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“The Hidden Demand for Flexibility: a Theory for Gendered Employment Dynamics”

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The Hidden Demand for Flexibility - A Theory of Gendered Employment Dynamics*

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Abstract

Empirical evidence highlights women's demand for flexible working hours as a critical cause of the persistent gender disparities in the labor market. We propose a theory of how hidden demand for flexibility drives gendered employment dynamics. We develop a dynamic contracting model between an employer and an employee whose time availability is stochastic and unverifiable. We model men and women only to differ in their probability of having low time availability, which we measure in the ATUS. We explore contracts designed specifically for each gender (*gender-tailored*) and the polar case where a *male-tailored* contract is given to both men and women. For the latter, we show that contracting frictions endogenously give rise to well-documented gendered labor market outcomes: (i) the divergence and non-convergence of gender earnings differentials over the life-cycle, and (ii) women's shorter job duration and weaker labor force attachment.

Keywords: Gender wage gap, child penalty, flexible working hours, recursive contracts.

JEL codes: J16, J22, J41, D82

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1 Introduction

Women's need for flexible working hours has emerged as the primary source of the remaining gender differences in the labor market (Goldin (2014)). While men and women have converged on many employment dimensions, a persistent gender gap in wages and working hours remains (Blau and Kahn (2017)). In the US, 70% of the gender earnings gap can be explained by the child penalty (Cortés and Pan (2020)). The burden of unpaid care responsibilities such as childcare, mostly carried out by women, creates unpredictable schedule changes and calls for fewer working hours. Empirically, the relationship between temporal flexibility and labor market outcomes has been widely studied.[1] However, there is no theoretical understanding of the way in which women's higher flexibility needs drive gendered employment dynamics.

In this paper, we propose an explanation for gender differences in the life-cycle dynamics of wages and employment based on a hidden – external time commitments are unverifiable – demand for flexible working hours. We develop a theoretical framework that takes the unpredictable and unverifiable nature of flexibility needs seriously and studies them in dynamic employment relationships. We model the demand for flexibility as stochastic and unverifiable shocks to time availability. To study gender differences, we allow men and women to differ only in their probability of having limited time availability, $p_{\text{men}} < p_{\text{women}}$, as we will measure in the data. Hence, gender is not encoded as an ad-hoc difference in preferences. This allows for between gender similarities and within gender differences in working conditions over time depending on their exposure to flexibility shocks.[2] We study the effects of gendered flexibility needs on wage and employment dynamics through the lens of a dynamic contracting problem between an employer and an employee. We explore two types of contracts: (i) contracts designed for each gender's flexibility needs (*gender-tailored*) and (ii) the polar case where a *male-tailored* contract is given to both men and women. When contracts do not internalize women's flexibility needs (i.e. under *male-tailored* contracts), contractual frictions endogenously give rise to gendered employment dynamics that have been empirically documented. In particular, the model can account for the divergence and non-convergence of earnings differentials over the life-cycle, and women's shorter job duration and weaker labor force attachment. By contrast, under gender-tailored contracts, while individual differences

[1] Mas and Pallais (2017), Mas and Pallais (2020), Wiswall and Zafar (2018).

[2] Our framework serves more generally to understand the consequences of flexibility needs on employment dynamics and is not per se gendered. It is also applicable to other sociodemographic groups, e.g. single fathers, that might experience unpredictable external time commitments.

may exist, the systematic disparities between men's and women's labor market outcomes are negligible.

Job flexibility covers various temporal aspects like hours worked, specific times, and work hour predictability. Reduced-hour arrangements alone don't fully meet the demand for flexibility since care and family responsibilities can be unpredictable[3]: consider a woman heading to work, only to receive a call that her child is sick and requires immediate attention. Despite the employer allowing her to work from home, frequent emergencies will make it too costly for the employer to accommodate her needs while maintaining current working conditions and raise doubts about her genuine need for flexibility. Over time, this can have long-lasting consequences, e.g. not getting promoted.

We study a dynamic contracting problem (in the spirit of Clementi and Hopenhayn (2006), Dovis (2019)) that aims to capture the mechanism of this story. A risk-neutral employer hires a risk-averse employee based on a contract that specifies a wage and working hours for every period. The employee's time availability is subject to i.i.d shocks. A low time availability shock corresponds to the case where e.g. a child unexpectedly needs to be picked up from school. Conversely, a high time availability shock coincides with the case where the employee can work as planned. Experiencing a low shock will make every hour worked more costly to that employee. Informational asymmetries arise as the employer can not observe the realization of the employee's time availability shock and thus does not know how costly it is for the employee to work (private information). Furthermore, the employee does not have to commit to staying in the contract (limited commitment) and can pursue an outside option. Likewise, the employer can also choose to terminate the contract.

We first characterize analytically the main properties of the optimal contract for compensation dynamics, working hours and termination probabilities as well as the region in the state space where termination may be optimal. The private information friction implies that in order to provide flexibility in hours, an employee with high (low) time availability must be rewarded (penalized). Moreover, because the employment relationship is dynamic and the employee is risk averse, it is optimal for the employer to smooth these rewards and penalties over time. Therefore, an employee who demands to work fewer hours due to low time availability will be penalized with lower wages in all future periods. When an employee experiences a sufficiently long sequence of low time availability, working conditions worsen.

[3]We focus on flexibility needs arising from unforeseeable family emergencies. A stable and predictable work schedule, i.e. not to be called in for work unexpectedly when they have scheduled care responsibilities, is equally important for women, see e.g. Ciasullo and Uccioli (2022). See Appendix F.2 on how our model can be extended to a setting where the employer seeks flexibility in working hours.

Pursuing an outside option becomes more attractive for the employee, which makes providing flexibility more costly. Eventually, terminating the employment relationship can become optimal.

To understand the gendered employment dynamics arising from these contracts we allow the probability of having low time availability to differ by gender, namely, $p_{men} < p_{women}$. This is the only parameter allowed to encode gender differences. We contrast two types of contracts. First, we study *gender-tailored* contracts. These contracts are designed for each gender specifically, meaning we solve the optimal contract for each p. Through a comparative statics exercise on p, we show that for a male and female employee that have the same compensation level, men experience larger wage penalties for demanding flexibility.[4] We find that when contracts fully internalize differences in flexibility needs, the average gender wage gap is constant over time. This is because the higher frequency of women's penalties is offset by men's higher penalties.

Second, we study *male-tailored* contracts. These are contracts initially designed for men but which then are also given to women. In practice, employers may not be able or allowed to offer gender-specific work arrangements. Hence, this exercise highlights the dynamic consequences of the incompatibility of women's flexibility needs in male-dominated work environments.[5] When employment relationships are designed to fit men's flexibility needs, women's average wages gradually start to diverge from men's. This is because women get both more frequent but also higher penalties. We also explore an intermediate case where the employer simultaneously employs men and women but is constrained to design a unique contract for both genders (*team-tailored* contract). For this case, we show that men's wages diverge upward and women's downward.

To directly compare wage paths and termination probabilities, the provision of temporal flexibility, and the flexibility penalties by gender, we also solve and simulate the model numerically. We combine the recursive Lagrangian method of Marcet and Marimon (2019) with a direct promised utility approach. We use the latter to solve the model in the termination region. We calibrate the model and use the American Time Use Survey (ATUS) to identify meaningful values for p_{men} and p_{women}. In the ATUS, we observe daily minutes spent on care activities during usual working hours. From this, we find that, for our baseline calibration, men have a frequency of limited time availability of $p_{\text{men}} = 6\%$, whereas women

[4]This is indeed an empirical finding with respect to part-time work, sometimes referred to as "flexibility stigma"(Coltrane *et al.* (2013), Golden (2020), Aaronson and French (2004), Dunn (2018), Wolf (2014), O'Dorchai *et al.* (2007)).

[5]Torre (2017), Mas and Pallais (2020), Patrick *et al.* (2016), Cha (2013).

are more than twice as likely to be interrupted with a chance of $p_{\text{women}} = 15\%$. We also show substantial heterogeneity in these probabilities across different socioeconomic groups and quantify the effect on wage dynamics in our model.

Comparing the numerical results of *male-tailored* and *gender-tailored* contracts allows us to understand what drives differences in employment dynamics. Our results shed light on the mechanisms behind two well-documented gendered labor market outcomes:

1. *Women's wages start to diverge from men's after childbirth but do not fully converge back to men's after children grow up.*

   A large and internationally diverse literature has shown that women and men have divergent earnings growth trajectories after childbirth, even when they were previously on the same career paths (Barth *et al.* (2021), Paul (2016)). This is partially explained by occupational sorting of women anticipating greater flexibility needs before having children (Kleven *et al.* (2019a), Mas and Pallais (2020), Cortés and Pan (2019)). However, mother's wages also diverge from men's within the same firm and occupation, mainly through the lack of promotions (Lucifora *et al.* (2021), Bronson and Thoursie (2019)). Over time, many women are being pushed out of current work arrangements due to parental demands and end up in predominantly lower-paying jobs (Patrick *et al.* (2016)).

   Importantly, despite children growing up and the gender wage gap narrowing, women's wages still never fully converge back to men's (Goldin *et al.* (2022)). Our model provides a potential underlying mechanism for both the divergence (within and across jobs) and non-convergence in wages. When contracts do not fully internalize differences in flexibility needs (i.e. *male-tailored* or *team-tailored* contracts), the average gender wage gap gradually grows over time and women that are particularly exposed to flexibility needs will be pushed out of the current employment relation. For employees that stay, in our main calibration, we are able to explain more than 40% of the within-firm divergence of wages. Lastly, in our model, demanding flexibility is penalized with both current and future wage cuts so that even when men's and women's flexibility needs are the same again after children grow up, wages will not converge back.

2. *Women's job duration is shorter and labor force attachment is weaker.*

   It is well documented that women's job spells are, on average, substantially shorter than men's (Hall (1982), Molloy *et al.* (2020), Munasinghe *et al.* (2008)). This is in part related to gender differences in job mobility patterns. A higher job turnover for women has been shown to contribute to the gender wage gap (Amano-Patiño *et al.* (2020)). Empirical evidence suggests that this is a result of women's care responsibilities leading also to a weaker labor force attachment.[6] Unlike static occupational choice models, our dynamic contracting setting is able to speak to the impact of gender differences in external demands on job duration and turnover. Under *male-tailored* contracts, a larger share of women end up with depressed working conditions due to their higher flexibility needs. This leads to higher termination rates and shorter job duration for women.

Finally, comparing *gender-tailored* and *male-tailored* (and *team-tailored*) contracts highlights the potential adverse consequences of non-discriminatory contracts. This is a practical concern, as policymakers may overlook the unfavorable effects of not allowing work arrangements to be targeted towards one gender. For example, Antecol *et al.* (2018) show that gender-neutral tenure clock extensions reduced women's tenure probability while increasing men's.

Related Literature. Our theory provides a conceptually new way of thinking about the link between flexibility needs and working conditions. We propose that a hidden demand for flexible working hours drives gender differences in employment dynamics. That is, they are a result of information and contracting frictions. The seminal work in the literature providing microfoundations on how gendered flexibility needs can explain wage differentials is Goldin (2014). By analyzing the convexity of the hour-wage relationship in a static model, she can explain differences in the gender pay gap across and within occupations.[7] Erosa *et al.* (2022) conduct a quantitative analysis of the Goldin (2014) theory and find that it can account for a large share of the gender gaps in occupational choice, wages, and hours. By contrast, our dynamic model gives insights into the divergence (within and across occupations) of wages after childbirth and the non-convergence of wages after children are grown up – which cannot be rationalized with static models.

The across-occupation wage gap has been studied by Flabbi and Moro (2012) and Mor-

[6]Married women are more likely to stay at home, find a new, more family-friendly job (Mas and Pallais (2020), Mas and Pallais (2017), Wiswall and Zafar (2018)) or pursue flexibility-oriented self-employment (Lim (2019), Gurley-Calvez *et al.* (2009), Bento *et al.* (2021)). Unmet needs for workplace flexibility push women into less profitable work arrangements or home production (Patrick *et al.* (2016)).

[7]Our theory disconnects wage from the marginal product of labor, which has important normative implications. If wage differentials are driven by a convex production function as in Goldin (2014), a reduction of the wage differential inside a family will entail a loss in production efficiency, which may be inefficient both socially and for the family. By contrast, if they are driven by contracting frictions, there may not be any production efficiency loss from a reduction in wage differentials.

chio and Moser (2024).[8] They quantify the effect of gender differences in preferences for occupational amenities, including job flexibility. In our model, differential flexibility needs create within-firm and within-occupation gender wage gaps and push women into lower-wage work arrangements that were suboptimal ex-ante.

Another strand of the literature uses models of human capital accumulation to explain the child penalty (Erosa *et al.* (2016), Amano-Patiño *et al.* (2020) and Barigozzi *et al.* (2023)). Due to maternity leave and lower working hours after returning to employment, women are able to accumulate less human capital on the job. These models hence generate (within-firm) divergence and non-convergences of wages. Our theory provides an alternative to generating these gendered wage dynamics that is based on contracting frictions where women can be paid less even if they are equally productive as men. In addition, human capital models remain silent about gender differences in job duration. Lastly, models of promotions (Lazear and Rosen (1990), Lommerud *et al.* (2015) and Bronson and Thoursie (2019)) can also give rise to within-firm gender wage gaps.[9]

Conceptually closest to our approach is Albanesi and Olivetti (2009).[10] They show how the combination of information frictions in the labor market (moral hazard and adverse selection) and intra-household home production decisions can generate self-fulfilling gendered equilibria. We abstract from home production decisions, take the gendered flexibility needs as given, and instead focus on its unpredictable nature and study the consequences for the life-cycle dynamics of wages and employment.

A growing empirical literature has tried to measure gender differences in external demands on time (Buzard *et al.* (2023)), including unexpected incidences during working hours (Cubas *et al.* (2021), Schoonbroodt (2018)). We use our model to study the effects of these unpredictable and gendered demands on time. Moreover, we provide our own model-consistent estimates using the ATUS.

The dynamic contracting problem we study combines private information (from the flexibility shocks being unobservable to the employer) and limited commitment (from the em-

[8]Relatedly, Le Barbanchon *et al.* (2021) show that women are more willing to tradeoff wages for a shorter commute time and study the consequences for the gender wage gap.

[9]In our model, we could interpret increases in wages as – current or as changes in consumption in anticipation of future – promotions. Hence, our model may shed light on how differences in expected demands on time can create gender differences in promotion opportunities.

[10]Relatedly, Albanesi *et al.* (2015) study gender differences in top executives' compensation through a moral hazard model but find that a model of "managerial power" is more consistent with empirical evidence. Models that feature self-fulfilling gendered equilibria have also been studied by Dolado *et al.* (2013) and Lommerud *et al.* (2015).

ployee being able to leave at any time) frictions. In this sense, the model is close to the sovereign debt model in Dovis (2019). As in Dovis (2019), the combination of the two frictions implies that the optimal contract may (temporarily) end up in a region with ex-post inefficiencies. In our case, this results from the fact that as the employee's value approaches the outside option, it is impossible to induce an employee with low time availability to work positive hours. Then the employer's cost of increasing the compensation is smaller than the gain of inducing an employee with limited time to work positive hours, which implies that the Pareto frontier is increasing. In addition to Dovis (2019), in this region, we allow the principal the option to terminate the contract, which will be optimal if its outside option is high enough.

Outline. The paper is organized as follows. Section 2 lays down the environment. Section 3 sets up the dynamic contracting problem and characterizes the main properties of the optimal contract, including optimality of termination. In Section 4 we show how differences in time availability can generate gendered employment dynamics. We explore *gender-tailored*, *male-tailored*, and *team-tailored* contracts. In Section 5, we explain how we use the American Time Use Survey to calibrate the model, present the parametrization and discuss the solution method. Section 6 presents the numerical results and Section 7 connects them to the aforementioned gendered labor market outcomes. Section 8 concludes.

2 Environment

Time is discrete and indexed by $t = 0, 1, ..., \infty$. There is one employer, the principal, and one employee, the agent. They contract on a long-term employment relation that specifies hours and wages for each period. The employee does not have to commit to staying in the contract and can pursue an outside option. Conversely, the employer can also choose to terminate the employment relation. Thus, the contract features two-sided limited commitment. The source of uncertainty is a shock to the time availability of the employee, which is not observable to the employer. We model the contracting relation between the employer and the employee as a message game. At time 0, the employer makes a take-it-or-leave-it offer to the employee. The offer consists of a contract whose terms can be contingent on all public information.

2.1 Preferences

The employer and the employee are infinitely lived. At every period, the employee's time availability f_t can take two values $f_t \in \{f^L, f^H\}$ with $0 < f^L < f^H < 1$. Time availability is independently and identically distributed over time with $\mathbb{P}(f_t = f^L) = p$. This is the only parameter that will differ by gender in the entire model. The employee values a stochastic sequence of wages $\{w_t\}_{t=0}^{\infty}$ and working hours $\{h_t\}_{t=0}^{\infty}$ according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(w_t, h_t; f_t), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor. The per-period utility is specified by

$$U(w, h; f) = u(w) - (1 - f)\psi(h), \tag{2}$$

where $u : \mathbb{R}^+ \to \mathbb{R}$ is strictly increasing and concave and $\psi : \mathbb{R}^+ \to \mathbb{R}$ is strictly increasing and convex. The employee has an outside option that gives a life-time value $\overline{v} \in \mathbb{R}$.[11]

The employer is risk-neutral, also discounts the future with $\beta \in (0,1)$ and has a per period profit of

$$\pi(w, h) = g(h) - w, \tag{3}$$

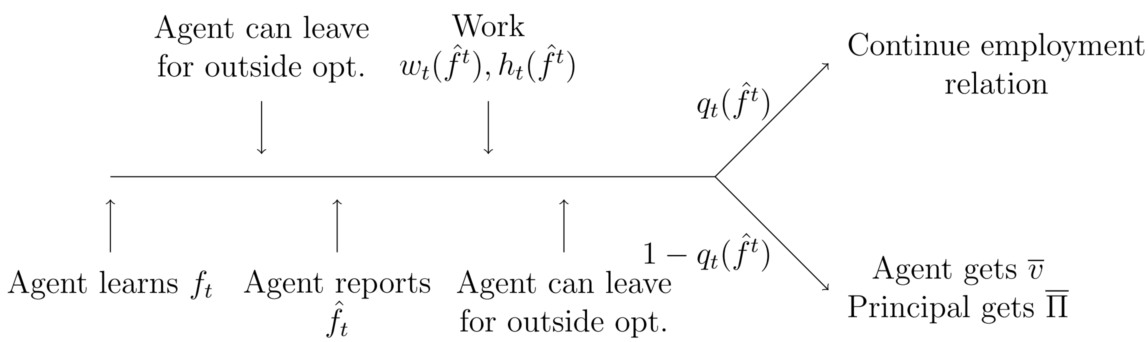
where $g : \mathbb{R}^+ \to \mathbb{R}^+$ is increasing, concave and satisfies the condition $\lim_{h\to 0} g_h(h) = +\infty$. The employer has an outside option that gives a value of $\overline{\Pi} \in \mathbb{R}$.

2.2 Information

The employer cannot observe the realization of f_t, the time availability of the employee in period t. The employer bases the contract on the time availability reported by the employee in each period, $\hat{f}_t$. We invoke the Revelation Principle to, without loss of generality, restrict attention to truth-telling mechanisms. This reduces the message space to $\{f^L, f^H\}$. A reporting strategy for the employee is given by $\hat{f} = \{\hat{f}_t(f^t)\}_{t=0}^{\infty}$, where $f^t = (f_1, ..., f_t)$.

[11] We leave the interpretation for the outside option open. It could, for example, include a more flexible work arrangement, home production, or self-employment.

Figure 1: Timing



2.3 Timing

The timing of the contract within each period is as depicted in Figure 1. At the beginning of the period, the employee learns their time availability $f_t \in \{f^L, f^H\}$ and then has the possibility to pursue their outside option. If the employee stays, they then report $\hat{f}_t \in \{f^L, f^H\}$ to the employer. Based on the (history of) report(s) the employer offers the employee to work $h_t(\hat{f}^t)$ for a wage $w_t(\hat{f}^t)$. The employee again has the option to leave and pursue the outside option. At the end of the period, the employer proposes the employment relation to continue with a probability $q_t(\hat{f}^t)$ and to terminate with a probability $1 - q_t(\hat{f}^t)$. In the latter case, the employee gets the outside option $\overline{v}$ and the employer gets $\overline{\Pi}$. The employee can choose to leave both after learning this period's time availability and at the end of the period.

2.4 Contract

The dynamic contract specifies employment policies which are contingent on all information provided by the employee. Letting $\hat{f}^t = (\hat{f}_1, ..., \hat{f}_t)$ denote the history of the employee's reports, the contract $\sigma = \{w_t(\hat{f}^t), h_t(\hat{f}^t), q_t(\hat{f}^t)\}$ specifies a contingent policy of termination probabilities q_t, hours h_t and wages w_t.

2.5 Feasible Contract

Definition 1 *A contract σ is feasible if $\forall t \geq 1$ and $\forall \hat{f}^t \in \{f^L, f^H\}^t$*

(i) $q(\hat{f}^t) \in [0, 1]$

(ii) $w(\hat{f}^t), h(\hat{f}^t) \geq 0$

We collect the assumptions made so far:

Assumption 1 *(i) The utility function* $U(w,h;f) = u(w) - (1-f)\psi(h)$ *is strictly increasing and concave in both arguments and satisfies the single crossing property* $U_{hf} > 0$. *The production function* g *is increasing, continuously differentiable and satisfies* $\lim_{h\to 0} g_h(h) = +\infty$.
(ii) The time availability shock can take on two values $f \in \{f^L, f^H\}$ *and is independent and identically distributed over time. Each period time availability can be low with probability* $p \in (0,1)$ *and high with probability* $1-p \in (0,1)$.

The following convention for notation is useful: a superscript H indicates an element in the contract designed for an employee that reports to be of high time availability, e.g. the wage specified for an employee that reports to have high time availability at time t is denoted by $w_t^H = w(f^{t-1}, f^H)$. Equivalently, the superscript L denotes an element in the contract designed for an employee that reports to be of low time availability in the current period.

2.6 A Benchmark: Contracts under Symmetric Information

As a first step, we consider the case of symmetric information, where the employer observes the employee's realization of time availability and the employee has full commitment.

Proposition 1 *In the first best, at every history* f^t*, the contract satisfies:*

1. *No distortions in hours worked, that is,*

$$g'(h_t(f^t)) = \frac{(1-f_t)\psi'(h_t(f^t))}{u'(w_t(f^t))}. \tag{4}$$

2. *Full insurance and perfect intertemporal wage smoothing, that is* $w = w_t(f^t)$.

3. *No termination, that is* $q_t(f^t) = 1$.

In the first best, the employer is able to set hours and wage optimally for both high and low time availability. In particular, this means hours are not distorted. The employee is perfectly insured against flexibility shocks and is thus allowed to reduce hours worked without a penalty on total compensation. In particular, wages for both employees with high and low time availability are the same and constant over all periods. Lastly, given that both parties agreed to enter the employment relation at time zero, it is never optimal for the employer to lay off the employee.

2.7 Model Discussion

We study a particular type of contract where wages are adjusted as frequently as flexibility needs arise. While such contracts may not have a direct counterpart in reality, employers may implicitly track attendance and adjust wages in the long run through, for example, reduced promotion opportunities. In this case, we can interpret the changes in current compensation as a result of the employees adjusting their private borrowing or saving in anticipation of future pay raises or wage cuts.

We purposely avoid putting structure on the outside option. We keep it the same for both genders throughout since we do not want our termination rates to be driven mechanically, but it is reasonable to expect this variable to be gendered. The outside option could capture, for example, the value of going into a higher flexibility and lower-wage occupation, dropping out of the labor force, or pursuing flexibility-oriented self-employment.

To focus on gender differentials in wage dynamics resulting from flexibility needs, we deliberately abstract from other drivers of wage dynamics over the life-cycle, such as human capital accumulation. For this reason, when women's wages diverge downwards in the model, we do not interpret it as women's wages decreasing over time. Instead, we view this as women's wage divergence relative to men's wage path.

Lastly, our framework focuses on flexibility needs that arise when employees need to adjust their working hours due to unpredictable family emergencies. As highlighted by Ciasullo and Uccioli (2022), a stable and predictable work schedule is equally important for women to be able to, for example, schedule care responsibilities. In Appendix F.2, we lay out a simple extension of our model where the employer can also ask to work unpredictable extra hours. We assume that the employee's cost of working these extra hours is also stochastic and unverifiable. All the main intuitions and results go through in that setting.

3 Optimal Contract

In this section, we define the recursive formulation of the (constrained) optimal contract with information asymmetry in f and limited commitment induced by the outside option. We characterize the properties of the optimal contract upon continuation and then show that termination may be optimal in some states. These results hold for a given probability p of having low time availability.

3.1 Recursive Formulation

The employee's continuation value from staying in the contract, denoted v_t, evolves according to

$$v_t = (1-p)\left(U(w^H, h^H; f^H) + \beta v^H_{t+1}\right) + p\left(U(w^L, h^L; f^L) + \beta v^L_{t+1}\right), \tag{5}$$

which is composed of the expected per period utility and the expected continuation value. The quantities v^H_{t+1} and v^L_{t+1} are the continuation values contingent on high and low time-availability report, respectively. Following Sargent and Ljungqvist (2000) and Clementi and Hopenhayn (2006), we present the problem in a recursive form, where the state variable of the problem is the employee's continuation value at the beginning of a period. In particular, the contract σ, introduced in Section 2.4, can now be decomposed into current allocations $(w_t(f^j), h_t(f^j))$ and a continuation value v^j_{t+1} contingent on reported f^j for $j \in \{H, L\}$.[12]

First, define the total value to the employee if she stays in the contract. The flow equation (5) now becomes the promise-keeping constraint, as the initial value v must be delivered to the employee by the continuation contract. Suppressing time subscript this gives:

$$v = (1-p)\left(U(w^H, h^H; f^H) + \beta v^H\right) + p\left(U(w^L, h^L; f^L) + \beta v^L\right). \tag{PK}$$

As usual, the only relevant *incentive constraint* is the one imposing truthful reporting in the high state:[13]

$$U(w^H, h^H; f^H) + \beta v^H \geq U(w^L, h^L; f^H) + \beta v^L. \tag{IC}$$

The incentive compatibility constraint (IC) captures the informational frictions in setting the contract. It ensures that the employee has no incentive to misreport. In particular, an employee with high time availability does not want to take the contract designed for someone with low time availability.

The lack of commitment of the employee imposes two constraints that are of different timing. First consider the standard *limited commitment constraints*:

$$v^H, v^L \geq \overline{v}. \tag{LC}$$

[12] In this section we focus on contracts without termination, thus continuation probabilities are one for both reported time availabilities, i.e. $q(f^H) = q(f^L) = 1$.

[13] This is shown formally in the proof of Proposition 2 in the Appendix.

These constraints impose that the employee's continuation value promised in the current contract must exceed their outside option, i.e. after working in a period they will still prefer to stay in the contract. Additionally, a contract should be sustainable in the following way:

$$\begin{aligned} U(w^H, h^H; f^H) + \beta v^H &\geq \overline{v}, \\ U(w^L, h^L; f^L) + \beta v^L &\geq \overline{v}. \end{aligned} \tag{SUST}$$

These *sustainability constraints* ensure that even before working and regardless of their time availability, employees will want to continue in the employment relationship. Hence, (LC) and (SUST) guarantee that the employee prefers staying in the employment relation to the outside option at different times of the contract.

We denote the employer's value contingent upon continuation as $\hat{\Pi}$. This is different from the value of the employer prior to the firing decision, which is denoted as Π. The choice variables are the wages and hours for the current period contingent on the reported time availability as well as the contingent continuation values v^H and v^L. The employer then solves:

$$\widehat{\Pi}(v) = \max_{\substack{w^H, h^H, v^H \\ w^L, h^L, v^L}} (1-p)\left[\pi(w^H, h^H) + \beta\Pi(v^H)\right] + p\left[\pi(w^L, h^L) + \beta\Pi(v^L)\right] \tag{6}$$

subject to (PK), (IC), (SUST), (LC). To recover the time zero problem, we can solve $\widehat{\Pi}(v_0)$ for a given starting promised utility v_0.

Contingent on the high or low time availability, the first term of the maximand corresponds to the current period's expected profits and the second term indicates the expected discounted total value of the employer prior to the firing the decision.

3.2 Termination Decision

We now turn to the termination decision of the employer. In some states, a stochastic firing decision may be optimal. Technically speaking, this is the case when the outside option for the employer implies that the Pareto frontier is not a convex set. Allowing for randomization over the termination decision is equivalent to assuming that the employer offers a lottery to the employee at the end of every period. The employment relation ends with probability $1-q$, in which case the employer receives $\overline{\Pi}$, and it continues with probability q. In the latter case the employee receives continuation value v_c. Then the function $\Pi(v)$ solves the following functional equation:

$$\Pi(v) = \max_{q\in[0,1],v_c} (1-q)\overline{\Pi} + q\widehat{\Pi}(v_c)$$
$$\text{subject to } (1-q)\overline{v} + qv_c = v.$$

3.3 Time Zero Contracting

We are agnostic about how the starting promised utility v_0 is determined and how it differs by gender. For employees to engage in the employment relation we need $v_0 \geq \overline{v}$. The choice of v_0 does not affect the results we present in the following sections.

3.4 Properties

We now present the main properties of the optimal contract in case of continuation. The results on optimal termination are discussed in Section 3.5.

Under symmetric information, the employer is able to fully insure the employee against having low time availability. As shown in the following proposition, under private information and lack of commitment the optimal contract no longer provides full insurance against flexibility shocks.

Proposition 2 *At every history f^t, the optimal allocation satisfies*

1. *Hours worked of an employee with high time availability are undistorted and satisfy equation (4). The hours of an employee with low time availability are distorted downwards and satisfy*
$$g'(h^L) > \frac{(1-f^L)\psi'(h^L)}{u'(w^L)} \tag{7}$$

2. *For $v > \overline{v}$, the employer compensates high time availability with a higher wage ($w^H > w^L$) and higher continuation utility $v^H > v^L$. Moreover, when the (SUST) and (LC) constraints do not bind, $v^H > v > v^L$.*

3. *If the (SUST) and (LC) constraints do not bind, the following Inverse Euler equation holds*
$$\frac{1}{u'(w_{t-1})} = p\frac{1}{u'(w_t^L)} + (1-p)\frac{1}{u'(w_t^H)}. \tag{8}$$

The presence of private information induces a higher compensation for working more hours for an employee with high time availability. Having low time availability will be penalized with a lower wage and accommodated with working less hours. Low time availability employees are offered contracts with working hours lower than predicted in the first best setting. This is directly evident by comparing equation (7) with (4). When an employee demands flexibility, hours are distorted below the first best optimum. Hence, the contract underworks the employees with low time availability. This is because, to correctly screen time availability, lying about f needs to be less attractive for the high time availability employee. Therefore, the optimal contract prescribes fewer hours than at the first best after low time availability to discourage the type with high time availability from claiming otherwise.

Because the employee is risk averse and the employment relationship dynamic, it is optimal for the employer to smooth the employee's compensation intertemporally. This means that high time availability not only is rewarded with a higher wage at t, but is also compensated with a higher continuation value, which implies that wages increase in all future periods. Wage premia are thus permanent. Similarly, the provision of flexibility comes at the cost of lower wages, which, due to the optimality of smoothing compensation, implies a lower continuation value and, therefore, lower wages in all future periods.

The third result of the proposition shows that when the (SUST) and (LC) constraints do not bind, the inverse marginal utilities follow a martingale, implying that the cross-sectional average along a large population of workers would be constant over time. This typically implies that average wages are also approximately constant over time, as we will show in the numerical simulations. With log utility ($u(c) = \log(c)$), the average wages are exactly constant, i.e.

$$w_{t-1} = pw_t^L + (1-p)w_t^H.$$

In Section 4, we explore the implications of this result for the dynamics of the gender wage gap.

3.5 Optimality of Termination

We now characterize when terminating the employment relation is optimal. Intuitively, this occurs when the employee has accumulated sufficiently many incidences of low time availability and the cost of providing flexibility is too high for the employer. If the outside option for the employer is high enough, termination can be optimal.

Formally, our approach to show this property of the optimal contract is the following.

First, we study a constrained problem where the principal is not allowed to terminate the contract. We show that the principal's value in the constrained problem $\Pi^c(v)$ is increasing in some region around $\overline{v}$. This implies that there must be some range of values for the outside option $\overline{\Pi}$ such that the (constrained) Pareto frontier $\{\overline{\Pi}, \Pi^c(v)\}$ is not a convex set. Then, Π^c and Π do not coincide, and terminating the contract with a positive probability is optimal. Intuitively, in the inefficient region where $\Pi^c(v)$ is increasing, both the employee and the employer would gain by increasing v, but this would violate the (PK) constraint. If $\overline{\Pi}$ is high enough, the principal can offer a lottery between terminating and continuing at a higher v such that the (PK) constraint is satisfied and the principal obtains a strictly higher value.

The value of the principal in the constrained problem Π^c satisfies the following Bellman equation:

$$\Pi^c(v) = \max_{\substack{w^H, h^H, v^H \\ w^L, h^L, v^L}} (1-p)[\pi(w^H, h^H) + \beta \Pi^c(v^H)] + p[\pi(w^L, h^L) + \beta \Pi^c(v^L)]$$

$$\text{subject to (PK), (IC), (SUST), (LC).}$$

The contract becomes inefficient as v approaches $\overline{v}$ because it is not possible to induce an employee with low time availability to work positive hours while satisfying the (IC) and (SUST) constraints. Intuitively, when $v = \overline{v}$ the (SUST) and (LC) constraints together imply that the continuation utility of both high and low time availability employees must be the same. Full insurance can only be incentive-compatible if the information rent given to the time-affluent employee is 0, which requires $h^L = 0$. The following lemma formalizes this result.

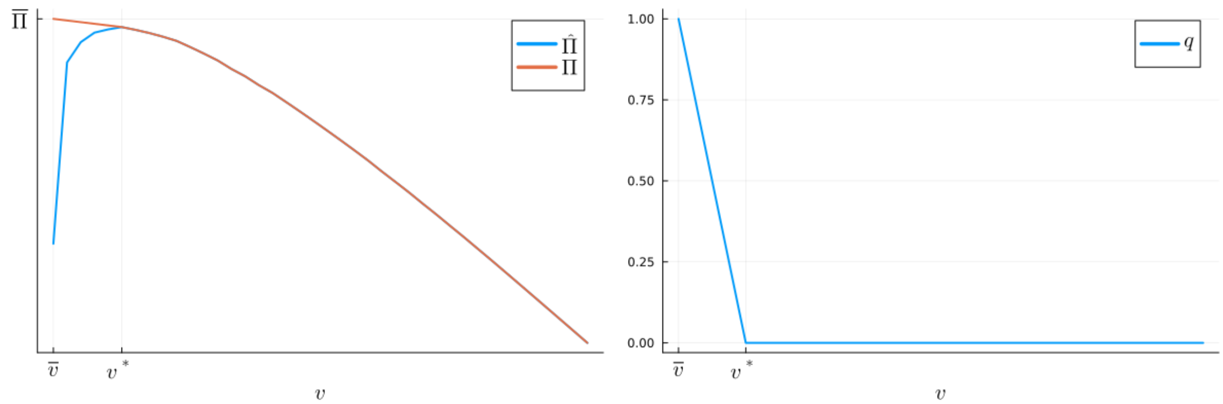
Lemma 1 *For an allocation satisfying (IC), (SUST) and (PK), h^L converge to 0 as v converges to $\overline{v}$.*

Relying on Lemma 1, we can now show that Π^c must be increasing in a region around $\overline{v}$. The result and proof are akin to Dovis (2019).

Proposition 3 *There exists a $\widetilde{v} > \overline{v}$ such that the (constrained) profit function $\Pi^c(v)$ is increasing over $[\overline{v}, \widetilde{v})$ and decreasing for $v > \widetilde{v}$.*

The proof relies on the fact that the condition $g'(0) = \infty$ implies that the gains from slightly increasing h^L must be larger than the cost of providing a higher expected utility

Figure 2: Pareto frontier and termination probabilities



(a) Principal's values. In red Π, in blue $\widehat{\Pi}$.

(b) Termination probabilities

to the employee. Finally, we characterize the optimality of termination in the following corollary.

Corollary 1 *If $\overline{\Pi} > \Pi^c(\overline{v})$, there exists a set of values $(\overline{v}, v^*)$ where a positive termination probability, i.e. $q < 1$, is optimal.*

Figure 2 depicts the results of Proposition 3 and Corollary 1. On the left, we see that there exist values v for which the principal's values are increasing. Hence, both parties would improve from increasing promised utilities v. On the right, we see that for promised utilities close enough to the outside option $\overline{v}$ termination occurs, i.e. termination probabilities are positive. Typically, v^* will be the point where the line going from $(\overline{v}, \overline{\Pi})$ to $(v^*, \Pi(v^*))$ is the tangent at $(v^*, \widehat{\Pi}(v^*))$. Below this point, increasing v would be beneficial for both the employer and employee. However, this would violate the (PK) constraint. This can be bypassed by offering a lottery between terminating and continuing at a higher v. Then, the promised utility upon continuation will be v^* and the termination probabilities will be given by

$$q = \frac{v - \overline{v}}{v^* - \overline{v}}. \tag{9}$$

This mechanism has an intuitive explanation in our setting. The employees that end up in the termination region are the ones that suffered from a long sequence of low time availability, permanently depressing continuation values. The closer the promised continuation value gets to the outside option, the costlier it is for the employer to provide flexibility. At $v = \overline{v}$, demanding flexibility would lead to a contract with zero hours $h^L = 0$ (Lemma 1) and correspondingly a low wage. From the employer's perspective it makes sense to either terminate the employment relationship or to increase the hours to a point where the gains from increased production are higher than the cost of providing a higher continuation value.

4 Generating Gendered Employment Dynamics

Up to this point we have studied the characteristics of the constrained optimal contract for a fixed probability of having low time availability. First, we have shown that low time availability is penalized with both current ($w_L < w_H$) and future ($v_L < v_H$) wage cuts. Second, a sufficiently long sequence of low time availability can drive employees into the termination region, making the outside option attractive. Hidden flexibility needs encoded as private information on time availability thus allow the model to generate employment dynamics.

We now introduce gender by comparing contracts with two different probabilities of low time availability. This will be the only parameter that will encode differences by gender with $p_{\text{men}} < p_{\text{women}}$. First, we consider *gender-tailored* contracts. These contracts are designed with the flexibility needs of each gender in mind. In practice, that means we solve the optimal contract twice, once for each p. Second, we consider *male-tailored* contracts. These contracts are designed with men's need for flexibility in mind but then given to both men and women. Third, we explore the wage dynamics in an intermediate case where the employer has to give the same contract to a team of men and women (*team-tailored* contract).

When contracts fully internalize flexibility needs (*gender-tailored* contracts), there are no systematic differences in employment dynamics by gender. In particular, this implies that the average wage gap is constant over time. We also show that men receive higher wage cuts when demanding flexibility which is consistent with the empirically documented flexibility stigma (Aaronson and French (2004)). Under *male-tailored* and *team-tailored* contracts, the wage gap gradually grows over time.

Finally, we show that men's and women's wages will not fully converge back after women's need for flexibility converges back to that of men (e.g. children are grown up). This is the case for each of the three types of contracts studied.

4.1 Comparative Statics in Gender-tailored Contracts

We now derive comparative statics results on how the optimal contract varies with p for a fixed continuation utility v. This allows us to understand how the optimal contract differs for a man and a woman who were promised the same compensation. The following proposition shows that for a male employee –i.e., lower p– the contract features lower hours when time-limited and lower current and future compensation for both high time availability (w^H and v^H) and low time availability (w^L and v^L).

Proposition 4 *For a fixed current promised utility, v, hours, h^L, wages, w^H and w^L, and promised utilities, v^H, and v^L, are all increasing in p. In particular, if $p_{men} < p_{women}$, men*

1. *earn smaller rewards for high time availability, i.e.*

$$w^{H,men} < w^{H,women} \text{ and } v^{H,men} < v^{H,women},$$

2. *experience larger penalties for low time availability, i.e.*

$$w^{L,men} < w^{L,women} \text{ and } v^{L,men} < v^{L,women},$$

3. *and work fewer hours after low time availability, i.e. $h^{L,men} < h^{L,women}$.*

The result is proven by considering an admissible perturbation of the optimal allocation that lowers h^L and wages (w^H and w^L) or promised utilities (v^H and v^L). Then, we show that the gains from this perturbation decrease with p, implying that in an optimal allocation hours h^L, wages and promised utilities are increasing in p for a fixed v.

The parameter p captures the employee's probability of experiencing low time availability and this is known by the employer. For example, this means that the employer understands that, on average, a young mother is more likely to ask for lower working hours than a single male employee. From the employer's perspective, a higher p increases the average costs of providing flexible working hours and increases the rewards and/or decreases the penalties required to deliver the same average compensation.

First, to understand why wages and promised utilities are higher for women, notice that a higher p increases the frequency of penalties (w^L and v^L) and decreases the frequency of rewards (w^H and v^H). Consequently, to deliver the same average compensation, the contract must offer an incentive-compatible combination of larger rewards and lower penalties.

Conversely, since men have to be penalized less often, the employer can achieve the same average compensation with lower rewards and higher penalties. This result is in line with the so-called flexibility stigma (Aaronson and French (2004), Golden (2020)): when men reduce work hours for family reasons, they are punished with higher wage cuts than women.[14]

To understand why h^L is lower for men than for women, recall from Proposition 2 that the employer provides flexibility but underworks the employee below the first best level. By doing so, the employer discourages an employee with high time availability from claiming low time availability, which, in turn, allows to provide more insurance against flexibility shocks.[15] For women, the expected direct cost (lower production) from reducing h^L is higher than for men due to the higher frequency of low time availability. Therefore, it is optimal to underwork the employee less. While this may appear counterintuitive, note that in Proposition 4, we study how optimal contracts differ by gender assuming that both were promised the same compensation level. However, in our dynamic model, the distribution of promised compensation levels is endogenous and, in particular, depends on the frequency of experiencing limited time availability.

The key insight from studying *gender-tailored* contracts follows from Proposition 2, where we showed that optimal contracts display approximately constant average wages over time. Recall that for *gender-tailored* contracts, we solve for the optimal contract twice, once for each p. This means, in particular, that if the (SUST) and (LC) constraints do not bind, an Inverse Euler equation holds for each $p \in \{p_{\text{women}}, p_{\text{men}}\}$:

$$\frac{1}{u'(w_{t-1})} = p\frac{1}{u'(w_t^L)} + (1-p)\frac{1}{u'(w_t^H)}. \tag{10}$$

Note that, with log utility, this implies that wages follow a martingale $w_{t-1} = pw_t^H + (1-p)w_t^L$ and averages wages would be constant over time. Therefore, if men and women entered the contract with the same initial wage, the gender gap in average wages would be approximately zero and constant over time. Intuitively, following from Proposition 4, the frequency offsets the size of the penalties. Women have lower penalties and higher rewards than men but at a higher frequency and vice versa. Hence, contracts that fully internalize the flexibility needs of each employee do not display systematic differences in wage dynamics.

[14] It also follows that h^H is decreasing in p because the hours of the time affluent employee are decreasing in wages due to income effects as shown in the optimality condition (4).

[15] Formally, by lowering h^L, the employer reduces information rent given to an employee with high time availability at the direct cost of reducing production. Lower information rents allow the employer to provide more insurance to employees with low time availability, which is profitable as the same v can now be delivered with lower average wages.

Finally, although men experience larger drops in wages for demanding flexibility, a direct implication of Proposition 4 is that the total utility penalty is actually larger for women. To understand this, notice that from the incentive constraints we can write:

$$\big(U(w^H, h^H; f^H) + \beta v^H\big) - \big(U(w^L, h^L; f^L) + \beta v^L\big) = (f^H - f^L)\psi(h^L).$$

Since h^L is larger for women (as shown in Propositon 4) the gap in utility between high and low time availability is also larger for women. Intuitively, because the women are less underworked with low time availability, the employer is not able to provide as much insurance against flexibility needs.

4.2 Male-tailored Contracts

So far we have considered optimal contracts catering to the individual's flexibility needs. In practice, it may not be feasible –legally or due to complexity reasons– for the employer to design multiple types of contracts for various employees. Our framework can be easily used to study employment dynamics with contracts designed only for one type of employee. In particular, we consider the case where both men and women are under a contract initially designed for men. We believe that this a useful polar case to study, because actual contracts may not be designed specifically to cater each employees' needs. Specifically, in male dominated working environments, women's higher need for temporal flexibility may not be taken into account.

In Section 4.1, we showed that there are no systematic differences in wage dynamics for *gender-tailored* contracts. Conversely, if a woman took a contract designed for men (*male-tailored* contract), she would experience the same penalty and reward structure as her male colleague, but she would still be penalized more often. By Proposition 4, she now suffers higher penalties and lower rewards than in her optimal contract. Equation (8) and $p_{\text{women}} > p_{\text{men}}$ imply

$$\frac{1}{u'(w_{t-1}^{\text{men}}(f^{t-1}))} > p_{\text{women}} \frac{1}{u'(w_t^{\text{men}}(f^{t-1}, f^L))} + (1 - p_{\text{women}}) \frac{1}{u'(w_t^{\text{men}}(f^{t-1}, f^H))}, \tag{11}$$

where superscript *men* denotes allocations under men's optimal contract. Women's wages diverge downwards under *male-tailored* contracts and the average wage gap grows gradually over time. Intuitively, women now bear both disadvantages, higher penalties/lower rewards as well as a higher chance of being time-limited. For the same reason, women's promised utility also diverges downwards under a *male-tailored* contract. This will generally result in larger termination rates for women.

Women's wage dynamics depend on the characteristics of men's contract and their probability of low time availability. The following proposition uses the Inverse Euler equation for men's contracts to characterize the growth of the gender wage gap under *male-tailored* contracts.

Proposition 5 *With log utility ($u(c) = \log(c)$), the expected growth rate of women's wages under a male-tailored contract is equal to:*

$$\mathbb{E}_{p_{women}}\left(\frac{w_t^{men}(f^t)}{w_{t-1}^{men}(f^{t-1})}\right) - 1 = \underbrace{\left(\frac{p_{women} - p_{men}}{1 - p_{men}}\right)}_{>0,\ \textit{Difference in ps}} \underbrace{\left(\frac{w_t^{men}(f^{t-1}, f^L)}{w_{t-1}^{men}(f^{t-1})} - 1\right)}_{<0,\ \textit{Men's penalty}} < 0, \tag{12}$$

where $\mathbb{E}_{p_{women}}$ denotes the expectation under the probability p_{women}.[16]

The growth rate of women's wages – and so of the gender wage gap– depends on two terms: the difference in the probability of low-time availability and the change in men's wages in case of low time availability. Hence, the wage gap will grow faster when the differences between p_{women} and p_{men} are larger and/or men are penalized more for low-time availability.

Finally, since continuation utilities for women under contracts designed for men do not coincide with men's continuation utility, it is not guaranteed that the contract remains incentive-compatible for women. In Appendix E, we provide conditions for incentive compatibility (for both types f^H and f^L) in this case and verify them numerically.

4.3 Team-tailored Contracts

A less polar case than the *male-tailored* contract is a scenario in which an employer hires a fraction $s \in (0, 1)$ of men and a fraction $(1 - s)$ of women but has to give the same contract to both genders. We refer to this intermediate case as *team-tailored* contracts. Again, we can show that the optimal contract satisfies an Inverse Euler equation but with the (history-contingent) average probability of low time availability. To this end, we denote by $P^{\text{men}}(f^t)$

[16] With the general utility function, we have a similar expression for the growth rate of the inverse marginal utilities:

$$\mathbb{E}_{p_{\text{women}}}\left(\frac{u'(w_{t-1}^{\text{men}}(f^{t-1}))}{u'(w_t^{\text{men}}(f^t))}\right) - 1 = \left(\frac{p_{\text{women}} - p_{\text{men}}}{1 - p_{\text{men}}}\right)\left(\frac{u'(w_{t-1}^{\text{men}}(f^{t-1}))}{u'(w_t^{\text{men}}(f^{t-1}, f^L))} - 1\right).$$

and $P^{\text{women}}(f^t)$ the probability measure over histories induced by the probabilities of low time availability p_{men} and p_{women}, respectively.

Proposition 6 *If an incentive-compatible team-tailored contract exists, at every history f^t, it satisfies the following Inverse Euler equation:*[17]

$$\frac{1}{u'(w_{t-1}^{avg}(f^t))} = p^{avg}(f^t)\frac{1}{u'(w_t^{avg,\ L}(f^t, f^L))} + (1 - p^{avg}(f^t))\frac{1}{u'(w_t^{avg,H}(f^t, f^H)))},$$

where

$$p^{avg}(f^t) \equiv \frac{sP^{men}(f^t)p_{men} + (1-s)P^{women}(f^t)p_{women}}{sP^{men}(f^t) + (1-s)P^{women}(f^t)},$$

and $p_{men} < p_{avg}(f^t) < p_{women}$ for all f^t.

Because $p_{\text{men}} < p_{\text{avg}}(f^t) < p_{\text{women}}$, in a *team-tailored* contract, men's wages always diverge upwards and women's wages downwards. If $p^{\text{avg}}(f^t)$ is close to p_{men}, i.e. when s is close to one and/or in histories f^t that are more likely for men, the contract approaches the *male-tailored* contract. Therefore, men's wages will be approximately constant over time, and women's wages diverge downwards faster. Conversely, if $p_{\text{avg}}(f^t)$ is close to p_{women}, women's wages are approximately constant, and men's wages diverge upwards faster.

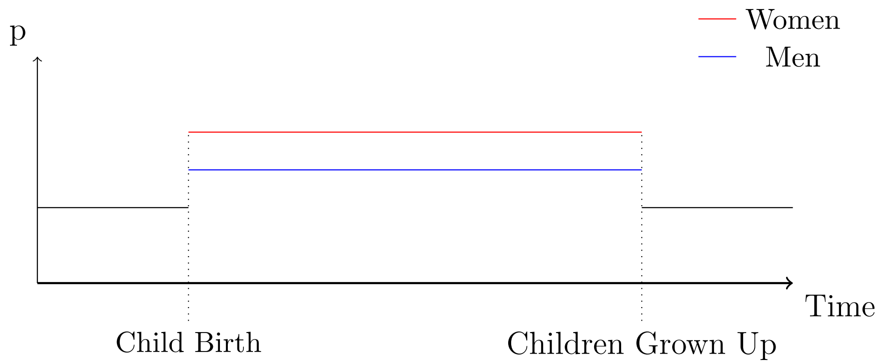
We can generally not show that an incentive-compatible *team-tailored* contract exists. However, if the *male-tailored* contract studied in the previous section is incentive-compatible for both men and women, there must also exist a team-tailored contract as the employer can always use the *male-tailored*.

4.4 Non-convergence of Wages

Goldin *et al.* (2022) document that women's wages do not converge back to the level of men after children grow up. In our model, we can interpret the children growing up as p_{women} converging to p_{men}. Figure 3 simplifies how we think our model fits into the life-cycle. Before childbirth, men and women have a similar external demand on time. After childbirth but before children grow up, women are more exposed to flexibility needs than men so

[17] In the proof of the proposition in Appendix B we also lay out the employer's problem. Solving for the optimal contract is challenging because the continuation utilities of men and women do not coincide, and, a priori, we do not know which incentive constraints bind. However, we can characterize the Inverse Euler equation in the proposition with a variational argument because uniform changes in utilities preserve the incentive and participation constraints of men and women.

Figure 3: Differences in p over time by gender.



$p_{\text{women}} > p_{\text{men}}$. Once children have grown up, men and women have a similar probability of limited time availability again.[18]

In this case, the wage dynamics in our model are consistent with the documented non-convergence of wages. Under *gender-tailored*, *male-tailored* as well as *team-tailored contracts*, the wages of women who have been penalized do not converge back to the level of men.

First, consider the *male-tailored* contract. If p_{women} converges to p_{men}, women's wage process converges to the same process as men's. By Proposition 2, inverse marginal utilities then follow a martingale, so average wages are approximately constant over time. Therefore, the wages would stop diverging downwards, but they do not converge to the same level as men's. The same logic applies for *team-tailored* contracts.

For *gender-tailored* contracts, Appendix F.3 studies an extension in which we allow the probability p to follow a time-varying and possibly stochastic process. Because p is observable, the employee is perfectly insured against changes in p. In particular, this implies that an employee who was penalized with low wages will not experience a sudden increase in wages if p increases.

5 Numerical Simulations

To connect our theoretical results with empirical regularities on gendered employment dynamics we solve the model numerically. We use the American Time Use Survey (ATUS) to calibrate the probabilities of being time limited by gender. We use these values to understand a reasonable range of values that our model is able to generate with respect to wage

[18]There may also be gender differentials before and after childbirth, e.g., caring for elderly parents.

gaps and exit rates.

5.1 Parametrization

We use the following isoelastic parametrization of the utility function

$$U(w,h;f) = \frac{w^{1-\sigma}}{1-\sigma} - (1-f)\frac{h^{1+\eta}}{1+\eta}, \tag{13}$$

where we set $\sigma = 2$ and $\eta = 2$. The Frisch elasticity, $1/\eta$, is thus equal to 0.5 following Chetty *et al.* (2011). For the numerical exercise, we consider a period to be equal to a quarter. Hence, we set the discount factor equal to $\beta = 0.987$. We believe this to be the best compromise between the high frequency of interruptions during a working week and the low frequency of wage adjustments observed in employment contracts.[19] We parameterize the production by

$$g(h) = \frac{h^\alpha}{\alpha},$$

and set $\alpha = 0.7$, which is the value commonly used to match the labor share.[20] In our model, α captures how costly it is for the employer to allow for volatility in the employee's working hours. We would thus expect this parameter to vary across occupations. If $\alpha = 1$, the production function is linear, and the employer only cares about expected hours. As α decreases, the employer prefers smoother working hours as the cost of providing flexibility increases. Qualitatively, our results do not rely on the specific value of α, except that decreasing returns ($\alpha < 1$) are needed to generate termination.

5.2 Identifying p, f^H and f^L

Our main parameter of interest is the probability of having low time availability p. This is the only parameter we use to differentiate men and women. Thus, we need to determine p_{men} and p_{women}. To identify meaningful values we turn to the American Time Use Survey (ATUS). The ATUS is a nationally representative U.S. time diary survey with detailed information on how many minutes at a certain time of the day respondents spent on different activities including work, care and leisure. We restrict our sample to full-time workers (excluding

[19]More generally, we may think that wages are adjusted at a lower frequency (e.g. yearly) but the employee can adjust its borrowing or saving in anticipation of future pay raises or wage cuts.

[20]For robustness, we do comparative statics on the value of α for our main result, see Figure 11 in Appendix A.

self-employed), aged between 20 and 65, and having at least one child below the age 12. We gather information on when and how many minutes a respondent spent on care activities, as well as the typical working hours in their current job.

Our goal is to determine gendered values for p that align with the reduction in hours implied by f^L. In the data, care activities are given in minutes, but in our model time availability is binary. To map the data to the model, we need to find a threshold of minutes spent on care to define an interruption. First, we construct the *Care-Work-Ratio*, which indicates the share of minutes of care activities during usual working time:

$$\text{Care-Work-Ratio} = \frac{\text{Minutes of Care Activities between 9am and 5pm}}{\text{Usual Minutes of Work}}.$$

We put the usual working minutes in the denominator so as not to inflate the care-work ratio by reducing the working time due to care activities. For more details on the sample selection and robustness of the following results, see Appendix D.

Second, we define the cutoff X such that, among all individuals with a Care-Work-Ratios above X, the average interruption comprises 25% of the working day:

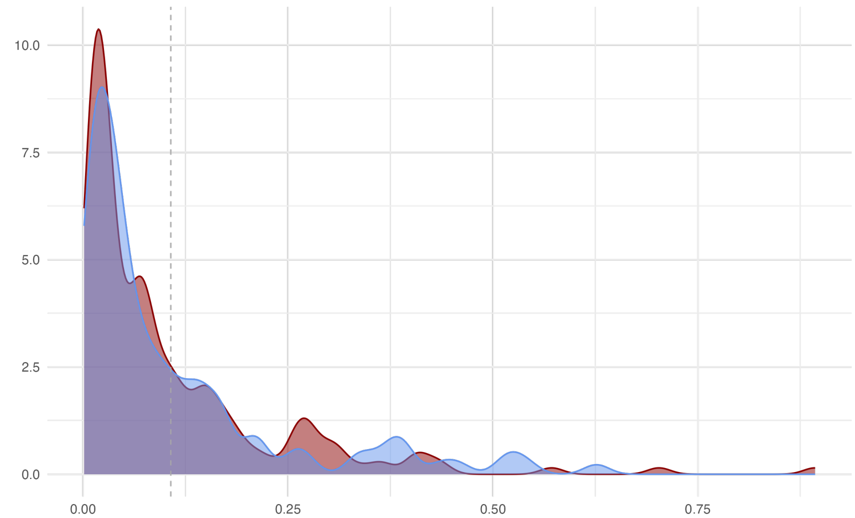
$$\mathbb{E}\left(\text{Care-Work-Ratio} \mid \text{Care-Work-Ratio} \geq X\right) = 0.25.$$

We do so to then choose f^L accordingly such that, in a fixed per-hour wage contract, an employee would reduce work time by 25%. We find a cutoff of X close to 11%. In our sample, 18% of men have a positive care-work ratio compared to 43% of women. The average care-work ratio in the whole sample is 2% for men and 4% for women. However, conditional on being positive, the average care-work ratio is similar: 11% for men and 10% for women. Figure 4 shows the distribution of all positive care-work ratios by gender. Figure 4 highlights that conditional on being positive, the distribution of care-work ratios for men and women is very similar. The size of most care interruptions during working hours is small, i.e. close to zero and below the cutoff X we find.

Hence, when we then translate the continuous minutes of care work during working hours into binary interruptions, we classify all care activities whose share of the working day is above 11% as an interruption. The probability of being hit with low time availability is then simply:

$$p_{\text{men}} = \frac{\text{Number of men with Care-Work-Ratio} \geq X}{\text{Number of men}}$$

Figure 4: Distribution of positive Care-Work-Ratios by gender



Note: Dashed line indicates cutoff X. Women in red, men in blue.

and

$$p_{\text{women}} = \frac{\text{Number of women with Care-Work-Ratio} \geq X}{\text{Number of women}}.$$

Using this approach we obtain $p_{\text{men}} = 0.06$ and $p_{\text{women}} = 0.15$. This means that while men have a 6% chance of having low time availability, women experience low time availability with a probability of 15%. Women are more than twice as likely to experience low time availability than men.

The cutoff X is arbitrary in the sense that we choose the size of the average interruption. In Table 3 in Appendix A, we present results for different cutoffs. As expected p_{men} and p_{women} increase when we reduce the average reduction in hours to 20% and decrease when we increase the average reduction in hours to 30%.

Moreover, in Table 1, we show further probabilities of limited time availability by gender across socioeconomic groups. While p_{men} remains fairly stable across groups, p_{women} varies substantially but always remains larger than p_{men}. Women without a college degree, below median family or weekly income are the most likely to be interrupted during working hours.

We are not the first to use the ATUS to document gender differences in external demands on time during working hours. Both Cubas *et al.* (2021) and Schoonbroodt (2018) analyze the effects on wages of parental childcare during working hours. In comparison to them, we document relatively small incidences of limited time availability, suggesting that our

Table 1: Probabilities of low time availability across different socioeconomic groups.

	p_{men}	p_{women}
Baseline	0.06 (0.012)	0.15 (0.022)
Non-College	0.07 (0.017)	0.16 (0.033)
College	0.06 (0.015)	0.11 (0.025)
Below Median Family Income	0.06 (0.020)	0.17 (0.041)
Above Median Family Income	0.05 (0.001)	0.11 (0.023)
Below Median Weekly Earnings	0.10 (0.027)	0.18 (0.031)
Above Median Weekly Earnings	0.05 (0.012)	0.09 (0.027)

Note: Standard errors in parenthesis.

results can be seen as a lower bound. Moreover our results are not directly comparable, since we map (continuous) minutes of care activities into our binary framework of high/low time availability. The most precise measure of differences in external demand on time can be found in Buzard *et al.* (2023). They conduct an experiment to measure how frequently schools contact mothers compared to fathers. They find that mothers are contacted 1.4 times more often. Since this addresses external time demands in a narrow context, their results can be considered a conservative estimate, as Buzard *et al.* (2023) argue.

To determine values for f^H and f^L, we start by normalizing the disutility of working when having high time availability such that the hours of the time affluent employee in the first best would equal 1/2.[21] The calibration of this parameter depends on p; using $p = p_{men}$ for both genders, we get $f^H = -5.2$. The value of f^L deserves some attention, as it determines how much a flexibility shock affects the disutility of working. We calibrate it by assuming that after a flexibility shock, if the employee was offered a fixed per-hour wage contract, she would want to work 25% less hours. Therefore, the size of an interruption in the model coincides with the one we used in the data to calibrate p. Under this assumption we obtain a value of $f^L = 1 - (4/3)^{\sigma+\eta}(1 - f^H) = -18.7$.[22]

[21] Note that in the first best, we have $v_0 = \frac{1}{1-\beta}\left(u(w^{FB}) - (1-p)(1-f^H)\psi(h^{H,FB}) - p(1-f^L)\psi(h^{L,FB})\right)$, which combined with (4) allows us to solve for the wage and hours.

[22] To derive this, notice that under a wage per-hour contract, for any $f \in \{f^L, f^H\}$, we have $w(wh)^{-\sigma} =$

5.3 Solution Method

We need to deal with two technical challenges to solve our dynamic contracting problem numerically. First, common to all recursive contracting problems, because the constraints are forward-looking, the transversality conditions may not hold. Marcet and Marimon (2019) provide a recursive Lagrangian formulation to solve this problem. However, a direct application of this method is known to fail when the Pareto frontier is not strictly concave (Cole and Kubler (2012)), which is the case in our model as shown in Section 3.5.

Our approach consists of using two different methods at different parts of the state space. From Section 3.5, we know that the constrained Pareto frontier Π^c is only concave when v is close enough to $\overline{v}$. Thus, for $v > v_{min}^{MM}$ with v_{min}^{MM} large enough, we first solve the model with the recursive Lagrangian following (Marcet and Marimon (2019)), which guarantees that the transversality condition holds. Then, for $v \in [\overline{v}, v_{min}^{MM}]$ we solve the model using a direct promised utility approach. A more detailed description of the solution method and algorithm can be found in Appendix C.

5.4 Outside Options

Finally, we also need to set values for the outside options of the employer and the employee, which, for the numerical exercise, we allow to be stochastic, as we explain next.

Stochastic Employee's Outside Option. To have a positive termination probability, the continuation value v must eventually reach the region with ex-post inefficiencies. Hence, we need $v^L < v$ at all points outside the region with positive termination probability. When the (SUST) constraint binds, a high cost of reaching the inefficient region can render $v^L > v$ optimal.[23] In our parametrization, with both p_{men} and p_{women}, we get $v^L > v$ as v approaches the termination region, so there is no exit in the optimal contract.

To circumvent this issue, we extend the model by letting the employee's outside option be stochastic. That is, we assume $\overline{v}$ can take values on a grid $\{\overline{v}_1, .., \overline{v}_i, ..., \overline{v}_I\}$ with corresponding probabilities $\{\overline{p}_i\}_{i=1}^{I}$. However, we maintain the assumption that this outside option is observable by the employer. Intuitively, this stochasticity smooths the employer's cost of

$(1-f)h^{\eta}$. So the ratio of hours is $\frac{h^H}{h^L} = \left(\frac{1-f^L}{1-f^H}\right)^{\frac{1}{\sigma+\eta}}$, then setting $\frac{h^H}{h^L} = \frac{4}{3}$ and solving for f^L we derive the expression above.

[23] Dovis (2019) derives conditions to have $v^L < v$ everywhere, but we find numerically that they typically are not sufficient in our model.

lowering v^L as the (SUST) constraint may not bind at $t+1$ when $\overline{v}_i$ is small. More details on the employer's problem with the stochastic outside option can be found in Appendix F.1.

Employer's Outside Options. Next, we need to assign a value to the employer's outside options in a contract with a man ($\overline{\Pi}^{\text{men}}$) and a woman ($\overline{\Pi}^{\text{women}}$). For every v, the employer value in a contract with a man $Q^{\text{men}}(v) \equiv \sum_{i=1}^{I} \overline{p}_i \Pi^{\text{men}}(v, \overline{v}_i)$ is generally much larger than $Q^{\text{women}}(v)$. Hence, if we set a common outside option $\overline{\Pi}^{\text{men}} = \overline{\Pi}^{\text{women}}$, the contract will typically deliver either no termination for men or termination at all v for women. For this reason, we pin down the outside options based on the highest value that the employer can attain by replacing the current employee with another one of the same gender.[24] We approximate the value of replacing the employee with the value functions in the constrained problem where the employer cannot terminate Π^c defined in Section 3.5.[25] That is, we set $\overline{\Pi}^{\text{men}} = \max_v \sum_{i=1}^{I} \overline{p}_i \Pi^{c,\text{men}}(v, \overline{v}_i)$ and $\overline{\Pi}^{\text{women}} = \max_v \sum_{i=1}^{I} \overline{p}_i \Pi^{c,\text{women}}(v, \overline{v}_i)$.

6 Numerical Results

Solving for the optimal contract numerically allows us to get a better picture on how gendered labor market outcomes arise from our theoretical framework of hidden demand for flexibility. We first consider wage dynamics and gender differences in penalties. Then, we assess differences in employment dynamics captured by job duration and termination probabilities. Comparing *gender-tailored* and *male-tailored* contracts allows us to emphasize the importance of employment contracts that account for differences in flexibility needs.

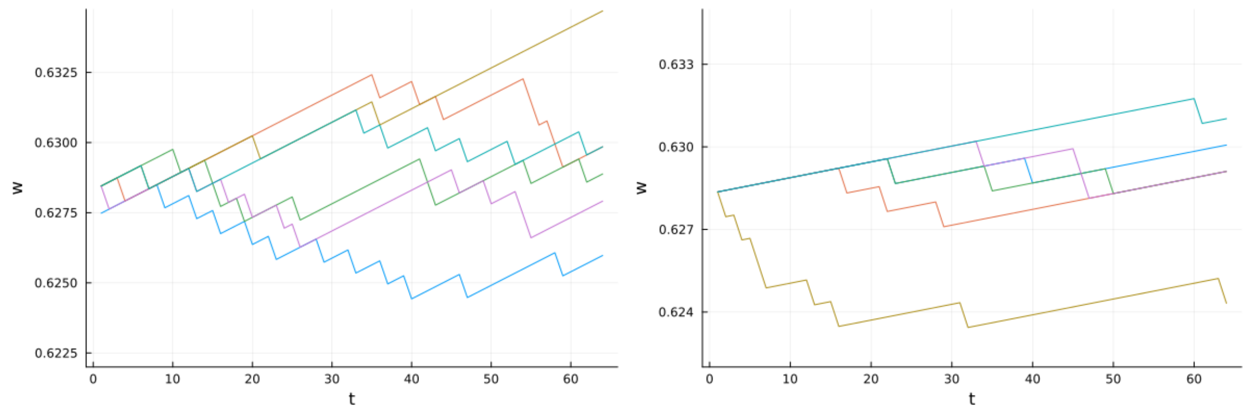
6.1 Wage Dynamics

Gender-tailored Contracts. Figure 5 shows five (randomly selected) sample paths of wages for women (left panel) and men (right panel) under *gender-tailored* contracts over 64 periods. To focus on the gender differences in wage dynamics, in this figure and throughout the section we set v_0^{men} and v_0^{women} such that men and women have the same average initial wages. Moreover, we solve the model at a v_0 far away from the termination region to focus on the effects of the private information friction. Each colored line represents the wage path

[24] This is a conservative value. If we assume, for example, that the employer may sometimes also hire an employee of the other gender, we would get higher termination probabilities for women.

[25] Otherwise, finding the outside options would require us to solve the fixed point problem $\overline{\Pi}^{\text{g}} = \max_v \sum_{i=1}^{I} \overline{p}_i \Pi^{c,\text{g}}(v, \overline{v}_i; \overline{\Pi}^{\text{g}})$ for each $g \in \{\text{men}, \text{women}\}$.

Figure 5: Wage paths gender-tailored contracts



(a) Wage Paths for Women (b) Wage Paths for Men

Note: Each colored line represents a randomly selected sampled path for women (left panel) and men (right panel).

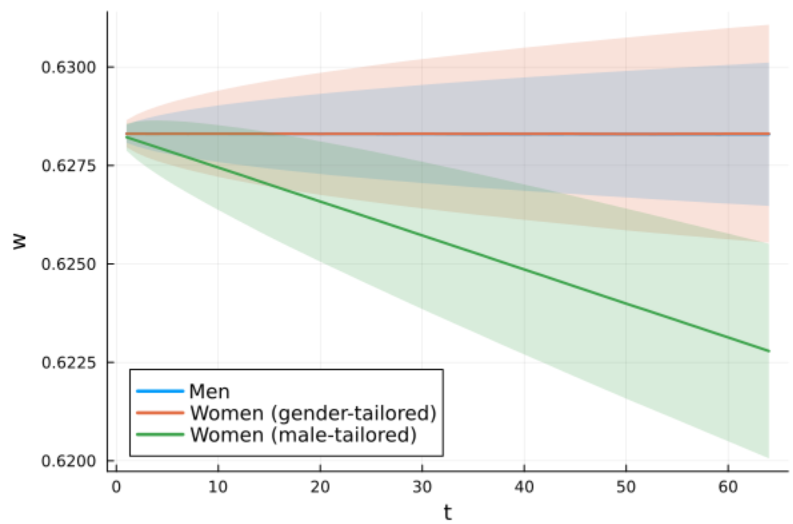
of an individual employee characterized by their respective sequence of time availability. Since, in our framework, gender is not mechanically encoded in the employee's preferences, both men and women could experience long sequences of low time availability. However, it is more likely for a woman to have such an extended series of low time availability. Hence, it is not the difference in p directly that the employer penalizes but the realizations of low time availability.

We observe that women suffer frequent and small wage penalties ($w^L_{\text{women}}/w_{t-1} - 1 = -0.11\%$), whereas men's wages feature slightly larger ($w^L_{\text{men}}/w_{t-1} - 1 = -0.12\%$) but less frequent penalties. In contrast, women experience larger rewards ($w^H_{\text{women}}/w_{t-1} - 1 = 0.019\%$) than men ($w^H_{\text{men}}/w_{t-1} - 1 = 0.007\%$), but less frequently.

Male-tailored Contracts. Figure 6 shows the average wage dynamics of men and women under *gender-tailored* and *male-tailored* contracts. The line indicates the average wage and the shaded area one standard deviation along the cross-section in each period. In blue (men) and red (women) we show wage dynamics under *gender-tailored* contracts, i.e with p_{women} and p_{men}. Average wages of men and women are the same – the red and the blue lines overlap – but women's wages feature an increasingly larger variance over time. For women under *male-tailored* contracts, the gender wage gap widens. Average wages between men and women gradually diverge, accompanied by an increasing volatility. After 16 years (64 periods), the wage gap grows to be 0.88%.

Given our normalization of a 25% reduction in working hours from an interruption, it is important to check the sensitivity of the resulting wage gap to this value. We recalibrate the parameters f^L, p^{women} and p^{men} by setting an interruption to involve 20% and 30% reductions in working hours (see Table 3 in Appendix A). We find that the ratio between p^{women} and p^{men} is larger with the 20% reduction, leading to a wage gap of 1.12%. However, this ratio is smaller with the 30% reduction, where we obtain a wage gap of 0.64%.

Figure 6: Average wage dynamics



Note: The lines denote the average wages along the cross-section each period and the shaded areas one standard deviation.

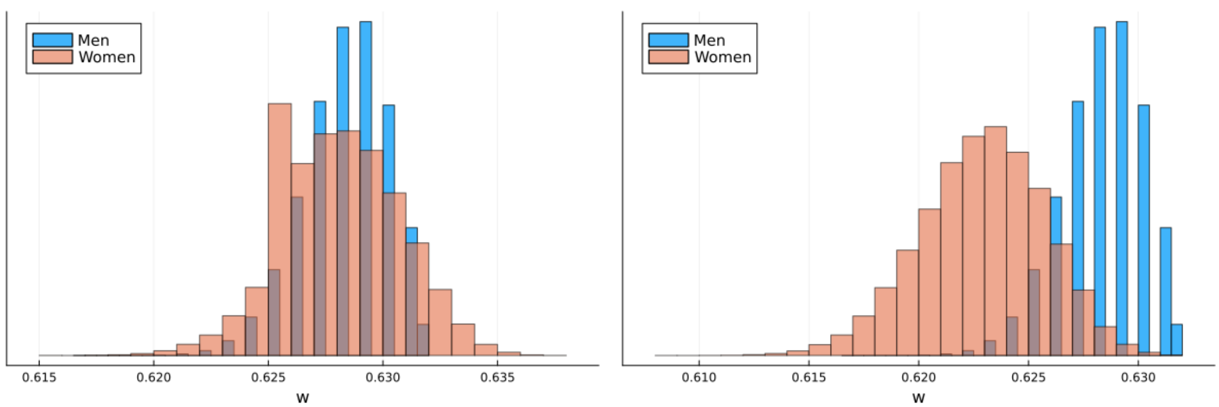
To put these numbers in perspective, note that in Figure 6, we are comparing the wage trajectory of equally productive men and women in the same occupation and firm that started with the same wages and that made no adjustments to their initial working arrangement. Moreover, by setting the same initial compensation the employer fully internalizes the higher cost of childcare responsibilities of women compared to men. In Section 7, we discuss how our mechanism explains a sizeable amount of the corresponding gender wage gap the empirical literature has found.

Figure 7 shows the distribution of wages after 16 years (64 periods), with men's wages displayed in blue and women's in red. The left panel shows the wage distribution for *gender-tailored* contracts. There is a higher mass of women with lower wages corresponding to female employees with a longer sequence of low time availability. However, we also see a larger mass of women with higher wages than men, resulting from the "lucky" women who suffered almost no low time availability and benefited from the higher rewards.

On the right panel, we compare the wage distributions of men with women under *male-tailored* contracts. Women's wage distribution is shifted to the left. Conversely, the distribution of men's wages is more skewed and shifted to the right. While men and women now experience the same penalty and reward structure, women get penalized more frequently than men.

Finally, we may also consider the case where men are under a women-tailored contract. In this scenario, men would be exposed to less frequent and smaller penalties, so their wages would diverge upwards (see Figure 9 in Appendix A). However, this result must be interpreted with caution because, as shown in Appendix E, the contract may not remain incentive-compatible.

Figure 7: Wage Distributions (at $t = 64$)



(a) Gender-tailored contract

(b) Male-tailored contract

Comparison across Socioeconomic Groups. Table 2 shows the average growth rate of women's wages as well as the average wage gap after 16 years across different socioeconomic groups. Differences in p between men and women across groups directly translate into differences in average growth rates and wage gaps. Our model predicts the smallest wage gap between men and women who earn more than the median weekly income, while the largest can be found among those whose family income is below the median. The wage gap depends on both men's and women's probability of limited time availability within each group. As discussed in Section 4.2, the average growth rate of women's wages under *male-tailored* contracts depends on the size of the penalties for men and the ratio of p_{men} and p_{women}. Compare, for example, college-educated and respondents with family income above the median: while p_{women} is the same, p_{men} is one percentage point higher for college-educated men. This relative difference translates into sizeable differences in wages after 16 years.

Table 2: Comparison of the wage gaps in male-tailored contracts across socioeconomic groups

	p_{men}	p_{women}	Avg. growth rate of womens' wages	Avg. wage gap after 16 years
Baseline	0.06	0.15	-0.014%	0.88%
Non-college	0.07	0.16	-0.014%	0.88%
College	0.06	0.11	-0.008%	0.49%
Below median weekly earnings	0.10	0.18	-0.012%	0.78%
Above median weekly earnings	0.05	0.09	-0.006%	0.39%
Below median family income	0.06	0.17	-0.018%	1.07%
Above median family income	0.05	0.11	-0.009%	0.58%

6.2 Employment Dynamics

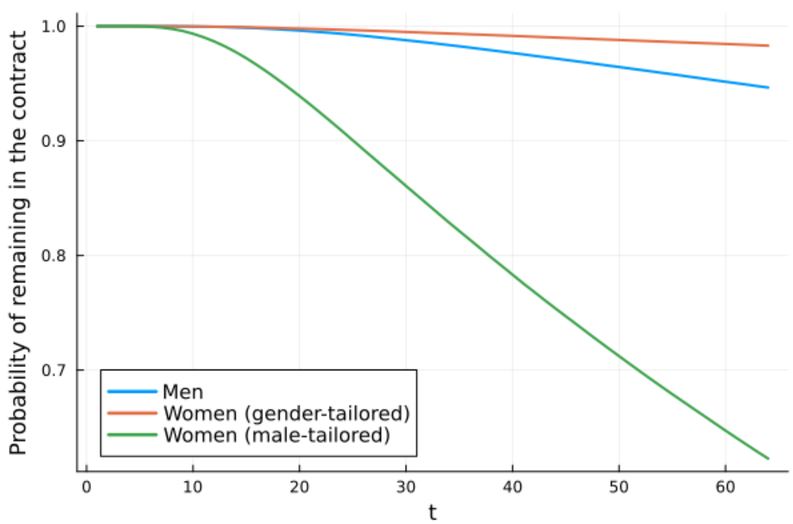
Figure 8 plots the probability of remaining on the contract over time for men and women under the *gender-tailored* contract and for women under the *male-tailored* contract. We set the same starting promised utility for men and women. Under *gender-tailored* contracts, the termination rates are slightly higher for men despite a higher mass of women being penalized. The intuition is the following. As compensation levels get pushed down, the contract requires that h^L goes to zero, which is costly for the employer. For men, on average, this cost is lower due to the lower probability of limited time availability. Hence, the cost of decreasing future compensation v^L as the contract approaches the termination region is lower for men. As a result, the penalized men end up reaching the termination region faster than women.

Under the *male-tailored* contract, similar to the wage dynamics, low time availability is now penalized with even lower future compensation pushing the contract faster to the termination region. Combined with the higher frequency of low time availability, this leads to larger termination rates for women.

After 16 years (64 periods), more than a third of women would have exited the contract, while under the *gender-tailored* contract, the share would be less than 2%. The share of women who terminate the contract is more than six times larger than for men. To put this result into context, note again that our model only captures ending employment relations due to the incompatibility of unpredictable flexibility needs and the current work arrangement. There are many other reasons why job duration rates vary by gender. For example, men are more likely to switch to other jobs for opportunity than for flexibility reasons, which is outside the scope of our model. In Section 7, we discuss to what extent our termination rates might even be smaller than their empirical counterpart.

Notice also that the termination probabilities are zero for some initial periods as it takes some time for the employees to reach the inefficient region. Whether termination is highest in the initial or later periods depends on how close the starting promised utility v_0 is to the outside option (see Figure 10 in Appendix A). However, regardless of the chosen starting value, it is always the case that the termination rates of men and women are similar under the *gender-tailored* contract but much larger for women under the *male-tailored* contract.

Figure 8: Termination dynamics



Note: Each line represents the fraction of men or women that remain in the contract at every period.

7 Recovering Stylized Facts

Our model simultaneously accounts for both wage divergence and non-convergence, as well as gender differences in labor force attachment. In this section, we contextualize our numerical results with empirical estimates and summarize the key mechanisms of our model.

7.1 Women's wages start to diverge from men's after childbirth but do not fully converge after children grow up

The bulk of the gender wage gap can be traced back to the child penalty. After childbirth, men and women's wages diverge even if they were on the same career paths before (Kleven *et al.* (2019b), Cortés and Pan (2023), Barth *et al.* (2021)).

Divergence. The empirical literature has found various estimates for the child penalty, comparing mothers to non-mothers (Yu and Hara (2021)), women to men (Blau and Kahn (2017)), mothers to fathers (Yu and Hara (2021), and within couples (Angelov *et al.* (2016)), revealing a range of wage gaps between 10% to more than 30%. Different studies emphasize different drivers of this penalty, but they consistently identify the most important factors to be reductions in hours (Angelov *et al.* (2016), Adda *et al.* (2017), Goldin (2014), Kleven *et al.* (2019b), Wasserman (2023), Erosa *et al.* (2016)), workforce interruptions (Adda *et al.* (2017), Bertrand *et al.* (2010)), pre- and post-selection into more flexible and lower-paying jobs (Blau and Kahn (2017), Adda *et al.* (2017), Yu and Hara (2021), Goldin *et al.* (2017), Morchio and Moser (2024), Felfe (2012), Wiswall and Zafar (2018), Card *et al.* (2016)) or leaving the labor force (Kleven *et al.* (2019b), Harkness *et al.* (2019), Adda *et al.* (2017)). Our focus, however, lies in understanding how women fare in the same job compared to male coworkers and how, over time, they get pushed out of an employment relation and into an ex-ante inefficient outside option.

We thus study wage trajectories of equally productive men and women in the same occupation and firm who started with the same wages and made no adjustments to their initial working arrangement. When ps are different, under *male-tailored* and *team-tailored* contracts, women's average wages gradually diverge downwards. In work arrangements that are designed with men's flexibility needs in mind, women bear both the higher penalties as well as the higher frequency of being time limited.

The closest empirical counterpart to the gender wage gap we are considering is thus within-firm, within-occupation differences. Here, estimates of the average wage gap range from 2% to 4% (Budig and England (2001), Yu and Hara (2021), Morchio and Moser (2024), Felfe (2012), Lucifora *et al.* (2021), Bronson and Thoursie (2019)). The main driver of this gap is predominantly forgone promotion opportunities. In our main calibration, we find a wage gap of 0.88% after 16 years, which is sizeable for the narrow context we are considering. In particular, this suggests that our mechanism can explain between 20% and 40% of the gap. This result does not take into account the wage cuts of women that end the employment relation. Our model can capture this too. Women's wages also diverge because some women - those with a sufficiently long sequence of limited time - are being pushed out of their current work employment. In particular, they leave for an outside option that has a lower continuation value than their current employment. While we are agnostic about what the outside option entails, we can interpret this as any type of lower wage work arrangement. Hence, the outside option could entail dropping out of the labor force, more flexible work arrangements or even self-employment. The literature has focused on modeling the wage gap across occupations as a result of an optimal tradeoff between wages and flexibility/amenities (Goldin (2014), Morchio and Moser (2024)). By contrast, our analysis shows how women can be ex-post pushed into lower-paying arrangements.

Non-convergence. Having children has long-lasting effects. Even when children are grown up women's wages do not fully converge back (Goldin *et al.* (2022), Angelov *et al.* (2016)).

Our model is able to capture the non-convergence of wages after women's flexibility needs converge back to men's.[26] As discussed in Section 4.4, because the penalties for demanding flexibility are permanent, the wages of women who experienced low time availability more often are permanently depressed and do not converge. This is true under any of the three types of contracts studied. One could interpret these long-lasting effects on wages as forgone pay raise opportunities. An example of this could be promotions, which are the predominant drivers of within-firm differences in wage trajectories (Lucifora *et al.* (2021), Bronson and Thoursie (2019)).

7.2 Women's job duration is shorter and labor force attachment is weaker

Women have been shown to have both shorter job duration and weaker labor force attachment than men, often stemming from child and elderly care responsibilities (Hall (1982), Molloy *et al.* (2020), Cortés and Pan (2023), Lundborg *et al.* (2017)). This is reflected in women being more likely to stay at home, find a new family-friendly job (Mas and Pallais (2020), Wiswall and Zafar (2018), Aaronson *et al.* (2021)), or pursue flexibility-oriented self-employment (Bento *et al.* (2021), Gurley-Calvez *et al.* (2009)). Unmet needs for workplace

[26] In our model, having a child can be interpreted as a change in external demands on time and, therefore, differences in p. As discussed in Section 4.4, we can also interpret the children growing up as p_{women} converging to p_{men}.

flexibility push women into less profitable work arrangements, including home production (Patrick *et al.* (2016)). Moreover, in male-dominated working environments, women's termination rates are relatively higher (Torre (2017), Cha (2013)).

Studies show a wide range in the employment rates of new mothers, from 20.9% to 70%, depending on e.g. the age of the child, the occupation or the mother's education level (Felfe (2012), Harkness *et al.* (2019), Erosa *et al.* (2016), Bertrand *et al.* (2010)). Moreover, while many women choose their occupations based on anticipated childcare needs, almost all change or adjust jobs (through hours) after returning to the labor force post-maternity leave (Felfe (2012), Adda *et al.* (2017), Bertrand *et al.* (2010), Hotz *et al.* (2018)).

In our model, we find that after 16 years over a third of women have exited their current employment relation. Since we are agnostic about the outside option it can entail both leaving the labor force or finding a new job. Taken together, while sizeable, our result may be in the lower end of the empirical estimates.

Our model highlights how working conditions that do not internalize differences in external demands on time can push women out of an employment relationship. As highlighted in Figure 8, when men and women are given contracts initially designed for them, both termination rates are similar. However, when contracts do not internalize differences in flexibility needs (*male-tailored*), a larger share of women exit the contract. Moreover, we are agnostic about the meaning of the outside option and keep it the same for both men and women. This might be a conservative choice as the value of outside options, such as home production, could significantly vary by gender.

8 Concluding Remarks

This paper proposes a theoretical framework to explain how gender differences in external demands on time can drive gendered employment dynamics. At the core of our theory is modeling the unpredictability and unverifiability of flexibility needs and studying its consequences in dynamic employment relationships. To study gender differences, we allow men and women to differ only in their probability of having limited time availability, which we find to be more than twice as large for women using survey data. This allows for between gender similarities and within gender differences in working conditions over time depending on their exposure to flexibility shocks. When contracts do not internalize women's flexibility needs (*male-tailored*), contractual frictions give rise to meaningful gendered labor market outcomes, including the divergence and non-convergence of gender earnings differentials over the life-cycle and women's shorter job duration and weaker labor force attachment. However, when contracts internalize women's flexibility needs (*gender-tailored*), there are no systematic gender differences in labor market outcomes.

The framework we propose is a useful starting point for understanding how gender differences in external demands (e.g., for parental involvement), can have permanent consequences on employees' working conditions. Inflexible workplaces can drive women out of breadwinner roles and simultaneously men out of caregiver roles, solely as a result of how unmet flexibility needs are penalized. Optimal contracting models like ours are also well suited to study policy implications as employment relations can endogenously respond to them.

Finally, the contrast between *male-tailored* and *gender-tailored* contracts illustrates how non-discriminatory contracts can have adverse consequences. In our model, gender differences in labor market outcomes result from the employer's inability to tailor contracts to each gender's flexibility needs. Well-intended policies that aim to reduce gender differences by imposing gender-neutral rules or contracts may have unintended consequences (see, for example, Antecol *et al.* (2018) on the effects of gender-neutral tenure clock extensions).

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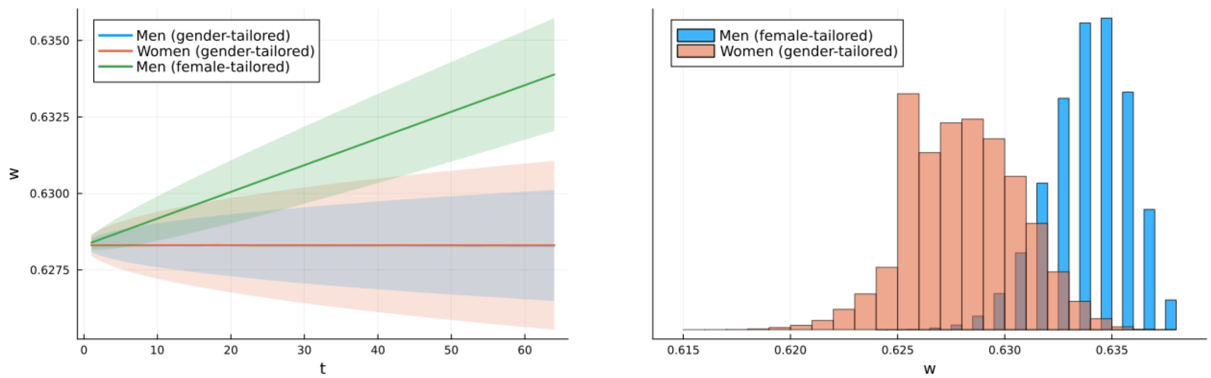
Appendix

A Extra Tables and Figures

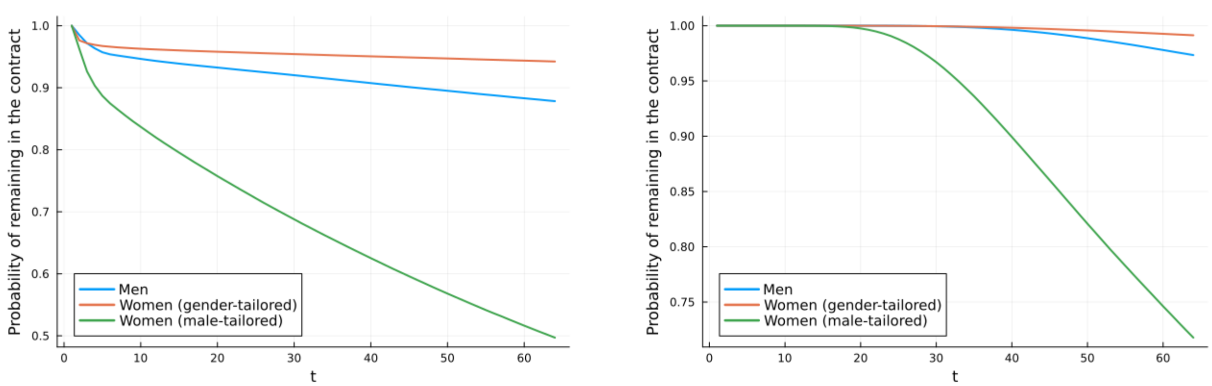
Figure 9: Men under women-tailored contract



(a) Average wage dynamics

(b) Wage distributions (at $t = 64$)

Figure 10: Termination dynamics with low and high v_0



(a) Low v_0

(b) High v_0

Figure 11: Comparative statics on α for women's wage dynamics under male-tailored contracts

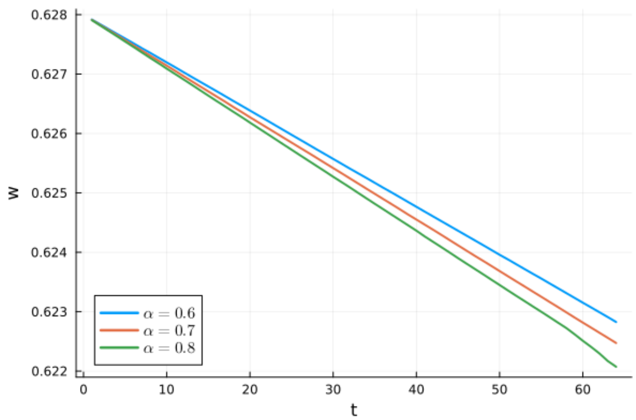


Table 3: Comparative statics with respect to the targeted reduction in hours

	p_{men}	p_{women}	Avg. growth rate of womens' wages	Avg. wage gap after 16 years
Baseline (25% reduction)	0.06	0.15	-0.014%	0.88%
20% reduction in hours	0.08	0.21	-0.018	-1.12%
30% reduction in hours	0.04	0.1	-0.01%	-0.64%

B Proofs

Throughout, we use the following notation for the continuation utilities:

$$\omega^H = U(w^H, h^H; f^H) + \beta v^H$$
$$\omega^L = U(w^L, h^L; f^L) + \beta v^L.$$

Proof of Proposition 1

In the simple case when the employer can observe the employee's time availability f and there is full commitment, the Lagrangian of the principal's problem writes as

$$\begin{aligned}\mathcal{L} &= (1-p)\left[g(h^H) - w^H + \beta\Pi(v^H)\right] + p\left[g(h^L) - w^L + \beta\Pi(v^L)\right] \\ &\quad + \lambda\left[(1-p)\left(u(w^H) - (1-f^H)\psi(h^H) + \beta vH\right) + p\left(u(w^L) - (1-f^L)\psi(h^L) + \beta vH\right) - v\right].\end{aligned}$$

Then, for any $j \in \{H, L\}$, the first order conditions are

$$w^j: \quad \frac{1}{u'(w^j)} = \lambda \tag{14}$$

$$h: \quad \frac{g'(h^j)}{(1-f)\psi'(h^j)} = \lambda \tag{15}$$

$$v^j: \quad \Pi(v^j) = -\lambda \tag{16}$$

and the envelope condition

$$\widehat{\Pi}'(v) = \lambda. \tag{17}$$

Combining (14) and (15) we get the first result of the proposition:

$$g'(h^j) = \frac{(1-f^j)\psi'(h^j)}{u'(w^j)}. \tag{18}$$

From (14) and (16), it also follows that contracts feature full insurance, i.e. $w^H = w^L$ and $v^H = v^L$. From (16) and (17), it also follows that $v = v^j$ so there is perfect intertemporal smoothing. Finally, because v is constant over time, the principal's value is also constant. Hence, it is never optimal to terminate the contract if it is optimal to enter it.

Proof of Proposition 2

We start establishing some preliminary results on the relevant constraints. We first show that (IC) constraint of the high type must bind in the optimal allocation. We assume throughout that $h^H \geq h^L$ and verify ex-post that this condition is indeed satisfied. In this case, incentive compatibility requires that either $u(w^H) \geq u(w^L)$, $v^H \geq v^L$, or both. By contradiction, assume that the high type's (IC) does not bind. Then the principal could lower the utility of the high type while satisfying (IC), (SUST) and (LC), and redistribute to the low type while satisfying the (PK) constraint. Because the cost of increasing the utility of the high type is smaller, the principal can obtain a direct resource gain from the perturbation.

Then, note that the (IC) constraints can be written as

$$\omega^H = \omega^L + (f^H - f^L)\psi(h^L) \tag{19}$$

$$\omega^L \geq \omega^H + (f^L - f^H)\psi(h^H). \tag{20}$$

Using $h^H \geq h^L$, it is easy that the high type's (IC) binding implies that the (IC) of the low type will not bind. For the sustainability constraint, because $h^L \geq 0$, we must have $\omega^H \geq \omega^L$. Hence, if the allocation is incentive-compatible, the (SUST) constraint of the high type ($\omega^H \leq \overline{v}$) is implied from the (SUST) constraint of the low type, and so it can be ignored. Finally, we ignore the (LC) of the high type and verify ex-post that it does not bind.

The Lagrangian of the principal's problem is

$$\begin{aligned}
\mathcal{L} = &(1-p)\left[g(h^H) - w^H + \beta\Pi(v^H)\right] + p\left[g(h^L) - w^L + \beta\Pi(v^L)\right] \\
&+ \lambda\left[(1-p)\left(u(w^H) - (1-f^H)\psi(h^H) + \beta v^H\right) + p\left(u(w^L) - (1-f^L)\psi(h^L) + \beta v^L\right) - v\right] \\
&+ \mu\left[u(w^H) - (1-f^H)\psi(h^H) + \beta\hat{v}^H - u(w^L) + (1-f^H)\psi(h^L) + \beta v^L\right] \\
&+ p\gamma\left[u(w^L) - (1-f^L)\psi(h^L) + \beta v^L - \overline{v}\right] + p\xi\beta\left[v^L - \overline{v}\right]
\end{aligned}$$

The first order conditions are:

$$h^H: \quad \frac{g'(h^H)}{(1-f^H)\psi'(h^H)} = \lambda + \frac{\mu}{1-p} \tag{21}$$

$$h^L: \quad \frac{g'(h^L)}{(1-f^L)\psi'(h^L)} = \lambda + \gamma - \frac{\mu}{p}\frac{(1-f^H)}{(1-f^L)} \tag{22}$$

$$w^H: \quad \frac{1}{u'(w^H)} = \lambda + \frac{\mu}{1-p} \tag{23}$$

$$w^L: \quad \frac{1}{u'(w^L)} = \lambda + \gamma - \frac{\mu}{p} \tag{24}$$

$$v^H: \quad -\Pi'(v^H) = \lambda + \frac{\mu}{1-p} \tag{25}$$

$$v^L: \quad -\Pi'(v^L) = \lambda + \gamma + \xi - \frac{\mu}{p} \tag{26}$$

And the envelope condition:
$$\widehat{\Pi}'(v) = -\lambda. \tag{27}$$

Moreover, combining the envelope condition and the first order condition for v in problem (6), it is easy to see that $\widehat{\Pi}'(v_c) = \Pi'(v)$.

Part (i): Combining the first order conditions (21) and (23):

$$g'(h^H) = \frac{(1-f^H)\psi'(h^H)}{u'(w^H)}.$$

Combining (22) and (24):

$$g'(h^L) = \frac{(1-f^L)\psi'(h^L)}{u'(w^L)}\left(1 + u'(w^L)\frac{\mu}{p}\frac{f^H - f^L}{(1-f^L)}\right) > \frac{(1-f^L)\psi'(h^L)}{u'(w^L)}.$$

Part (ii): First note that because $v > \overline{v}$ we have $h^L > 0$ (see Lemma 1), which implies $\omega^H > \omega^L$. Moreover, we also know that $h^H > h^L$. This implies that we must have either $u^H > u^L$, $v^H > v^L$ or both, we now show it must be both. Consider the case where the (LC) constraint does not bind, i.e. $\xi = 0$. Then because $\mu > 0$, the FOCs (23)-(26) directly imply $u^H > u^L$ and $v^H > v^L$. Now assume the (LC) binds, so $v^L = \overline{v}$. The constraint for the high type implies that $v^H \geq v^L$, which combined with the FOC requires $\lambda + \frac{\mu}{1-p} \geq \lambda + \gamma + \xi - \frac{\mu}{p}$. Because $\xi > 0$ from the FOC for v^H and v^L we deduce that $u^H > u^L$. The last step is to show that $v^H > v^L$. By contradiction, assume $v^H = v^L$ is optimal. Optimality implies that the allocations following f^H and f^L must be the same. Now consider a perturbation where we decrease u^H by $\varepsilon > 0$ and increase v^H by $\frac{1}{\beta}\varepsilon$ so that ω^H is kept constant. We increase v^H by increasing u^H_{t+1} by $\frac{1}{\beta(1-p)}\varepsilon$. The resource gain from this perturbation is

$$\frac{\Delta\widehat{\Pi}}{\varepsilon} \approx \frac{1}{u'(w^H_t)} - \frac{1}{u'(w^H_{t+1})}, \tag{28}$$

which is positive if $w^H_t > w^H_{t+1}$. The (SUST) and (IC) constraints imply that $\omega^H > \overline{v} = v^H$, which, because there is no distortion in hours for the high type, implies $w^H_t > w^H_{t+1}$.

Next, we show $v^H > v > v^L$ in the region without exp-post inefficiencies where constraints (SUST) and (*LC*) do not bind. Substituting the envelope condition (27) into the FOCs (25) and (26), we have
$$-\Pi'(v^H) = -\Pi'(v) + \frac{\mu}{1-p},$$

$$-\Pi'(v^L) = -\Pi'(v) - \frac{\mu}{p},$$

where we use $\Pi = \widehat{\Pi}$ because in this region there is no exit (see Proposition 3 and Corollary 1) and $\gamma = \xi = 0$ because the (SUST) and (LC) do not bind. Then, because Π is decreasing in this region (Proposition 3) and $\mu > 0$, it follows that $v^H > v$ and $v < v^L$.

Part (iii): Finally, when the (SUST) and (LC) do not bind, combining the FOC (23) and (24), and using sequential notation

$$\lambda_t = (1-p)\frac{1}{u'(w_t^H)} + p\frac{1}{u'(w_t^L)}.$$

Combining the FOC for the promised utility (equations (25) and (26)) and for wages, we observe that $-\Pi'(v^H) = \frac{1}{u'(w^H)}$ and $-\Pi'(v^L) = \frac{1}{u'(w^L)}$. Then, using the envelope condition implies $\lambda_t = \frac{1}{u'(w_{t-1})}$, which gives us the Inverse Euler equation in the proposition.

Proof of Lemma 1

First, notice that as $v \to \overline{v}$, the (PK) constraint converges to

$$(1-p)\omega^H + p\omega^L = \overline{v}. \tag{29}$$

Then the (SUST) constraint ($\omega^L \geq \overline{v}$) implies that we must have $\omega^H = \omega^L = \overline{v}$. Substituting into the (IC) constraint

$$\overline{v} = \overline{v} + (f^H - f^L)\psi(h^L), \tag{30}$$

which implies $h^L \to 0$.

Proof of Proposition 3

We first show that the (constrained) profit function Π^c is increasing in a neighborhood around $\overline{v}$. Starting from $v = \overline{v}$, consider perturbation where we increase promised utility by ε, i.e. $v = \overline{v} + \varepsilon$ and the hours of the low type by ε^h, i.e. $h^L = \varepsilon^h$ by Lemma (1). We now show that this perturbation can satisfy the (IC), (PK), (LC) and (SUST) constraints and deliver higher profits for the principal, i.e. $\Pi^c(\overline{v}) < \Pi^c(\overline{v} + \varepsilon)$. We keep v^L and v^H fixed at the optimal given $v = \overline{v}$, so the (LC) constraint holds. To satisfy the (SUST) constraint, we increase w^L to keep the low type's continuation utility, ω^L, constant:

$$\Delta w^L = \frac{(1-f^L)\psi'(\varepsilon^h)\varepsilon^h}{u'(w^L)}. \tag{31}$$

As h^L increases, the information rent that must be given to the high type to preserve incentive compatibility also increases. To this end, we increase the high's type wage w^H by:

$$\Delta w^H = \frac{(f^H-f^L)\psi'(\varepsilon^h)\varepsilon^h}{u'(w^H)}. \tag{32}$$

Finally, the (PK) constraint must also be satisfied. Since ω^L is kept fixed, we only need to make sure that Δw^H increases the high's type utility enough. Hence,

$$(1-p)(f^H-f^L)\psi'(\varepsilon^h)\varepsilon^h = \varepsilon, \tag{33}$$

which gives us the link between ε and ε^h. The next step is to show that for a small enough ε^h, this perturbation increases the principal's profits. The change in the principal's objective function is

$$\Delta\Pi^c \approx -(1-p)\Delta w^H - p\Delta w^L + pg'(\varepsilon^h)\varepsilon^h. \tag{34}$$

Substituting for the wage changes

$$\frac{\Delta\Pi^c}{\varepsilon^h} \approx -(1-p)\frac{(f^H-f^L)\psi'(\varepsilon^h)}{u'(w^H)} - p\frac{(1-f^L)\psi'(\varepsilon^h)}{u'(w^L)} + pg'(\varepsilon^h) \tag{35}$$

$$= -\left[(1-p)\frac{(f^H-f^L)}{u'(w^H)} + p\frac{(1-f^L)}{u'(w^L)}\right]\psi'(\varepsilon^h) + pg'(\varepsilon^h). \tag{36}$$

The first term inside the squared brackets is a bounded constant. Moreover, we have the Inada conditions $\lim_{\varepsilon^h\to 0}\psi'(\varepsilon^h) = 0$ and $\lim_{\varepsilon^h\to 0} g'(\varepsilon^h) = \infty$, so for a small enough ε^h, $\frac{\Delta\Pi}{\varepsilon^h} > 0$. Therefore $\Pi^c(\overline{v}+\varepsilon) \geq \Pi^c(\overline{v}) + \Delta\Pi^c\varepsilon^h > \Pi^c(\overline{v})$ which implies that Π^c must be increasing in a neighborhood around $\overline{v}$.

Finally, we show that Π^c must be decreasing at $v > \widetilde{v}$ for some $\widetilde{v} > \overline{v}$. Notice that for high enough $\widetilde{v}$, the (SUST) and (LC) constraints are not binding so $\gamma = \xi = 0$. Then combining the envelope condition (27), and adding up the FOCs (25) and (26) we obtain

$$\frac{\partial\Pi^c(v)}{\partial v} = -\left((1-p)\frac{1}{u'(w^H)} + p\frac{1}{u'(w^L)}\right) < 0,$$

which shows that Π^c is decreasing.

Proof of Proposition 4

We show the result first for wages. Then, we show it is straightforward to extend it for the promised utilities. Starting from the optimal allocation, we consider a variation where we decrease h^L, w^L and w^H while satisfying the (IC) and (PK) constraints and show that the resource gain is decreasing in p. For $\varepsilon > 0$ small, we lower the hours of the low type by $\Delta h^L = -\varepsilon$. We move along the indifference the low type's indifferent curve, i.e. we keep ω^L fixed, so u^L needs to be adjusted by

$$\Delta u^{L,\omega} = (1 - f^L)\psi'(h^L)(-\varepsilon) < 0 \tag{37}$$

This relaxes the RHS of the incentive constraint (19) by $(f^H - f^L)\psi'(h^L)(-\varepsilon) < 0$, which allows as to decrease u^H by

$$\Delta u^{H,IC} = (f^H - f^L)\psi'(h^L)(-\varepsilon) < 0. \tag{38}$$

Because ω^L is fixed, the ex-ante utility decreases by $(1-p)\Delta u^{H,IC} < 0$. To satisfy the (PK), we increase both types' wage utilities uniformly by $\Delta u^{PK} = (1-p)\Delta u^{H,IC}$, which preserves incentive compatibility. The resulting total changes in the wage utility of each type are

$$\Delta u^{H,TOT} = \Delta u^{H,IC} + \Delta u^{PK} = p(f^H - f^L)\psi'(h^L)(-\varepsilon) < 0 \tag{39}$$

$$\Delta u^{L,TOT} = \Delta u^{L,\omega} + \Delta u^{PK} = [(1 - f^L) - (1-p)(f^H - f^L)]\psi'(h^L)(-\varepsilon) < 0, \tag{40}$$

where the first inequality follows from $f^H > f^L$, and the second from $1 - f^L > 0$, $1 - p < 1$ and $f^H < 1$, which verifies that $\Delta w^L < 0$ and $\Delta w^H < 0$. The principal's gain from this perturbation is

$$\Delta\Pi \approx (1-p)\left(-\frac{1}{u'(w^H)}\Delta u^{H,TOT}\right) + p\left(g'(h^L)(-\varepsilon) - \frac{1}{u'(w^L)}\Delta u^{L,TOT}\right). \tag{41}$$

Substituting for the changes in the wage utility and rearranging

$$\frac{\Delta\Pi}{\varepsilon} \approx p(1-p)\left(\frac{1}{u'(w^H)} - \frac{1}{u'(w^L)}\right)(f^H - f^L)\psi'(h^L) \tag{42}$$

$$+ p\left(\frac{(1-f^L)\psi'(h^L)}{u'(w^L)} - g'(h^L)\right). \tag{43}$$

Differentiating with respect to p

$$\frac{\partial\frac{\Delta\Pi}{\varepsilon}}{\partial p} \approx (1-2p)\left(\frac{1}{u'(w^H)} - \frac{1}{u'(w^L)}\right)(f^H - f^L)\psi'(h^L) \tag{44}$$

$$+ \left(\frac{(1-f^L)\psi'(h^L)}{u'(w^L)} - g'(h^L)\right). \tag{45}$$

We need to show that $\frac{\partial\frac{\Delta\Pi}{\varepsilon}}{\partial p} < 0$. For $p < 1/2$, the result is not direct because in the optimal allocation $\frac{1}{u'(w^H)} - \frac{1}{u'(w^L)} > 0$. Using the FOCs (23) and (24),

$$\frac{1}{u'(w^H)} - \frac{1}{u'(w^L)} = \lambda + \frac{\mu}{1-p} - (\lambda - \frac{\mu}{p}) = \frac{\mu}{p(1-p)}. \tag{46}$$

Moreover, from the optimality condition (7),

$$g'(h^L) - \frac{(1-f^L)\psi'(h^L)}{u'(w^L)} > \frac{\mu}{p}(f^H - f^L)\psi'(h^L). \tag{47}$$

Hence, $\frac{\partial\frac{\Delta\Pi}{\varepsilon}}{\partial p} < 0$ if

$$\frac{\mu}{p}(f^H - f^L)\psi'(h^L) > (1-2p)\frac{\mu}{p(1-p)}(f^H - f^L)\psi'(h^L), \tag{48}$$

which is equivalent to $p > 0$ and completes the proof.

To show v^H and v^L are also decreasing, notice we can follow the same variation as above but with $\Delta v^{H,TOT} = \frac{1}{\beta}\Delta u^{H,TOT}$ and $\Delta v^{L,TOT} = \frac{1}{\beta}\Delta u^{L,TOT}$.[27] Then, the resource gain for the principal is

$$\Delta\Pi \approx (1-p)\Pi'(v^H)\Delta u^{H,TOT} + p\left(g'(h^L)(-\varepsilon) + \Pi'(v^L)\Delta u^{L,TOT}\right). \tag{49}$$

[27] In fact any variation where a fraction $\alpha \in [0,1]$ of the change in compensation is delivered through the flow utilities, u^H and u^L, and a fraction $1-\alpha$ trough promised utilities, v^H and v^L, would work.

Then using the FOC we can substitute $\Pi'(v^H) = -\frac{1}{u'(w^H)}$ and $\Pi'(v^L) = -\frac{1}{u'(w^L)}$ and follow the same steps as above.

Proof of Proposition 5

As discussed, with log-utility wages are a martingale under men's contract:

$$w_{t-1}^{\text{men}}(f^{t-1}) = p_{\text{men}} w_t^{\text{men}}(f^{t-1}, f^L) + (1 - p_{\text{men}}) w_t^{\text{men}}(f^{t-1}, f^H).$$

Then, rearranging terms we can write:

$$\frac{w_t^{\text{men}}(f^{t-1}, f^H)}{w_{t-1}^{\text{men}}(f^{t-1})} - 1 = -\left(\frac{p_{\text{men}}}{1 - p_{\text{men}}}\right)\left(\frac{w_t^{\text{men}}(f^{t-1}, f^L)}{w_{t-1}^{\text{men}}(f^{t-1})} - 1\right). \tag{50}$$

The expected wages for women under the *male-tailored* contract are

$$\mathbb{E}_{p_{\text{women}}}(w_t^{\text{men}}(f^t)) = p_{\text{women}} w_t^{\text{men}}(f^{t-1}, f^L) + (1 - p_{\text{women}}) w_t^{\text{men}}(f^{t-1}, f^H).$$

So, the expected growth rate is:

$$\begin{aligned}
\mathbb{E}_{p_{\text{women}}}\left(\frac{w_t^{\text{men}}(f^t)}{w_{t-1}^{\text{men}}(f^{t-1})}\right) - 1 &= p_{\text{women}}\left(\frac{w_t^{\text{men}}(f^{t-1}, f^L)}{w_{t-1}^{\text{men}}(f^{t-1})} - 1\right) + (1 - p_{\text{women}})\left(\frac{w_t^{\text{men}}(f^{t-1}, f^H)}{w_{t-1}^{\text{men}}(f^{t-1})} - 1\right) \\
&= \left(p_{\text{women}} - (1 - p_{\text{women}})\frac{p_{\text{men}}}{1 - p_{\text{men}}}\right)\left(\frac{w_t^{\text{men}}(f^{t-1}, f^L)}{w_{t-1}^{\text{men}}(f^{t-1})} - 1\right) \\
&= \left(\frac{p_{\text{women}} - p_{\text{men}}}{1 - p_{\text{men}}}\right)\left(\frac{w_t^{\text{men}}(f^{t-1}, f^L)}{w_{t-1}^{\text{men}}(f^{t-1})} - 1\right),
\end{aligned}$$

where the second equality uses equation (50).

Proof of Proposition 6

A recursive formulation of the problem is challenging as the continuation utilities for men and women under a common contract do not coincide. Hence, it is more convenient to work with the sequential problem. Without sustainability and limited commitment constraints, the employer's problem consists of maximizing

$$\Pi(v_0) = \sum_{t=0}^{\infty} \sum_{f^t} \beta^t \left[s P^{\text{men}}(f^t)\pi(w(f^t), h(f^t)) + (1 - s)P^{\text{women}}(f^t)\pi(w(f^t), h(f^t))\right] \tag{51}$$

subject to the incentive constraints for men and women:

$$\sum_{t=0}^{\infty}\sum_{f^t}\beta^t P^{\text{men}}(f^t)U(w(f^t),h(f^t);f^t) \geq \sum_{t=0}^{\infty}\sum_{f^t}\beta^t P^{\text{men}}(f^t)U(w(\hat{f}_t(f^t)),h(\hat{f}_t(f^t));f^t) \quad (52)$$

$$\sum_{t=0}^{\infty}\sum_{f^t}\beta^t P^{\text{women}}(f^t)U(w(f^t),h(f^t);f^t) \geq \sum_{t=0}^{\infty}\sum_{f^t}\beta^t P^{\text{women}}(f^t)U(w(\hat{f}_t(f^t)),h(\hat{f}_t(f^t));f^t) \quad (53)$$

for all type histories $f^\infty \in \{f^L, f^H\}^\infty$ and reporting strategies $\hat{f} : \{f^L, f^H\}^\infty \to \{\hat{f}^L, \hat{f}^H\}^\infty$, and the time-0 participation constraints:

$$\sum_{t=0}^{\infty}\sum_{f^t}\beta^t P^{\text{men}}(f^t)U(w(f^t),h(f^t);f^t) \geq v_0 \quad (54)$$

$$\sum_{t=0}^{\infty}\sum_{f^t}\beta^t P^{\text{men}}(f^t)U(w(f^t),h(f^t);f^t) \geq v_0. \quad (55)$$

Then, assuming that there exists a contract that satisfies constraints (52)-(55), we can derive the Inverse Euler equation in the proposition with the usual perturbation argument. Fix a history f^t, and consider a perturbation where we decrease the wage utility by $\delta u(f^t) = -\varepsilon$ for $\varepsilon > 0$ small, and we increase the wage utility in the following period of both types by $\delta u(f^t, f^L) = u(f^t, f^H) = \frac{\varepsilon}{\beta}$. Due to the uniform change in utilities, this perturbation preserves all the incentive and participation constraints. Then, using $P^{\text{avg}}(f^t) \equiv sP^{\text{men}}(f^t) + (1-s)P^{\text{women}}(f^t)$ the change in the employer's value is

$$\delta\Pi = \beta^t P^{\text{avg}}(f^t)\frac{1}{u'(f^t)}(-\varepsilon) + \beta^{t+1}\left(P^{\text{avg}}(f^t, f^L)\frac{1}{u'(f^t, f^L)}\frac{\varepsilon}{\beta} + P^{\text{avg}}(f^t, f^H)\frac{1}{u'(f^t, f^H)}\frac{\varepsilon}{\beta}\right).$$

In an optimal contract, the gains from this perturbation must be zero, setting $\delta\Pi = 0$ and collecting terms

$$\frac{1}{u'(f^t)} = \frac{P^{\text{avg}}(f^t, f^L)}{P^{\text{avg}}(f^t)}\frac{1}{u'(f^t, f^L)} + \frac{P^{\text{avg}}(f^t, f^H)}{P^{\text{avg}}(f^t)}\frac{1}{u'(f^t, f^H)},$$

where

$$
\begin{aligned}
\frac{P^{\text{avg}}(f^t, f^L)}{P^{\text{avg}}(f^t)} &= \frac{sP^{\text{men}}(f^t, f^L) + (1-s)P^{\text{women}}(f^t, f^H)}{sP^{\text{men}}(f^t) + (1-s)P^{\text{women}}(f^t)} \\
&= \frac{sP^{\text{men}}(f^t)p_{\text{men}} + (1-s)P^{\text{women}}(f^t)p_{\text{women}}}{sP^{\text{men}}(f^t) + (1-s)P^{\text{women}}(f^t)} \\
&= p_{\text{avg}}(f^t)
\end{aligned}
$$

and

$$
\frac{P^{\text{avg}}(f^t, f^L)}{P^{\text{avg}}(f^t)} = \frac{sP^{\text{men}}(f^t)(1-p_{\text{men}}) + (1-s)P^{\text{women}}(f^t)(1-p_{\text{women}})}{sP^{\text{men}}(f^t) + (1-s)P^{\text{women}}(f^t)} = 1 - p_{\text{avg}}(f^t).
$$

Finally, using $P^{\text{men}}(f^t), P^{\text{women}}(f^t) \in (0,1)$ for all $f^t \in \{f^L, f^H\}^t$, $s \in (0,1)$ and $p_{\text{men}} < p_{\text{women}}$ it follows that $p_{\text{men}} < p_{\text{avg}}(f^t) < p_{\text{women}}$ for all $f^t \in \{f^L, f^H\}^t$.

C Details Numerical Solution

There are two challenges to solving the optimal contract problem numerically. The first one, which is common to all dynamic contracting problems, is that the constraints in the dynamic programming problem are forward-looking, and as a result, the set of feasible promised utilities is not known ex-ante. This prevents using standard dynamic programming techniques. One solution is to follow Marcet and Marimon (2019), which consists of solving a recursive Lagrangian. The second challenge is that this approach is known to fail when the Pareto frontier is not strictly concave (Cole and Kubler (2012)), which is the case in the termination region.

Our approach consists of first solving the principal's value function Π with the recursive Lagrangian method in a region where the (SUST) constraint does not bind: $\{v^{MM}_{min}, ..., v^{MM}_{max}\}$ with $v^{MM}_{min} >> \overline{v}$. With this solution, we can then solve the problem with a direct promised utility approach (i.e. standard dynamic programming optimizing over promised utilities) in the region $\{\overline{v}, ..., v^{MM}_{min}\}$. Because the value function has already been computed with high promised utilities, we know that we lie in the range of feasible promised utilities.

C.1 Recursive Lagrangian

As discussed, for high values of promised utility where the (SUST) is far from binding, we solve the model following Marcet and Marimon (2019). Let $\mathbf{x} = \{h^H, h^L, w^H, w^L\}$ and denote by λ and μ the multipliers on the (PK) and (IC) constraints, respectively. We define the recursive Lagrangian as

$$\mathcal{L}(\lambda, \mu, \mathbf{x}) = (1-p)\left[g(h^H) - w^H + (\lambda + \frac{\mu}{1-p})\left(u(w^H) - (1-f^H)\psi(h^H)\right) + \beta W(\lambda + \frac{\mu}{1-p})\right] + p\left[g(h^L) - w^L + (\lambda - \frac{\mu}{p})u(w^L) - \left(\lambda(1-f^L) - (1-f^H)\frac{\mu}{p}\right)\psi(h^L) + \beta W(\lambda - \frac{\mu}{p})\right],$$

where W solves the saddle-point problem:

$$W(\lambda) = \min_{\mu \geq 0} \max_{\mathbf{x}} \mathcal{L}(\lambda, \mu, \mathbf{x}). \tag{56}$$

With standard value function iteration, we can solve W and compute the policy functions $\mathbf{x}$ on a grid $\{\lambda_{min}, ..., \lambda_{max}\}$. For every (λ, μ) we compute the policy functions using the FOCs (21)-(24). With the policy functions, we can then also compute Π and v by VFI on the grid for λ. Combining the two we can also compute Π on a grid $\{v^{MM}_{min}, ..., v^{MM}_{max}\}$. Finally, we verify that $v^{MM}_{min} = v(\lambda_{min})$ is high enough such that the (SUST) is far from binding. Otherwise, we increase λ_{min} and solve again.

C.2 Promised Utility Approach

We now have a solution for the principal's value function Π on a grid $\{v^{MM}_{min}, ..., v^{MM}_{max}\}$. The next step is to solve the problem in the region $(\overline{v}, ..., v^{MM}_{min})$ with a promised utility approach. We solve separately for both Π^c, i.e. when the principal is not allowed to terminate, and Π. The algorithm to solve for Π^c is the following:

Algorithm Π^c:

1. Guess Π^c on a grid $\{\overline{v}, ..., v^{MM}_{min}\}$.
2. For every $v \in \{\overline{v}, ..., v^{MM}_{min}\}$, optimize over (h^H, h^L, v^L).
   - The remaining policy variables are obtained from the constraints. For every h^H, use

the FOC (23) to solve:

$$w^H = \left(\frac{1-f^H}{z\alpha}(h^H)^{(\eta+1-\alpha)}\right)^{-\frac{1}{\sigma}}. \tag{57}$$

Then, at every point, combining the (IC) and (PK) constraints we can compute:[28]

$$w^L = u^{-1}\left(v - \beta v^L + \left[(1-f^L) + (1-p)(f^L - f^H)\right]\psi(h^L)\right) \tag{58}$$

$$v^H = \frac{1}{\beta}\left(u^L - u^H + (1-f^H)(\psi(h^H) - \psi(h^L))\right) + v^L. \tag{59}$$

- Check if the optimal policy x^* satisfies the (SUST) constraint. If it's not satisfied, add a large penalty in the objective.
- Compute the objective and find the point $x^* = (h^{H*}, h^{L*}, v^{L*})$ that maximizes the principal's value.

3. Update the value function, if it doesn't satisfy the tolerance go back to step 2.

Algorithm Π and $\widehat{\Pi}$: First, we check that the outside is such that $\overline{\Pi} > \Pi^c(\overline{v})$. Otherwise, there is no exit, and $\Pi^c = \Pi$. Then the algorithm proceeds as follows:

1. Start with a guess of the continuation value function $\widehat{\Pi}^{guess}$. We can use the solution of the constrained frontier Π^c from the previous part.[29]

2. Compute the guess for the value function Π^{guess}. For this, first find the grid point v^* such that the line from $(\overline{v}, \overline{\Pi})$ to $(v^*, \widehat{\Pi}^{guess}(v^*))$ is weakly above $\widehat{\Pi}^{guess}$. Compute the slope of this line as $b = \frac{\overline{\Pi} - \widehat{\Pi}^{guess}(v^*)}{v^* - \overline{v}}$. Then for $v \in \{\overline{v}, v^*\}$, compute

$$\Pi^{guess}(v) = \overline{\Pi} - b(v - \overline{v}) \tag{60}$$

and for $v \in \{v^*, v^{MM}_{min}\}$ set $\Pi^{guess}(v) = \widehat{\Pi}^{guess}(v)$.

3. Solve

$$\widehat{\Pi}^{pol}(v) = \max(1-p)\left[g(h^H) - w^H + \beta\Pi^{guess}(v^H)\right] + p\left[g(h^L) - w^L + \beta\Pi^{guess}(v^L)\right] \tag{61}$$

[28] If need to extrapolate, use the solution computed with the other method.

[29] As Π^c and Π are very similar in the region without termination. Π^c gives a good initial guess and the value function converges fast.

subject to (IC), (PK), (SUST) and (LC).[30] To solve this, repeat the step 2. of the algorithm for Π^c.

4. With $\widehat{\Pi}^{pol}$ follow the same procedure as step 2. to compute Π^{pol}. Check the distance (can do for both Π and $\widehat{\Pi}$), update guess and go back to 3. until convergence.

D Data

The American Time Use Survey (ATUS) is a nationally representative U.S. time diary survey with detailed information on how many minutes a certain time of the day respondents spent on various activities, including work, care and leisure. Our sample comprises information from 2003 to 2022. We focus on working population aged 20 to 65, excluding self-employed. We restrict information to time diaries of typical working days, Monday to Friday. In accordance with out framework, in the baseline calibration, we focus on parents who have a least one child below the age of 12. Further to not rely on time-diary entries on working time we keep observations with answers on usual working hours.[31] To capture full-time workers, we limit the usual working hours to be at least 35 hours per week and cap maximum 60 working hours per week. This leaves with a small sample of 1072 observations. Given this being a particularly selected sample of full time workers, the sample is comprised of 627 men and 445 women.

To construct our data set we rely on tools provided by IPUMS. In particular, IPUMS allows to easily construct customized time-use variables. We create variables for both care and work activities that indicate on an hourly basis how many minutes of that hour were dedicated to the corresponding task. In particular, we look at care activities between 9 am and 5 pm, which we assume to be typical working hours.[32] Table 4 shows summary statics for three key variables: age, number of children and the age of the youngest child. By construction the age of the youngest child is below 12 and respondents have on average 2 children.

Figure 12 shows the distribution of minutes of care work during 9 am and 5pm by gender. We see that women have a particularly high frequency of 5-20 minute interruptions, but are always more likely to do care work for any size of interruption.

[30] As before, if need to extrapolate, use the solution computed with the other method

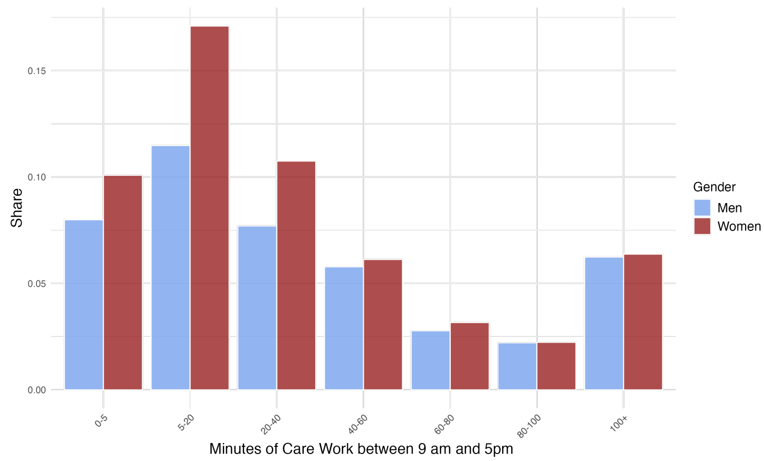
[31] We do so to not inflate the care-work ratio with lower working hours resulting from the care activities on that particular day.

[32] By doing so we most likely underestimate the results for our care-work ratio because many observations report more than 40 usual working hours per week.

Table 4: Summary Statistics for Key Variables

	Mean	Min	Max
Age	39	21	64
Number of Children	1.95	1	9
Age of Youngest Child	6.02	0	12

Figure 12: Share of men and women with different minutes of care activities.



The care-work ratio and probabilities p_{men} and p_{women} are constructed as explained in Section 5. Standard errors for the latter are computed using the bootstrap method with 1000 resamples.

E Incentive Compatibility in Male-tailored Contracts

In this section, we provide conditions for incentive compatibility in male-tailored contracts, i.e. when women take men's contract, and then verify numerically that they are satisfied. In a static model, this would be straightforward as the probability p would not show up in the incentive constraints. So, if a contract was incentive-compatible for men, it would also be incentive-compatible for women. However, in a dynamic setting, the continuation values will be different when women take men's contract because they depend on p. Hence, incentive compatibility does not follow directly.

We start with the IC constraint for type f^H. Let $v_w(v^H)$ and $v_w(v^L)$ denote the continuation utilities of a women taking a men's contract after f^H and f^L, respectively. Given a fixed v, the IC constraint of a woman under men's contract writes:

$$u(w^H) - (1 - f^H)\psi(h^H) + \beta v_w(v^H) \geq u(w^L) - (1 - f^H)\psi(h^L) + \beta v_w(v^L). \tag{62}$$

At the same time, the IC constraint for men (which binds) implies

$$v^H - v^L = \frac{1}{\beta}\left[u(w^L) - u(w^H) - (1 - f^H)(\psi(h^L) - \psi(h^H))\right]. \tag{63}$$

Combining the two incentive constraints, we get that incentive compatibility for women with high time availability is satisfied if:

$$v_w(v^H) - v_w(v^L) \geq v^H - v^L. \tag{64}$$

Second, we also need to verify that the constraint that prevents a woman with type f^L from reporting f^H, i.e.:

$$u(w^L) - (1 - f^L)\psi(h^L) + \beta v_w(v^L) \geq u(w^H) - (1 - f^L)\psi(h^H) + \beta v_w(v^H). \tag{65}$$

Using the incentive constraint for f^H of men (Equation 63) and collecting terms we get that the previous constraint is equivalent to:

$$v^H - v^L + \frac{1}{\beta}(f^H - f^L)(\psi(h^H) - \psi(h^L)) \geq v_w(v^H) - v_w(v^L) \tag{66}$$

Intuitively, both conditions will be satisfied ((64) and (66)) if the difference $f^H - f^L$ is sufficiently larger than the difference between p_{men} and p_{women} so that $v_w(v^H) - v_w(v^L)$ is close to $v^H - v^L$.

To check these conditions numerically, we approximate v_w on a grid for v with Montecarlo simulations over a sufficiently long time horizon. That is, for every v, we first get $v^H(v)$ and $v^L(v)$ from the men's policy functions. Then, we compute $v_w(v^H(v))$ and $v_w(v^L(v))$ with a Montecarlo simulation using the policies of a man with promised utilities $v^H(v)$ and $v^L(v)$, respectively, and the probability p_{women}. To make a more accurate comparison, we also approximate the men's values $v^H(v)$ and $v^L(v)$ using the same Montecarlo simulations. Figure 13 plots the differences in continuation utilities $v_w(v^H(v)) - v_w(v^L(v))$ and $v^H(v) - v^L(v)$ over a grid for the continuation utility v. The difference for women under men's contracts is always higher (blue line), which verifies that – in our calibration – the condition of equation (64) holds, and so incentive compatibility is preserved for type f^H. Figure 14 verifies that the IC constraint for f^L also holds. The green line is much higher (in our calibration $f^H - f^L = 13.5$), so the condition (66) should generally be very slack.

An equivalent condition is required for men under women's contract. That is, letting $v_m(v^H)$ and $v_m(v^L)$ denote the men's continuation utilities under a women's contract, incentive compatibility for the type with high time availability is preserved if: $v_m(v^H) - v_m(v^L) \geq v^H - v^L$. Figure 15 shows that this condition is not satisfied, so incentive compatibility is not preserved when men take women's contracts.

Figure 13: Verification IC for women under men's contract

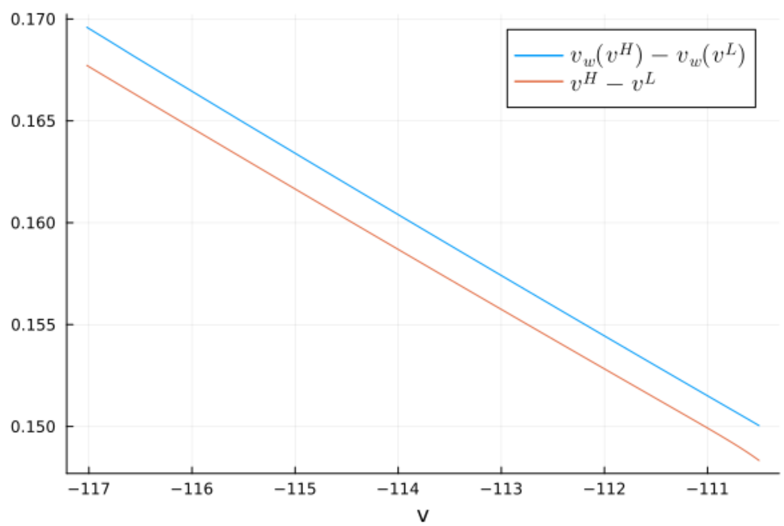


Figure 14: Verification IC for women under men's contract (lower and upper bounds)

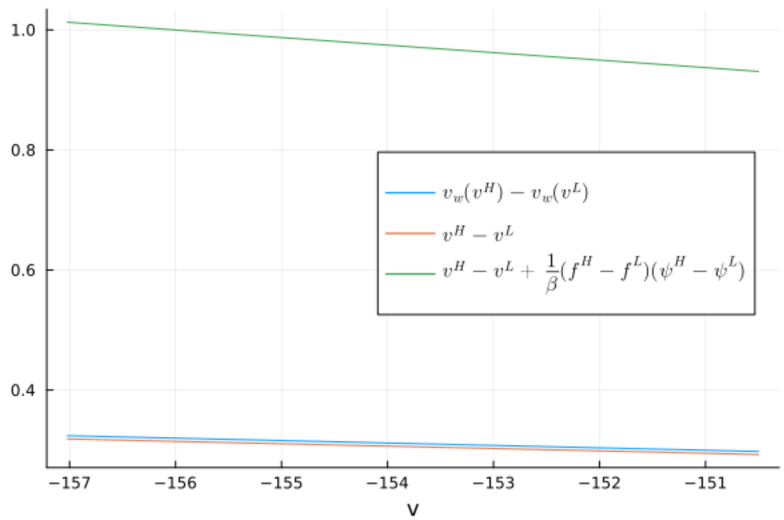
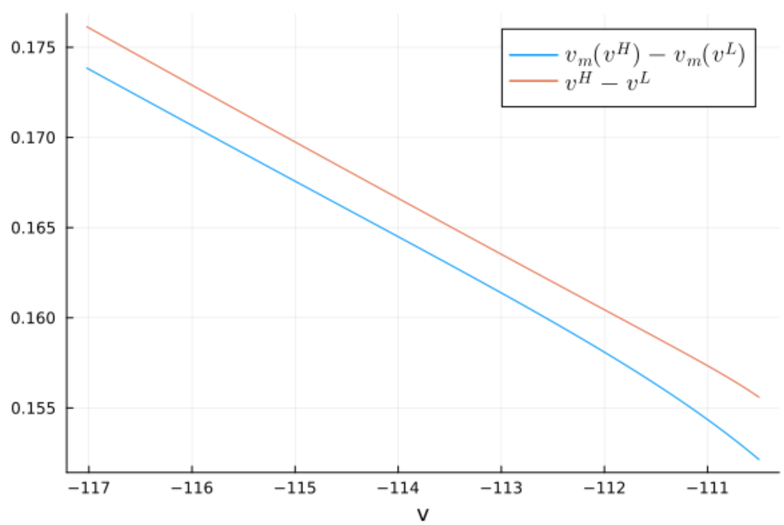


Figure 15: Verification IC for men under women's contract



F Extensions

F.1 Stochastic Outside Option

As discussed in the main text, we extend the model by allowing the outside option to be stochastic in order to generate termination. We assume $\overline{v}$ can take values on a grid

$\{\overline{v}_1, .., \overline{v}_i, ..., \overline{v}_I\}$ with corresponding probabilities $\{\overline{p}_i\}_{i=1}^I$. We also assume that this outside option is observable by the employer and realized at the end of the period but before the termination decision. Hence, the recursive problem of the employer following no termination and with realized outside option $\overline{v}_i$ writes

$$\widehat{\Pi}(v, \overline{v}_i) = \max_{\substack{w^H, h^H, v^H \\ w^L, h^L, v^L}} (1-p)\left[\pi(w^H, h^H) + \beta \sum_{j=1}^{I} \overline{p}_j \Pi(v^H, \overline{v}_j)\right] + p\left[\pi(w^L, h^L) + \beta \sum_{j=1}^{I} \overline{p}_j \Pi(v^L, \overline{v}_j)\right]$$

subject to the usual (PK) and (IC) constraints, the (SUST) constraints based on the current outside option

$$U(w^H, h^H; f^H) + \beta v^H \geq \overline{v}_i$$
$$U(w^L, h^L; f^L) + \beta v^L \geq \overline{v}_i,$$

and with the (LC) constraints based on the highest outside option

$$v^H, v^L \geq \overline{v}_I.$$

If the (LC) constraints were not based on the highest outside option, we could get that the promised utility is smaller than the outside option in some states. But then the allocation cannot satisfy simultaneously the (PK) and (SUST) constraints. Finally, the employer's problem before the termination decision and with realized outside option $\overline{v}_i$ is

$$\Pi(v, \overline{v}_i) = \max_{q \in [0,1], v_c} (1-q)\overline{\Pi} + q\widehat{\Pi}(v_c, \overline{v}_i)$$
$$\text{subject to } (1-q)\overline{v}_{i'} + q v_c = v.$$

F.2 Unpredictable Hours for the Employer

Our model focuses on the flexibility in working hours on the worker side. However, regular and predictable schedules have also been found to be equally as important for women Ciasullo and Uccioli (2022). To capture this, we now study an extension where the employer needs flexibility in hours in the sense that, with some probability, she would like the employee to work more hours than usual. Then, we assume that the cost of working these extra hours for the employee is stochastic –e.g. some days the employee can stay longer in the office because the other parent can pick up the child from school– and unverifiable. The private information of the cost of extra hours implies similar dynamics and that (qualitatively) all results go through.

Consider a version of the model where, with probability p^N, the employer asks the employee to work "regular" hours, but with probability $(1-p^N)$, the employer needs the employee to work "extra" hours. When the employer needs regular hours, the production function is given by $z^N g(h)$, but when the employer needs extra hours by $z^E g(h)$ with $z^E > z^N$.

To capture extra costs of working overtime, we assume that if hours are below a threshold h^*, the employee disutility is $(1-f^H)\varphi(h)$. However, if hours are higher than h^*, the disutility increases to $(1-f^L)\varphi(h)$ with probability p. We assume the following GHH utility function

$$U(w,h;f) = u(w-(1-f)\varphi(h)), \tag{67}$$

so that hours are always invariant to compensation. When $z = z^N$, the hours of work are always given by

$$z^N g'(h^N) = (1-f^H)v'(h^N) \tag{68}$$

if $h^N \leq h^*$ because there is no private information and no income effects. We assume that $h^N = h^*$, so the hours demanded will be higher than h^* whenever $z = z^E$.

For simplicity and to focus on the wage induced by the flexibility needs of the employer, we assume full commitment of the employee. Hence, we can drop the (SUST) and (LC) constraints and abstract from the termination decision. It is easy to show that the results on the optimal termination extend to this model. The employer's problem writes

$$\begin{aligned}\widehat{\Pi}(v) = \max p^N[z^N g(h^N) - w^N + \beta\Pi(v^N)]\\ + (1-p^N)[(1-p)(z^E g(h^{E,H}) - w^{E,H} + \beta\Pi(v^{E,H})) + p(z^E g(h^{E,L}) - w^{E,L}) + \beta\Pi(v^{E,L})]\end{aligned}$$

subject to

$$\begin{aligned}&p^N[U(w^N,h^N,f^H) + \beta v^N] +\\ &(1-p^N)[(1-p)(U(w^{E,H},h^{E,H},f^H) + \beta v^{E,H}) + p(U(w^{E,L},h^{E,L},f^L) + \beta v^{E,L})] = v\end{aligned}$$

and

$$U(w^{E,H},h^{E,H},f^H) + \beta v^{E,H} \geq U(w^{E,L},h^{E,L},f^H) + \beta v^{E,L}.$$

Notice that we only need to consider the incentive constraint for the case where $z = z^E$

because if $z = z^N$, the disutility is not private information. We let λ be the multiplier on the promise-keeping constraint and $(1 - p^N)\mu$ the multiplier on the incentive constraint. Combining the FOC for w^N, v^N, and the envelope condition

$$u_t'(N) = u_{t+1}'(N). \tag{69}$$

So, the compensation is constant over time when the employer needs regular hours. However, when $z = z^E$, we get similar results as before. Combining the FOC $w^{E,H}$ and $h^{E,H}$ we get

$$z^E g'(h^{E,H}) = (1 - f^H)v'(h^{E,H}) \tag{70}$$

and

$$z^E g'(h^{E,L}) = (1 - f^L)\psi'(h^{E,L})\left(1 + u'(w^{E,L})\frac{\mu}{p}\frac{f^H - f^L}{1 - f^L})\right) > (1 - f^L)\psi'(h^{E,L}). \tag{71}$$

Finally, the FOC for $v^{E,H}$ and $v^{E,L}$ give

$$-\hat{\Pi}'(v^{E,H}) = \lambda + \frac{\mu}{1 - p} \tag{72}$$

$$-\hat{\Pi}'(v^{E,L}) = \lambda - \frac{\mu}{p}. \tag{73}$$

Then, it is easy to see that we obtain similar dynamics with $v^{E,H} > v^{E,L}$. Therefore, when the employer needs extra hours, the employee is penalized in case of low time availability for the extra hours. It is also easy to verify that all the results and comparisons of *gender-tailored*, *male-tailored* and *team-tailored* contracts also go through in this model.

F.3 Stochastic and time-varying p and the non-convergence of wages

In this section, we extend the model to allow for a stochastic and time-varying process for the probability of a low time availability p. We assume $p_t \in \{p^1, ..., p^I\} \equiv \mathcal{P}$ follows a time-dependent Markov process with transition probabilities $Q_t(p_t|p_{t-1})$. Notice that this formulation nests a deterministic process where $p_{\text{women}} < p_{\text{men}}$ for T periods and $p_{\text{women}} = p_{\text{men}}$ afterwards (e.g. when the children grow up). However, we maintain the assumption that p is observable for the employer.

For this section, we assume full commitment of the agent so that we can drop the (SUST) and (LC) constraints and abstract from the termination decision. Before the realization of p_t, the principal's state variables are (v_t, p_{t-1}, t) and its objective is

$$\Pi_t(v_t, p_{t-1}) = \max \sum_{p_t \in \mathcal{P}} Q_t(p_t|p_{t-1})$$
$$\times\{(1-p_t)\left[\pi_{t+1}(w^H(p_t), h^H(p_t)) + \beta\Pi_t(v^H_{t+1}(p_t), p_t)\right] + p_t\left[\pi(w^L(p_t), h^L(p_t)) + \beta\Pi_{t+1}(v^L_{t+1}(p_t), p_t)\right]\}$$

It will be convenient to split the promise-keeping constraints and denote by $\widetilde{v}(p_t)$ the ex-post utility after the realization of p_t. That is, we have the following constraints

$$\sum_{p_t \in \mathcal{P}} Q_t(p_t|p_{t-1})\widetilde{v}_t(p_t) = v_t \tag{74}$$

and for all p_t

$$(1-p_t)\left(U(w^H(p_t), h^H(p_t); f^H) + \beta v^H_{t+1}(p_t)\right) + p_t\left(U(w^L(p_t), h^L(p_t); f^L) + \beta v^L_{t+1}(p_t)\right) = \widetilde{v}_t(p_t). \tag{75}$$

We place multiplier λ_t on constraint (74) and multipliers $Q_t(p_t|p_{t-1})\widetilde{\lambda}_t(p_t)$ on constraints (75). The incentive constraints are as before, but we have one constraint for each p_t, and we place multipliers $Q_t(p_t|p_{t-1})\mu(p_t)$. The FOCs are:

$\widetilde{v}_t(p_t)$:

$$\lambda_t = \widetilde{\lambda}_t(p_t) \tag{76}$$

$w^H(p_t)$:

$$\widetilde{\lambda}_t(p_t) + \frac{\mu(p_t)}{1-p_t} = \frac{1}{u'(w^H(p_t))} \tag{77}$$

$w^L(p_t)$:

$$\widetilde{\lambda}_t(p_t) - \frac{\mu(p_t)}{p_t} = \frac{1}{u'(w^L(p_t))} \tag{78}$$

Adding up the two FOCs and using (76), we get that for all p_t:

$$\lambda_t = (1-p_t)\frac{1}{u'(w^H(p_t))} + p_t\frac{1}{u'(w^L(p_t))}. \tag{79}$$

Hence, the employee is insured against changes in p_t because the expected inverse marginal utilities are equalized across all realizations. With log-utility ($u(c) = \log(c)$), this implies that expected wages are the same for all p_t,

$$\lambda_t = (1-p_t)w^H(p_t) + p_t w^L(p_t). \tag{80}$$

In fact, this result holds for all future periods. The FOCs for $v^H_{t+1}(p_t)$ and $v^L_{t+1}(p_t)$ give

$$\left(-\frac{\partial \Pi_{t+1}(v^H_{t+1}(p_t), p_t)}{\partial v^H_{t+1}(p_t)}\right) = \widetilde{\lambda}_t(p_t) + \frac{\mu(p_t)}{1-p_t} \tag{81}$$

$$\left(-\frac{\partial \Pi_{t+1}(v^L_{t+1}(p_t), p_t)}{\partial v^L_{t+1}(p_t)}\right) = \widetilde{\lambda}_t(p_t) - \frac{\mu(p_t)}{p_t}. \tag{82}$$

Adding up the two FOCs and using (76), for all p_t we have

$$\begin{aligned}\lambda_t &= (1-p_t)\left(-\frac{\partial \Pi_{t+1}(v^H_{t+1}(p_t), p_t)}{\partial v^H_{t+1}(p_t)}\right) + p_t\left(-\frac{\partial \Pi_{t+1}(v^L_{t+1}(p_t), p_t)}{\partial v^L_{t+1}(p_t)}\right)\\ &= (1-p_t)\lambda^H_{t+1} + p_t\lambda^L_{t+1}.\end{aligned}$$

where the second line substitutes the envelope conditions. Iterating forwards and using history notation, we have, for all $\tau \geq 1$

$$\lambda_t = \mathbb{E}\left[\lambda_{t+\tau}(f^{t+\tau})|f^t\right] \tag{83}$$

$$= \mathbb{E}\left[\frac{1}{u'(w_{t+\tau}(f^{t+\tau}))}|f^t\right] \tag{84}$$

and assuming log utility

$$\lambda_t = \mathbb{E}_{t-1}\left[w_{t+\tau}(f^{t+\tau})|f^t\right], \tag{85}$$

which holds for all p_t.