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Mechanism Design and Innovation Incentive for an Ad-Funded Platform*

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Abstract

We study a mechanism design problem of a monopoly platform that matches content of varying quality, ads with different ad revenues, and consumers with heterogeneous tastes for content quality. The optimal mechanism balances revenue from advertising and revenue from selling access to content: Increasing advertising revenue requires serving content to more consumers, which may reduce access revenue. Contrary to the standard monopolistic screening, the platform may serve content to consumers with negative virtual values while, to reduce information rents, limiting their access to higher-quality content. Then, an increase in ad profitability reduces its incentive to invest in content quality.

Keywords: ad-funded platform, mechanism design, matching, innovation
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1 Introduction

Digital platforms generate enormous benefits and shape our daily lives. However, concerns about big platforms' market power have been mounting, as illustrated by the adoption of the Digital Markets Act (2022) in the European Union. When evaluating consumer harm from a platform's market power, it is important to account for its business model (Scott-Morton et al., 2019). Existing reports on digital platforms pay particular attention to advertising-supported platforms and agree that consumer harm from their market power can manifest in lower quality, innovation, and privacy.[1]

Specifically for news media platforms, there are concerns generated by their increasing reliance on online intermediaries such as search engines, social media, and news aggregators (OECD, 2021; Ofcom, 2022; Latham, Burnik, Durán, and Khaki, 2022).[2] One concern is that financial links between these intermediaries and news outlets could distort the positioning and display given to different news outlets. For example, Google may have incentives to distort its search results to favor news outlets that generate more revenue for its ad services. According to Latham et al. (2022)[p.50], "such incentives can encourage news outlets towards an ad-funded business model which might have knock-on effects for media plurality by, for example, reducing incentives for investment in content generation."

In this paper, we study a mechanism design problem of a monopoly platform, which incorporates three sides: consumers, ads, and content with heterogeneous quality. The optimal mechanism highlights how the platform's trade-off between revenue from advertising and revenue from selling access to content shapes the matching pattern among consumers, content, and ads. We explore its implications for the platform's incentive to invest in content quality and study when increased ad profitability reduces the investment incentive.

Specifically, we consider a platform that hosts a unit mass of content items—such as news articles—and ads. Items vary in quality, and ads generate different levels of advertising revenue. Consumers have heterogeneous values for quality—or heterogeneous types—and also incur a type-independent attention cost of consuming items

[1] See Competition & Market Authority (2020), Crémer et al. (2019) and Stigler Committee on Digital Platforms (2019).

[2] According to Reuters Institute Digital News Report (2024), across all 47 markets from six continents, search and aggregators, taken together (33 percent), are a more important gateway to news than social media (29 percent) and direct access (22 percent).

and viewing ads. For each type of consumer, a mechanism determines a one-to-one matching between items and ads, allocates a set of item-ad pairs, and specifies a monetary transfer. The platform's goal is to maximize total revenue, which is the sum of monetary transfers and ad revenues.

Increasing advertising revenues requires the platform to allocate consumers as many items as possible, but reducing their information rents requires the platform to restrict low-value consumers' access to high-quality items (Mussa and Rosen, 1978). To highlight this trade-off, we first solve for the optimal mechanism when all ads generate the same revenue. When the advertising revenue is below a threshold, the optimal mechanism resembles the standard monopolistic screening: Consumers with negative Myersonian virtual values are excluded, and consumers with positive virtual values receive items whose qualities exceed some type-dependent threshold, which we call top-down allocations. In contrast, when the advertising revenue is high, the platform allocates all items to consumers with positive virtual values. Furthermore, consumers with negative virtual values receive items whose qualities are below some type-dependent threshold, which we call bottom-up allocations. Bottom-up allocations reduce the information rents of higher-value consumers while enabling the platform to earn advertising revenue from lower-value consumers.

Both kinds of allocations are relevant in practice. For example, a media website may limit the number of articles non-subscribers can read (e.g., *The New York Times* and *The Washington Post*). Such a policy would generate top-down allocations as consumers will read only content whose (subjective) qualities are high. Alternatively, a news outlet may prevent non-subscribers from accessing premium content (e.g., *Le Monde* and *Le Figaro*), leading to bottom-up allocations.

We then characterize the optimal mechanism for the main specification in which ads generate different revenues. In general, the platform prefers to match more profitable ads with items that are allocated to a larger number of consumers. Regarding the consumers with positive virtual values, higher-quality items are allocated to more consumers. Thus, the platform matches higher-quality items with more profitable ads, leading to a positive assortative matching. Conversely, regarding the consumers with negative virtual values, the platform matches lower-quality items with more profitable ads, leading to a negative assortative matching.

Building upon this, we study the implications of the optimal mechanism on the platform's incentive to invest in content quality. Specifically, we allow the platform to choose the distribution of content quality at a cost and study how this choice depends on the distribution of advertising revenues. We provide a condition (respectively, another condition) under which increased profitability of ads—captured by a stochastic increase in the ad revenue distribution—reduces (respectively, raises) the platform's incentive to invest in content quality. In particular, when ads are so profitable that the platform allocates items to consumers with negative virtual values, further increase in the profitability of ads crowds out the incentive to invest in quality. Intuitively, to increase ad revenues further, the platform should allocate higher-quality items to consumers with negative virtual values, which increases the information rents of higher-value consumers. As a result, the provision of higher-quality items becomes more costly, which curtails the platform's incentive to improve content quality.

The platform's quality choice is non-monotone in the profitability of ads. Consequently, the platform's greater reliance on ads can either increase or decrease consumer surplus. The platform hosting more profitable ads will allocate more items to consumers, contributing to consumer surplus. However, the platform may also have a lower incentive to invest in quality. For example, if ads are sufficiently profitable, the platform allocates all items to all consumers to maximize advertising revenue. In this case, the platform's revenue comes only from ads, which removes the platform's incentive to improve quality. Consequently, compared to a platform without ads, an ad-funded platform could offer lower-quality items, serve more consumers, and generate a lower consumer surplus.

This paper contributes to the literature on non-linear pricing in two-sided markets (e.g., Damiano and Li (2007), Johnson (2013), Choi, Jeon, and Kim (2015), Sato (2019), Lin (2020), Jeon, Kim, and Menicucci (2022), Gomes and Pavan (2016, 2019), Corrao, Flynn, and Sastry (2023)). The literature studies a mechanism design problem that could apply to a platform that matches consumers and advertisers. Our main contribution is to study the platform's design problem that incorporates three sides—consumers, advertisers, and content—and to explore its implications for the platform's incentive to innovate.[3] To our knowledge, no one has shown how the classical trade-off between rent extraction and efficiency may affect an ad-funded platform's incentive to innovate.

[3]While our baseline model does not incorporate advertisers as strategic players, our model is equivalent to the one in which the platform faces advertisers who are privately informed of the value of an impression. See the discussion in Section 3.2.

The literature on platform design has studied the incentives for platforms to design policies such as the quality of its service, the quality of sellers it hosts, and the degree of competition within a platform (e.g., Casner (2020), Liu, Yildirim, and Zhang (2022) Teh (2022), Johnen and Somogyi (2024), Madio and Quinn (2024)). In particular, Etro (2021) demonstrates that a purely ad-funded platform underinvests in quality, unlike a device-funded platform, and Choi and Jeon (2023) find that an ad-funded platform charging consumers has design incentives more aligned with consumer interests than a purely ad-funded platform. In contrast to these papers, we adopt a mechanism-design approach and study how the classical trade-off between efficiency and rent extraction interacts with the platform's incentive to improve content quality. More broadly, our work also relates to Anderson and Coate (2005) and many other works that study when advertising may lower social welfare.

2 Example

We begin with an example that highlights our main insights. An ad-funded platform sells a unit of content to a unit mass of consumers. If a consumer buys the content at price p, her utility is $\theta q - a - p$, where θ is the consumer's private taste for quality, $q \geq 0$ is the quality of the content, and $a \geq 0$ is an exogenous attention cost of consuming the content and the associated ad. Type θ is distributed according to the uniform distribution on $[0, 1]$, denoted by $U[0, 1]$. If a consumer does not purchase the content, she receives a utility of 0.

The platform earns an advertising revenue of $r > 0$ whenever the content is consumed. Thus, the platform's revenue comes from the sale of the content and the advertising revenue. Moreover, the platform chooses the quality q of the content at cost $\frac{cq^2}{2}$ with $c > 0$.

If the platform chooses quality q and price p, consumers purchase the content if and only if $\theta q - a - p \geq 0$, or $\theta \geq \frac{a+p}{q}$. Given $\theta \sim U[0, 1]$, the platform's problem reduces to:

$$\max_{p,q \geq 0} (p + r) \left[1 - \frac{a + p}{q}\right] - \frac{cq^2}{2}. \tag{1}$$

We solve problem (1) in two steps: First, we solve for the platform's optimal price p for any given quality level q. We then maximize the resulting profit with respect to q.

First, fix any quality level q. The standard logic of mechanism design (e.g., Myerson (1981)) implies that the platform's revenue from *the optimal price* equals the efficient virtual surplus obtained by replacing each type θ with its virtual type, $v(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$, and efficiently allocating the content according to the virtual type. The platform's profit then equals the maximized virtual surplus net of the investment cost:

$$\Pi(q,r) := \int_0^1 \max(v(\theta)q + r - a, 0)\ \mathrm{d}\theta - \frac{cq^2}{2}, \tag{2}$$

where $v(\theta) := 2\theta - 1$ as $\theta \sim U[0,1]$. To simplify notation, let $F^*(x) = \Pr(v(\theta) \le x)$ denote the distribution function of a virtual type. Then we can rewrite (2) as

$$\Pi(q,r) = \int_{\frac{a-r}{q}}^1 xq + r - a\ \mathrm{d}F^*(x) - \frac{cq^2}{2}. \tag{3}$$

The maximized virtual surplus (3) implies that, given the optimal quality level $q^* \in \arg\max_{q \ge 0} \Pi(q,r)$, the platform allocates the content to consumers whose virtual types $x = v(\theta) = 2\theta - 1$ exceed $\frac{a-r}{q^*}$, or equivalently, to types above $\theta^* = \frac{1}{2}\left(1 + \frac{a-r}{q^*}\right)$. The optimal price solves $\theta^* q^* - a - p^* = 0$, or $p^* = \frac{1}{2}(q^* - a - r)$.

Our example offers two insights: (i) in contrast to the standard monopolistic screening, the platform may allocate the content to consumers who have negative virtual values, which occurs when ad revenue r is larger than attention cost a; and (ii) for this reason, when $r > a$, an increase in ad revenue reduces the platform's incentive to invest in quality.

First, note that the platform sells content to consumers whose virtual values exceed the threshold $\frac{a-r}{q}$, which decreases with r. As r increases, the platform sells the content to a larger number of consumers. In particular, when $r > a$, the platform sells the content to consumers with negative virtual values; the platform sacrifices the revenue from selling content to earn more advertising revenue.

Second, the impact of a greater advertising revenue on the quality choice is non-monotone. To see this, note that the marginal impact of quality improvement on the platform's profit (3) is

$$\frac{\partial \Pi}{\partial q} = \int_{\frac{a-r}{q}}^1 x\,\mathrm{d}F^*(x) - cq, \tag{4}$$

which is non-monotone and single-peaked at $r = a$. Thus, the optimal quality choice is also non-monotone and single-peaked at $r = a$. Figure 1 illustrates the optimal quality choice as a function of r. When $r < a$, an increase in r induces the platform to serve more consumers among those who have positive virtual values. As a result,

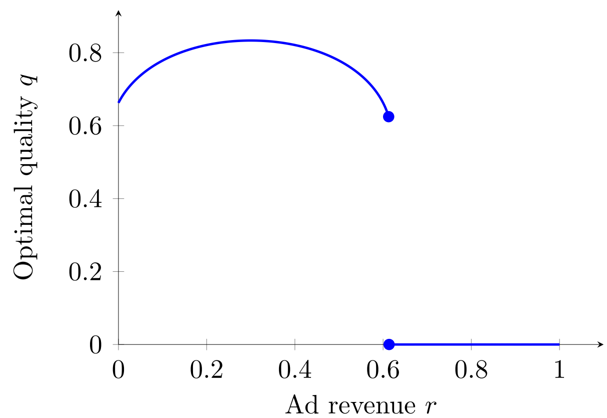

Figure 1: The optimal quality choice as a function of ad revenue $r \in [0, 1]$ when $a = c = 0.3$ and $\theta \sim U[0, 1]$. The optimal choice is not unique at r slightly above 0.6, around which it discontinuouly drops from $q \approx 0.6$ to 0.

the platform's marginal revenue from quality investment (i.e., the integral term of (4)) increases with r, resulting in a higher quality level. In contrast, when $r > a$, a further increase in r induces the platform to serve more consumers who have negative virtual values. Consequently, the platform's marginal revenue from quality investment decreases with r, resulting in a lower quality level.

A special case of this example is when the consumer's utility takes the standard form as in Mussa and Rosen (1978), i.e., $a = 0$ and the utility is given by $\theta q - p$. In this case, the platform always allocates the content to some consumers with negative virtual values, and the optimal quality level is unambiguously decreasing in $r \geq 0$.

In practice, platforms such as news outlets host heterogeneous content and ads. In the next section, we study the mechanism design problem of such a platform and generalize these insights.

3 Model

A monopoly platform faces a unit mass of consumers and hosts a unit mass of content items and ads. Each item is identified with its quality $q \in [0, \overline{q}]$ with $\overline{q} > 0$. The quality is distributed according to a continuous probability distribution $G \in \Delta[0, \overline{q}]$ that has a full support on $[0, \overline{q}]$.[4] For now, we assume that G is exogenous, but later

[4]Let ΔS denote the set of all probability distributions on a set S. The assumption that G is continuous and has a full support is unnecessary for our results; however, it greatly simplifies exposition.

allow the platform to choose it at a cost. Each ad is identified with its advertising revenue $r \in [0, \overline{r}]$ with $\overline{r} > 0$, which is distributed according to an arbitrary probability distribution, $H \in \Delta[0, \overline{r}]$.

The platform matches items and ads. Let $X := [0, \overline{q}] \times [0, \overline{r}]$ be the set of possible pairs of item quality and ad revenue.[5] A matching policy $m \in \Delta X$ is a joint distribution of item quality q and ad revenue r such that the first and second margins of m are G and H, respectively.[6] Let M denote the set of all matching policies. By extension, we call a pair of random variables $(\tilde{q}, \tilde{r})$ a matching policy if $(\tilde{q}, \tilde{r}) \sim m$ for some $m \in M$, i.e., $(\tilde{q}, \tilde{r})$ has distribution m.

Any pair $(q, r) \in X$ is called a content pair. Any set $B \subseteq X$ of content pairs is called a content bundle. Given a matching policy m, the gross quality of content bundle B is $Q = \int_B q \, \mathrm{d}m(q, r)$; its size is $N = m(B)$; and its advertising revenue is $R = \int_B r \, \mathrm{d}m(q, r)$. A tuple $(Q, N, R) \in [0, \overline{q}] \times [0, 1] \times [0, \overline{r}]$ is *feasible* if these equalities hold under some matching policy and some content bundle. A given content bundle B may lead to different values of (Q, N, R) depending on the matching policy m, because the frequency of content pairs that appear in B depends on m.

Consumers value items and have different tastes for quality. The type θ of each consumer is distributed according to $F \in \Delta\Theta$ where $\Theta := [\underline{\theta}, \overline{\theta}]$ with $0 \leq \underline{\theta} < \overline{\theta} < +\infty$. Type distribution F has a positive density f on Θ and a strictly increasing virtual value, $v(\theta) := \theta - \frac{1-F(\theta)}{f(\theta)}$. To simplify exposition, we assume that there exists a unique interior type $\theta^0 \in (\underline{\theta}, \overline{\theta})$ such that $v(\theta^0) = 0$. The type distribution is commonly known, but the type of each consumer is her private information.

The platform chooses a (direct) mechanism, $\{(m(\theta), B(\theta), T(\theta))\}_{\theta \in \Theta}$, which for each type θ of consumer specifies a matching policy $m(\theta)$, a content bundle $B(\theta)$, and a transfer $T(\theta)$ from the consumer to the platform. Let $(Q(\theta), N(\theta), R(\theta))$ denote the gross quality, size, and revenue of content bundle $B(\theta)$ under matching policy $m(\theta)$. The platform's profit is

$$\int_\Theta (T(\theta) + R(\theta)) \, \mathrm{d}F(\theta).$$

[5]The distribution $G \times H$ may not have a full support on X because we do not assume H to have a full support on $[0, \overline{r}]$.

[6]Formally, $m(A \times [0, \overline{r}]) = G(A)$ for every measurable $A \subseteq [0, \overline{q}]$, and $m([0, \overline{q}] \times A) = H(A)$ for every measurable $A \subseteq [0, \overline{r}]$.

The payoff of a consumer with type θ is

$$\theta Q(\theta) - aN(\theta) - T(\theta),$$

where $a > 0$ captures the attention cost of consuming an item and an ad. Cost a is exogenous and common across consumers.

3.1 Classes of Allocations and Matching Policies

The following allocations and matching policies are relevant to the optimal mechanism. Given a mechanism, we say that a consumer receives a *top-down allocation* if her content bundle is written as $\{(q, r) \in \operatorname{supp} m : q \geq q^*\}$ for some cutoff q^*, i.e., a consumer receives all items whose qualities exceed some threshold.[7] Similarly, a consumer receives a *bottom-up allocation* if her content bundle is $\{(q, r) \in \operatorname{supp} m : q \leq q^*\}$ for some q^*.

A matching policy m is said to be *positively assortative* if $(\tilde{q}, H^{-1}(G(\tilde{q}))) \sim m$, where H^{-1} is the generalized inverse of H.[8] A matching policy m is said to be *negatively assortative* if $(\tilde{q}, H^{-1}(1 - G(\tilde{q}))) \sim m$. For example, under a negatively assortative matching, an item with quality q is matched with an ad with revenue $H^{-1}(1 - G(q))$. If the quality of an item is in the p-percentile, it is matched with an ad whose revenue is in the $1 - p$ percentile. Figure 2 depicts the two matching policies.

[7] Here, $\operatorname{supp} m$ is the support of distribution m.

[8] The generalized inverse of distribution H is defined as $H^{-1}(t) = \inf\{x : H(x) > t\}$.

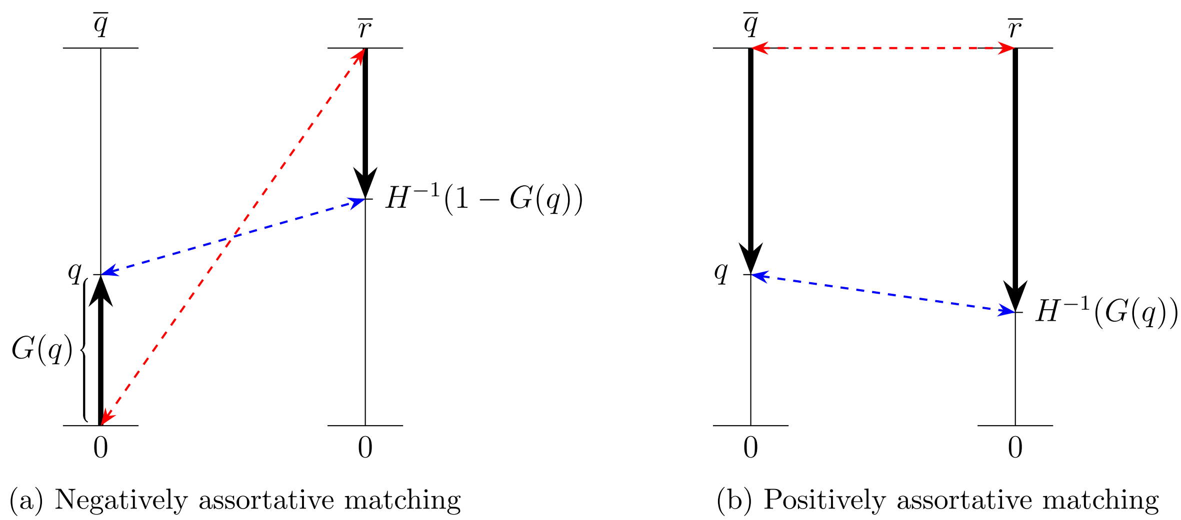

Figure 2: Assortative matching

3.2 Discussion

Two Interpretations of Advertising Revenue r. We can interpret advertising revenue r in two ways. In the main text, we interpret r as the revenue that accrues to the platform when the ad is shown to a consumer, or more broadly, the revenue that the platform can earn by monetizing attention and data generated by content consumption. Alternatively, we interpret r as the virtual type of an advertiser who is privately informed of the value of an impression. This interpretation is relevant when advertisers are explicitly modeled as strategic players, and the platform offers screening contracts to both consumers and advertisers. The Supplemental Appendix shows that such a model reduces to our baseline model once we view r as an advertiser's virtual type.

Negative Price and No Moral Hazard. We assume that (i) the platform can make monetary transfers to consumers by setting $T(\theta) < 0$, and (ii) consumers cannot ignore assigned content pairs. Both assumptions are restrictive: For example, if the platform offers some monetary rewards in exchange for consuming low-quality content, users might receive the reward and ignore the content. At the same time, our main insights are robust to an alternative specification. In the Supplemental Appendix, we show that our main results hold in a model where G and H are degenerate (as in Section 2) and the assumptions (i) and (ii) are absent.

Matching Between Items and Ads. The platform must show one ad for each item, which prohibits the platform from allocating items without ads. While this assumption may appear restrictive, it is without loss of generality. In the Supplemental Appendix, we consider an extension in which the platform can allocate items without ads, and ads impose extra disutilities on consumers in addition to a. This extension becomes equivalent to the baseline model such that a positive measure of ads generates zero revenue (i.e., $H(0) > 0$). In the corresponding baseline model, matching items with $r = 0$ plays the same role as allocating items without ads in the extension.

4 Optimal Mechanism

Given a mechanism $\{(m(\theta), B(\theta), T(\theta))\}_{\theta\in\Theta}$, the payoff-relevant components of $(m(\theta), B(\theta))$ are the associated gross quality, size, and revenue. As a result, we can write the platform's design problem as follows:

$$\max_{\{(Q(\theta),N(\theta),R(\theta),T(\theta))\}_{\theta\in\Theta}} \int_\Theta (T(\theta) + R(\theta))\,\mathrm{d}F(\theta)$$

$$\text{subject to} \quad \theta Q(\theta) - aN(\theta) - T(\theta) \geq \theta Q(\theta') - aN(\theta') - T(\theta'), \forall \theta, \theta' \in \Theta \tag{IC}$$

$$\theta Q(\theta) - aN(\theta) - T(\theta) \geq 0, \forall \theta \in \Theta \tag{IR}$$

$$(Q(\theta), N(\theta), R(\theta)) \text{ is feasible for each } \theta \in \Theta.$$

Inequalities (IC) and (IR) are the incentive-compatibility and participation constraints. Recall that a virtual value is denoted by $v(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$. By the standard argument (e.g., Myerson (1981)), we can rewrite the platform's problem as the maximization of virtual surplus subject to the constraint that the gross quality is increasing in types:

$$\max_{\{(Q(\theta),N(\theta),R(\theta))\}_{\theta\in\Theta}} \int_\Theta v(\theta)Q(\theta) - aN(\theta) + R(\theta)\,\mathrm{d}F(\theta) \tag{P}$$

$$\text{subject to} \quad Q(\theta) \text{ is increasing in } \theta \in \Theta; \text{ and} \tag{M}$$

$$(Q(\theta), N(\theta), R(\theta)) \text{ is feasible for each } \theta \in \Theta.$$

A special case of Problem (P) is when advertising revenue is constant, i.e., distribution H places probability 1 on some $r^* \geq 0$. In this case, the problem of matching items with ads is trivial. Thus, we omit the description of matching policies and describe each content bundle in terms of the set of item qualities. The following result characterizes the platform's optimal mechanism in this special case. Recall that type θ^0 uniquely solves $v(\theta^0) = 0$, and a consumer has a positive or negative virtual value if $\theta > \theta^0$ or $\theta < \theta^0$, respectively.

Claim 1 (Constant advertising revenue). *Suppose that the advertising revenue is constant and equals r^*. The optimal mechanism has the following properties:*

1. *If $r^* > a$, any type $\theta > \theta^0$ receives all items. Any type $\theta < \theta^0$ receives bottom-up allocation $[0, \frac{a-r^*}{v(\theta)}]$, where the cutoff $\frac{a-r^*}{v(\theta)} > 0$ is increasing in θ.*
2. *If $r^* < a$, any type $\theta < \theta^0$ receives no item. Any type $\theta > \theta^0$ receives top-down allocation $[\frac{a-r^*}{v(\theta)}, \overline{q}]$, where the cutoff $\frac{a-r^*}{v(\theta)} > 0$ is decreasing in θ.*

Proof. The contribution of item q to the virtual surplus in Problem (P) is $v(\theta)q + r^* - a$. The optimal mechanism allocates items to type θ whenever its contribution to the virtual surplus is nonnegative. Thus, type θ consumes the set of items, $\{q \in [0, \overline{q}] : v(\theta)q + r^* - a \geq 0\}$, which results in the item allocation described in the statement. □

Part 1 of Claim 1 highlights a difference between the standard monopolistic screening and our model. When ads are profitable (Part 1), the platform allocates items to consumers with negative virtual values. At the same time, the platform excludes these consumers from accessing high-quality content. Intuitively, allocating items generates advertising revenues even when consumers have negative virtual values, but it increases information rents of higher types and reduces the platform's revenue from selling items. Moreover, the reduction of the revenue from selling items is larger when the platform allocates higher-quality items. Balancing this trade-off, the platform allocates negative virtual types the items whose qualities are below some threshold. As the type increases (within $(0, \theta^0)$), the threshold increases as the platform finds it less costly to allocate higher-quality items in terms of information rents.

A special case of a constant r^* is when $r^* = 0$, i.e., the platform does not host any ads. In such a case, we say that the platform adopts the *subscription-funded business model.* The platform with the subscription-funded business model excludes types below θ^0 and allocates some top-down allocation to types above θ^0. The platform may implement such an allocation policy by imposing a cap $1-G\left(\frac{a}{v(\theta)}\right)$ on the number of items a consumer can access, e.g., a news website may impose a cap on the number of articles a user can read. Facing this cap, a user will optimally consume items whose quality exceeds $\frac{a}{v(\theta)}$. A consumer with a higher type will face a higher cap and thus access to more items.

We now turn to the main specification with general distributions of quality and advertising revenue. The optimal mechanism combines an allocation policy similar to Claim 1 and a type-dependent matching policy.[9]

Proposition 1. *The following mechanism maximizes the platform's profit:*

1. *For any type with a negative virtual value (i.e., $\theta < \theta^0$), the platform matches the items and ads negatively assortatively. Type θ receives a bottom-up allocation whose quality cutoff $q^-(\theta)$ is increasing on $[\underline{\theta}, \theta^0]$. If $q^-(\theta) \in (0, \overline{q})$, it solves*

$$v(\theta)q^-(\theta) + H^{-1}(1 - G(q^-(\theta))) - a = 0. \tag{5}$$

2. *For any type with a positive virtual value (i.e., $\theta > \theta^0$), the platform matches the items and ads positively assortatively. Type θ receives a top-down allocation, whose quality cutoff $q^+(\theta)$ is decreasing on $[\theta^0, \overline{\theta}]$. If $q^+(\theta) \in (0, \overline{q})$, it solves*

$$v(\theta)q^+(\theta) + H^{-1}(G(q^+(\theta))) - a = 0. \tag{6}$$

Proof. We first ignore the monotonicity constraint (M) and solve Problem (P). Under any matching policy, the platform offers content pair (q, r) to type θ whenever $v(\theta)q + r - a \geq 0$. Thus, the contribution of (q, r) to the virtual surplus generated by type θ is

$$\hat{V}_\theta(q, r) := (v(\theta)q + r - a)\, \mathbf{1}_{\{v(\theta)q + r - a \geq 0\}}, \tag{7}$$

where $\mathbf{1}_{\{.\}}$ is the indicator function. The virtual surplus generated by type θ under

[9]The optimal mechanism is not unique: The platform's profit remains the same so long as the set of allocated items and ads remains the same for each consumer type. The optimal mechanism described in Proposition 1 enables us to cleanly characterize the optimal matching policy because the matching depends only on the sign of a consumer's virtual type.

matching policy m is thus

$$\mathbb{E}_{(\tilde{q},\tilde{r})\sim m}\left[\hat{V}_\theta(\tilde{q},\tilde{r})\right] = \int_X (v(\theta)q + r - a)\,\mathbf{1}_{\{v(\theta)q+r-a\geq 0\}}\,\mathrm{d}m(q,r). \tag{8}$$

To show Part 1, take any $\theta < \theta^0$ and any $q_L, q_H \in [0, \overline{q}]$ such that $q_H > q_L$. We have

$$\hat{V}_\theta(q_H, r) - \hat{V}_\theta(q_L, r) = \begin{cases} 0 & \text{if} \quad r < a - v(\theta)q_L \\ -v(\theta)q_L - r + a & \text{if} \quad a - v(\theta)q_L \leq r \leq a - v(\theta)q_H \\ v(\theta)(q_H - q_L) & \text{if} \quad r > a - v(\theta)q_H, \end{cases}$$

which is overall decreasing in r. Thus $\hat{V}_\theta(q,r)$ is submodular in (q,r). As a result, the negative assortative matching of items and ads maximizes $\mathbb{E}_{(\tilde{q},\tilde{r})\sim m}\left[\hat{V}_\theta(\tilde{q},\tilde{r})\right]$, or equivalently, item q is matched with ad $r = H^{-1}(1 - G(q))$ (e.g., Galichon, 2018, Theorem 4.3). Because the platform allocates type θ the items that generate positive virtual surplus, the highest quality $q^-(\theta)$ that type θ will receive solves equation (5) whenever $q^-(\theta) \in (0, \overline{q})$. Indeed, given that $v(\theta) < 0$ and $H^{-1}(1 - G(\cdot))$ is decreasing, virtual surplus $v(\theta)q + H^{-1}(1 - G(q)) - a$ is positive if and only if $q < q^-(\theta)$. By the symmetric argument, we conclude that the allocation that maximizes the virtual surplus from type $\theta > \theta^0$ is as described in Part 2.

We now verify the monotonicity of the allocation. On each of intervals $[\underline{\theta}, \theta^0]$ and $[\theta^0, \overline{\theta}]$, a higher type receives a larger set of items in the set inclusion. As a result, gross quality $Q(\theta)$ is increasing on each interval. Thus, it suffices to show that $Q(\theta)$ does not decrease when it crosses θ^0. Because $v(\theta^0) = 0$, we have $G(q^+(\theta^0)) = H(a) = 1 - G(q^-(\theta^0))$, or $1 - G(q^+(\theta^0)) = G(q^-(\theta^0))$. As the type increases and crosses θ^0, the set of items allocated switches from $[0, q^-(\theta^0)]$ to $[q^+(\theta^0), \overline{q}]$. These two sets of qualities have the same mass because $1 - G(q^+(\theta^0)) = G(q^-(\theta^0))$. Moreover, the average quality of $[q^+(\theta^0), \overline{q}]$ is higher than $\int_0^{\overline{q}} q\,\mathrm{d}G(q)$, which is greater than the average quality of $[0, q^-(\theta^0)]$. Noting that the gross quality of a set of items equals the product of its average quality and mass, we conclude that the gross quality of $[q^+(\theta^0), \overline{q}]$ is greater than the gross quality of $[0, q^-(\theta^0)]$. Therefore, the monotonicity condition holds. □

Figure 3 depicts the optimal allocation of items when q and r are uniformly distributed on $[0, 1]$ and the attention cost is $a = 0.5$. The striped red area depicts the set of qualities allocated to consumers with negative virtual values. As a consumer's type increases, the platform includes higher-quality items in the bundle, resulting in a greater size and average quality. However, no types below θ^0 will have access to items with quality above 0.5. The blue area depicts the set of qualities allocated to consumers with positive virtual values. In this case, the platform allocates items whose qualities exceed the type-dependent cutoff $q^+(\theta)$. As a consumer's type increases, the platform includes lower-quality items in the bundle, resulting in a greater size and a lower average quality.

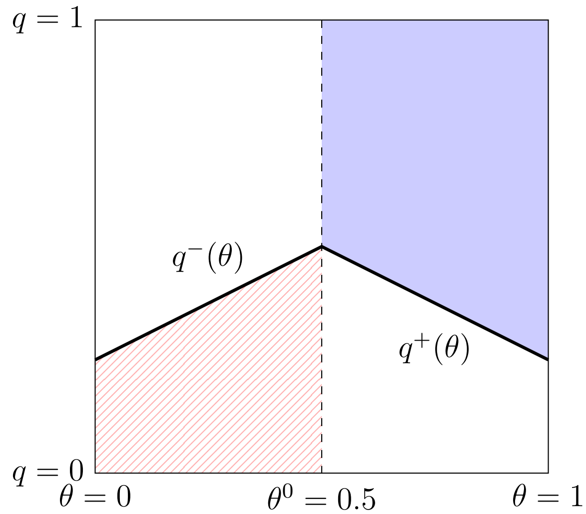

Figure 3: The optimal allocation of items when $G = H = U[0,1]$ and $a = 0.5$. The striped red area depicts the allocation to types with negative virtual values; and the blue area depicts the allocation to types with positive virtual values.

Proposition 1 also states that the optimal matching between items and ads depends on the sign of a consumer's virtual value. In general, the platform prefers to match more profitable ads with items that have higher demand. If consumers have positive virtual values, the items shown to more consumers are higher-quality items, resulting in positive assortative matching. In contrast, for negative virtual values, the platform prioritizes allocating lower-quality items to reduce information rents. As a result, more profitable ads are matched with lower-quality items and consumed by more consumers.

In the next section, we will show that the increased profitability of ads may strengthen or weaken the platform's incentive to improve content quality. Here, we note that for a fixed distribution of item quality, the increase in the profitability of ads benefits everyone.

Corollary 1. *Fix any distribution G of item quality. Suppose that the distribution H of advertising revenue increases in the first-order stochastic dominance. Then, the payoff of each consumer type and the platform's profit increase under the optimal mechanism.*

Proof. Suppose that H stochastically increases to H'. The platform's profit increases even without changing the mechanism because the ad revenue associated with each item increases both under positive and negative assortative matching. Thus, the profit increases further when the platform adopts a mechanism that is optimal under H'. For consumers, Proposition 1 implies that the stochastic increase in H raises $q^{-}(\theta)$ and decreases $q^{+}(\theta)$ for each relevant θ. As a result, each type receives a larger set of items and thus a higher gross quality. By the standard argument of mechanism design, the expected utility of each type θ equals $\int_{\underline{\theta}}^{\theta} Q(x)\,\mathrm{d}x$. Thus, a higher gross quality implies a larger payoff for each consumer type. □

5 Platform's Innovation Incentive

The platform's optimal mechanism has implications on its incentive to invest in content quality. To formally investigate this, we extend the model by incorporating the platform's choice of quality distribution. Let $\mathcal{G}$ denote the set of quality distributions the platform can choose from. The elements of $\mathcal{G}$ are totally ordered according to the first-order stochastic dominance, denoted by $\geq_{FOSD}$. Let $C(G) \geq 0$ be the cost of choosing distribution $G \in \mathcal{G}$.[10]

Let $P(G, H)$ denote the profit from the optimal mechanism given quality distribution G and advertising revenue distribution H. The platform's problem of choosing a quality distribution is as follows:

$$\max_{G \in \mathcal{G}} P(G, H) - C(G). \tag{Q}$$

For expositional simplicity, we assume that the solution to this problem is unique.

We provide a condition under which the platform that faces higher advertising revenues will choose a higher or lower content quality. In practice, the advertising revenue distribution may exhibit a stochastic increase when the platform gains access to better-targeting technology or better information about advertisers' willingness to

[10]Although it is natural to assume that $C(\cdot)$ is increasing in $\geq_{FOSD}$, we do not impose any restriction on $C(\cdot)$ unless otherwise noted.

pay for impressions.[11]

To conduct comparative statics, we first study when the platform's expected profit exhibits increasing or decreasing differences in (G, H). Let $M(G, H)$ denote the set of all matching policies given (G, H).

Lemma 1. *Given a function $\hat{V} : X \to \mathbb{R}$, define*

$$V(G, H) := \max_{m \in M(G,H)} \int_X \hat{V}(q, r) \, \mathrm{d}m(q, r).$$

Take any $\overline{H}$, $\underline{H}$, $\overline{G}$, and $\underline{G}$ such that $\overline{H} \geq_{FOSD} \underline{H}$ and $\overline{G} \geq_{FOSD} \underline{G}$.

1. *If $\hat{V}$ is submodular, then V has decreasing differences in (G, H), i.e., $V(\overline{G}, \overline{H}) - V(\overline{G}, \underline{H}) \leq V(\underline{G}, \overline{H}) - V(\underline{G}, \underline{H})$.*

2. *If $\hat{V}$ is supermodular, then V has increasing differences in (G, H), i.e., $V(\overline{G}, \overline{H}) - V(\overline{G}, \underline{H}) \geq V(\underline{G}, \overline{H}) - V(\underline{G}, \underline{H})$.*

Proof. The submodularity of $\hat{V}$ implies that[12]

$$V(G, H) = \int_0^1 \hat{V}(G^{-1}(u), H^{-1}(1-u)) \, \mathrm{d}u.$$

Because $\overline{G} \geq_{FOSD} \underline{G}$, we have $\overline{G}^{-1}(u) \geq \underline{G}^{-1}(u)$ for any $u \in [0, 1]$. Similarly, $\overline{H} \geq_{FOSD} \underline{H}$ implies $\overline{H}^{-1}(1-u) \geq \underline{H}^{-1}(1-u)$. The submodularity of $\hat{V}$ implies

$$\begin{aligned}
&\hat{V}(\overline{G}^{-1}(u), \overline{H}^{-1}(1-u)) - \hat{V}(\overline{G}^{-1}(u), \underline{H}^{-1}(1-u)) \\
\leq &\hat{V}(\underline{G}^{-1}(u), \overline{H}^{-1}(1-u)) - \hat{V}(\underline{G}^{-1}(u), \underline{H}^{-1}(1-u)).
\end{aligned}$$

Integrating both sides with respect to $u \in [0, 1]$, we conclude that V has decreasing differences. Part 2 follows from the symmetric argument. □

To describe the next lemma, define two sets of advertising revenue distributions as

[11] In the Supplemental Appendix, we formally show that the platform's access to information that eliminates advertisers' private information causes a stochastic increase in H.

[12] See Theorem 4.3 of Galichon (2018).

follows:

$$\mathcal{H}^- := \{H \in \Delta[0, \overline{r}] : H(a) = 1\}$$
$$\mathcal{H}^+ := \{H \in \Delta[0, \overline{r}] : H(a) = 0\}.$$

If $H \in \mathcal{H}^-$, any ad generates a revenue less than attention cost a. In contrast, if $H \in \mathcal{H}^+$, any ad generates a revenue greater than a.

Lemma 2. *Take any $\overline{H}$, $\underline{H}$ $\overline{G}$, and $\underline{G}$ such that $\overline{H} \geq_{FOSD} \underline{H}$ and $\overline{G} \geq_{FOSD} \underline{G}$.*

1. *Within $\mathcal{H}^+$, greater advertising revenue decreases the platform's incentive to improve content quality, i.e., if $\underline{H} \in \mathcal{H}^+$, $P(\overline{G}, \overline{H}) - P(\underline{G}, \overline{H}) \leq P(\overline{G}, \underline{H}) - P(\underline{G}, \underline{H})$.*

2. *Within $\mathcal{H}^-$, greater advertising revenue increases the platform's incentive to improve content quality, i.e., if $\overline{H} \in \mathcal{H}^-$, $P(\overline{G}, \overline{H}) - P(\underline{G}, \overline{H}) \geq P(\overline{G}, \underline{H}) - P(\underline{G}, \underline{H})$.*

Proof. We prove Part 1. The virtual surplus that type θ generates is given by (7). If $r > a$ with probability 1, then for any $\theta \geq \theta^0$, $\hat{V}_\theta(q, r) = v(\theta)q + r - a$, which is trivially submodular in (q, r). For any $\theta < \theta^0$, $\hat{V}_\theta(q, r)$ is submodular as we showed in the proof of Proposition 1. Thus for any $\theta \in \Theta$, Lemma 1 implies that

$$V_\theta(G, H) = \max_{m \in M(G,H)} \int_X \hat{V}_\theta(q, r) \, \mathrm{d}m(q, r)$$

is submodular in (G, H) in the sense defined in the lemma. Therefore the platform's optimal profit, $P(G, H) = \int_\Theta V_\theta(G, H) \, \mathrm{d}F(\theta)$, is submodular in (G, H). The symmetric argument applies to Part 2. □

This lemma, combined with the standard argument of monotone comparative statics, implies the following:

Proposition 2. *The solution to problem (Q) satisfies the following.*

1. *Within $\mathcal{H}^+$, a stochastically greater advertising revenue distribution induces the platform to choose a stochastically smaller quality distribution.*

2. *Within $\mathcal{H}^-$, a stochastically greater advertising revenue distribution induces the platform to choose a stochastically greater quality distribution.*

The intuition for Part 1 is as follows. When all ads generate revenues above attention cost a, any type with a positive virtual value will consume all items. Therefore, a further increase in advertising revenue only changes the item allocation by having the platform allocate more items to consumers with negative virtual values (i.e., $\theta < \theta^0$). However, this change in allocation makes it more costly for the platform to improve quality, because items allocated to types below θ^0 increase information rents and reduce profits, and this effect is severe when the platform hosts many high-quality items. As a result, greater advertising revenues endogenously increase the cost of offering high-quality content, and reduce the platform's incentive to improve content quality.

In contrast, when the condition of Part 2 holds, consumers with negative virtual values are always excluded. Then, higher advertising revenues incentivize the platform to allocate more items to those with positive virtual values. As a result, the platform has a greater incentive to improve quality.

Consumer Surplus. If ad revenues are initially low (Part 2 of Proposition 2), then a stochastic increase in $H \in \mathcal{H}^-$ will increase consumer welfare, because the platform will introduce higher-quality items and allocate them to more consumers. In contrast, the welfare impact of greater advertising revenue is ambiguous in the case described in Part 1: If H stochastically increases within $\mathcal{H}^+$, the platform will offer lower-quality items but allocate them to more consumers. Whether this change increases the gross quality allocated to each type, which determines consumer welfare, depends on which effect dominates. However, if the advertising revenues are sufficiently high, we can conclude that the ad-funded model is harmful to consumers.

Claim 2. *Assume that $\underline{\theta} = 0$ and H satisfies $H\left(a + \frac{\overline{q}}{f(0)}\right) = 0$. The platform's optimal profit $P(G, H)$ becomes independent of the quality distribution. Thus, if the investment cost is minimized by a quality distribution $G_0 \in \mathcal{G}$ that is degenerate at $q = 0$, the platform chooses G_0 in Problem (Q), resulting in zero consumer surplus.*

Proof. The platform allocates items to type θ if and only if $v(\theta)q + r - a \geq 0$. Because $r > a$ with probability 1, the platform allocates any item to types with nonnegative virtual values. Moreover, $\underline{\theta} = 0$ and $r > a + \frac{\overline{q}}{f(0)}$ implies that $v(0)\overline{q} + r - a \geq 0$, which implies $v(\theta)q + r - a \geq 0$ for any $\theta < \theta^0$ and $q \in [0, \overline{q}]$. To sum up, under the stated condition, the platform allocates all items to all consumers, regardless of their types.

The platform's optimal profit is then written as

$$\int_0^{\overline{r}} \int_0^{\overline{q}} \int_0^{\overline{\theta}} (v(\theta)q + r - a) \,\mathrm{d}F(\theta) \,\mathrm{d}G(q) \,\mathrm{d}H(r) = \int_0^{\overline{r}} (r - a) \,\mathrm{d}H(r),$$

where the equality follows from

$$\int_0^{\overline{\theta}} v(\theta) \,\mathrm{d}F(\theta) = \int_0^{\overline{\theta}} \theta \,\mathrm{d}F(\theta) - \int_0^{\overline{\theta}} f(\theta) \cdot \frac{1 - F(\theta)}{f(\theta)} \,\mathrm{d}\theta = 0.$$

Because the platform's profit $P(G, H)$ is independent of the quality distribution, in Problem (Q), the platform will choose $G \in \arg\min_{G' \in \mathcal{G}} C(G')$. The solution to this problem is G_0 by assumption. □

The result captures the following intuition: When ad revenues are very high, the platform allocates content to many consumers, including those with a low willingness to pay. However, because of the consumers' incentive constraints, such a policy prevents the platform from charging higher prices to consumers with high willingness to pay. As a result, the platform's profit becomes less sensitive to content quality, which lowers the platform's incentive to improve content quality. Claim 2 captures this intuition in an extreme way by focusing on the case in which the ads are so profitable that the platform's allocation policy becomes independent of consumers' types.

6 Conclusion

We study the mechanism design problem of a monopoly platform that matches consumers, content, and ads. The optimal mechanism is determined by a trade-off between two revenue sources: Increasing advertising revenue requires allocating content to a larger number of consumers, which may reduce the revenue from selling access to content. In contrast to the standard monopolistic screening, the platform may not exclude consumers with negative virtual values. However, to reduce information rents, the platform allows such consumers to access only lower-quality content. An increase in the profitability of advertising—which could come from improved targeting technology—may further increase the demand for lower-quality content, decrease the platform's incentive to invest in content quality, and possibly consumer surplus. Our result implies that the increasing importance of ad revenue for news media may negatively affect their incentive to invest in journalism.

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Supplemental Appendix

Appendix A: Privately Informed Advertisers

In this appendix, we first describe the extension in which the platform faces privately informed advertisers in addition to consumers. We then prove that such a model is equivalent to the baseline model in which r is the virtual value of an advertiser.

We modify the baseline model as follows. In addition to the platform and consumers, the model has a unit mass of advertisers. Each advertiser is identified with its type, $\alpha \in [\underline{\alpha}, \overline{\alpha}]$ with $\overline{\alpha} > \underline{\alpha} \geq 0$, which is the revenue the advertiser earns when its ad is shown to a consumer. Advertisers' types are distributed according to distribution $\hat{H} \in [\underline{\alpha}, \overline{\alpha}]$, which has a positive density $\hat{h}$ and an increasing virtual type, $u(\alpha) := \alpha - \frac{1-\hat{H}(\alpha)}{\hat{h}(\alpha)}$.

A mechanism determines allocations and prices based on the reported types of consumers and advertisers. Specifically, for each consumer type θ, the mechanism determines the matching between items and advertisers, a content bundle, and a monetary transfer. Note that these objects determine the mass of consumers, denoted by $D(\alpha)$, who will see the ad of each advertiser type α. In addition, the mechanism determines the price $P(\alpha)$ that each advertiser α pays. If an advertiser has type α but reports $\hat{\alpha}$, its payoff is $\alpha D(\hat{\alpha}) - P(\hat{\alpha})$. The mechanism has to satisfy the incentive compatibility and participation constraints of consumers and advertisers.

Suppose that the platform chooses a mechanism under which an advertiser of type α pays $P(\alpha)$ and has its ad shown to mass $D(\alpha)$ of consumers. Then the platform's advertising revenue is $\int_{\underline{\alpha}}^{\overline{\alpha}} P(\alpha) \, \mathrm{d}\hat{H}(\alpha)$. Using the local incentive compatibility of advertisers, we can rewrite the advertising revenue as $\int_{\underline{\alpha}}^{\overline{\alpha}} u(\alpha) D(\alpha) \, \mathrm{d}\hat{H}(\alpha)$. We can further rewrite it as

$$\int_{\underline{\alpha}}^{\overline{\alpha}} u(\alpha)D(\alpha)\ \mathrm{d}\hat{H}(\alpha) = \int_\Theta \int_{B(\theta)} u(\alpha)\ \mathrm{d}m_\theta(q,\alpha)\, \mathrm{d}F(\theta) = \int_\Theta R(\theta)\ \mathrm{d}F(\theta),$$

where $R(\theta) = \int_{B(\theta)} u(\alpha)\ \mathrm{d}m_\theta(q,\alpha)$, and m_θ is the matching policy for type θ. Therefore, the model reduces to the baseline model such that $r = u(\alpha)$ and H being the distribution of virtual type $u(\alpha)$ with $\alpha \sim \hat{H}$.

One caveat is that $u(\alpha)$ may take a negative value for some α, while we assume $r \geq 0$ in the baseline model. However, in the extended model, advertisers with negative virtual values are never matched with any items. Therefore, when we consider H in the baseline model that replicates the extended model, we can select H such that the mass of advertisers with $r = 0$ equals the mass of advertisers (in the extended model) who have negative virtual values.

The optimal mechanism in Proposition 1 has the property that more profitable ads are shown to more consumers. In the extended model, this would mean that an ad associated with a higher virtual type is shown to more consumers. Because the virtual type of an advertiser is increasing in type α, we conclude that an ad that is associated with a higher type is shown to more consumers, i.e., the monotonicity of allocation for advertisers holds.

In the main text, we have conducted monotone comparative statics with respect to H. Based on the above extension, one way in which H increases in the first-order stochastic dominance (FOSD) is when the platform gains access to advertisers' willingness to pay. Eliminating advertisers' private information means that the platform's advertising revenue is calculated based on α instead of $u(\alpha)$, where the distribution of α dominates that of $u(\alpha)$ in FOSD. In particular, if $\underline{\alpha}$ is sufficiently large so that even the lowest virtual type is above a, then by Proposition 2.1, the platform's access to information that eliminates the advertiser's private information will induce the platform to choose a lower content quality distribution.

Appendix B: Allocating Items Without Ads

In our baseline model, we assume that the platform must show an ad along with each item. In this appendix, we allow the platform to allocate items without ads and show that such a model reduces to our baseline model with a particular choice of H.

Consider the following extension: As in the baseline model, the platform hosts a unit mass of items and a unit mass of ads. But the platform can now allocate an item without any ad. If the platform allocates item q without an ad, its contribution to a consumer's utility is $\theta q - a$. If the platform allocates item q with an ad, its contribution to a consumer's utility is $\theta q - a - d$, where $d > 0$ is the disutility from an ad. Let $\hat{H}$ denote the advertising revenue distribution and assume it is continuous at d.

In this extended model, if the platform displays an ad with revenue r', the advertising revenue increases by r', but the sales from content decrease by d, compared to the case in which the platform allocates the same item without an ad. Hence, we can treat an ad in the extended model as an ad of our baseline model where $r = r' - d$. Moreover, replacing ads that have negative revenues (i.e., $r' < d$) with ads that have zero revenues does not affect the platform's optimal mechanism. Therefore, the extended model reduces to the baseline model in which $H(0) = \hat{H}(d)$ and $H(r) = \hat{H}(r + d)$. Matching an item with ad $r = 0$ is interpreted (in the extended model) as displaying no ad, and matching an item with ad r is interpreted as matching the item with ad $r + d$.

Appendix C: Optimal Mechanism Under Degenerate G and H

In this appendix, we assume that the ad revenue distribution and the feasible quality distribution are degenerate. Let r denote the unique ad revenue and q denote the quality level the platform has chosen. The platform can choose q at cost $C(q) \geq 0$, which is strictly increasing, strictly convex, continuously differentiable, and $C'(0) = 0$.

The purpose of this exercise is to show that our main insight—that the platform may choose a lower content quality when ads become more profitable—is robust when the platform cannot use negative prices and consumers are free to ignore the allocated content and ads (see the discussion in Section 3.2 for detail). Figure A1 presents the platform's optimal quality choice when it can only use a nonnegative price under the same parametrization as Figure 1 of the main text. The optimal quality choice remains non-monotone, and for an intermediate range of r, a higher ad revenue leads to a lower quality level.

Below, we analytically show the non-monotonicity of the optimal quality level with respect to r. We first solve for the platform's optimal mechanism when the platform can use a negative price, and then study the optimal mechanism and quality choice when only nonnegative prices are allowed.

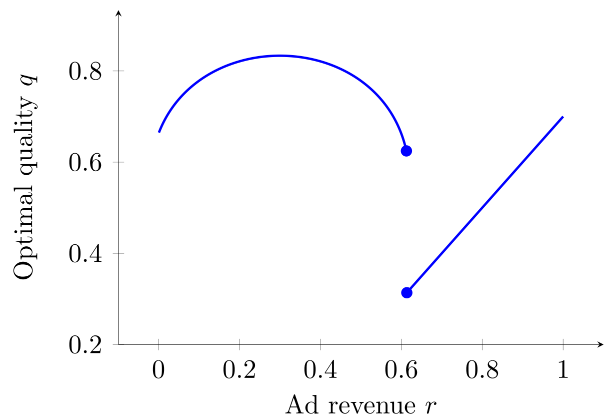

Figure A1: The optimal quality choice as a function of $r \in [0, 1]$ when the platfrom can only use a nonnegative price, with $a = c = 0.3$ and $\theta \sim U[0, 1]$. The optimal choice is not unique at r slightly above 0.6, around which it discontinuouly drops from $q \approx 0.6$ to $q \approx 0.3$.

C.1 Optimal Mechanism when Negative Prices are Feasible

When G and H are degenerate, the matching between ads and items are trivial. In the main text, we have shown that the platform's profit from the optimal mechanism is written as its virtual surplus:

$$\int_{\underline{\theta}}^{\overline{\theta}} (v(\theta)q + r - a)\, \mathbf{1}_{\{v(\theta)q+r-a\geq 0\}}\, \mathrm{d}F(\theta). \tag{9}$$

Let F^* denote the distribution of $v(\theta)$ with $\theta \sim F$. Then we can write (9) as

$$\int_{\frac{a-r}{q}}^{\overline{\theta}} xq + r - a \, \mathrm{d}F^*(\theta). \tag{10}$$

The platform chooses q to maximize

$$\Pi(q, r) := \int_{\frac{a-r}{q}}^{\overline{\theta}} xq + r - a \, \mathrm{d}F^*(x) - C(q). \tag{11}$$

Let $S_q(r)$ be the set of quality levels that maximize $\Pi(q, r)$ for a given r.

Claim 3. *On $r \in [0, a)$, $S_q(r)$ is increasing in the strong set order. On $r \in [a, \infty)$, $S_q(r)$ is decreasing in the strong set order. Given the chosen quality q^*, the optimal mechanism is to sell the item at a (possibly negative) price of $v^{-1}(\frac{a-r}{q^*})q^* - a$.*

Proof. We have

$$\frac{\partial \Pi}{\partial r} = 1 - F^* \left(\frac{a-r}{q} \right), \tag{12}$$

which is increasing in q for $r \leq a$ and decreasing for $r \geq a$. Thus, we obtain the part of the claim regarding comparative statics. A posted price of $v^{-1}(\frac{a-r}{q})q - a$ ensures that consumers with types above $v^{-1}(\frac{a-r}{q})$, or equivalently, those with virtual types above $\frac{a-r}{q}$ will buy the item. These consumers generate nonnegative virtual surplus $v(\theta)q^* + r - a$ by construction. This verifies the optimality of the posted price mechanism. □

C.2 The Case of Non-Negative Monetary Transfer

The assumption of nonnegative price implies that consumers are willing to consume the allocated content even if they could potentially ignore the content to save the attention cost. Indeed, the consumer's participation constraint is $\theta q - a - p \geq 0$, which implies $\theta q - a \geq 0$ whenever $p \geq 0$. This property may fail under a negative price when $\theta q - a - p \geq 0$ but $\theta q - a < 0$, i.e., consumers prefer to accept the contract, receive the monetary transfer, and then ignore the content and ads.

First, we derive the optimal price. Suppose that the platform chooses quality level q. The platform's profit, written as virtual surplus, is as follows:

$$\int_{\underline{\theta}}^{\overline{\theta}} (v(\theta)q + r - a)\, x(\theta)\, \mathrm{d}F(\theta). \tag{13}$$

where $x(\theta)$ is the probability with which type θ receives the item, whose quality is q.

For any given q, the platform's solution will take one of the two forms. First, suppose that any $v(\theta)q + r - a \geq 0$ implies $v(\theta) \geq v(a/q)$, or equivalently, $\frac{a-r}{q} \geq v(a/q)$. In this case, the platform can implement the unconstrained optimal mechanism with a nonnegative posted price of $v^{-1}(\frac{a-r}{q})q - a$. Given this price, consumers purchase the item if and only if $v(\theta)q + r - a \geq 0$.

Second, suppose that $\frac{a-r}{q} < v(a/q)$. In this case, the price $v^{-1}(\frac{a-r}{q})q - a$, which induces all types with nonnegative virtual surplus to purchase the item, will be negative. The optimal mechanism for the platform is to post a price of 0 to allocate the item to as many types with nonnegative virtual surplus as possible. In this case, consumers purchase the item if and only if $\theta q - a \geq 0$.

In terms of the consumer's virtual type, the platform sells the item to all types whose virtual types are above $\max\left(\frac{a-r}{q}, v\left(\frac{a}{q}\right)\right)$. Using the notation F^* for the distribution of virtual types, we can write the platform's maximal profit at quality q as

$$\int_{\max\left(\frac{a-r}{q}, v\left(\frac{a}{q}\right)\right)}^{\overline{\theta}} xq + r - a \,\mathrm{d}F^*(x) - C(q). \tag{14}$$

To state the result, we impose the following assumption, which holds, for example, if $\theta \sim U[0,1]$ and a is small enough.

Assumption 1. Type distribution F has an increasing generalized hazard rate, $\frac{xf(x)}{1-F(x)}$. The primitives satisfy

$$0 \geq v\left(\frac{a}{(C')^{-1}\left(\int_0^{\overline{\theta}} x \,\mathrm{d}F^*(x)\right)}\right). \tag{15}$$

The following result describes how the platform's quality choice depends on its advertising revenue. In particular, for an intermediate range of r, a greater r leads to a lower quality.

Claim 4. *Let $S_q(r) \subset \mathbb{R}_+$ be the set of quality levels that maximize the platform's profit (14) given ad revenue r. There exist $r_1 \in (0, a)$ and $r_2 > a$ such that:*

1. *On $r \in [0, r_1)$, $S_q(r)$ is increasing in the strong set order.*
2. *On $r \in [a, r_2]$, $S_q(r)$ is decreasing in the strong set order.*

Proof. First, we show Point 2. We begin with showing that there is a unique finite $r_2 > a$ such that the nonnegative price constraint $\frac{a-r}{q} \geq v\left(\frac{a}{q}\right)$ holds if $r \in [a, r_2]$ and is violated if $r \in (r_2, \infty)$. The proof consists of three steps.

Step 1. The constraint $\frac{a-r}{q} \geq v\left(\frac{a}{q}\right)$ is violated at some $r \geq a$. Within $r \in [a, \infty)$, the platform's unconstrained quality choice is decreasing in r (by Claim 3), so we have $\frac{a-r}{q} \to -\infty$ as $r \to \infty$. At the same time, $v\left(\frac{a}{q}\right)$ increases as q decreases. Therefore, there must be some r above which the constraint is violated.

Step 2. If the constraint is violated at some $r_2 \geq a$, we must have $r_2 > a$. To prove this, it suffices to show that the unconstrained quality choice satisfies the constraint at $r = a$. Plugging $r = a$ into (11), we can simplify the platform's unconstrained problem as

$$\int_0^{\overline{\theta}} xq \,\mathrm{d}F^*(x) - C(q). \tag{16}$$

The first-order condition is

$$\int_0^{\overline{\theta}} x \,\mathrm{d}F^*(x) - C'(q) = 0.$$

The optimal quality choice is $(C')^{-1}\left(\int_0^{\overline{\theta}} x \,\mathrm{d}F^*(x)\right)$. The nonnegative price constraint, which is now written as $0 \geq v(a/q)$ given $r = a$, holds at this quality if

$$0 \geq v\left(\frac{a}{(C')^{-1}\left(\int_0^{\overline{\theta}} x \,\mathrm{d}F^*(x)\right)}\right),$$

which holds by Assumption 1.

Step 3. Finally, we show that if the unconstrained quality choice violates the constraint $\frac{a-r}{q} \geq v\left(\frac{a}{q}\right)$ at $r > a$, it continues to do so for any $r' > r$. Suppose that $a - r < qv\left(\frac{a}{q}\right)$ at some $r > a$. Note that

$$qv\left(\frac{a}{q}\right) = a - \frac{1 - F\left(\frac{a}{q}\right)}{\frac{1}{q} f\left(\frac{a}{q}\right)},$$

which is decreasing in q when F has an increasing generalized hazard rate (Assumption 1). The unconstrained quality choice is decreasing in r when $r > a$. Therefore, if $a - r < qv\left(\frac{a}{q}\right)$ at some $r > a$, then $a - r' < q'v\left(\frac{a}{q'}\right)$ for any $r' > r$, where q and q' denote the optimal quality choices under r and r', respectively.

To sum up, as r increases from a, there is a unique $r_2 > a$, with the following properties: (i) if $r \in [a, r_2]$, the platform's unconstrained quality choice and the resulting optimal mechanism satisfy the nonnegative price constraint; and (ii) if $r \in (r_2, \infty)$, the platform's unconstrained quality choice and the resulting optimal mechanism violate the nonnegative price constraint.

Point 2 holds because if $r \in [a, r_2]$, the platform's optimal choice coincides with the unconstrained choice in Claim 3.

For Point 1, note that at $r = 0$, the constraint $a/q > v(a/q)$ always holds. Thus in a neighborhood of $r = 0$, we can apply Claim 3 to obtain Point 1. $\square$