WORKING PAPERS

N° 1554

July 2024

"Discounting"

Christian Gollier

Toulouse School of Economics

Discounting

Christian Gollier[1]

Toulouse School of Economics, University of Toulouse-Capitole

June 2024

Prepared for the *Handbook of the Economics of Climate Change*

Abstract

This document is a newcomer guide to the economic theory of discounting, with applications to climate change and sustainability. It borrows ingredients from public economics, decision theory, and asset pricing theory, without any prerequisites beyond microeconomics 101. Aiming at sustainability issues, I focus the analysis on the valuation of intergenerational impacts. The starting point is the Ramsey rule, now more than a century old: We discount the future because we are inequality-averse and because we are used to believe that future generations will be wealthier than us. From this trivial but fundamental insight, I explore the role of uncertainty, a key ingredient for any realistic representation of the distant future of humanity on this planet.

Keywords: discounting, asset price, carbon price, sustainability, climate change, deep uncertainty, Ramsey rule.

[1] This research has also been funded by the French Agence Nationale de la Recherche under grant ANR-17-EURE-0010 (Investissements d'Avenir program).

Introduction

Our collective failure to reduce our emissions of greenhouse gases over the last three decades is a clear illustration of the free-riding problem associated with the climate externality. But it also raises the issue of whether human beings are short-termists by ignoring the grave consequences of their behaviour on the welfare of their peers who will live on this planet in the future. Whether this statement is true or not depends upon what are our individual and collective responsibilities towards future generations, and upon how these responsibilities should be translated into our collective values system. I refer here to the valuation of our natural assets (biodiversity, renewable and non-renewables resources, ...), or of our emissions of persistent pollutants such as CO_2 and mercury. The estimation of the carbon value, which economists have been referring to as the social cost of carbon, has been a focal issue for climate economists since the beginning of this century, following the seminal work by Bill Nordhaus. At the core of this experts' debate lies the problems of cost-benefit analysis and discounting.

One of the most basic missions of economists is to evaluate actions from the point of view of a predefined objective, most often utility or welfare. For an act to be desirable, one should obviously ensure that the welfare benefit of this act exceeds its welfare cost. One key difficulty of this evaluation comes from the observation that most human, corporate and governmental acts entail benefits and costs that are scattered over a potentially long duration. Typically, an investment imposes an immediate cost and a flow of delayed benefits. Even if all benefits and costs can be valued under a one-dimensional monetary scheme, they remain different by nature, because we typically don't consider that one future dollar has the same value as an immediate dollar, even in the absence of inflation. In this cost-benefit analysis, most people penalize delayed dollars just because they cannot be spent immediately. They discount the future.

When one discounts a sure benefit of 100 materializing in one year at a constant discount rate of 5%, its present value should be 100/1.05= 95.24. With such a discount rate, a project that costs x today and that generates a sure benefit of 100 next year is socially desirable if and only if x is smaller than 95.24, i.e., its Net Present Value (NPV) -x+100/1.05 is positive. But if this benefit of 100 materializes only in a century, its present value should be reduced to $100/1.05^{100}$=0.76. With such a discount rate, one should not be ready to pay more than 0.76 to get 100 in one hundred years. This simple exercise illustrates the crucial role of discounting in the valuation of long-dated impacts. Even a small discount rate compresses the present values of distant impacts, sometimes up to the point where these impacts can be completely ignored in the traditional cost-benefit analyses used by economists to evaluate public and private investments and policies. This is why economists are often at odd with other scientists and ecologists on various sustainability issues. Remember that until the end of 2023, the official discount rate used in the United States to evaluate public policies was set at the incredibly high real rate of 7% (with an alternative rate at 3% for sensitive analysis, whatever that means). This rate has been reduced since then to 2% (OMB, 2023). In this survey, I try to answer the question of where this rate should be set, and on what morale basis.

This survey has three parts. I first determine the efficient discount rate in the artificial context without any uncertainty. In the second part of the survey, I reevaluate this analysis in the context of risk and uncertainty. Finally, in Part III, I discuss different implementation of the theory of discounting in different countries, and for its application to climate change.

Part I: Discounting under certainty

I.1 Why do we discount the future?

In the absence of any uncertainty, economists have identified three reasons for why benefits materializing in the more distant future should be more heavily penalized in their valuation process: impatience, temporal arbitrage, and inequality aversion.

The celebrated "marshmallow test" in psychology is an impressive illustration of human beings' preferences for immediate rewards compared to delayed ones. In short, people are impatient. Since Samuelson (1937), economists have recognized this trait of human behaviour by describing individual intertemporal preferences as a discounted sum of the flow of temporal utility. Thus, an increase in future utility has a lower impact on welfare today than the same increase in current utility. It is often assumed that this rate of impatience, or "rate of pure preference for the present", is constant over time, yielding an exponential discount factor.[2] But climate economists have extended the model from an intertemporal framework to an intergenerational one. By doing this, they have transformed individual impatience, a perfectly reasonable human trait, into collective racism towards future generations, a morally unacceptable attitude. It is ethically

[2] Exponential discounting is important for the intertemporal consistency of the decision process. The alternative "hyperbolic discounting" framework generates a time-consistency problem: A dynamic strategy considered as optimal today may become suboptimal with the passing of time, even in the absence of any new information. The anticipation of switching strategy in the future makes the immediate action incoherent with future choices (Laibson, 1997). For example, with hyperbolic discounting, it may be intertemporally optimal to smoke one last cigarette today if one stop smoking tomorrow. But as tomorrow becomes today, the same argument can be applied to smoke one "last" cigarette again. This consumption is intertemporally incoherent.

unacceptable to measure intergenerational welfare with a lower weight for more distant generations. How could one punish people for being born later? In this survey, I will ignore this unethical "agism" by setting the rate of pure preference for the present to zero. The intergenerational welfare will be measured by the unweighted sum of the flow of generational utility.[3] This is supported by a large family of public economists, as documented in Broome (1992) and Gollier (2013).

A second argument to discount the future is based on the opportunity cost of capital. Keep in mind here that the discount rate of a project can be interpreted as the minimum rate of return required to invest in that project. Keep also in mind that an action aimed at improving the future should not be evaluated in a vacuum. There exists a myriad of other possible actions to make the world better in the future: beyond the reduction of emissions of CO_2 which will reduce climate damages in the future, one could also reduce public debt, or make various investments that will improve our ability to create valuable goods in the future. If there exists a scalable investment in the economy that generates a sure payoff of 4 dollars in 35 years from now for every dollar invested today,[4] this means that one can secure a sure rate of return of 4% per year for safe investments with a 35-year maturity. Therefore, it would make no sense to invest in any safe project with a 35-year maturity yielding a rate of return smaller than 4%. This is particularly true if funding this alternative project must be made by divesting from the scalable one, as such a substitution would destroy future value for zero benefit today. The simple way to observe the rate of return of the best safe investment opportunity in the economy is to look at the interest rate. If financial markets are efficient, the interest rate is also the cost of capital of risk-free corporations, which are therefore incentivized to invest in their safe investments up to the point where their marginal rate of return equals this cost of capital. In short, this paragraph justifies using the interest rate as the rate at which risk-free benefits and costs should be discounted. The new official public discount rate of 2% used in the United States relies on this argument (OMB, 2023).

The third argument to discount the future is based on both our collective beliefs in long-term economic growth and inequality aversion, as described in the next section.

---

[3] Mathematical economists have often criticized this approach by the unbounded nature of this welfare function in any infinite horizon model. But we know that the lifetime of Earth is limited by the one of the Sun, which is finite.

[4] Something growing at 4% has an accumulated value of $\exp(0.04t)$ or 1.04^t depending upon whether one uses a continuous-time or a discrete-time approach. I prefer the continuous-time approach, yielding $4 \simeq \exp(0.04 \times 35)$.

I.2 A model-free argument for discounting

When examining an action that yields a sacrifice for someone and a benefit for someone else, as is the case for fighting climate change, there is a clear conflict of interest between them, yielding a social dilemma. This is why many classical philosophers such as John Locke, Jean-Jacques Rousseau, Immanuel Kant and John Rawls have recommended using the concept of the veil of ignorance to determine an ethically reasonable attitude to this kind of social dilemma. Let us put people under a veil of ignorance about who will be on the sacrifice side and who will be on the benefit side of the action. This creates a sense of impartiality when determining whether this action should be performed or not.

Let us apply this resolution of social dilemma to the following example, inspired by Okun (1975). Croesus consumes twice as many goods and services as John. One contemplates a policy aimed at increasing the aggregate prosperity of the community. The problem is that the policy is detrimental to the poor John, and beneficial for the wealthy Croesus. More specifically, the policy generates $x>1$ additional dollars for Croesus for each dollar lost by John. Clearly, the policy makes the community more prosperous, but at the same time more unequal. It is also clear that one would reject such a policy if $x=1$, as it would not even raise GDP in this case. This rejection defines the concept of inequality aversion. Okun was not only interested whether one is inequality-averse. He was also interested in measuring the intensity of our inequality aversion. In this framework (which reverses Okun's "leaky bucket" experiment where income is transferred from the wealthy to the poor), the respondents to this experiment under the veil of ignorance are requested to elicit the minimum value of x that would make them willing to accept the policy. Inspired by estimations of the degree of inequality aversion existing in the literature (see for example Gollier (2013)), suppose that we come to a consensus that, to be socially acceptable under the veil of ignorance, the minimum benefit accruing to Croesus for each dollar lost by John should be $x=4$ dollars. This is a measure of our collective degree of inequality aversion.

How is this analysis linked to discounting? Consider an economy which grows at 2% per year. This means that consumption will double every 35 years. This is typically what happened in the western world over the last century. Now, contemplate an investment that would generate a single sure benefit of x dollars in 35 years for each dollar invested today. Under the veil of ignorance, what should be the minimum benefit x in the future to compensate for the dollar lost today? You certainly recognize here the analogy with the issue examined in the previous paragraph, as one asks a current John to sacrifice something for the benefit of a future Croesus who will consume twice as much as the present John. Under the consensus of x=4, I can claim that the value today of generating 4 dollars in 35 years is one dollar. In other words, the discounted value of 4 dollars in 35 years should be 1 dollar. As seen in the previous section, this corresponds to a discount rate of 4%. This is the discount rate that should be used in an economy growing at a sure annual rate of 2%.

Observe that this morale argument supporting a discount rate of 4% if the economy is certain to grow at 2% per year is model-free. It just relies on the impartiality concept of the veil of ignorance and on a degree of inequality aversion which is experimentally estimated. We discount the future because future generations will be wealthier than us, so that investing for the future makes our society more intergenerationally unequal. The socially desirable level of the discount rate should be interpreted as the minimum rate of return of safe projects that compensates for the fact that implementing them would raise intergenerational inequalities.

I.3 Discounted expected utility and the Ramsey rule

Since Ramsey (1928), economists have argued that discounting is rational in a growing economy under (intertemporal) inequality aversion. In fact, Ramsey provided a model to support the intuition presented in the previous section. I hereafter describe this model and its implicit and explicit moral foundations. Observe first that the veil of ignorance substitutes a context of (intertemporal) inequality into a context of risk. Without knowing the generation to which one will belong, the unequal levels of consumption between generations make one's situation uncertain. This raises the deep question of how to evaluate welfare under uncertainty, a fundamental issue examined by decision theorists over the last three centuries (Gollier, 2001). Although this classical decision theory often fails to pass the test of observed human behaviours, the Expected Utility (EU) theory has a strong normative appeal associated with its underlying "independence axiom". This axiom states that if one prefers outcome X to outcome Y under certainty, one also prefers a lottery that gives X with probability p over lottery that gives Y with probability p, independent of the alternative Z that materializes with probability $1-p$ in these two lotteries. This axiom forces the additivity of the welfare functional with respect to the states of nature.

Under EU, if John and Croesus consume respectively C_0 and C_t, then under the veil of ignorance, before knowing who will be Croesus, they will both enjoy a welfare equalling

$$V_0 = \frac{1}{2}U(C_0) + \frac{1}{2}U(C_t),$$

where $U(C)$ is the utility level attained by an individual consuming C. I assume that U is increasing and concave. The concavity of U, a key property usually referred to as decreasing marginal utility, is synonymous to risk/inequality aversion. Indeed, if C_t is larger than C_0, a marginal transfer of consumption from t to 0 reduces risk/inequality and increases welfare V_0 if and only if U is concave. This is the definition of risk/inequality aversion. Economists typically use the family of power utility functions $U(C)=C^{1-\gamma}/(1-\gamma)$, in which $\gamma>0$ is an index of relative risk/inequality aversion.

Observe that under the veil of ignorance, the indifference towards a policy that generates 4 dollars to Croesus consuming C_t for each dollar extracted from John consuming C_0 means that

$$U'(C_0) = 4U'(C_t). \tag{1}$$

Under the power specification, if Croesus consumes twice as much as John, this means that $1=4\times 2^{-\gamma}$, or $\gamma=2$. Thus, the thought experiment presented in the previous section is compatible with this EU model if we calibrate it with a degree of relative inequality aversion of 2.

In this model, it is interesting to measure the marginal rate of substitution between consumption in the two states/dates/individuals 0 and t. Let

$$PV = \frac{\partial V_0}{\partial C_t} / \frac{\partial V_0}{\partial C_0} = \frac{U'(C_t)}{U'(C_0)} \tag{2}$$

denote the willingness to sacrifice consumption in 0 to increase consumption in t by 1, i.e., the present value of a sure unit benefit materializing at t. Replacing $U'(C)$ by $C^{-\gamma}$ and C_t by C_0e^{gt} where g is the growth rate of consumption over time, we can replace equation (2) by

$$PV = \exp(-\gamma g t). \tag{3}$$

Since the present value of 1 in t years is related to the discount rate through equation $PV = \exp(-r_f t)$, this means that the efficient discount rate is

$$r_f = \gamma g. \tag{4}$$

This is the Ramsey rule: The socially desirable discount rate is equal to the product of the degree of inequality aversion γ by the growth rate of consumption *g*. We recognize here the two ingredients that I used in the previous section to build a normative argument to discount the future. If $\gamma=2$ and $g=2\%$, we get a discount rate of 4%. Notice that in a degrowth economy, the inequality aversion argument goes in reverse, yielding a negative discount rate. In the economic literature, the Ramsey rule is more generally written as $r_f=\delta+\gamma g$, where δ is the rate of pure preference for the present which corresponds to the rate at which the flow of utility is discounted. As explained earlier, it should be set to zero in its intergenerational implementation.

I.4 Extensions

Thus, intertemporal inequalities are critically important to justify discounting, which in turn plays a critical role in the practice of sustainable policy evaluation. It is therefore a puzzle that most policy evaluations are performed with a very limit integration of *intra*-generational inequalities. This puzzle can be explained by the limited understanding by most decision-makers about the role of inequalities to justify discounting. As we discount the prosperous future, we should also impose lower weights to value benefits accruing to wealthier people within the same period, in the spirit of my original Croesus and John story: one dollar of benefit accruing to Croesus should be valued as four dollars accruing to John. Coherence requires to use the same intensity of inequality aversion in proceeding to the selection of the discount rate and the intragenerational weights for benefit-cost analysis.

In this simple model, I implicitly assumed that people consume a single good, say rice or potatoes. Hopefully, the real world is of course more sophisticated. This is because one believes that more rice will be available in the future that one kilogram of rice in the future is valued less than one kilogram of rice today. But consider for example the value of the service of an ecosystem that is expected to shrink in the future. In that context, one should value more one future unit of this service in the future than in the present. In order to discount this time-transfer of units of specific goods and services, one could use Ramsey rule (4) where *g* would be the growth rate of consumption of the specific good under scrutiny, which could be positive or negative. The idea that each good or service in the economy could have its own discount rate was first developed by Malinvaud (1953), and then extended by Guesnerie (2004), Hoel and Sterner (2007), Sterner and Persson (2008), and Gollier (2010) in the case of uncertainty.

However, the growth rate of the consumption of a specific good or service is not the only ingredient to determine its specific discount rate. We may well have less of the service of an ecosystem in the future, but this service could be substituted by something else so that the future value of the service does not grow over time. For example, if one expects that a specific future benefit will see its societal valuation grows at a specific rate *x*, for example because of the anticipation of an increase in its relative scarcity with limited substitutability, then one could use a corrected rate r_f-x to discount this future benefit, in exchange for ignoring its intrinsic value growth. A recent paper by Drupp et al. (2024) illustrates this idea in the case of ecosystem services. Potentially because of economic growth or because of pollution or excessive extraction of natural resources associated with these services, the relative scarcity of some of these services are expected to grow, yielding an increase in their societal value. A key ingredient of this argument is the income-elasticity of the demand for these services, which is associated to their degree of substitutability.

Part II: Discounting under uncertainty

II.1 Stochastic discount factor

The Ramsey rule has a clear and transparent normative foundation. Under certainty, it offers a simple framework to evaluate sustainability policies. Things are more complex in the case of uncertainty. We need to recognize two sources of uncertainty, the one surrounding future consumption C_t and the one surrounding the future benefit B_t of the project under scrutiny. Under EU and the veil of ignorance about both the state of nature and the generation to which everyone will belong, the present value PV of this future uncertain benefit must be such that

$$\frac{1}{2}u(C_0+PV)+\frac{1}{2}Eu(C_t)=\frac{1}{2}u(C_0)+\frac{1}{2}Eu(C_t+B_t), \tag{5}$$

so that one would be indifferent to get PV immediately or lottery B_t in the future. At the margin, that is for a marginal project, this condition is equivalent to

$$PV=E\left[e^{-\rho_t t}B_t\right] \text{ with } e^{-\rho_t t}=m_t=\frac{u'(C_t)}{u'(C_0)}, \tag{6}$$

where $\rho_t=\gamma g_t$ is the stochastic discount rate, m_t is the stochastic discount factor (SDF), and

$$g_t=\frac{1}{t}\ln\left(\frac{C_t}{C_0}\right) \tag{7}$$

is the stochastic annualized growth rate of consumption between 0 and t. Equation (6) is ubiquitous in the modern asset pricing literature, with m_t denoting the stochastic discount factor, the price kernel or the Arrow-Debreu price. Under uncertainty, pricing a future payoff consists in using the Ramsey rule state by state to determine the present state-specific value of the future benefit. The value today of the future lottery B_t is just equal to the expectation of this stochastic present value. As soon as C_t is uncertain, the discount factor m_t and the discount rate ρ_t are stochastic, i.e., contingent to the state. But the message is exactly the same as in the first part of this paper, except for the fact that the underlying valuation procedure needs to be implemented state by state.

Let me illustrate this approach with a simple example. Assume that γ equals 2 and that there are only possible states of nature. The growth rate is constant but unknown. In the normal state, the growth rate is g_n=2.33%, whereas in the degrowth state, the growth rate is g_d=-1%. The probability of degrowth is p=10%, so that the expected growth rate is 2%. In this context, let us value a benefit that takes value B_{tn} and B_{td} respectively in states n and d. The SDF approach consists in discounting each of these two contingent benefits at the Ramsey rates ρ_t of 4% and -2% respectively. The two contingent present values PV_n and PV_d are then weighted by their respective probability to obtain the present value of the project. In Table 1, I calculated the present value of various projects that differ on their maturity and risk profile by using formula (6).

Consider first a project (project 1) that generates for sure 1000 dollars next year. We see in Table 1 that its present value of this project equals 961 dollars. This corresponds to discounting the future benefit at a risk-free rate of 3.98%. For a maturity of 100 years (project 2), this sure future benefit of 1000 dollars has a contingent PV of 9 dollars in the normal state and of 7389 dollars in the degrowth state, yielding a present value of 747 dollars. This is equivalent to using a risk-free discount rate equals 0.29%. Interestingly enough, these risk-free discount rates are decreasing with maturity and are smaller than the Ramsey rate of 4% that would have been obtained by using the expected growth of 2% in the Ramsey rule. We will see why in the next section.

	Project 1	Project 2	Project 3	Project 4
t	1	100	100	100
B_{tn}	1000	1000	500	1100
B_{td}	1000	1000	5500	100
PV_n	954	9	5	10
PV_d	1020	7389	40640	739
PV	961	747	4068	83
R_t	3.98%	0.29%	-1.40%	2.49%

Table 1: Present value PV and risk-adjusted discount rates R_t for four different projects that differ on their maturity t and on their contingent payoffs (B_{tn},B_{tc}). They all have an expected future value of \$1000. We assume that there is a normal scenario with a consumption growth rate of 2.33% forever and one degrowth scenario with a consumption growth rate of -1% forever. The probability of degrowth is 10%.

In projects 3 and 4, the future benefit is contingent to the state, but its expected future value is 1000 dollars as in the first two projects. In Project 3, the future benefit is much larger in the bad state of nature than in the good one, so that it provides some insurance against the macroeconomic catastrophe described by the degrowth state. Because of its huge PV contingent to the bad state, the present value of project 3 (4068 dollars) is much larger than its expected future benefit (1000 dollars). If one defines the risk-adjusted discount rate R_t as the rate that should be used to discount the future expected benefit to get PV, this risk-adjusted discount rate is here equal to -1.40%. Finally, Project 4 exhibits a positive correlation between its future benefit and future consumption. The value of Project 4 today is a small 83 dollars, which is compatible with a risk-adjusted discount rate of 2.49%. In the next section, I provide an economic logic to these large discrepancies in present values and discount rates.

These 4 projects demonstrate that, although the value today of a future benefit is the expectation of its stochastic present value, which suggests risk-neutrality, the investor's risk attitude plays a critical role in its valuation. They also illustrate the simplicity of the generalization of the intuitive Ramsey rule and methodology to the case of uncertainty.

II.2 Risk-adjusted discount rates: Prudence and risk-aversion

In the asset-pricing literature and in the practice of finance and capital budgeting, it is a tradition to measure the present value of an uncertain future benefit as the expectation of that future benefit discounted at a risk-adjusted rate R_t. This has already been used in the example described in Table 1. This risk-adjustment can be obtained by rewriting formula (6) as follows:

$$PV = e^{-R_t t} E\left[B_t\right] \text{ with } R_t = \frac{-1}{t}\ln\left(\frac{E\left[B_t e^{-\rho_t t}\right]}{E\left[B_t\right]}\right). \tag{8}$$

This approach with a risk-adjusted discount rate R_t that is specific to the project seems to be more complex than the stochastic discount factor (SDF) approach. In particular, it perverts the intuitive simplicity of the Ramsey rule that prevails in the SDF approach. But as we will see in the next section, there exists a special case in which the risk-adjusted discount rate approach is in fact much easier than the SDF one.

From formula (8), the rate R_{ft} to be used in this uncertainty context to evaluate any risk-free benefit equals

$$R_{ft} = \frac{-1}{t}\ln\left(E\left[e^{-\rho_t t}\right]\right). \tag{9}$$

It is easy to check that, for any time horizon $t>0$, this risk-free rate R_{ft} is smaller than the expected contingent Ramsey rate $R_{f0} = E\left[\rho_t\right]$. When t tends to zero, the risk-free discount rate tends to R_{f0}, so that for small maturities, the uncertainty surrounding the risk-free discount rate is unaffected by the uncertainty surrounding consumption growth. On the contrary, for large maturities, R_{ft} tends to the smallest possible discount rate among the different contingent ρ_t. These results are general as they do not depend upon the distribution of the contingent discount/growth rates. They are described in Weitzman (1998, 2001).

The property that the uncertainty surrounding future consumption reduces the rate at which risk-free benefits should be discounted relies on the notions of prudence and precautionary savings. From formula (6), the present value of a unit benefit materializing in t years equals $Eu'(C_t)/u'(C_0)$. This is larger than $u'(EC_t)/u'(C_0)$, i.e. uncertainty reduces the risk-free discount rate, if and only if marginal utility is a convex function of consumption (Leland (1968), Drèze and Modigliani (1972)), a property called prudence (Kimball, 1990). Notice that all concave power utility functions have a convex marginal utility. Prudence is supported by the observation that an increase in the uncertainty surrounding someone's future incomes raises her optimal (precautionary) saving, as stated by John M. Keynes. The reduction of the risk-free discount rate is the asset-pricing version of this idea: The uncertainty surrounding the future should induce us to invest more for that future. This is done by reducing the risk-free discount rate. This explains why the discount rates of projects 1 and 2 in Table 1 are less than 4%. This concludes our discussion about how the macroeconomic uncertainty affect the risk-free discount rate.

Let me now turn to the issue of how the uncertainty surrounding the net benefit should affect the rate at which its expected value should be discounted. Formulas (6) and (8) clearly show that the riskiness of the future benefit affects its present value only if it is statistically linked to the uncertainty surrounding the stochastic discount rate ρ_t, thus to the uncertainty surrounding future aggregate consumption C_t. Said differently, if B_t is uncertain but $B_t|C_t$ is independent of C_t, then its present value is equal to its expected future value discounted at the risk-free rate R_{ft}. Risk that are not statistically linked to the aggregate risk should not be priced. This is due to the property of the EU model that risk is a second-order effect (Segal and Spivak, 1990). In other words, when B_t and C_t are independent, the risk premium of kB_t is proportional to k^2 in the small.

Suppose alternatively that C_t and B_t covary positively, which implies that ρ_t and B_t covary positively too, under risk aversion. This implies that

$$R_t = \frac{-1}{t}\ln\left(\frac{E\left[B_t e^{-\rho_t t}\right]}{E\left[B_t\right]}\right) \geq \frac{-1}{t}\ln\left(\frac{E\left[B_t\right]E\left[e^{-\rho_t t}\right]}{E\left[B_t\right]}\right) = R_{ft}. \tag{10}$$

Under risk aversion, the risk-adjusted discount rate R_t should be larger than the risk-free rate whenever C_t and B_t covary positively. This condition defines a project that raises the aggregate risk borne by the representative agent. This undesirable feature justifies penalizing this project by discounting the expected benefit at a risk-adjusted rate R_t which is larger than the risk-free rate R_{ft}. If C_t and B_t covary negatively, we get that the associated project specific discount rate R_t should be smaller than the risk-free rate R_{ft}. In practice, because aggregate consumption is itself generated by a portfolio of different investments (in human and physical capital), it happens that most projects covary positively with C_t, as in Project 4 in Table 1, so that the risk adjustment of the project-specific discount rate is in general positive. But counterexamples are easy to find, such as gold, military defence, intensive care units, and most investments in self-insurance (earthquake-resistant building norms, adaptation to climate change, …). Project 3 in Table 1 shares this insurance feature. It explains why its risk-adjusted discount rate (-1.40%) is smaller than the risk-free discount rate (0.29%) associated with maturity *t=100* years.

II.2 Consumption-based Capital Asset Pricing Model (CCAPM)

Under the risk-adjusted discount rate approach summarized in formula (8), the present value of future benefit B_t is equal to its expected future value discounted at a rate R_t which is specific to its statistical link with g_t. This sensitivity of the discount rate to the risk profile of the benefit is a key insight of this asset pricing model. In order to specify this relationship, it is useful to calibrate the above model in the special case where the income-elasticity of benefit B_t is a constant β, so that $B_t = \xi_t C_t^{\beta}$ with $\beta \in \mathbb{R}$ and ξ_t is an independent random variable. A positive (negative) β signals a project that increases (decreases) the aggregate risk. Under that specification, we obtain that

$$R_t = -\frac{1}{t}\ln\left(\frac{E\left[e^{(\beta-\gamma)g_t t}\right]}{E\left[e^{\beta g_t t}\right]}\right). \qquad (11)$$

Observe that this discount rate is a function of the project-specific β. Let's further assume that g_t is normally distributed with mean μ_t and volatility σ_t. In this framework, using the SDF approach would be difficult given the infinite number of states of nature. But using Stein's Lemma,[5] equation (11) is implies that

$$R_t = -\left(\left((\beta-\gamma)\mu_t + 0.5(\beta-\gamma)^2 t\sigma_t^2\right) - \left(\beta\mu_t + 0.5\beta^2 t\sigma_t^2\right)\right) = R_{ft} + \beta\pi_t \qquad (12)$$

with

[5] It states that if x is normally distributed with mean μ and standard deviation σ, then $E\left[\exp(kx)\right]$ equals $\exp(k\mu + 0.5k^2\sigma^2)$ for all $k \in \mathbb{R}$.

$$R_{ft} = \gamma\mu_t - 0.5\gamma^2 t\sigma_t^2 \quad \text{and} \quad \pi_t = \gamma t\sigma_t^2. \tag{13}$$

Observe that we recover here the two basic properties of asset pricing: the risk-free rate is reduced by the uncertainty surrounding the future aggregate consumption, and the risk-adjustment of the project-specific discount rate has the same sign as the project's beta.

The calibration of the CCAPM pricing rules (13) requires information on the mean and the standard deviation of g_t, the annualized growth rate of consumption between 0 and t defined by equation (7). This calibration is particularly challenging in the case of maturities relevant for sustainability issues such as climate change. Christensen, Gillingham and Nordhaus (2018) used 13 probabilistic expert forecasts to perform this task for the growth of the global real per-capita GDP between 2010 and 2100. Their procedure to aggregate these 13 beliefs implies a normal distribution for g_{90} with mean $\mu_{90}=2.1\%$ and standard deviation $\sigma_{90}=1.1\%$. Plugging these parameters into (13) and using $\gamma=2$ as before, we get

$$R_{f90} = 2.4\% \quad \text{and} \quad \pi_{90} = 1.8\%. \tag{14}$$

Under this CCAPM framework, the evaluator with a project maturing in 90 years should be given these two tutelar discounting parameters to evaluate the risk-adjusted discount rate using the estimation of the project's beta. Notice that, by definition, the beta of a project is nothing else than the income-elasticity of its net benefit. Sectoral betas could be estimated by an OLS regression of the log added value of the sector over the log GDP/cap. For example, using US data between 1997 and 2018, Gollier, van der Ploeg and Zheng (2023) estimated a sectoral beta of the rail transportation sector of 2.27, and a sectoral beta for hospitals of -0.32. Any expected benefit materializing in 90 years should be discounted at a rate of 6.49% and 1.82% respectively for railways and for hospitals.

II.3 Classical CCAPM

We could reproduce the same procedure for other maturities in order to generate a term structure for the risk-free discount rate R_{ft} and the aggregate risk premium π_t. This requires determining the risk profile of C_t or g_t. The most standard calibration assumes that C_t follows a geometric Brownian motion with trend μ and volatility σ. It implies that g_t is normally distributed with mean $\mu_t=\mu$ and standard deviation $\sigma_t=\sigma/t$. In this special case, the CCAPM formula (13) simplifies to

$$R_{ft} = \gamma\mu - 0.5\gamma^2\sigma^2 \quad \text{and} \quad \pi_t = \gamma\sigma^2. \tag{15}$$

This continuous-time CCAPM formula has been pioneered by Rubinstein (1976), Lucas (1978) and Breeden (1979). An interesting feature of this version of the CCAPM is that it yields risk-free discount rates and aggregate risk premia that are independent of the time-horizon, i.e., it yields flat term structures of risk-adjusted discount rates and risk premia. This is due to the fact that the variance of $\ln(C_t / C_0)$

grows proportionally with the time horizon t in the case of a geometric Brownian process for consumption. More generally, when $\ln(C_{t+1}/C_t)$ is an i.i.d. process (not necessarily gaussian), the term structures of the risk-free discount rates and of the aggregate risk premia are flat. Since this pioneering standard CCAPM innovation of the 70s, the asset pricing literature has generalized the above formula to other stochastic processes for C_t, such as auto-regressive models allowing for business cycles, and more generally to all affine models. They all generate special cases of the CCAPM formula (13).

II.4 Asset pricing puzzles and their implications in the discounting debate

The strong ethical foundation of the CCAPM makes it an attractive benchmark for policy evaluation. However, the standard version of the CCAPM presented in the previous section failed to explain observed asset prices on financial markets. Indeed, over the last century or so, the annual growth rate of real GDP per capita in the western world had a mean around μ=2% and a volatility around σ=3%. Combining these parameters with γ=2 in the calibration of formula (15) yields

$$R_{ft} = 3.82\% \quad \text{and} \quad \pi_t = 0.18\%. \tag{16}$$

Observe first that the precautionary motive reduces the Ramsey discount rate of 4% to 3.82%. In other words, the macroeconomic uncertainty has a very small impact on our willingness to invest in risk-free projects. The risk-free rate puzzle (Weil, 1989) comes from the comparison between this risk-free discount rate and the real interest rate observed on markets in the past, which has been around 1% in the western world during the last century or so (Kocherlakota (1996), Jorda et al. (2019)). This huge discrepancy between the efficient discount rate and the observed interest rate is a puzzle because at equilibrium, the interest rate is the cost of capital of risk-free corporations. To maximize their profit, they will use it as a discount rate to evaluate their own risk-free investment projects. The fact that risk-free projects have been evaluated by financial market players at such a small rate of 1% means that they have been much more long-termist over the period than what would have been socially desirable. Why did our great grand-parents save and invest so much in such fast-growing economies for the benefit of future generations that happened to be so much wealthier than them anyway? The ethical approach yielding a discount rate for risk-free project at 3.82% clashes with the approach based on the opportunity cost of capital that yields a risk-free discount rate around 1%. Much of the discounting debate over the last 3 decades has been about which is right and which is wrong among these two approaches, the predicted rate by the model or the observed rate on the market.

The estimation (16) of an aggregate risk premium of $\pi_t = 0.18\%$ is also puzzling. It means that a claim on aggregate consumption ($\beta = 1$) should be penalized for the risk that it bears by adjusting the discount rate upward to 4% from the risk-free rate $R_{ft} = 3.82\%$. This penalty is surprisingly low. It happens that a portfolio of diversified equity has a beta around 3 (Bansal and Yaron, 2004), so that we should observe an expected excess return compared to the interest rate, which is referred to as the equity premium, around 0.54%. Kocherlakota (1996) and Jorda et al. (2019) estimated that the observed equity premium in the western world has been 6% on average. This means that financial markets suggest an aggregate risk premium of 2%, which is more than 10 times larger than its value of 0.18% predicted by the model. This large discrepancy is usually referred to in the literature as the equity premium puzzle. Contrary to my conclusion above for safe assets, markets seem to have been much more short-termist than socially desirable when contemplating risky investments.

II.5 Infrequent macro catastrophes

As shown by Barro (2006, 2009), the two asset pricing puzzles that prevail when the CCAPM is calibrated using a geometric Brownian process for consumption can be solved by adding to that process the possibility of infrequent macroeconomic catastrophes. In continuous time, this can be done by mixing a geometric Brownian process with a Poisson process. In discrete time, this can be done by assuming that each year, the annual change in log consumption is $N(\mu,\sigma^2)$ with probability $1-p$, which corresponds to business-as-usual, or is equal to $ln(1-k)$ with probability p, which corresponds to a catastrophe. Parameter k is the loss expressed as a share of aggregate consumption. Because the probability p of catastrophe is small, these events may not be in the data. Maybe, in spite of two world wars and a great depression, we have been lucky enough over the last century to have escaped the kind of catastrophes that could prevail in the future: civilizational crisis, climate change, outbreak of a new epidemic without a vaccine, and so on. These potential events are in our mind, but they are not in the data. This magnifies the macroeconomic risk. It reduces the risk-free rate because of the precautionary investment motive, and it raises the risk premium because of risk aversion. This has thus the potential to solve both the risk-free rate puzzle and the equity premium puzzle.

Because changes in log consumption are i.i.d., the term structure of risk-adjusted discount rates R_t are flat. Using equation (11), we obtain that

$$e^{-R_t} = \frac{(1-p)e^{(\beta-\gamma)\mu+0.5(\beta-\gamma)^2\sigma^2} + p(1-k)^{\beta-\gamma}}{(1-p)e^{\beta\mu+0.5\beta^2\sigma^2} + p(1-k)^{\beta}}. \tag{17}$$

To illustrate, let me calibrate this discounting system with *γ=2, μ=2%, σ=3%* as before, with the additional parameters of the catastrophe: *p=1%* and *k=50%*. With a 1% probability every year, consumption drops by 50%. The plausibility of this catastrophe reduces the expected growth rate of consumption to 1.33%. The risk-free rate also goes down to $R_{ft} = 0.71\%$, and the risk premium goes up to $\pi_t = 2.45\%$. We see that in spite of the low probability of catastrophes, their plausibility has a dramatic effect on the efficient discounting system. Basically, if one could agree on the realism of this calibration, the two asset-pricing puzzles are solved: equilibrium market returns are efficient and can be predicted by using the CCAPM incremented with infrequent macro catastrophes. Macroeconomic catastrophes are a fact of human history, and extreme events are much more frequent than what is assumed under the assumptions of the classical CCAPM with a Brownian motion. For these reasons, this classical calibration of the CCAPM should be definitely discarded to calibrate an efficient discounting system.

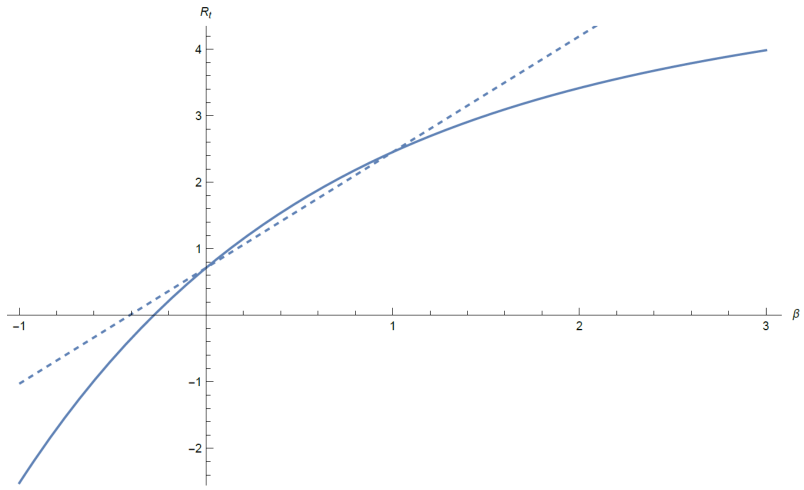

Figure 1: The risk-adjusted discount rate R_t based on the Barro model (17) as a function of the income elasticity β of the net benefit of the project, calibrated with *γ=2, μ=2%, σ=3%*, *p=1%* and *k=50%*. The dashed curve corresponds to the linear approximation using the risk-free rate and the risk-adjusted discount rate for β=1.

We have seen that when one calibrates pricing rule (11) with a normal distribution for the annualized growth rate g_t , the risk-adjusted discount rate R_t is a linear function of the income-elasticity β of the net benefit of the project. Equation (17) illustrates the fact that this linearity is specific to this gaussian framework. In Figure 1, I draw the non-linear relationship between R_t and β under the calibration described in the previous paragraph. Martin (2013) links the shape of this curve to the higher cumulants of the distribution of g_t. The concavity of $R_t(\beta)$ that emerges from this calibration is due to the negative skewness of the distribution of the annualized growth rate of consumption. Using the linear approximation based on the risk-free rate and the aggregate risk premium would have the effect of potentially vastly overestimating the risk-adjustment of the discount rate for betas larger than 1.

II.6 Parametric uncertainty

Shaping a stochastic process that fits our collective beliefs about our future prosperity for the decades and centuries to come is an extraordinary challenge on which this whole exercise of characterizing the term structures of risk-adjusted discount rates is based. This process is by nature ambiguous and deeply uncertain. In other words, our future risks are uncertain. How should this second layer of risk affect our estimation of short, medium and long term discount rates? I explored the problem of making the parameters of the growth process uncertain in Gollier (2016). Consider the simplest example in which the annual growth rate of consumption is binary and i.i.d. conditional to parameter p. Every year, with probability $1-p$, consumption will grow at 2%, and with probability p, it will go down by 1%. What is the effect on the macroeconomic risk of introducing some parametric uncertainty on the loss probability p? Because expected (marginal) utility is linear in probabilities, a white noise on p does not affect $Eu'(C_1)$, so it does not affect the efficient discount rates for a one-year maturity. Notice however that for longer maturities, this white noise on p affects the distribution of C_2. Compare for example two contexts, one in which p=0.5 with certainty, and one in which p is either 0.9 or 0.1 with equal probabilities. In the first context, the probability getting two consecutive positive (negative) growth years is 25%. If you do the math in the case with an uncertain p, you will obtain that the probability of this event is increased to 41%. It illustrates the fact that observing a low-growth outcome in the first period is bad news for C_1, but it is also bad news for the future because of the Bayesian update of the probability distribution, for the worse. This parametric uncertainty increases the fatness of the tails of the distribution of future log consumption. This point has been made first by Halevy and Feltkamp (2005). By raising the long-run macro uncertainty, it reduces the long-term risk-free discount rate under prudence and it raises the long-term aggregate risk premium under risk aversion.

The simplest illustration of this effect is obtained by assuming that conditional to μ, $ln(C_{t+1}/C_t)$ is i.i.d. $N(\mu,\sigma^2)$. Suppose also that our prior beliefs on μ are $N(\mu_0,\sigma_\mu^2)$. In that case, equation (11) can be solved by using Stein's Lemma twice to obtain

$$
\begin{aligned}
R_t &= -\frac{1}{t}\ln\left(\frac{E\left[e^{(\beta-\gamma)g_t t}\right]}{E\left[e^{\beta g_t t}\right]}\right) = -\frac{1}{t}\ln\left(\frac{E\left[e^{(\beta-\gamma)\mu t+0.5(\beta-\gamma)^2\sigma^2 t}\right]}{E\left[e^{\beta\mu t+0.5\beta^2\sigma^2 t}\right]}\right) \\
&= -\frac{1}{t}\ln\left(\frac{e^{(\beta-\gamma)\mu_0 t+0.5(\beta-\gamma)^2\sigma_\mu^2 t^2+0.5(\beta-\gamma)^2\sigma^2 t}}{e^{\beta\mu_0 t+0.5\beta^2\sigma_\mu^2 t^2+0.5\beta^2\sigma^2 t}}\right) \\
&= R_{ft} + \beta\pi_t
\end{aligned}
\tag{18}
$$

with

$$
R_{ft} = \gamma\mu_0 - 0.5\gamma^2(\sigma^2 + t\sigma_\mu^2) \quad \text{and} \quad \pi_t = \gamma(\sigma^2 + t\sigma_\mu^2). \tag{19}
$$

Observe that the risk-free discount rate and the aggregate risk premium are respectively linearly decreasing and linearly increasing with maturity t. Risk-adjusted discount rates are thus asymptotically unbounded. Observe also that parametric uncertainty does not affect short-term discount rates. These results are related to Weitzman (1998, 2001) who showed that parametric uncertainty leads to an asymptotic risk-free discount rate that is equal to the smallest plausible conditionally efficient discount rate. Here, using the Ramsey rule, the smallest plausible discount rate is obtained when assuming that μ is $-\infty$, in which case the conditionally efficient discount rate R_{ft} equals $-\infty$ too.

II.7 Other social welfare functions

Under the veil of ignorance and recognizing the merits of the independence axiom, there exists a strong scientific and morale basis for using EU as the intergenerational welfare function. But, following for example Olijslagers and van Wijnbergen (2024), let me consider two prominent extensions of the EU model for discounting: ambiguity aversion and Epstein-Zin preferences. Both preference functionals violate the independence axiom as they are non-linear in probabilities.

The Ellsberg paradox (Ellsberg, 1961) suggests that even in a static framework, many people value a lottery yielding \$100 with an unknown probability p that is equally likely to be either 0.9 or 0.1 less than a lottery that yields \$100 with a sure probability of 0.5. This "ambiguity aversion" has been formalized in the literature in different ways, but its most convincing version has been provided by Klibanoff, Marinacci and Mukerji (KMM, 2006). As in the previous section, suppose that the distribution of C_t is ambiguous in the sense that its distribution is parametrized by θ whose distribution F represents our prior beliefs on this second-order risk. The smooth ambiguity aversion model of KMM takes the following form:

$$V_0 = \frac{1}{2}U(C_0) + \frac{1}{2}\phi^{-1}\left(\int \phi\left(E\left[U(C_t)\middle|\theta\right]\right)dF(\theta)\right), \tag{20}$$

where the concavity of function $\phi(.)$ measures the degree of ambiguity-aversion. Gierlinger and Gollier (2011) characterized the conditions under which the ambiguity surrounding future consumption growth reduces the risk-free discount rate, as suggested by the intuition. This model violates the compounding of probabilities that is so fundamental to probability theory. Although real people may violate it in some circumstances, I find it hard for policy-makers to make the same trivial mistake to shape optimal policies.

The other alternative to EU initiated in the 70s is aimed at disentangling risk aversion from inequality aversion. Epstein and Zin (1989) offered a definitive representation of the so-called "recursive preferences". In the simpler case of two consumption dates, it simplifies to

$$V_0 = \frac{1}{2}U(C_0) + \frac{1}{2}U(\chi_t) \quad \text{with} \quad V(\chi_t) = EV(C_t). \tag{21}$$

Under this representation, the concavity of function U measures inequality aversion, whereas the concavity of function V measures risk aversion. When U and V are not identical, this model violates the impartiality principle of the veil of ignorance by treating differently the economic uncertainty surrounding C_t and the generational uncertainty surrounding the generation to which one will belong. The typical calibration of the two utility functions are such that $U(C) = C^{1-\rho}/(1-\rho)$ and $V(C) = C^{1-\gamma}/(1-\gamma)$, where ρ and γ are the indices of respectively relative inequality aversion and relative risk aversion. In the geometric Brownian case with trend μ and volatility σ, the CCAPM formula (15) must be generalized as follows in that case:

$$R_{ft} = \rho\mu - 0.5\left(\rho\gamma + \gamma - \rho\right)\sigma^2 \quad \text{and} \quad \pi_t = \gamma\sigma^2. \tag{22}$$

As anticipated, the aggregate risk premium is governed by risk aversion, whereas the Ramsey term in the risk-free rate is governed by inequality aversion, the precautionary term mixing the two ingredients. But again, under the impartiality principle implemented by the veil of ignorance, there is no reason to differentiate ρ from γ. Bansal and Yaron (2004) generalized this model by examining the case of mean-reverting processes for μ and σ.

II.8 Summary of reasonable calibrations of the efficient discounting system

To sum up, I described in this survey four science-based calibrations of the efficient risk-sensitive discounting system, as documented in Table 2. If one eliminates the classical CCAPM for the reasons mentioned above, we have a relatively convergent consensus for an aggregate risk premium around 2%. The only remaining issue is related to the choice of the risk-free discount rate. A risk-free rate between 1.00% and 2.50% seems to be reasonable.[6] For very long maturities, there is an argument relying on parametric uncertainties that supports using a smaller risk-free discount rate and a larger aggregate risk premium.

	Risk-free rate	Risk premium
Classical CCAPM	$R_{ft} = 3.82\%$	$\pi_t = 0.18\%$
CCAPM calibrated using Christensen et al. (2018)	$R_{f90} = 2.40\%$	$\pi_{90} = 1.80\%$
CCAPM with catastrophes à la Barro (2006, 2009)	$R_{ft} = 0.71\%$	$\pi_t = 2.45\%$
Financial markets	$R_{ft} = 1.00\%$	$\pi_t = 2.00\%$

Table 2: Three calibrations of the CCAPM discounting system (12), and their comparison with observed asset returns on financial markets. The "classical CCAPM" is greyed because of its absence of realism in using a geometric Brownian representation of the macroeconomic uncertainty.

III. Policy issues

III.1 Public policy evaluation in practice

Arrow and Lind (1970) have demonstrated that public investments should be discounted at the risk-free interested rate when their net benefits are uncorrelated to aggregate consumption. This paper was published a few years before the emergence of the modern asset pricing literature initiated by Rubinstein (1976) and Lucas (1978). Many experts from the early 70s to today took the Arrow-Lind theorem literally and ignored its restricted domain of applicability to zero CCAPM beta projects. This may explain why most countries continue to use a single discount rate for public policy evaluation. Gollier, van der Ploeg and Zheng (2023) document that 24% of respondents in a sample of around a thousand economic experts around the world continue to support using a single discount rate. Without any additional adjustment for the risk profile of each project, using a single discount rate generates a vastly

[6] Personally, I support a solution in which the short-term risk-free discount rate is indexed on the short-term return of the sovereign debt, or on the Ramsey rule using the prediction of the short-term growth rate of GDP/cap. This would have the additional benefit to make the public investment policy more Keynesian.

inefficient allocation of public capital, with an overinvestment in risky project, and an underinvestment of safer ones, in particular in projects that reduce aggregate risks.

The tradition to use a single discount rate for public project evaluation contributed to the many misunderstandings surrounding discounting (Lucas, 2014). In particular, it is not clear which single discount rate should be used. The unicity of the public discount rate is incompatible with the multiplicity of expected rates of return that prevails in the real world. Using the market interest rate, or its predicted level from consumption-based models characterized by the Ramsey rule, would underestimate the average efficient risk-adjusted discount rate because the average beta of public projects is positive and probably close to 1. Using this approach is likely to generate too many public projects passing the test of a positive net present value. Given the limitation of public funds, this will require using a (random) rationing scheme, a very inefficient allocation rule. Other countries, such as the U.S. until recently, preferred using the average cost of capital in the economy as the reference for their public discount rate.

In the United Kingdom, the discount rate is based on the Ramsey rule, with a calibration containing a rate of pure preference for the present of 1.5%, an inequality aversion of 1 and a growth rate of 2%, yielding a discount rate of 3.5% (Treasury, 2018). A smaller discount rate is used in the health sector to take account of the expected increase in the relative value of life. For maturities greater than 30 years, the guidance specifies a stepwise decreasing discount rate motivated by Weitzman (2001). The discount rate is 3.0% for years 31-75 and it falls to 1% for maturities larger than 300 years.

In the United States, the official discount rate has been 7% over the last three decades, with an alternative rate of 3% that should be used for sensitivity analysis (OMB, 2003). The Office of Management and Budget has recently imposed an extraordinary reduction of the official discount rate to 2% (OMB, 2023). This rate is based on the average real risk-free interest rate over the last 30 years. It should be revised every three years on this basis. The OMB recommends taking care of the riskiness of the project by discounting at 2% the flow of certainty equivalent net benefits. This strategy is based on the following version of equation (8):

$$PV = e^{-R_{ft}t} B_t^c \quad \text{with} \quad B_t^c = \frac{E\left[B_t e^{-\rho_t t}\right]}{E\left[e^{-\rho_t t}\right]}, \tag{23}$$

where $B_t^c \in \mathbb{R}$ is the certainty equivalent net benefit, i.e., at the margin, $Eu(C_t + B_t^c) = Eu(C_t + B_t)$. Observe that this OMB guidance is not in the culture of public and private investment capital budgeting and investment valuation. It is unrelated to the tradition of asset pricing. There is a considerable risk of incoherence between the choice of the risk-free discount rate $R_{ft} = E\left[e^{-\rho_t t}\right] = 2\%$, and the estimation of the certainty equivalent B_t^c based on the statistical relation between the net benefit B_t and the stochastic Ramsey discount rate ρ_t. I have shown that a risk-free rate of 2% is compatible with observed economic growth only if low-frequency catastrophe events are included in the probabilistic description of the future. If evaluators are not required to include them, it is likely that they will be confronted to the well-known equity premium puzzle, with certainty-equivalent net benefits almost equal to their expectations, an outcome incompatible with observed risk pricing in the real world. The jury is still out.

As a last example, let me consider the case of France. France is currently the only country in the world to impose a public discounting system based on the CCAPM formula $R_t = R_{ft} + \beta\pi_t$. For short maturities, the current tutelar risk-free rate is $R_{ft} = 1.2\%$ and the tutelar aggregate risk premium is $\pi_t = 2\%$ (Guesnerie, 2023). However, in practice, many evaluators find it difficult to estimate the beta of their projects, so that most of them use the average rate of R_t=3.2%. Guidelines to estimate the income-elasticity of projects' net benefits should be improved (Cherbonnier and Gollier, 2021).

III.2 Application to climate change

The debate about the efficient discounting system reemerged two decades ago in the context of climate change and the necessity to estimate the impact of our climate inaction. The Social Cost of Carbon (SCC) is defined as the present value of the flow of marginal climate damages generated by emitting one ton of CO_2 today. Because of the long maturity of this flow, the choice of the discount rate is crucial for the determination of the SCC. For example, EPA (2023) estimated the SCC for 2020 at \$120, \$190 and \$340 for a reference discount rate of 2.5%, 2% and 1.5%, respectively. In the early years of the development of climate economics, the debate of the discount rate to be used for the SCC was mostly focused on the Ramsey rule. It culminated in 2007 and the years after the publication of the Stern report (Stern, 2006). Nicholas Stern used the EU framework to calibrate an intergenerational welfare function under certainty with an index of relative inequality aversion of 1 and a growth rate of consumption of 1.3%. Stern also used a rate of pure preference for the present of 0.1%. Thus, from the Ramsey rule, he implicitly used a discount rate of 1.4%, yielding a SCC estimate that was one order of magnitude larger than previously estimated. Although his DICE model was run under certainty, Bill Nordhaus typically used a discount rate around 4-5%, on the basis of the opportunity cost of (risky) capital. More recently, as documented above, EPA mostly sided for a discount rate à la Stern, and a SCC around \$200 per tCO_2.

Relying on the Ramsey rule for the climate discount rate is fundamentally misleading. As claimed in this survey, using the Ramsey rule to estimate the SCC is correct only if the "climate beta", i.e., the income-elasticity of the marginal climate damage, is zero. But abating CO_2 emissions would have an insurance effect if this damage is negatively correlated with GDP, i.e., when the climate beta is negative. In that context, abating CO_2 would not only reduce the expected damage. It would also have a larger social benefit in the worst states of nature (low GDP), yielding a hedging value for the action. This insurance benefit should be recognized in that case by using a climate discount rate smaller than the risk-free discount rate. There are some reasons to believe in this negative climate beta story. Indeed, if the main source of aggregate risk is due to the climate damage function or to the climate sensitivity parameter, then a worse climate scenario yields at the same time a low future GDP (due to the large average climate damage), and a large benefit of the abating effort today (due to the large marginal climate damage). This negative climate beta would raise the SCC.

But there is another effect that goes in the opposite direction as it generates a positive climate beta. Suppose that the main source of aggregate risk comes from the unknown future productivity growth. In that context, there will be a positive statistical relationship between the future benefit of decarbonation and GDP. Indeed, a larger positive shock on productivity will generate at the same time a larger GDP and more emissions. Because the damage function is convex, this generates a larger marginal climate damage, and thus a larger future benefit to the mitigation effort today. This yields a positive climate beta, a larger climate discount rate, and a smaller SCC. In that context, fighting climate change raises the macroeconomic risk. Its valuation should be penalized.

Given the fact that uncertainty surrounds both the intensity of the climate problem and the productivity growth, the sign of the climate beta is ambiguous. Dietz, Gollier and Kessler (2018) were the first to address this question. They simulated the DICE model by randomizing various parameters of this integrated assessment model (IAM). They regressed the log marginal damage with respect to the log GDP/cap to estimate the climate beta assuming that DICE is a correct representation of the climate/economic dynamics of the world. They obtained a climate beta that is positive and close to 1. The main reason for this result is that DICE, as many other IAMs, is based on the hypothesis that climate damages are proportional to GDP. This is an obvious channel for a climate beta equalling 1. Notice also that much of the impacts of climate change are related to human life, health, morbidity and life expectancy. The income-elasticity of the value of statistical life being close to 1 (Hammitt and Robinson, 2011), so is the climate beta. However, one must be aware of the possibility for the climate beta to be much smaller than 1, in particular if one magnifies the uncertainty surrounding the climate damage function and the climate sensitivity parameter.

In their survey, Gollier, van der Ploeg and Zheng (2023) documented that most economists continue to believe that the rate at which climate damages should be discounted to measure the SCC should be close to the risk-free discount rate. This suggests a consensus for a zero climate beta. This is inconsistent with what is modelled in our IAMs. More research efforts should be made in the coming years to estimate the climate beta and to rationalize the climate discount rate in order to estimate the social cost of carbon.

IV. Conclusion

The reader of this survey may feel some disappointment at the end of this journey in the discounting kingdom. Already in 1968, William Baumol commented that "*few topics in our discipline rival the social rate of* discount *as a subject exhibiting simultaneously a very considerable degree of knowledge and a very substantial level of ignorance*" (Baumol, 1968). Although some progresses have been made in knowledge, such as a new scientific consensus for using risk-adjusted discount rates, the discipline is still struggling to determine the rate at which one should discount climate damages, ecological impacts, or the benefit generated by transportation, education, energy and health infrastructures. The recent decision of the OMB to continue to recommend a single public discount rate in the United States illustrates that lack of achievement perfectly. The consequences are dire, in particular for the misallocation of our portfolio of actions and efforts to improve the future and for making our world more sustainable.

Bibliographical references

Arrow, K.J., and R.C. Lind, (1970), Uncertainty and the evaluation of public investment decision, *American Economic Review*, 60, 364–378.

Bansal, R. and A. Yaron, (2004), Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Barro, R.J., (2006), Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics*, 121, 823–866.

Barro, R.J., (2009), Rare disasters, asset prices, and welfare costs, *American Economic Review*, 99, 243–264.

Baumol, W.J., (1968), On the social rate of discount, *American Economic Review* 58, 788–802.

Drupp, M.A., *et al., (2024),* Accounting for the increasing benefits from scarce ecosystems, *Science* 383, 1062-1064.

Breeden, D., (1979), An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–296.

Broome, J., (1992), *Counting the cost of global warming*, Cambridge: White House Press.

Cherbonnier, F., and C. Gollier, (2021), Risk-adjusted social discount rates, *The Energy Journal* 43, 19-41.

Christensen, P., K. Gillingham, and W. Nordhaus, (2018), Uncertainty in forecasts of long-run economic growth, *Proceedings of the National Academy of Sciences* 115, 5409-5414.

Dietz, S., C. Gollier, and L. Kessler, (2018), The climate beta, *Journal of Environmental Economics and Management* 87, 258-274.

Drèze, J.H., and F. Modigliani, (1972), Consumption decisions under uncertainty, *Journal of Economic Theory* 5, 308–335.

Ellsberg, D., (1961), Risk, ambiguity, and the savage axioms. Quarterly Journal of Economics 75, 643–669.

EPA, (2023), *Supplementary Material for the Regulatory Impact Analysis for the Final Rulemaking, "Standards of Performance for New, Reconstructed, and Modified Sources and Emissions Guidelines for Existing Sources: Oil and Natural Gas Sector Climate Review",* EPA Report on the Social Cost of Greenhouse Gases: Estimates Incorporating Recent Scientific Advances, U.S. Environmental Protection Agency.

Epstein, L. and S. Zin, (1989), Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Gierlinger, J., and C. Gollier, (2011), Socially efficient discounting under ambiguity aversion, TSE Discussion Paper.

Gollier, C., (2001), *The economics of risk and time*, MIT Press.

Gollier, C., (2010), Ecological Discounting, *Journal of Economic Theory* 145, 812-829.

Gollier, C., (2013), *Pricing the planet's future. The economics of discounting in an uncertain world*, Princeton University Press.

Gollier, C., (2016), Evaluation of long-dated assets : The role of parameter uncertainty, *Journal of Monetary Economics* 84, 66-83.

Gollier, C., R. van der Ploeg and J. Zheng, (2023), The discounting premium puzzle: Survey evidence from professional economists, *Journal of Environmental Economics and Management* 122.

Guesnerie, R., (2004), Calcul économique et développement durable, *Revue Economique* 55, 363–382.

Guesnerie, R., (2023), *Guide de l'évaluation socioéconomique des investissements publics*, France Stratégie.

Halevy, Y., and V. Feltkamp, (2005), A Bayesian approach to uncertainty aversion, *Review of Economic Studies* 72, 449–466.

Hammitt, J.K., and L.A. Robinson, (2011), The income elasticity of the value per statistical life: Transferring estimates between high and low income populations, *Journal of Benefit-Cost Analysis* 2, 1-27.

Hoel, M., and T. Sterner, (2007), Discounting and relative prices, *Climatic Change* 84 (2007) 265–280.

Jordà, O., K. Knoll, D. Kuvshinov, M. Schularick, A.M. Taylor, (2019), The rate of return on everything, 1870–2015, *The Quarterly Journal of Economics* 134, 1225–1298

Klibanoff, P., M. Marinacci, and S. Mukerji, (2005), A smooth model of decision making under ambiguity. Econometrica, 73(6), 1849{1892.

Kocherlakota, N. R. (1996), The equity premium: it's still a puzzle, *Journal of Economic Literature*, 34, 42–71.

Kimball, M.S., (1990), Precautionary saving in the small and in the large, *Econometrica* 58, 53-73.

Laibson D., (1997), Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics* 112(2), 443–477.

Leland, H.E., (1968), Saving and uncertainty: The precautionary demand for saving, *Quarterly Journal of Economics*, 465–473.

Lucas, R., (1978), Asset prices in an exchange economy, *Econometrica*, 46, 1429–46.

Lucas, D. (2014): Rebutting Arrow and Lind: why governments should use market rates for discounting, *Journal of Natural Resources Policy Research*, 6, 85–91.

Malinvaud, E., (1953), Capital accumulation and efficient allocation of resources, *Econometrica* 21 (2), 233–268.

Martin, I., (2013), Consumption-based asset pricing with higher cumulants, *Review of Economic Studies*, 80, 745-773.

Nordhaus, W., (2013), *The climate casino: risk, uncertainty, and economics for a warming world*, Yale University Press.

Okun, A.M., (1975), *Equality and Efficiency: The Big Tradeoff*, Brookings Institution, Washington DC.

OMB, (2003), Circular N. A-4 To the Heads of Executive Department Establishments, Subject: Regulatory Analysis, Tech. rep., Washington: Executive Office of the President.

OMB, (2023), *Circular No. A-4*, U.S. Office of Management and Budget, November 9, 2023.

Olijslagers, S., and S. van Wijnbergen, (2024), Discounting the future: On climate change, ambiguity aversion and Epstein–Zin preferences, *Environmental and Resource Economics* 87, 683–730.

Ramsey, F.P. , (1928), A mathematical theory of saving, *The Economic Journal* 38, 543-559.

Rubinstein, M., (1976), The valuation of uncertain income streams and the pricing of options, *The Bell Journal of Economics* 7(2), 407–425.

Samuelson, P.A., (1937), A note on the measurement of utility, *Review of Economic Studies* 4, 155-161.

Segal, U., and A. Spivak, (1990), First order versus second order risk aversion, *Journal of Economic Theory* 51, 111-125.

Stern, N., (2006), *The economics of climate change: The Stern Review*, Cambridge University Press.

Sterner, T., and M. Persson, (2008), An even Sterner report: Introducing relative prices into the discounting debate, *Review of Environmental Econonomics and Policy* 2 (1), 61-76.

Treasury, (2018), *Green book: Central government guidance on appraisal and evaluation*, Technical report, UK Treasury.

Weil, P., (1989): The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics*, 24, 401–421.

Weitzman, M. L., (1998), Why the far-distant future should be discounted at its lowest possible rate, *Journal of Environmental Economics and Management* 36 (3): 201–208.

Weitzman, M.L., (2001), Gamma discounting, *American Economic Review*, 91, 260-271.