecting their pricing behavior, then entry should decrease with overlapping ownership.[55] Finally, if the common ownership hypothesis is correct (*i.e.*, common ownership affects firm conduct in both ways), then entry is expected to either depend negatively on common ownership or have an inverted-U relationship with it.

Future research could study the effects of overlapping ownership through entry (into a new market) when possible entrants interact in multiple (possibly interdependent) markets. This opens the gate to potential collusive strategies.

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[55] This could be the case if entry decisions are made by top executives who internalize the (common) shareholders' interests, while pricing decisions are made by low-level management who are not incentivized to do so. For instance, Ruiz-Pérez (2019) finds that in the U.S. airline industry, common ownership matters for entry decisions but not for pricing behavior. On the other hand, Park and Seo (2019) find that common ownership makes U.S. airlines internalize the effects of their pricing decisions on the competitors' profits.

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Online Appendix

Free entry in a Cournot market with overlapping ownership

Xavier Vives and Orestis Vravosinos

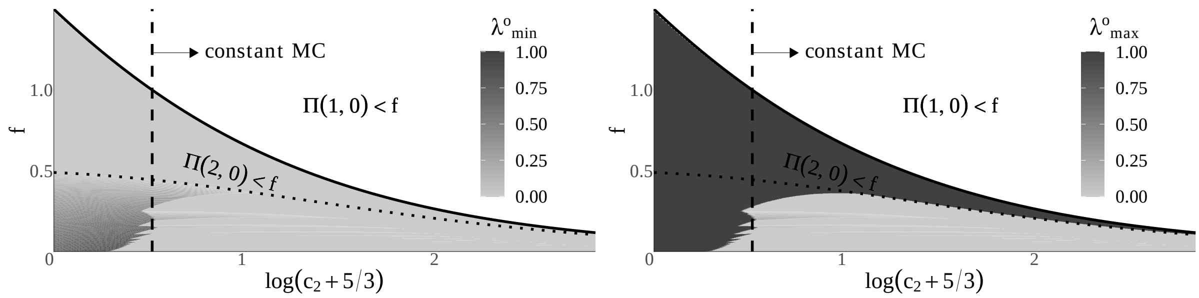
A Main appendix

A.1 Additional simulation results

Figure 6 plots the socially optimal level(s) $\lambda^o \coloneqq \arg\max_{\lambda\in[0,1]} \mathrm{TS}_{n^*(\lambda)}$ of overlapping ownership under linear demand and linear-quadratic costs as a function of the entry cost f and the level c_2 of decreasing, constant, or increasing MC.[56]

Figure 6: Socially optimal level(s) $\lambda^o \coloneqq \arg\max_{\lambda\in[0,1]} \mathrm{TS}_{n^*(\lambda)}$ of overlapping ownership under linear demand and linear-quadratic costs as a function of the entry cost f and the level c_2 of decreasing, constant, or increasing MC

(a) Minimum socially optimal λ, λ^o_{min} **(b)** Maximum socially optimal λ, λ^o_{max}



Note: $a = 3$, $b = c_1 = 1$, $c_2 \in [-2/3, 15]$. For better readability, an increasing transformation is applied on the x-axis (*i.e.*, c_2). On the dashed line, there is constant MC (*i.e.*, $c_2 = 0$). On the left (resp. right) of the line, marginal cost is decreasing (resp. increasing). In the white region above the solid line, the net monopoly profit is negative (*i.e.*, $\Pi(1,0) < f$), and thus no firm enters for every λ. Above the dotted line, the net duopoly profit is negative (*i.e.*, $\Pi(2,0) < f$). Thus, in the region between the solid and the dotted line, only one firm enters absent overlapping ownership (*i.e.*, for $\lambda = 0$). In the region on the left of (and including) the dashed line (and below the solid line), one firm enters if $\lambda = 1$ (to see why, look at the derivation of Remark 4.2 in the proof of Proposition 5). In some cases, λ^o is not a singleton. In these cases, every optimal $\lambda \in \lambda^o$ induces entry by only one firm, in which case the exact value of λ does not matter. Panel (a) plots the minimum among all optimal levels of overlapping ownership, while panel (b) plots the maximum one. For example, in the region between the dotted and solid lines, both $0 \in \lambda^o$ and $1 \in \lambda^o$ lead to entry by a single firm, maximizing total surplus.

[56] It can be checked that $\mathrm{sgn}\{\widehat{n}^*(\lambda) - \widehat{n}^o(\lambda)\}$, $\Pi(n,\lambda)$, TS_n, and $\widehat{n}^*(\lambda)$ depend on a, c_1, and f only through $(a - c_1)^2/f$ (see the discussion in the Derivation of Numerical result 2 in Appendix B). Thus, $a = 3$, $c_1 = 1$ in Figure 2 and Figure 6 in the appendix is, modulo the integer constraint on n, without loss of generality in the following sense. If plotted without the integer constraint, then for any a,c_1,f such that $(a - c_1)^2/f = 80$, Figure 2 will give the same result about when entry is excessive or insufficient. Similarly, Figure 6 without the integer constraint (*i.e.*, for $\widehat{\lambda}^o \coloneqq \arg\max_{\lambda\in[0,1]} \mathrm{TS}_{\widehat{n}^*(\lambda)}$) will give the same result for any a and c_1 with the scaling of the f-axis adjusted. For instance, if a is equal to 2 instead of 3, then the f-axis scale will be divided by $(3 - 1)^2/(2 - 1)^2 = 2$ (e.g., where 0.5 under $a = 3$ on the f-axis, we will have 0.25 under $a = 2$).

A.2 Some commonly used conditions

In the proofs to come, it will be useful to remember that if $\Delta > 0$ (resp. $\Delta < 0$), then

$$
\begin{aligned}
& (1+\lambda+\Delta/n)/H_n = 1 + H_n^{-1} - \Lambda_n^{-1} C''(q)/P'(Q) \\
\overset{\text{(resp. } \geq)}{\leq} & (1+\lambda+\Delta/\Lambda_n)/H_n = 1 + H_n^{-1} + \left[(1-\lambda)(1-H_n) - C''(q)/P'(Q)\right]/(\Lambda_n H_n) \\
\overset{\text{(resp. } \geq)}{\leq} & (1+\lambda+\Delta)/H_n = \left(2 - C''(q)/P'(Q)\right)/H_n,
\end{aligned}
$$

where $\Lambda_n \coloneqq 1 + \lambda(n-1) = nH_n$. Also, $E_{P'}(Q) < \frac{1+\lambda+\Delta(Q,Q_{-i})}{1-(1-\lambda)(1-s_i)}$ on L implies that for any $n \in [1, +\infty)$ and any $Q < \overline{Q}$, $E_{P'}(Q) < (1+\lambda+\Delta(Q,(n-1)Q/n))/H_n$. Thus, part (ii) of the maintained assumption implies that when $\Delta < 0$, $E_{P'}(Q)$ is also lower than $(1+\lambda+\Delta/\Lambda_n)/H_n$ and $(1+\lambda+\Delta/n)/H_n$ in the symmetric equilibrium.

A.3 Proofs of section 2

Where clear we may simplify notation (e.g., omitting the subscript n).

Proof of Proposition 1 Wlog we can constrain attention to quantity profiles $\boldsymbol{q} \in \left\{\boldsymbol{x} \in [0,\overline{q}]^n : \sum_{i \in \mathcal{F}} x_i \leq \overline{Q}\right\}$. Also, the best response of firm i depends on $\boldsymbol{q}_{-i}$ only through Q_{-i}. Denote by $r(Q_{-i})$ the best response correspondence of a firm (the same for all firms). If it is a differentiable function, its slope is given by $r_i'(Q_{-i}) = -1 + \Delta(Q,Q_{-i})/[1 + \lambda + \Delta(Q,Q_{-i}) - (s_i + \lambda(1-s_i))E_{P'}(Q)]$, for $q_i = r(Q_{-i})$. The proof is then similar to that of Theorem 2.1 in Amir and Lambson (AL; 2000).[57]

Case $\boldsymbol{\Delta} > \mathbf{0}$: We first prove statement (a).

Existence of symmetric equilibrium: Firm i's problem is equivalent to choosing the total quantity to be given by the correspondence $R : [0,\overline{Q}] \to [0,\overline{Q}]$ defined as

$$R(Q_{-i}) \coloneqq \underset{Q \in [Q_{-i}, Q_{-i}+\overline{q}]}{\arg\max} \left\{P(Q)\left[Q - (1-\lambda)Q_{-i}\right] - C(Q - Q_{-i})\right\} = r(Q_{-i}) + Q_{-i}.$$

taking Q_{-i} as given. The maximand above is strictly supermodular since $\Delta > 0$, so by Theorem A.1 in AL every selection from $R(Q_{-i})$ is non-decreasing in Q_{-i}. Thus, every selection of the correspondence $B : [0,(n-1)\overline{q}] \rightrightarrows [0,(n-1)\overline{q}]$ given by $B(Q_{-i}) \coloneqq (n-1)R(Q_{-i})/n$ is also non-decreasing in Q_{-i}. By Tarski's intersection point theorem (Theorem A.3 in AL), B has a fixed point, which is a symmetric equilibrium.

Non-existence of asymmetric equilibria: Suppose by contradiction that an asymmetric equilibrium exists, and denote it by $\widetilde{\boldsymbol{q}}$. Then, any permutation of $\widetilde{\boldsymbol{q}}$ should also be an equilibrium, and since $\widetilde{\boldsymbol{q}}$ is asymmetric there exists a permutation $\widehat{\boldsymbol{q}}$ with a firm i such that $\widehat{q}_i > \widetilde{q}_i$. But $\widetilde{Q} = \widehat{Q}$, so $\widehat{Q}_{-i} < \widetilde{Q}_{-i}$. Thus, $R\left(\widehat{Q}_{-i}\right) = R\left(\widetilde{Q}_{-i}\right) = \widetilde{Q} \geq \widetilde{Q}_{-i} > \widehat{Q}_{-i} \implies R\left(\widehat{Q}_{-i}\right) >$

[57]The proof of uniqueness under $\Delta > 0$ is not considered in AL but is also an extension of standard results.

$\widehat{Q}_{-i}$, so $\widetilde{Q} = R\left(\widehat{Q}_{-i}\right)$ makes the first derivative of the firm's objective non-negative, that is $P\left(\widetilde{Q}\right) + P'\left(\widetilde{Q}\right)\left[\widetilde{Q} - (1-\lambda)\widehat{Q}_{-i}\right] - C'(\widetilde{Q} - \widehat{Q}_{-i}) \geq 0$. Also, since the firm's action space is not bounded from above, it trivially holds that $P\left(\widetilde{Q}\right) + P'\left(\widetilde{Q}\right)\left[\widetilde{Q} - (1-\lambda)\widetilde{Q}_{-i}\right] - C'(\widetilde{Q} - \widetilde{Q}_{-i}) \leq 0$. The last two inequalities imply

$$-(1-\lambda)P'\left(\widetilde{Q}\right) - \frac{C'(\widetilde{Q} - \widetilde{Q}_{-i}) - C'(\widetilde{Q} - \widehat{Q}_{-i})}{\widetilde{Q}_{-i} - \widehat{Q}_{-i}} \leq 0. \tag{5}$$

Last, since every selection from $R(Q_{-i})$ is non-decreasing in Q_{-i}, it follows from $R\left(\widehat{Q}_{-i}\right) = R\left(\widetilde{Q}_{-i}\right) = \widetilde{Q}$ that $R(Q_{-i}) = \widetilde{Q}$ for all $Q_{-i} \in [\widehat{Q}_{-i}, \widetilde{Q}_{-i}]$. Therefore, in (5) we can let $\widehat{Q}_{-i} \to \widetilde{Q}_{-i}$, which gives $\Delta(\widetilde{Q}, \widetilde{Q}_{-i}) \leq 0$, a contradiction.

For part (b) it remains to show that at most one symmetric equilibrium exists. $E_{P'} < (1+\lambda+\Delta)/H_n$ on L—which holds given that $E_{P'} < (1+\lambda+\Delta/n)/H_n$ and $\Delta > 0$ on L—implies that $\partial^2\left(\pi_i + \lambda\sum_{j\neq i}\pi_j\right) \big/ (\partial q_i)^2 < 0$, so that $r(Q_{-i})$ is a differentiable function. At a symmetric quantity profile we have $r'(Q_{-i}) = -1 + \Delta(Q, Q_{-i})/(1+\lambda+\Delta(Q,Q_{-i}) - H_n E_{P'}(Q))$. Symmetric equilibria are solutions to $g(q) \equiv r((n-1)q) - q = 0$. Thus, there will be at most one symmetric equilibrium if $g' < 0$, that is, if for any $q \in [0, \overline{Q}/n)$,

$$-\frac{1+\lambda - H_n E_{P'}(nq)}{1+\lambda+\Delta(nq,(n-1)q) - H_n E_{P'}(nq)} < \frac{1}{n-1} \iff E_{P'}(nq) < \frac{1+\lambda+\Delta(nq,(n-1)q)/n}{H_n}$$

which is true, since by assumption it is true on L.

Case $\boldsymbol{\Delta} < \mathbf{0}$: We first prove part (a) for $m = n$. $\Delta < 0$ and $E_{P'}(Q) < \frac{2 - C''(Q-Q_{-i})/P'(Q)}{1-(1-\lambda)(1-s_i)}$ implies that the objective function of each firm is strictly concave in its quantity (in the part where $P(Q) > 0$). Thus, for Q_{-i} such that $r(Q_{-i}) > 0$, $r(Q_{-i})$ is a differentiable function with slope $r_i'(Q_{-i}) = -1 + \Delta/(2 - C''(q_i)/P'(Q) - (s_i + \lambda(1-s_i))E_{P'}(Q)) < -1$ given $\Delta < 0$. Thus, again $g' < 0$ since $r' < -1 < (n-1)^{-1}$ for every $n \geq 2$. Also, $g(0) \geq 0$ and $\lim_{q\to\infty} g(q) = -\infty$, so by continuity of g there exists a unique symmetric equilibrium.

We now prove part (a) for $m < n$. Let q_m be the symmetric equilibrium quantity produced by each firm when m firms are in the market. The m firms are clearly best-responding by producing q_m each. Also, $r'(Q_{-i}) < -1$ (when $r(Q_{-i}) > 0$) implies that $r(mq_m) = r((m-1)q_m + q_m) \leq \max\{r((m-1)q_m) - q_m, 0\} = 0$, since by definition of q_m, $r((m-1)q_m) = q_m$. Thus, the non-producing firms are also best-responding.

To show part (b) assume by contradiction that there is an equilibrium $\widetilde{\boldsymbol{q}}$ of a different type. Then there exist firms i and j such that $\widetilde{q}_i \neq \widetilde{q}_j$, $\widetilde{q}_i > 0$, $\widetilde{q}_j > 0$ in that equilibrium. Wlog let $\widetilde{q}_i > \widetilde{q}_j$. Given that $R'(Q_{-i}) = r'(Q_{-i}) + 1 < 0$ (when $R(Q_{-i}) > Q_{-i}$) it follows that $R(\widetilde{Q}_{-i}) = R(\widetilde{Q}_{-j}) \implies \widetilde{Q}_{-i} = \widetilde{Q}_{-j} \implies \widetilde{q}_i = \widetilde{q}_j$, a contradiction. **Q.E.D.**

Note: The second order of differentiability of $P(Q)$ in Proposition 1 is inessential. However, it simplifies the arguments and interpretation and emphasizes the tension between the assumption $\Delta < 0$ and the one on $E_{P'}(Q)$. The latter guarantees that π_i is strictly concave in q_i whenever $P(Q) > 0$. Decreasing MC is needed for $\Delta < 0$ but at the same time tends to violate profit concavity.[58]

Proof of Corollary 1.1 $\Delta(Q,Q_{-i}) = 1 - \lambda + c_2/b$, constant over L. $E_{P'}(Q) = 0$, also constant. Last, we have that $1 + \lambda + \Delta(Q,Q_{-i}) = 2 + c_2/b$. The result then follows from Proposition 1. Notice also that $Q_n = (a - c_1)/[b(H_n + 1) + c_2/n]$, which is positive since $a > c_1$ and $c_2 > -2bc_1/a > -2b$. $\Pi(n,\lambda) = (a - c_1)^2 (bnH_n + c_2/2) / [bn(H_n + 1) + c_2]^2$ is also positive. Last, $C'(q_n) = [bc_1(H_n + 1) + ac_2/n]/[b(H_n + 1) + c_2/n]$ is positive given $c_2 > -2bc_1/a$, so in equilibrium marginal cost is positive. **Q.E.D.**

Proof of Proposition 2 (i) From the pricing formula (1) the Implicit Function Theorem gives $\partial Q_n/\partial\lambda = -(n-1)Q/[n + \Lambda - C''(Q/n)/P'(Q) - \Lambda E_{P'}(Q)] < 0$. For fixed n, total surplus changes with λ in the same direction as total quantity: $d\,\mathrm{TS} = P(Q)dQ - \sum_{i=1}^{n} C'(q)dq = (P(Q) - C'(q))\, dQ$. Differentiating $\Pi(n,\lambda)$ with respect to λ we get

$$\frac{\partial\Pi(n,\lambda)}{\partial\lambda} = P'(Q_n)\frac{Q_n}{n}\frac{\partial Q_n}{\partial\lambda} + (P(Q_n) - C'(q_n))\frac{\partial Q_n}{\partial\lambda}\frac{1}{n} = P'(Q_n)\frac{Q_n}{n}\frac{\partial Q_n}{\partial\lambda}\frac{n - \Lambda_n}{n},$$

which is positive for $\lambda < 1$, where the second equality follows from the pricing formula (1).

(ii) Using the pricing formula (1) we get

$$\begin{aligned}\frac{\partial\Pi(n,\lambda)}{\partial n} &= P'(Q_n)\frac{Q_n}{n}\frac{\partial Q_n}{\partial n} - Q_n P'(Q_n) H_n \frac{n\frac{\partial Q_n}{\partial n} - Q_n}{n^2} \\ &\propto -\left[(1-\lambda)\left(H_n^{-1} - 1\right) + n + \Lambda_n - H_n^{-1} C''(q_n)/P'(Q_n) - \Lambda_n E_{P'}(Q_n)\right] < 0,\end{aligned}$$

where the inequality follows from what we have seen in section A.2.

(iii) $\partial q_n/\partial n = \partial(Q_n/n)/\partial n = n^{-1}\partial Q_n/\partial n - Q_n/n^2 \propto -(1 + \lambda - H_n E_{P'}(Q))$.

(iv) From the pricing formula (1) the Implicit Function Theorem gives $\partial Q_n/\partial n = q_n\Delta/(n(1 + \lambda + \Delta/n - H_n E_{P'}(Q_n))) \propto \Delta$. **Q.E.D.**

A.4 Proofs of sections 3 and 4

Proof of Proposition 3 The derivative of $\Psi(n,\lambda)$ with respect to n is equal to

$$\frac{\partial\Psi(n,\lambda)}{\partial n} = \lambda(\Pi(n,\lambda) - \Pi(n-1,\lambda)) + \Lambda_n\frac{\partial\Pi(n,\lambda)}{\partial n} - (\Lambda_n - 1)\left.\frac{\partial\Pi(\nu,\lambda)}{\partial\nu}\right|_{\nu=n-1}$$

[58] In the $\Delta > 0$ case, for $\lambda = 0$ we recover the condition $C'' - P' > 0$, under which AL show that a symmetric equilibrium exists and there are no asymmetric equilibria (Theorem 2.1). In the $\Delta < 0$ case, the assumption on $E_{P'}$ guarantees that the firm's objective is quasiconcave in its quantity, under which condition AL show the same result. For $\lambda = 1$, increasing MC is necessary for the uniqueness of the (symmetric) equilibrium. To see why, notice for example that with constant MC, there are infinitely many equilibria (the symmetric one included), all with the same total quantity arbitrarily distributed across firms, since each firm maximizes aggregate industry profits. Analogously, with $C'' < 0$ it is an equilibrium for firms to concentrate all production in one firm to take advantage of the decreasing MC, as indicated in part (ii-a) of the proposition.

$$\propto E_{\Delta\Pi,n} - \left(\frac{\Lambda_n - 1}{\Lambda_n} + \frac{n-1}{\Lambda_n} \frac{\left.\frac{\partial \Pi(\nu,\lambda)}{\partial \nu}\right|_{\nu=n-1}}{\Pi(n,\lambda) - \Pi(n-1,\lambda)} \right) < 0,$$

and the result obtains given Proposition 1. **Q.E.D.**

Proof of Proposition 4 The derivative of $\Psi(n,\lambda)$ with respect to λ is given by

$$\begin{aligned}\frac{\partial \Psi(n,\lambda)}{\partial \lambda} &= (n-1)\left(\Pi(n,\lambda) - \Pi(n-1,\lambda)\right) + \Lambda_n \frac{\partial \Pi(n,\lambda)}{\partial \lambda} - (\Lambda_n - 1)\frac{\partial \Pi(n-1,\lambda)}{\partial \lambda} \\ &\propto -\frac{\lambda\left(\frac{\partial \Pi(n,\lambda)}{\partial \lambda} - \frac{\partial \Pi(n-1,\lambda)}{\partial \lambda}\right)}{\Pi(n,\lambda) - \Pi(n-1,\lambda)} - \frac{1}{\lambda}\frac{\lambda \frac{\partial \Pi(n,\lambda)}{\partial \lambda} \Big/ \Pi(n,\lambda)}{\frac{\Pi(n,\lambda)-\Pi(n-1,\lambda)}{\Pi(n,\lambda)}(n-1)} - 1.\end{aligned}$$

The result follows by the Implicit Function Theorem given Proposition 3. **Q.E.D.**

Proof of Proposition 5 We have $\partial\,\mathrm{TS}_n/\partial n = \Pi(n,\lambda) - f - \Lambda_n Q_n P'(Q_n)\partial q_n/\partial n$. Given $\Psi(\widehat{n}^*(\lambda),\lambda) = f$, $d\,\mathrm{TS}_n/dn|_{n=\widehat{n}^*(\lambda)}$ is equal to (denote $\Pi_n(n,\lambda) \equiv \partial\Pi(n,\lambda)/\partial n$)

$$-\phi\left(\widehat{n}^*(\lambda),\lambda\right)\lambda\widehat{n}^*(\lambda)\Pi_n\left(\widehat{n}^*(\lambda),\lambda\right) - \Lambda_{\widehat{n}^*(\lambda)}Q_{\widehat{n}^*(\lambda)}P'\left(Q_{\widehat{n}^*(\lambda)}\right)\left.\frac{\partial q_n}{\partial n}\right|_{n=\widehat{n}^*(\lambda)},$$

and the result follows from single-peakedness of total surplus in n (and given $P' < 0$), if we substitute in $\Pi(n,\lambda)/\partial n$ and $\partial q_n/\partial n$ from the proof of Proposition 2. For Remark 4.2, notice that $\Delta < 0$ on L implies $C''(q) < 0$ for every $q < \overline{Q}$. By Proposition 2, Q_n is decreasing in n, and thus, so is consumer surplus. Also, $n\Pi(n,\lambda) \equiv P(Q_n)Q_n - nC(q_n) < P(Q_n)Q_n - C(Q_n) \leq P(q_1)q_1 - C(q_1) = \Pi(1,\lambda)$, where the first inequality follows from $C'' < 0$, and the second from q_1 being the monopolist's optimal quantity. Thus, both consumer surplus and industry profits are maximized for $n = 1$, so $n^o(\lambda) = 1$. Last, $n\Pi(n,\lambda) < \Pi(1,\lambda)$ for $n = 2$ and $\lambda = 1$ implies that $\Psi(2,1) = 2\Pi(2,1) - \Pi(1,1) < 0$, so $n^*(1) = 1$. Proposition 7 and Remark B.1 in the appendix also show that $\Psi(n,1) = n\Pi(n,1) - (n-1)\Pi(n-1,1) < 0$ for every $n \geq 2$, so a single firm entering is the unique equilibrium. **Q.E.D.**

Proof of Proposition 6 We have that $d\widehat{n}^*(\lambda)/df = \left.\left(\partial\Psi(n,\lambda)/\partial n\right)^{-1}\right|_{n=\widehat{n}^*(\lambda)}$, and part (ii) follows if we take the directional derivative of $d\widehat{n}^*(\lambda)/df$. **Q.E.D.**

B Additional material

Where clear we may simplify notation, for example omitting the subscript n,λ for equilibrium objects. We may also write for example Q_n instead of $Q_{n^*(\lambda)}$, n instead of $n^*(\lambda)$. Also, we write $\Pi_n(n,\lambda) \equiv \partial\Pi(n,\lambda)/\partial n$, $\Pi_\lambda(n,\lambda) \equiv \partial\Pi(n,\lambda)/\partial\lambda$, $\Pi_{n\lambda}(n,\lambda) \equiv \partial^2\Pi(n,\lambda)/(\partial n\partial\lambda)$, $\Pi_{nn}(n,\lambda) \equiv \partial^2\Pi(n,\lambda)/(\partial n)^2$.

B.1 Individual firm's objective function under overlapping ownership

Here we briefly describe settings of common and cross ownership which can give rise to the Cournot-Edgeworth λ oligopoly model that we study.

B.1.1 Firm objectives under common ownership

The objective function that we use can arise under common ownership. One example of an ownership structure that gives rise to it is described in section B.10.1. For additional examples, see López and Vives (2019; Table 1 and Online Appendix) and Azar and Vives (2021).

B.1.2 Firm objectives under cross ownership

Firm objectives under cross ownership are also described in Gilo et al. (2006) and López and Vives (2019). Assume that we start with each firm i being held by shareholders who do not hold shares of any of the other firms. Then, each *firm* i buys share $\alpha \in [0, 1/(N-1))$ of every other firm $k \in \mathcal{F} \setminus \{i\}$ *without control rights*. In other words, each firm i acquires a claim to share α of the *total earnings* of every other firm. The total earnings of each firm i now include the profit directly generated by firm i and firm i's earnings from its claims over the other firms' total earnings. It can be shown that this gives rise to our symmetric model with $\lambda \coloneqq \alpha/[1-(N-2)\alpha] \in [0,1)$.

B.2 Pricing-stage equilibria under parametric assumptions

CESL demand is of the form

$$P(Q) = \begin{cases} a + bQ^{1-E} & \text{if } E > 1 \\ \max\{a - b\ln Q, 0\} & \text{if } E = 1 \\ \max\{a - bQ^{1-E}, 0\} & \text{if } E < 1 \end{cases}$$

for parameters $a \geq 0$ and $b > 0$. For $E = 0$ this reduces to linear demand, while for $a = 0$ and $E > 1$ it reduces to constantly elastic demand with elasticity $\eta = (E-1)^{-1}$.

Claim 1 provides the equilibria under parametric assumptions on the demand and cost functions. The total quantity is decreasing in the level of overlapping ownership, λ.

Claim 1. Under CESL demand and constant returns to scale the total equilibrium quantity in the pricing stage is

$$Q_n = \begin{cases} \left[\frac{b(1-H_n(E-1))}{c-a}\right]^{\frac{1}{E-1}} & \text{if } E \in (1,2) \text{ and } c > a \\ e^{\frac{a-c-bH_n}{b}} & \text{if } E = 1 \\ \left[\frac{a-c}{b(1+H_n(1-E))}\right]^{\frac{1}{1-E}} & \text{if } E < 1 \text{ and } a > c, \end{cases}$$

where $H_n \coloneqq \Lambda_n/n$, $\Lambda_n \coloneqq 1 + \lambda(n-1)$. Under linear demand and linear-quadratic costs, it is $Q_n = \frac{a-c_1}{b(1+H_n)+c_2/n}$.

Proof of Claim 1 Under CESL demand and constant marginal costs the pricing formula $P(Q_n) - C'(q_n) = -H_n Q_n P'(Q_n)$ gives

$$\begin{aligned} a + b(Q_n)^{1-E} - c = H_n b(E-1)(Q_n)^{1-E} \quad & \text{if } E > 1 \\ a - b \ln Q_n - c = H_n b \quad & \text{if } E = 1 \\ a - b(Q_n)^{1-E} - c = H_n b(1-E)(Q_n)^{1-E} \quad & \text{if } E < 1 \end{aligned}$$

and the result follows. In the case $E > 1$, $E < 2$ and $c > a$ guarantee that there is an interior equilibrium. Notice that if $a > c$, then the profit per unit $P(Q) - AC(q) \geq a - c > 0$ is positive and bounded away from zero for every $Q \geq q \geq 0$, and thus there is no equilibrium. In the case $E < 1$, if $a \leq c$, then in the unique equilibrium $Q_n = 0$.

For linear demand and linear-quadratic costs the pricing formula $P(Q_n) - C'(q_n) = -H_n Q_n P'(Q_n)$ gives $a - bQ_n - c_1 - c_2(Q_n/n) = H_n b Q_n$, and the result follows. **Q.E.D.**

B.3 Additional comparative statics of pricing stage equilibrium

We now study how aggregate industry profits depend on the number of firms.

$$\mu_n \coloneqq 1 - \frac{\partial Q_n}{\partial n}\frac{n}{Q_n}\frac{1-H_n}{\eta(Q_n) - H_n}.$$

Proposition 7. The following statements hold:

(i) if $\mu_n \leq 0$, aggregate industry profits are decreasing in n,

(ii) if $\mu_n > 0$, aggregate industry profits are decreasing (resp. increasing) in n if $E_C(q_n) \overset{(\text{ resp. } >)}{<} \mu_n^{-1}$,

(iii) if $C''(q) < 0$ for every $Q \in [0, Q_n]$, then monopoly maximizes aggregate industry profits, $\Pi(1,\lambda) > n\Pi(n,\lambda)$.

Proof of Proposition 7 (i-ii) Given what we see in the proof of Proposition 8, for aggregate industry profits we have that

$$\begin{aligned} \frac{\partial [n\Pi(n,\lambda)]}{\partial n} &= P(Q_n)\frac{Q_n}{n} - C(q_n) + nP'(Q_n)\left(\frac{Q_n}{n}\right)^2 \left[\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}(1-H_n) + H_n\right] \\ &\propto -\left[\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}(1-H_n) - \eta(Q_n)\frac{P(Q_n) - C'(q_n) + C'(q_n)\frac{E_C(q_n)-1}{E_C(q_n)}}{P(Q_n)} + H_n\right] \\ &\overset{(1)}{=} -\left[\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}(1-H_n) - \eta(Q_n)\left(1 - \frac{H_n}{\eta(Q_n)}\right)\frac{E_C(q_n)-1}{E_C(q_n)}\right] \end{aligned}$$

$$\propto E_C(q_n)\left(1 - \frac{\partial Q_n}{\partial n}\frac{n}{Q_n}\frac{1 - H_n}{\eta(Q_n) - H_n}\right) - 1,$$

where $H_n < \eta(Q_n)$ from the pricing formula (1).

(iii) We have that

$$n\Pi(n,\lambda) \equiv P(Q_n)Q_n - nC(q_n) \overset{C''<0}{<} P(Q_n)Q_n - C(Q_n) \le P(q_1)q_1 - C(q_1) = \Pi(1,\lambda),$$

where the last inequality follows by the definition of q_1 being the monopolist's optimal quantity.

To see why Remark B.2 holds notice that for $\lambda = 1$

$$\begin{aligned}\frac{\partial\left[n\Pi\left(n,\lambda\right)\right]}{\partial n} &= P(Q_n)\frac{Q_n}{n} - C(q_n) + nP'(Q_n)\left(\frac{Q_n}{n}\right)^2 \\ &\overset{C''>0}{>} P(Q_n)\frac{Q_n}{n} - C'(q_n)q_n + P'(Q_n)\frac{Q_n^2}{n} \propto \frac{P(Q_n) - C'(q_n)}{P(Q_n)} - \frac{1}{\eta(Q_n)} = 0,\end{aligned}$$

where the equality follows from the pricing formula (1). **Q.E.D.**

Remark B.1. If $\Delta < 0$, then $\partial Q_n/\partial n > 0$, so $\mu_n \le 1$, and thus, aggregate industry profits are decreasing in n if $E_C(q_n) < 1$. If for example $C'' < 0$ globally (consistent with $\Delta < 0$), then indeed $E_C(q_n) < 1$.

Remark B.2. If $\lambda = 1$ and $C''(q) > 0$ for every $q \in [0,q_n]$, aggregate industry profits are increasing in n.

Consider the extreme case of $\lambda = 1$ and notice the following. Condition $\Delta > 0$ requires decreasing returns to scale, so that aggregate gross profits increase with n (*i.e.*, $n\Pi(n,1) > (n-1)\Pi(n-1,1)$ for any n) due to savings in variable costs as production is distributed across more firms, even though the total quantity increases (see Proposition 2), and thus price decreases with the number of firms. Intuitively, aggregate gross profits increasing in n for $\lambda = 1$ is tied to the uniqueness of the (symmetric) equilibrium in the pricing stage. Since firms jointly maximize aggregate profits, the latter should increase with n for firms to strictly prefer to spread production evenly. On the other hand, under constant returns to scale aggregate profits are constant in n; increasing the number of firms simply changes how the firms can jointly produce the fixed level of total output that maximizes joint profits.[59] Last, under increasing returns to scale it is an equilibrium for all production to be concentrated in a single firm.

[59]As argued already, in this case, the are infinitely many equilibria of the pricing stage, all with the same total quantity.

B.4 When $\Psi(n,\lambda)$ is not (globally) decreasing in n

Our assumption that $\Psi(n,\lambda)$ is (globally) decreasing in n may fail—yet without strongly affecting our results. For example, under constant MC, (gross) aggregate industry profits are independent of the number of firms when $\lambda = 1$, so $\Psi(n,1) = 0$ for every $n \geq 2$, and thus $n^*(1) = 1$. It can also be seen that under decreasing MC, $\Psi(n,1) = n\Pi(n,1) - (n-1)\Pi(n-1,1) < 0$ for every $n \geq 2$ (see Proposition 7 and Remark B.1), so $n^*(1) = 1$. Thus, under $c_2 < 0$ and $\lambda = 1$, only one firm entering is the unique free entry equilibrium, even though $\Psi(n,1)$ need not be globally decreasing in n. In Figure 3c, where there is increasing MC, $n^*(1) = 5$.

B.5 Concavity of total surplus in the number of firms

Lemma 1. TS_n is globally strictly concave in n if for every n

$$\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}\left[1-\lambda-H_n\left(\left(\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}-1\right)(1-E_{P'}(Q_n))+\frac{\partial^2 Q_n}{(\partial n)^2}\left(\frac{\partial Q_n}{\partial n}\right)^{-1}n-1\right)\right]>\frac{1-\lambda}{n}.$$

Under constant marginal costs and $E_{P'}(Q_n) < 2$ for every n, this is true if $E'_{P'}(Q) \equiv \partial E_{P'}(Q)/\partial Q$ is not too high; particularly, $E'_{P'} \leq 0$ is sufficient, and thus so is CESL demand.

Proof of Lemma 1 We have seen that the first derivative of equilibrium total surplus with respect to n is given by

$$\frac{d\,\mathrm{TS}_n}{dn} = \Pi(n,\lambda) - f - \Lambda_n Q_n P'(Q_n)\frac{\frac{\partial Q_n}{\partial n}-q_n}{n},$$

so if we denote $\Pi_n(n,\lambda) \equiv \partial\Pi(n,\lambda)/\partial n$, the second derivative is given by

$$\begin{aligned}\frac{d^2\,\mathrm{TS}_n}{(dn)^2} =&\Pi_n(n,\lambda) - \lambda Q_n P'(Q_n)\frac{\frac{\partial Q_n}{\partial n}-q_n}{n} - \Lambda_n\frac{\partial Q_n}{\partial n}P'(Q_n)\frac{\frac{\partial Q_n}{\partial n}-q_n}{n}\\ &- \Lambda_n Q_n P''(Q_n)\frac{\partial Q_n}{\partial n}\frac{\frac{\partial Q_n}{\partial n}-q_n}{n} - \Lambda_n Q_n P'(Q_n)\frac{\left(\frac{\partial^2 Q_n}{(\partial n)^2}-\frac{dq_n}{\partial n}\right)n-\frac{\partial Q_n}{\partial n}+q_n}{n^2}\\ \propto& -\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}\left[1-\lambda-H_n\left(\left(\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}-1\right)(1-E_{P'}(Q_n))+\frac{\partial^2 Q_n}{(\partial n)^2}\left(\frac{\partial Q_n}{\partial n}\right)^{-1}n-1\right)\right]\\ &+\frac{1-\lambda}{n}.\end{aligned}$$

Under constant marginal costs

$$\frac{\partial Q_n}{\partial n} = \frac{1-\lambda}{n+\Lambda-\Lambda E_{P'}(Q_n)}\frac{Q_n}{n} \implies$$

$$\frac{\partial^2 Q_n}{(\partial n)^2} = (1-\lambda)\left(-\frac{1+\lambda-\lambda E_{P'}(Q_n)-\Lambda E'_{P'}(Q_n)\frac{\partial Q_n}{\partial n}}{(n+\Lambda-\Lambda E_{P'}(Q_n))^2}\frac{Q_n}{n}+\frac{\frac{\frac{\partial Q_n}{\partial n}n-Q_n}{n^2}}{n+\Lambda-\Lambda E_{P'}(Q_n)}\right),$$

$$\frac{\partial^2 Q_n}{(\partial n)^2}\left(\frac{\partial Q_n}{\partial n}\right)^{-1} n = -n\frac{1+\lambda-\lambda E_{P'}(Q_n)-\Lambda E'_{P'}(Q_n)\frac{\partial Q_n}{\partial n}}{n+\Lambda-\Lambda E_{P'}(Q_n)}+\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}-1$$

so that

$$\frac{d^2\,\mathrm{TS}_n}{(dn)^2} \propto -\left\{\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}\left[1-\lambda-H_n\left(\begin{array}{r}\left(\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}-1\right)(2-E_{P'}(Q_n))\\ -n\frac{1+\lambda-\lambda E_{P'}(Q_n)-\Lambda E'_{P'}(Q_n)\frac{\partial Q_n}{\partial n}}{n+\Lambda-\Lambda E_{P'}(Q_n)}-1\end{array}\right)\right]-\frac{1-\lambda}{n}\right\}$$

$$\propto -\left\{\begin{array}{r} n(1-\lambda)(n+\Lambda_n-\Lambda_n E_{P'}(Q_n))-(n+\Lambda_n-\Lambda_n E_{P'}(Q_n))^2\\ -\Lambda_n\left(\begin{array}{r}-(n+\Lambda_n-(1-\lambda)-\Lambda_n E_{P'}(Q_n))(2-E_{P'}(Q_n))\\ -n\left(1+\lambda-\lambda E_{P'}(Q_n)-\Lambda_n E'_{P'}(Q_n)\frac{\partial Q_n}{\partial n}\right)-(n+\Lambda_n-\Lambda_n E_{P'}(Q_n))\end{array}\right)\end{array}\right\}.$$

The partial derivative of the expression in the brackets with respect to $E_{P'}(Q_n)$ is given by

$$\begin{aligned}&-\Lambda n(1-\lambda)+2\Lambda(n+\Lambda-\Lambda E_{P'}(Q_n))\\ &-\Lambda(\Lambda(2-E_{P'}(Q_n))+n+\Lambda-(1-\lambda)-\Lambda E_{P'}(Q_n)+\Lambda-(1-\lambda)+\Lambda)\\ \propto&\lambda n-(3\Lambda-2(1-\lambda))=-(2\Lambda-(1-\lambda))<0,\end{aligned}$$

so that, given $E_{P'}(Q_n)<2$, for $d^2\,\mathrm{TS}_n/(dn)^2$ to be negative it is sufficient that

$$\begin{aligned} &n(1-\lambda)(n+\Lambda-2\Lambda)-(n+\Lambda-2\Lambda)^2\\ -\Lambda&\left(-n\left(1+\lambda-2\lambda-\Lambda E'_{P'}(Q_n)\frac{\partial Q_n}{\partial n}\right)-(n+\Lambda-2\Lambda)\right)\geq 0 \Longleftrightarrow\\ &1-\lambda-\frac{\Lambda}{n}E'_{P'}(Q_n)Q_n\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}\geq-\frac{(\Lambda+1-\lambda)(n-\Lambda)}{\Lambda n},\end{aligned}$$

which is true for $E'_{P'}$ not too high. **Q.E.D.**

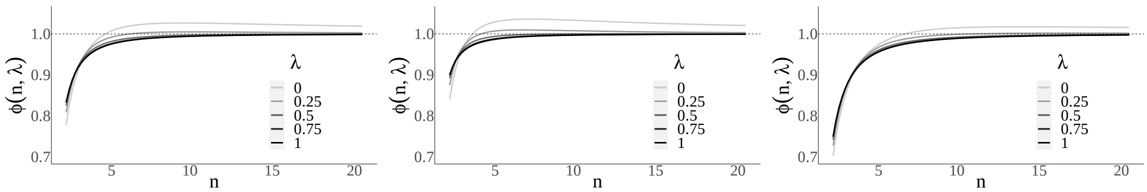
Remark B.3. More generally, all else constant, the condition of Lemma 1 is satisfied if the elasticity of the slope of Q_n with respect to n, $\frac{\partial^2 Q_n}{(\partial n)^2}\left(\frac{\partial Q_n}{\partial n}\right)^{-1} n$, is not too high. Also, remember that $\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}\in(0,1)$ under the assumptions of Proposition 2(iii-a), so all else constant, in that case the condition is satisfied if $E_{P'}(Q)$ is not too high.

B.6 Numerical results showing that ϕ is close to 1

The numerical results of Figure 7 verify that $\phi(n,\lambda)$ is indeed close to 1, especially for $n\geq 3$.

Figure 7: $\phi(n,\lambda)$ under linear demand and linear-quadratic costs

(a) $b = 1,\ c_2 = 1$ **(b)** $b = 2,\ c_2 = 1$ **(c)** $b = 1,\ c_2 = 2$



Note: It can be checked that $\phi(n,\lambda)$ is invariant with respect to c_1 and the demand parameter a.

B.7 Derivation of Numerical Results

Under CESL demand and constant returns to scale, given Claim 1 we find that

$$\Pi(n,\lambda) = \begin{cases} \frac{1}{n}\left[a + b\left[\frac{b(1-H_n(E-1))}{c-a}\right]^{\frac{1-E}{E-1}} - c\right]\left[\frac{b(1-H_n(E-1))}{c-a}\right]^{\frac{1}{E-1}} & \text{if } E \in (1,2) \text{ and } c > a \\ \frac{1}{n}\left[a - b\ln\left(e^{\frac{a-c-bH_n}{b}}\right) - c\right]e^{\frac{a-c-bH_n}{b}} & \text{if } E = 1 \\ \frac{1}{n}\left[a - b\left[\frac{a-c}{b(1+H_n(1-E))}\right]^{\frac{1-E}{1-E}} - c\right]\left[\frac{a-c}{b(1+H_n(1-E))}\right]^{\frac{1}{1-E}} & \text{if } E < 1 \text{ and } a > c, \end{cases}$$

$$= \begin{cases} \frac{H_n(E-1)b^{\frac{1}{E-1}}}{n}\left[\frac{1-H_n(E-1)}{c-a}\right]^{\frac{2-E}{E-1}} & \text{if } E \in (1,2) \text{ and } c > a \\ \frac{bH_n}{n}e^{\frac{a-c-bH_n}{b}} & \text{if } E = 1 \\ \frac{H_n(1-E)}{nb^{\frac{1}{1-E}}}\left[\frac{a-c}{1+H_n(1-E)}\right]^{\frac{2-E}{1-E}} & \text{if } E < 1 \text{ and } a > c, \end{cases}$$

Derivation of Numerical Result 1 Parameters a, b and c only affect the magnitudes of $d\widehat{n}^*(\lambda)/d\lambda$ and $dQ_{\widehat{n}^*(\lambda)}/d\lambda$, and not their signs. The result then is obtained in a way analogous to the one described in the Derivation of Numerical Result 3. ■

Derivation of Numerical Result 2 Notice that, given a fixed n, $\phi(n,\lambda)$ is independent of a, c_1, and f. Also, $E_{P'}(Q_n) = 0$ and $\Delta = 1 - \lambda + c_2/b$ always (independently of a, c_1, and f). Thus, the expressions in Proposition 5 depend on a, c_1, and f only through their effect on $\widehat{n}^*(\lambda)$. Also, if we look at the expression for $\Pi(n,\lambda)$ in the proof of Corollary 1.1, it is easy to see that $\widehat{n}^*(\lambda)$ depends on a, c_1, and f only though $(a - c_1)^2/f$.[60] Further, $\Pi(2,0) \geq f$ if and only if $(a - c_1)^2/f \geq 2(3b + c_2)^2/(2b + c_2)$, so the values that b and c_2 can take that make the net monopoly profit non-negative also depend on a, c_1, and f only through $(a - c_1)^2/f$. Finally, $\Delta \geq 0$ is satisfied for every λ if and only if $c_2 \geq 0$, which does not depend on a, c_1, or f. Thus, without loss of generality, we can let $a = 1$, $c_1 = 0$ and only vary b, c_2, and f in the simulations. We then numerically check that for every $(b,c_2,f) \in \{(b,c_2,f) : \exists(t_1,t_2,t_3) \in \{0,1,\ldots,9\}^3$ such that $b = 0.01 + 1.11t_1, c_2 = 100t_2/9, f = 0.001 + [(2b + c_2)/(2(3b + c_2)^2) - 0.001]t_3/9\}$ (*i.e.*, for

[60]Namely, $\widehat{n}^*(\lambda)$ is increasing in $(a - c_1)^2/f$.

1,000 parametrizations) there exists a threshold $\overline{\lambda}$ as claimed by solving for $\widehat{n}^*(\lambda)$ and $\widehat{n}^o(\lambda)$ for every $\lambda \in \{0, 0.05, 0.1, \ldots, 0.95\}$. ■

Derivation of Numerical Result 3 It is easy to see that the signs of derivatives of $\Psi(n,\lambda)$ are independent of a, b and c. Thus, we can wlog set (i) $a = b = 1$ and $c = 2$ for the case $E \in (1,2)$, and (ii) $a = 2$, $b = c = 1$ for the case $E < 1$. We numerically find that

$$\min_{(n,\lambda,E)\in[2,7]\times[0,1]\times[1.001,1.7]} \frac{\Psi(n,\lambda)}{\partial\lambda\partial n} \approx 2.31 \cdot 10^{-6} > 0,$$

which is reached for $n = 7$, $\lambda = 0$ and $E = 1.001$. In the case of $E < 1$, we similarly find that

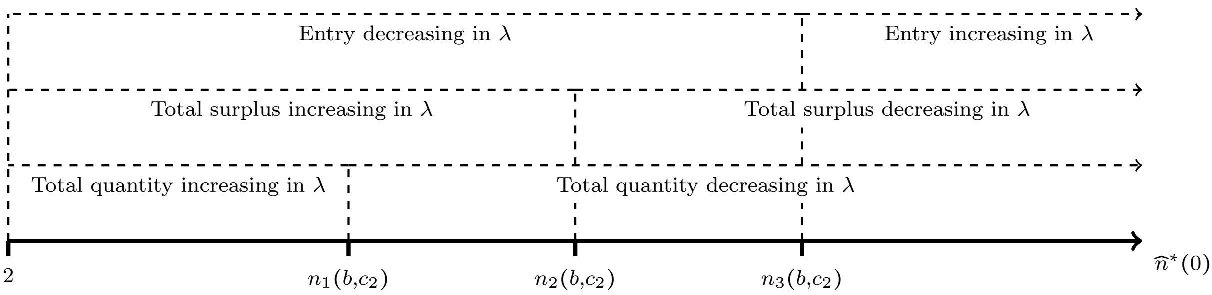
$$\min_{(n,\lambda,E)\in[2,8]\times[0,1]\times[-1000,0.999]} \frac{\Psi(n,\lambda)}{\partial\lambda\partial n} \approx 1.11 \cdot 10^{-7} > 0,$$

which is reached for $n = 8$, $\lambda = 0$ and $E = 0.999$. In additional simulations, allowing E to be even lower than -1000 does not change the result. ■

B.8 Additional results on the linear-quadratic model

Claim 2 studies how entry, the total quantity and total surplus change with overlapping ownership around $\lambda = 0$. Figure 8 summarizes the results.

Figure 8: Comparative statics around $\lambda = 0$ under linear demand and linear-quadratic cost



Note: See Claim 2 for precise statement. $n_1(b,c_2) > 2$ only under significantly decreasing MC.

Claim 2. Ignore the integer constraint on n (so that entry is given by $\widehat{n}^*(\lambda)$). Let demand be linear and cost be linear-quadratic with $a > c_1 \geq 0$, $-b \neq c_2 > -2bc_1/a$, and assume $\widehat{n}^*(0) \geq 2$. Then, there exist thresholds $n(b,c_2) \in \mathbb{R}^3$ (that depend on b and c_2) with $n_3(b,c_2) > n_2(b,c_2) > \max\{n_1(b,c_2), 2\}$ such that starting from $\lambda = 0$:

(i) entry is locally increasing (resp. decreasing) in λ if $\widehat{n}^*(0) \overset{(\text{resp. } <)}{>} n_3(b,c_2)$,

(ii) the total surplus is locally increasing (resp. decreasing) in λ if $\widehat{n}^*(0) \overset{(\text{resp. } >)}{<} n_2(b,c_2)$,

(iii) if $c_2 > -3b/2$, then $n_1(b,c_2) < 2$ and the total quantity is locally decreasing in λ,

(iv) if $c_2 < -3b/2$, then $n_1(b,c_2) > 2$ and the total quantity is locally increasing (resp. decreasing) in λ if $\widehat{n}^*(0) \overset{\text{(resp. >)}}{<} n_1(b,c_2)$,

Proof of Claim 2 The total derivative of $\widehat{n}^*(\lambda)$ at $\lambda = 0$ is

$$\frac{d\widehat{n}^*(\lambda)}{d\lambda} = (n-1)\frac{b\left[(n-1)(bn+c_2)^2 - (n+1+c_2/b)(b+c_2/2)(b(2n+1)+2c_2)\right]}{2(b+c_2/2)(bn+c_2)^2}\Bigg|_{n=\widehat{n}^*(0)},$$

where the denominator is positive and the numerator is a third-degree polynomial in n. In part (i), n_3 is the unique real root of the polynomial, which has a negative discriminant. In part (ii), the discriminant is positive, and the result follows with n_3 the highest of the three real roots of the polynomial equation above. Also,

$$\frac{dQ_{\widehat{n}^*(\lambda)}}{d\lambda} = \frac{\partial Q_n}{\partial\lambda} + \frac{\partial Q_n}{\partial n}\frac{d\widehat{n}^*(\lambda)}{d\lambda} = \frac{Q_{\widehat{n}^*(\lambda)}}{n+1+c_2/b}\left[(1+c_2/b)\frac{d\widehat{n}^*(\lambda)}{d\lambda}\frac{1}{n} - (n-1)\right]\Bigg|_{n=\widehat{n}^*(0)}$$

and for $n_1 \equiv (-2b^2 - 5bc_2 - c_2^2)/(2b^2) + \sqrt{(6b^3c_2 + 11b^2c_2^2 + 6bc_2^3 + c_2^4)/b^4}/2$ the corresponding results follow. For $\lambda = 0$, $\Psi(\widehat{n}^*(\lambda),\lambda) = \Pi(\widehat{n}^*(\lambda),\lambda) = f$, we get $d\,\mathrm{TS}_{\widehat{n}^*(\lambda)}/d\lambda \propto dQ_{\widehat{n}^*(\lambda)}/d\lambda - q_{\widehat{n}^*(\lambda)}d\widehat{n}^*(\lambda)/d\lambda$ and for $n_2 \equiv (2b - c_2 + \sqrt{8b^2 + 6bc_2 + c_2^2})/(2b)$ the corresponding result follows. It can be checked that $n_3 > n_2 > n_1$. **Q.E.D.**

Part (i) of the Corollary extends our finding that if without overlapping ownership many (resp. few) firms enter, then marginally increasing overlapping ownership will increase (resp. decrease) entry.

Part (ii) shows that marginally increasing λ above 0 increases total surplus if and only if entry is low. Particularly, the direct (negative) effect of an increase in λ on total surplus is dominated by the alleviation of excessive entry (since for $\lambda = 0$ entry is excessive) due to the increase in λ. We thus obtain another sufficient condition: if absent overlapping ownership, entry would be low, then a planner that regulates overlapping ownership (but not entry) should choose a positive level of it.

Parts (iii) and (iv) show that introducing a small amount of overlapping ownership may only increase the total quantity when MC is significantly decreasing (which means that the Cournot market is quasi-anticompetitive) and entry is low. In that case, the softening of pricing competition due to the increase in overlapping ownership is dominated by the concurrent decrease in entry—which tends to increase the total quantity since the market is quasi-anticompetitive. This yields a sufficient condition for consumer surplus to be maximized by some $\lambda > 0$. As shown in Figure 3d, this condition is not necessary, since with decreasing MC a positive level of overlapping ownership can be optimal under a consumer surplus standard even when overlapping ownership decreases the total quantity around $\lambda = 0$.

B.9 Free entry under pre-entry overlapping ownership and the presence of maverick firms

This section presents a model of free entry with pre-entry overlapping ownership under the presence of maverick firms.

For simplicity, model the maverick firms as a competitive fringe that in the first stage (where oligopolists enter) submit an aggregate supply schedule. Namely, there is a set $\mathcal{F}_m$ of infinitesimal firms. Firm $i \in \mathcal{F}_m$ chooses to either be inactive or produce one (infinitesimal) unit of the good at cost $\chi(i)$.[61] Thus, the aggregate supply function by the maverick firms in the third stage $S : \mathbb{R}_+ \to \mathbb{R}_+$ is given by $S(p) \coloneqq \int_{i\in\mathcal{F}_m} I\left(\chi(i) \le p\right) di$. $S : \mathbb{R}_+ \to \mathbb{R}_+$ with $S(p) = 0$ for every $p \in [0,\underline{p}]$ and $S'(p) > 0$ for every $p > \underline{p}$ where $\underline{p} \ge 0$. Then, the price $p > 0$ in the competitive equilibrium among the maverick firms will be implicitly given by $P^{-1}(p) = Q + S(p)$, where Q is the total quantity produced by the oligopolists.[62] This means that in the second stage, the oligopolists are essentially faced with inverse demand $\widetilde{P} : \mathbb{R}_+ \to \mathbb{R}_+$ given by

$$\widetilde{P}(Q) = \begin{cases} P\left(Q + \omega^{-1}(Q)\right) \in \left(\underline{p}, P(Q)\right) & \text{if } P(Q) > \underline{p} \\ P(Q) & \text{if } P(Q) \le \underline{p} \end{cases}$$

where $\omega : \mathbb{R}_{++} \to \mathbb{R}_+$ is given by $\omega(y) \coloneqq P^{-1} \circ S^{-1}(y) - y$.[63] $\omega^{-1}(Q)$ gives the quantity supplied in the competitive equilibrium among the maverick firms when the oligopolists produce Q. For example, in the case of (i) linear demand $P(Q) = \max\{a - bQ, 0\}$, (ii) linear maverick aggregate supply schedule $S(p) = \max\{(p - \underline{p})/b_m, 0\}$ with $b_m > 0$ and $\underline{p} \ge 0$, and (iii) constant MC (for the oligopolists), $C(q) = cq$, with $a > c \ge \underline{p}$,[64] for any $Q \in [0, (a-c)/b]$, $\widetilde{P}$ is given by[65]

$$\widetilde{P}(Q) = \overbrace{a - \frac{a - \underline{p}}{1 + b_m/b}}^{<a} - \overbrace{\frac{b}{1 + b/b_m}}^{<b} Q.$$

The (prospect of) entry by maverick firms essentially changes the demand faced by the commonly-owned firms by depressing it and making it more elastic. If in the paper wherever P we read $\widetilde{P}$, the results on the effects of overlapping ownership on entry and

[61] This cost can be thought to include any applicable entry costs. Since maverick firms are infinitesimal and each supply an infinitesimal quantity, their entry cost is also infinitesimal.

[62] We assume that $S(p) > P^{-1}(p)$ for p large enough.

[63] To see this substitute $p = P\left(Q + \omega^{-1}(Q)\right)$ in $P^{-1}(p) = Q + S(p)$, which gives

$$Q + \omega^{-1}(Q) = Q + S \circ P\left(Q + \omega^{-1}(Q)\right) \iff P^{-1} \circ S^{-1} \circ \omega^{-1}(Q) - \omega^{-1}(Q) = Q,$$

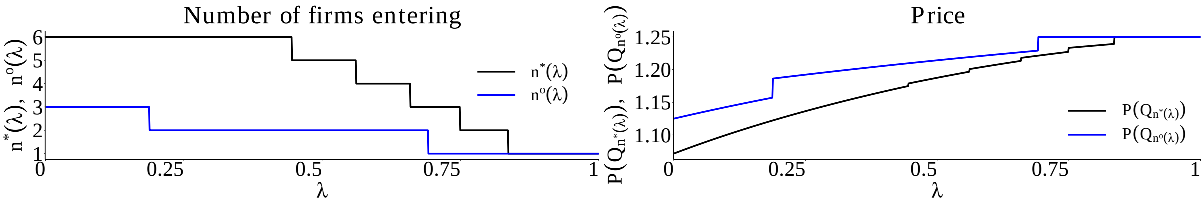
which is true by definition of ω.

[64] For $c = \underline{p}$, the most efficient maverick firms is as efficient as the oligopolists.

[65] The inverse demand $\widetilde{P}$ for higher Q does not play a role since the commonly-owned firms will never produce more than $(a-c)/b$. To derive $\widetilde{P}$, solve for it in $(a - \widetilde{P}(Q))/b = Q + (\widetilde{P}(Q) - \underline{p})/b_m$.

the price continue to hold (with the number of firms n not counting maverick firm entry). A comparison of Figure 9 with Figure 3a in the paper (the two figures use the same parametrization but in the former maverick firms are added) shows entry to be less sensitive to overlapping ownership due to the presence of the maverick firms, as argued in the paper.

Figure 9: Equilibrium with pre-entry overlapping ownership under the presence of maverick firms for varying λ



Note: Lines represent values in equilibrium; linear demand, constant MC: $a = 2$, $b = c = 1$, $f = 0.01$; linear maverick aggregate supply schedule: $b_m = \underline{p} = 1$.

Last, the total surplus $\widetilde{\mathrm{TS}}(\boldsymbol{q})$ now includes the maverick firms' surplus, where $\boldsymbol{q}$ still is the quantity profile of the oligopolists. Denote by $\widetilde{\mathrm{TS}}_n$ the pricing stage equilibrium total surplus when n commonly-owned firms enter. Equation (4) also applies in the case with maverick firms but with $\widetilde{\Xi}(n,\lambda) \coloneqq (n-1)\left(\widetilde{\Pi}\left(n-1,\lambda\right) - \widetilde{\Pi}\left(n,\lambda\right)\right)$, $\widetilde{\Pi}(n,\lambda) \coloneqq \widetilde{P}(Q_n)q_n - C(q_n)$ and $\widetilde{P}$ replacing Ξ, Π and P. Q_n, q_n are still the quantities produced by the commonly-owned firms in the pricing stage equilibrium where n of them enter. $\widehat{n}^*(\lambda)$ is now pinned down by $\widetilde{\Pi}(\widehat{n}^*(\lambda),\lambda) - \lambda\widetilde{\Xi}(\widehat{n}^*(\lambda),\lambda) = f$.

Provided $\widetilde{P}(Q) \geq \underline{p}$ or equivalently $P(Q) \geq \underline{p}$,[66] total surplus now includes the maverick firms' surplus and is thus given by

$$\widetilde{\mathrm{TS}}(\boldsymbol{q}) \coloneqq \overbrace{\int_0^{Q+S(\widetilde{P}(Q))} \left(P(X) - \widetilde{P}(Q)\right) dX}^{\text{consumer surplus}} + \overbrace{\int_{\underline{p}}^{\widetilde{P}(Q)} S(p)dp}^{\text{maverick firms' surplus}} + \overbrace{\widetilde{P}(Q)Q - \sum_{i=1}^{n} C(q_i) - nf}^{\text{commonly-owned firms' profits}}$$

$$= \mathrm{TS}(\boldsymbol{q}) + \underbrace{\int_Q^{Q+S(\widetilde{P}(Q))} P(X)dX - S\left(\widetilde{P}(Q)\right)\widetilde{P}(Q)}_{\geq 0;\text{ consumer surplus “due to” maverick firms' production}} + \underbrace{\int_{\underline{p}}^{\widetilde{P}(Q)} S(p)dp}_{\geq 0;\text{ maverick firms' surplus}}$$

where $\boldsymbol{q}$ still the quantity profile of the oligopolists and $\mathrm{TS}(\boldsymbol{q}) \equiv \int_0^Q P(X)dX - \sum_{i=1}^n C(q_i) - nf$ the total surplus without maverick firms. For any fixed quantity profile of the oligopolists, total surplus is higher when the maverick firms are present (and produce) compared to when they are not. We have that

$$\frac{d\widetilde{\mathrm{TS}}_n}{dn} = P(Q_n)\left(n\frac{\partial q_n}{\partial n} + q_n\right) - C(q_n) - nC'(q_n)\frac{\partial q_n}{\partial n} - f$$

[66] Otherwise, wherever $\widetilde{P}(Q)$ substitute $\underline{p}$, and the equation reduces to $\widetilde{\mathrm{TS}}(\boldsymbol{q}) = \mathrm{TS}(\boldsymbol{q})$.

$$+\begin{bmatrix} \left(1+S'(\widetilde{P}(Q_n))\widetilde{P}'(Q_n)\right)P\left(Q_n+S(\widetilde{P}(Q_n))\right)-P(Q_n) \\ -S'(\widetilde{P}(Q_n))\widetilde{P}'(Q_n)\widetilde{P}(Q_n)-S(\widetilde{P}(Q_n))\widetilde{P}'(Q_n)+S(\widetilde{P}(Q_n))\widetilde{P}'(Q_n) \end{bmatrix}\frac{\partial Q_n}{\partial n}$$
$$=\widetilde{\Pi}(n,\lambda)-f-(1+\lambda(n-1))\,Q_n\widetilde{P}'(Q_n)\frac{\partial q_n}{\partial n},$$

where $\widetilde{\mathrm{TS}}_n$ is the pricing stage equilibrium total surplus when n commonly-owned firms enter, $\widetilde{\Pi}(n,\lambda) \coloneqq \widetilde{P}(Q_n)q_n - C(q_n)$, and Q_n, q_n are still the quantities produced by the commonly-owned firms in the pricing stage equilibrium where n of them enter.

Whether there is excessive or insufficient entry by commonly-owned firms will depend on the same forces identified in the previous section but with adjusted magnitude since P is replaced by $\widetilde{P}$. Notice that excessive or insufficient entry is based on a planner that controls the entry of oligopolists and allows them and the maverick firms to produce freely. Importantly, given the production decisions of the oligopolists, the maverick firms' production level maximizes total surplus since the maverick firms are perfect competitors.

B.10 Free entry under post-entry overlapping ownership

In the last section overlapping ownership develops before entry, thus directly affecting the incentives of firms to enter. In this section we study the case where potential entrants have no prior overlapping ownership, but after they enter the market and before they pick quantities in the second stage they develop overlapping ownership, so that they have an Edgeworth coefficient of effective sympathy $\lambda \in [0,1]$. Now, the only channel through which overlapping ownership affects entry is by increasing profits in the post-entry game. Firms expect this and therefore entry increases with overlapping ownership.

This can be interpreted as a long-run equilibrium whereby start-up firms (or already existing firms but without overlapping ownership) enter the industry and then develop overlapping ownership through time. Appendix B.1 describes explicitly how post-entry overlapping ownership can arise. Also, given that the extent to which overlapping ownership affects corporate conduct is an open empirical question, this section can also be interpreted as studying pre-entry overlapping ownership when it affects pricing but does *not* cause firms to internalize their entry externality.

The exogeneity of λ is important with post-entry overlapping ownership, since the incentives of firms to allow for ownership ties after entry are not modeled. For instance, if the number of shares that investors buy from the entrepreneurs depended on the extent of entry—since the latter affects profits, then λ would be a function of n. Although the exogeneity of λ is restrictive, if firms become publicly traded after entry (at least in the long-run), they indeed have limited control over their ownership ties, since for instance investment funds are free to buy shares of all firms.

B.10.1 An example of post-entry overlapping ownership

Post-entry overlapping ownership can for example arise in the form of common ownership as described below. There is a finite set $\mathcal{J}$ of investors. Let all firms be newly-established and the set of investors $\mathcal{J}$ be partitioned into $\{J_0\} \cup \cup_{i\in\mathcal{F}} \{J_i\}$ with $|J_i| = |J_0| = m$ for every $i \in \mathcal{F}$. Before entry each firm i is (exclusively) held by the set J_i of entrepreneurs with $\beta_{ji} = 1/m$ for every $j \in J_i$; there is no common ownership before entry, so when considering entry, the entrepreneurs of each firm unanimously agree to maximize their own firm's profit.[67] After entry, the set J_0 of investors, who previously held no shares of any firm, buy firm shares. Each investor $j \in J_0$ now holds share $\beta'_{ji} = \sigma/m$ of each firm i that has entered, and each entrepreneur $j \in J_i$ holds share $\beta'_{ji} = (1-\sigma)/m$ of her firm for some $\sigma \in [0,1]$. That is, after entry each entrepreneur sells the same amount of shares to the investors, who are now uniformly invested in all firms in the industry. Consider the O'Brien and Salop (2000) model, who assume that the manager of firm i maximizes a weighted average of the shareholders' portfolio profits (*i.e.*, she maximizes $\sum_{j\in\mathcal{J}} \gamma_{ji} u_j(\boldsymbol{q})$, where γ_{ji} captures the extent of j's control over firm i and $u_j(\boldsymbol{q}) \coloneqq \sum_{i\in\mathcal{F}} \beta_{ji}\pi_i(\boldsymbol{q})$ is j's total portfolio profit) and for every firm i that has entered let $\gamma'_{ji} = \widetilde{\gamma}/m$ be the control each investor $j \in J_0$ has over firm i for some $\widetilde{\gamma} \in [0,1]$, and $\gamma'_{ji} = (1-\widetilde{\gamma})/m$ the control each entrepreneur $j \in J_i$ has over her firm i.[68] After entry, it is easy to see that the manager of each firm i maximizes

$$\pi_i(\boldsymbol{q}) + \lambda \sum_{k\neq i} \pi_k(\boldsymbol{q}), \quad \text{where } \lambda = \frac{1}{1 + (\widetilde{\gamma}^{-1} - 1)(\sigma^{-1} - 1)} \in [0,1].$$

Here λ is increasing in the common owners' level of holdings σ and control $\widetilde{\gamma}$. Under proportional control $\sigma = \widetilde{\gamma}$, and $\lambda = \left[1 + (\sigma^{-1} - 1)^2\right]^{-1}$.

B.10.2 The entry stage

Each firm only looks at its own profit to decide whether to enter as there is no overlapping ownership when it does so.[69] $\boldsymbol{q}_n$ is a free entry equilibrium production profile if and only if

$$\Pi(n,\lambda) \geq f > \Pi(n+1,\lambda)$$

as in Mankiw and Whinston (1986). If overlapping ownership develops only after firms enter, it affects the incentives of firms to enter only through its effect on product market outcomes. We assume that there exists n such that $\Pi(n,\lambda) < f$ for any λ.

[67] This relies on the fact that a firm's entrepreneurs only hold shares of their firm both before and after entry. Common ownership develops after entry not through a firm's entrepreneurs investments in other firms but because outside investors invest in multiple firms.

[68] For every other pair of entrepreneur j and firm i, $\beta'_{ji} = \gamma'_{ji} = 0$.

[69] Formally, if a firm does not enter, its payoff is 0; if it does, it is $(1 + \lambda(n-1))(\Pi(n,\lambda) - f)$. Thus, it is optimal for an n-th firm to enter if and only if $\Pi(n,\lambda) \geq f$.

B.10.3 Existence and uniqueness of equilibrium

Proposition 8 studies the existence and uniqueness of a free entry equilibrium.

Proposition 8. $\Pi(n,\lambda)$ is decreasing in n and a unique free entry equilibrium exists.

Proof of Proposition 8 Given that $\Pi(n,\lambda)$ is decreasing in n by Proposition 2, the result follows given that $\Pi(n,\lambda) < f$ for n large. **Q.E.D.**

In equilibrium, firms enter until profits have fallen so much that if an additional firm enters, gross profit will no longer cover the entry cost. $\widehat{n}^*(\lambda)$ is uniquely pinned down by $\Pi(\widehat{n}^*(\lambda),\lambda) = f$ and $n^*(\lambda) = \max\{n \in \mathbb{N} : \Pi(n,\lambda) \geq f\} = \lfloor \widehat{n}^*(\lambda) \rfloor$.

B.10.4 Overlapping ownership effects

Proposition 9 studies the effects of overlapping ownership.

Proposition 9. Ignore the integer constraint on n (so that entry is given by $\widehat{n}^*(\lambda)$). Then

(i) the number of firms entering is increasing in λ,

(ii) individual quantity, total quantity, and total surplus are decreasing in λ,

(iii) if $C'' \geq 0$, then the MHHI is increasing in λ.

Proof of Proposition 9 Given $\Pi(\widehat{n}^*(\lambda),\lambda) = f$, the Implicit Function Theorem gives

$$\frac{d\widehat{n}^*(\lambda)}{d\lambda} = \frac{(n-1)(H_n^{-1}-1)}{1 + H_n + \Lambda_n^{-1}\left[(1-\lambda)(1-H_n) - C''(q_n)/P'(Q_n)\right] - H_n E_{P'}(Q_n)} > 0,$$

where the inequality follows from what we have seen in section A.2.

(ii) The total derivative of the total quantity is then proportional to

$$\begin{aligned}\frac{dQ_{\widehat{n}^*(\lambda)}}{d\lambda} &\propto \frac{\partial Q_n}{\partial \lambda}\frac{\Lambda_n(1+\lambda) + 1 - \lambda - C''(q_n)/P'(Q_n) - \Lambda_n^2 E_{P'}(Q_n)/n}{(n-1)(n-\Lambda_n)} + \frac{\partial Q_n}{\partial n} \\ &= \frac{Q_n\begin{bmatrix} -\left(\Lambda_n(1+\lambda) + \Delta - \Lambda_n^2 E_{P'}(Q_n)/n\right) \\ +(n-\Lambda_n)\Delta/n\end{bmatrix}}{(n-\Lambda_n)\left(n + \Lambda_n - C''(q_n)/P'(Q_n) - \Lambda_n E_{P'}(Q_n)\right)} = -\frac{\Lambda_n Q_n}{n(n-\Lambda_n)} < 0,\end{aligned}$$

so total quantity decreases with λ, and thus so does individual quantity since the number of firms increases with λ. The total derivative of the total surplus is

$$\begin{aligned}\frac{d\,\mathrm{TS}_{\widehat{n}^*(\lambda)}}{d\lambda} &= P(Q_n)\frac{dQ_{\widehat{n}^*(\lambda)}}{d\lambda} - \frac{d\widehat{n}^*(\lambda)}{d\lambda}C(q_n) - nC'(q_n)\left(\frac{dQ_{\widehat{n}^*(\lambda)}/d\lambda}{n} - \frac{q_n}{n}\frac{d\widehat{n}^*(\lambda)}{d\lambda}\right) - \frac{d\widehat{n}^*(\lambda)}{d\lambda}f \\ &= \frac{dQ_{\widehat{n}^*(\lambda)}}{d\lambda}\left(P(Q_n) - C'(q_n)\right) - \left(P(Q_n) - C'(q_n)\right)q_n\frac{d\widehat{n}^*(\lambda)}{d\lambda} < 0.\end{aligned}$$

where the second equality follows from $\Pi(\widehat{n}^*(\lambda),\lambda) = f$.

(iii) Last, the total derivative of $\text{MHHI}^* = H_{n^*}$ is

$$\frac{d\,\text{MHHI}\left(\boldsymbol{q}_{\widehat{n}^*(\lambda)}\right)}{d\lambda} = \frac{n-1}{n} + \left(\frac{\lambda}{n} - \frac{\Lambda_n}{n^2}\right)\frac{d\widehat{n}^*(\lambda)}{d\lambda} \propto \frac{n-1}{n}\left(\frac{d\widehat{n}^*(\lambda)}{d\lambda}\right)^{-1} + \frac{\lambda n - \Lambda_n}{n^2}$$
$$\propto \frac{\left[1 + \lambda + \Delta(Q_n,(n-1)q_n)/n + \left(\frac{1}{n} - \frac{1}{\Lambda_n}\right)\frac{C''(q_n)}{P'(Q_n)}\right]/H_n - E_{P'}(Q_n)}{n(n-\Lambda_n)} > 0,$$

where the inequality is implied by $C'' \geq 0$ combined with the maintained assumption (ii) that requires $E_{P'}(Q_n) < (1 + \lambda + \Delta(Q_n,(n-1)q_n)/n)/H_n$. **Q.E.D.**

Remark B.4. There exists a set of thresholds $\mathcal{L} := \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$, $\lambda_1 < \lambda_2 < \cdots < \lambda_k$, such that

(a) for every $\lambda \in \mathcal{L}$, $\Pi(n^*(\lambda),\lambda) = f$, and $n^*(\lambda) = \widehat{n}^*(\lambda)$,

(b) for λ between two consecutive thresholds $n^*(\lambda)$ remains constant and everything behaves as in the Cournot game with a fixed number of firms.

When we take into account the integer constraint, the number of firms is a step function of λ, and individual quantity decreases with jumps down. The total quantity has a decreasing trend with jumps up (resp. down) for the values of λ at which an extra firm enters under $\Delta > 0$ (resp. $\Delta < 0$). Also, total surplus tends to decrease with λ.[70]

Importantly, even when there is free entry of firms—so that increases in λ lead to the entry of new firms as incumbents suppress their quantities, if the entering firms develop overlapping ownership after entering (up to the level the incumbents have), consumer and total surplus tend to decrease with λ, as in the symmetric case with a fixed number of firms. Also, if one looks at HHI, it will seem as if competition rises as λ increases, which can even be the case with MHHI, although the latter will increase with λ if we slightly strengthen our assumptions. Last, for appropriate levels of λ a small increase in λ can spur the entry of an extra firm causing the total quantity to rise.

The fact that the price increases with λ is to be expected. Remember that an increase in λ is met with an increase in n so that the zero profit condition $\Pi(\widehat{n}^*(\lambda),\lambda) = f$ is satisfied. When the Cournot market is quasi-anticompetitive ($\Delta < 0$), both the increase in λ and the increase in n cause the price to increase. When the Cournot market is

[70]To compare total surplus under the integer constraint on n, $\text{TS}_{n^*(\lambda)}$, to its value when we ignore the integer constraint, $\text{TS}_{\widehat{n}^*(\lambda)}$, notice the following. For λ between two consecutive thresholds, $\lambda \in (\lambda_k,\lambda_{k+1})$, it holds that $\widehat{n}^*(\lambda) > n^*(\lambda)$. Thus, given that total surplus is single-peaked in n, if there is (weakly) excessive entry under the integer constraint, ignoring the integer constraint exacerbates excess entry. Therefore, between two λ thresholds $\text{TS}_{\widehat{n}^*(\lambda)} < \text{TS}_{n^*(\lambda)}$, and for λ equal to a thresholds $\text{TS}_{n^*(\lambda)}$ has a jump down. But if under the integer constraint entry is insufficient by 1 firm (which is possible), $n^*(\lambda) = n^o(\lambda) - 1$, then the above does not follow.

quasi-competitive ($\Delta > 0$), the increase in λ tends to increase the price, while the increase in n tends to decrease it. The former effect dominates. For example, assume non-increasing MC and by contradiction that after an increase in λ enough additional firms enter the market to keep the price at its level before the increase in λ (or even make it lower). Then, after the increase in λ (i) each firm has a lower share of the market, (ii) the price has not increased, and (iii) the average (variable) cost of production has not decreased (due to non-increasing MC and individual quantity has decreased). Thus, individual profit has decreased, violating the zero profit condition. The result still holds under increasing MC, since under $\Delta > 0$,

$$\left| \frac{\overbrace{\partial \Pi\left(n,\lambda\right)/\partial \lambda}^{+}}{\underbrace{\partial \Pi\left(n,\lambda\right)/\partial n}_{-}} \right| = \left| \frac{\overbrace{\left(1-H_n\right)\frac{\partial Q_n}{\partial \lambda}}^{-}}{\underbrace{\left(1-H_n\right)\frac{\partial Q_n}{\partial n}}_{+} + \underbrace{H_n \frac{Q_n}{n}}_{+}} \right| < \left| \frac{\partial Q_n/\partial \lambda}{\partial Q_n/\partial n} \right| = \left| \frac{\overbrace{dP\left(Q_n\right)/d\lambda}^{+}}{\underbrace{dP\left(Q_n\right)/dn}_{-}} \right|.$$

This means that for individual profit to stay unchanged after an increase in λ, fewer firms need to enter compared to the number of firms that need to enter for the price to remain unchanged after the increase in λ.

The mechanism behind the effect of λ on entry is akin to the impact of collusion on entry in the dynamic stochastic oligopoly model of Fershtman and Pakes (2000), where firms freely enter, set prices, and invest in quality. In their model, for example, a potential entrant only looks at its profit to decide whether to enter foreseeing the possibility of future collusion with an incumbent monopolist. This possibility increases entry incentives (*i.e.* it increases the threshold of quality that the incumbent needs to achieve to deter entry) compared to the equilibrium without collusion. This in turn causes the incumbent monopolist to invest more in quality when future collusion is possible. Overall, the collusive equilibrium features on average higher prices but also more entry and higher qualities and consumer surplus.

B.10.5 Entry cost effect on entry

Proposition 10 studies the effect of the entry cost on entry, as well as how this effect depends on the extent of overlapping ownership. It mirrors Proposition 6 with the role of internalized profit $\Psi(n,\lambda)$ now assumed by profit $\Pi(n,\lambda)$.

Proposition 10. Ignore the integer constraint on n (so that entry is given by $\widehat{n}^*(\lambda)$). Then

(i) entry is decreasing in the entry cost,

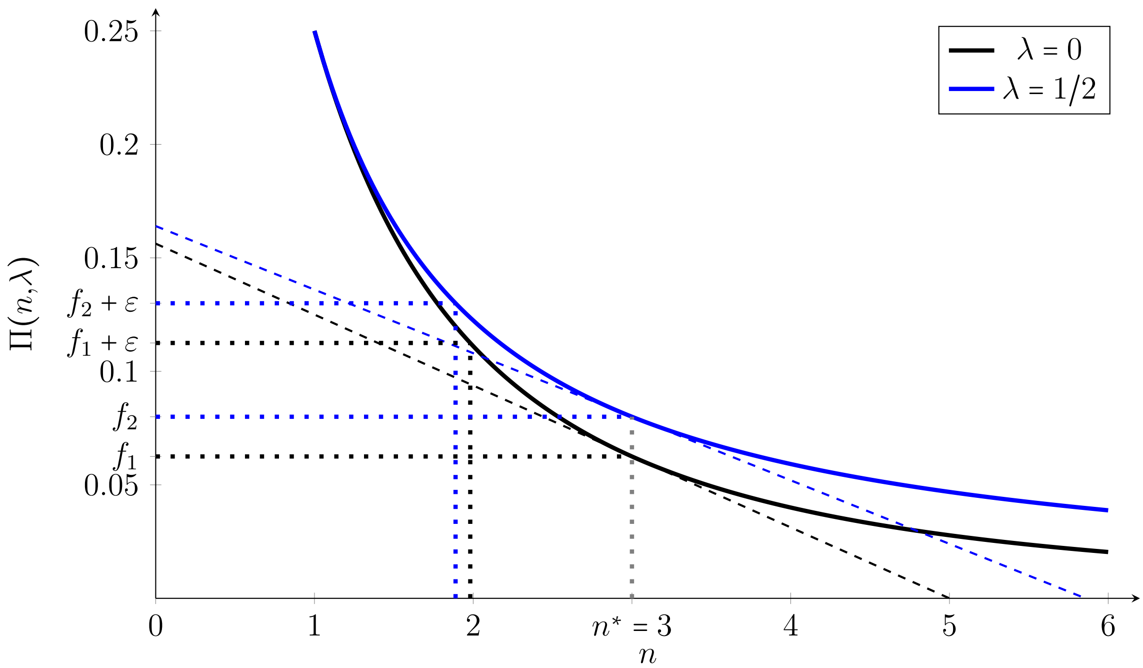
(ii) if λ increases and other parameters x (e.g., demand, cost) change infinitesimally so that $\widehat{n}^*(\lambda)$ stays fixed and $\partial^2\Pi(n,\lambda)/(\partial x \partial n) = 0$ (e.g., (f,λ) changes in direction $\mathbf{v} \coloneqq (-(d\widehat{n}^*(\lambda)/d\lambda)/(d\widehat{n}^*(\lambda)/df),1))$, then $|d\widehat{n}^*(\lambda)/df|$ changes in direction given by $\text{sgn}\{\partial^2\Pi(n,\lambda)/(\partial\lambda\partial n)|_{n=\widehat{n}^*(\lambda)}\}$.

Proof of Proposition 10 We have that $d\widehat{n}^*(\lambda)/df = \left(\partial\Pi\left(n,\lambda\right)/\partial n\right)^{-1}\Big|_{n=\widehat{n}^*(\lambda)}$, and part (ii) follows if we take the directional derivative of $d\widehat{n}^*(\lambda)/df$. **Q.E.D.**

As long as individual profit is decreasing in n, the results of Proposition 10 are not specific to Cournot competition. Part (ii) says that if an increase in λ makes individual profit in the pricing stage equilibrium more (resp. less) strongly decreasing in the number of firms, then an increase in the entry cost needs to be met with a smaller (resp. larger) increase in the number of firms for the zero profit entry condition to continue to hold.

Figure 10 explains the reasoning behind this result. There are initially $n^* = 3$ firms in equilibrium, which can be a result of $\lambda = 0$ and $f = f_1$, or $\lambda = 1/2$ and $f = f_2 > f_1$. Also, for $n \leq 3$, an increase of λ from 0 to 1/2 makes profit less strongly decreasing in n (*i.e.*, $\partial^2\Pi\left(n,\lambda\right)/(\partial\lambda\partial n) > 0$). Thus, an increase in the entry cost by ε will decrease entry by more when $\lambda = 1/2$ (and initially $f = f_2$) compared to when $\lambda = 0$ (and initially $f = f_1$).

Figure 10: Entry cost effect on entry mediated by λ under linear demand and constant MC



Note: $a = 2$, $b = 1$, $c = 1$. The black and blue solid lines represent $\Pi(n,0)$ and $\Pi(n,1/2)$, respectively. The black and blue dashed lines are tangent to the corresponding solid lines at $n = n^*$.

Claim 3 provides sufficient conditions for the cross derivative of $\Pi(n,\lambda)$ to be negative (resp. positive), which by Proposition 10 implies that overlapping ownership alleviates (resp. exacerbates) the negative effect of the entry cost on entry.

Claim 3. Assume constant MC.

(i) If $\partial E_{P'}(Q)/\partial Q \geq 0$, $E_{P'}(Q_n) \in [0{,}1]$ and $n \geq 5 + E_{P'}(Q_n)$, then $\partial^2 \Pi(n,\lambda)/(\partial\lambda\partial n) < 0$ for every $\lambda \in (0{,}1)$.

(ii) If $\partial E_{P'}(Q)/\partial Q \leq 0$, $E_{P'}(Q_n) \leq 0$ and $n \leq 6/(2 - E_{P'}(Q_n))$, then $\partial^2 \Pi(n,\lambda)/(\partial\lambda\partial n) > 0$ for every $\lambda \in (0{,}1)$.

Proof of Claim 3 We have $\partial \Pi(n,\lambda)/\partial n = P'(Q_n) q_n^2 \left[\frac{\partial Q_n}{\partial n}\frac{n}{Q_n}(1 - H_n) + H_n\right] < 0$, so

$$\frac{\partial^2 \Pi(n,\lambda)}{\partial n \partial \lambda} \propto \left\{ \begin{array}{r} \left[-(1 - E_{P'}(Q_n))\dfrac{\partial Q_n}{\partial n}\dfrac{n}{Q_n} - (2 - E_{P'}(Q_n))\dfrac{\Lambda_n}{n - \Lambda_n}\right]\dfrac{\partial Q_n}{\partial \lambda}\dfrac{1}{Q_n} \\ -\dfrac{\partial^2 Q_n}{\partial n \partial \lambda}\dfrac{n}{Q_n} - \dfrac{n-1}{n - \Lambda_n}\left(1 - \dfrac{\partial Q_n}{\partial n}\dfrac{n}{Q_n}\right) \end{array} \right\}.$$

Denote $E'_{P'}(Q) \equiv \partial E_{P'}(Q)/\partial Q$. Under constant marginal costs

$$\frac{\partial^2 Q_n}{\partial n \partial \lambda} = \frac{\left\{ \begin{array}{c} \left(-Q_n + (1-\lambda)\dfrac{\partial Q_n}{\partial \lambda}\right)(n + \Lambda - \Lambda E_{P'}(Q_n)) \\ -(1-\lambda) Q_n \left[n - 1 - (n-1) E_{P'}(Q_n) - \Lambda E'_{P'}(Q_n)\dfrac{\partial Q_n}{\partial \lambda}\right] \end{array} \right\}}{(n + \Lambda - \Lambda E_{P'}(Q_n))^2}\frac{1}{n}, \quad \text{so that}$$

$$\begin{aligned} \frac{\partial^2 \Pi(n,\lambda)}{\partial n \partial \lambda} &\propto \left\{ \begin{array}{c} 2\Lambda^2 (E_{P'}(Q_n))^2 + \left[n\Lambda(n - \Lambda - 1) - 2n^2 - \Lambda^2\right] E_{P'}(Q_n) \\ -n(n - \Lambda)(n + \Lambda - 6) - \dfrac{\Lambda(n-\Lambda)^2 Q_n E'_{P'}(Q_n)}{n + \Lambda - \Lambda E_{P'}(Q_n)} \end{array} \right\} \\ &< E_{P'}(Q_n)\left[2\Lambda^2 E_{P'}(Q_n) - n\Lambda - 2n^2 - \Lambda^2\right] \leq 0, \end{aligned}$$

where the first (resp. second) inequality follows from $n \geq 5 + E_{P'}(Q_n)$, $\lambda \in (0{,}1)$, $E_{P'}(Q_n) \leq 1$, $E'_{P'} \geq 0$ (resp. $0 \leq E_{P'}(Q_n) \leq 1$). Similarly follows part (ii). **Q.E.D.**

Claim 3 encompasses CESL demand. Therefore, under CESL demand with $E \in [0{,}1]$ and constant MC, in markets with not too low entry ($n \geq 6$ is sufficient), overlapping ownership makes entry less strongly decreasing in the entry cost. This means that as long as it does not induce firms to internalize the entry externality, overlapping ownership could alleviate the negative macroeconomic implications of rising entry costs documented by Gutiérrez, Jones and Philippon (2021) in the U.S. over the past 20 years. The sufficient condition of part (ii) requires $n \leq 3$, as is the case in Figure 10.

The conditions in part (i) of Claim 3 overlap with those of Numerical result 3, which deals with the case of pre-entry overlapping ownership. Thus, under the same parameterization, whether overlapping ownership exacerbates or alleviates the negative effect of the entry cost on entry will depend on the form of overlapping ownership. If overlapping ownership is present prior to entry thus making firms internalize the entry externality, then it exacerbates the effect. If it develops after entry, it alleviates the effect.

B.10.6 Equilibrium entry versus the socially optimal level of entry

The derivative of equilibrium total surplus with respect to n is given by

$$\begin{aligned}\frac{d\,\mathrm{TS}_n}{dn} &= P(Q_n)\left(n\frac{\partial q_n}{\partial n} + q_n\right) - C(q_n) - nC'(q_n)\frac{\partial q_n}{\partial n} - f \\ &= \Pi(n,\lambda) - f + n\left(P(Q_n) - C'(q_n)\right)\frac{\partial q_n}{\partial n},\end{aligned}$$

and therefore

$$\left.\frac{d\,\mathrm{TS}_n}{dn}\right|_{n=\widehat{n}^*(\lambda)} = \overbrace{\Pi\left(\widehat{n}^*(\lambda),\lambda\right) - f}^{=0} + n\left(P(Q_n) - C'(q_n)\right)\left.\frac{\partial q_n}{\partial n}\right|_{n=\widehat{n}^*(\lambda)} \propto \left.\frac{\partial q_n}{\partial n}\right|_{n=\widehat{n}^*(\lambda)},$$

so that with TS_n single-peaked in n, under business-stealing (resp. business-enhancing) competition entry is excessive (resp. insufficient). The results of Mankiw and Whinston (1986) and Amir, Castro and Koutsougeras (2014) generalize to the case of post-entry overlapping ownership. Proposition 11 shows that indeed with business-stealing competition and under the integer constraint, entry is never insufficient by more than one firm.

Proposition 11. The following statements hold:

(i) if $\Delta > 0$ and $E_{P'}(Q) < 2$ on L, then $n^*(\lambda) \geq n^o(\lambda) - 1$,

(ii) if $\Delta < 0$, then $n^*(\lambda) \geq n^o(\lambda) = 1$.

Proof of Proposition 11 Part (i): If $n^o(\lambda) \leq 2$, we are done since $n^*(\lambda) \geq 1$ given that monopoly profit is positive. For $n^o(\lambda) \geq 3$ keep in mind that $E_{P'}(Q) < 2$ on L implies that for every $n \in [2, +\infty)$, $E_{P'}(Q) < (1+\lambda)/H_n$ on L. The proof follows the proof of part (a) of Proposition 1 in Amir, Castro and Koutsougeras (ACK; 2014). By definition, $\mathrm{TS}_{n^o(\lambda)} \geq \mathrm{TS}_{n^o(\lambda)-1}$, which implies $\int_{Q_{n^o(\lambda)-1}}^{Q_{n^o(\lambda)}} P(X)dX - n^o(\lambda)C\left(q_{n^o(\lambda)}\right) + \left(n^o(\lambda)-1\right)C\left(q_{n^o(\lambda)-1}\right) \geq f$, which then gives $\Pi\left(n^o(\lambda)-1,\lambda\right) - f \geq P\left(Q_{n^o(\lambda)-1}\right)q_{n^o(\lambda)-1} - \int_{Q_{n^o(\lambda)-1}}^{Q_{n^o(\lambda)}} P(X)dX + n^o(\lambda)\left(C\left(q_{n^o(\lambda)}\right) - C\left(q_{n^o(\lambda)-1}\right)\right)$, which given $P' < 0$ and that in the Cournot game total quantity is increasing in n, implies

$$\begin{aligned}\Pi\left(n^o(\lambda)-1,\lambda\right) - f &> P\left(Q_{n^o(\lambda)-1}\right)\left(q_{n^o(\lambda)-1} + Q_{n^o(\lambda)-1} - Q_{n^o(\lambda)}\right) \\ &\quad + n^o(\lambda)\left(C\left(q_{n^o(\lambda)}\right) - C\left(q_{n^o(\lambda)-1}\right)\right) \implies \\ \Pi\left(n^o(\lambda)-1,\lambda\right) - f &> n^o(\lambda)\left(P\left(Q_{n^o(\lambda)-1}\right) - C'\left(\widetilde{q}\right)\right)\left(q_{n^o(\lambda)-1} - q_{n^o(\lambda)}\right),\end{aligned}$$

for some $\widetilde{q} \in \left[q_{n^o(\lambda)-1} - q_{n^o(\lambda)}\right]$, where the implication follows by the mean value theorem. As $R(Q_{-i})$ is non-decreasing in Q_{-i}, it follows as in the proof in ACK that there exists $\widetilde{Q}_{-i} \in \left[(n^o(\lambda)-2)q_{n^o(\lambda)-1}, (n^o(\lambda)-1)q_{n^o(\lambda)}\right]$ such that $\widetilde{q} \in r\left(\widetilde{Q}_{-i}\right)$ with $R\left(\widetilde{Q}_{-i}\right) \geq Q_{n^o(\lambda)-1}$ and $P\left(R(\widetilde{Q}_{-i})\right) \geq C'(\widetilde{q})$, so that $P\left(Q_{n^o(\lambda)-1}\right) \geq P\left(R(\widetilde{Q}_{-i})\right) \geq C'(\widetilde{q})$.

Given $E_{P'} < (1+\lambda)/H_n$, Proposition 2 implies that $q_{n^o(\lambda)-1} > q_{n^o(\lambda)}$, which combined with the above gives $\Pi\left(n^o(\lambda)-1,\lambda\right) - f \geq 0$. Also, by Proposition 2 $\Pi(n,\lambda)$ is decreasing in n, so it must be $n^*(\lambda) \geq n^o(\lambda) - 1$ for the entry condition to be satisfied.

Part (ii): Since $\Pi(1,\lambda) > f$, $n^*(\lambda) \geq 1$. Also, $\Delta < 0$ on L implies that $C''(q) < 0$ for every $q < \overline{Q}$. By Proposition 2 Q_n is decreasing in n, and thus, so is consumer surplus. Also, $n\Pi(n,\lambda) \equiv P(Q_n)Q_n - nC(q_n) < P(Q_n)Q_n - C(Q_n) \leq P(q_1)q_1 - C(q_1) = \Pi(1,\lambda)$, where the first inequality follows from $C'' < 0$. Thus, both consumer surplus and industry profits are maximized for $n = 1$, so $n^o(\lambda) = 1$. **Q.E.D.**

Remark B.5. Under a consumer surplus standard

(i) if $\Delta > 0$, then $n^o(\lambda) = \infty$ (since Q_n is increasing in n), so $n^*(\lambda) < n^o(\lambda)$,

(ii) if $\Delta < 0$, then $n^o(\lambda) = 1$ (since Q_n is decreasing in n), so $n^*(\lambda) \geq n^o(\lambda)$.

Under a consumer surplus standard, entry is insufficient (resp. excessive) when returns to scale are at most mildly increasing (resp. sufficiently increasing).

B.11 Free entry with pre-entry overlapping ownership: a more tractable framework

In this section, we make the free entry model with pre-entry overlapping ownership more tractable by ignoring the integer constraint on n. The way we do this is *not* just by letting (2) hold with equality. Instead, now each "infinitesimal" firm considers whether to enter or not examining a differential version of (3).[71] Consider firm i of "size" $\varepsilon > 0$ and let $n \in \mathbb{R}_+$ be the number of other firms entering. Firm i's payoff if it enters is $(\varepsilon + \lambda n)\left(\Pi(n+\varepsilon,\lambda) - f\right)$, while if it does not, it is $\lambda n\left(\Pi(n,\lambda) - f\right)$. The difference is

$$\varepsilon\Pi(n+\varepsilon,\lambda) + \lambda n\left[\Pi(n+\varepsilon,\lambda) - \Pi(n,\lambda)\right] - \varepsilon f.$$

Notice that for $\varepsilon = 1$ we recover the case with an integer number of firms. Dividing this expression by ε and letting $\varepsilon \to 0$ gives

$$\Pi(n,\lambda) + \lambda n \frac{\partial \Pi(n,\lambda)}{\partial n} - f.$$

Therefore, $\boldsymbol{q}_n$ is a free entry equilibrium if

$$\underbrace{\Pi(n,\lambda)}_{\substack{\text{own profit}\\ \text{from entry}}} + \underbrace{\lambda n \frac{\partial \Pi(n,\lambda)}{\partial n}}_{\substack{\text{entry externality}\\ \text{on other firms}}} = \underbrace{f}_{\substack{\text{entry}\\ \text{cost}}} \qquad \text{and} \tag{6}$$

[71] Of course, the firm is infinitesimal only for the purpose of the algebra. The firm understands the (marginal) effect of its entry on market outcomes, and in the pricing stage firms still compete à la Cournot but with the symmetric equilibrium solution extended to $n \in \mathbb{R}_{++}$.

$$(1+\lambda)\frac{\partial\Pi(n,\lambda)}{\partial n}+\lambda n\frac{\partial^2\Pi(n,\lambda)}{(\partial n)^2}<0. \tag{7}$$

Naturally, we only consider the free entry equilibrium and planner's solution with $n \in \mathbb{R}_+$; we denote the number of firms in the two solutions by $n^*(\lambda)$ and $n^o(\lambda)$, respectively. The entry externality is now measured by $n\partial\Pi(n,\lambda)/\partial n$. (6) says that the marginal firm entering is exactly indifferent between entering or not. (7) guarantees that an extra infinitesimal firm does not want to enter, and given that $\partial\Pi(n,\lambda)/\partial n<0$, can equivalently be written as

$$1+\lambda-\lambda E_{\partial\Pi/\partial n,n}(n,\lambda)>0, \qquad \text{where} \quad E_{\partial\Pi/\partial n,n}(n,\lambda):=-\frac{\frac{\partial^2\Pi(n,\lambda)}{(\partial n)^2}}{\frac{\partial\Pi(n,\lambda)}{\partial n}}n$$

is the elasticity of the slope of individual profit with respect to n. Also, given that $\partial\Pi(n,\lambda)/\partial n<0$, $\lambda>0$ implies through (6) that the entering firms make positive net profits in equilibrium. For $\lambda=0$, (6) reduces to the standard zero profit condition.

Provided that (7) holds for every n, the (unique) equilibrium level of entry $n^*(\lambda)$ is pinned down by

$$\Pi\left(n^*(\lambda),\lambda\right)+\lambda n^*(\lambda)\left.\frac{\partial\Pi(n,\lambda)}{\partial n}\right|_{n=n^*(\lambda)}=f.$$

More tractable results, where discrete differences are replaced by differentials, analogous to those derived in the paper can be derived in this model. For example, Proposition 5 holds with ϕ exactly equal to 1.

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