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"Optimal Investment in Network Infrastructures"

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Optimal Investment in Network Infrastructures*

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Abstract

We analyze the optimal investment in a common infrastructure in a market with network externalities. Taking a dynamic mechanism design perspective, we contrast the level of investment and the associated payments across firms that a budget-constrained welfare-maximizing principal would set to those emerging in an unregulated market. We consider two market scenarios: first, a nascent market in which only one firm operates and an entrant may arrive at a later stage; second, a more mature market in which two firms already operate. In these settings, the principal needs to set access fees so as to provide enough incentives to invest in the infrastructure, while also avoiding wasteful investment. At the same time, the principal needs to coordinate investment and usage of the shared network given the various externalities that each firm exerts. We highlight the relative importance of these two aspects and how regulation can be designed so as to improve social welfare. We also highlight how the optimal timing of investment depends crucially on the regulator's coordination power.

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1 Introduction

In many industries such as telecom and payments, service providers need to operate on network infrastructures, whose development requires significant investments. These infrastructures are *non-rival*, the access to a given network may be shared with other providers. By investing in a possibly common infrastructure, each provider creates a positive externality on *current and future* users. Moreover, the benefits from using the network typically depend on the number of users using the same network; that is, we have (positive) *network externalities*. This raises coordination issues, on the timing of the investment and on the selection of the infrastructure on which to operate. Both dimensions are further complicated by the fact that new firms may arrive at a later stage and may benefit from existing infrastructures, which introduces a non-trivial dynamic component in determining how to manage the investment and the access to the infrastructure.

In this paper, we take a mechanism design approach to investigate the optimal investment in a network infrastructure, which we view as a problem of dynamic public-good provision. We consider as a running application the market for digital payments; our logic, however, can be extended to other industries in which both network externalities and investment in non-rival infrastructures are important.

The main ingredients of our model are the following. First, investment in infrastructures has a public good component. Second, all else equal, the payoff that each firm enjoys from using a given infrastructure increases in the number of other firms using the infrastructure. Third, firms' value from operating in the common infrastructure depends both on an idiosyncratic and on a common shock. The former captures the firm-specific valuation attached to the common infrastructure, which is private information. The latter describes some aggregate demand uncertainty, which can be resolved only if one firm invests and operates on the infrastructure. This creates an additional externality: the investment of one firm reveals some payoff relevant information to the other firms. Lastly, investment in the common infrastructure may occur over time, either because firms arrive sequentially in the market or because one firm may be induced to invest early so as to resolve the aggregate demand uncertainty.

We incorporate these ingredients in a simple two-periods model in which two firms can operate on two independent small-scale infrastructures or invest in a large-scale common infrastructure. While operating in the large-scale infrastructure is costlier, each firm may potentially derive extra benefits from it. These benefits depend on the firm's private valuation, on whether the other firm also joins the common infrastructure, and on the state of the aggregate demand. We are interested in comparing two different settings. In the first case, which can be viewed as a market at an early stage of development, a monopolist operates at small-scale in the first period and should decide whether to invest in the large infrastructure, knowing that a second firm would arrive in the second period and possibly join the common infrastructure (if it has been built). In the second setting, which can describe instead a more developed market, the two firms are already operating at small scale in the first period, and simultaneously decide whether to invest in a common infrastructure. In both cases, upon investment in the common infrastructure, firms can set access fees, possibly based on the reported private valuations.

Our main objective is to analyze how, in these two settings, unregulated contracting between the two service providers can lead to inefficient outcomes in terms of investment and usage of the common infrastructure. We then consider how a public authority may intervene and alleviate these inefficiencies. Specifically, we consider a principal who designs a mechanism to maximize social surplus subject to the budgetary constraints. We focus on two key roles that the principal may play. First, the principal can regulate access fees so as to provide enough incentives to invest in the infrastructure, while at the same time avoiding wasteful investment. As we show, this puts a limit on how much of the total surplus each firm can extract. Second, given the various externalities that each firm exerts on the other firm, the principal needs to coordinate investment and usage of the shared network. We highlight how one aspect or the other (or both) can be important and how regulation can be designed so as to improve social welfare.

Indirectly, our analysis also speaks to the optimal timing of common investment. We show how depending on our parameters, and specifically the cost of the investment and the uncertainties related to it, the public authority may push towards an early investment, when only one firm is on the market, or a late investment, when both the incumbent and the entrant operate. We also highlight how the optimal timing of investment depends crucially on the capacity (or lack thereof) of the regulator to induce both firms to operate on the common infrastructure; that is, on its coordination power.

More specifically, our first setting features sequential entry: firm 1 operates in period 1, while firm 2 arrives in period 2. In this case, the mechanism determines the probability of investment in period 1 only based on firm 1's type. In order to set a simple benchmark, we define our parameter values such that, in the first-best scenario with no budget balance constraints, it is optimal to invest in period 1 irrespective of firm 1's type, and to grant usage to both firms if and only if the aggregate demand turns out to be high and at least one firm has a high valuation.

Suppose that the market is not regulated and firm 1 designs a mechanism so as to maximize its expected revenues, with no private information and full commitment. We show that when the opportunity cost of operating in the large infrastructure is sufficiently large, firm 1 may invest but at the same time commit to exclude firm 2 if it reports low valuation. While socially wasteful, this threat increases the incentive for firm 2 to truthfully report its valuation and so the required payment to firm 1 in case of high valuation. In this way, firm 1 can extract all the surplus from firm 2. This mechanism leads to an inefficiently low usage of the infrastructure and it is reminiscent of monopoly pricing, in which a monopolist prefers to increase prices at the expenses of cutting quantity. In our case, moreover, wasteful investment in the large infrastructure is more likely to arise precisely when the cost of investment is large.

It is also immediate to notice that the inefficiency would not be resolved by simply giving all the bargaining power to firm 2. As we show, in this case, a classic hold-up problem arises: knowing that firm 2 is less likely to contribute in period

2, firm 1 has lower incentives to invest in the first place, leading to an inefficiently low level of investment. Under this perspective, the key role of the principal is to regulate access fees, providing sufficient incentives to invest, which limits the payments that firm 2 can expect, while at the same time avoiding wasteful investments, which instead puts a cap on the payments that firm 1 can expect. These trade-offs determine the optimal mechanism that a budget-constrained principal would design in order to maximize social welfare.

In the second part of our analysis, we consider a setting in which both firms are available in period 1, and so the mechanism can be designed based on the joint distribution of firms' valuations. Relative to the sequential entry case, this improved information may allow for more efficient outcomes. At the same time, however, the fact that both firms could invest in their own infrastructure opens the possibility of a game in which firms and consumers need to coordinate (or to bargain) on which infrastructure they operate on, which may be socially wasteful. The main role of a regulator here is to coordinate firms' decisions so as to induce an efficient equilibrium at the investment and usage stages.

We start by considering an unregulated market. In this case, the main source of inefficiency is again due to the fact that, in order to maximize its expected revenues, a firm with high valuation may commit not to invest in the common infrastructure if the other firm reports a low valuation. As in the previous analysis with sequential entry, this is socially wasteful but allows extracting further surplus from the other firm.

Suppose instead we introduce a principal who can elicit both firms' valuations and coordinate their joint investments. The mechanism can now induce more efficient outcomes than in the setting with sequential entry considered above. When both firms are available in period 1, the probability of inefficient investment is strictly lower, as the principal can condition on both firms' type and so avoid investing if both firms report low valuations. At the same time, the probability of efficient investment is strictly larger, as the risk that the investment cost cannot be recovered in period 2 is lower and so the budget constraint is less likely to bind. This highlights the potential efficiency gains of investing in a more mature market.

The efficiency gains of late investment come from the combination of two effects. First, the principal can elicit both firms' valuations and design the investment probabilities based on them. Second, the principal can coordinate the investment between the two firms and choose which firm needs to make the investment. In order to highlight the value of information, we consider a setting in which, as in the case of sequential entry, only one firm is able to incur the investment (for example for technological reasons), but at the same time the principal can still condition his recommendations based on both firms' reports. In this case, the outcome is equivalent in terms efficiency to the one in which both firms can invest. That is, the key efficiency gains induced by the principal arise from the possibility to communicate with both firms, rather than from the technological aspect as to which firm can invest.

In order to highlight the value of coordination, we consider a setting in which the market is regulated by a weaker principal, which we call "mediator." While having the same objective function as the principal, the mediator does not have the power to license firms' ability to operate on a given network. As the mediator cannot prevent deviating firms from using their own infrastructure, firms' outside option is now potentially more attractive. In case of deviation, the firm can not only operate on its own small-scale infrastructure, but also invest in a large infrastructure and try to induce the other firm to operate on it by optimally setting an access fee. This implies that the mediator can induce lower payments when firms participate in the common infrastructure, which makes the budget constraint more likely to bind. In turn, this reduces the probability of investment even when this would be socially efficient.

The resulting efficiency losses can be substantial. As we show, absent coordination power, the principal may derive little benefits from having more information. In fact, the welfare gains obtained in a setting with a mediator can be *lower* than those with sequential entry. That is, if the principal lacks coordination power, it may be more efficient to opt for an early investment, when only one firm is on the market, rather than waiting that both firms operate, since the efficiency gains from having more information would be more than outweighed from the efficiency losses from the severe coordination problems.

These results highlight that "regulatory capacity" is a key dimension to consider when assessing the optimal timing of investment in the common infrastructure. Interventions in more developed markets require enough regulatory capacity to mandate that all firms operate on the infrastructure; the benefits of late interventions are much lower, and can even turn negative, when the regulator cannot prevent firms from building and operating on their own separate network. These arguments resonate well with recent developments of digital payment infrastructures, in Brazil and Bangladesh for example. One of the key determinants of the massive expansion of the instant payment platform PIX in Brazil has been the central bank's ability to mandate that all the largest institutions participate in the scheme from its launch. Conversely, in Bangladesh, a public digital payment scheme has been introduced without securing the participation of all major providers, which has severely limited its success.

Broadly, our paper belongs to the mechanism design literature on public-goods, in which a principal (e.g., a government) typically plans to provide a non-rival and/or non-excludable good in case agents (e.g., residents) attach enough values to it. However, the agents' valuations are their private information and the principal is budget-concerned, so that the celebrated VCG mechanism (Vickrey (1961), Clarke (1971), and Groves (1973)) or its dynamic counterpart (Bergemann & Välimäki (2002)) are not available.[1] The novelty of our paper relative the existing work is in the combination of two key elements. First, we consider a network good, where each agent's payoff depends on how many other agents are available; second, we consider a dynamic environment where some agents may arrive late and

[1]For example, Myerson (1982) study a bilateral-trading model, which can be reinterpreted as a two-agent public-good problem. Their result shows a fundamental impossibility of the first-best welfare under fairly general conditions. They also characterize the second-best mechanism, which our dynamic analysis builds on. Mailath & Postlewaite (1990) and Hellwig (2003) study the public-good problem with many agents. With more agents, free-riding would become severer, and Mailath & Postlewaite (1990) show that, under certain conditions, little surplus can be enjoyed in such a setting. Hellwig (2003) shows that, under different conditions, some non-negligible surplus can still be enjoyed in the optimal mechanism with many agents. Moulin & Shenker (1992) and Moulin (1994) study different variations of the public-good problems (corresponding to different types of goods) and derive the properties of desirable mechanisms.

aggregate uncertainty can be resolved only if one agent invests. This combination makes the social decision of public goods provision unique and non-trivial.

In static environments, the most related work is Shichijo & Fukuda (2021) who study public goods with network externality. Even in a static setting, network externality introduces a coordination-game aspect to the problem, because each agent's incentive to participate in a mechanism and reporting truthfully may depend on her expectation about other agents' participation. In our dynamic environment, the problem is further complicated by the fact that some agents may only be available in later periods. This creates uncertainty on whether the current investment would create any value for the existing and future-arriving agents.

In a dynamic setting, Athey & Segal (2013) propose a version of an AGV mechanism (d'Aspremont & Gérard-Varet (1979)) which satisfy desirable incentive and budget properties. In our setting, the aggregate demand uncertainty and the potential unavailability of some agents are important, especially with the network externality of the public good: It is socially better to invest in a public good if aggregate demand is high enough and late-coming agents have high-enough valuations, but that can only be known later. Those elements make the possibility result of Athey & Segal (2013) inapplicable, and our second-best mechanism accommodates some inefficiency as a consequence of those frictions.

A second stream of related literature is the one studying public-good provision problems as a dynamic game among voluntary contributors (see, for example, Admati & Perry (1991), Marx & Matthews (2000), and Battaglini et al. (2014)). In these papers, the main frictions come from the specificity of the considered game forms.[2] Under these "frictional" game forms, they obtain inefficiency in the public good provision even under complete information and no late arrivals. In our case, without incomplete information and no late arrivals, the first-best efficiency would trivially be possible, because the principal can design the game form optimally. Rather, our focus is on the role of incomplete information and dynamic arrival of the agents in shaping the optimal mechanism and implying possible sources of

[2]Specifically, Admati & Perry (1991) consider an alternate-move game, Marx & Matthews (2000) consider a multi-stage simultaneous-move game, Battaglini et al. (2014) study the difference between irreversible and reversible investment environments.

inefficiency.

Finally, a substantial literature in IO and competition policy has investigated the effects of various regulatory interventions on firms' incentives to invest in network infrastructures (see e.g. Vogelsang (2003); Cambini & Jiang (2009); Briglauer et al. (2014) for comprehensive reviews). While this literature typically focuses on firms' strategic interactions and on how specific policies can reduce inefficiencies, we take a complementary view and consider a mechanism design problem. Our main focus is on the role of a budget-constrained principal (e.g. the government) which designs welfare-maximizing policies in a rather general space.

2 Basic ingredients

Let us first introduce the set of players and explain their payoffs. The timing and information structure of the game are explained later, as we consider different possibilities.

There exist two ex ante symmetric firms $i = 1, 2$ (also called, agents) interpreted as a potential service providers. Time is discrete, $t = 1, 2$, and no one discounts the future. Each firm may enter the market either at $t = 1$ or $t = 2$ (different timing structures are considered in the following sections).

Each firm can operate on its own network, which is incompatible across firms, and the payoff of this small-scale service provision is normalized to 0 throughout. Alternatively, firms can operate on a common and fully interoperable infrastructure (henceforth, just referred to as the infrastructure) and enjoy a possibly larger payoff by exploiting the network externalities. The degree of this additional payoff is heterogeneous across firms, and there is some common uncertainty regarding say the demand side of the service.

More formally, let v_i denote the benefit of operating on the common infrastructure (recall that we normalize the value of operating in small-scale to 0). This additional revenue is privately known by the firm and it also depends on the *demand state* $\Theta \in \{H, L\}$, and on the number of firms operating on the infrastructure. We assume that $v_i \in [0, 1]$ if $\Theta = H$ and both firms are operating on the infrastructure (*network externality*), while it is 0 otherwise (i.e., either if $\Theta = L$ or if only one firm operates on the infrastructure).[3]

Let θ denote the probability that $\Theta = H$. This demand state Θ is initially uncertain, although it is revealed publicly at the end of $t = 1$ if at least one firm operates on the infrastructure in $t = 1$. This can create a social benefit of investing early so as to learn the demand state.

At the same time, it is more costly for each firm to operate on the common infrastructure. We denote with $\gamma > 0$ the (exogenously given) additional cost each firm must bear in each period when operating on the infrastructure, relative to the small-scale one. We assume no fixed investment cost in the infrastructure (this is mainly for notational simplicity).

From these assumptions, it is not clear if it is socially desirable to invest in the common infrastructure. If $v_i < \gamma$ for both i, or if the demand state is known to be $\Theta = L$, then it is socially wasteful to operate on the infrastructure. If $v_i > \gamma$ for both i and $\Theta = H$, then it is socially valuable to do so. In the intermediate case where one has $v_i > \gamma$ but the other has $v_j < \gamma$ (and $\Theta = H$), then it depends on $v_1 + v_2$.

Finally, depending on the specifications, we assume that the market can be regulated by a public authority (also called, principal), that wishes to maximize social welfare. The authority can commit to a *mechanism*, comprising a decision rule of investment and a monetary transfer rule, as a function of the agents' reported values v_i (hence, appreciating their incentive compatibility constraints). We will contrast this situation with an unregulated market in which firms operate in a decentralized way. We will analyze the inefficiencies arising in the latter case, hence highlighting the possible scope for a regulatory authority.

[3]The fact that the state is revealed at the end of period 1 can be interpreted as requiring some time for consumers to learn about the service and so for their demand to be realized. This assumption will simplify some of our next analysis.

3 Emerging Markets: Regulating Prices

In this section, we assume that only firm 1 is available at $t = 1$; firm 2 is available only at $t = 2$. This can thought as a situation in which the market is at an early stage, with just one firm operating. We first solve the principal's problem of total surplus maximization; we then consider an unregulated market.

3.1 Timing and information structure

The timing and information structure of the game is as follows.

At $t = 1$, firm 1 reports its value type v_1 to the mechanism (recall that v_1 is the additional revenue of firm 1 if both firms operate on the common infrastructure and $\Theta = H$). Firm 2 is unavailable at this point, so the mechanism recommends firm 1's investment in the infrastructure with probability $q(v_1)$, only as the function of v_1 but not of v_2.

If firm 1 does not invest, then we proceed to the next period, $t = 2$. If it does, then firm 1 may operate on it already at $t = 1$, which would require paying the cost $\gamma > 0$ and it would bring no additional revenue. However, it would allow the society to learn the state Θ.

Now consider $t = 2$, where both firm 1 and 2 are available. If no investment has been made at $t = 1$, it is not optimal to invest in the infrastructure since Θ would be revealed at the end of $t = 2$, that is too late to enjoy network externalities. Also, even if firm 1 invested in and operated on the infrastructure, if that revealed that $\Theta = L$, then it would be socially optimal that no firm operates on it at $t = 2$.

Thus, only if firm 1 invested in and operated on the infrastructure, and if that revealed that $\Theta = H$, it may be socially valuable that both firms operate on it at $t = 2$ (and obviously, the network externality implies that it should be either both firms operating on it or neither).

In that last case, having observed what has happened, firm 2 reports its value type v_2 to the mechanism. Assume that firm 2 does not observe firm 1's report of v_1 at this point.[4] Given the report of v_1 (made at $t = 1$) and v_2, the mechanism

[4] Although different assumptions are possible, in light of the revelation principle, it is optimal

specifies the probability that both firms operate on the infrastructure, $\tilde{r}(v) \in [0,1]$; and it also specifies the monetary transfer from each i to the principal, $\tilde{p}_i(v)$. To be clear, $\tilde{r}(v)$ is the probability *conditional on the investment at* $t = 1$ *and* $\Theta = H$. Let $r(v) = \theta q(v)\tilde{r}(v) \in [0, \theta q(v)]$ be the probability *unconditional* on those events. Similarly, let $p_i(v) = \theta q(v)\tilde{p}_i(v) \in \mathbb{R}$ denote i's expected payment *unconditional* on those events.

If the principal is not budget-constrained, then the first-best outcome is possible based on the idea of Vickrey–Clarke–Groves mechanisms. In that mechanism, each firm's payment function is designed so that, with that payment function, each firm's objective is essentially fully aligned with the social surplus. However, in the context of public goods, it is known that the principal always makes a loss in expectation. Therefore, in the realistic case where the principal is budget-constrained, it is far less trivial what the desirable mechanism is.

3.2 Optimal Mechanism

Assume that $v_i \in \{0,1\}$ and it is i.i.d. across agents, with $\Pr(v_i = 1) = \pi$. By focusing on a simple binary-type case, we can better highlight the properties of the optimal mechanism.

The optimal mechanism is given by:

$$\begin{aligned}
\max \quad & \mathbb{E}_v[q(v_1)(-\gamma) + \theta q(v_1)\tilde{r}(v)(v_1 + v_2 - 2\gamma)] \\
\text{sub. to} \quad & r(v) = \theta q(v_1)\tilde{r}(v) \leq \theta q(v_1),\ \mathbb{E}_v[p_1(v) + p_2(v)] \geq 0, \\
& \mathbb{E}_{v_2}[q(v_1)(-\gamma) + \theta q(v_1)\{\tilde{r}(v)(v_1 - \gamma) - \tilde{p}_1(v)\}] \\
& \ \geq \max\{0, \mathbb{E}_{v_2}[q(v_1')(-\gamma) + \theta q(v_1')\{\tilde{r}(v_1', v_2)(v_1 - \gamma) - \tilde{p}_1(v_1', v_2)\}]\} \\
& \mathbb{E}_{v_1|Invest\ at\ t\,=\,1}[\theta\{\tilde{r}(v)(v_2 - \gamma) - \tilde{p}_2(v)\}] \\
& \ \geq \max\{0, \mathbb{E}_{v_1|Invest\ at\ t\,=\,1}[\theta\{\tilde{r}(v_1, v_2')(v_2 - \gamma) - \tilde{p}_2(v_1, v_2')\}]\},
\end{aligned}$$

where the second constraint is the budget-balance constraint, and the last two con-

for the principal not to reveal the report of v_1 to firm 2. On the other hand, the assumption that firm 2 observes the investment at $t = 1$, is innocuous: even if firm 2 does not observe it, the result would not change at all.

ditions are about the agents' participation and incentive-compatibility constraints. For firm 1's constraint, firm 1 computes its expected payoff without knowing v_2, nor Θ. Firm 2 instead knows whether firm 1 has invested at $t = 1$ or not, and the constraints above are when firm 1 has invested. Obviously, if no investment has been made at $t = 1$, firm 2's continuation payoff is 0. Observe that this last constraint can be rewritten as follows:

$$\mathbb{E}_{v_1}[r(v)(v_2 - \gamma) - p_2(v)] \;\; \geq \;\; \max\{0, \mathbb{E}_{v_1}[r(v_1, v_2')(v_2 - \gamma) - p_2(v_1, v_2')]\},$$

because its left-hand side is $\mathbb{E}_{v_1}[q(v_1)]$ times the left-hand side of the original constraint (i.e., $\mathbb{E}_{v_1 | Invest\ at\ t\,=\,1}[\theta\{\tilde{r}(v)(v_2 - \gamma) - \tilde{p}_2(v)\}]) + (1 - \mathbb{E}_{v_1}[q(v_1)]) \cdot 0$, and similarly for the right-hand side.

Therefore, the optimal mechanism is given by:

$$\begin{aligned}
\max \quad & \mathbb{E}_v[q(v_1)(-\gamma) + r(v)(v_1 + v_2 - 2\gamma)] \\
\text{sub. to} \quad & r(v) \leq \theta q(v_1),\ \mathbb{E}_v[p_1(v) + p_2(v)] \geq 0, \\
& \mathbb{E}_{v_2}[q(v_1)(-\gamma) + r(v)(v_1 - \gamma) - p_1(v)] \\
& \qquad \geq \max\{0, q(v_1')(-\gamma) + r(v_1', v_2)(v_1 - \gamma) - p_1(v_1', v_2)\} \\
& \mathbb{E}_{v_1}[r(v)(v_2 - \gamma) - p_2(v)] \\
& \qquad \geq \max\{0, \mathbb{E}_{v_1}[r(v_1, v_2')(v_2 - \gamma) - p_2(v_1, v_2')]\}.
\end{aligned}$$

By the standard argument, the participation constraint is binding for the low type $v_i = 0$, while the incentive-compatibility constraint is binding for the high type $v_i = 1$. Let $U_i(v_i)$ be agent i's expected payoff under truth-telling given its type v_i:

$$\begin{aligned}
U_1(v_1) &= q(v_1)(-\gamma) + \mathbb{E}_{v_2}[r(v_1, v_2)](v_1 - \gamma) - P_1(v_1) \\
U_2(v_2) &= \mathbb{E}_{v_1}[r(v_1, v_2)](v_2 - \gamma) - P_2(v_2),
\end{aligned}$$

where $P_i(v_i) = \mathbb{E}_{v_{-i}}[p_i(v_i, v_{-i})]$. The binding constraints imply:

$$\begin{aligned}
-P_1(0) - \gamma q(0) - \gamma \mathbb{E}_{v_2}[r(0, v_2)] &= 0 \\
-P_2(0) - \gamma \mathbb{E}_{v_2}[r(v_1, 0)] &= 0 \\
-P_1(1) - \gamma q(1) + \mathbb{E}_{v_2}[r(1, v_2)](1-\gamma) &= -P_1(0) - \gamma q(0) + \mathbb{E}_{v_2}[r(0, v_2)](1-\gamma) \\
&= \mathbb{E}_{v_2}[r(0, v_2)] \\
-P_2(1) + \mathbb{E}_{v_1}[r(v_1, 1)](1-\gamma) &= -P_2(0) + \mathbb{E}_{v_1}[r(v_1, 0)](1-\gamma) \\
&= \mathbb{E}_{v_1}[r(v_1, 0)],
\end{aligned}$$

and thus:

$$\begin{aligned}
P_1(0) &= -\gamma q(0) - \gamma[\pi r(01) + (1-\pi) r(00)] \\
P_2(0) &= -\gamma[\pi r(10) + (1-\pi) r(00)] \\
P_1(1) &= -\gamma q(1) + (1-\gamma)[\pi r(11) + (1-\pi) r(10)] - [\pi r(01) + (1-\pi) r(00)] \\
P_2(1) &= (1-\gamma)[\pi r(11) + (1-\pi) r(01)] - [\pi r(10) + (1-\pi) r(00)].
\end{aligned}$$

In what follows, it is clearly optimal to set $r(00) = 0$, and hence we omit this term from here on.

The budget-balance constraint becomes:

$$\begin{aligned}
0 \leq BB &\equiv (1-\pi)(P_1(0) + P_2(0)) + \pi(P_1(1) + P_2(1)) \\
&= (1-\pi)\{-\gamma q(0) - \gamma \pi (r(10) + r(01))\} \\
&\quad + \pi\{-\gamma q(1) + (1-\gamma)[2\pi r(11) + (1-\pi)(r(10) + r(01))] - \pi(r(10) + r(01))\} \\
&= -\gamma(\pi q(1) + (1-\pi) q(0)) \\
&\quad + \pi^2 r(11)(2 - 2\gamma) + \pi(1-\pi)(1-2\gamma)(r(10) + r(01)) - \pi^2(r(10) + r(01)).
\end{aligned}$$

We remark that, when $BB = 0$, maximizing the social surplus (that is, $-\gamma(\pi q(1) + (1-\pi) q(0)) + \pi^2 r(11)(2-2\gamma) + \pi(1-\pi)(1-2\gamma)(r(10) + r(01))$) is equivalent to maximizing $\pi^2(r(10) + r(01))$. Indeed,we will find that the optimal mechanism sets $r(10) + r(01)$ as large as possible, which as we will show gives $r(10) = \theta$ and

$r(01) = \theta q(0)$, where q(0) is maximized conditional on the budget constraints.

The Lagrangian is:

$$
\begin{aligned}
L &= -\pi\gamma q(1) - (1-\pi)\gamma q(0) \\
&\quad +\pi^2 r(11)(2-2\gamma) + \pi(1-\pi)(r(10)+r(01))(1-2\gamma) \\
&\quad +\lambda \cdot BB \\
&= (1+\lambda)\{-\gamma(\pi q(1) + (1-\pi)q(0)) + \pi^2 r(11)(2-2\gamma) + \pi(1-\pi)(1-2\gamma)(r(10)+r(01))\} \\
&\quad -\lambda\pi^2(r(10)+r(01))
\end{aligned}
$$

with the remaining constraint $q(v_1) \geq r(v)$ for all v.

From the Lagrangian, it is clear that, fixing $q(1)$, it is optimal to set $r(11)$ as large as possible; that is, $r(11) = \theta q(1)$. Similarly, fixing $r(01)$, it is optimal to set $q(0)$ as small as possible: that is, $q(0) = \frac{r(01)}{\theta}$.

Thus, the Lagrangian becomes:

$$
\begin{aligned}
L &= (1+\lambda)\{-\gamma(\pi q(1) + (1-\pi)q(0)) + \pi^2\theta q(1)(2-2\gamma) + \pi(1-\pi)(1-2\gamma)(r(10)+\theta q(0))\} \\
&\quad -\lambda\pi^2(r(10)+\theta q(0)).
\end{aligned}
$$

To reduce the number of cases and focus on interesting parameter regions, let us assume:

Assumption 1. $\theta\pi(1-2\gamma) > \gamma$.

The assumption implies that it is optimal to set $q(0) = 1$ if budget-unconstrained (i.e., $\lambda = 0$). It also implies that $q(1) = 1$ is optimal, as the coefficient for $q(1)$ in the Lagrangian is proportional to $\pi\theta(2-2\gamma) - \gamma > 0$. Thus:

$$
\begin{aligned}
L &= (1+\lambda)\{-\gamma(\pi + (1-\pi)q(0)) + \pi^2\theta(2-2\gamma) + \pi(1-\pi)(1-2\gamma)(r(10)+\theta q(0))\} \\
&\quad -\lambda\pi^2(r(10)+\theta q(0)).
\end{aligned}
$$

It further implies that $q(0) > 0$ in the optimal mechanism. To see this, suppose contrarily that $q(0) = 0$ in the optimal mechanism. It must be that $\lambda > 0$ by the above assumption, i.e., the budget constraint must be binding. However, with

$q(0) = 0$, we have:

$$BB = -\gamma\pi + \pi^2\theta(2 - 2\gamma) + \pi(1 - \pi)(1 - 2\gamma)r(10) - \pi^2 r(10),$$

which is strictly positive, because (a) with $r(10) = 0$ we have:

$$BB = \pi(-\gamma + \pi\theta(2 - 2\gamma)) > 0,$$

and (b) with $r(10) = \theta$ we have:

$$BB = \pi(-\gamma + \theta(1 - 2\gamma)) > 0,$$

and (c) by linearity, $BB > 0$ with any in-between $r(10)$.

The coefficient for $r(10)$ in the Lagrangian is proportional to:

$$(1 + \lambda)(1 - \pi)(1 - 2\gamma) - \lambda\pi,$$

while that for $q(0)$ is proportional to:

$$(1 + \lambda)\{-\gamma\frac{1 - \pi}{\pi\theta} + (1 - \pi)(1 - 2\gamma)\} - \lambda\pi,$$

and thus, in the optimal mechanism, $q(0) > 0$ necessarily implies $r(10) = \theta$.

For $q(0)$, either $q(0) = 1$ (if it is budget-feasible), or $q(0) \in (0, 1)$ satisfies the budget constraint with equality. First, $q(0) = 1$ (together with $q(1) = 1$, $r(11) = r(10) = r(01) = \theta$) is optimal if:

$$BB = \pi\{-\gamma + \theta((2 - \pi)(1 - 2\gamma) - \pi)\} - (1 - \pi)\gamma \geq 0, \tag{1}$$

that is,

$$\gamma \leq \gamma_1 \equiv \frac{2\pi\theta(1 - \pi)}{2\pi\theta(2 - \pi) + 1}. \tag{2}$$

If $\gamma > \gamma_1$, then the optimal mechanism sets an interior $q(0)$ in order to satisfy the budget constraint with equality:

$$q(0) = \frac{\pi(-\gamma + \theta(1-2\gamma))}{(1-\pi)(\gamma - \pi\theta(1-2\gamma)) + \pi^2\theta} \in (0,1), \tag{3}$$

together with $q(1) = 1$, $r(11) = r(10) = \theta$ and $r(01) = \theta q(0)$.[5]

Regarding the payments, we have:

$$P_2(0) \;=\; -\gamma\pi\theta < 0 \tag{4}$$

and

$$P_2(1) \;=\; (1-\gamma)\pi\theta + \{(1-\gamma)(1-\pi) - \pi\}\theta q(0) > 0, \tag{5}$$

and thus, agent 2 is subsidized if $v_2 = 0$ (in expectation with respect to v_1), while it pays if $v_2 = 1$. The first case (with $v_2 = 0$) is because agent 2 brings a positive externality through the network externality in this economy: Recall that, given $v_2 = 0$, the infrastructure is used if and only if $v_1 = 1$. That is, agent 2 enjoys no value from it, while agent 1 can enjoy it only if agent 2 uses it too. Therefore, agent 2 must be incentivized to use the infrastructure. The second case (with $v_2 = 1$) is different: Now with $v_2 = 1$, agent 2 enjoys a positive value from the infrastructure. Moreover, agent 1 with $v_1 = 0$ must be subsidized in order for him to invest at $t = 1$ (recall $q(0) > 0$). By the budget-balance requirement, its source must be agent 2 with $v_2 = 1$.

For agent 1, we have:

$$P_1(0) = -\gamma q(0) - \gamma\pi\theta q(0) < 0, \tag{6}$$

that is, it is paid if $v_1 = 0$, otherwise it would not invest at $t = 1$. If agent 1 has

[5] That $q(0) \in (0,1)$ is because:

$$\begin{aligned}(1-\pi)(\gamma - \pi\theta(1-2\gamma)) + \pi^2\theta \;&=\; \pi\{-\gamma + \theta(1-2\gamma)\}\\ &\quad +(1-\pi)\gamma - \pi\{-\gamma + \theta((2-\pi)(1-2\gamma) - \pi)\},\end{aligned}$$

and the last term is positive when $BB < 0$.

$v_1 = 1$, then:

$$P_1(1) = -\gamma + (1-\gamma)\theta - \pi\theta q(0), \tag{7}$$

which is positive or negative, depending on the parameters. More specifically, suppose that γ is large and so q(0) is close to 0 (that is, given the expression of q(0), γ is close to $\theta/(1+2\theta)$). Then using Assumption 1 one can conclude that $P_1(1)$ is negative. That is intuitive: when the opportunity cost of undertaking the investment is large, firm 1 needs to be subsidized. Suppose instead that γ is small and so q(0) is close to 1 (that is, given the expression of q(0), γ is close to $2\theta(1-\pi)/(1+2\theta)(2-\pi)$). In this case, it is immediate to show that $P_1(1)$ is positive if and only if $\pi < 2\theta/(1+2\theta)$; that is, when π is sufficiently small relative to θ. Again this is intuitive: when θ is small, so is the probability that firm 1 is compensated in period 2 (which only occurs when the state turns out to be H) and so firm 1 needs to be subsidized in order to invest in period 1. When π is small, so is the probability that firm 2 will be of high type. Since firm 2 is likely to need a subsidy to operate, and since the budget constraint needs to balance, firm 1 needs to contribute a positive amount.

We summarize our findings in the next proposition.

Proposition 1. Under Assumption 1, the optimal mechanism for the principal is such that: $q(1) = 1, r(11) = r(10) = \theta, r(00) = 0$ and $r(01) = \theta q(0)$, where $q(0) = 1$ if $\gamma \leq \gamma_1$, and $q(0)$ is given by Equation 3 if $\gamma > \gamma_1$. The corresponding transfers are given in Equations 4-7.

3.2.1 First-Best Outcomes

In the first-best allocation, the principal does not have to worry about the budget-balance requirement, because she can set each firm's payment exactly in the way that each firm breaks even; as long as the investment and usage occur only if it is socially valuable (which is indeed the case), the principal can always make a non-negative revenue. Thus, the first-best allocation would set $r(11) = r(10) = \theta$ with $q(1) = 1$; $r(01) = \theta q(0)$ and $r(00) = 0$ where $q(0)$ maximizes the expected welfare:

$$-\gamma(\pi + (1-\pi)q(0)) + \pi^2\theta(2-2\gamma) + \pi(1-\pi)(1-2\gamma)(r(10) + \theta q(0));$$

equivalently, $q(0) = 1$ if $-\gamma + \pi(1-2\gamma)\theta > 0$, which is indeed the case given Assumption 1. Accordingly, the probability of investment is always weakly higher in the first-best outcome.

3.3 Unregulated Market

Now, consider the case without the principal. Although there can be several ways to formulate that case, we wish to remain as close as possible to the previous setting. Let us imagine a situation where one of the agents, say firm 1, writes a mechanism on its own at the "ex ante" stage where no one has private information yet. This may be interpreted as the case where firm 1 has a full bargaining power and can commit to the proposed mechanism. For comparison, let us continue to assume Assumption 1.

As in the previous analysis, we consider mechanisms which induce firms to truthfully reveal their types. For firm 1, as we will see, the associated IC constraint would not bind. Inducing truthful revelation of firm 2's type is a way for firm 1 to maximize its expected revenues. A potential concern is that, by designing a given mechanism, firm 1 may reveal some private information. The literature on mechanism design by an informed principal has shown that i) firm 1 should (without loss of generality) choose the same mechanism irrespective of its type (so that it does not convey any information to firm 2), while at the same time eliciting firm 1's own report and make the allocation contingent on firm 1's type (*the inscrutability principle*), and that (ii) firm 1's incentive compatibility is typically not binding so irrespective of its type firm 1 may maximize its own expected payoff subject only to firm 2's incentive compatibility (and other kinds of feasibility) constraints.[6]

[6]Myerson (1982) provides a classic treatment; Maskin & Tirole (1990) and Mylovanov & Tröger (2014) consider private values, most closely to our setting.

As we will show, even if the setting is close to the one with the principal, having firm 1 designing the mechanism introduces some distortion in the decisions and payments. For example, $r(01)$ could be lower than socially optimal (recall that, under Assumption 1, the principal-optimal mechanism sets $r(01) = \theta$).

The optimal mechanism in view of firm 1 is given as follows:

$$\begin{aligned}
\max \quad & \mathbb{E}_v[q(v_1)(-\gamma) + r(v)(v_1 - \gamma) - p_1(v)] \\
\text{sub. to} \quad & r(v) \leq \theta q(v_1),\ p_1(v) + p_2(v) \geq 0,\ \forall v; \\
& \mathbb{E}_{v_2}[q(v_1)(-\gamma) + r(v)(v_1 - \gamma) - p_1(v)] \\
& \qquad \geq \max\{0, q(v_1')(-\gamma) + r(v_1', v_2)(v_1 - \gamma) - p_1(v_1', v_2)\},\ \forall v_1 \\
& \mathbb{E}_{v_1}[r(v)(v_2 - \gamma) - p_2(v)] \\
& \qquad \geq \max\{0, \mathbb{E}_{v_1}[r(v_1, v_2')(v_2 - \gamma) - p_2(v_1, v_2')]\}, \forall v_2.
\end{aligned}$$

Regarding firm 2, again, IR for type 0 and IC for type 1 must bind in the optimal mechanism. Thus:

$$\begin{aligned}
P_2(0) &= -\gamma[\pi r(10) + (1-\pi)r(00)] \\
P_2(1) &= (1-\gamma)[\pi r(11) + (1-\pi)r(01)] - [\pi r(10) + (1-\pi)r(00)].
\end{aligned}$$

For firm 1, for now, let us ignore its IR and IC constraints. They can be shown to be satisfied in the optimal mechanism. Moreover, the budget constraint must be binding (otherwise, firm 1 can reduce $p_1(\cdot)$ and get better off). Also, let us set $r(00) = 0$ as it is never optimal to set $r(00) > 0$.

Therefore, the problem becomes:

$$\begin{aligned}
\max \quad & \mathbb{E}_v[q(v_1)(-\gamma) + r(v)(v_1 - \gamma) + p_2(v)] \\
\text{sub. to} \quad & r(v) \leq \theta q(v_1)\ \forall v,
\end{aligned}$$

where:

$$\begin{aligned}
& \mathbb{E}_v[q(v_1)(-\gamma) + r(v)(v_1 - \gamma) + p_2(v)] \\
= \quad & -\gamma(\pi q(1) + (1-\pi)q(0)) + \pi(1-\gamma)(\pi r(11) + (1-\pi)r(10)) + (1-\pi)\pi r(01) \\
& +(1-\pi)(-\gamma\pi r(10)) + \pi\{(1-\gamma)[\pi r(11) + (1-\pi)r(01)] - \pi r(10)\},
\end{aligned}$$

where the second line on the right hand side corresponds to the payment from firm 2.

As before, it is optimal to set $q(1) = 1$, $r(11) = \theta$, and $r(01) = \theta q(0)$ (the logic is the same and hence omitted). Thus, the objective becomes

$$\begin{aligned}
& -\gamma(\pi + (1-\pi)q(0)) + \pi(1-\gamma)(\pi\theta + (1-\pi)r(10)) + (1-\pi)\pi\theta q(0) \\
& +(1-\pi)(-\gamma\pi r(10)) + \pi\{(1-\gamma)\theta[\pi + (1-\pi)q(0)] - \pi r(10)\} \\
= \quad & -\gamma\pi + \pi^2\theta(2-2\gamma) + q(0)(1-\pi)\{-\gamma + \pi\theta(2-\gamma)\} + r(10)\pi\{(1-\pi)(1-2\gamma) - \pi\}
\end{aligned}$$

As opposed to the case with the principal, now, the coefficient for $r(10)$ is not necessarily greater than that of $q(0)$. Indeed, although Assumption 1 implies $-\gamma + \pi\theta(2-\gamma) > 0$ and hence $q(0) = 1$ in the optimal mechanism for firm 1, it is possible that $(1-\pi)(1-2\gamma) - \pi$ is non-positive (for example, imagine the case with $\gamma \simeq 0$ and $1 - 2\pi < 0$).

Intuitively, the reason is that firm 1 is more revenue-oriented than the principal. Notice that $r(10)$ is the probability that both firms operate on the infrastructure introduced at $t = 1$ (recall $q(1) = 1$ in the optimal mechanism). Given that the facility is there, the principal would let the firms use it for sure, in order to yield the positive social surplus. However, given that $v_2 = 0$, firm 1 cannot expect much payment from firm 2: indeed, $P_2(0)$ is negative, that is firm 2 with $v_2 = 0$ is rather subsidized. Due to incentive compatibility for type $v_2 = 1$, this also means that the revenue from firm 2 with $v_2 = 1$ cannot be so high (as otherwise firm 2 would have an incentive to mimic the low-value type). By setting $r(10) = 0$ and hence wasting the infrastructure in case of $v_2 = 0$, firm 1 can expect a higher fee revenue.

Notice also that, in the case with the principal, the mechanism always sets

$q(1) = 1$ while potentially reducing $q(0)$ in order to satisfy the budget constraint. This is because $q(1)$ affects both $r(11)$ and $r(10)$ (recall that $r(11) = r(10) = \theta q(1)$) while $q(0)$ only affects $r(01)$; hence, increasing $q(1)$ is socially more valuable than increasing $q(0)$. On the other hand, firm 1 would set $q(0) = 1$, because it can expect revenue from firm 2 with $v_2 = 1$.

Let us compute the payment in case $r(10) = 0$, that is, when

$$\gamma > \gamma_2 \equiv \frac{1 - 2\pi}{2 - 2\pi}. \tag{8}$$

We have:

$$P_2(0) = 0 \tag{9}$$

$$P_2(1) = (1 - \gamma)\theta. \tag{10}$$

Regarding type $v_2 = 0$, as explained above, it is supposed to be subsidized in the principal-optimal mechanism, but here, it receives 0, as type $v_2 = 0$ never uses the infrastructure. As a consequence, $P_2(1)$ is such that type $v_2 = 1$ receives no rent, that is, firm 2 is fully extracted.

Conversely, if $\gamma \leq \gamma_2$, we have $r(10) = \theta$ and payments are given by:

$$P_2(0) = -\gamma\pi\theta \tag{11}$$

$$P_2(1) = (1 - \gamma)\theta - \pi\theta. \tag{12}$$

In this case, firm 1 can extract less surplus from firm 2 as it is not optimal to commit to deny the usage of the infrastructure in case firm 2 reports $v_2 = 0$, which increases the (minimal) subsidy required by firm 2 for using the infrastructure if $v_2 = 0$ and decreases the (maximal) payment required to firm 2 for using the infrastructure if $v_2 = 1$.

Proposition 2. The optimal mechanism for firm 1 is such that: $q(1) = q(0) = 1, r(11) = r(01) = \theta$, and $r(00) = 0$. If $\gamma > \gamma_2$, $r(10) = 0$ and payments are given in Equations (9) and (10). If $\gamma \leq \gamma_2$, $r(10) = \theta$ and payments are given in Equations (11) and (12).

Relatively to the case with the principal, we now have a larger probability of investment (since the firm always sets $q(0) = 1$), which brings the outcome closer to the first best (which given Assumption 1 is to have $q(0) = 1$). At the same time, we have a lower probability of usage conditional on investment (since we may have $r(10) = 0$). This is reminiscent of monopoly pricing in which the monopolist prefers to cut quantity (here, usage) and increase prices (here, transfers). This is clearly inefficient since it involves paying the investment cost γ and at the same time not using the infrastructure. Moreover, this is more likely to happen precisely when the cost of such inefficiency is large (that is, when $\gamma \geq \gamma_2$).

3.3.1 Hold Up Problems

The assumption that firm 1 has full bargaining power can be seen as extreme. On the other hand, leaving full bargaining power to firm 2 may also generate inefficiencies. Knowing that it will be fully extracted, firm 1 may refrain from investing in period 1, thereby resulting in too little investment relative to the case with the principal.

To illustrate this most simply in our setting, suppose that the mechanism is instead designed by firm 2; that is, maximizing firm 2's expected payoffs, subject to firm 1's participation and incentive compatibility constraints. Firm 1 would choose its investment in period 1 anticipating the mechanism proposed by firm 2 in period 2. Following the same logic as above, let us define

$$\gamma_3 \equiv \frac{\pi\theta(1-2\pi)}{(1-\pi)(2\pi\theta-\pi+1)}, \tag{13}$$

and show the following proposition.[7]

Proposition 3. The optimal mechanism for firm 2 is such that: $q(1) = 1, r(11) = r(10) = \theta$, and $r(00) = 0$. If $\gamma > \gamma_3$, $q(0) = r(01) = 0$; if $\gamma \leq \gamma_3$, $q(0) = 1$ and $r(01) = \theta$.

Relative to the case in which firm 1 designs the mechanism, we observe as intuitive a lower probability of investment in period 1 due to a classic hold-up

[7] The proof repeats the one in the previous section and it is hence omitted.

problem. In terms of efficiency, however, the overall effect is not clear since firm 2 would never prevent usage after the investment has been made (while firm 1 may set r(10)=0).

Remark. Our modeling of the bargaining equilibrium outcome as maximizing the weighted sum of each firm's payoffs is in the same spirit as Loertscher & Marx (2022), who formalize this idea in a static bargaining problem. As in Loertscher & Marx (2022), the outcome tends to be more inefficient when the Pareto weights are more biased toward one agent, and especially at the extreme case where all the bargaining power is given to one of the agents. Interestingly, in our setting with sequential arrival, different kinds of inefficiencies arise depending on which firm has the bargaining power. If the incumbent firm has the full bargaining power, it behaves like in the usual monopoly problem, investing in the public goods in some unnecessary states; while if the newly arriving firm has it, it is more like a hold-up problem, resulting in too little investment by the incumbent firm.

4 Mature Markets: Coordinating Firms

While in the previous section we have considered an emerging market in which firms enter sequentially, we here consider a setting corresponding, say, to a more mature market in which both firm 1 and 2 are available at $t = 1$. In principle, the investment can be made at either $t = 1$ or $t = 2$ (or never), but given our assumption on the resolution of the uncertainty about Θ, it is better to never invest than to invest at $t = 2$.

That both agents are available at $t = 1$ creates a possibility of both investing in their own infrastructures at the same time, which is also socially wasteful. In case both invest at $t = 1$, then at $t = 2$, one of them is to be used. More specifically, we can imagine a coordination game where each firm (and maybe consumers too) decide which one to use. There are typically many equilibria of this game, some of which would be socially inefficient, and the role of the principal is to coordinate firms' decisions so as to induce an efficient equilibrium at the investment and usage stages. We will contrast this situation with a setting in which the principal has a more limited power, and in particular it cannot prevent firms from using their own infrastructure if they decide to deviate from the principal's recommendation. This allows us to highlight in the simplest way the role of the principle as a coordinator.

4.1 Timing and information structure

The timing and information structure of the game is modified as follows.

At $t = 1$, both firm 1 and 2 (simultaneously) report their value types v_1 and v_2 to the mechanism. The mechanism recommends each firm to invest in a infrastructure with probability, $q_1(v)$ and $q_2(v)$, respectively. In principle, it is possible that both firms invest at the same time; however, let us focus on the case where, on the equilibrium path, they do not invest at the same time. That is, $q_i(v)$ denotes the probability that *only* i invests in a infrastructure, which implies $q_1(v) + q_2(v) \leq 1$. This assumption can be shown without loss: any mechanism with overlapping investment can be dominated by another mechanism without it.[8]

If no investment happens at $t = 1$, then nor at $t = 2$, and hence, the end of the game. Similarly, even if an investment is made at $t = 1$, if that reveals $\Theta = L$, then no firm operates on the infrastructure, and hence, the end of the game. Only if an investment has been made at $t = 1$ revealing $\Theta = H$, firms may operate on the infrastructure at $t = 2$: in this case, given that both types are reported at $t = 1$, it is without loss to assume that both firms operate on the infrastructure *for sure*.[9]

A complication arises in case a firm invests in a infrastructure even if it is not recommended to do so. For now, we assume that the principal can coordinate on the pure equilibrium where no firm operates on it at $t = 2$. We examine the role of the principal as a coordinator in this sort of situation, by later considering an alternative story where such a coordination is not possible. The other ingredients

[8]Notice also that having both firms operating in a common infrastructure already at $t = 1$ is not optimal given that this would require that both firms incur the cost γ already at $t = 1$ while network externalities can only be enjoyed at $t = 2$.

[9]More precisely, any mechanism such that an investment is made at $t = 1$ with some v while it is not used at $t = 2$ with the same v is dominated by another mechanism where no investment is made with this v.

of the game are the same as in Section 3.

4.2 Unregulated Market

We first consider the case where, as in Section 3.3, one of the firms (say, firm 1) designs a contract instead of the principal, corresponding to the case where this firm has the full bargaining power over the other firm.

Let $q_i(v)$ denote the probability that firm i invests in the infrastructure at $t = 1$.[10] The problem of maximizing firm 1's ex ante expected payoff is as follows:

$$\begin{aligned}
\max \quad & \mathbb{E}_v[q_1(v)(-\gamma) + q(v)\theta(v_1 - \gamma) + p_2(v)] \\
\text{sub. to} \quad & \mathbb{E}_{v_1}[q_2(v_1,0)(-\gamma) + q(v_1,0)\theta(-\gamma) - p_2(v_1,0)] \geq 0 \\
& \mathbb{E}_{v_1}[q_2(v_1,1)(-\gamma) + q(v_1,1)\theta(1-\gamma) - p_2(v_1,1)] \\
& \quad \geq \mathbb{E}_{v_1}[q_2(v_1,0)(-\gamma) + q(v_1,0)\theta(1-\gamma) - p_2(v_1,0)],
\end{aligned}$$

where $q(v) = q_1(v) + q_2(v)$, the first constraint is firm 2's IR if $v_2 = 0$, and the second constraint is its IC if $v_2 = 1$; as in the previous cases, all the other constraints do not bind.

Making the two constraints binding, we obtain:

$$\begin{aligned}
P_2(0) &= \mathbb{E}_{v_1}[q_2(v_1,0)(-\gamma) + q(v_1,0)\theta(-\gamma)] \\
&= -\gamma[\pi(q_2(10) + q(10)\theta] \\
P_2(1) &= \mathbb{E}_{v_1}[q_2(v_1,1)(-\gamma) + q(v_1,1)\theta(1-\gamma)] - \theta\mathbb{E}_{v_1}[q(v_1,0)] \\
&= \pi[q_2(11)(-\gamma) + q(11)\theta(1-\gamma) - q(10)\theta] \\
&\quad +(1-\pi)[q_2(01)(-\gamma) + q(01)\theta(1-\gamma) - q(00)\theta].
\end{aligned}$$

[10]From the argument expressed in Section 4.1, there is no gain in both firms' simultaneously investing in the infrastructure, as only one of them is to be used at $t = 2$. Here, $q_i(v)$ should be interpreted as the probability of i's investment without $-i$'s.

Plugging them in the objective and rearranging it, the problem reduces to:

$$\begin{aligned}\max \quad & \pi^2 q(11)(-\gamma+\theta(2-2\gamma))+(1-\pi)\pi q(01)(-\gamma+\theta(1-2\gamma)) \\ & +\pi(1-\pi)q(10)(-\gamma+\theta(1-2\gamma))-\pi^2\theta q(10) \\ & +(1-\pi)^2 q(00)(-\gamma+\theta(-2\gamma))-\pi(1-\pi)\theta q(00).\end{aligned}$$

Clearly, it is optimal to set $q(11) = q(01) = 1$ and $q(00) = 0$. Thus, the objective becomes:

$$\begin{aligned}& \pi^2(-\gamma+\theta(2-2\gamma))+(1-\pi)\pi(-\gamma+\theta(1-2\gamma)) \\ & +\pi q(10)[(1-\pi)(-\gamma+\theta(1-2\gamma))-\pi\theta].\end{aligned}$$

Recall that, in Section 3.3, if $(1-\pi)(1-2\gamma)-\pi < 0$, then firm 1 would set $r(10) = 0$, thereby investing in the infrastructure at $t = 1$ while at the same time setting a very high fee, which prevents firm 2 to use the infrastructure when $v_2 = 0$. Here, in a similar way, firm 1 may also set $q(10) = 0$ and the same phenomenon occurs when $(1-\pi)(-\gamma+\theta(1-2\gamma))-\pi\theta < 0$, that is when

$$\gamma > \gamma_4 \equiv \frac{\theta(1-2\pi)}{(1+\pi)(1+2\theta)}. \tag{14}$$

Notice that $\gamma_4 < \gamma_2$, that is the condition is a weaker than in Section 3.3. The difference comes from the fact that, here, firm 1's decision at $t = 1$ can depend on firm 2's type. In Section 3.3, firm 1 with $v_1 = 1$ invests at $t = 1$ hoping that firm 2 has $v_2 = 1$, while it may turn out to be a waste; here, firm 1 with $v_1 = 1$ invests at $t = 1$ if and only if $v_2 = 1$. In both cases, firm 1 sets a high fee so that firm 2 with $v_2 = 1$ is fully extracted. In both cases, the outcome is inefficient. Here, the probability of inefficiency is larger (since the condition for having $q(10) = 0$ is weaker than the one for having $r(10) = 0$ in Section 3.3). At the same time, in Section 3.3, the inefficiency is costlier as it involves paying the investment cost γ, and moreover as mentioned, the inefficiency is more likely to arise when γ is large. We can summarize as follows:

Proposition 4. The optimal mechanism for firm 1 is such that: $q(11) = q(01) = 1$ and $q(00) = 0$. If $\gamma \leq \gamma_4$, $q(10) = 1$; if $\gamma > \gamma_4$, $q(10) = 0$.

4.3 Optimal Mechanism

Consider now the optimal mechanism with a principal. As described above, in case two firms invest in an infrastructure, the principal can choose which one to coordinate on. This means that, even though a firm can always invest at $t = 1$, the principal can negate it at $t = 2$ by selecting an equilibrium where no one operates on that infrastructure. Therefore, the firm's outside option, that is, the right-hand side of its participation constraint, is given by operating on its small-scale network, whose payoff is normalized to 0.

The principal's optimal mechanism is given by:

$$\begin{aligned}
\max \quad & \mathbb{E}_v[q(v)(-\gamma + \theta(v_1 + v_2 - 2\gamma))] \\
\text{sub. to} \quad & \mathbb{E}_v[p_1(v) + p_2(v)] \geq 0, \\
& \mathbb{E}_{v_2}[q_1(v)(-\gamma) + \theta q(v)(v_1 - \gamma) - p_1(v)] \\
& \qquad \geq \max\{0, \mathbb{E}_{v_2}[q_1(v_1', v_2)(-\gamma) + \theta q(v_1', v_2)(v_1 - \gamma) - p_1(v_1', v_2)]\} \\
& \mathbb{E}_{v_1}[q_2(v)(-\gamma) + \theta q(v)(v_2 - \gamma) - p_2(v)] \\
& \qquad \geq \max\{0, \mathbb{E}_{v_1}[q_2(v_1, v_2')(-\gamma) + \theta q(v_1, v_2')(v_2 - \gamma) - p_2(v_1, v_2')]\},
\end{aligned}$$

where $q(v) = q_1(v) + q_2(v)$.

Let $P_i(v_i) = \mathbb{E}_{v_{-i}}[p_i(v_i, v_{-i})]$. By the standard argument, the low type's IR and the high type's IC constraints are binding, and thus:

$$\begin{aligned}
P_i(0) &= \mathbb{E}_{v_{-i}}[q_i(0, v_{-i})](-\gamma) + \theta \mathbb{E}_{v_{-i}}[q(0, v_{-i})](-\gamma), \\
P_i(1) &= \mathbb{E}_{v_{-i}}[q_i(1, v_{-i})](-\gamma) + \theta \mathbb{E}_{v_{-i}}[q(1, v_{-i})](1 - \gamma) - \theta \mathbb{E}_{v_{-i}}[q(0, v_{-i})].
\end{aligned}$$

Therefore, the budget-balance constraint becomes:

$$\begin{aligned} 0 \leq BB &= \pi P_1(1) + \pi P_2(1) + (1-\pi)P_1(0) + (1-\pi)P_2(0) \\ &= \mathbb{E}_v[q(v)(-\gamma + \theta(v_1 + v_2 - 2\gamma))] \\ &\quad -\pi\theta((1-\pi)2q(00) + \pi(q(01) + q(10))), \end{aligned}$$

and therefore, the corresponding Lagrangian is:

$$\begin{aligned} L &= (1+\lambda)\mathbb{E}_v[q(v)(-\gamma + \theta(v_1 + v_2 - 2\gamma))] \\ &\quad -\lambda\theta[2\pi(1-\pi)q(00) + \pi^2(q(01) + q(10))]. \end{aligned}$$

It is easy to see that the solution must satisfy $q(11) = 1$ and $q(00) = 0$. Regarding $q(01), q(10)$, we have (i) $q(10) = q(01) = 1$ if it is budget-feasible:

$$0 \leq \pi^2[-\gamma + \theta(2 - 2\gamma))] + 2\pi(1-\pi)[-\gamma + \theta(1 - 2\gamma)] - 2\pi^2\theta; \tag{15}$$

while (ii) otherwise, any pair of $(q(10), q(01))$ with $q(10) + q(01) = Q(< 2)$ is optimal, where:

$$0 = \pi^2[-\gamma + \theta(2 - 2\gamma)] + \pi(1-\pi)[-\gamma + \theta(1 - 2\gamma)]Q - \pi^2\theta Q. \tag{16}$$

We can summarize in the following proposition.

Proposition 5. Under Assumption 1, the optimal mechanism for the principal is such that: $q(11) = 1, q(00) = 0$ and $q(01) + q(10) = Q$. If Equation 15 holds, we have $Q = 2$; otherwise, Q is given by Equation 16.

In order to understand the role of the principal in this setting, it is useful to compare the optimal mechanism relative to the one in which only firm 1 is available in period 1, as described in Section 3.2. First, we notice that the principal's budget constraint is more likely to bind in Section 3.2. Comparing the condition for having $q(0) = q(1) = 1$ in Section 3.2 (that is, Equation 1) to the one for having $q(10) = q(01) = 1$ here (that is, Equation 15), we notice that the latter is more likely to be satisifed, the difference being equal to $\gamma(1-\pi)^2$. This is intuitive as it corresponds to the probability that, in Section 3.2, both firms turn out to be of low type and in that case the investment cost cannot be recovered.

We then observe that outcomes are more efficient when both firms are available in period 1. Given our Assumption 1, it is optimal to invest if at least one firm has high valuation. When both firms are available, we have a lower probability of inefficient investment (occurring when both firms turn out to have low type); this probability being zero relative to $(1-\pi)^2 q(0)$ in Section 3.2. At the same time, the probability of efficient investment is larger, being equal to $\pi^2+\pi(1-\pi)Q$, relative to $\pi^2+\pi(1-\pi)(1+q(0))$ in Section 3.2, where it can be shown with simple algebra that $Q > 1+q(0)$ unless $Q = 2$ and $q(0) = 1$ (i.e., the case of corner solutions).[11]

The difference in the total surplus is a weighted sum of these two components: letting W_3 denote the surplus in the optimal mechanism in Section 3.2, and W_4 denote the one in this section, we have:

$$W_4 - W_3 = \gamma(1-\pi)^2 q(0) + \pi(1-\pi)\{\theta(1-2\gamma)-\gamma\}(Q-1-q(0)). \qquad (17)$$

The first term in the r.h.s. of the equation represents the expected cost of inefficient investment; the second term represents the expected benefit of efficient investment. One may question how these terms depend on our underlying parameters. Indirectly, this would shed light on how the effects of investing in the common infrastructure would differ between emerging and mature markets and so under which conditions the benefits of waiting that the market develops (as in Section 4) would be large relative to the case of an early investment (as in Section 3).[12]

[11]Observe that $Q-1-q(0)$ equals:

$$\frac{\pi(-\gamma+\theta(2-2\gamma))\{(1-\pi)(\gamma-\pi(\theta)(1-2\gamma))+\pi^2\theta\}-[(1-2\pi)\gamma+\pi^2\theta(2-2\gamma)]\{(1-\pi)(\gamma-\theta(1-2\gamma))+\pi\theta\}}{\{(1-\pi)(\gamma-\pi(\theta)(1-2\gamma))+\pi^2\theta\}\{(1-\pi)(\gamma-\theta(1-2\gamma))+\pi\theta\}}$$

which simplifies to

$$\frac{(1-\pi)^2\gamma\{\theta(1-2\gamma)-\gamma\}}{\{(1-\pi)(\gamma-\pi(\theta)(1-2\gamma))+\pi^2\theta\}\{(1-\pi)(\gamma-\theta(1-2\gamma))+\pi\theta\}} > 0.$$

[12]These comparisons are only suggestive of the possible dynamic trade-offs. As we mention in the conclusion, we view a full treatment of these trade-offs as an important avenue for future

At this level of generality, comparative statics are difficult, because a change in a parameter affects many terms at the same time (recall that Q and $q(0)$ are complicated functions of the parameters). Hence, let us focus on some sub-cases where we have clearer comparative statics. Suppose that γ is sufficiently small or π is sufficiently large, implying $Q = 2$ and $q(0) = 1$. In this case, $W_4 - W_3 = \gamma(1-\pi)^2$, and thus, only the part of the investment cost matters. When γ is sufficiently small or π is sufficiently large, however, this part is small, implying a small difference in surplus between the two cases.

Next, consider the case of interior solutions: $Q < 2$ and $q(0) < 1$. A change in a parameter affects the surplus difference in a complex manner (as it affects $W_4 - W_3$ directly and also indirectly through Q and $q(0)$). In order to gain some insights, focus on the case where $x \equiv \theta(1-2\gamma) - \gamma \simeq 0$, that is, in case one firm has a high valuation while the other has a low valuation, there is almost no expected gain from investment. Because both $q(0)$ and $Q - 1 - q(0)$ are proportional to x, they are close to 0.[13] Therefore, $W_4 - W_3$ is close to 0 too. Intuitively, this is the case where any parameter affects $W_4 - W_3$ only indirectly through $q(0)$ and $Q - 1 - q(0)$ (more precisely, through x), as its direct effect is 0: indeed, denoting $W_4 - W_3 = xf$, where $f(> 0)$ is a (complicated) function of the parameters, we have:

$$\frac{\partial}{\partial x}(W_4 - W_3) = f + x\frac{\partial f}{\partial x} \simeq f > 0,$$

that is, $W_4 - W_3$ increases in x. Intuitively, when $x \simeq 0$ (and hence $q(0)$ and $Q - 1 - q(0)$ are close to 0), there is not much gain in waiting for the principal, because the efficiency gain of it is limited (that is, $W_4 - W_3$ is close to 0). An increase in x means that the efficiency increases more when both firms are present

analysis.

[13]Recall that:

$$\begin{aligned} q(0) &= \frac{\pi(\theta(1-2\gamma)-\gamma)}{(1-\pi)(\gamma-\pi\theta(1-2\gamma))+\pi^2\theta} \\ Q-1-q(0) &= \frac{(1-\pi)^2\gamma\{\theta(1-2\gamma)-\gamma\}}{\{(1-\pi)(\gamma-\pi(\theta)(1-2\gamma))+\pi^2\theta\}\{(1-\pi)(\gamma-\theta(1-2\gamma))+\pi\theta\}}. \end{aligned}$$

(W_4) than when only one firm exists (W_3).

Finally, in order to further analyze the differences between the market scenarios in Sections 3 and 4, notice that the efficiency gains that the principal can induce when both firms are available can be driven by two effects. First, the principal can coordinate the investment between the two firms. Second, the principal can elicit both firms' valuations and design the investment probabilities based on them. In the next analysis (Sections 4.4 and 4.5), we highlight the contribution of these two dimensions, assessing the value of information and of coordination in our framework.

4.4 The Value of Information

The analyses in Section 3.2 and 4.3 uncover the advantage of having both firms available at the initial stage. Indeed, the principal can achieve at least weakly higher expected welfare if both are available. The fact that both firms are available implies two differences from the case where only one is available: first, the principal can communicate with both firms, and hence the investment decision can be a function of both v_1 and v_2 instead of just v_1; and second, the principal can choose which firm to invest in the infrastructure. A natural question is which aspect contributes more to the welfare improvement. To see this, in this section, we consider a hypothetical situation where, as in Section 3.2, only firm 1 can invest while at the same time allowing the principal to communicate with both firms at $t = 1$. This can be considered as an intermediate case between Section 3.2 and 4.3. Comparing the outcomes in this case with those in Section 3.2 allows to highlight the value of information in our setting.

Recall that, in the analysis in Section 4.3, the only variable that matters is $q(v) = q_1(v) + q_2(v)$ for each v, rather than individual $q_i(v)$. In other words, two allocations $(q_1(\cdot), q_2(\cdot))$ and $(q_1'(\cdot), q_2'(\cdot))$ attain the same surplus (where the corresponding P and P' are determined by the binding IR and IC constraints) as long as they satisfy $q_1(v) + q_2(v) = q_1'(v) + q_2'(v)$ for all v. In particular, letting $q^*(\cdot)$ denote the investment probabilities (summed across two firms) in the optimal mechanism, we can achieve the same welfare by simply setting $q_1(v) = q^*(v)$ and

$q_2(v) = 0$ for all v. In other words, from an ex-ante perspective, what is crucial is the fact that the principal can communicate with both (and hence the investment decision can be contingent on v), rather than the technological aspect as to which firm can invest.

4.5 The Value of Coordination

Suppose there is no principal who can coordinate on the equilibrium selection at $t = 2$ in case multiple infrastructures are invested at $t = 1$. To highlight this coordination value, in the "no-principal" scenario, we assume that the mechanism itself is still designed by a surplus-maximizing entity, call him a *mediator* to avoid confusion. That is, this mediator, much like the principal in the previous analysis, designs the probability of each firm's investment and the corresponding transfers to maximize the total surplus subject to the budget constraint, each firm's participation and incentive compatibility constraint. The key difference with the principal is that while the principal could license firms' ability to operate on a given network and so coordinate on the network's usage, the mediator does not have such power.

Each firm can decide whether or not to participate to the mechanism; that is, to report its type to the mediator and comply with the ensuing recommendations. If at least one of the firm decides not to participate, the mediator has no further role. Differently from the principal, the mediator cannot control firms' behaviors if they decide not to comply. Each firm i can decide to invest in its own infrastructure at $t = 1$ and optimally set an access fee, in case the other firm $-i$ wishes to operate on firm i's infrastructure at $t = 2$. This implies that, relative to the case with the principal, each firm's participation constraint is not relative to zero payoff, but relative to the payoff of investing in an infrastructure on its own and of setting the fee optimally.

Formally, the mediator's problem is as follows:

$$\begin{aligned}
\max \quad & \mathbb{E}_v[q(v)(-\gamma + \theta(v_1 + v_2 - 2\gamma))] \\
\text{sub. to} \quad & \mathbb{E}_v[p_1(v) + p_2(v)] \geq 0, \\
& \mathbb{E}_{v_2}[q_1(v)(-\gamma) + \theta q(v)(v_1 - \gamma) - p_1(v)] \\
& \qquad \geq \max\{x_1(v_1), \mathbb{E}_{v_2}[q_1(v_1', v_2)(-\gamma) + \theta q(v_1', v_2)(v_1 - \gamma) - p_1(v_1', v_2)]\} \\
& \mathbb{E}_{v_1}[q_2(v)(-\gamma) + \theta q(v)(v_2 - \gamma) - p_2(v)] \\
& \qquad \geq \max\{x_2(v_2), \mathbb{E}_{v_1}[q_2(v_1, v_2')(-\gamma) + \theta q(v_1, v_2')(v_2 - \gamma) - p_2(v_1, v_2')]\},
\end{aligned}$$

where $q(v) = q_1(v) + q_2(v)$, and $x_i(v_i)$ represents firm i's non-participation payoff given its type v_i.

Let us discuss $x_i(v_i)$ more in detail. There are potentially multiple ways to define the non-participation payoff and, in principle, $x_i(v_i)$ can be derived from a game in which firms simultaneously decide their investment and transfers, and possibly bargain in order to select on which infrastructure to operate. Here, in order to focus our analysis, we take a reduced form and simply assume that in case both firms decide to invest, one infrastructure is selected at random, and both firms (and consumers) operate on the selected infrastructure. We further exploit the symmetry of our setting and set this probability equal to 1/2.

If firm i does not participate, because it is an off-path event, we can freely select $-i$'s belief about i's type. Let $-i$ believe that firm i has the high type for sure, so that $-i$ finds it optimal to invest in its own infrastructure, and set the fee of $1 - \gamma$ in case his infrastructure is selected at $t = 2$. Hence, in this case, i's payoff is 0.

In case firm i's infrastructure is selected, then i can optimally set its access fee. If $v_i = 0$, it is optimal for firm i to charge the fee $1 - \gamma$ so that only the high type of firm j would accept it. Therefore:

$$x_i(0) = -\gamma + \frac{1}{2}\theta\pi(1 - 2\gamma). \tag{18}$$

If $v_i = 1$, then firm i's optimal fee can be either (i) $1 - \gamma$ so that only the high type of firm j accepts it (as above), or (ii) $-\gamma$ so that both types of firm j accept it. That is:

$$x_i(1) = -\gamma + \frac{1}{2}\theta\pi(2 - 2\gamma), \tag{19}$$

which is assumed to exceed $-\gamma + \frac{1}{2}\theta(1 - 2\gamma)$ (that is, $\pi > (1 - \pi)(1 - 2\gamma)$). This assumption is basically just for notational simplicity.

In the rest of this section, let us assume that $x_i(0) \geq 0$ (and hence $x_i(1) \geq 0$) in order to focus on the interesting parameter region:

Assumption 2. $-\gamma + \frac{1}{2}\theta\pi(1 - 2\gamma) \geq 0$.

This assumption is slightly stronger than Assumption 1. By the same logic as in the previous cases, it can be shown that the optimal mechanism sets $q_1(1,1) + q_2(1,1) = 1$ and $q_1(0,0) = q_2(0,0) = 0$, which we assume from here on.

As before, the participation constraint of the low-value type holds with equality:

$$P_i(0) = \mathbb{E}_{v_{-i}}[-\gamma q_i(0, v_{-i}) + \theta q(0, v_{-i})(-\gamma)] - x_i(0). \tag{20}$$

The constraints for the high-value type becomes:

$$\mathbb{E}_{v_{-i}}[-\gamma q_i(1, v_{-i})] + \theta\mathbb{E}_{v_{-i}}[q(1, v_{-i})(1 - \gamma)] - P_i(1) = \max\{x_i(1), \theta\mathbb{E}_{v_{-i}}[q(0, v_{-i}) + x_i(0)]\};$$

In order to define which term in r.h.s. of the previous equation binds, let $i = 1$ and observe that:

$$\begin{aligned} x_1(1) &= -\gamma + \theta\pi(1 - \gamma) \\ \theta\mathbb{E}_{v_2}[q(0, v_2)] + x_1(0) &= -\gamma + \theta\pi(q(01) + \frac{1}{2} - \gamma), \end{aligned}$$

and thus, $x_1(1) \leq \theta\mathbb{E}_{v_2}[q(0, v_2)] + x_1(0)$ iff $\frac{1}{2} \leq q(01)$. The next lemma shows that indeed this inequality holds.

Lemma 1. $q(01), q(10) \geq \frac{1}{2}$ in the optimal mechanism.

Proof. If $q(01) < \frac{1}{2}$ in some optimal mechanism, then IR binds for $v_1 = 1$ and IC does not. Then, increasing $q(01)$ is always beneficial for the principal: It strictly increases the objective (because $q(01)$ has a positive coefficient there), and it increases the left-hand side of each binding constraint (note that IC for $v_1 = 1$ is not binding). Therefore, it contradicts that the mechanism is optimal. The same logic also shows $q(10) \geq \frac{1}{2}$. □

From Lemma 1, we have that $x_1(1) \leq \theta \mathbb{E}_{v_2}[q(0, v_2)] + x_1(0)$ and so the IC constraint when $v_1 = 1$ writes as:

$$P_i(1) = \mathbb{E}_{v_{-i}}[-\gamma q_i(1, v_{-i})] + \theta \mathbb{E}_{v_{-i}}[q(1, v_{-i})(1-\gamma) - q(0, v_{-i})] - x_i(0). \qquad (21)$$

The budget constraint becomes:

$$\pi^2(-2\gamma + \theta(2-2\gamma)) + \pi(1-\pi)(-2\gamma + \theta(1-2\gamma))(q(10) + q(01))$$
$$-\pi^2\theta(q(10) + q(01)) - 2(-\gamma + \theta\pi(\frac{1}{2} - \gamma)) \geq 0.$$

We obtain that either $q(10) = q(01) = 1$ if that does not violate the budget constraint, or they solve the above budget balance constraint with equality. That is, recalling that $Q = q(01) + q(10)$, we have $q(10) = q(01) = 1$ if it is budget-feasible:

$$\pi^2(-2\gamma - 2\theta\gamma) + 2\pi(1-\pi)(-2\gamma + \theta(1-2\gamma)) - 2(-\gamma + \theta\pi(\frac{1}{2} - \gamma)) \geq 0. \qquad (22)$$

while otherwise, we have

$$Q = \frac{\pi^2(-2\gamma + \theta(2-2\gamma)) - 2(-\gamma + \theta\pi(\frac{1}{2} - \gamma))}{\theta\pi(2\pi - 1) + \gamma\pi(1-\pi)(2+2\theta)}. \qquad (23)$$

Proposition 6. Under Assumption 2, the optimal mechanism for the mediator is such that: $q(11) = 1, q(00) = 0$ and $q(01) + q(10) = Q$. If Equation 22 holds, we have $Q = 2$; otherwise, Q is given by Equation 23.

Comparing with the solution in Section 4.3, here we obtain a smaller probability of the efficient investment. While, as the principal, the mediator is able to condition on both firms' valuation and so avoid inefficient investment (when both firms report low type), it also faces a tighter budget constraint than the principal, thereby reducing the investment probability. Specifically, the difference in the budget constraint from the case with the principal is given by the last term in the previous equation, which corresponds to $2x_i(0)$, as expressed in Equations 20 and 21. Since $x_i(0) > 0$, the participation constraints are harder to satisfy and so expected payments must be lower.

Interestingly, however, these effects are weaker when γ is larger: as shown in Equation 18, firms' outside option $x_i(0)$ decreases in the investment cost γ. As a result, the l.h.s. of Equation 22 may increase in γ, implying that the budget constraint may be less likely to bind, and so the probability of investment may be larger when the investment cost is *larger*. In particular, the subsidy received by a low type firm is lower (i.e., the payment in Equation 20 is closer to zero) as γ increases, which allows relaxing the budget constraint.

The comparison with the unregulated case is however more subtle. Conditional on being away from the first best, inefficiencies are smaller in the setting with the mediator. The reason is that, as seen in Section 4.2, firm 1 may set $q(01) = 1$ and $q(10) = 0$ if γ is large enough. Instead, the mediator would always set $q(01) + q(10) > 1$. This can be easily seen by noticing that, when $q(01) = 1$ and $q(10) = 0$, the above budget constraint is slack, implying that the mediator would set larger investment probabilities. At the same time, the probability of being away from full efficiency (i.e., from q(01)=1 and q(10)=1) is not necessarily smaller in a setting with a mediator relative to an unregulated case. The reason is that, while in terms of objective function the mediator is fully aligned with the principal, in terms of commitment (or coordination) power it is not. In fact, the commitment power is larger (hence, closer to the case with the principal) in an unregulated setting, and it is a priori not clear which effect dominates.

Finally, it is useful to compare the case with the mediator to the one in Section 3.2 in which firms sequentially enter the market. On the one hand, the mediator has more information than the principal in Section 3.2 (whose investment decision are based only on firm 1's type), and this potentially leads to more efficient outcomes. On the other hand, the mediator faces coordination issues, as it cannot prevent firms from building their own infrastructure, which may be lead to efficiency losses. We show that the latter effect may dominate; that is, absent coordination power, the mediator cannot take advantage of the increased information and in fact outcomes can be less efficient than in Section 3.2 with sequential entry. To see this, notice that the welfare in the mediator case writes as

$$\pi^2(-\gamma + \theta(2 - 2\gamma)) + Q\pi(1 - \pi)(-\gamma + \theta(1 - 2\gamma)),$$

while the welfare with sequential entry in Section 3.2 writes as

$$-\gamma(\pi + (1 - \pi)q(0)) + \pi^2\theta(2 - 2\gamma) + \pi(1 - \pi)(1 - 2\gamma)\theta(1 + q(0)).$$

Taking the difference between the latter and the former, we have:

$$(1 - Q)\pi(\theta(1 - 2\gamma) - \gamma) + q(0)(\pi\theta(1 - 2\gamma) - \gamma),$$

which can be positive when $q(0)$ is large relative to Q (recall that $\pi\theta(1-2\gamma)-\gamma > 0$ from Assumption 1). For example, suppose that $\pi \to 1$, we have from Equation (23) that $Q \to 1$ and from Equation (3) that $q(0) > 0$. Hence, in this case, welfare gains are larger with sequential entry than with the mediator.

5 Conclusion

We have analyzed the optimal investment in a common infrastructure in a market characterized by network externalities. We have compared the level of investment and the associated payments in a setting in which a principal can design a mechanism specifying investment probabilities and prices to an unregulated case, both in a nascent market in which only one firm operates and another firm may arrive at a later stage, and in a more mature market in which two firms already operate. Our analysis has highlighted the value of regulation both in terms of price setting and in terms of coordination.

Our model is deliberately very simple; other important elements are worth considering. While our analysis indirectly speaks about the optimal timing of investment (showing under which conditions regulatory interventions are more efficient in more mature markets), we have not modeled explicitly the option value of waiting (say, to have a better sense of the aggregate demand) against its potential costs (say, as firms discount the future). In addition, one may introduce further uncertainty, not only on whether a potential entrant will show up in a later period but also on the number of potential entrants. In a market with several firms, the optimal mechanism may not only exploit network externalities, but also induce competition among firms so as to possibly reduce information rents. We view these as interesting avenues for future research.

References

Admati, A. R. & Perry, M. (1991), 'Joint projects without commitment', *Review of Economic Studies* **58**(2), 259–276.

Athey, S. & Segal, I. (2013), 'An efficient dynamic mechanism', *Econometrica* **81**(6), 2463–2485.

Battaglini, M., Nunnari, S. & Palfrey, T. R. (2014), 'Dynamic free riding with irreversible investments', *American Economic Review* **104**(9), 2858–2871.

Bergemann, D. & Välimäki, J. (2002), 'Information acquisition and efficient mechanism design', *Econometrica* **70**(3), 1007–1033.

Briglauer, W., Frübing, S. & Vogelsang, I. (2014), 'The impact of alternative public policies on the deployment of new communications infrastructure–a survey', *Review of Network Economics* **13**(3), 227–270.

Cambini, C. & Jiang, Y. (2009), 'Broadband investment and regulation: A literature review', *Telecommunications Policy* **33**(10-11), 559–574.

Clarke, E. H. (1971), 'Multipart pricing of public goods', *Public Choice* **11**(1), 17–33.

d'Aspremont, C. & Gérard-Varet, L.-A. (1979), 'Incentives and incomplete information', *Journal of Public Economics* **11**(1), 25–45.

Groves, T. (1973), 'Incentives in teams', *Econometrica* **41**(4), 617–31.

Hellwig, M. F. (2003), ‘Public-good provision with many participants’, *Review of Economic Studies* **70**(3), 589–614.

Loertscher, S. & Marx, L. M. (2022), ‘Incomplete information bargaining with applications to mergers, investment, and vertical integration’, *American Economic Review* **112**(2), 616–649.

Mailath, G. J. & Postlewaite, A. (1990), ‘Asymmetric information bargaining problems with many agents’, *Review of Economic Studies* **57**(3), 351–367.

Marx, L. M. & Matthews, S. A. (2000), ‘Dynamic voluntary contribution to a public project’, *Review of Economic Studies* **67**(2), 327–358.

Maskin, E. & Tirole, J. (1990), ‘The principal-agent relationship with an informed principal: The case of private values’, *Econometrica* **58**(2), 379–409.

Moulin, H. (1994), ‘Serial cost-sharing of excludable public goods’, *Review of Economic Studies* **61**(2), 305–325.

Moulin, H. & Shenker, S. (1992), ‘Serial cost sharing’, *Econometrica* **60**(5), 1009–1037.

Myerson, R. B. (1982), ‘Optimal coordination mechanisms in generalized principal–agent problems’, *Journal of Mathematical Economics* **10**(1), 67–81.

Mylovanov, T. & Tröger, T. (2014), ‘Mechanism design by an informed principal: Private values with transferable utility’, *Review of Economic Studies* **81**(4), 1668–1707.

Shichijo, T. & Fukuda, E. (2021), ‘Cost-sharing mechanism for excludable goods with generalized non-rivalry’, *Journal of Economic Theory* **193**, 105233.

Vickrey, W. (1961), ‘Counterspeculation, auctions, and competitive sealed tenders’, *Journal of Finance* **16**(1), 8–37.

Vogelsang, I. (2003), ‘Price regulation of access to telecommunications networks’, *Journal of Economic Literature* **41**(3), 830–862.