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Inter-Sectoral Knowledge Diffusion and Scale Effects in Schumpeterian Growth Models

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Abstract

We formalize inter-sectoral knowledge diffusion in a standard fully endogenous Schumpeterian growth model. Each sector is simultaneously sending and receiving knowledge; thereby, to produce new knowledge, the research and development activity of each sector draws from a pool of knowledge which stems from this diffusion. This enables us to revisit the scale effects issue by revealing how this property (inconsistent with empirical evidence) relates with knowledge diffusion (the importance of which is empirically highlighted). We show that suppressing knowledge diffusion across sectors is a sufficient but not necessary condition for obtaining scale-invariancy. Then, we identify several sets of assumptions which enable us to obtain models which are reasonably consistent with empirical evidence both on scale effects and how knowledge diffuses in the economy. Specifically, these models do not exhibit scale effects (or at least not significant ones) while considering various scope of knowledge diffusion (including possible occurrence of general-purpose technologies).

Keywords. Schumpeterian growth theory / Scale effects / Knowledge diffusion / Knowledge spillovers / Non rivalry / Technological distance

JEL Classification. O30 / O31 / O33 / O40 / O41

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1 Introduction

The seminal models of endogenous growth theory based on innovation, as initiated by Romer (1990), Grossman & Helpman (1991), or Aghion & Howitt (1992), all have in common to predict that the economy's long-run per capita growth rate increases in its size, measured by the population level. The presence of this scale effects property is strongly inconsistent with twentieth century observed stylized facts. Indeed, empirical evidence both for the United States (e.g., Backus, Kehoe & Kehoe 1992) and for OECD countries (e.g., Jones 1995a) have invalidated the fact that the larger the scale of the economy is, the stronger growth will be. Furthermore, the scale effects property entails a theoretical problem if one aims at considering population growth; in particular, if population is assumed to grow at a positive and constant rate, the economy's per capita growth rate increases exponentially over time and eventually becomes infinite in the steady-state.[1]

A large body of literature has analyzed the issue of scale effects (see, for instance, the surveys in Jones 1999, Laincz & Peretto 2006, Dinopoulos & Sener 2007, Ha & Howitt 2007, or Bond-Smith 2019). The presence of scale effects in innovation-based growth models is generally related to the non rivalry property of knowledge. Indeed, each unit of knowledge produced in any given sector *can potentially* be used simultaneously and infinitely by any agent in the economy without precluding its use by any other agent, notably by the research and development (R&D) activity of any other sector. However, this does not necessarily mean that this unit *is effectively* used by all of these agents. In particular, some sectors' R&D activity may not use it; this, for various reasons. Obviously, some units of knowledge are of no use for some sectors, and usability may require some technological, or sectorial, proximity. Besides, institutional factors - such as international trade agreements, degree of patent protection granted by intellectual property law, degree of easiness of doing business - can also determine whether the knowledge produced in a given sector diffuses to other sectors, or not. This means that, beyond the non rivalry property of knowledge, it appears necessary to formalize explicitly how knowledge diffuses across sectors.

In the present paper, we develop a Schumpeterian endogenous growth model *à la* Aghion & Howitt (1992, 1998) in which the pools of knowledge used by R&D activities stem from inter-sectoral knowledge diffusion.[2] This formalization enables us to provide new insights on

[1] Two forms of scale effects can be identified in the literature: "strong scale effects", as in the first generation of endogenous growth models, and "weak scale effects" as in the semi-endogenous growth models. We provide a few more details on this distinction below. One can also refer to Jones (2005) for a thorough discussion on this subject. In the present paper, we focus on "strong scale effects" and, as it is generally done, we refer to them simply as "scale effects".

[2] The term "diffusion" has also been used to refer to the phenomenon involving that "there is a lag between the appearance of a technology and its peak usage" (Chari & Hopenhayn 1991). The link between this temporal dimension of knowledge diffusion and scale effects has been studied for instance by Schulstad (1993); he shows that "the introduction of a diffusion process through which new technology is gradually incorporated into the economy-wide stock of knowledge can reduce and eliminate the dependence of the growth rate upon the size of the economy". In this paper, we abstract away from this temporal dimension of knowledge diffusion by considering instantaneous diffusion, and we focus on inter-sectoral knowledge diffusion. The introduction of a lag involved by technology adoption remains to be explored within our model and is left for further research (one

the issue of scale effects. Let us briefly present our main contributions before developing them in more detail hereinafter.

First, a simple key message of the paper is the following. If the scope of knowledge diffusion is large (the limit case being the one in which each sector's R&D receives the whole knowledge produced in any other one), scale effects are likely to occur. On the contrary, if the scope of knowledge diffusion is narrow (the limit case being the one in which each sector uses only its own knowledge, that is the case of no inter-sectoral knowledge diffusion), it is likely that there are no scale effects. Then, this enables us to show that the assumptions on the technologies introduced in the related literature in order to develop scale-invariant endogenous growth models are formally equivalent to assuming that there is no inter-sectoral diffusion of knowledge (see points a. and b. in Subsection 3.1).

Second, deepening the analysis of the link between knowledge diffusion and scale effects, we show how an apparent paradox can be overcome. In theoretical models, inter-sectoral knowledge diffusion seems to imply scale effects. Besides, various empirical studies highlight the existence of inter-sectoral knowledge diffusion (see, for instance, Griliches 1992 and 1995; or Hall, Mairesse & Mohnen 2010) but, as mentioned above, there is no empirical evidence of important scale effects. We investigate whether it is possible to develop a growth model that maintains inter-sectoral knowledge diffusion while remaining in accordance with the empirical facts regarding scale effects. Eventually, we identify a set of assumptions consistent with empirical evidence on knowledge diffusion such that the results given by the model are also relevant regarding the scale effects property.

It is commonly agreed in the literature that the undesirable property of scale effects is fundamentally linked to the technologies considered in the models, in particular to the production function of innovations, in which one generally considers two types of inputs: rival ones (e.g., labor, capital) and a stock of knowledge. For instance, Romer (1990) considers that the R&D activity uses the whole stock of knowledge to produce a new variety of good/a new sector (this in a linear form). In other words, it is implicitly assumed that there is global knowledge diffusion; this explains why this model exhibits scale effects. Among others, Eicher & Turnovsky (1999) or Jones (1999, 2005), explain that production possibilities are likely to be characterized by increasing returns to scale because of knowledge spillovers. That is why, as the population size increases, knowledge is used by more agents, thus leading the economy to grow at a higher rate. Thenceforth, many models have eliminated scale effects by modifying the technologies initially introduced. Synthetic expositions of the various functional forms of knowledge production adopted, as well as quite exhaustive surveys of the literature on scale effects, can be found in Jones (1999, 2005), in Li (2000, 2002), in Laincz & Peretto (2006), in Dinopoulos & Sener (2007), in Ha & Howitt (2007), or in Bond-Smith (2019). Two major approaches to suppress scale effects have been identified.

A first range of scale-invariant models - including Jones (1995b), Kortum (1997), Segerstrom (1998), among others - has given birth to the *"semi-endogenous growth"* literature. The major

could for instance consider that the more distant two sectors are, the longer the lag in technology adoption).

drawback of semi-endogenous growth models lies in the fact that economic policies have no incidence on the long-run growth rates.

An alternative range of models - often referred to as "*fully* endogenous growth models without scale effects" - appeared through the impulse of Aghion & Howitt (1998), Dinopoulos & Thompson (1998), Peretto (1998), Young (1998), Howitt (1999), Peretto & Smulders (2002), among others. These models restore the effect of economic policies on long-term growth, without displaying the scale effects property. In the present paper, we develop a model which belongs to this branch.

The approach of the semi-endogenous growth literature is based on the notion of "diminishing technological opportunities"; it provides scale-invariant growth models in which the long-run growth rate of the economy is proportional to the exogenous rate of population growth. More precisely, these models exhibit "weak scale effects": scale effects are still present in the determination of the variables levels but no longer of their growth rates. Moreover, in the absence of population growth, the growth rate of the economy is nil. Finally, economic policies - especially subsidies to R&D - turn to have an impact only on the levels of economic variables, not on the long-run growth rates. In semi-endogenous growth models, returns to scale are still increasing, and (strong) scale effects are suppressed by assuming *decreasing returns in the stock of knowledge*.[3]

Contrary to semi-endogenous growth models, scale-invariant *fully* endogenous growth models consider *constant returns in the stock of knowledge*. Scale effects are eliminated through a "variety expansion mechanism." This mechanism is in line with Young (1998)'s insight that, as population grows, the proliferation of sectors reduces the efficiency of R&D activities in improving the quality of an existing product because the R&D effort is diluted in more sectors. As explained by Dinopoulos & Sener (2007), "horizontal product differentiation takes the form of variety accumulation and removes the scale effects property from these models [...]. Vertical product differentiation takes the form of quality improvements or process innovations and generates endogenous long-run growth." The model developed in the present paper builds upon this literature while introducing explicitly knowledge diffusion. Notably, this enables us to show that scale-invariancy is typically achieved by wiping out inter-sectoral knowledge diffusion; more fundamentally, we show that this is a sufficient but not necessary condition. Eventually, we provide a scale-invariant *fully* endogenous growth model with some inter-sectoral knowledge diffusion.

Successive reviews have given rise to a debate regarding the respective relevancy of using semi-endogenous or fully endogenous models. Li (2000), for instance, argues that the former methodology is more general than the latter. Ha & Howitt (2007) maintain that fully endogenous growth is more accurate; like Madsen (2008), they argue that empirical evidences are more supportive of fully endogenous Schumpeterian growth theory than they are of semi-endogenous

[3]Even though this paper focuses on fully endogenous growth theory, we derive a law of knowledge accumulation (see Lemma 1) which could be easily modified in order to consider decreasing returns in the stock of knowledge (see point c. in Subsection 2.1). This would enable us to introduce explicitly inter-sectoral knowledge diffusion in a semi-endogenous growth framework.

growth theory. Cozzi (2017) considers a hybrid model and shows that "if population is increasing fast enough the semi-endogenous growth approach characterizes the long-run, while if population grows less, is constant, or shrinks, the fully endogenous approach eventually becomes dominant." These two ranges of the literature both basically focus on how technologies determine scale effects. Obviously, besides these approaches, the literature has also studied other ways that can limit scale effects, in particular via market structure. For instance, Peretto & Connolly (2007) introduce fixed costs for product lines to cancel scale effects. Laincz & Peretto (2006) provide support for the product proliferation mechanism later used in Peretto (2018) to develop a scale-invariant Schumpeterian growth model with endogenous market structure in which the overall production structure does not need to be linear in the growth-driving factor and yet generates endogenous growth.

In the present paper, as mentioned above, we show that it is possible to develop a *fully* endogenous Schumpeterian growth model which, on the one hand does not exhibit scale effects (or, at least, not important ones), and on the other hand allows a certain inter-sectoral knowledge diffusion to be taken into account. The key point here is to provide a formalization describing how knowledge may (or may not) diffuse across sectors. For that purpose, we introduce a circle *à la* Salop (1979) within a standard Schumpeterian model *à la* Aghion & Howitt (1992, 1998). Then, we use this formalization to provide new insights on the issue of scale effects. Formally, we assume that a continuum of sectors is located on the circle and that each sector is simultaneously a sender and a receiver of knowledge. Furthermore, we assume that the knowledge produced by the R&D activity of each sector can diffuse more or less over the circle. More precisely, we formalize the following idea. The knowledge created in any given sector diffuses towards a set of other sectors; the R&D activity of each sector creates knowledge using the knowledge produced by a set of other sectors; and the size of these sets depends on the scope of knowledge diffusion (*i.e.* on the extent to which the R&D activity of any given sector interacts with the R&D activity of any other sector). Let us now examine our key results in more details.

First, we show why assuming no inter-sectoral knowledge diffusion (assumption which obviously appears as counterfactual) is a sufficient condition to have a scale-invariant fully endogenous growth model.

Second, we prove that it is not a necessary condition; indeed, we provide a set of basic assumptions on knowledge diffusion such that there are no scale effects, public policies have an impact on the growth rate, and there still remains some inter-sectoral knowledge diffusion. These assumptions may however appear questionable insofar as they impose some restrictions on the way knowledge diffuses which seem to be counterintuitive. Specifically, we first assume that, as the size of the economy increases, the scope of diffusion remains unchanged (*i.e.* the subset of sectors reached by each innovation remains unchanged). However, some innovations indubitably entail knowledge that is likely to impact a wide range of sectors of the economy (potentially all sectors) and to impact new sectors as they come into being. One can think in particular of *"General-Purpose Technologies" (GPTs)* which are not considered under these simple assumptions.[4]

Third, we relax these simplifying assumptions and investigate if the model can nevertheless comply with most of the commonly agreed empirical facts regarding growth models - namely the absence of significant scale effects, the effects of public policies, and somehow realistic interactions among sectors R&D activities. First, we consider possible arrival of GPTs. This implies scale effects; however, we show that their extent is small because it depends on the probability of occurrence of GPTs which is low, as argued by Lipsey, Carlaw & Bekar (2005). Second, we consider that the scope of knowledge diffusion changes as the economy expands. We derive the following series of results. Assuming that the scope of knowledge diffusion expands with the economy implies scale effects; but, under assumptions of increasing complexity, the extent of these scale effects decreases as the economy expand and/or these scale effects asymptotically vanish. On the contrary, assuming that the scope of knowledge diffusion contracts as the size of the economy increases (the underlying idea would be that there is some specialization effect at stake), there are negative scale effects. Ultimately, we manage to provide a *fully* endogenous Schumpeterian growth model in accordance with the main empirical facts related to innovation-based growth theory.

Our paper is thus a contribution to the *fully* endogenous growth without scale effects theory. In this literature - as argued in Jones (1999, 2005), Peretto & Smulders (2002), Laincz & Peretto (2006), Dinopoulos & Sener (2007), Ha & Howitt (2007), Aghion & Howitt (2009, Ch.4), or Bond-Smith (2019) - scale effects are eliminated by means of two assumptions. The first one consists in assuming that the size of the economy, as measured by the size of its population, has an impact on the number of sectors (or equivalently the variety of goods). The second one has to do with the way the process of knowledge accumulation is formalized.

The first assumption has been widely debated by the related literature and a consensus has been reached: most models consider that an increase in population size results in a proportionate increase in the number of sectors (e.g., Peretto 1998; Young 1998; Aghion & Howitt 1998, Ch. 12; Dinopoulos & Thompson 1998; Howitt 1999). This feature has been justified empirically (see, for instance, Laincz & Peretto 2006). Besides, it can be derived from market-based mechanisms as explained, for instance, by Dinopoulos & Sener (2007). This first assumption introduces a form of "variety expansion mechanism" and thus neutralizes a possible channel for scale effects insofar as the increase in R&D effort made possible by a growing population is diluted by the increasing number of sectors. In fact, one could think that this assumption could be sufficient to suppress scale effects from models initially exhibiting this undesirable property. In these models, the number of sectors is fixed; hence, as the population increases, so does the quantity of labor available for R&D activity in each sector. Consequently, each sector is likely to produce more innovations, which eventually fosters growth. Introducing an assumption whereby the population size and the number of sectors increase proportionally cancels out this effects. Indeed, as the population increases, the R&D effort spreads over a growing number

[4] A few examples of GPTs are writing, electricity, or computer science. For more details on the theory and applications of GPTs, we refer the reader to Helpman (1998), to Lipsey, Carlaw & Bekar (2005), or to Aghion & Howitt (2009, Ch. 9).

of sectors; thereby, the effect on the overall rate of productivity growth dispels. Nevertheless, this assumption may not be sufficient to obtain scale invariancy because there is a second more intricate mechanism at stake. Knowledge diffusion may imply that, as the number of sectors increases, not only each sector receives knowledge from more sectors, but each sector produces knowledge which diffuses to a growing number of sectors. Accordingly, there is an increasing number of sectors, each of which having access to more knowledge, and thus producing more knowledge, which then diffuses to a growing number of sectors. In short, if knowledge diffusion is sufficiently broad, an increase in the population - because it goes along with an increase in the number of sectors - can give rise to a "snowball effect" leading to a higher growth rate. In the present paper, we aim to delve further into this issue. For that purpose, we also make this proportionality assumption in order to be able to focus on a second possible channel of scale effects, namely knowledge diffusion.

The second assumption - which relates to the formalization of the process of knowledge accumulation - is implicitly connected to knowledge diffusion. Specifically, it relates to how the input knowledge is considered in the technology formalizing the production of innovations. The surveys mentioned above identify two types of formalization. a) It can be assumed that, in each sector, R&D activity produces new knowledge by using merely the knowledge previously accumulated within this sector (e.g., Segerstrom 1998; Peretto 1999; Acemoglu 2009, Ch. 14; Aghion & Howitt 2009, Ch. 4). In other words, they explicitly consider the absence of inter-sectoral diffusion of knowledge. b) Alternatively, it can be assumed that, in each sector, R&D activity produces new knowledge by using the average knowledge across all sectors (e.g., Dinopoulos & Thompson 1998; Peretto 1998; in Howitt 1999; Li 2000 and 2003). The framework introduced in the present paper enables us to show that assuming that the pool of knowledge used by R&D activity in each sector consists in the average knowledge also amounts to considering no inter-sectoral knowledge diffusion; this seems somewhat counterintuitive given the assumption initially made here.

This more or less explicit assumption of absence of inter-sectoral knowledge diffusion is a way to neutralize a second channel of scale effects: "as discussed in detail in Aghion and Howitt (1998), Peretto (1998) and Peretto & Smulders (2002), as long as the knowledge aggregator does not rise with the number of sectors/goods, there is no scale effect." (Connolly & Peretto 2007). However, this assumption not only removes inter-sectoral knowledge diffusion from growth models originally highlighting interaction between sectors (e.g., Romer 1990; Aghion & Howitt 1998; Jones 2005), it also goes against the fact that many empirical studies have shown the significance of the interactions between sectors (e.g., Griliches 1992 and 1995; Hall 2004; Hall, Mairesse & Mohnen 2010).

This is a key point on which the present paper stands out from the related literature. The explicit formalization of knowledge diffusion in a standard Schumpeterian growth model enables us to consider a variety of pools of knowledge in which R&D activities draw from to produce new knowledge. Thereby, we are able to complete the results of the related literature, and to study the hereinabove paradox.

The paper is organized as follows. In Section 2, we develop a *fully* endogenous Schumpeterian growth model in which we formalize explicitly knowledge diffusion and how it gives rise to the pools of knowledge used by R&D activities. Besides, we provide the basic intuitions regarding the link between scale effects and knowledge diffusion. Finally, we fully characterize the decentralized equilibrium *à la* Aghion & Howitt (1992). In Section 3, we use this model to revisit the issue of scale effects. We conclude in Section 4. All computations are provided in Appendix - Section 5.

2 A Fully Endogenous Schumpeterian Growth Model With Explicit Knowledge Diffusion

In this section, we develop a fully endogenous Schumpeterian growth model in which we explicitly formalize knowledge diffusion. In particular, we explain how inter-sectoral knowledge diffusion shapes the pools of knowledge in which R&D activities draw from to produce new knowledge. In Subsection 2.1, we present the assumptions on which the process of knowledge creation rely, and we derive the resulting general law of knowledge accumulation: in each sector, knowledge is produced using two complementary factors, labor and a pool of knowledge. This enables us to give some basic intuitions on the origin of scale effects. In Subsection 2.2, we formalize how these pools stem from knowledge diffusion. In Subsection 2.3, we present the remaining assumptions. Finally, in Subsection 2.4 we define and characterize the Schumpeterian equilibrium *à la* Aghion & Howitt (1992).

In all what follows, g_{z_t} denotes the rate of growth, $\dot{z}_t/z_t$, of any variable z_t. There is a continuum Ω_t, of measure N_t, of intermediate sectors uniformly distributed on a circle in the spirit of Salop (1979). At each date t, each sector ω, $\omega \in \Omega_t$, is characterized by a stock of knowledge $\chi_{\omega t}$ and by an intermediate good ω, produced in quantity $x_{\omega t}$, which embodies this stock of knowledge. Assuming that knowledge is homogenous, the whole stock of knowledge in the economy at date t is

$$\mathcal{K}_t = \int_{\Omega_t} \chi_{\omega t}\, d\omega. \tag{1}$$

In this paper, we aim to develop a Schumpeterian model in direct line with the fully endogenous growth models without scale-effects (e.g., Aghion & Howitt 1998; Dinopoulos & Thompson 1998; Peretto 1998; Young 1998; Howitt 1999; Segerstrom 2000). These quality improving innovations models do not exhibit the undesirable property of scale effects while maintaining the effects of public policies. As underlined by the literature reviews in Jones (1999), Laincz & Peretto (2006), Dinopoulos & Sener (2007), Ha & Howitt 2007), Aghion & Howitt (2009, Ch. 4), or Bond-Smith (2019), scale effects are removed through a "variety expansion mechanism". As stated by Bond-Smith (2019), in these models "the entry of variety expanding ideas in response to population growth mitigates increasing returns to innovation by spreading research effort over a wider variety of ideas" and "constrains the impact of increasing returns from a growing population on innovation." In other words, the increase in R&D effort made possible by the growing population is diluted by the expanding set of sectors. In that respect, these papers all have in common to somehow consider that the number of sectors increases as the population level does so. In the present paper, we aim to investigate the link between knowledge diffusion and scale effects within this range of the literature. In that respect, we also introduce a "variety expansion mechanism" assumption in order to turn off the channel between the intensity of R&D within each sector and scale effects, which enables us in turn to focus on the channel between knowledge diffusion and scale effects. Formally, we consider the following standard set of assumptions.

Assumption 1. *The measure of the set of sectors N_t and the size of the population L_t are proportional: $N_t = \gamma L_t$, $\gamma > 0$. Population grows at constant rate $g_{L_t} = n, n \geqslant 0$, and its initial size, L_0, is normalized to one.*

In what follows, we will refer to L_t or indifferently N_t as to the "size of the economy". Besides, for simplicity's sake and through misuse of language, we will often refer to N_t as to the "number of sectors", even though the set of sectors is a continuum.

There are several ways to justify a linear relation between the number of sectors and the population level.[5] The main argument put forward by Laincz & Peretto (2006) is empirical. They show that, even though this relation "might induce one to conclude that this class of models requires another *'knife-edge'* condition in that one needs to assume that the number of firms is exactly proportional to population", the number of establishments is indeed proportional to employment according to their data. This relation can also be justified theoretically; as stated by Dinopoulos & Sener (2007), "the linear relationship between the number of varieties and the level of population can be derived from market-based mechanisms with solid micro foundations." For instance, in Young (1998), it is derived under the standard assumptions of fixed-entry costs and monopolistic competition. Howitt (1999) or Segerstrom (2000), consider both vertical (quality improving) and horizontal (variety expanding) R&D activities; linearity is then obtained as a general equilibrium result. Peretto (2018) presents a Schumpeterian growth model with endogenous market structure that allows the overall production structure to be more than linear in the growth-driving factor, and in which linearity is achieved in the steady state. In Aghion & Howitt (1998, Ch. 12, and 2009, Ch. 4), the set up is quite straightforward; it can easily be adapted in the present paper to obtain the linear relation of Assumption 1. Assume that the probability of inventing a new intermediate good at date t is a linear function of the population size and that, at each date t, an exogenous fraction of intermediate goods becomes obsolete and vanishes. Then, the variation of the number of sectors at date t is given by $\dot{N}_t = \kappa L_t - \xi N_t$, where κ and ξ are positive parameters, and where $L_t = e^{nt}$. The solution of this differential equation is $N_t = \frac{\kappa}{n+\xi}\left(e^{nt} - e^{-\xi t}\right), \forall t$; dividing both sides by L_t gives

[5] Jones (1999) provides a discussion on the more general relation $N_t = L_t^{\beta}, \beta \gtreqless 1$. This assumption could be introduced in our model in replacement of Assumption 1 to generalize our analysis and more specifically to investigate how this channel of growth intertwine with knowledge diffusion to possibly generate scale effects; this issue is left for further research.

$\frac{N_t}{L_t} = \frac{\kappa}{n+\xi}\frac{e^{nt}-e^{-\xi t}}{e^{nt}} = \frac{\kappa}{n+\xi}\left(1 - e^{-(\xi+n)t}\right), \forall t$. Consequently, since $\xi + n > 0$, the ratio number of intermediate sectors over population level will eventually stabilize at a steady-state value, $(N_t/L_t)^{ss} = \kappa/(n+\xi) \equiv \gamma$.

One might think that such a proportionality assumption may be sufficient to cancel scale effects. Indeed, part of the issue in the Schumpeterian growth models initially exhibiting scale effects relates to the fact that the set of sectors in the economy is constant. Accordingly, assuming an increasing population implies that the quantity of labor devoted to R&D in each sector is also likely to increase, thus leading to the production of more innovations in each sector, and therefore to higher growth. Then, introducing an assumption according to which the size of the population impacts the size of the set of sectors could offset this source of scale effects. One could indeed infer that, as the population increases, sectors proliferation would dilute R&D effort in a larger set of sectors, thus dissipating its effect on the overall rate of productivity growth. However, as it will be underlined in this paper, this assumption is not sufficient to cancel scale effects. In fact, an expanding set of sectors may result in a more intricate issue which has to do with the nature of knowledge spillovers, and more precisely with the way knowledge diffuses across sectors. This can be summarized as follows. Given that an increase in the population size goes along with an expansion of the set of sectors, if knowledge spillovers are sufficiently strong (*i.e.* if inter-sectoral knowledge diffusion is broad enough), not only the knowledge produced in each sector will diffuse to an expanding set of sectors, but each sector will receive knowledge from an expanding set of sectors. In brief, there will be a growing number of sectors, each of which having access to more knowledge and thus producing more innovations; this "snowball effect" may ultimately lead to higher growth. Therefore, this framework may exhibit scale effects insofar as an increase in the population size is likely to lead to a higher growth rate.

As detailed in Section 3, we delve further into this issue by analyzing precisely to what extent inter-sectoral knowledge diffusion may generate scale effects. Beforehand, let us present the process of knowledge creation and some simple intuitions regarding the fact that the presence (or the absence) of scale effects is fundamentally linked to the size of the pool of knowledge used by the R&D activity of each sector. Obviously, as it will be detailed further, the formation of these pools and their size are determined by the extent to which knowledge diffuses across sectors, that we coin the *"scope of knowledge diffusion"*, and that we will formally define in Subsection 2.2.

2.1 Knowledge accumulation and simple intuitions

In line with the standard literature discussed above, we formalize precisely the idea along which the R&D activity of any sector produces knowledge (innovations) using two types of inputs: rival goods (e.g., labor, physical capital, final good) and a non rival good (a stock of knowledge previously created). For that purpose, we consider that the mechanism at the source of the creation of knowledge relies on two core assumptions. First, we assume stochastic arrival of innovations as initially introduced in Grossman & Helpman (1991) or in Aghion & Howitt

(1992). Second, in order to take into account the fact that each R&D activity creates new knowledge making use of previously created knowledge, we introduce a new assumption along which the R&D activity of each sector ω draws from a specific *pool of knowledge* $\mathcal{P}_{\omega t}$. Formally, we assume that, for any intermediate good ω, $\omega \in \Omega_t$, if an innovation occurs at date t, the increase in knowledge $\Delta\chi_{\omega t}$ (*i.e.* the jump on the quality ladder, or the quality improvement of the intermediate good) depends positively on the current size of this pool of knowledge.

Assumption 2. *If $l_{\omega t}$ is the amount of labor devoted to R&D at date t in any intermediate sector ω, $\omega \in \Omega_t$, to move on to the next quality of intermediate good ω, innovations occur randomly with a Poisson arrival rate $\lambda l_{\omega t}$, $\lambda > 0$.*

Assumption 3. *For any intermediate sector ω, $\omega \in \Omega_t$, if an innovation occurs at date t, the increase in knowledge is $\Delta\chi_{\omega t} = \sigma\mathcal{P}_{\omega t}$, $\sigma > 0$.*

From these two assumptions, we show that the expected knowledge in any intermediate sector ω, $\mathbb{E}\left[\chi_{\omega t}\right]$, is a differentiable function of time, and we derive its law of motion. By language abuse we call "law of motion of knowledge" what is in fact the law of motion of expected knowledge. Formally, we write $\dot{\chi}_{\omega t} \equiv \frac{d\,\mathbb{E}[\chi_{\omega t}]}{dt}$. This enables us to write the following lemma.

Lemma 1. *Under Assumptions 2 and 3, the law of knowledge accumulation in any sector ω is*

$$\dot{\chi}_{\omega t} = \lambda\sigma l_{\omega t}\mathcal{P}_{\omega t}, \forall\omega \in \Omega_t. \tag{2}$$

Proof. See Appendix 5.1.

Note that, in order to obtain properties that are standard in endogenous growth theory, only the input $l_{\omega t}$ (*resp.* the input $\mathcal{P}_{\omega t}$) appears in Assumption 2 (*resp.* Assumption 3); moreover, the relations considered in these two assumptions are assumed to be linear. More generally, one could assume that the Poisson arrival rate in Assumption 2 is a function $\lambda\left(l_{\omega t}, \mathcal{P}_{\omega t}\right)$ and that the increase in knowledge, $\Delta\chi_{\omega t}$, in Assumption 3 is a function $\sigma\left(l_{\omega t}, \mathcal{P}_{\omega t}\right)$. Accordingly, one would obtain the following law of knowledge accumulation: $\dot{\chi}_{\omega t} = \lambda\left(l_{\omega t}, \mathcal{P}_{\omega t}\right)\sigma\left(l_{\omega t}, \mathcal{P}_{\omega t}\right)$, $\forall\omega \in \Omega_t$. This law is rather general and encompasses most of the ones used in the literature as illustrated by the following few examples.

a. It is interesting to note that this knowledge production function relates to the one originally introduced in the seminal paper of Romer (1990). Indeed, if one assumes $\lambda\left(l_{\omega t}, \mathcal{P}_{\omega t}\right) = \lambda l_{\omega t}$, $\sigma\left(l_{\omega t}, \mathcal{P}_{\omega t}\right) = \sigma\mathcal{P}_{\omega t}$, and $\mathcal{P}_{\omega t} = \mathcal{K}_t$, one gets $\dot{\chi}_{\omega t} = \lambda\sigma l_{\omega t}\mathcal{K}_t, \forall\omega \in \Omega_t$. Furthermore, assuming $N_t = N$ (*i.e.* $\Omega_t = \Omega$) and summing on Ω, one obtains $\dot{\mathcal{K}}_t = \lambda\sigma L_t^R\mathcal{K}_t$, where $L_t^R = \int_\Omega l_{\omega t}\, d\omega$ is the overall quantity of labor used in R&D in the economy. Hence, the law of knowledge accumulation which is here derived from assumptions made in a model with stochastic vertical innovations leads to a law of motion of the whole knowledge formally identical to the one considered in endogenous growth models with horizontal innovations as initially introduced by Romer.

b. Consider that $\lambda(l_{\omega t}, \mathcal{P}_{\omega t}) = \lambda l_{\omega t}$ (as in Assumption 2), and that $\sigma(l_{\omega t}, \mathcal{P}_{\omega t}) = \sigma \mathcal{P}_{\omega t}$ (as in Assumption 3).

- If one assumes furthermore that $\mathcal{P}_{\omega t} = \chi_{\omega t}$, one gets $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \chi_{\omega t}, \forall \omega \in \Omega_t$. This law of knowledge accumulation is similar to the ones often considered in endogenous growth models with vertical innovations as for instance in Grossman & Helpman (1991), in Segerstrom (1998), in Peretto (1999), in Acemoglu (2009, Ch. 14), or in Aghion & Howitt (2009, Ch. 4).

- Assuming that $\mathcal{P}_{\omega t} = \chi_t^{max} \equiv \max\{\chi_{\omega t}, \omega \in \Omega_t\}$, one gets $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \chi_t^{max}, \forall \omega \in \Omega_t$; a law similar to the one introduced initially in Aghion & Howitt (1992).

c. One last example is the following. Assuming $\lambda(l_{\omega t}, \mathcal{P}_{\omega t}) = \lambda l_{\omega t}$, $\sigma(l_{\omega t}, \mathcal{P}_{\omega t}) = \sigma {\mathcal{P}_{\omega t}}^{\Phi}$, with $\sigma > 0, \Phi < 1$, and $\mathcal{P}_{\omega t} = \mathcal{K}_t$, one gets $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} {\mathcal{K}_t}^{\Phi}$, $\forall \omega \in \Omega_t$. Furthermore, assuming $N_t = N$ (*i.e.* $\Omega_t = \Omega$) and summing on Ω, one obtains $\dot{\mathcal{K}}_t = \lambda \sigma L_t^R {\mathcal{K}_t}^{\Phi}$, where $L_t^R = \int_\Omega l_{\omega t} \, d\omega$. This law - which considers decreasing returns in the stock of the whole knowledge available in the economy - is formally identical to those assumed in the semi-endogenous growth theory. Indeed, in Jones (1999), in Laincz & Peretto (2006), in Dinopoulos & Sener (2007), in Ha & Howitt (2007), in Acemoglu (2009, Ch. 13), or in Bond-Smith (2019), this theory is presented using a similar law of knowledge accumulation.

Finally, it remains to explain what exactly these pools of knowledge $\mathcal{P}_{\omega t}$ consist of, and how they are shaped. This is done in Subsection 2.2 below, in which we formalize the fact that the stock of knowledge which constitutes each of these pools stems from knowledge diffusion. In particular, we explain how knowledge can diffuse on a more or less large subset of sectors: it will be made clear that the broader the scope of knowledge diffusion is in average, the larger the pools are. Beforehand, let us give basic intuitions by presenting succinctly the two polar cases.

i. The case in which the knowledge produced in any sector ω diffuses across the whole set of sectors Ω_t (we will refer to this type of knowledge diffusion as to "global inter-sectoral knowledge diffusion"). Here, all sectors use the same pool of knowledge which comprises the whole stock of knowledge available in the economy (in each sector, the R&D activity uses the knowledge accumulated in this sector as well as in any other sector): $\mathcal{P}_{\omega t} = \mathcal{K}_t, \forall \omega \in \Omega_t$.

ii. The case in which the knowledge produced in each sector ω does not diffuse to other sectors (there is no inter-sectoral knowledge diffusion). Here, in each sector ω, the pool of knowledge used by the R&D activity comprises only the knowledge accumulated within this sector: $\mathcal{P}_{\omega t} = \chi_{\omega t}, \forall \omega \in \Omega_t$.

Considering the law of knowledge accumulation (2) in these two polar case enables us to provide the basic intuitions regarding the link between knowledge diffusion and the presence or the absence of scale effects. In order to obtain these intuitions as directly as possible, let us use some standard assumptions and basic results which are usual in endogenous growth models. These results will be rigourously proven in Subsection 2.4 below, once the presentation of the whole model is complete.

Let us consider the commonly made assumption of symmetry across sectors.[6] In the present paper, this amounts to assuming that the quantity of labor and the level of knowledge characterizing each sector are both identical across sectors: $l_{\omega t} = l_t$, and $\chi_{\omega t} = \chi_t, \forall \omega \in \Omega_t$. Furthermore, computing the Schumpeterian equilibrium presented further in Definition 1, one will obtain the two following results (see Appendix 5.2 and Lemma 6). First, the quantity of labor allocated to R&D activity in each sector is constant over time and independent of L_t ($l_t = l$); therefore, in each sector ω, the law of knowledge accumulation (2) is $\dot{\chi}_{\omega t} = \lambda \sigma l \mathcal{P}_{\omega t}, \forall \omega \in \Omega_t$. Second the growth rate of per capita consumption is $g_{c_t} = g_{\chi_t} + n$. Now back to the two polar cases.

***i.* Global inter-sectoral knowledge diffusion: $\mathcal{P}_{\omega t} = \mathcal{P}_t = \mathcal{K}_t, \forall \omega \in \Omega_t$.** Using Assumption 1 and the symmetry assumption, (1) writes $\mathcal{K}_t = N_t \chi_t = \gamma L_t \chi_t$; thus, in any sector $\omega \in \Omega_t$, the law of knowledge accumulation (2) is $\dot{\chi}_t = \lambda \sigma \gamma l L_t \chi_t$. Then the growth rate of per capita consumption is $g_{c_t} = g_{\chi_t} + n = \lambda \sigma \gamma l L_t + n$; it depends on L_t. Accordingly, global inter-sectoral knowledge diffusion goes along with the presence of scale effects.

***ii.* No inter-sectoral knowledge diffusion: $\mathcal{P}_{\omega t} = \chi_{\omega t}, \forall \omega \in \Omega_t$.** From the symmetry assumption, in each sector ω, the pool of knowledge is $\mathcal{P}_{\omega t} = \mathcal{P}_t = \chi_t, \forall \omega \in \Omega_t$, and the law of knowledge accumulation (2) is $\dot{\chi}_{\omega t} = \dot{\chi}_t = \lambda \sigma l \chi_t, \forall \omega \in \Omega_t$. Then the growth rate of per capita consumption is $g_{c_t} = g_{\chi_t} + n = \lambda \sigma l + n$; it is independent of L_t. There are no scale effects in the absence of inter-sectoral knowledge diffusion.

The brief analysis of these two polar cases enables us to understand intuitively that the presence or the absence of scale effects is basically linked with the composition of the pool of knowledge used by the R&D activity of each sector. Moreover, since it is legitimate to think that these pools are shaped by the way knowledge diffuses across sectors, this analysis illustrates the key part played by inter-sectoral knowledge diffusion in the understanding of the scale effects property. In particular, it appears that suppressing inter-sectoral knowledge diffusion is a sufficient condition to remove scale effects. Is it a necessary condition? To explore these issues further, it appears necessary to formalize how knowledge diffusion shapes the pools of knowledge.

After having completed the presentation of our fully endogenous Schumpeterian growth model in the remaining of Section 2, we will establish and study rigourously the link between knowledge diffusion and scale effects in Section 3.

[6]The assumption of symmetry across sectors is commonly made in endogenous growth theory. It is required at some point to compute the first-best social optimum as well as the decentralized equilibrium; see, for instance, Aghion & Howitt (1992 or 1998 - Ch. 3), Peretto & Smulders (2002), or Peretto (2018). For more details on this issue, one can refer to Peretto (1998, 1999), Laincz & Peretto (2006) or to Cozzi, Giordani & Zamparelli (2007) in which the relevancy of the symmetric equilibrium is discussed.

2.2 Knowledge diffusion and pools of knowledge

In Subsection 2.1, we have described the knowledge accumulation process (see Lemma 1) without providing any particular specification to the pool of knowledge $\mathcal{P}_{\omega t}$ used by R&D activity in each sector ω. Now, we propose a mechanism formalizing how the composition of each of these pools relies on the influence that R&D activities have on each other. In that respect, we explicitly introduce a process of knowledge diffusion across sectors' R&D activities. The resulting framework encompasses the two polar cases described above, as well as all intermediary cases of inter-sectoral knowledge diffusion.

Many empirical studies highlight the fact that R&D performed in one sector may produce positive spillovers effects on other sectors (e.g., Griliches 1992 and 1995; Hall 2004; Hall, Mairesse & Mohnen 2010). Furthermore, as stated by Hall et al., "such spillovers are all the more likely and significant as the sender and the receiver are closely related." Consistent with these stylized facts, we consider that any given sector ω, $\omega \in \Omega_t$, is simultaneously a potential *sender* and a potential *receiver* of knowledge. As a sender, this sector's R&D activity produces knowledge $\chi_{\omega t}$ which diffuses toward other sectors. As a receiver, this sector's R&D activity uses a pool of knowledge $\mathcal{P}_{\omega t}$ which comprises the knowledge produced in this sector so far, $\chi_{\omega t}$, and potentially knowledge diffused from other sectors.

2.2.1 Sending knowledge: the scope of knowledge diffusion

We first intend to provide a formalization allowing us to consider simultaneously several types of knowledge diffusion ranging from no inter-sectoral diffusion (only intra-sectoral knowledge diffusion) to global inter-sectoral knowledge diffusion. More precisely, we want to present a model with the following two fundamental features. First, at any date t, *i)* some sectors produce knowledge that diffuses across the whole set of sectors, *ii)* some sectors produce knowledge that does not diffuse to any other one, and finally *iii)* some sectors produce knowledge that diffuses toward a more or less wide set of sectors. Second, we want the weights of the three types of sectors to be able to vary between zero and one, such that any case comprised between the two polar cases mentioned above may be encompassed. Formally, we consider the following framework in which all three types of sectors are homogeneously spread over the circle.

i. A proportion p_G of sectors are such that the knowledge produced in each of these sectors diffuses across the whole set of sectors in the economy (we will refer to *"global inter-sectoral knowledge diffusion"*). We therefore consider the possible arrival of a type of knowledge diffusion that echoes to the concept of "General-Purpose Technologies" (GPTs) as coined by Bresnahan & Trajtenberg (1995), who argue that some particular innovations "are characterized by pervasiveness [...], inherent potential for technical improvements." One can cite as examples of such innovations, in ancient times, writing and printing, and in more recent times, electricity, microchip, information and communication technologies (ICTs), or artificial intelligence (AI).

ii. A proportion p_s of sectors are such that the knowledge produced in each of them diffuses only within this sector. In this case without any inter-sectoral knowledge diffusion, we will refer to *"sector specific knowledge"*.

iii. A proportion p_m of sectors are such that the knowledge they produce diffuses over a more or less wide set of sectors. In this intermediate case, we will refer to the diffusion of *"medium knowledge"* or to "partial inter-sectorial knowledge diffusion". To simplify the analysis, we assume that if a sector ω produces medium knowledge, this knowledge diffuses symmetrically over the circle Ω_t from the location of sector ω on a subset of sectors of measure θ_t; formally, knowledge $\chi_{\omega t}$ diffuses on the subset of sectors $[\omega - \theta_t/2; \omega + \theta_t/2]$. In other words, θ_t is the "number" of receivers of knowledge $\chi_{\omega t}$; thus it is assumed to be smaller than the total "number" of sectors: $\theta_t < N_t$. We also assume that $\theta_t > 1$; the reason for this is given in footnote 7.

Furthermore, we assume that θ_t is a function of N_t, the measure of the set of sectors Ω_t (or, by abuse of language, the "number" of sectors in the economy). The reason for this lies in that as the number of sectors increase, the number of potential receivers of knowledge is affected in two possible ways. At first glance, one could consider that it increases; however, one could also consider that as the number of sectors increase, each sector tends to specialize, thus limiting the potential interactions between sectors. Moreover, since $N_t = \gamma L_t$, it is equivalent to assume that θ_t is a function of L_t. Formally, we assume $\theta_t = \theta(L_t)$, where $\theta(.)$ is a monotonous function of class C^2, bounded below by one and above by N_t. In Section 3 below, we will make some additional assumptions on this function, in particular on the signs of $\theta'(L_t)$ and $\theta''(L_t)$, and we will study the consequence of these different assumptions on scale effects.

Let us denote by $\Theta_{\omega t}$ the *"scope of knowledge diffusion"* of the stock of knowledge $\chi_{\omega t}$, which is defined as the measure of the subset of sectors of Ω_t which receive and use the knowledge $\chi_{\omega t}$ produced in sector ω. Then, since the proportions p_G, p_s, and p_m (with $p_G + p_s + p_m = 1$) can also be interpreted as the probabilities that a given sector ω produces one particular type of knowledge, $\Theta_{\omega t}$ is a random variable which can take three values: $N_t = \gamma L_t$ with probability p_G, 0 with probability p_s, and $\theta(L_t)$ with probability p_m. One gets the following lemma.

Lemma 2. *At each date t, for any sector ω, the average scope of knowledge diffusion is*

$$\mathbb{E}[\Theta_{\omega t}] \equiv \mathbb{E}[\Theta_t] = p_G \gamma L_t + p_m \theta(L_t), \forall \omega \in \Omega_t.$$

According to the framework developed so far, $\mathbb{E}[\Theta_{\omega t}]$ is the expected measure of the subset of sectors of Ω_t using the knowledge produced in sector ω. Nevertheless, an alternative interpretation of $\mathbb{E}[\Theta_{\omega t}]$ - which will be useful in the remaining of the paper - can be given. Because we assume that all three types of sectors are uniformly distributed over the circle and that, for each sector, knowledge diffuses symmetrically from its position, $\mathbb{E}[\Theta_{\omega t}]$ can also be considered as the expected measure of the subset of sectors that produce knowledge which is effectively received by sector ω. This will be made explicit below in 2.2.2. Note that the average scope of knowledge diffusion, $\mathbb{E}[\Theta_t]$, can also be related to the concept of *"technological distance"* introduced in Peretto & Smulders (2002). Indeed, they explain that the "extent to which a firm can take advantage of the public knowledge created by other firms decreases with the technological distance between the creator and the user of such knowledge." In our framework, the wider $\mathbb{E}[\Theta_t]$ is, the more likely firms are to interact: accordingly, a technological distance could be obtained as a decreasing function of $\mathbb{E}[\Theta_t]$.

2.2.2 Receiving knowledge: how are the pools of knowledge shaped?

Now, we aim to provide a formal expression of the pool of knowledge $\mathcal{P}_{\omega t}$ used by R&D activity in each sector ω. In that respect, given the formalization introduced in 2.2.1, we investigate how these pools are shaped by knowledge diffusion. Specifically, one has to determine the impact of each of the three types of knowledge diffusion on any given sector.

Let us consider the standard symmetric case in which $\chi_{\omega t} = \chi_t$, $\forall \omega \in \Omega_t$. The R&D activity of any sector ω, $\omega \in \Omega_t$, uses knowledge produced in three disjoint subsets of sectors of Ω_t. Firstly, the R&D activity of sector ω always uses the stock of knowledge $\chi_{\omega t} = \chi_t$ it produces. Secondly, the R&D activity of sector ω can also use the stock of knowledge produced in sectors that are located in the neighborhood $[\omega - \theta(L_t)/2; \omega + \theta(L_t)/2]$. On this subset of measure $\theta(L_t)$, the knowledge that could potentially be used by the R&D activity of sector ω and which comes from other sectors is equal to $[\theta(L_t)\chi_t - \chi_t]$ (the stock $\chi_{\omega t} = \chi_t$ is subtracted since it has already been taken into account). The knowledge effectively used by the R&D activity of sector ω is either diffused from sectors producing *medium knowledge* which are close enough to sector ω (*i.e.* located in the neighborhood), or from any sector producing GPTs also located in this neighborhood. Since the proportions of these sectors are respectively p_m and p_G, the total stock of knowledge diffused from this neighborhood and effectively used is thus $(p_m + p_G)[\theta(L_t)\chi_t - \chi_t]$. Thirdly, the R&D activity of sector ω can finally use the stock of knowledge produced in sectors located outside the neighborhood $[\omega - \theta(L_t)/2; \omega + \theta(L_t)/2]$, that is located on the subset of Ω_t of measure $N_t - \theta(L_t)$. The knowledge diffused from this subset that could be used by the R&D activity of sector ω is $[N_t - \theta(L_t)]\chi_t$. Since within this subset only the knowledge diffused from sectors producing GPTs can reach sector ω, and since the proportion of such sectors is p_G, the total stock of knowledge diffused from this subset and effectively used is thus $p_G[N_t - \theta(L_t)]\chi_t$. Finally, the pool of knowledge used by the R&D activity of sector ω is $\mathcal{P}_t = \chi_t + (p_m + p_G)[\theta(L_t)\chi_t - \chi_t] + p_G[N_t - \theta(L_t)]\chi_t$. By assumption one has $p_s + p_m + p_G = 1$ and $N_t = \gamma L_t$; after simplification, one gets

$$\mathcal{P}_t = p_s\chi_t + p_m\theta(L_t)\chi_t + p_G\mathcal{K}_t, \text{with } \mathcal{K}_t = N_t\chi_t = \gamma L_t\chi_t.$$

This expression can be rewritten using Lemma 2. One obtains the following lemma which summarizes the main results related to knowledge diffusion and to the pools of knowledge that stem from it.[7]

[7]We do not allow $\theta(L_t) < 1$ for consistency. Consider the intermediate case in which $p_m = 1$. Then, one has $\mathbb{E}[\Theta_t] = \theta(L_t)$, and thus $\mathcal{P}_t = \theta(L_t)\chi_t$. Assuming that $\theta(L_t) > 1$ guarantees that this pool is necessarily greater than the pool in the polar case without inter-sectoral knowledge diffusion (*i.e.* in which $p_s = 1$), $\mathcal{P}_t = \chi_t$.

Lemma 3. *At each date t, for any sector ω, the pool of knowledge used by its R&D activity is*

$$\mathcal{P}_{\omega t} \equiv \mathcal{P}_t = (p_s + \mathbb{E}[\Theta_t])\, \chi_t, \forall \omega \in \Omega_t, \text{ where } \mathbb{E}[\Theta_t] = p_G \gamma L_t + p_m \theta(L_t).$$

In Section 3, we will consider successively different assumptions on the probabilities p_G, p_s and p_m, and on the derivatives $\theta'(L_t)$ and $\theta''(L_t)$. Depending on the assumptions considered, we will get a model exhibiting the scale effects property, or not. For instance, the three following results will be rigorously proven. If $p_G = 0$ and $\theta'(L_t) = 0$, there are no scale effects. If $p_G > 0$ and/or $\theta'(L_t) > 0$, there are scale effects. If $p_G = 0$ and $\theta'(L_t) > 0$ there are scale effects, but if one assumes moreover that $\theta''(L_t) < 0$, the extent of these scale effects decreases over time.

Before completing the presentation of the model, let us return to the two polar cases presented in Subsection 2.1. From Lemmas 1, 2, and 3, one obtains the following results.

i. **Global inter-sectoral knowledge diffusion.** The first polar case corresponds to an economy in which there would only be GPTs; it is obtained by assuming $p_G = 1$. The average scope of knowledge diffusion would be maximum: from Lemma 2, one has $\mathbb{E}[\Theta_t] = \gamma L_t = N_t$. The pool of knowledge used in each sector would comprise the whole stock of knowledge in the economy: from Lemma 3, one has $\mathcal{P}_{\omega t} = \mathcal{P}_t = N_t \chi_t = \mathcal{K}_t$, $\forall \omega \in \Omega_t$. Hence, in any sector ω, the law of knowledge accumulation (2) would be $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \mathcal{K}_t$, $\forall \omega \in \Omega_t$.

ii. **No inter-sectoral knowledge diffusion.** The second polar case corresponds to an economy in which there would be only "sector specific knowledge", that is only intra-sectoral knowledge diffusion; it is obtained by assuming $p_s = 1$. The average scope of knowledge diffusion would be minimum: from Lemma 2, one has $\mathbb{E}[\Theta_t] = 0$. The pool of knowledge used in each sector would comprise solely the knowledge accumulated within the sector so far: from Lemma 3, one has $\mathcal{P}_{\omega t} = \mathcal{P}_t = \chi_t$, $\forall \omega \in \Omega_t$. Therefore, in any sector ω, the law of knowledge accumulation (2) would be $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \chi_{\omega t}$, $\forall \omega \in \Omega_t$.

2.3 Remaining assumptions

In the two previous subsections, we presented the way knowledge accumulates, the key part played by the pools of knowledge in this process, and we formalized how these pools arise from knowledge diffusion. Now, we present the remainder of the assumptions, all of which are standard in Schumpeterian growth theory.

Each household is modelled as a dynastic family whose intertemporal preferences are represented by the discounted utility $\mathcal{U} = \int_0^\infty L_t u(c_t) e^{-\rho t} dt$, where $\rho > n$ is the common subjective discount rate and $u(c_t)$ is the individual instantaneous utility at date t,[8] which is given by $u(c_t) = \ln(c_t)$.[9] Given Assumption 1, the population of workers in the economy at date t is

[8]Barro & Sala-i-Martin (1995, Ch. 2) provide more details on this formulation of the households behavior within the context of the Ramsey model of growth. See also Segerstrom (1998).

[9]The results are robust if one considers a C.E.S. instantaneous utility function of parameter $\varepsilon, u(c_t) = \frac{c_t^{1-\varepsilon}}{1-\varepsilon}$.

$L_t = e^{nt}$. Thus one has

$$\mathcal{U} = \int_0^\infty \ln(c_t) e^{(n-\rho)t} dt. \tag{3}$$

At each date t, each of the L_t identical households is endowed with one unit of labor that is supplied inelastically. The total quantity of labor L_t is used to produce the final good and in R&D activities. Hence, the labor constraint is

$$L_t = L_t^Y + \int_{\Omega_t} l_{\omega t}\, d\omega. \tag{4}$$

Besides labor, the production of the final good requires the use of all available intermediate goods, each of which is associated with its own level of knowledge. The final good production function is

$$Y_t = (L_t^Y)^{1-\alpha} \int_{\Omega_t} \chi_{\omega t}(x_{\omega t})^\alpha d\omega \ , \ 0 < \alpha < 1. \tag{5}$$

The final good has two competing uses. Firstly, it is used in the production of intermediate goods along with

$$x_{\omega t} = \frac{y_{\omega t}}{\chi_{\omega t}} \ , \ \omega \in \Omega_t, \tag{6}$$

where $y_{\omega t}$ is the quantity of final good used to produce $x_{\omega t}$ units of intermediate good ω. This technology illustrates the increasing complexity in the production of intermediate goods: as the quality of a given intermediate good increases, its production requires more resources. Secondly, it is consumed by the representative household in quantity c_t. One gets the following constraint on the final good market:

$$Y_t = L_t c_t + \int_{\Omega_t} y_{\omega t} d\omega. \tag{7}$$

This concludes the presentation of our model which is based on assumptions commonly used in fully endogenous Schumpeterian growth models; the novelty being that we explicitly formalize how knowledge diffusion gives rise to the pools of knowledge used by R&D activities. Let us now compute the Schumpeterian equilibrium.

2.4 Schumpeterian equilibrium

We consider a decentralized economy with creative destruction which is in direct line with the one introduced by Aghion & Howitt (1992).[10] Once an innovation occurs in a given sector, the innovator is granted an infinitely-lived patent, and monopolizes the production and sale of the intermediate good (which embodies knowledge) until replaced by the next innovator. This equilibrium involves two market failures.

The first one, which results from the presence of monopolies, can be corrected by an *ad valorem* subsidy ψ on each intermediate good demand. The second one relates to the externality

[10] We implicitly consider the case of drastic innovations. One could characterize the condition on the parameters, in particular the ones related to knowledge diffusion, under which innovations are drastic or non drastic (see, for instance, Gray & Grimaud 2016).

triggered by the fact that there is no market for knowledge; it can be corrected by a public tool φ which can consist in a subsidy or in a tax on the profits of R&D activities. Normalizing the price of final good to one, and denoting the rate of return on assets, the wage, and the price of intermediate good ω at date t by r_t, w_t, and $q_{\omega t}$ ($\omega \in \Omega_t$), respectively, the set of Schumpeterian equilibria is defined as follows.

Definition 1. *At each vector of public policy tools (ψ, φ) is associated a particular Schumpeterian equilibrium. It consists of time paths of set of quantities*

$$\left\{\left(c_t(\psi,\varphi), Y_t(\psi,\varphi), \{l_{\omega t}(\psi,\varphi)\}_{\omega\in\Omega}, L_t^Y(\psi,\varphi), \{x_{\omega t}(\psi,\varphi)\}_{\omega\in\Omega}, \{\chi_{\omega t}(\psi,\varphi)\}_{\omega\in\Omega}\right)\right\}_{t=0}^{\infty}$$

and of prices

$$\left\{\left(r_t(\psi,\varphi), w_t(\psi,\varphi), \{q_{\omega t}(\psi,\varphi)\}_{\omega\in\Omega}\right)\right\}_{t=0}^{\infty},$$

such that: the representative household maximizes its utility; firms maximize their profits; the final good market, the financial market, and the labor market are perfectly competitive and clear; on each intermediate good market, the innovator is granted an infinitely-lived patent and monopolizes the production and sale until replaced by the next innovator; and there is free entry on each R&D activity (i.e. the zero profit condition holds for each R&D activity).

Lemma 6 in Appendix 5.2 presents exhaustively the variables at the steady state equilibrium (all computations are provided in this appendix). In particular, one gets the growth rates of total output and of per capita consumption:

$$g_{Y_t}(\psi,\varphi) = g_{\mathcal{K}_t}(\psi,\varphi) + n \text{ and } g_{c_t}(\psi,\varphi) = g_{\mathcal{K}_t}(\psi,\varphi) = g_{\chi_t}(\psi,\varphi) + n,$$
$$\text{where } g_{\chi_t}(\psi,\varphi) = \lambda\sigma\left(p_s + \mathbb{E}[\Theta_t]\right) l(\psi,\varphi) \text{ and } l(\psi,\varphi) = \frac{1}{\gamma} - \frac{\lambda/\gamma + \rho}{\lambda\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)}. \quad (8)$$

It is noteworthy that all the equilibrium variables depend on the vector of public policy tools (ψ, φ). In particular, R&D public policies have an impact on the growth rates. In this respect, this model is in line with the ones developed in the *fully* endogenous growth theory. This literature has indeed provided endogenous growth models in which public policies affect the economy; besides, they did so while suppressing the scale effects property.

Clearly, our model might exhibit scale effects: as it will be detailed in Section 3, the growth rate of per capita consumption, $g_{c_t}(\psi,\varphi)$, can potentially depend on the size of the economy, as measured by the population level L_t, or equivalently by the measure of the set of sectors (the "number of sectors" in the economy) N_t. Indeed, $g_{c_t}(\psi,\varphi)$ is determined by $g_{\chi_t}(\psi,\varphi)$, which depends on the average scope of knowledge diffusion, $\mathbb{E}[\Theta_t]$, which itself might depend on L_t.[11]

[11] More precisely, the model might exhibit *strong* scale effects (*i.e.* $g_{c_t}(\psi,\varphi)$ might depend on the population size L_t); and it exhibits *weak* scale effects (*i.e.* $g_{c_t}(\psi,\varphi)$ depends on the population growth rate n), as long as $n > 0$. As mentioned above in the introduction (see footnote 1), this paper focuses on the issue of strong scale effects; accordingly we refer to them simply as to "scale effects". Note that, contrary to semi-endogenous growth models, the growth rate is still positive even if $n = 0$.

3 Scale Effects Property Revisited

Because it explicitly formalizes knowledge diffusion, the *fully* endogenous Schumpeterian growth model developed in Section 2 enables us to revisit the issue of scale effects by studying how this undesirable property is fundamentally linked with inter-sectoral knowledge diffusion. For that purpose, the following lemma recapitulates the relevant key results obtained when computing the set of Schumpeterian equilibria *à la* Aghion & Howitt (see Lemma 6 in Appendix 5.2 for the full characterization). To lighten the notations in this section, the argument (ψ, φ) is dropped in all variables.

Lemma 4. *Consider the Schumpeterian equilibrium presented in Definition 1 and computed in Lemma 6. One has the following results. In each sector ω, the quantity of labor devoted to R&D activity is*

$$l_{\omega t} = l = \frac{1}{\gamma} - \frac{\lambda/\gamma + \rho}{\lambda\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)}, \forall \omega \in \Omega_t; \tag{9}$$

and the pool of knowledge used by R&D activity is

$$\mathcal{P}_{\omega t} = \mathcal{P}_t = \left(p_s + \mathbb{E}\left[\Theta_t\right]\right)\chi_t, \forall \omega \in \Omega_t. \tag{10}$$

The average scope of knowledge diffusion in the economy is

$$\mathbb{E}\left[\Theta_t\right] = p_G \gamma L_t + p_m \theta\left(L_t\right), \tag{11}$$

where $p_G \geq 0$, $p_s \geq 0$, $p_m \geq 0$, and $\theta(.)$ is a monotonous function of class C^2, bounded below by one and above by N_t.

Then, in each sector ω, the law of knowledge accumulation is

$$\dot{\chi}_{\omega t} = \dot{\chi}_t = \lambda\sigma l \mathcal{P}_t, \forall \omega \in \Omega_t; \tag{12}$$

and the growth rate of knowledge is

$$g_{\chi\omega t} = g_{\chi t} = \lambda\sigma\left(p_s + \mathbb{E}\left[\Theta_t\right]\right) l, \forall \omega \in \Omega_t. \tag{13}$$

Finally, the growth rate of per capita consumption is

$$g_{c_t} = g_{\chi t} + n = \lambda\sigma\left(p_s + \mathbb{E}\left[\Theta_t\right]\right) l + n = \lambda\sigma\left[p_s + p_G\gamma L_t + p_m\theta\left(L_t\right)\right] l + n. \tag{14}$$

As mentioned above, the model may or may not exhibit scale effects insofar as the growth rate of per capita consumption, g_{c_t}, given in (14), may or may not depend on the size of the population L_t.

First and foremost, as shown by (9), the R&D effort (*i.e.* the "number of researchers") in each sector, l, does not depend on the size of the population, L_t. This results from the fact that in the present framework two main opposite effects exactly compensate each others. On the one hand, when L_t increases, the number of potential researchers increases; this tends to imply that l increases in L_t. On the other hand, there is a "variety expansion mechanism": when the “number of sectors” N_t increases, there is a potential dilution of the “number of researchers” available for each sector; since L_t and N_t are assumed to be proportional, this tends to imply that l decreases in L_t. Because of the two commonly made assumptions of linearity between L_t and N_t, and of symmetry across sectors, these two effects cancel each other out; thereby, l is independent of L_t and N_t.

Consequently, the presence of any scale effects cannot be due to the fact that an increase in L_t would imply a rise in the quantity of labor used in the R&D activity of each sector, l, leading to a higher growth rate. This feature - which results from assumptions commonly made in the related literature - enables us to suppress a potential channel of scale effects in order to focus on the part played by knowledge diffusion. In the present framework, the transmission channel for scale effects necessarily involves that an increase in L_t leads to an increase in the marginal productivity of labor in R&D.

As shown in Lemma 5 below, a key determinant of this marginal productivity is obviously the size of the pool of knowledge available; more fundamentally, this lemma proves that the marginal productivity of labor in R&D might depend intricately on L_t since it depends on $\mathbb{E}[\Theta_t]$ (which, as seen in (11), may depends on L_t) and on χ_t (which itself depends on $\mathbb{E}[\Theta_t]$).

Lemma 5. *In each sector, the marginal productivity of labor in R&D activity is*

$$\frac{\partial \dot{\chi}_t}{\partial l} = \lambda\sigma \left(1 + l\lambda\sigma \int_0^t \left(p_s + \mathbb{E}[\Theta_u]\right) du\right) \mathcal{P}_t,$$

where $\mathcal{P}_t = \left(p_s + \mathbb{E}[\Theta_t]\right)\chi_t$, $\chi_t = \chi_0 e^{\lambda\sigma l \int_0^t (p_s + \mathbb{E}[\Theta_u]) du}$, *and* $\mathbb{E}[\Theta_t] = p_G \gamma L_t + p_m \theta(L_t)$.

Proof. See Appendix 5.3.

As shown by (14), scale effects can only stem from the fact that the average scope of knowledge diffusion $\mathbb{E}[\Theta_t]$ depends on L_t. More precisely, g_{c_t} can only depend on L_t through two fundamental terms, each of which corresponding to a specific type of knowledge diffusion. The first term, $p_G \gamma L_t$, relates to the presence of general-purpose technologies (GPTs). The second one, $p_m \theta(L_t)$, relates to the presence of “medium knowledge” and how its diffusion across sectors is impacted by L_t.

To go further in the analysis of the link between knowledge diffusion and scale effects, we introduce a measure of the impact of L_t on the growth rate of per capita consumption:

Definition 2. *The measure of scale effects is*

$$\mathcal{S}_t = \frac{\partial g_{c_t}}{\partial L_t} = \lambda\sigma l \frac{\partial \mathbb{E}[\Theta_t]}{\partial L_t} = \lambda\sigma l \left[p_G \gamma + p_m \theta'(L_t)\right]. \tag{15}$$

Clearly, as mentioned above, two types of inter-sectoral knowledge diffusion have an influence on the extent of scale effects:

1. Scale effects can result from the presence of GPTs: $\mathcal{S}_t$ depends on the probability p_G of occurrence of GPTs.

2. Scale effects can also be a consequence of the presence of medium knowledge (*i.e.* knowledge impacting a more or less wide range of sectors): $\mathcal{S}_t$ depends both on the probability p_m of arrival of this type of knowledge, and on the derivative $\theta'(L_t)$, which measures to what extent the diffusion of this knowledge is affected by an increase in the size of the population L_t, or equivalently in the measure of the set of sectors (the "number of sectors") N_t (since $N_t = \gamma L_t$). Regarding the sign of this derivative, we argue that it depends on two opposite effects.

   (a) On the one hand, one could argue that, as the "number of sectors" N_t increases, the knowledge produced in any given sector can potentially influence more sectors. This *"expanding effect"* tends to increase $\theta(L_t)$, that is the measure of the subset of sectors that use medium knowledge $[\omega - \theta(L_t)/2; \omega + \theta(L_t)/2]$.

   (b) On the other hand, it could also be argued that, as its size increases, the economy becomes more complex, which implies that sectors specialize and become less likely to interact. This *"specialization effect"* tends to reduce $\theta(L_t)$.

   If the *expanding effect* outweighs the *specialization effect*, then the derivative $\theta'(L_t)$ is positive. Conversely, if the *specialization effect* outweighs the *expanding effect*, then $\theta'(L_t)$ is negative.

In Subsection 3.1, we study three particular cases of knowledge diffusion within the *fully* endogenous Schumpeterian growth model developed in Section 2. Studying in turn only sector specific knowledge, global inter-sectoral knowledge diffusion, and only constant partial inter-sectoral knowledge diffusion enables us to understand the fundamental relation between knowledge diffusion and scale effects. Then, in Subsection 3.2, we consider more general cases in which several types of knowledge diffusion coexist, and in which partial inter-sectoral knowledge diffusion may rely on more sophisticated assumptions. In these more sophisticated cases, scale effects may be present, but we identify under which assumptions they could nevertheless be cancelled, or at least mitigated. Eventually, not only do we provide a richer model but also a model more in line with empirical facts both from the point of view of scale effects and of knowledge diffusion.

3.1 Knowledge diffusion and scales effects: the main insights

In the following proposition we consider three particular cases of knowledge diffusion and, for each of them, we give the key results related to scale effects. Each of these three cases consider only one type of knowledge diffusion at once (this assumption will be relaxed in Subsection 3.2). Case 1 and Case 2 correspond to the two polar cases presented above, namely to polar case ii. (in which there is no inter-sectoral knowledge diffusion) and to polar case i. (in which there is global inter-sectoral knowledge diffusion), respectively. Case 3 consists in an intermediate case in which partial inter-sectoral knowledge diffusion is considered in the simplest way insofar as the scope of diffusion of medium knowledge remains constant as the economy expands (this assumption will also be relaxed in Subsection 3.2).

Proposition 1. *The link between knowledge diffusion and scales effects is illustrated by the following three elementary cases.*

Case 1. *If $p_s = 1$ (i.e. $p_G = p_m = 0$), then $\mathbb{E}[\Theta_t] = 0$ and $\mathcal{S}_t = 0$.*

There are no scale effects due to the absence of inter-sectoral knowledge diffusion.

Case 2. *If $p_G = 1$ (i.e. $p_s = p_m = 0$), then $\mathbb{E}[\Theta_t] = \gamma L_t = N_t$ and $\mathcal{S}_t = \lambda\sigma l\gamma > 0$.*

There are scale effects due to the presence of global inter-sectoral knowledge diffusion.

Case 3. *If $p_m = 1$ (i.e. $p_s = p_G = 0$) and $\theta(L_t) = \theta > 1$, then $\mathbb{E}[\Theta_t] = \theta$ and $\mathcal{S}_t = 0$.*

In spite of the fact that there is some inter-sectoral knowledge diffusion, there are no scale effects. Scale invariancy results from the absence of GPTs and from the fact that the expanding and specialization effects exactly compensate each other (i.e. $p_G = 0$ and $\theta'(L_t) = 0$, respectively; see Definition 2 above).

Proof. For each of the three cases, the expressions of $\mathbb{E}[\Theta_{\omega t}]$ and $\mathcal{S}_t$ are straightforwardly derived from Lemma 4 and from (15) by taking particular values for p_G, p_s, and p_m; and, for Case 3, by assuming furthermore that $\theta(L_t)$ is constant. □

The analysis of these three cases of *fully* endogenous growth models (R&D public policies have an impact on the growth rates) is going to enable us to highlight and to understand the link between inter-sectoral knowledge diffusion and scale effects. Beforehand, let us recall some essential points. Determining whether a model exhibits scale effects, or not, basically amounts to studying the impact of an increase in the population size, L_t, on the growth rate of per capita consumption, g_{c_t}. As shown in Lemma 4 , g_{c_t} depends on the growth rate of knowledge in each sector: $g_{c_t} = g_{\chi t} + n$. It is therefore necessary to study the impact of an increase in L_t on $g_{\chi t}$.

As shown in Lemma 4, the creation of new knowledge in each sector, $\dot{\chi}_t$, involves two complementary inputs. Indeed, as seen in (12), the R&D activity of each sector uses labor in quantity l - which, as discussed above, is independent of L_t (see (9)) - as well as the pool of knowledge $\mathcal{P}_t$ given in (10). Thus, the growth rate of knowledge, $g_{\chi t} = \dot{\chi}_t/\chi_t$ depends on l and on the ratio $\mathcal{P}_t/\chi_t$. Consequently, the analysis of scale effects comes down to the following questions: do the pools, and more precisely the ratio $\mathcal{P}_t/\chi_t$, depend on L_t? and why is it so?

From (10), we know that $\mathcal{P}_t/\chi_t = p_s + \mathbb{E}[\Theta_t]$; hence, for each of the three cases presented in Proposition 1, we need to understand what the impact of an increase in L_t on the average scope of knowledge diffusion $\mathbb{E}[\Theta_t]$ is. In what follows, for each of the three cases, we will compute the marginal productivity of labor in R&D (see Lemma 5 for its general expression), and we will check whether it depends on L_t or not.

Case 1 - No inter-sectoral knowledge diffusion, absence of scale effects.

In this case, it is assumed that $p_s = 1$, and thus that $p_G = p_m = 0$. In any sector, the R&D activity produces neither GPTs nor medium knowledge, but only sector specific knowledge. Hence, the average scope of knowledge diffusion is nil: $\mathbb{E}[\Theta_t] = 0$. Since the knowledge produced in each sector thus diffuses solely within this sector, the pool of knowledge used by each sector R&D activity is limited to the stock of knowledge accumulated within this sector; formally, $\mathcal{P}_{\omega t} = \mathcal{P}_t = \chi_t, \forall \omega \in \Omega_t$. Since $\mathcal{P}_t/\chi_t = 1$, one has $g_{\chi_t} = \lambda\sigma l$, and thus $g_{c_t} = \lambda\sigma l + n$, both of which do not depend on L_t. There are no scale effects: from (15), one has $\mathcal{S}_t = 0$.

When the economy expands (*i.e.* when the size of the population L_t and thus the measure of the set of sectors (the "number of sectors") $N_t = \gamma L_t$ increase), the R&D activity of each sector keeps operating without being affected by the growing "number of sectors" producing knowledge, precisely because this knowledge does not diffuse. Each sector continues to produce its knowledge using exclusively its own stock of knowledge: $\dot{\chi}_t = \lambda\sigma l \chi_t$.

The basic reason for the absence of scale effects in this case lies in that the marginal productivity of labor in R&D is independent of L_t: from Lemma 5, one has $\frac{\partial \dot{\chi}_t}{\partial l} = \lambda\sigma\left(1 + l\lambda\sigma t\right)\chi_t$, with $\chi_t = \chi_0 e^{\lambda\sigma l t}$. That is why the increase in L_t has no effect on the R&D activity of each sector, hence no effect on the growth rate of the knowledge produced by this sector, and eventually no effect on the growth rate of per capita consumption.

Case 2 - Global inter-sectoral knowledge diffusion, presence of scale effects.

In this case, it is assumed that $p_G = 1$: the knowledge produced in any sector diffuses across the whole set of sectors. Therefore, the average scope of knowledge diffusion includes all sectors: $\mathbb{E}[\Theta_t] = N_t$. Consequently, the pool of knowledge used by R&D activity in each sector is the whole stock of knowledge in the economy (*i.e.*, it includes the knowledge accumulated in all sectors so far): $\mathcal{P}_{\omega t} = \mathcal{P}_t = \mathcal{K}_t = N_t\chi_t = \gamma L_t \chi_t, \forall \omega \in \Omega_t$. Each of these pools clearly depends on L_t. Besides, one has $\mathcal{P}_t/\chi_t = \gamma L_t$, $g_{\chi_t} = \lambda\sigma l \gamma L_t$, and thus $g_{c_t} = \lambda\sigma l\gamma L_t + n$. This case exhibits scale effects: from (15), one has $\mathcal{S}_t = \lambda\sigma\gamma l > 0$.

Here, when L_t - or equivalently N_t - increases, the R&D activity of each sector is directly affected. This impact comes from the interaction between the formation of the pools of knowledge and the production of new knowledge which gives rise to a kind of "snowball effect".

In the presence of global inter-sectoral knowledge diffusion, the pool used by the R&D activity in each sector ($\mathcal{P}_t$) grows faster than the sector's own stock of knowledge (χ_t). Formally, in Case 2, one has $g_{\mathcal{P}_t} = g_{\chi_t} + g_{N_t} = g_{\chi_t} + n$, while in the absence of inter-sectoral diffusion (Case 1), one has $g_{\mathcal{P}_t} = g_{\chi_t}$. The reason for this is twofold. First, because in Case 2 the knowledge produced in each sector diffuses to all sectors, the R&D activity of each sector receives knowledge from a subset of sectors of measure N_t, which furthermore increases with L_t (since $N_t = \gamma L_t$); conversely, this measure is nil in Case 1. Second, in Case 2, the stock of knowledge emitted by each of these sectors also increases with L_t: indeed, from Lemma 5 one has

$$\chi_t = \chi_0 e^{\lambda\sigma l \int_0^t N_u du} = \chi_0 e^{\lambda\sigma l \int_0^t \gamma L_u du} = \chi_0 e^{\frac{\lambda\sigma\gamma l}{n}(e^{nt}-1)} = \chi_0 e^{\frac{\lambda\sigma\gamma l}{n}(L_t-1)}. \tag{16}$$

Consequently, in each sector, the R&D activity uses a pool of knowledge which is an increasing function of L_t: $\mathcal{P}_t = \gamma L_t \chi_t = \gamma L_t \chi_0 e^{\frac{\lambda\sigma\gamma l}{n}(L_t-1)}$.

Moreover, since the knowledge produced by each sector, $\dot{\chi}_t$, is a linear function of the pool $\mathcal{P}_t$, and since this pool is increasing in L_t, each sector produces a flow of knowledge which increases with L_t: $\dot{\chi}_t = \lambda\sigma l \mathcal{P}_t = \lambda\sigma l\gamma L_t \chi_0 e^{\frac{\lambda\sigma\gamma l}{n}(L_t-1)}$. Then, the quantity of knowledge diffused from each sector increases as L_t increases, resulting in more knowledge received by each of the sectors of the economy, which in turn will produce more knowledge, and so on and so forth. Finally, the fact that $\mathcal{P}_t$ increases faster than χ_t basically results from this "snowball effect".

As in Case 1, this result can be understood by looking at the marginal productivity of labor in R&D. In Case 2, Lemma 5 writes

$$\begin{aligned}\frac{\partial \dot{\chi}_t}{\partial l} &= \lambda\sigma\left(1 + \lambda\sigma l \int_0^t \gamma L_u du\right)\mathcal{P}_t = \lambda\sigma\left(1 + \lambda\sigma l\gamma \int_0^t e^{nu} du\right)\mathcal{P}_t \\ &= \lambda\sigma\left(1 + \frac{\lambda\sigma l\gamma}{n}(e^{nt}-1)\right)\mathcal{P}_t, \text{ where } \mathcal{P}_t = \gamma L_t\chi_t \text{ and } \chi_t \text{ given in (16).}\end{aligned}$$

Therefore, one has

$$\frac{\partial \dot{\chi}_t}{\partial l} = \lambda\sigma\gamma\left(1 + \frac{\lambda\sigma l\gamma}{n}(L_t-1)\right)L_t\chi_0 e^{\frac{\lambda\sigma l\gamma}{n}(L_t-1)}.$$

Here, the presence of scale effects goes through the fact that, unlike in Case 1, the marginal productivity of labor in R&D increases rapidly because it depends on L_t and on χ_t (which itself depends on L_t).

The results obtained in Cases 1 and 2 prove rigourously - within a general equilibrium Schumpeterian growth model - the intuitions given at the end of Subsection 2.1 which were derived from a brief analysis of the two polar cases. Especially, we have now shown that suppressing any type of inter-sectoral knowledge diffusion (*i.e.* assuming that the R&D activity of each sector uses a pool of knowledge which consist in the stock of knowledge accumulated in this sector only) is a sufficient condition for a standard fully endogenous growth model not to exhibit scale effects. In fact, this result echoes to the way the scale effects property has been removed in most fully endogenous growth models; indeed, as it will be argued below, scale-invariancy has often been achieved by wiping out inter-sectoral knowledge diffusion.

As mentioned above (see Assumption 1), surveys by Jones (1999), Laincz & Peretto (2006), Dinopoulos & Sener (2007), Ha & Howitt (2007), or Bond-Smith (2019) explain that a "variety expansion mechanism" has been introduced in order to remove scale effects from endogenous growth models while maintaining the effects of public policies. The basic underlying idea in these fully endogenous growth models follows from Young (1998)'s insight that, as population grows, the proliferation of sectors reduces the efficiency of R&D activities in improving the quality of an existing product because the R&D effort is diluted in more sectors. The formalization typically considered relies on two assumptions. The first one is the proportionality between the size of the population and the "number of sectors"; in the present paper, we also make this assumption (see Assumption 1 and its associated comments), so as to focus on the second assumption, which relates to the way the process of knowledge accumulation is formalized. As detailed in the surveys mentioned above, two types of formalization can a priori be distinguished. Let us present both types using the framework introduced in the present paper, and show that they both boil down to considering implicitly that there is no inter-sectoral knowledge diffusion.

a. In Segerstrom (1998), Peretto (1999), Acemoglu (2009, Ch. 14), or Aghion & Howitt (2009, Ch. 4), among others, it is implicitly assumed that, in each sector, the pool of knowledge in which R&D activity draws from to produce new knowledge comprises solely the knowledge previously accumulated within this sector. Using our notations, the considered knowledge production function in each sector is $\dot{\chi}_{\omega t} = \lambda\sigma l_{\omega t}\mathcal{P}_{\omega t}$, where $\mathcal{P}_{\omega t} = \chi_{\omega t}, \forall \omega \in \Omega_t$. Therefore, in this first type of scale-invariant fully endogenous growth models, it is implicitly assumed that there is no inter-sectoral knowledge diffusion; exactly as in Case 1 of our model presented above.

b. A second type of scale-invariant fully endogenous growth models - such as the models presented in Dinopoulos & Thompson (1998), in Peretto (1998), in Howitt (1999), or in Li (2000 and 2003) - consider firm-specific knowledge production functions such that, as stated by Laincz & Peretto (2006), "spillovers depend on average knowledge".[12] In other words, as argued in the surveys mentioned hereinbefore, in each sector, new knowledge is produced using the *average knowledge across all sectors*. For instance, Laincz & Peretto (2006) formalize this assumption in equation 9 of their paper. One can equivalently refer to equations 7 and 9 in Jones (1999), to equations 13 and 14 in Dinopoulos & Sener (2007), to equation 5 in Ha & Howitt (2007), to equation 7 in Bond-Smith (2019), or to the framework used in Aghion & Howitt (2009, Ch. 4). Using our notations, this assumption amounts to considering the following knowledge production function in each sector:

$$\dot{\chi}_{\omega t} = \lambda\sigma l_{\omega t}\mathcal{P}_{\omega t}, \text{ where } \mathcal{P}_{\omega t} = \int_{\Omega_t} \frac{\chi_{ht}}{N_t} dh = \frac{\mathcal{K}_t}{N_t}, \forall \omega \in \Omega_t. \tag{17}$$

[12]The argument commonly put forward to justify this assumption follows from the idea that the number of sectors is a measure of R&D difficulty. Then, assuming that each sector uses only the average knowledge is a way to account for the fact that, as R&D difficulty increases, a given level of R&D investment is going to generates fewer innovations. As stated by Peretto & Smulders (2002), "R&D productivity depends on some measure of accumulated public knowledge that is independent of the number of firms and hence of the scale of the economy. This independence may stem from the assumption that (a) spillovers across firms are absent (e.g., Peretto 1999), that (b) spillovers depend on average knowledge (e.g., Smulders & Van de Klundert 1995; Peretto 1998; Dinopoulos & Thompson 1998), or that (c) spillovers depend on the knowledge of the most advanced firm (e.g., Young 1998; Aghion & Howitt 1998; Howitt 1999). All these models have the property that a large economy replicates the structure of a small economy. [...] Moreover, although they allow for spillovers, all these models assume that a larger number of firms undertaking independent R&D projects does not support a larger aggregate stock of public knowledge."

Actually, assuming that the pool of knowledge used by R&D activity in each sector consists in the average knowledge in the economy is equivalent to considering that this pool is limited to the stock of knowledge produced within this sector. Indeed, consider the usual symmetric case - assumption required to compute the equilibrium - in which $\chi_{\omega t} = \chi_t, \forall \omega \in \Omega_t$. The whole stock of knowledge (1) then writes $\mathcal{K}_t = N_t \chi_t$. Consequently, from (17), one has $\mathcal{P}_{\omega t} = \chi_t, \forall \omega \in \Omega_t$. Here too, it is implicitly assumed that there is no inter-sectoral knowledge diffusion as in Case 1 of our model.

To sum up, the formalization of knowledge diffusion and of the resulting pools of knowledge introduced in the present paper has enabled us to highlight that many scale-invariant fully endogenous models consider laws of knowledge accumulation in which it is eventually assumed that there is no inter-sectoral knowledge diffusion. Just like in the particular Case 1 of our model, these models implicitly suppose that $\mathbb{E}[\Theta_t] = 0$ (or equivalently that $p_s = 1$).

More fundamentally, the analysis carried out so far has revealed that under the assumption of linearity between the size of the population and the "number of sectors", a sufficient condition to suppress scale effects is to assume that there is no inter-sectoral knowledge diffusion. However, wiping out inter-sectoral knowledge diffusion from models originally considering some type of interaction between sectors appears to be in contradiction with the common view on how knowledge springs into existence. Indeed, these interactions have been strongly emphasized in growth theory (e.g., Romer 1990; Aghion & Howitt 1998; Jones 2005): it is generally agreed that new pieces of knowledge "diffuse gradually, through a process in which one sector gets ideas from the research and experience of others." (Aghion & Howitt 1998, Ch. 3). Moreover, the significance of the interactions between sectors has also been highlighted by many empirical studies (e.g., Griliches 1992 and 1995; Hall 2004; Hall, Mairesse & Mohnen 2010).

Thereby, this leads us to wonder whether assuming no inter-sectoral knowledge diffusion is a necessary condition to obtain a scale-invariant fully endogenous growth model. For the purpose of studying this question, let us now consider an intermediate case to Cases 1 and 2, the analysis of which constitutes a first basic step in the study of the apparent paradox sketched above.

Case 3 - Constant partial inter-sectoral knowledge diffusion: a fully endogenous growth model with inter-sectoral knowledge diffusion but without scale effects.

We now assume $p_m = 1$ and $\theta(L_t) = \theta > 1$. Here, unlike Case 2, there is no global inter-sectoral knowledge diffusion (*i.e.* we do not consider possible occurrence of GPTs since $p_G = 0$). However, contrary to Case 1, there is nevertheless some inter-sectoral knowledge diffusion. Furthermore, this diffusion is assumed to be constant (θ is independent of L_t); as explained above, this may be interpreted as a situation in which the expanding effect and the specialization effect exactly compensate each other. The knowledge produced in each sector diffuses to a subset of sectors the measure of which is constant. Accordingly, the average scope of knowledge diffusion is also constant: $\mathbb{E}[\Theta_t] = \theta$. Consequently, each sector receives knowledge from a subset of sectors (which includes this sector itself) the measure of which remains constant as L_t increases. Then, the pool of knowledge used by the R&D activity of each sector is $\mathcal{P}_{\omega t} = \mathcal{P}_t = \theta\chi_t, \forall\omega \in \Omega_t$. Therefore, the ratio $\mathcal{P}_t/\chi_t = \theta$ is independent of L_t. Finally, one has $g_{\chi_t} = \lambda\sigma l\theta$ and thus $g_{c_t} = \lambda\sigma l\theta + n$. This case does not exhibit scale effects: from (15), one has $\mathcal{S}_t = 0$. In fact, Case 3 is close to Case 1 hereinabove insofar as in both cases, the growth rate of knowledge in any sector, g_{χ_t}, is independent of L_t. The difference between these two cases lies in the fact that there is inter-sectoral knowledge diffusion in Case 3. However, this diffusion is circumscribed to a neighborhood of sectors which remains constant as the economy expands. Note that, the greater the value of θ, the higher the growth rate g_{χ_t}. Because of this inter-sectoral diffusion, the growth rate of knowledge in each sector is higher in Case 3 than in Case 1. Nevertheless, like in Case 1, the basic reason for the absence of scale effects is also related to the fact that the marginal productivity of labor in R&D is independent of L_t; indeed, from Lemma 5, one has $\frac{\partial \dot{\chi}_t}{\partial l} = \lambda\sigma\left[1 + \lambda\sigma l\theta t)\right]\mathcal{P}_t$, where $\mathcal{P}_t = \theta\chi_t$ and $\chi_t = \chi_0 e^{\lambda\sigma l\theta t}$. That is the reason why the increase in L_t has no effect on the R&D activity of each sector, no effect on the growth rate of the knowledge produced by this sector, and finally no effect on the growth rate of per capita consumption. In spite of the fact that there is some inter-sectoral knowledge diffusion, there are no scale effects.

We have seen through Case 1 that the absence of inter-sectoral knowledge diffusion is a sufficient condition to cancel scale effects in a standard fully endogenous Schumpeterian growth model; Case 3 proves that it is not a necessary condition.

The framework developed in Case 3 is in line with the *fully* endogenous Schumpeterian growth theory insofar as it does not exhibit scale effects while maintaining the impact of public policies on the growth rate. However, contrary to the previous literature, this particular case of our model preserves some inter-sectoral knowledge diffusion. Nevertheless, the simplicity of the considered diffusion of knowledge can appear questionable. Indeed, only one type of diffusion is considered at once; furthermore, it is assumed that knowledge diffusion is never expanding (absence of GPTs and restrictions on the way medium knowledge diffuses since the expanding effect and the specialization effect exactly compensate each other). Let us now return to cases which considers simultaneously several types of knowledge diffusion (including possibly GPTs) and more general assumptions on partial inter-sectoral knowledge diffusion.

3.2 Knowledge diffusion and scale effects: more insights. Expanding effect, complexity effect, specialization effect, GPTs, and Scale effects

Our objective is now to study the consequence on the scale effects property of the occurrence of GPTs, and of the fact that medium knowledge diffuses to a subset of sectors the measure of which varies with L_t (*i.e.* of the fact that the expanding and specialization effects do not exactly compensate each other in general). Eventually, we investigate under which reasonable assumptions on inter-sectoral knowledge diffusion one can obtain a Schumpeterian growth model in adequation with commonly agreed empirical facts; that is a model in which *public policies have an impact on the growth rate*, in which *GPTs can occur*, and in which *medium knowledge diffusion may be affected by the fact that the economy expands*, and which *does not exhibit scale effects* (or, if it does, in which it is not a significant problem).

Foremost, regarding the impact of public policies; as explained previously, this is not an issue in the present framework. Indeed, all the equilibrium variables depend on the vector of public policy tools (ψ, φ), and this for any assumption on knowledge diffusion (see Lemma 6, Appendix 5.2); in particular, as seen in (8), R&D public policies have an effect on the growth rates.

The study of the three cases presented in the following proposition enables us to tackle the issue related to knowledge diffusion and scale effects. We proceed in two steps. In a first step, we study independently the consequences on the scale effects property on the one hand of GPTs (Case 4) and on the other hand of the fact that medium knowledge diffusion relies on broader assumptions (Case 5). In a second step, we move on to Case 6, which considers the richest set of assumptions as it merges Cases 4 and 5.

Proposition 2. *The links between expanding effect, complexity effect, specialization effect, GPTs, and Scale effects are illustrated by the three following cases, all of which consider simultaneously several types of knowledge diffusion.*

Case 4. *If $p_G > 0$, $p_s > 0$, $p_m > 0$, and $\theta_t = \theta(L_t) = \theta > 1$ then $\mathbb{E}[\Theta_t] = p_G \gamma L_t + p_m \theta$, and*

$$\mathcal{S}_t = \lambda \sigma l p_G \gamma > 0.$$

There are scale effects due to the presence of GPTs only.

Case 5. *If $p_G = 0$, $p_s > 0$, $p_m > 0$, and $\theta_t = \theta(L_t)$, where $\theta(.)$ is a monotonous function of class C^2, bounded below by one and above by N_t, then $\mathbb{E}[\Theta_t] = p_m \theta(L_t)$, and*

$$\mathcal{S}_t = \lambda \sigma l p_m \theta'(L_t). \tag{18}$$

There might be scale effects, negative scale effects, or even no scale effects, depending on the sign of the derivative $\theta'(L_t)$, which depends on the relative extent of the expanding and specialization effects.

Sub case 5.1. *If $\theta'(L_t) > 0$, then $\mathcal{S}_t > 0$. There are scale effects because the expanding effect overcomes the specialization effect.*

Sub case 5.2. *If $\theta'(L_t) < 0$, then $\mathcal{S}_t < 0$. There are negative scale effects because the specialization effect overcomes the expanding effect.*

Sub case 5.3. *If $\theta'(L_t) = 0$, then $\mathcal{S}_t = 0$. There are no scale effects because the specialization effect and the expanding effect neutralize each other.*

Case 6. *If $p_G > 0$, $p_s > 0$, $p_m > 0$, and $\theta_t = \theta(L_t)$, where $\theta(.)$ is a monotonous function of class C^2, bounded below by one and above by N_t, then $\mathbb{E}[\Theta_t] = p_G\gamma L_t + p_m\theta(L_t)$, and*

$$\mathcal{S}_t = \lambda\sigma l \left[p_G\gamma + p_m\theta'(L_t)\right].$$

The presence of GPTs tends to imply scale effects. Besides, the consequence of the presence of medium knowledge on scale effects depends on the sign of the derivative $\theta'(L_t)$. Eventually, there might be positive, negative, or even no scale effects.

Proof. For each case, the expressions of $\mathbb{E}[\Theta_{\omega t}]$ and $\mathcal{S}_t$ are straightforwardly derived from Lemma 4 and from (15). □

Let us comment Proposition 2. Each of these three cases may exhibit scale effects; our aim is to try to find out how they could nevertheless be cancelled, or at least mitigated. For that purpose, we investigate how additional assumptions on the probability of arrival of GPTs, p_G, and on the scope of diffusion of medium knowledge, θ_t, affect the scale effects property.

Case 4 - Constant partial inter-sectoral knowledge diffusion and GPTs.

Here, we assume $p_G > 0$, $p_s > 0$, $p_m > 0$, and $\theta(L_t) = \theta > 1$. We consider simultaneously the three types of knowledge diffusion. Besides, we assume that medium knowledge diffuses to a subset of sectors the measure of which is independent of L_t: like in Case 3, the expanding and specialization effects exactly compensate each other. But, unlike in Case 3, we now allow for the possible arrival of GPTs. Thus, it is clear that in this case, there are scale effects only because of the occurrence of GPTs. The basic reason for this is that their scope of knowledge increases proportionally to the size of the economy.

Because the existence of GPTs is not questionable, one could think that one faces here a paradox in the sense that the presence of GPTs leads to a property of the model (scale effects) which is at odds with well established empirical facts (no significant evidence of scale effects). In fact, this contradiction is only apparent. The measure of scale effects is $\mathcal{S}_t = \lambda\sigma l p_G\gamma$. It is indeed strictly positive; yet, it depends on p_G, the probability of occurrence of GPTs.

As emphasized by the literature on GPTs (e.g., Helpman 1998, or Lipsey, Carlaw & Bekar 2005), we know that it is empirically reasonable to assume that, in the large mass of discoveries, GPTs are quite rare (one can think, for instance, of writing and printing, electricity, microchip, or AI); in the present framework, this amounts to assuming that p_G is low. Therefore, even if the presence of GPTs implies scale effects, their measure $\mathcal{S}_t$ is small; moreover, one has $\lim_{P_G \to 0} \mathcal{S}_t = 0$. The main intuition behind these results is obtained by analyzing the expression of the marginal productivity of labor obtained from Lemma 5:

$$\frac{\partial \dot{\chi}_t}{\partial l} = \lambda\sigma \left(1 + l\lambda\sigma \int_0^t \left(p_s + \mathbb{E}[\Theta_u]\right) du\right) \mathcal{P}_t,$$

where $\mathcal{P}_t = \left(p_s + \mathbb{E}[\Theta_t]\right)\chi_t$, $\chi_t = \chi_0 e^{\lambda\sigma l \int_0^t (p_s + \mathbb{E}[\Theta_u]) du}$, and $\mathbb{E}[\Theta_t] = p_G\gamma L_t + p_m\theta$.

The key point here lies in that, since the probability p_G is low, the impact of L_t on the marginal productivity of labor is weak, thus scale effects are not substantial. Moreover, one has $\lim_{P_G \to 0} \frac{\partial \dot{\chi}_t}{\partial l} = \lambda\sigma \left(1 + l\lambda\sigma \int_0^t (p_s + p_m\theta)\, du\right)(p_s + p_m\theta)\,\chi_t$, where $\chi_t = \chi_0 e^{\lambda\sigma l \int_0^t (p_s + p_m\theta) du}$; if GPTs are extremely rare, then the marginal productivity tends to be independent of L_t. In the end, for p_G very small, Case 4 boils down to a case similar to Case 3 (in fact, it corresponds to the merger of Cases 1 and Case 3, in which one would have simultaneously sector specific knowledge and medium knowledge).

To sum up, the occurrence of GPTs in the present framework does not necessarily contradict empirical findings according to which there is no significant evidence of the presence of scale effects.

Case 5 - Non-constant partial inter-sectoral knowledge diffusion: expanding effect, complexity effect, and specialization effect.

Here, we assume $p_G = 0$, $p_s > 0$, $p_m > 0$, and $\theta_t = \theta(L_t)$, where $\theta(.)$ is a monotonous function of class C^2, bounded below by one and above by N_t. In order to focus on the part played by the diffusion of medium knowledge, we now assume that there are no GPTs, and we consider more general assumptions on how inter-sectoral knowledge diffusion expands, or contracts, when the size of the economy, L_t, increases.

Until now, the diffusion of medium knowledge was assumed to be independent of L_t: we have considered so far that the expanding and specialization effects were neutralizing each other. This can obviously be disputable. Contrary to all the above, we now relax this simplifying assumption by considering that medium knowledge diffuses on a subset of sectors of measure $\theta_t = \theta(L_t)$, and we study how this affects scale effects.[13] Specifically, we make additional assumptions on the signs of $\theta'(L_t)$ and $\theta''(L_t)$, and we investigate under which reasonable set of assumptions, one can obtain a model in adequation with empirical evidences that there are no (substantial) scale effects.

The measure of scale effects is $\mathcal{S}_t = \lambda\sigma l p_m \theta'(L_t)$. Clearly, it depends on $\theta'(L_t)$, which represents the extent to which the diffusion of medium knowledge is affected by an increase in the size of the population L_t. In other words, $\mathcal{S}_t$ is determined by how the scope of diffusion of medium knowledge evolves as the economy expands. In Cases 5.1, 5.2, and 5.3, we study how the sign of the derivative $\theta'(L_t)$ influences the scale effects property.

Sub case 5.1 - Expanding effect and complexity effect.

If $\theta'(L_t) > 0$, the expanding effect overcomes the specialization effect, and this case exhibits scale effects ($\mathcal{S}_t > 0$) because medium knowledge diffusion expands with the size of the economy. When the "number of sectors" increases, each sector receives knowledge from an increasing "number of sectors". Hence, the size of the pool of knowledge used in each sector increases. Thereby, each sector produces more knowledge. Each sector receiving an increasing amount

[13] In fact, we study the general case of diffusion of medium knowledge presented above in Subsection 2.2.1-*iii*).

of knowledge from an increasing "number" of sectors is thus going to produce more knowledge which will then diffuse to an increasing "number of sectors". Accordingly, this snowball effect enhance the marginal productivity of labor in R&D. That is the reason why there are scale effects.

However, the following could be considered. As the size of the economy (and thus the "number of sectors") increases, the knowledge produced in each sector impacts a growing "number of sectors" (because of expanding knowledge diffusion). Nevertheless, one could think that a larger "number of sectors" makes interactions between sectors more complex. It is then reasonable to think that this increasing complexity implies that the proportion of sectors impacted within the expanding economy could decrease and eventually tend to zero. This could explain the presence of scale effects, but also why they may progressively vanish as the economy expands.

The following corollary formalizes these ideas, and shows that adding some extra assumptions on the function $\theta\left(L_t\right)$ allows us to mitigate scale effects while maintaining expanding diffusion.

Corollary 1. ***Introducing a "complexity effect".***

Assume $p_G = 0$, $p_s > 0$, $p_m > 0$, and $\theta_t = \theta\left(L_t\right)$, where $\theta\left(.\right)$ is a monotonous function of class C^2, bounded below by one and above by N_t, and $\theta'\left(L_t\right) > 0$.

1. *If $\theta''\left(L_t\right) < 0$, then $\frac{\partial \mathcal{S}_t}{\partial L_t} < 0 \Leftrightarrow \dot{\mathcal{S}}_t < 0$: the measure of scale effects decreases as the size of the economy increases, or, equivalently, is decreasing over time.*

2. *If $\lim_{L_t \to \infty} \theta'\left(L_t\right) = 0$, then $\lim_{t \to \infty} \mathcal{S}_t = 0$: scale effects asymptotically vanish.*

Proof. The derivative of (18) with respect to L_t writes

$$\frac{\partial \mathcal{S}_t}{\partial L_t} = \frac{\partial \lambda \sigma l p_m \theta'\left(L_t\right)}{\partial L_t} = \lambda \sigma l p_m \theta''\left(L_t\right) < 0.$$

Besides, since $L_t = e^{nt}$, the derivative of (18) with respect to time writes

$$\dot{\mathcal{S}}_t = \frac{\partial \lambda \sigma l p_m \theta'\left(L_t\right)}{\partial t} = \frac{\partial \lambda \sigma l p_m \theta'\left(e^{nt}\right)}{\partial t} = \lambda \sigma l p_m n e^{nt} \theta''\left(e^{nt}\right) = \lambda \sigma l p_m n L_t \theta''\left(L_t\right) < 0.$$

This proves the first point. The proof of the second point is straightforward; writing the limit of (18) given that $L_t = e^{nt}$ and that $\lim_{L_t \to \infty} \theta'\left(L_t\right) = 0$, one gets $\lim_{t \to \infty} \mathcal{S}_t = 0$. □

Corollary 1 shows that scale effects can be alleviated by considering that, even if knowledge diffusion expands, it is slowed down by the fact that the economy expansion goes along increasing complexity. Formally, this can be achieved by assuming that $\theta\left(L_t\right)$ is concave, or/and by assuming that the scope of diffusion of medium knowledge becomes asymptotically constant. If $\theta''\left(L_t\right) < 0$, the expansion of the scope of knowledge diffusion is curbed down by the proliferation of sectors (recall that $N_t = \gamma L_t$). Because the scope of knowledge diffusion does not expand as fast as the size of the economy, each sector interacts with a growing subset of sectors but the measure of this subset decreases relatively to the measure of the whole expanding set of sectors. Thereby, increasing complexity somehow dilutes scale effects: the measure of scale effects decreases as L_t increases ($\frac{\partial \mathcal{S}_t}{\partial L_t} < 0$), or equivalently decreases over time ($\dot{\mathcal{S}}_t < 0$). If $\lim_{L_t \to \infty} \theta'(L_t) = 0$, the set of sectors on which knowledge diffuses becomes constant asymptotically; because of increasing complexity, scale effects asymptotically vanish ($\lim_{t \to \infty} \mathcal{S}_t = 0$).

To sum up, even if there are scale effects induced by the fact that the expanding effect overcomes the specialization effect ($\theta'(L_t) > 0$), they can be mitigated by the introduction of a *complexity effect* ($\theta''(L_t) < 0$ or/and $\lim_{L_t \to \infty} \theta'(L_t) = 0$), allowing us to match empirical evidences and to have a non explosive growth rate.

Sub case 5.2 - Specialization effect.

Now, if $\theta'(L_t) < 0$, the expanding effect is outweighed by the specialization effect, and this case exhibits negative scale effects ($\mathcal{S}_t < 0$). In other words, the growth rate of the economy is impacted by the size of the economy, but negatively. Why is it so? From Assumption 1, we know that the "number of sectors" increases as the population level increases. Even if there is inter-sectoral knowledge diffusion, this diffusion tends to contract ($\theta'(L_t) < 0$). Specialization implies that, as sectors proliferate, each sector is likely to interact with a decreasing "number of sectors"; hence, each sector is able to use fewer stocks of knowledge.[14] This implies that, in each sector, the pool of knowledge in which R&D activity draws from tends to shrink as the economy expands; thereby weakening the marginal productivity of labor in R&D.[15]

Sub case 5.3 - Back to constant partial inter-sectoral knowledge diffusion.

If $\theta'(L_t) = 0$, the expanding and specialization effects cancel each other, and the resulting model is scale-invariant. One gets back to a case similar to Case 3, but in which one would have simultaneously sector specific knowledge and diffusion of medium knowledge.

In Cases 4 and 5, we investigated how to obtain a fully endogenous Schumpeterian growth model in which one makes reasonable assumptions on knowledge diffusion (firstly, in Case 4, on the presence of GPTs and secondly, in Case 5, on the influence of the economy expansion on the scope of diffusion of medium knowledge) while conforming to empirical evidence on the non significance of scale effects. Let us now study the most general cases of our framework which merge Cases 4 and 5; basically, we get back to the initial specification of our model presented

[14]Note that, in this paper, knowledge diffusion formalization lies on the fact that sectors share a more or less large "number" of *stocks of knowledge*, each of which being produced by a given sector. One could adapt the formalization to consider that it is the flow of knowledge inherent in each innovation that diffuses and not the accumulated stock. On the distinction between these flows and stocks of knowledge, see Gray & Grimaud (2016).

[15]Negative scale effects appears in other frameworks for other reasons. In Jones (1999), they occur as a result of the fact that "the number of sectors grows less than proportionally with population." In Peretto & Connolly (2007), fixed operating costs are introduced in an endogenous growth model with horizontal and vertical innovation; negative scale effects result from the fact that at equilibrium, the number of firms increases more than the aggregate demand.

in Section 2,

Case 6 - Expanding effect, complexity effect, specialization effect, and GPTs.

We assume $p_G > 0$, $p_s > 0$, $p_m > 0$, and $\theta_t = \theta(L_t)$, where $\theta(.)$ is a monotonous function of class C^2, bounded below by one and above by N_t. We consider simultaneously the occurrence of GPTs and non-constant partial inter-sectoral knowledge diffusion; accordingly, the average scope of knowledge diffusion in the economy is $\mathbb{E}[\Theta_t] = p_G \gamma L_t + p_m \theta(L_t)$. Our objective is to analyze the effects of GPTs and of the diffusion of medium knowledge together. The measure of scale effects is $\mathcal{S}_t = \lambda \sigma l [p_G \gamma + p_m \theta'(L_t)]$; it depends both on p_G (like in Case 4) and on $\theta'(L_t)$ (like in Case 5). The following corollary merges the results obtained when analyzing Cases 4 and 5, and illustrates how the scale effect property is affected by the presence of GPTs together with, on the one hand, an expanding knowledge diffusion and a complexity effect, and, on the other hand, a specialization effect.

Corollary 2. *Assume $p_G > 0$, $p_s > 0$, $p_m > 0$, and $\theta_t = \theta(L_t)$, where $\theta(.)$ is a function of class C^2.*

Case 6.1. *If $\theta'(L_t) \geq 0$, then $\mathcal{S}_t > 0$. The model exhibits scale effects because of the occurrence of GPTs and also potentially because the scope of medium knowledge diffusion may be expanding (if $\theta'(L_t) > 0$). However, these scale effects are not necessarily an issue for two reasons.*

- *The impact of GPTs can be considered as weak, since p_G may be assumed small.*
- *In the presence of an expanding scope of diffusion of medium knowledge, its impact can be mitigated by introducing a complexity effect, that is by assuming $\theta''(L_t) < 0$ or/and $\lim_{L_t \to \infty} \theta'(L_t) = 0$.*

Case 6.2. *If $\theta'(L_t) < 0$, then $\mathcal{S}_t \gtreqless 0$. The scale effects generated by GPTs may be more or less offset by the specialization effect that contracts the diffusion of medium knowledge.*

Corollary 2 exhibits two versions of our fully endogenous Schumpeterian growth model both complying with several commonly agreed key empirical facts. Public policies have an impact on the growth rates, GPTs can occur, and there is no significant problem of scale effects; moreover, both models consider that the scope of diffusion of medium knowledge evolves as the economy expands.

In Case 6.1. the scope of diffusion of medium knowledge stretches as the economy expands ($\theta'(L_t) \geq 0$). Then, for the scale effects issue to be mitigated, two additional assumptions on inter-sectoral knowledge diffusion are required. First, innovations involving knowledge diffusing across the whole set of sectors (GPTs) should not be too frequent (p_G must be small); since this is likely the case empirically, the impact of GPTs on scale effects can be considered as negligible. Second, the function $\theta(L_t)$ should display a complexity effect.

In Case 6.2, the scope of diffusion of medium knowledge contracts as the economy expands ($\theta'(L_t) < 0$). When the presence of GPTs is combined with the fact that the specialization effect overcomes the expanding effect, there are two opposite mechanisms at stake as the economy expands. On the one hand, each GPT enter the pool of knowledge of a growing set of sectors. On the other hand, the specialization effect contracts the subset of sectors on which medium knowledge diffuses. Eventually, the measure of scale effects $\mathcal{S}_t$ may be positive, nil, or even negative, depending on the sign of $p_G\gamma + p_m\theta'(L_t)$.

In Section 3, we revisited the issue of scale effects in light of knowledge diffusion within a *fully* endogenous Schumpeterian growth model. Firstly, we showed that the absence of inter-sectoral knowledge diffusion ($\mathbb{E}[\Theta_t] = 0$, or equivalently $p_G = p_m = 0$) is a sufficient condition to eliminate scale effects (Case 1), and that global inter-sectoral knowledge diffusion (any innovation involves knowledge which diffuses across the whole set of sectors) undoubtedly entails scale effects (Case 2). Secondly, we proved in Case 3 that the absence of inter-sectoral knowledge diffusion is not a necessary condition for scale-invariancy. In fact, as illustrated by Cases 3, 4, 5, and 6, there is no contradiction between the presence of inter-sectoral knowledge diffusion and the absence of scale effects as long as the overall scope of knowledge diffusion is not too "broad", which is the case under relatively reasonable assumptions (GPTs should not occur too frequently; and, if the scope of knowledge diffusion expands, it should do so at a lower pace than the one of the economy).

4 Conclusion

This paper develops the idea that there is a close link between the fact that fully endogenous growth models exhibit (or not) the undesirable scale effects property and the scope of knowledge diffusion considered in such models.

This link clearly appears at first when studying two polar cases of fully endogenous Schumpeterian growth models. On the one hand, a model assuming knowledge spillovers across all sectors (*i.e.* considering that the R&D activity of each sector uses a pool of knowledge that comprises all the knowledge accumulated in the economy) exhibits scale effects. On the other hand, a model assuming no inter-sectoral knowledge spillovers (*i.e.* considering that the R&D activity of each sector uses a pool that consists only of the knowledge accumulated within this sector) does not display scale effects.

The underlying reason of the link between scale effects and knowledge diffusion is found in the impact of the pools of knowledge on the marginal productivity of labor in R&D. The basic insights are as follows. The wider inter-sectoral knowledge diffusion, the larger the pool of knowledge used by each sector's R&D activity. Then, the more knowledge diffusion spreads with the size of the economy (as measured equivalently by the number of sectors or by the population level), the more likely an increase in the size of the economy will lead to larger pools of knowledge, implying a higher marginal productivity of labor in R&D activity, more innovations, and thus a higher growth rate. Hence, the more knowledge diffusion spreads along with the size of the economy, the more likely the model will be displaying scale effects.

Accordingly, it becomes obvious that a sufficient condition to have a scale-invariant fully endogenous growth model is to assume no inter-sectoral knowledge diffusion. In fact, this has been a channel often used in the fully endogenous growth literature to remove scale effects while maintaining the effect of R&D policies on the growth rate. However, the point is that the assumption according to which there would be no significant knowledge spillovers is not reasonable. Many papers have indeed pointed out that knowledge produced in a given sector may be used by the R&D activities of other sectors (e.g., Romer 1990; Scotchmer 1991; Griliches 1992, 1995; Aghion & Howitt 1998; Weitzman 1998; Hall 2004; Jones 2005; Hall, Mairesse & Mohnen 2010). Furthermore, as detailed for instance in Bresnahan & Trajtenberg (1995) or in Helpman (1998), the occurrence of general-purpose technologies (GPTs) seems indubitable. Similarly, as argued for instance by Coe & Helpman (1995), Coe, Helpman & Hoffmaister (1997 and 2009), or Bournakis, Christopoulos & Mallick (2018), there is also evidence of international R&D Spillovers.

We therefore face the following paradox. There is (some) knowledge diffusion across sectors (including the one resulting from GPTs) but there are no scale effects (or at least they are not empirically significant); and, at first glance, inter-sectoral knowledge diffusion seems to generate scale effects. In this paper, we investigate whether this paradox can be overcome and we show that it is in fact only apparent.

In that respect, we use a new methodology: we introduce explicitly knowledge diffusion in a standard fully endogenous Schumpeterian growth model. In particular, the formalization we provide explains how knowledge diffusion shapes the pools of knowledge used by R&D activities and thus determines the significance of scale effects. The first basic result we obtain within this framework confirms that if there is no inter-sectoral knowledge diffusion, there are no scale effects, precisely because the size of the pools is minimum.

This leads us to tackle the aforementioned paradox. We first build a model in which scale effects are cancelled while maintaining some inter-sectoral knowledge diffusion. Even if it solves the paradox, this model is somehow restrictive on the way knowledge diffusion is considered: it does not allow for the occurrence of GPTs, and it assumes that the scope of diffusion of knowledge remains constant as the economy expands (*i.e.* is not impacted by the size of the economy). Then, we isolate the impact of GPTs on scale effects and we show that, even if a model that considers the possible arrival of GPTs displays scale effects, these effects are not important since their strength depends on the probability of occurrence of GPTs which can be considered as low (Lipsey, Carlaw & Bekar 2005). Finally, we introduce more general assumptions on how knowledge diffusion expands (or contracts) as the economy expands and we determine under which sets of assumptions one can obtain Schumpeterian growth models that comply with most of the commonly agreed empirical facts - namely the absence of important scale effects, the impact of public policies on the growth rate, and somehow realistic interactions among sectors R&D activities (including the occurrence of GPTs).

5 Appendix

5.1 Law of knowledge accumulation - Proof of Lemma 1

Consider any given sector ω, $\omega \in \Omega_t$, and a time interval $(t, t+\Delta t)$. The level of knowledge in this sector at date t is $\chi_{\omega t}$. Let k, $k \in \mathbb{N}$, be the number of innovations that occur over $(t, t+\Delta t)$. Given Assumptions 2 and 3, the level of knowledge at date $t+\Delta t$, $\chi_{\omega\, t+\Delta t}$, is a random variable taking the values $\{\chi_{\omega t} + k\sigma\mathcal{P}_{\omega t}\}_{k\in\mathbb{N}}$ with associated probabilities $\left\{\frac{\left(\int_t^{t+\Delta t}\lambda l_{\omega u}du\right)^k}{k!}e^{-\int_t^{t+\Delta t}\lambda l_{\omega u}du}\right\}_{k\in\mathbb{N}}$. Therefore, the expected level of knowledge at date $t+\Delta t$ is

$$\mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] = \sum_{k=0}^{\infty}\frac{\left(\int_t^{t+\Delta t}\lambda l_{\omega u}du\right)^k}{k!}e^{-\int_t^{t+\Delta t}\lambda l_{\omega u}du}\left[\chi_{\omega t} + k\sigma\mathcal{P}_{\omega t}\right]. \tag{19}$$

Let $\mathcal{I}_{\omega u}$ denote a primitive of $l_{\omega u}$ with respect to the time variable u. Thus, one has

$$\int_t^{t+\Delta t}\lambda l_{\omega u}du = \lambda\left(\mathcal{I}_{\omega t+\Delta t} - \mathcal{I}_{\omega t}\right) \equiv \Lambda.$$

Accordingly, (19) rewrites

$$\begin{aligned}
\mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] &= \sum_{k=0}^{\infty}\frac{\Lambda^k}{k!}e^{-\Lambda}\left[\chi_{\omega t}+k\sigma\mathcal{P}_{\omega t}\right] = e^{-\Lambda}\left[\chi_{\omega t}\sum_{k=0}^{\infty}\frac{\Lambda^k}{k!} + \sigma\mathcal{P}_{\omega t}\sum_{k=0}^{\infty}k\frac{\Lambda^k}{k!}\right] \\
&= e^{-\Lambda}\left[\chi_{\omega t}\sum_{k=0}^{\infty}\frac{\Lambda^k}{k!} + \sigma\mathcal{P}_{\omega t}\sum_{k=1}^{\infty}k\frac{\Lambda^k}{k!}\right] = e^{-\Lambda}\left[\chi_{\omega t}\sum_{k=0}^{\infty}\frac{\Lambda^k}{k!} + \sigma\mathcal{P}_{\omega t}\sum_{k=1}^{\infty}\frac{\Lambda^k}{(k-1)!}\right] \\
&= e^{-\Lambda}\left[\chi_{\omega t}\sum_{k=0}^{\infty}\frac{\Lambda^k}{k!} + \sigma\mathcal{P}_{\omega t}\Lambda\sum_{k=1}^{\infty}\frac{\Lambda^{k-1}}{(k-1)!}\right] = e^{-\Lambda}\left[\chi_{\omega t}\sum_{k=0}^{\infty}\frac{\Lambda^k}{k!} + \sigma\mathcal{P}_{\omega t}\Lambda\sum_{k'=0}^{\infty}\frac{\Lambda^{k'}}{k'!}\right].
\end{aligned}$$

The Maclaurin series expansion for e^{Λ} is given by $e^{\Lambda} = \sum_{k=0}^{\infty}\frac{\Lambda^k}{k!}$. Therefore, one has

$$\mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] = e^{-\Lambda}\left[\chi_{\omega t}e^{\Lambda} + \sigma\mathcal{P}_{\omega t}\Lambda e^{\Lambda}\right] = \chi_{\omega t} + \sigma\mathcal{P}_{\omega t}\Lambda = \chi_{\omega t} + \sigma\mathcal{P}_{\omega t}\lambda\left(\mathcal{I}_{\omega t+\Delta t} - \mathcal{I}_{\omega t}\right).$$

Then, one can express the Newton's difference quotients of $\mathbb{E}\left[\chi_{\omega t}\right]$ and of $\mathcal{I}_{\omega t}$:

$$\frac{\mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] - \chi_{\omega t}}{\Delta t} = \lambda\sigma\frac{\mathcal{I}_{\omega t+\Delta t} - \mathcal{I}_{\omega t}}{\Delta t}\mathcal{P}_{\omega t}.$$

Finally, letting Δt tend to zero, one has $\frac{d\,\mathbb{E}[\chi_{\omega t}]}{dt} = \lambda\sigma l_{\omega t}\mathcal{P}_{\omega t}$.

Therefore, the expected knowledge at date t in any sector ω is a differentiable function of time; we thus obtain the law of accumulation of $\mathbb{E}\left[\chi_{\omega t}\right]$. By abuse of notation, we assimilate the evolution of the random variable $\chi_{\omega t}$ to the evolution of its expected value $\mathbb{E}\left[\chi_{\omega t}\right]$. Formally, we write $\dot{\chi}_{\omega t} \equiv \frac{d\,\mathbb{E}[\chi_{\omega t}]}{dt} = \lambda\sigma l_{\omega t}\mathcal{P}_{\omega t}$. This proves Lemma 1.

5.2 Schumpeterian equilibrium: characterization and computation

This appendix provides the detailed analysis of the decentralized economy presented in Definition 1. We derive the time paths of set of prices and of quantities and fully characterize the set of equilibria as functions of the public tools vector (ψ, φ).

In the **final sector**, the competitive firm maximizes its profit

$$\pi_t^Y = (L_t^Y)^{1-\alpha} \int_{\Omega_t} \chi_{\omega t}(x_{\omega t})^\alpha d\omega - w_t L_t^Y - \int_{\Omega_t} (1-\psi) q_{\omega t} x_{\omega t} d\omega.$$

The first-order conditions with respect to L_t^Y and $x_{\omega t}$ give respectively

$$w_t = (1-\alpha)\frac{Y_t}{L_t^Y} \text{ and } q_{\omega t} = \frac{\alpha (L_t^Y)^{(1-\alpha)} \chi_{\omega t}(x_{\omega t})^{\alpha-1}}{1-\psi}, \forall \omega \in \Omega_t. \tag{20}$$

Given the production function (6), in each **intermediate good sector** ω, $\omega \in \Omega_t$, the incumbent monopoly maximizes its profit

$$\pi_t^{x_\omega} = q_{\omega t} x_{\omega t} - y_{\omega t} = (q_{\omega t} - \chi_{\omega t}) x_{\omega t}, \tag{21}$$

where the demand for intermediate ω is given in (20). The first-order condition with respect to $x_{\omega t}$ gives $\frac{\alpha^2 (L_t^Y)^{(1-\alpha)} \chi_{\omega t}(x_{\omega t})^{\alpha-1}}{1-\psi} - \chi_{\omega t} = 0, \forall \omega \in \Omega_t$. Hence, one gets the usual result of symmetry in the use of intermediate goods:

$$x_{\omega t} = x_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y, \forall \omega \in \Omega_t. \tag{22}$$

The final good production function (5) can be rewritten using (22) together with the definition of the whole disposable knowledge in the economy (1); one gets

$$Y_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{K}_t. \tag{23}$$

Log-differentiating (23) with respect to time gives

$$g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t}. \tag{24}$$

The final good resource constraint (7) can be rewritten using (22) together with (1) and (6); one gets $Y_t = L_t c_t + [\alpha^2/(1-\psi)]^{\frac{1}{1-\alpha}} L_t^Y \mathcal{K}_t$. Dividing both sides by Y_t and using (23), one gets

$$\frac{L_t c_t}{Y_t} = 1 - \frac{\alpha^2}{1-\psi} \tag{25}$$

Log-differentiating (25) gives $g_{L_t} + g_{c_t} - g_{Y_t} = 0$. Since $g_{L_t} = n$ from Assumption 1, one gets

$$g_{Y_t} = g_{c_t} + n. \tag{26}$$

The wage and the price of intermediate goods given in (20) can be rewritten using (23) and (22), respectively:

$$w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t \text{ and } q_{\omega t} = \frac{\chi_{\omega t}}{\alpha}, \forall\omega \in \Omega_t. \tag{27}$$

From (22) and from the marked-up price of intermediate good ω given in (27), one can rewrite (21), the instantaneous monopoly profit on the sale of each intermediate good ω, as

$$\pi_t^{x_\omega} = \frac{1-\alpha}{\alpha}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y \chi_{\omega t}, \forall\omega \in \Omega_t. \tag{28}$$

Let us now consider any **R&D activity** ω, $\omega \in \Omega_t$, and derive the innovators' arbitrage condition. Given the governmental intervention on behalf of R&D activities, the incumbent innovator having successfully innovated at date t receives, at any date $\tau > t$, the net profit $\tilde{\pi}_\tau^{x_\omega} = (1+\varphi)\pi_\tau^{x_\omega}$ with probability $e^{-\int_t^\tau \lambda l_{\omega u} du}$ (*i.e.* provided that there is no innovation upgrading intermediate good ω between t and τ). The sum of the present values of the incumbent's expected net profits on the sale of intermediate good ω, at date t, is therefore $\tilde{\Pi}_t^{x_\omega} = \int_t^\infty \tilde{\pi}_\tau^{x_\omega} e^{-\int_t^\tau (r_u + \lambda l_{\omega u})du} d\tau$. Differentiating this expression with respect to time gives the arbitrage condition in each R&D activity ω:

$$r_t + \lambda l_{\omega t} = \frac{\dot{\tilde{\Pi}}_t^{x_\omega}}{\tilde{\Pi}_t^{x_\omega}} + \frac{\tilde{\pi}_t^{x_\omega}}{\tilde{\Pi}_t^{x_\omega}}, \forall\omega \in \Omega_t. \tag{29}$$

The free-entry condition (*i.e.* zero profit condition) in each R&D activity ω is $w_t = \lambda\tilde{\Pi}_t^{x_\omega}$, where w_t is the unit cost of labor, given in (27), and where $\lambda\tilde{\Pi}_t^{x_\omega}$ is the expected revenue when one unit of labor is invested in R&D.[16] This gives

$$\tilde{\Pi}_t^{x_\omega} = \tilde{\Pi}_t^{x} = \frac{1-\alpha}{\lambda}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t, \forall\omega \in \Omega_t. \tag{30}$$

Log-differentiating (30) with respect to time gives $\dot{\tilde{\Pi}}_t^{x_\omega}/\tilde{\Pi}_t^{x_\omega} = g_{\mathcal{K}_t}$. Moreover, from (28) and (30), one has

$$\frac{\tilde{\pi}_t^{x_\omega}}{\tilde{\Pi}_t^{x_\omega}} = \frac{(1+\varphi)\frac{1-\alpha}{\alpha}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y \chi_{\omega t}}{\frac{1-\alpha}{\lambda}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t} = \frac{(1+\varphi)\lambda\alpha L_t^Y \chi_{\omega t}}{(1-\psi)\,\mathcal{K}_t}, \forall\omega \in \Omega_t.$$

Accordingly, the arbitrage condition (29) writes

$$r_t + \lambda l_{\omega t} = g_{\mathcal{K}_t} + \frac{(1+\varphi)\lambda\alpha L_t^Y \chi_{\omega t}}{(1-\psi)\mathcal{K}_t}, \forall\omega \in \Omega_t. \tag{31}$$

[16]Indeed, innovations in sector ω are assumed to occur with a Poisson arrival rate of $\lambda l_{\omega t}$: for one unit of labor is invested in R&D activity ω, the probability to obtain one innovation at date t is thus λ. Moreover, its value, taking into account the R&D public policy, is $\tilde{\Pi}_t^{x_\omega}$.

The **representative household** maximizes intertemporal utility (3) subject to the standard budget constraint $\dot{b}_t = w_t + r_t b_t - c_t - nb_t - T_t/L_t$, where b_t is the per capita financial asset and T_t is a lump-sum tax charged by the government.

The Hamiltonian of the problem writes $\mathcal{H} = \ln(c_t)e^{(n-\rho)t} + \mu_t \left[w_t + r_t b_t - c_t - nb_t - \frac{T_t}{L_t}\right]$, where μ_t is the co-state variable associated with the household's budget constraint. The first-order conditions and the transversality condition are

$$\frac{\partial \mathcal{H}}{\partial c_t} = 0 \Leftrightarrow \frac{1}{c_t}e^{(n-\rho)t} = \mu_t; \tag{32}$$

$$\frac{\partial \mathcal{H}}{\partial b_t} = -\dot{\mu}_t \Leftrightarrow \mu_t (r_t - n) = -\dot{\mu}_t \Leftrightarrow \frac{\dot{\mu}_t}{\mu_t} = n - r_t; \tag{33}$$

$$\text{and } \lim_{t\to\infty} \mu_t b_t = 0. \tag{34}$$

Log-differentiating (32) with respect to time gives $-g_{c_t} + n - \rho = \frac{\dot{\mu}}{\mu_t}$; then, using (33), Hence, one gets the usual Keynes-Ramsey condition:

$$r_t = g_{c_t} + \rho. \tag{35}$$

Besides, at date t, in intermediate sector ω, there is one fixed intangible asset stemming from the latest innovation (a patent); its value, $\tilde{\Pi}_t^{x_\omega}$, is given in (30). Accordingly, one has

$$b_t = \frac{1}{L_t}\int_{\Omega_t} \tilde{\Pi}_t^{x_\omega}\, d\omega = \frac{N_t}{L_t}\tilde{\Pi}_t^{x} = \gamma\tilde{\Pi}_t^{x}. \tag{36}$$

As commonly done in the literature (see footnote 6), we make the standard symmetry assumption, in which $l_{\omega t} = l_t$ and $\chi_{\omega t} = \chi_t$, $\forall \omega \in \Omega_t$. One gets the following results:

- The labor constraint (4) (using $N_t = \gamma L_t$ from Assumption 1) becomes

$$L_t = L_t^Y + \int_{\Omega_t} l_{\omega t}\, d\omega = L_t^Y + N_t l_t = L_t^Y + \gamma L_t l_t. \tag{37}$$

- The whole disposable knowledge (1) becomes $\mathcal{K}_t = N_t \chi_t$. Besides, since one has $N_t = \gamma L_t$ and $g_{L_t} = n$ (Assumption 1), one obtains

$$g_{\mathcal{K}_t} = g_{\chi_t} + n. \tag{38}$$

- In any sector ω, the pool of knowledge and the law of knowledge accumulation (Lemmas 1, 2 and 3) rewrite $\mathcal{P}_{\omega t} = \mathcal{P}_t = (p_s + \mathbb{E}[\Theta_t])\chi_t, \forall \omega \in \Omega_t$, and $\dot{\chi}_{\omega t} = \dot{\chi}_t = \lambda\sigma l_t \mathcal{P}_t, \forall \omega \in \Omega_t$, respectively. Consequently, the growth rate of knowledge in any sector ω is

$$g_{\chi_{\omega t}} = g_{\chi_t} = \lambda\sigma (p_s + \mathbb{E}[\Theta_t])\, l_t\ ,\ \forall \omega \in \Omega_t. \tag{39}$$

- The arbitrage condition in any R&D activity ω, $\omega \in \Omega_t$, (31) becomes $r_t + \lambda l_t = g_{\mathcal{K}_t} + \frac{(1+\varphi)\lambda\alpha L_t^Y \chi_t}{(1-\psi)N_t\chi_t}$. Furthermore, using (38), (39) and $N_t = \gamma L_t$ (Assumption 1), one gets

$$r_t + \lambda l_t = \lambda\sigma (p_s + \mathbb{E}[\Theta_t])\, l_t + n + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)\gamma L_t}. \tag{40}$$

The following system summarizes the equations that enable us to compute the equilibrium:

$$\begin{cases} x_{\omega t} = x_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y, \forall \omega \in \Omega_t & (22) \\ Y_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{K}_t & (23) \\ g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t} & (24) \\ g_{Y_t} = g_{c_t} + n & (26) \\ w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t \text{ and } q_{\omega t} = \frac{\chi_{\omega t}}{\alpha}, \forall \omega \in \Omega_t & (27) \\ r_t = g_{c_t} + \rho & (35) \\ L_t = L_t^Y + \gamma L_t l_t & (37) \\ g_{\mathcal{K}_t} = g_{\chi_t} + n & (38) \\ g_{\chi_{\omega t}} = g_{\chi_t} = \lambda\sigma\left(p_s + \mathbb{E}\left[\Theta_t\right]\right) l_t \ , \ \forall \omega \in \Omega_t & (39) \\ r_t + \lambda l_t = \lambda\sigma\left(p_s + \mathbb{E}\left[\Theta_t\right]\right) l_t + n + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)\gamma L_t} & (40) \end{cases}$$

From 35) and (40), one gets

$$g_{c_t} + \rho + \lambda l_t = \lambda\sigma\left(p_s + \mathbb{E}\left[\Theta_t\right]\right) l_t + n + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)\gamma L_t}. \tag{41}$$

Using (24), (38) and (39), one has $g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t} = g_{L_t^Y} + g_{\chi_t} + n = g_{L_t^Y} + \lambda\sigma\left(p_s + \mathbb{E}\left[\Theta_t\right]\right) l_t + n$. Then, from (26), one obtains

$$g_{c_t} = g_{L_t^Y} + \lambda\sigma\left(p_s + \mathbb{E}\left[\Theta_t\right]\right) l_t. \tag{42}$$

Combining (41) and (42) gives $g_{L_t^Y} + \rho + \lambda l_t = n + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)\gamma L_t}$. Using (37), and rearranging the terms, one gets

$$g_{L_t^Y} - \frac{\lambda}{\gamma L_t}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right] L_t^Y = n - \rho - \frac{\lambda}{\gamma}. \tag{43}$$

In order to solve this differential equation in L_t^Y, we use a variable substitution: let $X_t = 1/L_t^Y$. Log-differentiation with respect to time gives $g_{X_t} = -g_{L_t^Y}$. Substituting into (43) gives the following first-order linear differential equation in X_t:

$$-g_{X_t} - \frac{\lambda}{\gamma L_t}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\frac{1}{X_t} = n - \rho - \frac{\lambda}{\gamma} \Leftrightarrow \dot{X}_t - \left(\frac{\lambda}{\gamma} + \rho - n\right) X_t = -\frac{\lambda}{\gamma}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right] e^{-nt}.$$

The solution of this Ricatti equation is

$$X_t = e^{\left(\frac{\lambda}{\gamma}+\rho-n\right)t}\left[X_0 - \frac{\frac{\lambda}{\gamma}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)}{\left(\frac{\lambda}{\gamma}+\rho-n\right) - (-n)}\right] + \frac{\frac{\lambda}{\gamma}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)}{\left(\frac{\lambda}{\gamma}+\rho-n\right) - (-n)} e^{-nt}$$

$$\Leftrightarrow X_t = e^{\left(\frac{\lambda}{\gamma}+\rho-n\right)t}\left[X_0 - \frac{\lambda}{\lambda+\gamma\rho}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)\right] + \frac{\lambda}{\lambda+\gamma\rho}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right) e^{-nt}$$

$$\Leftrightarrow X_t = e^{-nt}\left\{e^{\left(\frac{\lambda}{\gamma}+\rho\right)t}\left[X_0 - \frac{\lambda}{\lambda+\gamma\rho}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)\right] + \frac{\lambda}{\lambda+\gamma\rho}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)\right\}.$$

Accordingly, one gets

$$L_t^Y = \frac{1}{X_t} = \frac{e^{nt}}{e^{\left(\frac{\lambda}{\gamma}+\rho\right)t}\left[\frac{1}{L_0^Y} - \frac{\lambda}{\lambda+\gamma\rho}\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)\right] + \frac{\lambda}{\lambda+\gamma\rho}\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)}.$$

From Assumption 1, one has $L_t = e^{nt}$ (L_0 has been normalized to one), thus one has

$$L_t^Y = \frac{(\lambda+\gamma\rho)\,L_t}{e^{\left(\frac{\lambda}{\gamma}+\rho\right)t}\left[\frac{\lambda+\gamma\rho}{L_0^Y} - \lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)\right] + \lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)}. \tag{44}$$

Now, using the transversality condition (34), let us show that it is necessarily the case that L_t^Y immediately jumps to its steady-state level. Using (30), (32) and (36), one gets

$$\mu_t b_t = \frac{1}{c_t} e^{(n-\rho)t}\gamma\tilde{\Pi}_t^x = \frac{1}{c_t} e^{(n-\rho)t}\gamma\frac{1-\alpha}{\lambda}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t;$$

and using (23) and (25), one gets

$$L_t c_t = \left(1-\frac{\alpha^2}{1-\psi}\right)Y_t = \left(1-\frac{\alpha^2}{1-\psi}\right)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} L_t^Y\mathcal{K}_t.$$

Thus, one obtains

$$\mu_t b_t = \frac{e^{(n-\rho)t}\gamma\frac{1-\alpha}{\lambda}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t}{\left(1-\frac{\alpha^2}{1-\psi}\right)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}}\frac{L_t^Y\mathcal{K}_t}{L_t}} = \frac{\frac{1-\alpha}{\lambda}e^{(n-\rho)t}\gamma L_t}{\left(1-\frac{\alpha^2}{1-\psi}\right)L_t^Y}.$$

Then, using (44), one has

$$\mu_t b_t = \frac{\frac{1-\alpha}{\lambda}e^{(n-\rho)t}\gamma L_t}{\left(1-\frac{\alpha^2}{1-\psi}\right)}\cdot\frac{e^{\left(\frac{\lambda}{\gamma}+\rho\right)t}\left[\frac{\lambda+\gamma\rho}{L_0^Y} - \lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)\right] + \lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)}{(\lambda+\gamma\rho)\,L_t}$$

$$\Leftrightarrow \mu_t b_t = \frac{(1-\alpha)\gamma}{\lambda\left(1-\frac{\alpha^2}{1-\psi}\right)(\lambda+\gamma\rho)}\left(e^{\left(\frac{\lambda}{\gamma}+n\right)t}\left[\frac{\lambda+\gamma\rho}{L_0^Y} - \lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)\right] + e^{(n-\rho)t}\lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)\right).$$

Since $\rho > n$, one has $\lim_{t\to\infty} e^{(n-\rho)t}\lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right) = 0$. Hence the transversality condition (34) holds if and only if

$$\lim_{t\to\infty} e^{\left(\frac{\lambda}{\gamma}+n\right)t}\left[\frac{\lambda+\gamma\rho}{L_0^Y} - \lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)\right] = 0 \Leftrightarrow \frac{\lambda+\gamma\rho}{L_0^Y} - \lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right) = 0.$$

Consequently, (44) gives

$$L_t^Y = \frac{(\lambda+\gamma\rho)\,L_t}{\lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)} = L_0^Y L_t, \text{ and thus } g_{L_t^Y} = n. \tag{45}$$

We can now fully characterize the set of Schumpeterian equilibria described in Definition 1; that is the quantities and prices as functions of the public policy tools vector (ψ,φ). The equilibrium labor partition is characterized by (37) and (45):

$$L_t^Y(\psi,\varphi)=\frac{1+\frac{\rho\gamma}{\lambda}}{1+\frac{1+\varphi}{1-\psi}\alpha}L_t \text{ and } l_t(\psi,\varphi)=\frac{1}{\gamma}-\frac{L_t^Y(\psi,\varphi)}{\gamma L_t}=\frac{1}{\gamma}-\frac{\lambda/\gamma+\rho}{\lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)}. \qquad (46)$$

From (22) and (46), one gets the equilibrium quantity of intermediate good ω, $x_{\omega t}(\psi,\varphi)=x_t(\psi,\varphi)$, $\forall\omega\in\Omega_t$. From (39) and (46), one obtains the growth rate of knowledge in each intermediate sector ω at equilibrium, $g_{\chi_{\omega t}}(\psi,\varphi)=g_{\chi_t}(\psi,\varphi)=\lambda\sigma\left(p_s+\mathbb{E}\left[\Theta_t\right]\right)l(\psi,\varphi),\forall\omega\in\Omega_t$, and the equilibrium growth rate of the economy $g_{c_t}(\psi,\varphi)=g_{Y_t}(\psi,\varphi)-n=g_{\mathcal{K}_t}(\psi,\varphi)=g_{\chi_t}(\psi,\varphi)+n\equiv g_t(\psi,\varphi)$. The equilibrium prices are derived from (35) and (27). The interest rate is $r_t(\psi,\varphi)=g_t(\psi,\varphi)+\rho$, the wage is $w_t(\psi,\varphi)=(1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}}\mathcal{K}_t(\psi,\varphi)$, and the prices of intermediate goods are $q_{\omega t}(\psi,\varphi)=q_t(\psi,\varphi)=\frac{\mathcal{K}_t(\psi,\varphi)}{\alpha\gamma L_t}$, $\forall\omega\in\Omega_t$, where $\mathcal{K}_t(\psi,\varphi)=\mathcal{K}_0e^{\int_0^t g_s(\psi,\varphi)ds}$. Finally, one gets the following lemma.

Lemma 6. *At each date t, the set of Schumpeterian equilibria* à la *Aghion & Howitt is characterized as follows.*
Quantities (levels and growth rates). *The quantity of labor used in the R&D activity of each sector ω is*

$$l_{\omega t}(\psi,\varphi)=l(\psi,\varphi)=\frac{1}{\gamma}-\frac{\lambda/\gamma+\rho}{\lambda\left(1+\frac{1+\varphi}{1-\psi}\alpha\right)},\forall\omega\in\Omega_t;$$

The quantity of labor and the quantity of each intermediate good ω used in the final good production are

$$L_t^Y(\psi,\varphi)=L_t-N_t l(\psi,\varphi)=\left[1-\gamma l(\psi,\varphi)\right]L_t$$

$$\text{and } x_{\omega t}(\psi,\varphi)=x_t(\psi,\varphi)=\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}}L_t^Y(\psi,\varphi),\forall\omega\in\Omega_t.$$

The growth rate of knowledge in each sector ω is

$$g_{\chi_{\omega t}}(\psi,\varphi)=g_{\chi_t}(\psi,\varphi)=\lambda\sigma\left(p_s+\mathbb{E}\left[\Theta_t\right]\right)l(\psi,\varphi),\forall\omega\in\Omega_t,\ \text{where } \mathbb{E}\left[\Theta_t\right]=p_m\theta(L_t)+p_G\gamma L_t.$$

The growth rate of the whole knowledge in the economy is

$$g_{\mathcal{K}_t}(\psi,\varphi)=g_{\chi_t}(\psi,\varphi)+n.$$

The growth rate of the total output is

$$g_{Y_t}(\psi,\varphi)=g_{\mathcal{K}_t}(\psi,\varphi)+n.$$

The growth rate of per capita consumption is

$$g_{c_t}(\psi,\varphi)=g_{\mathcal{K}_t}(\psi,\varphi)=g_{\chi_t}(\psi,\varphi)+n.$$

Prices. *The equilibrium interest rate, wage and prices of intermediate goods (price of final good is normalized to one) are respectively*

$$r_t(\psi,\varphi) = g_t(\psi,\varphi) + \rho,\ \ w_t(\psi,\varphi) = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t(\psi,\varphi),$$

$$q_{\omega t}(\psi,\varphi) = q_t(\psi,\varphi) = \frac{\mathcal{K}_t(\psi,\varphi)}{\alpha\gamma L_t},\ \forall\omega\in\Omega_t,\ \ \textit{where}\ \mathcal{K}_t(\psi,\varphi) = \mathcal{K}_0 e^{\int_0^t g_s(\psi,\varphi)ds}.$$

5.3 Marginal productivity of labor in R&D - Proof of Lemma 5

In each sector, the marginal productivity of labor in R&D activity As shown in Lemma 4, at equilibrium, the law of knowledge accumulation - that is the innovations production function - is $\dot{\chi}_t = \lambda\sigma l\mathcal{P}_t = \lambda\sigma l\left(p_s + \mathbb{E}\left[\Theta_t\right]\right)\chi_t$. Besides, denoting by χ_0 the initial stock of knowledge in any sector, one has $\chi_t = \chi_0 e^{\int_0^t g_{\chi_u}du} = \chi_0 e^{\lambda\sigma l\int_0^t (p_s+\mathbb{E}[\Theta_u])du}$. Then, at equilibrium, the marginal productivity of labor in R&D activity is

$$\frac{\partial\dot{\chi}_t}{\partial l} = \lambda\sigma\left(p_s + \mathbb{E}\left[\Theta_t\right]\right)\left(\chi_t + l\frac{\partial\chi_t}{\partial l}\right),$$

$$\text{with } \frac{\partial\chi_t}{\partial l} = \left(\lambda\sigma\int_0^t\left(p_s + \mathbb{E}\left[\Theta_u\right]\right)du\right)\chi_0 e^{\lambda\sigma l\int_0^t (p_s+\mathbb{E}[\Theta_u])du} = \left(\lambda\sigma\int_0^t\left(p_s + \mathbb{E}\left[\Theta_u\right]\right)du\right)\chi_t.$$

Therefore, since $\mathcal{P}_t = \left(p_s + \mathbb{E}\left[\Theta_t\right]\right)\chi_t$, one obtains

$$\frac{\partial\dot{\chi}_t}{\partial l} = \lambda\sigma\left(p_s + \mathbb{E}\left[\Theta_t\right]\right)\left(\chi_t + l\left(\lambda\sigma\int_0^t\left(p_s + \mathbb{E}\left[\Theta_u\right]\right)du\right)\chi_t\right)$$

$$= \lambda\sigma\left(1 + l\lambda\sigma\int_0^t\left(p_s + \mathbb{E}\left[\Theta_u\right]\right)du\right)\left(p_s + \mathbb{E}\left[\Theta_t\right]\right)\chi_t = \lambda\sigma\left(1 + l\lambda\sigma\int_0^t\left(p_s + \mathbb{E}\left[\Theta_u\right]\right)du\right)\mathcal{P}_t.\square$$

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