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Abstract

Electricity is consumed continuously night and day and is not storable at large scale. Consequently, in an electricity industry organized and managed efficiently, demand should be tightly responsive to time-varying prices. We explore the consequences of the limited ability of electricity consumers to use price signals in their decisions to withdraw energy from the grid and the advantages of an assistance service that can correct this bias. Depending on the statistical distribution of price misperception types, we determine the allocation of assistance that allows to decrease total consumption and the outcome of different market structures. Because of the impossibility of distinguishing between consumers who underestimate and those who overestimate electricity prices, we show that it may be suboptimal to organize a market for assistance. We also show that it is less efficient to rely on a private integrated monopoly than on two separate private monopolies, one for assistance, the other for energy.

JEL codes: C72, D24, D47, L23, L94

Key words: demand response, electricity, energy saving, quasi rationality, consumers screening, smart appliances, market design.

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1 Introduction

The consumer of electricity who is able to adapt to scarcity signals, ideally to wholesale prices, is better off than when he is constrained to buy at the market average price. Additionally, this rational consumer[1] provides social advantages, first because he allows to save on production, storage and transmission capacities[2], second because he generates environmental byproducts since the production units necessary to supply inflexible customers at peak hours burn fossil fuels that emit both greenhouse gases and local pollutants. This explains why making electricity consumers responsive to price variations is often encouraged by public authorities. For example 'Directive (EU) 2019/944 of the European Parliament and of the Council of 5 June 2019 on common rules for the internal market for electricity' states that "*All consumers should be able to benefit from directly participating in the market, in particular by adjusting their consumption according to market signals and, in return, benefiting from lower electricity prices or other incentive payments*". The European authorities consider that the benefits of such active participation are likely to increase over time, as the awareness of otherwise passive consumers is raised about their possibilities to be active and as the information on the possibilities of active participation becomes more accessible and better known.[3] They add that *"Consumers should have the possibility of participating in all forms of demand response. They should therefore have the possibility of (...) choosing to have a smart metering system and a dynamic electricity price contract.[4] This should allow them to adjust their consumption according to real-time price signals that reflect the value and cost of electricity or transportation in different time periods, while Member States should ensure the reasonable exposure of consumers to wholesale price risk".*

During the 20th century, the electricity industry has been organized with the constraint of being flexible enough on the supply side in order to satisfy price-inelastic energy demand.[5] Given the development of intermittent renewables and the banishment of flexible fossil fuels to fight climate change, given the limited and costly possibilities offered by the technologies of energy storage, with the

[1] "*Homo rationalis is the species that always acts both purposefully and logically, has well-defined goals, is motivated solely by the desire to approach these goals as closely as possible, and has the calculating ability required to do so.*" (Aumann, 1985, p. 35)

[2] As explained by Boom and Schwenen (2021), they produce a public good.

[3] A similar policy is followed by the California Energy Commission (CEC): https://www.energy.ca.gov/news/2022-10/cec-adopts-standards-help-consumers-save-energy-peak-times

[4] The Directive defines a dynamic electricity price contract as "*a contract between a supplier and a final customer that reflects the price variation in the spot markets, including in the day-ahead and intraday markets, at intervals at least equal to the market settlement frequency*".

[5] Fabra et al. (2021) show that the short-run elasticity of Spanish households to changes in real-time prices (RTP) is not significantly different from zero. On RTP and its quasi synonyms peak-load pricing, scarcity pricing, dynamic pricing, etc., see Boiteux (1949), Kahn (1970), Joskow and Wolfram (2012). On the flexibility of production plants, see Crampes and Renault (2019, 2021).

aforementioned Directive the European authorities tried to give the consumer an active role in efficient mechanisms to reach real-time equilibrium in energy exchanges. Consumers would find in each Member State at least one supplier, and every supplier that has more than 200 000 final customers should propose one dynamic contract. However, "*Suppliers shall obtain each final customer's consent before that customer is switched to a dynamic electricity price contract.*" (*ibid.* article 11). In other words, exposure to price risk shall be an opt-in decision. Being billed with a dynamic-pricing contract should not be mandatory.

More recently, following the energy crisis in 2021 and the attack of Ukraine by Russia in 2022, the European Commission urged Member States to undertake concrete steps to reduce the exposure of consumers to the volatility of energy prices. (European Commission, 2021, 2022a, 2022b).[6] In Spain, the regulated tariff used to set the daily price of electricity on the basis of the 24 hourly energy auctions. As a result, in 2022, consumers on the regulated tariff experienced historic spikes in electricity prices, while those on the free market saw no change in the price they had agreed with their supplier, usually a year in advance. A reform approved in June 2023 gradually reduces the weight of the daily market in setting the price by combining it with futures markets. The weight of the daily market in the calculation of the regulated electricity tariff will be reduced from 75% in 2024 to 45% in 2025. In France, a recent law (n°2024-330 of 11 April 2024) authorises even very small businesses and small local authorities (less than 36 kVA) to take out a regulated tariff contract designed by the government.

In this paper we analyze the biased perception of wholesale prices by electricity consumers and how this bias can be reduced. We first consider the qualities that final consumers should have to be efficient participants in electricity markets (section 2). In section 3, we analyze the consequences of spot price misperception, that is the inability to withdraw energy from the grid in accordance with prices that vary 24 times a day. Section 4 is devoted to two solutions to improve the well-being of electricity consumers *i*) the standard fixed-price contract signed with a retailer (subsection 4.1), and *ii*) the reliance on an assistance service helping the consumer to use the information on prices more accurately (subsection 4.2) in order to decrease energy consumption (subsection 4.3).

In section 5, we determine the energy market equilibrium when an oligopoly supplies electricity to consumers with three types of price misperception (as detailed in subsection 5.1). Using the results of the Cournot equilibrium (subsection 5.2), we move backward to analyze the supply of assistance emphasizing the difficulties of screening among consumers who underestimate and those who overestimate electricity prices (section 6). We compare the performance of different market structures in terms of consumption reduction. Section 7 concludes.

[6] In France, the government introduced a tariff shield in February 2022. The aim of this shield was to limit the increase in regulated electricity tariffs to 4%. It applied to residential and small business consumers in mainland France and to all consumers in non-interconnected areas who have opted for regulated tariffs.

2 From consumer's theory to the real world

When it comes to electricity consumption, it is highly unlikely to observe perfectly rational behavior, in particular concerning households.[7] This is due to an accumulation of factors that we explore below: electricity must be combined with specific equipment to provide services, it is produced and consumed as a continuous flow, and consumers do not have the cognitive and procedural skills necessary to optimize their electricity uses. Before going through these successive concepts, we recall the main outcomes from field and lab experiments on the weak responsiveness of energy consumers to wholesale prices.

2.1 Empirical evidences

Most recent papers on dynamic pricing in the electricity industry are empirical. They evaluate the costs and benefits of an exposure to wholesale prices, either by a direct pass-through of the spot market price (real-time pricing) or under softer formats[8]. They all find positive but limited effects (e.g. US $10 per year per household in Allcott 2011).[9] They also find that a strong "default effect" sticks most consumers to their current contract (Fowlie et al. 2017), which justifies that, contrary to the Spanish rule (Fabra et al. 2021), dynamic pricing should only be offered as an opt-in option, in particular to poor and less educated households.[10]

Researchers also insist on the necessity of helping consumers by installing smart equipment (Blonz et al. 2021, Bollinger and Hartmann 2018, Gillan 2017, Jessoe and Rapson 2014). This necessity of a complementary technical assistance is an indirect recognition of the bounded rationality of consumers: economic agents actually use heuristics (i.e. simple cognitive shortcuts) to take decisions (Simon, 1982; Kahneman, 2003; Pollitt and Shaorshadze, 2011; Frederiks et al., 2015).[11] Additionnally, the capacity of electricity consumers to choose efficiently between suppliers is limited:[12] Wilson and Waddams (2010) show that between a

[7] There is now a substantial literature on behavioural biases in the field of electricity retailing. See Schneider and Sunstein (2017), and, more recently, Bailey et al. (2024). For a more general view, see Köszegi (2013) who presents a survey of psychological-and-economics research in contract theory.

[8] Examples of softer exposure are Time-of-Use, Critical-Peak-Pricing, Peak-Time-Rebates, Load-shedding, Load-shifting, Priority service, and all possible combinations of these mechanisms (Borenstein, 2005; Astier and Léautier, 2021).

[9] However, Ito et al. (2023) have recently shown that well-calibrated public incentives can attract those consumers who are most sensitive to price variations. This self-selection increases social welfare.

[10] For an econometric analysis of the regressive effects of Real Time Pricing, see Cahana et al. (2022).

[11] Some countries explicitly recognize the citizen's right to make mistakes, which concretely means the right to correct mistakes made in good faith in order to avoid penalties. See for example https://www.economie.gouv.fr/igpde-editions-publications/thearticle_n6

[12] Littlechild (2021) provides an overview of the hurdles on the way to opening electricity retailing to full competition.

fifth and a third of UK consumers actually lost surplus as a result of switching, and in aggregate, switching consumers appropriated less than half of the maximum gains available to them. In a lab experiment, Mayol and Staropoli (2021) find that subjects prefer simple tariffs over complex ones because they get confused by the latter.

These field and lab evidences show the necessity to investigate the consequences of "bounded rationality" at the time where the European authority and others consider that consumers' flexibility is an essential part of the future design of electric systems. Flexibility requires strong rationality, that is customers being informed about time-varying prices and knowing how to use this information to control their electric appliances, for example to set the temperature of space heaters or air conditioners. Can they do so?

2.2 Intermediary good

Electricity is not a final consumption good. This is obvious in industrial processes. It is also true at the household level. Indeed, what we consume is services derived from the combination of power and appliances. For example fresh beers come out of a fridge motored by electric energy. A realistic representation of the consumer's decision should then entail a "local production function" transforming electricity into services for a given mix of equipment in the dwelling or the factory (Crampes et al. 2023). To solve this production problem, the consumer should know the number of kWh necessary for each task (e.g. 2.3 kWh per load for a clothes washer, warm wash, cold rinse) and the kWh prices along the day (assuming that the price of water is not time-varying). Actually, most households have no idea what the electricity price and quantity consumed are.[13] Consequently they will start the washing machine without precisely taking the energy cost into consideration, maybe by an automatic programming at night when energy is cheap but with the risk of disturbing their neighbors because of the noise. And for ironing clothes we can exclude that they will get out of bed during the night to save some cents. If energy storage becomes cheaper and cheaper at home, charging batteries for ironing at day with electricity stored at night can become profitable. But the equipment needs space and, for now, it is too costly.

2.3 Time dimension

Contrary to most goods, we withdraw electricity from the grid and/or the domestic generators or batteries continuously night and day, for heating, cooking, cooling, lighting, and so on. On power exchanges, electricity is mainly traded and priced on a hourly base in day-ahead markets.[14] As consumers we can use smartphones to obtain this information on tomorrow prices. Actually, we do so

[13]This is not the case of large industrial consumers who need to minimize their energy bill.

[14]Day-ahead markets are cleared by fixing, whereas intraday operations are a continuous matching of supply and demand. See for example https://www.epexspot.com/en/market-data.

at scarce moments, and most of us never do. The large majority of electricity consumers are only interested in the cost of electricity once, twice or twelve times a year, upon receiving the bill from their provider, and we know that in this bill, energy only accounts for one part, the two other parts being transmission and taxes. This, coupled with the intermediary nature of electricity evoked formerly, means that small and medium consumers are not prepared to react to price variations at short notice. Nudges such as colored lights can at least send signals to limit consumption at critical hours. This requires specific signaling equipment that comes with the contract. An example is Option Tempo, a time-of-use pricing proposed by the French operator EDF.[15]

2.4 Limited ability

In addition to the technical and time constraints mentioned above, behavioral limits explain why consumers are not perfect rational agents regarding electricity. We rather are in the Thaler's world of "quasi-rationality" where mental accounting procedures lead agents to behave in economically inconsistent ways. Broadly speaking, consumers have a limited capacity in acquiring, processing and applying information efficiently. For example they are handicapped by limited attention, that is the incomplete consideration of some elements in their choice set, particularly quality indicators and prices.[16] Limited attention has been analyzed by Lee and Malmendier (2011) in auctions, Lacetera et al. (2012) in the used car market, Stango and Zinman (2014) in the field of bank overdraft fees, Andor et al. (2017) regarding energy labels, Bach et al (2022) in postal platform pricing. Harding and Sexton (2017) synthesizes evaluations of experiments on time-varying electricity rates in the context of a theory of consumer inattention and adjustment costs. In Kahn-Lang (2022), there are two demand-side market distortions: heterogeneous search frictions and inattention-based inertia. It results that some consumers with "high search frictions" sign up with marketers at a high price because they find it costly to search for the best price.

Another difficulty is the "framing" effects, i.e. when the form is more important than the substance: subjects presented with the same budget set described in different ways change their responses (Tversky and Kahneman, 1981). In a lab experiment, Verhagen et al. (2012) found that framing effects exist in the domain of energy tariff selection, and that risky choice framing and attribute framing are powerful tools in convincing energy consumers to switch from traditional to renewable energy.

[15] On Tempo, see https://en.selectra.info/energy-france/suppliers/edf and https://www.kelwatt.fr/guide/tempo.

[16] In the neuroscience field, Christie and Schrater (2015) view "*the allocation of metabolic resources as a control problem with the concrete resource limitation given by the dynamics of glycogen storage and use*". They provide a "*rational explanation for a wide range of troublesome biases and patterns in decision making*". For an historical perspective of the attention concept in economics, see Festré and Garrouste (2015).

With consumers limited in their ability to understand the variations in power prices and, anyway, limited in their capacity to respond to price signals, one can argue that most of us will not choose the ex ante least cost contract, and will not switch on and off our appliances to minimize the energy bill.[17]

2.5 Risk aversion and its retinue

The concept of *risk aversion* is key in the analysis of markets where demand conditions (e.g. temperature) and supply conditions (e.g. renewables' capacity factors) are highly fluctuating. Denoting by q the random consumption of electricity and $S(q)$ the gross surplus from this consumption, we know after the Arrow and Pratt works that risk aversion is characterized by a concave valuation $S''(q) < 0$. This is often considered as the main reason why electricity consumers prefer to sign flat tariff contracts, but we will see in subsection (3.2) that it is an incomplete interpretation. Behavioral economics have provided additional characteristics to reinforce this reluctance to time-varying outcomes. *Loss aversion* which means that the response to losses is stronger than the response to corresponding gains (Kahneman and Tversky 1992), resulting in a dissymmetric $S(q)$ function for positive and negative values of q. *Prudence,* modelled by $S'''(q) \geq 0$, means the propensity to forearm oneself in the face of uncertainty (Kimball, 1990; Brunette and Jacob, 2019). Before being locked in a specific energy contract, the prudent agent would invest in specific appliances to adapt its consumption to the time profile of the electricity prices. And if appliances are costly, prudence rather suggests to sign a contract with a time-invariant price. Another concept is *temperance,* modelled by $S''''(q) < 0$. According to Kimball (1992), a *temperate* agent desires to moderate total exposure to risk. Then, he "*may respond to an unavoidable risk by reducing exposure to other risks even when the other risks are statistically independent of the first*". A consequence could be that the household exposed to risks at his working place and during travels could prefer to avoid the additional risk of highly fluctuating energy bills at home.

This risk reluctance is likeley to play a role in the refusal to sign price-varying contracts. It also explain why "energy retailing is essentially a financial service" (Cerre, 2022). In the following section we rather focus on a modeling of the inability to process correctly the price signal and its consequences.

3 Price unawareness

Models of market mechanisms applied to the electricity industry rely on two polar cases as regards consumers. The first one, favored by the economist, assumes perfect reactivity to wholesale prices (Léautier, 2018, chapter 5). In a nutshell,

[17] On the consequences of limited rationality of both consumers and regulators, see Auriol et al. (2021, p. 366-369). In the energy industry, the British regulator refers to behavioral economics to explain why consumer engagement is hampered (Ofgem, 2011).

denoting by q the quantity demanded and p the unit wholesale price, demand is the function $q(p)$ obtained by solving the optimization problem $\max_q S(q) - pq$, where $S(q)$ is the (locally increasing and concave) surplus function that measures in monetary units the satisfaction derived from the purchase of q. The other polar case, favored by the engineer, is no reactivity at all ($q'(p) \equiv 0 \ \forall p$), in particular in dispatch models where timescales are very short (Hirth 2017).[18] The reasons of this lack of response to price variations can be informative (no access to market data and/or ignorance of how to react to information on prices), behavioral (usage standards, habits), or transactional (adaptation costs, distance to the consumption spot, lack of smart appliances and smart meters[19]). Other reasons are organizational, for example the presence of retailers who bridge the gap between consumers and the wholesale market (fixed price contracts, insurance against price variations) and public intervention to fix regulated tariffs. More general models combine the two polar cases (e.g. Ambec and Crampes, 2021).

Actually, many consumers do react to price variations (Cahana et al. 2022), at least to large ones, but their reaction is imperfect, both for information reasons (cognitive bias) and for interpretation reasons (procedural bias). In the next subsections, we first define the type of quasi-rationality we will focus on, and then we evaluate the surplus lost by the consumer because of his informative imperfection.

3.1 A quasi rational electricity consumer

Following Thaler (1994), quasi rationality refers to cases in which people deviate from the behavior dictated by net surplus maximization and these deviations are somewhat predictable. To keep things simple we will assume that there is no technical constraint on the consumption of electricity. It is only characterized by a saturation threshold $\overline{q}$ such that the gross marginal surplus satisfies

$$S'(q) \lesseqqgtr 0 \text{ as } q \gtreqqless \overline{q}$$

where $S''(q) < 0$. We assume that the lack of rationality comes from a misjudgment on the price. Let $h(p)$ represent the consumer's valuation of the price when the actual value is p. Depending on the degree of quasi rationality of a given consumer, we can have $0 \leq h(p) \lesseqqgtr p$. In the following we will assume that rationality is still strong enough for $h'(p) \geq 0$.[20] We keep the hypothesis that he wants to maximize net surplus given this biased information. The consumer will

[18] It is also implicit in building the load-duration curve, obtained by stacking up forecasted demanded quantities in decreasing order without reference to any price sensibility; see Léautier, 2018, page 18.

[19] Joskow and Tirole (2006) analyze the implications of load profiling of consumers whose traditional meters do not allow for measurement of their real time consumption.

[20] $h'(p) < 0$ could arise because of an exogeneous additional shock. For example the consumer could be encouraged to increase the house temperature when it is freezing outside (so that p is high) by government promises of future lower prices or upon receiving an energy check.

choose the quantity $q_h(p)$ determined by solving $\max_q S(q) - h(p)q$, that is

$$q_h(p) = \arg\left[S'(q) = h(p)\right]. \tag{1}$$

From the differentiation of the first order condition, demand is non-increasing:

$$q_h'(p) \overset{def}{=} \frac{dq_h(p)}{dp} = \frac{h'(p)}{S''(q_h(p))} \leq 0. \tag{2}$$

and the second derivative is

$$q_h^{''}(p) = \frac{h^{''}(p)S^{''}(q_h(p)) - h'(p)S^{'''}(q_h(p))q_h'(p)}{(S''(q_h(p)))^2}. \tag{3}$$

The curvature depends on the shape of the surplus and valuation functions. For example $h^{''}(p) < 0$[21] and $S^{'''}(q) \geq 0$ (prudence) are sufficient conditions for $q_h^{''}(p) > 0$ so that demand is less reactive in periods of high prices than in periods of low prices. This is illustrated in Figure 1 for $S'(q) = a - q$ (with $a > 1$) and $h_1(p) = \sqrt{p}$, so that the rational demand function is $q(p) = a - p$ and the distorted demand is $q_{h1}(p) = a - \sqrt{p}$. As long as $p < 1$, $h_1(p) > p$ and consequently $q_{h1}(p) < q(p)$. The opposite holds for $p > 1$. In periods of peak prices, consumption is larger than under the perfect rationality hypothesis, in particular in the interval $p \in (a, a^2)$ where it is positive whereas it should be nil.

We also have drawn the symmetric case $h_2(p) = p^2$ resulting in $q_{h2}(p) = a - p^2$. If the perceived price is $h_2(p)$, demand is restricted when the price is high, and excessive when it is low. For $p \in (\sqrt{a}, a)$, purchases are nil whereas they should be positive.

There are many possibilities of specification for the function $h(p)$. For example, each consumer can be more or less sensible to a combination of the spot price and its mean: $h(p) = \xi p + (1-\xi)\mathbb{E}p$ with $1 \geq \xi \geq 0$. With this function, consumer ξ only partially takes the spot price into account, but he reacts well on average since $\mathbb{E}h(p) = \mathbb{E}p$. As we will see in section 4.1, the case where $\xi = 0$ is similar to a retail contract at a non contingent price.[22]

In the following, we will use the proportional valuation $h_\lambda(p) = \lambda p$ where all prices are over-estimated when $\lambda > 1$ and under-estimated when $\lambda < 1$. Combining it with a linear marginal gross surplus $S'(q) = a - q$, demand is $q_\lambda(p) = a - \lambda p \geq 0$[23]. For $\lambda > 1$ (resp. $\lambda < 1$) the consumer buys less (resp.

[21] When $h^{''}(p) < 0$, for the values of p where $h(p) < p$ (resp. $h(p) > p$), the consumer beliefs on prices are less (resp. more) biased for small values than for large values.

[22] There is a strand of literature where price misperception is due to the unability of consumers who face non-linear prices (for example $T(q) = pq + p_0$) to adapt to the marginal price p instead of the average price $T(q)/q$. For example Martimort and Stole (2020) analyze the pricing strategy of a monopoly when its clients have this type of price misperception. The same bias explains why increasing-block tariffs do not perform as well as expected to reduce demand: see Ito (2014) for electricity retailing.

[23] We do not consider the case of prosumers who can have a negative net demand because they prefer to sell to the grid when the price is high.

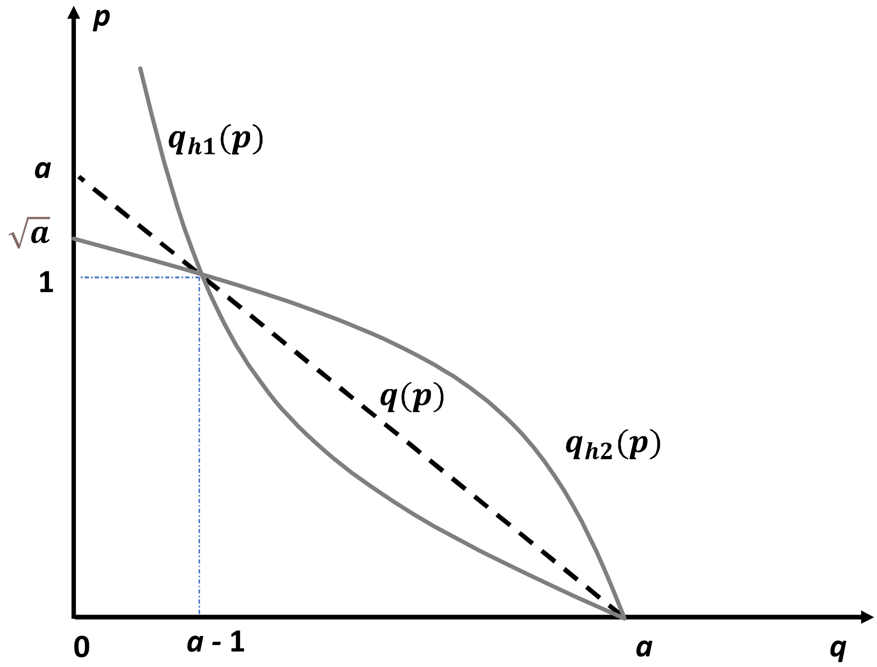

Figure 1: Optimal and distorted demand functions when surplus is quadratic

more) than what he should do, except at $p = 0$ where the distorted and optimal demands are equal. The price-elasticity of demand $-\frac{dq}{dp}\frac{p}{q} = \frac{\lambda p}{a-\lambda p}$ is an increasing function of λ.

3.2 Inclination or reluctance to dynamic price contracts

As mentioned in subsection 2.5, with $S^{''} < 0$ the consumer is reluctant to risks on *quantity*. Yet, the canonical model $\max_q S(q) - pq$ represents a consumer who is generically better off when he adopts a contract with time-varying *price* and he can adapt his consumption to the price signal. To see this, let us note the optimized net surplus

$$\widetilde{NS}(p) = S(q(p)) - pq(p) \tag{4}$$

where $q(p)$ is the solution to $S'(q) = p$. Then we can compute

$$\widetilde{NS}'(p) = S'(q(p))q'(p) - pq'(p) - q(p) = -q(p) \tag{5}$$

so that

$$\widetilde{NS}^{''}(p) = -q'(p)$$

Since $q'(p) < 0$ by $S^{''} < 0$, the second derivative wrt price of the consumer's net surplus $\widetilde{NS}^{''}(p)$ is positive, which means that he is a risk lover when uncertainty is on prices and he can perfectly adapt his consumption. Indeed, on average he is better off when adjusting his consumption to the observed price than if he buys at a flat price equal to the expected prices:

$$\mathbb{E}[\widetilde{NS}(p)] = \mathbb{E}\max_q[S(q)) - pq] > \max_q \mathbb{E}[S(q)) - pq] = \max_q S(q) - qE \quad (6)$$

where $E \stackrel{def}{=} \mathbb{E}[p]$.

With a model based on the hypothesis of perfect rationality, all consumers have an incentive to switch to contracts with time-varying prices. Since they actually don't switch, it is reasonable to consider that the model is incomplete. One missing piece is that, even if consumers are rational, switching is costly and this cost (e.g. the acquisition of smart appliances) must be explicitly added to the model. Another -not exclusive- reason is that energy consumers are not perfectly rational, they are just quasi-rational. In the following, we analyze the sign of the second derivative of the net surplus function with respect to the price as it is key to understand which type of contract is best for the consumer: ex ante average price vs. ex post random price?

The well-being of a quasi-rational consumer using biased information on prices can be evaluated in two different ways: either before or after he receives the electricity bill.

3.2.1 Before billing

The net surplus perceived by the quasi-rational consumer before he receives the energy bill is

$$\widetilde{NS}_h(p) = S(q_h(p)) - h(p)q_h(p) \quad (7)$$

where $q_h(p)$ is the solution to the first order condition (1). Substituting $S'(q_h) = h(p)$, we can calculate

$$\widetilde{NS}'_h(p) = -h'(p)q_h(p) \leq 0 \quad (8)$$

and

$$\widetilde{NS}''_h(p) = -h''(p)q_h(p) - h'(p)q'_h(p) \quad (9)$$

In the case of proportional valuation, since $h''_\lambda(p) = 0$ we still observe a preference for price variability. This preference is reinforced if the valuation is concave, $h''(p) < 0$, because the consumer has a perception of the distribution of peak prices narrower than the true distribution. By contrast, the consumer who overestimates large prices by $h''(p) > 0$ is less incline to prefer time-varying prices, and he can be averse to price randomness if $h(p)$ is strongly convex. In other words, $h''(p) \leq 0$ is sufficient but not necessary to a preference for time-varying prices and $h''(p) > 0$ is necessary but not sufficient to a preference for flat prices.

3.2.2 After billing

Actually, the energy bill is calculated by the retailer using the exact market prices. Then, instead of (7), after reading the bill the consumer's net surplus is

$$NS_h(p) = S(q_h(p)) - pq_h(p) \quad (10)$$

with

$$NS_h'(p) = (h(p) - p)q_h'(p) - q_h(p) \tag{11}$$

and

$$NS_h''(p) = (h'(p) - 2)q_h'(p) + (h(p) - p)q_h''(p) \tag{12}$$

This second derivative is quite different from the one obtained with the ex ante valuation (9), in particular because of the role played by the curvature of the demand function given by (3). In order to get more insights on the incentives to choose one contract, we compare the two second derivatives in the case of a proportional price distortion and quadratic utility.

3.2.3 Proportional price distortion

Assume that $h(p) = \lambda p$ (where $\lambda > 0$) and the gross surplus is quadratic, $S(q) = (a - \frac{q}{2})q$ so that demand is $q_\lambda(p) = a - \lambda p \geq 0$, $q_\lambda'(p) = -\lambda$ and $q_\lambda''(p) = 0$. Then (9) becomes

$$\widetilde{NS}_\lambda''(p) = \lambda^2 \tag{13}$$

and (12) becomes

$$NS_\lambda''(p) = (2 - \lambda)\lambda \tag{14}$$

Whereas all consumers behave as if they were risk lovers ($\widetilde{NS}_\lambda''(p) > 0$), when they receive the electricity bill those with a large overestimation of the price ($\lambda > 2$) become risk averse ($NS_\lambda''(p) < 0$). More generally, when they learn about their electricity bill all those with $\lambda > 1$ become less incline to accept a contract with uncertain prices ($NS_\lambda''(p) < \widetilde{NS}_\lambda''(p)$).

4 Helping the disappointed consumer

From the above analysis, we see that upon receiving the energy bill, the consumer can change his mind concerning the energy contract he signed. Depending on his biased valuation of prices, reading the bill can make him positively or negatively surprised. Indeed, combining (7) and (10), the ex post net surplus is

$$NS_h(p) = \widetilde{NS}_h(p) + q_h(p)(h(p) - p)$$

The pessimistic consumer ($h(p) > p$) is quite happy when receiving the energy bill. Inversely, for the optimistic (or careless) consumer, with $h(p) < p$ the bill reports bad news, all the more unpleasant as the quantity consumed is large. This can be corrected in two ways:

- the consumer can opt out of the time-varying contract and sign for a contract at flat unit price $\overline{p}$ with an entry fee p_0 that ensures a non random net surplus

$$\widetilde{NS}(\overline{p}) = S(q(\overline{p})) - \overline{p}q(\overline{p}) - p_0 \tag{15}$$

where $q(\overline{p})$ is the solution to $S'(q) = \overline{p}$;

- the alternative solution is to correct the information gap on varying prices by the purchase of a smart appliance or a service provision[24] s allowing the perceived price to be closer to p. In the following, we will generically call this service "*assistance*".[25]

4.1 Fixed-price option

We assume that technical and/or legal reasons prevent any form of price discrimination by retailers. The retailer proposes a non random contract with two-part pricing: $\overline{p}q + p_0$. If the consumer with price distortion $h(p)$ accepts the fixed-price contract, he will consume $q(\overline{p})$ and obtain the non random net surplus $\widetilde{NS}(\overline{p}, p_0) = S(q(\overline{p})) - \overline{p}q(\overline{p}) - p_0$.

The flat-price option is preferred to the time-varying price iff

$$\widetilde{NS}(\overline{p}, p_0) \geq \mathbb{E}NS_h(p) \tag{16}$$

where the right hand side is the expected value of (10). To keep things simple, we suppose that the retailer is risk neutral. She can buy energy on the spot market so that her average unit cost is $\mathbb{E}p = E$, and she will receive a zero default payment if the contract is not signed. Then, her net profit from the contract is $q(\overline{p})\overline{p} + p_0 - q(\overline{p})E \geq 0$.

The two price components $\overline{p}$ and p_0 are determined in a Nash bargaining process:

$$\max_{\overline{p}, p_0} (S(q(\overline{p})) - \overline{p}q(\overline{p}) - p_0 - \mathbb{E}NS_h(p))^{\beta} (q(\overline{p})\overline{p} + p_0 - q(\overline{p})E)^{1-\beta}$$

where β (resp. $1 - \beta$) is the bargaining power of the consumer (resp. retailer), $0 \leq \beta \leq 1$. Combining the first order condition wrt p_0

$$-\frac{\beta}{S(q(\overline{p})) - \overline{p}q(\overline{p}) - p_0 - \mathbb{E}NS_h(p)} + \frac{1 - \beta}{q(\overline{p})\overline{p} + p_0 - q(\overline{p})E} = 0$$

and the first order condition wrt $\overline{p}$

$$\frac{\beta\left[S'(q(\overline{p}))q'(\overline{p}) - \overline{p}q'(\overline{p}) - q(\overline{p})\right])}{S(q(\overline{p})) - \overline{p}q(\overline{p}) - p_0 - \mathbb{E}NS_h(p)} + \frac{(1 - \beta)[\overline{p}q'(\overline{p}) + q(\overline{p}) - q'(\overline{p})E]}{q(\overline{p})\overline{p} + p_0 - q(\overline{p})E} = 0,$$

[24] Examples are plug and play appliances, telephone calls, audio alert or flashing LED, SMS, applications on smartphone, ON/OFF switch delegated to a service provider. On curtailable electricty contracts, see for instance Harold et al. (2021). Bailey et al. (2024) conduct a field experiment with residential electricity customers to evaluate the effectiveness of utility-initiated vs customer-initiated demand response; in the latter, consumers were rewarded to initiate electricity reductions themselves upon receiving a price signal, some helped by a technology to remotely control devices in their home, others who needed to manually reduce consumption among their home appliances.

[25] As noticed by Joskow and Tirole (2006), having a Real Time Pricing meter is not sufficient for having consumption perfectly reactive to real time prices. An additional equipment is necessary to provide what they name "communication" allowing to control appliances.

using $S'(q(\overline{p})) = \overline{p}$ and after simplifying we obtain

$$\overline{p} = E, \qquad p_0 = (1-\beta)\left[S(q(E)) - \mathbb{E}NS_h(p) - q(E)E\right] \tag{17}$$

The retailer and the consumer agree on the maximization of the contract surplus $[S(q(\overline{p})) - \mathbb{E}NS_h(p) - q(\overline{p})E]$ by fixing $\overline{p} = E$ and they share it in proportion to their bargaining power. To illustrate how large the market for flat-price contracts can be, let us consider the case where

- the surplus function of the consumer is quadratic: $S(q) = (a - \frac{q}{2})q$, $q(E) = a - E$ and from (10) $NS_h(p) = \frac{1}{2}\mathbb{E}((a - h(p))(a + h(p) - 2p))$
- the price distortion is proportional: $h(p) = \lambda p$.

Combining these two specifications, we can write the contract surplus as

$$\begin{aligned} S(q(E)) - \mathbb{E}NS_h(p) - q(E)E &= \frac{1}{2}(a-E)^2 - \frac{1}{2}\mathbb{E}((a-\lambda p)(a+\lambda p) - 2p(a-\lambda p)) \\ &= \frac{1}{2}[E^2 + \lambda(\lambda-2)(\sigma^2+E^2)] \end{aligned}$$

where $\sigma^2 = \mathbb{E}\left[p^2\right] - E^2$ is the variance of the spot price p.

Then the potential market for flat price contracts is made of consumers λ such that $E^2 + \lambda(\lambda-2)(\sigma^2+E^2) > 0$. After some computation, we can rewrite the condition under the format[26]

$$(\lambda-1)^2 > \frac{\sigma^2}{\sigma^2+E^2}. \tag{18}$$

Clearly it is impossible to convince a fully rational consumer ($\lambda = 1$) to switch to a flat price, and, by continuity, it remains true if the error on the price is small. As we can see in Figure 2, only those who strongly deviate from the correct perception of the price, positively or negatively, will be better off with a flat price contract.

Observe that the threshold $\frac{\sigma^2}{\sigma^2+E^2}$ is increasing in the variance and decreasing in the average of the spot price. The latter can be viewed as an income effect: the more costly energy on average, the larger the incentive to secure the bill with a flat price contract. The former comes from the proportional specification that makes quasi-rational consumers behaving like risk-lovers (see subsection 3.2.3): when price randomness increases, consumers not too far away from the correct

[26]Proof: $E^2 > \lambda(2-\lambda)(\sigma^2+E^2)$ is equivalent to $\lambda^2(\sigma^2+E^2) - 2\lambda(\sigma^2+E^2) + E^2 > 0$. We have an equation of degree 2 in λ. The discriminant is $\Delta = 4(\sigma^2+E^2)^2 - 4E^2(\sigma^2+E^2) = 4(\sigma^2+E^2)\sigma^2 > 0$, so there are 2 real roots λ_1 and λ_2, and the expression is positive outside the interval joining the roots. We find $\lambda_1 = \frac{2(\sigma^2+E^2)+2\sqrt{\sigma^2}\sqrt{\sigma^2+E^2}}{2(\sigma^2+E^2)} = 1 + \sqrt{\frac{\sigma^2}{\sigma^2+E^2}}$ and $\lambda_2 = 1 - \sqrt{\frac{\sigma^2}{\sigma^2+E^2}}$. So consumer λ is better off with a flat price contract if and only if $\lambda \notin [\lambda_2, \lambda_1]$, i.e. if and only if $|\lambda - 1| > \sqrt{\frac{\sigma^2}{\sigma^2+E^2}}$.

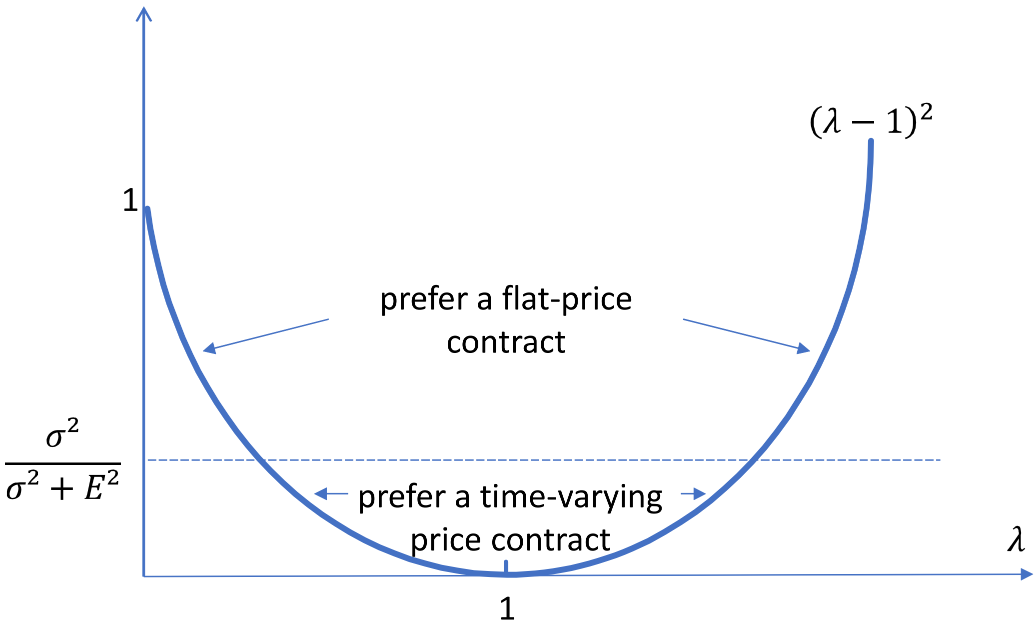

Figure 2: Flat vs. varying-price contracts

price valuation are better off when adapting their energy withdrawals to spot prices.

From (17), the flat price contract gives consumer h with bargaining power β

$$\widetilde{NS}_{h\beta}(E, p_0) = \beta \left[S(q(E)) - \mathbb{E} NS_h(p) - q(E)E \right]$$

For example, if the retailer is a monopolist ($\beta = 0$), she takes all the contract surplus through the fixed fee p_0, and the consumer is indifferent between signing the flat-price contract and buying energy at market prices. For any $\beta > 0$, the consumer is better off buying from the retailer. In particular, under harsh competition among retailers and/or a high monopsony power, β is close to 1 and the consumer keeps the largest share of the contract surplus. We are not aware of econometric evidences on a correlation between consumers' good knowledge of price ($h(p)$ close to p) and bargaining capacity (β close to 1), but one can conjecture that this is the case for large industrial and business customers. Some of them use their bargaining power to sign Power Purchase Agreements directly with producers. By contrast, small consumers, in particular households, have very low β and they gain little from a contract with a retailer.[27] Since in this paper we focus on price misperception, in the following we will discard the flat-price option and only analyse the consumer's choice between buying or not buying some assistance service.

[27] The risk of abuse of market power by energy retailers (β close to 0) is not purely theoretical. See for example Crampes and Laffont (2016) and Kahn-Lang (2022).

4.2 Assistance option

Let $g(p,s)$ represent the "information function" that bridges the gap between the erroneous price information $h(p)$ and the true value p. For example, $g(p,s) = sp + (1-s)h(p)$, with $s \in [0,1]$. In the following, we only consider the case where $s = 1$ after paying the price p_s.[28] Consequently, in the quadratic surplus case, using (4) the consumer has the choice between paying p_s for the service, then obtaining

$$\mathbb{E}\widetilde{NS}(p) - p_s = \frac{\mathbb{E}(a-p)^2}{2} - p_s \tag{19}$$

or not buying it, which gives the expected before-bill net surplus

$$\mathbb{E}\widetilde{NS}_h(p) = \frac{\mathbb{E}(a-h(p))^2}{2} \tag{20}$$

and the expected after-bill net surplus

$$\mathbb{E}NS_h(p) = \mathbb{E}[\frac{(a-h(p))^2}{2} + (a-h(p))(h(p)-p)] \tag{21}$$

For the optimistic or oblivious consumer, (i.e. such that $h(p) < p$), comparing (19) and (20) it is clear that $\mathbb{E}\widetilde{NS}_h(p) > \mathbb{E}\widetilde{NS}(p) - p_s$ whatever the energy prices p. Then it can be profitable to pay for assistance $i)$ only after reading the bill, **and** $ii)$ if $\mathbb{E}NS_h(p) \leq \mathbb{E}\widetilde{NS}(p) - p_s$, that is

$$p_s \leq \mathbb{E}\frac{[p-h(p)]^2}{2} \tag{22}$$

By contrast, if the consumer is pessimistic or wary (i.e. such that $h(p) > p$), the energy bill reports good news so that (21) is larger than (20). Then if p_s satisfies (22), the consumer is interested in the assistance service even before receiving the bill.

With the proportional specification $h(p) = \lambda p$, (22) writes

$$(\lambda - 1)^2 \geq \frac{2p_s}{\sigma^2 + E^2}. \tag{23}$$

This condition has some common features with (18) except that the threshold (i.e. the right-hand part of (23)) is now decreasing in the variance of the spot price since the device allows to adapt withdrawals to the exact price, so that more consumers are interested in buying it when spot prices are more varying.

Finally, observe that if $p_s > \frac{\sigma^2+E^2}{2}$, consumers with $0 \leq \lambda < 1$ are not interested in buying the assistance service. It takes a very low assistance price to incentivize negligent consumers to receive accurate signals on energy prices.

[28] Like in the former subsection, we assume the price discrimination is not feasible.

4.3 Assistance and energy savings

As already mentioned in footnote 24, there is a wide variety of assistance services, material or immaterial, distant or local, supplied by energy retailers or by independent firms. With the development of Artificial Intelligence, one can conjecture that this variety will increase (Lee et al. 2022). Like in all industries, some operators will try to gain market shares, and then market power, particularly by taking advantage of network externalities. Conversely, R&D can produce an increasing number of cheap digitized devices so that this market may become and remain competitive. It is also important to adress the question of vertical integration. Indeed, from a pure technological point of view, an energy seller is best placed to sell assistance since it has both skilled employees and the relevant information on consumers' behavior, notably from smart meters. Since we cannot predict the evolution of the assistance market, in the following we consider that firms compete "à la Cournot" on the two markets, which allows to cover a large range of market structures, from monopoly to perfect competition.

How to gauge the social performance of these market structures? Whereas in standard Industrial Organization they are evaluated by comparing their outcome with the allocation that maximizes the net social surplus, when agents are imperfectly rational it makes more sense to use a less demanding standard. In what follows, we will focus on one of the pillars of the policy to combat global warming, namely the reduction of energy consumption. In the European Union, the guiding principle that complements the objectives in the areas of sustainability, climate neutrality and green growth is "Energy Efficiency First".[29] The revised Energy Efficiency Directive of 20 September 2023 significantly raises the EU's ambition on energy efficiency, in particular as regards the reduction of demand for energy (European Parliament, 2023).

5 Energy market structure

To determine the outcomes of various market structures and measure their performance in terms of energy saving, we will use a three-type version of our former model of energy demand (subsection 5.1). Then, we determine the Cournot equilibrium on the energy market when n firms face this demand (subsection 5.2). We will anayze the market for assistance in section 6.

[29] "*While taking full account of security of supply and market integration, the Energy Efficiency First principle aims to ensure that only the energy really needed is produced, investments in stranded assets are avoided, demand for energy is reduced and managed in a cost-effective way.*" https://energy.ec.europa.eu/topics/energy-efficiency/energy-efficiency-targets-directive-and-rules/energy-efficiency-first-principle_en

For a comparative analysis of the costs and benefits of Energy Efficiency Obligations, see Rosenow and Bayer (2017).

5.1 Energy demand

How the assistance industry will be organised is tightly dependent on the distribution of consumer types in terms of bounded rationality. Even though one can obtain some results by using continuous distributions of probabilities of price misperception, in the following we will only rely on a discrete distribution with three types of consumers that are extreme features of the three cases screened by inequality (23), i.e. consumers who strongly underestimate the electricity prices, those who strongly overestimate them and, in between, consumers who approximate them correctly. From now on, we assume that a fraction $w_1 \geq 0$ correctly estimates the price ($\lambda = 1$), a fraction $w_0 \geq 0$ does not mind about prices ($\lambda = 0$), and a fraction $w_\infty \geq 0$ drasticaly overestimates the price ($\lambda = +\infty$), with $w_0 + w_1 + w_\infty = 1$. This trichotomy is obviously a caricature, but it does illustrate some realistic characteristics of electricity consumption behaviour.: i) $\lambda = 0$ represents households and small business operators who switch their electrical appliances on and off without the slightest idea of what the price might be; ii) by contrast, $\lambda = 1$ stands for industrial consumers and large service sector companies (e.g. hospitals and supermarkets); these often are already equipped with smart devices, some have interruptible contracts and/or intervene on the wholesale power market; iii) finally, $\lambda = +\infty$ evokes a small but increasing fraction of consumers who prefer not to buy from the grid; these off-grid people rely on other sources of energy and use their own electrical equipment as prosumers or belong to energy communities. This division into three groups could be refined but it is justified by the cost of screening a large variety of unobservable behaviors. With the assumptions of an identical quadratic surplus for all consumers: $S(q) = (a - q/2)q$, the individual demand for energy in each of the three groups is $q_0 = aw_0, q_1 = (a - p)w_1$, and $q_\infty = 0$ repectively. Notice that this breakdown also allows to contrast the welfare maximization objective that would consist in increasing the number of perfectly rational consumers ($\Delta w_0 < 0, \Delta w_1 > 0, \Delta w_\infty < 0$) and the energy saving objective aimed at decreasing not only the number of consumers in group 0 but also those pertaining to group 1. ($\Delta w_0 < 0, \Delta w_1 < 0, \Delta w_\infty > 0$). In both cases, decreasing the number of type-0 consumers who underestimate the electricity prices is a priority.

5.2 The energy market without assistance

Before an assistance is provided, the three groups count w_0, w_1 and w_∞ consumers respectively. Given the presence of inelastic consumers, it is necessary to introduce a price cap. With the quadratic surplus function, the natural candidate is a. Then total demand is $Q(p) = aw_0 + (a - p)w_1$ if $p < a$ and $Q(p) = aw_0$ for $p = a$. Fixing $p > a$ is forbidden.

The cost of energy c is random. It is distributed on the support $[c_L, c_H]$ according to the cfd $G(c)$. The expectation is $\bar{c}$ and the variance σ_c^2. We will write $e^2 = \bar{c}^2 + \sigma_c^2 = I\!\!E(c^2)$ and we assume $a > c_H$.[30]

Given the price cap and the direct demand function defined above, the inverse demand is

$$p(Q) = \min\{a + \frac{1}{w_1}(aw_0 - Q), a\}.$$

Assume that n identical firms compete in quantities. In the following lemmas we compute the electricity price, consumption and surplus at the Cournot equilibria, restricting attention to pure equilibria. The proofs are in the Appendix.

Lemma 5.1. ***Case A:*** *In states of nature where $c < a(1 - \frac{w_0}{nw_1})$, there exists a unique Cournot equilibrium where each firm $i = 1, ..., n$ produces $q_i^C = \frac{1}{n+1}(a(w_0 + w_1) - cw_1)$. Groups 0 and 1 are supplied. Their total consumption and the unit price are:*

$$Q_{01}^C = \frac{n}{n+1}(a(w_0 + w_1) - cw_1), \quad p_{01}^C = \frac{1}{w_1(n+1)}(a(w_0 + w_1) + cnw_1). \tag{24}$$

The after-billing net surplus of a type-0 consumer is

$$NS_0 = -\frac{a}{2}\left(2c\frac{n}{n+1} + a(\frac{w_1(1-n) + 2w_0}{(n+1)w_1})\right) \lesseqgtr 0. \tag{25}$$

The after-billing net surplus of a type-1 consumer is

$$NS_1 = \frac{1}{2(n+1)^2}\left(a(n - \frac{w_0}{w_1}) - cn\right)^2,$$

and the net surplus of a consumer with type ∞ is $NS_\infty = 0$ since he does not consume (from the grid) and receives no bill.

Notice that p_{01}^C increases with the ratio w_0/w_1, since when a large fraction of the population is willing to buy the maximal quantity at any price then the equilibrium price is high. The equilibrium price p_{01}^C naturally decreases with the number of firms n (recall that $a > c$). It is also interesting to observe in (25) that $NS_0 < 0$ when energy is supplied by a monopoly ($n = 1$) and it can become positive when n is large enough. By contrast, rational consumers have a positive net surplus ($NS_1 > 0$) whatever the market structure.

Case A corresponds to states of the world where the current electricity cost c is small, so that both types 0 and 1 consume electricity. We now consider the complementary case B where the cost of electricity is so high that only type 0-consumers do buy a positive quantity.

[30] During energy crises (e.g. war, drought, very cold weather), the unit cost of energy c can be very high. However, in developped countries, the dependency to electricity is so strong that the willingness to pay a is still higher.

Lemma 5.2. ***Case B:*** *In states of nature where $c \geq a(1-\frac{w_0}{nw_1})$, at every Cournot equilibrium the price is $p_0^C = a$ and the total consumption $Q_0^C = aw_0$. Only type-0 consumers buy electricity. Their after billing net surplus is $NS_0 = -a^2/2 < 0$, and the net surplus of other types is nil: $NS_1 = NS_\infty = 0$.*

The equilibria are exactly the profiles where the total production is aw_0 and where each firm produces at least the quantity $w_1(a-c)$. There exists a unique symmetric Cournot equilibrium where each firm i produces $q_i = \frac{aw_0}{n}$.

From the two lemmas we can derive the average values of consumption and surplus.

Proposition 5.3. ***When n firms compete in quantities on the electricity market, the average consumption at each equilibrium is:***

$$\mathbb{E}\mathrm{Q}^{\mathrm{C}} = \int_{\mathrm{c_L}}^{\mathrm{a}(1-\frac{\mathrm{w_0}}{\mathrm{nw_1}})} \frac{\mathrm{n}}{\mathrm{n}+1}(\mathrm{a}(\mathrm{w_0}+\mathrm{w_1}) - \mathrm{cw_1})\mathrm{dG}(\mathrm{c}) + \int_{\mathrm{a}(1-\frac{\mathrm{w_0}}{\mathrm{nw_1}})}^{\mathrm{c_H}} \mathrm{aw_0dG}(\mathrm{c}).$$

The average surplus of a consumer of type 0 is

$$\mathbb{E}NS_0 = \int_{c_L}^{a(1-\frac{w_0}{nw_1})} -\frac{a}{2}\left(2c\frac{n}{n+1} + a(\frac{w_1(1-n)+2w_0}{(n+1)w_1})\right)dG(c) - \int_{a(1-\frac{w_0}{nw_1})}^{c_H} \frac{a^2}{2}dG(c), \tag{26}$$

the average surplus of a type-1 consumer is

$$\mathbb{E}NS_1 = \int_{\mathrm{c_L}}^{\mathrm{a}(1-\frac{\mathrm{w_0}}{\mathrm{nw_1}})} \frac{1}{2(\mathrm{n}+1)^2}\left(\mathrm{a}(\mathrm{n}-\frac{\mathrm{w_0}}{\mathrm{w_1}}) - \mathrm{cn}\right)^2 \mathrm{dG}(\mathrm{c}),$$

and the average surplus of a type-∞ consumer is $\mathbb{E}NS_\infty = 0$.

Unsurprisingly, we have that $\mathbb{E}(NS_1) > \mathbb{E}(NS_0)$ and $\mathbb{E}(NS_1) > \mathbb{E}(NS_\infty)$. Indeed, behaving rationally gives a better outcome than the other behaviors. One can check that w_0 and n being fixed, $\mathbb{E}(NS_0)$ and $\mathbb{E}(NS_1)$ are increasing in w_1, or equivalently decreasing in w_∞. The reason is that the more numerous type-1 consumers, the more adapted to their profile the bids of the sellers: energy price is lower and consumption is higher on average. This obviously benefits type 0.

In the particular case where $w_0 = w_\infty = 0$ and $w_1 = 1$, since $a > c_H$ we have that $\mathbb{E}Q^C = \frac{n}{n+1}(a - \bar{c})$.

If $n = +\infty$ (perfect competition on the electricity market), only case A applies and we get: $\mathbb{E}Q^C = aw_0 + (a-\bar{c})w_1$, $\mathbb{E}(NS_0) = \frac{a}{2}(a-2\bar{c})$, $\mathbb{E}(NS_1) = \frac{1}{2}\mathbb{E}(a-c)^2$ and $\mathbb{E}(NS_\infty) = 0$.

6 The provision of assistance

Assume now an assistance (to switch to type $\lambda = 1$) is sold at price $p_s > 0$. If some consumers buy the assistance, the distribution of types evolves from $w = (w_0, w_1, w_\infty)$ to a new distribution $w(p_s) = (w_0(p_s), w_1(p_s), w_\infty(p_s))$. In all the following, we assume that the consumers do not anticipate the future changes of energy prices due to the assistance sales.[31]

A consumer with type $\lambda = 1$ will never buy the assistance, a consumer with type $\lambda = 0$ will buy the assistance if and only if $\mathbb{E}(NS_1) - p_s \geq \mathbb{E}(NS_0)$, and a consumer with type $\lambda = \infty$ will buy the assistance if and only if $\mathbb{E}(NS_1) - p_s \geq \mathbb{E}(NS_\infty) = 0$.

In (26), it is important to notice that $\mathbb{E}(NS_0)$ can be greater or lower than 0, leading to two very different cases. These consumers always consume the maximum quantity of electricity regardless of the prices. Therefore, high energy prices may end out with a negative after-billing surplus ($\mathbb{E}(NS_0) < 0$). In this case $\mathbb{E}(NS_0) < \mathbb{E}(NS_\infty)$, which implies that if the assistance price p_s is such that type-∞ consumers buy the service, then type-0 consumers also buy it. On the contrary, when the electricity prices are low enough so that $\mathbb{E}(NS_0) > 0$, the average net surplus after billing of the type-0 consumers is positive. In this case $\mathbb{E}(NS_0) > \mathbb{E}(NS_\infty)$, which implies that if the price p_s is such that type-0 consumers buy the assistance, then type-∞ consumers also buy it.

Given the objective to decrease the total average consumption of electricity, the best policy is to provide assistance to type-0 consumers (who consume too much) and not to type-∞ consumers (who consume too little), but this may be impossible in an assistance market where all consumers are offered the same price. We will first determine the optimal provision of assistance in a market where all consumers are offered to buy an assistance (to switch to type $\lambda = 1$) at a unique price p_s and then we discuss the possibility to implement it depending on the number of operators. In this section, we present an overview of the results. The details of the analysis are in Appendix 8.3.

6.1 Optimal Provision of assistance

Selling the assistance only to type-∞ consumers will increase the total consumption of electricity, since it increases the consumption of each type of consumers as explained at the end of subsection 5.2 (recall that $\mathbb{E}(NS_0)$ and $\mathbb{E}(NS_1)$ are increasing in w_1). On the other hand, it is not obvious that selling the assist-

[31] In (23) the number of consumers who buy the assistance service depends on the distribution of energy prices by $\sigma^2 + E^2 = \mathbb{E}(p^2)$. However, this is the distribution expected by consumers at the time they must make a decision as regards the assistance service. Different assumptions can be used to adress this question. We could assume that consumers have some degree of rationality to forecast how the future energy prices can influence the pricing policy of assistance operators. However, since from the very beginning we have been assuming that consumers have a biased view of the wholesale price, it is more reasonable to consider that they are myopic as regards this relationship.

ance to type 0 consumers will lead to a reduction of the total consumption of electricity. First, because if $\mathbb{E}(NS_0) > 0$ it is not possible to sell the assistance service to type 0 without selling it to type ∞, and these type-∞ consumers will start to consume positive quantities. Second, if w_0 is high enough so that $a(1 - \frac{w_0}{nw_1}) < c_H$, then $\mathbb{E}(NS_1) = 0$ (there are so many type 0-consumers that the price of electricity is high ($p_0^C = a$) and type 1 consumers are excluded from the market). Consequently, selling assistance to type-0 consumers will decrease the consumption of this type of consumers, but will induce a new repartition of types implying that type 1 consumers will now consume a positive quantity of electricity.

To identify the different cases, we define $\bar{Q}_1 = \frac{n}{n+1}(a - \bar{c})$ as the average consumption when there are only type-1 agents, and $\bar{Q}_3 = \int_{c_L}^{a(1-\frac{w_0}{nw_1})} \frac{n}{n+1}(a(w_0 + w_1) - cw_1)dG(c) + \int_{a(1-\frac{w_0}{nw_1})}^{c_H} aw_0 dG(c)$ the average consumption corresponding to the initial distribution of types (we will see in subsection 8.3.1 how these equilibrium quantities are determined). We can establish that:

case 1) if $\mathbb{E}(NS_0) > 0$ and $\bar{Q}_3 > \bar{Q}_1$, it is optimal to sell the assistance to both types 0 and $+\infty$.

case 2) if $\mathbb{E}(NS_0) > 0$ and $\bar{Q}_3 < \bar{Q}_1$, it is optimal to sell no assistance.

case 3) if $\mathbb{E}(NS_0) < 0$, it is optimal to sell the assistance to type 0-consumers only.

When $c_H \leq a(1-\frac{w_0}{nw_1})$, we can write $\mathbb{E}(NS_0) = -\frac{a}{2}\left(2\bar{c}\frac{n}{n+1} + a(\frac{w_1(1-n)+2w_0}{(n+1)w_1})\right)$, $\mathbb{E}(NS_1) = \frac{1}{2(n+1)^2}\mathbb{E}\left(a(n - \frac{w_0}{w_1}) - cn\right)^2$, $\bar{Q}_3 = \frac{n}{n+1}(a(w_0 + w_1) - \bar{c}w_1)$, and the condition $\bar{Q}_3 > \bar{Q}_1$ is equivalent to:

$$\bar{c}w_0 > (a - \bar{c})w_\infty. \tag{27}$$

We see that the optimal provision of assistance aimed at decreasing the consumption of energy is dependent on the statistical distribution of consumers' types and the energy cost, but also on the structure of the energy market. For example, if $n = +\infty$ (perfect competition on the electricity market) and $a > 2\bar{c}$, then $\mathbb{E}(NS_0) = \frac{a}{2}(a - 2\bar{c}) > 0$, $\mathbb{E}(NS_1) = \frac{1}{2}\mathbb{E}(a - c)^2 > 0$ and either case 1) or case 2) applies depending on whether (27) holds or not. If $\frac{w_0}{w_\infty}$ is sufficiently large, providing assistance to the whole society optimally decreases the total electricity consumption. By contrast, if $n = 1$ (monopoly on the electricity market), $\mathbb{E}(NS_0) < 0$ by (26), so that case 3) applies.

Recall that it is never optimal to sell the assistance to type ∞-consumers only. If we are in case 3, the best pricing policy for assistance would be to discriminate among type 0 and type ∞. In the following, we assume that price discrimination is impossible for legal or technical reasons.

6.2 Implementation with an assistance market

Which market structure for assistance is compatible with the three optimal sales.of cases 1), 2), and 3)? We consider a Cournot oligopoly with m identical firms with unit cost c_s on the assistance market, disjoint from the n firms that compete à la Cournot on the electricity market.

1) Consider case 1) where $\mathbb{E}(NS_0) > 0$ and $\bar{Q}_3 > \bar{Q}_1$ We know that it is optimal to sell the assistance to both types 0 and $+\infty$. We show in proposition 8.2 that at Cournot equilibrium the assistance is sold optimally if and only if :

$$c_s \leq \mathbb{E}(NS_1) - \mathbb{E}(NS_0) \text{ and } (w_0 + \frac{w_\infty}{m})\mathbb{E}(NS_0) < w_0(\mathbb{E}(NS_1) - c_s).$$

We obtain in particular that if $n = +\infty$, $a > 2\bar{c}$ and $\bar{c}w_0 > (a - \bar{c})w_\infty$, then the assistance is sold optimally if and only if:

$$c_s \leq \frac{1}{2}e^2 \text{ and } \frac{w_\infty}{m}a(a - 2\bar{c}) + 2w_0 c_s < w_0 e^2. \tag{28}$$

Recall that $e^2 = \bar{c}^2 + \sigma_c^2 = \mathbb{E}(c^2)$. This holds in particular when c_s is small and m is large enough. In words, when $\mathbb{E}(NS_0) > 0$ because downstream competition lowers electricity prices, selling assistance to both types of consumers necessitates a very low assistance price, that is a low cost c_s and fierce upstream competition (large m).

2) Consider now case 2) where $\mathbb{E}(NS_0) > 0$ and $\bar{Q}_3 < \bar{Q}_1$, which holds in particular when $n = +\infty$, $a > 2\bar{c}$ and $\bar{c}w_0 < (a - \bar{c})w_\infty$. Optimality commands to provide no assistance (given that price discrimination is impossible). We show in proposition 8.2 that there exists a Cournot equilibrium without sales on the assistance market if and only if ($c_s \geq \mathbb{E}(NS_1)$ or ($w_\infty = 0$ and $c_s \geq \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$)). So if c_s is small enough, each Cournot equilibrium of the assistance market will lead to an inefficient strict increase of the electricity consumption. This holds irrespective of the value of m.

3) Consider case 3) where $\mathbb{E}(NS_0) < 0$ so that it is optimal to sell the assistance to type 0-consumers only. Having a small number of firms m or a cost c_s not so small will lead to a high price of assistance, which will help the market to reach the optimal outcome. In Propositions 8.7 and 8.8 we obtain the following:

* if $c_s > \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$, there are no sales at equilibrium.

* if $\mathbb{E}(NS_1) < c_s < \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$, the unique Cournot equilibrium on the assistance market will lead to the optimal decrease of average electricity consumption (irrespective of the value of m).

* if $c_s \leq \mathbb{E}(NS_1)$ and m is large enough, at the unique Cournot equilibrium both types 0 and ∞ will buy the assistance, which is sub-optimal given the objective of a decrease in electricity consumption.

* if $c_s \leq \mathbb{E}(NS_1)$, the unique Cournot equilibrium induces the optimal decrease of average electricity consumption if and only if:

$$-w_0\mathbb{E}(NS_0) > w_\infty(m\mathbb{E}(NS_1) - (m-1)E(NS_0) - mc_s).$$

This is easier if m is small: decreasing m increases the market price of the assistance and helps to keep away type ∞-consumers from the electricity market. In particular when $m = 1$, the above inequality reduces to $-w_0\mathbb{E}(NS_0) > w_\infty(\mathbb{E}(NS_1) - c_s)$.

This applies in particular when $n = 1$ (monopoly on the electricty market). Indeed, when $n = 1$, if $c_H \leq a(1 - \frac{w_0}{w_1})$, the formulas simplify to $\mathbb{E}(NS_0) = -\frac{a}{2}(\bar{c} + a\frac{w_0}{w_1})$ and $\mathbb{E}(NS_1) = \frac{1}{8}\mathbb{E}(a(1 - \frac{w_0}{w_1}) - c)^2$. We also show that when $n = m = 1$ and costs are nil $c = c_s = 0$, the unique Cournot equilibrium induces the optimal decrease of average electricity consumption if and only if:

$$w_0 \geq w_1 \text{ or } (w_0 < w_1 \text{ and } 4w_0^2 w_1 > w_\infty(w_1 - w_0)^2).$$

4) We finally consider the case of an integrated monopoly.

There is a single firm which sells both the assistance and the electricity. We have $n = 1$ so that $\mathbb{E}(NS_0) < 0$ and it is optimal to sell the assistance to type 0-consumers only. On the one hand the integrated monopoly is in competition with itself and this may decrease the sales of assistance. On the other hand, the integrated monopoly might be able to sell assistance at a loss to attract consumers and increase its profit on the electricity market.

We show that the optimal sales can be achieved with a profit maximizing integrated monopoly if and only if $f_2 \geq \max\{f_1, f_3\}$, where f_1, f_2 and f_3 are defined in section 8.3.3.

We prove that when this condition $f_2 \geq \max\{f_1, f_3\}$ holds, then $c_s < \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$ (no assistance is sold at a loss) and $-w_0\mathbb{E}(NS_0) > w_\infty(\mathbb{E}(NS_1) - c_s)$. This implies that having two distinct monopolies (as we saw in case 3) with $n = m = 1$) would also lead to optimal sales. It is then more demanding to achieve the optimal sales with an integrated monopoly (where the assistance department is in competition with the energy department). We finally show that when costs are nil $c = c_s = 0$ and $0 < w_0 < w_1$, an integrated monopoly does not induce the optimal decrease of electricity consumption.

6.3 In a nutshell

The sign of the quantity $\mathbb{E}(NS_0)$ plays an important role in the decision to organize an assistance market.

If $\mathbb{E}(NS_0) > 0$ (in particular when there is perfect competition on the electricity market and $a \geq 2\bar{c}$), $c_H \leq a(1 - \frac{w_0}{nw_1})$ and $\bar{c}w_0 > (a - \bar{c})w_\infty$, to decrease the total electricity consumption, the assistance market should aim at providing assistance to both types 0 and ∞-consumers. This can be done with an independent assistance oligopoly with a large number of firms and small assistance costs. If $\mathbb{E}(NS_0) > 0$, $c_H \leq a(1 - \frac{w_0}{nw_1})$ and $\bar{c}w_0 < (a - \bar{c})w_\infty$, no independent assistance market can lead to a decrease of the total electricity consumption.

If $\mathbb{E}(NS_0) < 0$, to decrease the total electricity consumption the assistance market should aim at providing assistance to type 0 consumers only. When the cost of assistance is small, this is possible if and only if m is small enough so that $-w_0\mathbb{E}(NS_0) > w_\infty(m\mathbb{E}(NS_1) - (m-1)E(NS_0) - mc_s)$, which implies in particular $-w_0\mathbb{E}(NS_0) > w_\infty(\mathbb{E}(NS_1) - c_s)$.

It is more difficult to induce the optimal provision of assistance with an integrated monopoly than with two independent monopolies.

7 Conclusion

The transition to a carbon-free economy necessitates not only drastic changes in the supply of energy but also an adaptation of consumption patterns to the intermittence of green sources of electricity production. Time dependent prices for electricity are an efficient tool when consumers are able to adapt their behavior to these scarcity signals. Actually, except for large business and industrial customers who do react to variations of wholesale prices[32], most consumers behave routinely, because they ignore the true value of prices and/or they cannot adapt their energy withdrawals in real time. In the near future, we can expect more from "behind the meter" appliances to make electricity demand more flexible, in particular for charging electrified vehicles (Andrey and Hautie, 2013).

In this paper, we have considered that consumers have a biased knowledge on the electricity prices but they can pay service providers to obtain the exact information on prices The strategy of assistance supplier(s) obviously vary with the statistical distribution of the price misperception among consumers. We have contrasted several market structures and emphasized the opportunistic behavior of a private monopoly selling both energy and an assistance service aimed at transmitting exact price signals to individual consumers. The firm has an incentive to sell an assistance service only if it allows to increase its sales of energy. Actually, this opportunism will be challenged both by software developpers, such as the so-called GAFAM, as they do not supply energy, and by competition authorities that will not accept abuses of dominance. The development of these assistance services creates new problems. Current digital technologies rely heavily on the use of personal data (Crampes and Lefouili, 2021). Additionally, Artificial Intelligence (Lee et al., 2022) reduces the consumer's freedom of choice. Both raise privacy concerns that hinder their social acceptability

[32]Even large consumers are far from being fully reactive to scarcity signals. For example in CRE (2023), the French Regulator for Energy notes that "*commercial buildings represent an untapped reservoir of efficiency and flexibility: today, only 6% of commercial buildings over 1,000 m^2 are equipped with an energy management system, and the buildings that are equipped do not systematically use the potential of these systems. Few have an electricity supply that is differentiated according to time of year, encouraging people to modulate their consumption to avoid peaks.*

The BACS (Building Automation & Control Systems) decree, published in 2020 and reinforced in 2023, established a regulatory framework for the deployment of energy management solutions in commercial buildings larger than 1,000 m2, although implementation to date seems limited."

The model can be extended in various directions. On the technology side, instead of a drastic assistance that allows to become perfectly informed on the energy prices, we can imagine that the level of assistance is chosen endogeneously by each type of consumer. One can also consider other market structures such as a mix of vertically integrated firms and independent firms If the firms can identify the consumer's type, we can also have an assistance oligopoly with some firms targeting consumers with large λ and others targeting low λ. Another extension is that, instead of considering that consumers can choose between dynamic contracts and flat price contracts as we did in section 4.1 and the case where they have to choose between being or not being assisted for dynamic contracts developped all along the paper, we can imagine that consumers face the three options, assistance being also proposed by retailers. For this more complex choice, consumers need an additional layer of rationality.

Finally, note that we have focused on the energy savings target, which is missed because many consumers underestimate the electricity price. In fact, combating energy poverty is another important aspect of public policy. In the United States, 16% of households experience energy poverty defined as spending more than 6% of household income on energy expenditures (Scheier and Kittner, 2022). In addition to low income and the poor state of housing and electrical equipment, there are behavioural reasons for this excessive expenditure on energy which penalises households with a low level of education, unable to make rational decisions. It is not certain that price reading aids would solve this problem. The remote direct control of the quantities consumed, thanks in particular to AI, would probably be a more effective tool, in line with the superiority of the centralized control of consumption reductions established in the field experiment of Bailey et al. (2024).

References

[1] Allcott H. (2011), "Rethinking real-time electricity pricing", Resource and Energy Economics 33, 820–842

[2] Ambec S. and C. Crampes (2021), "Real-time electricity pricing to balance green energy intermittency", Energy Economics, vol. 94, n° 105074, February.

[3] Andor M., A. Gerster, and S. Sommer (2017), "Consumer Inattention, Heuristic Thinking and the Role of Energy Labels", RUHR Economic Papers, n° 671, http://dx.doi.org/10.4419/86788778

[4] Andrey C. and A. Haurie (2013), "The Economics of Electricity Dynamic Pricing and Demand Response Programmes. Application to Controlling BEVs and PHEVs Charging and Storage", April, (6)

[5] Astier N. and T-O. Léautier (2021), "Demand Response: Smart Market Designs for Smart Consumers", The Energy Journal, Vol. 42, No. 3, 153-175

[6] Aumann, R. (1985), "What is game theory trying to accomplish?" in K. Arrow and S. Honkapohja, eds, "Frontiers in economics", Basil Blackwell, 28–76.

[7] Auriol E., C. Crampes and A. Estache (2021), "Regulating Public Services. Bridging the Gap between Theory and Practice", Cambridge University Press, October.

[8] Bach C., R. Edwards and C. Jaag (2022), "Postal Platform Pricing with Limited Consumer Attention", April, 11th bi-annual Postal Economics Conference on E-commerce, Digital Economy and Delivery Services, Toulouse School of Economics.

[9] Bailey M., D. P. Brown, B. Shaffer, and F.A. Wolak (2024), "Centralized vs Decentralized Demand Response: Evidence from a Field Experiment", http://web.stanford.edu/group/fwolak/cgi-bin/sites/default/files/Centralized_vs_Decentralized_paper-6-5-2024.pdf

[10] Blonz J., K. Palmer, C.J. Wichman, and D.C. Wietelman (2021), "Smart Thermostats, Automation, and Time-Varying Prices", Resources for the Future, Working Paper 21-20, July, https://media.rff.org/documents/WP_21-20.pdf

[11] Boiteux M. (1949), "La tarification des demandes en pointe", Revue Générale de I'Electricité, 58, 321-340, translated as "Peak-load pricing" in Nelson J. R. (ed.) (1964) Marginal cost pricing in practice, pp. 59- 89, Engelwood Cliffs, NJ: Prentice Hall.

[12] Bollinger B.K and W.R. Hartmann (2018), "Information versus Automation and Implications for Dynamic Pricing", September, http://web.stanford.edu/~wesleyr/AutomationElasticities.pdf

[13] Boom, A and S. Schwenen (2021), "Is real-time pricing smart for consumers?". J Regul Econ 60, 193–213, https://doi.org/10.1007/s11149-021-09440-5

[14] Borenstein, S. (2005), "Time-Varying Retail Electricity Prices: Theory and Practice" in J.M. Griffin and S.L. Puller (Eds), "Electricity Deregulation: Choices and Challenges", U of Chicago.

[15] Brunette M. and J. Jacob (2019), "Risk aversion, prudence and temperance: an experiment in gain and loss", Research in Economics, Elsevier, 73 (2), pp.174-189.

[16] Cahana M., N. Fabra, M. Reguant and J. Wang (2022), "The Distributional Impacts of Real-Time Pricing", Centre for Economic Policy Research, April, Discussion Paper DP17200.

[17] Cerre (2022), "Retail energy markets under stress. Lessons learnt for the future of market design", Issue Paper, July, https://cerre.eu/wp-content/uploads/2022/07/220713_CERRE_Issue-Paper_Retail-Energy-Markets-Under-Stress.pdf

[18] Christie S.T. and P. Schrater (2015), "Cognitive cost as dynamic allocation of energetic resources", Front Neurosci. 9: 289, doi: 10.3389/fnins.2015.00289

[19] Crampes C, N. Ladoux and J.M. Lozachmeur (2023), "Pricing energy consumption and residential energy-efficiency investment: An optimal tax approach", Journal of Public Economic Theory, June, https://doi.org/10.1111/jpet.12655

[20] Crampes C. and Y. Lefouili (2021), "Green Energy Pricing for Digital Europe", Enjeux numériques - Annales des Mines, n° 15, September.

[21] Crampes C. and M. Laffont (2016), "Retail Price Regulation in the British Energy Industry", Competition and Regulation in Network Industries, vol. 17, n° 3-4.

[22] Crampes C. and J. Renault (2019), "How many markets for wholesale electricity when supply is partially flexible?", Energy Economics, vol. 81, June, p. 465–478.

[23] Crampes C. and J. Renault (2021), "Flexibility and Risk Transfer in Electricity Markets", TSE Working Papers, November.

[24] CRE (2023), "Rapport de la Commission de régulation de l'énergie sur le pilotage des bâtiments tertiaires, September, https://www.cre.fr/content/download/27756/file/230911_Rapport_Pilotage_Batiments_Tertiaires.pdf

[25] European Commission (2021), "Communication on tackling rising energy prices: a toolbox for action and support", COM(2021) 660 final

[26] European Commission (2022a), "Communication on REPowerEU: Joint European Action for more affordable, secure and sustainable energy", COM(2022) 108 final

[27] European Commission (2022b), "Security of supply and affordable energy prices: Options for immediate measures and preparing for next winter", COM(2022) 138 final.

[28] European Parliament (2023), "Directive 2023/1791 on energy efficiency and amending Regulation (EU) 2023/955 (recast)", 13 September, https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32023L1791.

[29] Fabra N., D. Rapson, M. Reguant, and J. Wang (2021), "Estimating the Elasticity to Real Time Pricing: Evidence from the Spanish Electricity Market", American Economic Association Papers & Proceedings, 111, 425-29

[30] Festré A. and P. Garrouste (2015), "The 'Economics of Attention': A History of Economic Thought Perspective", OEconomia. https://journals.openedition.org/oeconomia/1139

[31] Fowlie M., C. Wolfram, C. A. Spurlock, A. Todd-Blick, P. Baylis and P. Cappers (2017), "Default effects and follow-on behavior: evidence from an electricity pricing program", NBER WP 23553, DOI 10.3386/w23553, June.

[32] Frederiks E.R., K. Stenner and E.V. Hobman (2015), "Household energy use: Applying behavioural economics to understand consumer decision-making and behaviour" Renewable and Sustainable Energy Reviews, 41, 1385–1394.

[33] Gillan, J.M. (2017), "Dynamic Pricing, Attention, and Automation: Evidence from a Field Experiment in Electricity Consumption", https://are.berkeley.edu/sites/are.berkeley.edu/files/job-candidates/paper/JMP_Gillan_most_recent_draft.pdf

[34] Harding, M. and S. Sexton (2017), "Household Response to Time-Varying Electricity Prices", Annual Review of Resource Economics, October, Vol. 9, p. 337-359

[35] Harold J.,V. Bertsch and H. Fell (2021), "Preferences for curtailable electricity contracts: Can curtailment benefit consumers and the electricity system?", Energy Economics, Volume 102, October 2021, 105454

[36] Hirth L. (2017), "The European Electricity Market Model EMMA. Model documentation, Version 2017-07-12, http://neon-energie.de/emma

[37] Ito K. (2014). "Do consumers respond to marginal or average price? Evidence from nonlinear electricity pricing. American Economic Review, 104 537–563.

[38] Ito K., T Ida, and M. Tanaka (2023), "Selection on Welfare Gains: Experimental Evidence from Electricity Plan Choice", American Economic Review, 113(11): 2937–2973, https://doi.org/10.1257/aer.202101502937

[39] Jessoe K. and D. Rapson (2014), "Knowledge is (Less) Power: Experimental Evidence from Residential Energy Use", American Economic Review, vol. 104, n°4, April, 1417-38.

[40] Joskow P, and J. Tirole (2006), "Retail Electricity Competition", The RAND Journal of Economics, Vol. 37, No. 4, pp. 799-815

[41] Joskow P. and C.D. Wolfram (2012), "Dynamic Pricing of Electricity", American Economic Review, May, Vol. 102, No. 3, 381-385

[42] Kahn A.E. (1970), "The Economics of Regulation: Principles and Institutions", Volume 1. New York: John Wiley & Sons

[43] Kahn-Lang J. (2022), "Competing for (In)attention: Price Discrimination in Residential Electricity Markets", WP 333, November, Energy Institute at Haas.

[44] Kahneman K. (2003), "Maps of Bounded Rationality: Psychology for Behavioral Economics", The American Economic Review, 93(5), pp. 1449-1475, December

[45] Kahneman D. and A. Tversky (1992), "Advances in prospect theory: Cumulative representation of uncertainty". Journal of Risk and Uncertainty. 5 (4): 297–323

[46] Kimball M. S. (1990), "Precautionary Saving in the Small and in the Large", Econometrica, Vol. 58, pp. 53-73.

[47] Kimball M.S. (1992), "Precautionary motives for holding assets" in P. Newman, M. Milgate and J. Falwell (eds), The new palgrave dictionary of money and finance. MacMillan, London.

[48] Köszegi B. (2013),. "Behavioral contract theory", Journal of Economic Literature, 52(4): 1075-1118.

[49] Lacetera N., D. Pope, and J. Sydnor (2012), “Heuristic Thinking and Limited Attention in the Car Market”, American Economic Review, Vol. 102, No. 5, pp. 2206-2236

[50] Léautier T.-O (2018),"Imperfect Markets and Imperfect Regulation. An Introduction to the Microeconomics and Political Economy of Power Markets", MIT Press.

[51] Lee Y-H. and U. Malmendier (2011), "The Bidder's Curse", American Economic Review 101 (2): 749–787.

[52] Lee D., Y-T. Chen and S-L Chao (2022), "Universal workflow of artificial intelligence for energy saving", Energy Reports 8, 1602–1633.

[53] Littlechild, S.(2021), "The evolution of competitive retail electricity markets", p. 111-155 in J. M Glachant, P. L. Joskow and M. G. Pollitt (2021) "Handbook on Electricity Markets", Edward Elgar Oublishing, Cheltenham, UK.

[54] Martimort, D and Stole L.A (2020), "Nonlinear Pricing with Average-Price Bias", American Economic Review: Insights", vol. 2, n°3, September, 375-96.

[55] Mayol, A. and C. Staropoli (2021), "Giving consumers too many choices: a false good idea? A lab experiment on water and electricity tariffs", European Journal of Law and Economics 51 (2), pp.383-410.

[56] Ofgem (2011), "What can behavioural economics say about GB energy consumers", 21 March, https://www..gov.uk/sites/default/files/docs/2011/03/behavioural_economics_gbenergy_1.pdf

[57] Pollitt M.G. and I. Shaorshadze (2011), "The Role of Behavioural Economics in Energy and Climate Policy", EPRG Working Paper 1130, Cambridge Working Paper in Economics 1165

[58] Rosenow J. and E. Bayer (2017), "Costs and benefits of Energy Efficiency Obligations: A review of European programmes", August, Energy Policy 107:53-62.

[59] Scheier E. and N. Kittner (2022), "A measurement strategy to address disparities across household energy burdens", Nat Commun 13, 288. https://doi.org/10.1038/s41467-021-27673-y

[60] Schneider I. and C.R. Sunstein (2017), "Behavioral Considerations for Effective Time-Varying Electricity Prices, October", Behavioral Public Policy, Volume 1 , Issue 2 , November.

[61] Simon H. (1982), "Models of bounded rationality: Behavioral economics and business organization (Vol. 1 and 2), The MIT Press

[62] Stango V. and J. Zinman (2014), "Limited and Varying Consumer Attention: Evidence from Shocks to the Salience of Bank Overdraft Fees", Review of Financial Studies, vol. 27, issue 4, 990-1030

[63] Thaler R. H. (1994), "Quasi rational economics", Russell Sage Foundation

[64] Tversky A. and D. Kahneman (1981) "The Framing of Decisions and the Psychology of Choice", Science, vol. 211, 30 January, 453-458

[65] Verhagen E., L. Rook, J. van Dalen and W. Ketter (2012), “The Impact of Framing on Consumer Selection of Energy Tariffs”, December, International Conference on Smart Grid Technology, Economics and Policies (SG-TEP), doi: 10.1109/SG-TEP.2012.6642391

[66] Wilson C.M. and C. Waddams Price (2010), "Do Consumers Switch to the Best Supplier?", Oxford Economic Papers, New Series, vol. 62, no. 4, October, 647-668.

8 Appendix

8.1 Proof of Lemma 5.1

We assume $c < a(1 - \frac{w_0}{nw_1})$. Suppose we have an equilibrium $(q_1, ..., q_n)$ with price $p < a$ and total consumption $Q = q_1 + ... + q_n$. Since $p < a$, we have $p = a + \frac{1}{w_1}(aw_0 - Q)$.

The equilibrium profit of a firm $i = 1, .., n$ is $\pi_i(q_i, q_{-i}) = (p - c)q_i$. Considering deviations of firm i to q'_i such that the new price $p' = p'(q'_i, q_{-i})$ is still strictly smaller than a, we must have $\frac{\partial \pi_i(q_i, q_{-i})}{\partial q_i} = 0$, so $(a + \frac{1}{w_1}(aw_0 - Q) - c) - \frac{q_i}{w_1} = 0$, and then $q_i + Q = a(w_0 + w_1) - cw_1$. Consequently, q_i does not depend on i and $q_i = \frac{1}{n+1}(a(w_0 + w_1) - cw_1)$. The total production and the equilibrium price are $Q = \frac{n}{n+1}(aw_0 + (a-c)w_1)$ and $p = \frac{1}{n+1}(a\frac{(w_0+w_1)}{w_1} + nc) = \frac{1}{n+1}(a + a\frac{w_0}{w_1} + nc)$. Notice that $p < a$ by the assumption $c < a(1 - \frac{w_0}{nw_1})$.

To check we indeed obtained an equilibrium, it remains to consider deviations of a firm i from q_i to $q'_i = q_i - \varepsilon$ inducing a total production so small that the new price p' is a. Then $\varepsilon > 0$. If this deviation was profitable, it would also be profitable in the game without a price cap at a, and this is not possible since π is a concave polynomial in q_i and $\frac{\partial \pi(q_i, q_{-i})}{\partial q_i} = 0$.

So we have obtained an equilibrium where $q_i = \frac{1}{n+1}(a(w_0 + w_1) - cw_1)$ for each i.

To check that there is no other equilibrium, assume we have an equilibrium $(q_1, ..., q_n)$ with price $p \geq a$. Then the total consumption $Q = q_1 + ... + q_n$ is at most aw_0. If $Q < aw_0$, a small increase of production by a firm will not change the electricity price, so the deviation would be profitable. We obtain $Q = aw_0$. There exists i such that firm i produces $q_i \leq \frac{aw_0}{n}$, and the profit at this firm is $\Pi_i = q_i(a - c)$. Suppose firm i deviates to a production $q'_i = q_i + \varepsilon$ with $\varepsilon > 0$ small. The new price p' will satisfy $p' = a + \frac{1}{w_1}(aw_0 - Q - \varepsilon) = a - \frac{\varepsilon}{w_1} < a$. The profit of firm i induced by the deviation is:

$$\Pi'_i = q'_i(p' - c) = (q_i + \varepsilon)(a - c - \varepsilon/w_1) = \Pi_i + \varepsilon(a - c - q_i/w_1) - \varepsilon^2/w_1.$$

We have $a - c - q_i/w_1 \geq a - c - \frac{aw_0}{nw_1} = a(1 - \frac{w_0}{nw_1}) - c > 0$ by assumption. So for $\varepsilon > 0$ small, $\Pi'_i > \Pi_i$ and the deviation is profitable. This is a contradiction, so no equilibrium price can be equal to a. This concludes the proof of Lemma 5.1.

8.2 Proof of Lemma 5.2

We assume $c \geq a(1 - \frac{w_0}{nw_1})$. Suppose we have an equilibrium $(q_1, ..., q_n)$.

If the equilibrium price p is such as $p < a$, like in the proof of Lemma 5.1 we obtain $p = \frac{1}{n+1}(a + a\frac{w_0}{w_1} + nc) \geq a$, hence a contradiction. So $p = a$, which implies $Q \leq aw_0$. If $Q < aw_0$, any firm could slightly increase its production without affecting the price. This would be a profitable deviation, hence a contradiction. So necessarily $Q = aw_0$. Types 1 and ∞ do not consume any electricity, so

$S_1 = S_\infty = 0$. A consumer of type 0 consumes a quantity a at price a, hence has a net surplus after billing of $NS_0 = (a - a/2)a - a^2 = -a^2/2$.

Let us now consider the equilibrium conditions. The profit of a firm $i = 1, ..., n$ is $\Pi_i = q_i(a - c)$. A deviation to $q'_i < q_i$ is not profitable since it does not affect the price. Consider now a deviation from q_i to $q'_i = q_i + \varepsilon$, with $\varepsilon > 0$. The new price p' will satisfy $p' = a + \frac{1}{w_1}(aw_0 - Q - \varepsilon) = a - \frac{\varepsilon}{w_1}$, hence the deviation will lead firm i to a profit of $\Pi'_i = (q_i + \varepsilon)(a - c - \frac{\varepsilon}{w_1}) = \Pi_i + \varepsilon(a - c - \frac{q_i}{w_1}) - \frac{\varepsilon^2}{w_1}$. If $a - c - \frac{q_i}{w_1} > 0$, the deviation is profitable for some small $\varepsilon > 0$. So necessarily $q_i \geq w_1(a - c)$, and if $q_i \geq w_1(a - c)$ then $\Pi'_i \leq \Pi_i$ for all $\varepsilon \geq 0$.

We obtain that the Cournot equilibria in case B are exactly the profiles $(q_1, ..., q_n)$ such that for each i, $q_i \geq w_1(a - c)$, and $q_1 + ... + q_n = aw_0$ (notice that $aw_0 \geq nw_1(a - c)$ by assumption). And there is a unique *symmetric* equilibrium where each firm produces $q_i^C = \frac{aw_0}{n}$.

8.3 The provision of assistance to myopic consumers. Detailed analysis.

In this appendix, we first compute the Cournot equilibria of a general assistance market when demand is discrete, and we state proposition 8.1 whose results will be used in the following subsections to deal with the various cases identified. Subsection 8.3.1 is dedicated to the case where energy prices are low, so that $\mathbb{E}(NS_0) > 0$, and subsection 8.3.2 to the case of high energy prices so that $\mathbb{E}(NS_0) < 0$. Finally, in subsection 8.3.3 we consider the case of a vertically integrated monopoly selling both assistance and energy.

Consider a market where the assistance is sold by a Cournot oligopoly of $m \geq 1$ identical assistance providers with positive unit cost c_s, and with the following discrete demand:[33]

$$Q_s(p_s) = \begin{cases} 1 & \text{if} \quad p_s = 0 \\ Q_1 & \text{if} \quad 0 < p_s \leq p_1 \\ Q_2 & \text{if} \quad p_1 < p_s \leq p_2 \\ 0 & \text{if} \quad p_s > p_2 \end{cases}$$

for any parameters that satisfy $p_2 > p_1 > 0$, $Q_1 \geq Q_2 \geq 0$ and $Q_1 > 0$. We consider pure Cournot equilibria and use the following inverse demand function, where $P(0)$ is any real number greater than p_2 (its exact value does not matter):

$$P_s(q) = \begin{cases} P(0) & \text{if} \quad q = 0 \\ p_2 & \text{if} \quad 0 < q \leq Q_2 \\ p_1 & \text{if} \quad Q_2 < q \leq Q_1 \\ 0 & \text{if} \quad q > Q_1 \end{cases}$$

We have $Q(P(q)) = q$ for $q \in \{0, Q_2, Q_1, 1\}$, and $P(Q(p)) = p$ for $p \in \{0, p_1, p_2, P(0)\}$.

[33] We will use $Q_1 = w_0 + w_\infty$, $Q_2 = w_\infty$ in subsection 8.3.1, and $Q_1 = w_0 + w_\infty$, $Q_2 = w_0$ in subsection 8.3.2.

Proposition 8.1. ***Cournot equilibria on the assistance market***

There always exists a Cournot equilibrium, and at each equilibrium the total quantity of sales Q is 0, Q_2 or Q_1.

a) There exists a Cournot equilibrium with sales Q_1 if and only if:

$$c_s \leq p_1 \text{ and } Q_2(p_2 - p_1) \leq (Q_1 - Q_2)\left((m-1)p_2 + p_1 - mc_s\right). \tag{29}$$

b) There exists a Cournot equilibrium with sales Q_2 if and only if:

$$c_s \leq p_2 \text{ and } Q_2(p_2 - p_1) \geq m(Q_1 - Q_2)(p_1 - c_s). \tag{30}$$

c) There exists a Cournot equilibrium with no sales if and only if ($c_s \geq p_2$) or ($Q_2 = 0$ and $c_s \geq p_1$).

It results that:

- if $c_s > p_2$, there is a unique equilibrium with $Q = 0$.

- if $c_s = p_2$, there are two equilibrium sales with $Q = 0$ or $Q = Q_2$.

- if $p_1 < c_s < p_2$, the equilibrium sales at each equilibrium are $Q = Q_2$.

- if $c_s \leq p_1$ and $Q_2(p_2 - p_1) > (Q_1 - Q_2)((m-1)p_2 + p_1 - mc_s)$, the sales at each equilibrium are $Q = Q_2$.

- if $c_s \leq p_1$ and $Q_2(p_2 - p_1) < m(Q_1 - Q_2)(p_1 - c_s)$, the sales at each equilibrium are $Q = Q_1$.

- if $c_s \leq p_1$ and $m(Q_1 - Q_2)(p_1 - c_s) \leq Q_2(p_2 - p_1) \leq (Q_1 - Q_2)\left((m-1)p_2 + p_1 - mc_s\right)$, there are two equilibrium sales with $Q = Q_1$ or $Q = Q_2$.

Proof of Proposition 8.1.

Consider a Cournot equilibrium where each firm $i = 1, ..., m$ produces $q_i \geq 0$. The total quantity $Q = q_1 + ... + q_m$ is either Q_1, Q_2 or 0. Q_1. It cannot be in the open intervals $(0, Q_2)$ or (Q_2, Q_1) since a small increase of production will not affect the price, hence will constitute a profitable deviation. It cannot be greater than Q_1 since it induces a 0 price.

a) Assume there is a Cournot equilibrium with sales Q_1. We have $Q = Q_1$, and the price is $p = p_1$. The profit of each firm i is $\Pi_i = q_i(p_1 - c_s) \geq 0$, so $c_s \leq p_1$. Consider a deviation of firm i to $q_i - (Q_1 - Q_2)$, inducing a new price p_2. This deviation is possible and profitable if and only if

$$(q_i - (Q_1 - Q_2))(p_2 - c_s) > q_i(p_1 - c_s). \tag{31}$$

This is equivalent to:

$$(Q_1 - Q_2)c_s + (q_i - (Q_1 - Q_2))p_2 > q_i p_1. \tag{32}$$

Assume that $(Q_1 - Q_2)c_s + (\frac{Q_1}{m} - (Q_1 - Q_2))p_2 > \frac{Q_1}{m}p_1$. This is equivalent to assuming $Q_2(p_2 - p_1) > (Q_1 - Q_2)((m-1)p_2 + p_1 - mc_s)$. Consider firm i producing the largest quantity, then $q_i \geq \frac{Q_1}{m}$, so inequality (32) is satisfied (implying $q_i \geq Q_1 - Q_2$). This is a profitable deviation, contradicting the equilibrium property.

Assume now that $c_s \leq p_1$ and $(Q_1 - Q_2)c_s + (\frac{Q_1}{m} - (Q_1 - Q_2))p_2 \leq \frac{Q_1}{m} p_1$. Consider the symmetric strategy profile where each firm i produces $q_i = \frac{Q_1}{m}$. Then for each i, inequality (32) is not satisfied, so the deviation from q_i to $q_i - (Q_1 - Q_2)$ is not profitable. And *a fortiori*, no deviation to $q_i' < q_i - (Q_1 - Q_2)$ is profitable. No deviation to $q_i' > q_i$ is profitable since it induces a 0 price. And no deviation from q_i to $q_i' \in (q_i - (Q_1 - Q_2), q_i)$ can be profitable since it will not increase the price. So the symmetric strategy profile inducing $Q = Q_1$ is a Cournot equilibrium.

b) Assume there is a Cournot equilibrium with sales Q_2. We have $Q = Q_2$, and the price is $p = p_2$. The profit of each firm i is $\Pi_i = q_i(p_2 - c_s) \geq 0$, so $c_s \leq p_2$. Consider a deviation of firm i to $q_i + Q_1 - Q_2$, inducing a new price of p_1. This deviation is profitable if and only if

$$(q_i + Q_1 - Q_2)(p_1 - c_s) > q_i(p_2 - c_s). \tag{33}$$

This is equivalent to:

$$(Q_1 - Q_2)c_s + q_i(p_2 - p_1) < p_1(Q_1 - Q_2). \tag{34}$$

Assume that $(Q_1 - Q_2)c_s + \frac{Q_2}{m}(p_2 - p_1) < p_1(Q_1 - Q_2)$. Consider firm i producing the smallest quantity, we have $q_i \leq \frac{Q_2}{m}$ which implies that inequality (34) holds. Firm i has a profitable deviation, so there is no such equilibrium.

Assume now that $c_s \leq p_2$ and $(Q_1 - Q_2)c_s + \frac{Q_2}{m}(p_2 - p_1) \geq p_1(Q_1 - Q_2)$. Consider the symmetric strategy profile where each firm i produces $q_i = \frac{Q_2}{m}$. Then inequality (34) is not satisfied, and no deviation from q_i to $q_i + Q_1 - Q_2$ is profitable. What about other deviations ? A deviation to $q_i' \in (q_i, q_i + Q_1 - Q_2)$ will lead to the same price as the deviation to $q_i + Q_1 - Q_2$ but with fewer sales, so is *a fortiori* not profitable. A deviation to $q_i' < q_i$ decreases the production of firm i and will not increase the price, so cannot be profitable either. Finally, a deviation to $q_i' > q_i + Q_1 - Q_2$ will induce a price of 0 and is also not profitable either. So the symmetric strategy profile inducing $Q = Q_2$ is a Cournot equilibrium.

c) Assume there is a Cournot equilibrium with sales $Q = 0$. Then $q_i = 0$ for each firm i, and all profits are 0. A small increase of q_i will lead to a price of p_2, so necessarily $c_s \geq p_2$. And if $c_s \geq p_2$ it is easy to check that each firm producing 0 is a Cournot equilibrium.

Finally, we check that at least one of the cases a), b), c) holds. If $c_s \leq p_1$, then $c_s \leq p_2$ and since $mp_1 \leq (m-1)p_2 + p_1$, (41) or (42) holds, implying a) or b). If $c_s \geq p_2$, c) holds. And if $p_1 < c_s < p_2$ then (42) holds, implying b).

8.3.1 The case $\mathbb{E}(NS_0) > 0$

Optimal sales of assistance The case $\mathbb{E}(NS_0) > 0$ occurs when the electricity prices are low enough so that consumers who behave as if energy is for free still derive a positive net surplus expectation. There are three cases for the sales of assistance:

1. If $p_s \leq \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$, both types 0 and ∞ buy the assistance, so all consumers become of type 1 and the new average consumption is

$$\bar{Q}_1 := \frac{n}{n+1}(a - \bar{c}).$$

2. If $\mathbb{E}(NS_1) - \mathbb{E}(NS_0) < p_s \leq \mathbb{E}(NS_1)$, only type ∞ buy the assistance. The new distribution of types is $w(p_s) = (w_0, w_1 + w_\infty, 0)$ and the new average consumption is

$$\bar{Q}_2 := \int_{c_L}^{a(1-\frac{w_0}{nw_1})} \frac{n}{n+1}(a - cw_1)dG(c) + \int_{a(1-\frac{w_0}{nw_1})}^{c_H} aw_0 dG(c).$$

3. If $p_s > \mathbb{E}(NS_1)$, nobody buys the assistance. Then the distribution of types is unchanged and the average consumption remains

$$\bar{Q}_3 := \int_{c_L}^{a(1-\frac{w_0}{nw_1})} \frac{n}{n+1}(a(w_0 + w_1) - cw_1)dG(c) + \int_{a(1-\frac{w_0}{nw_1})}^{c_H} aw_0 dG(c).$$

In case 2, the provision of assistance would increase electricity consumption since $\bar{Q}_2 \geq \bar{Q}_3$. As the difference $\bar{Q}_3 - \bar{Q}_1$ may be positive or negative, given the objective to decrease the average consumption of electricity, we have two subcases:

- if $\bar{Q}_1 < \bar{Q}_3$, it is optimal to sell the assistance to both types 0 and ∞ consumers, and this can happen at price $p_s = \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$.
- if $\bar{Q}_1 > \bar{Q}_3$, it is optimal not to sell assistance.

The intuition is the following: to decrease the electricity consumption, we would like type-0 consumers (who consume too much) to buy the assistance, but not type ∞ since these consumers do not buy energy from the grid. If $w_0 = 0$, then $\bar{Q}_1 = \frac{n}{n+1}(a - \bar{c}) > \frac{n}{n+1}(a - \bar{c})w_1 = \bar{Q}_3$. By continuity, if w_0 is small then $\bar{Q}_1 > \bar{Q}_3$ will hold and it is not worth attracting type 0-consumers.

Notice that if the electricity costs are small enough so that $c_H \leq a(1 - \frac{w_0}{nw_1})$, then $\bar{Q}_3 = \frac{n}{n+1}(a(w_0 + w_1) - \bar{c}w_1)$, and in this case:

$$\bar{Q}_3 > \bar{Q}_1 \iff \bar{c}w_0 > (a - \bar{c})w_\infty.$$

So if $\frac{w_0}{w_\infty}$ is sufficiently large, equipping the whole society with assistance decreases the total electricity consumption.

The assistance market Can we implement the optimal decrease in energy consumption with an oligopoly market for assistance when $\mathbb{E}(NS_0) > 0$? Suppose that assistance is sold by a Cournot oligopoly of $m \geq 1$ identical providers with positive unit cost c_s. The demand function is:

$$Q_s(p_s) = \begin{cases} 1 & \text{if} \quad p_s = 0 \\ w_0 + w_\infty & \text{if} \quad 0 < p_s \leq \mathbb{E}(NS_1) - \mathbb{E}(NS_0) \\ w_\infty & \text{if} \quad \mathbb{E}(NS_1) - \mathbb{E}(NS_0) < p_s \leq \mathbb{E}(NS_1) \\ 0 & \text{if} \quad p_s > \mathbb{E}(NS_1) \end{cases}$$

Using Proposition 8.1 with $Q_1 = w_0 + w_\infty$, $Q_2 = w_\infty$, $p_1 = \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$ and $p_2 = \mathbb{E}(NS_1)$, we can state the following Proposition:

Proposition 8.2. *There always exists a Cournot equilibrium, and at each equilibrium the total sales Q are either $w_0 + w_\infty$ (both consumers with type 0 and type ∞ buy the assistance), w_∞ (only type ∞ consumers buy the assistance) or 0 (no assistance is sold).*

Moreover,

a) There exists a Cournot equilibrium where both types 0 and ∞ buy the assistance if and only if:

$$c_s \leq \mathbb{E}(NS_1) - \mathbb{E}(NS_0) \text{ and } (w_0 + w_\infty)\mathbb{E}(NS_0) \leq m w_0(\mathbb{E}(NS_1) - c_s). \quad (35)$$

b) There exists a Cournot equilibrium where only type-∞ buys the assistance if and only if:

$$c_s \leq \mathbb{E}(NS_1) \text{ and } (m w_0 + w_\infty)\mathbb{E}(NS_0) \geq m w_0(\mathbb{E}(NS_1) - c_s). \quad (36)$$

c) There exists a Cournot equilibrium where no assistance is sold if and only if:

$$(c_s \geq \mathbb{E}(NS_1)) \text{ or } (w_\infty = 0 \text{ and } c_s \geq \mathbb{E}(NS_1) - \mathbb{E}(NS_0)). \quad (37)$$

It follows from the proposition that:

- if $c_s > \mathbb{E}(NS_1)$, there is a unique equilibrium with $Q = 0$.

- if $c_s = \mathbb{E}(NS_1)$, there are two equilibrium quantities of sales with $Q = 0$ or $Q = w_\infty$.

- if $\mathbb{E}(NS_1) - \mathbb{E}(NS_0) < c_s < \mathbb{E}(NS_1)$, there is a unique equilibrium quantity of sales $Q = w_\infty$.

- if $c_s \leq \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$ and $(w_0 + w_\infty)\mathbb{E}(NS_0) > m w_0(\mathbb{E}(NS_1) - c_s)$, there is a unique equilibrium of sales $Q = w_\infty$.

- if $c_s \leq \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$ and $(m w_0 + w_\infty)\mathbb{E}(NS_0) < m w_0(\mathbb{E}(NS_1) - c_s)$, there is a unique equilibrium quantity with $Q = w_0 + w_\infty$.

- if $c_s \leq \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$ and $(w_0 + w_\infty)\mathbb{E}(NS_0) \leq m w_0(\mathbb{E}(NS_1) - c_s) \leq (m w_0 + w_\infty)\mathbb{E}(NS_0)$, there are two equilibrium quantities of sales with $Q = w_0 + w_\infty$ or $Q = w_\infty$.

Since the objective is to decrease the average consumption of electricity, if $\bar{Q}_1 < \bar{Q}_3$ we would like to induce an equilibrium on the assistance market with $Q = w_0 + w_\infty$, and if $\bar{Q}_1 > \bar{Q}_3$ we would like no assistance to be sold.

Application: perfect competition on the electricity market In this section, we consider the case where $n = +\infty$. To limit the number of cases to be reviewed, we also assume that $a > 2\bar{c}$. Using the results from section 5.2, only case A applies and we get: $\bar{Q} = aw_0 + (a - \bar{c})w_1$, $\mathbb{E}(NS_0) = \frac{a}{2}(a - 2\bar{c})$, $\mathbb{E}(NS_1) = \frac{1}{2}\mathbb{E}(a - c)^2 = \frac{1}{2}(a^2 - 2a\bar{c} + e^2)$ and $\mathbb{E}(NS_\infty) = 0$. As we assume $a > 2\bar{c}$, we have $\mathbb{E}(NS_0) > 0$ and we can apply the results of proposition 8.2 with $\mathbb{E}(NS_1) - \mathbb{E}(NS_0) = \frac{1}{2}e^2$. We obtain:

Proposition 8.3. *There always exists a Cournot equilibrium, and at each equilibrium the total sales Q are either $w_0 + w_\infty$ (consumers with types 0 or ∞ buy the assistance), w_∞ (only type ∞ consumers buys the assistance) or 0 (no assistance is sold). Moreover,*

a) There exists a Cournot equilibrium where both types 0 and ∞ buy the assistance if and only if:

$$c_s \leq \frac{1}{2}e^2 \text{ and } \frac{w_\infty}{m}a(a - 2\bar{c}) + 2w_0c_s \leq w_0(e^2 + \frac{m-1}{m}a(a - 2\bar{c})). \tag{38}$$

b) There exists a Cournot equilibrium where only types ∞ buy the assistance if and only if:

$$c_s \leq \frac{1}{2}(a^2 - 2a\bar{c} + e^2) \text{ and } \frac{w_\infty}{m}a(a - 2\bar{c}) + 2w_0c_s \geq w_0e^2. \tag{39}$$

c) There exists a Cournot equilibrium where no assistance is sold if and only if ($c_s \geq \frac{1}{2}(a^2 - 2a\bar{c} + e^2)$) or ($w_\infty = 0$ and $c_s \geq \frac{1}{2}e^2$)

It follows from the proposition that:

- if $c_s > \frac{1}{2}(a^2 - 2a\bar{c} + e^2)$, there is a unique equilibrium with $Q = 0$.
- if $c_s = \frac{1}{2}(a^2 - 2a\bar{c} + e^2)$, there are two equilibrium quantities of sales with $Q = 0$ or $Q = w_\infty$.
- if $\frac{1}{2}e^2 < c_s < \frac{1}{2}(a^2 - 2a\bar{c} + e^2)$, there is a unique equilibrium quantity of sales $Q = w_\infty$.
- if $c_s \leq \frac{1}{2}e^2$ and $\frac{w_\infty}{m}a(a - 2\bar{c}) + 2w_0c_s > w_0(e^2 + \frac{m-1}{m}a(a - 2\bar{c}))$, there is a unique equilibrium of sales $Q = w_\infty$.
- if $c_s \leq \frac{1}{2}e^2$ and $\frac{w_\infty}{m}a(a - 2\bar{c}) + 2w_0c_s < w_0e^2$, there is a unique equilibrium quantity with $Q = w_0 + w_\infty$.
- if $c_s \leq \frac{1}{2}e^2$ and $w_0e^2 \leq \frac{w_\infty}{m}a(a - 2\bar{c}) + 2w_0c_s \leq w_0(e^2 + \frac{m-1}{m}a(a - 2\bar{c}))$, there are two equilibrium quantities of sales with $Q = w_0 + w_\infty$ or $Q = w_\infty$.

Since the objective is to decrease the average consumption of electricity, if $\bar{c}w_0 > (a - \bar{c})w_\infty$ we would like to induce an equilibrium on the assistance market with $Q = w_0 + w_\infty$, and if $\bar{c}w_0 < (a - \bar{c})w_\infty$ we would like no assistance to be sold.

If $c_s < \frac{1}{2}(a^2 - 2a\bar{c} + e^2)$ and $\bar{c}w_0 < (a - \bar{c})w_\infty$, it is not possible to have an equilibrium where no assistance is sold, so the introduction of the assistance market (whatever m) will increase the total average consumption of electricity. We have obtained:

Proposition 8.4. *Assume $\bar{c}w_0 < (a-\bar{c})w_\infty$. If $c_s < \frac{1}{2}(a^2 - 2a\bar{c} + e^2)$, each Cournot equilibrium of the assistance market will lead to a strict increase of the average electricity consumption.*

The intuition is that when the cost of assistance is small enough, Cournot competition will lead to positive sales of assistance to type-∞ consumers who will start consuming positive quantities of electricity from the grid. And since $\bar{c}w_0 < (a-\bar{c})w_\infty$, the proportion of such consumers is high enough to increase the average consumption.

We assume in the sequel that $\bar{c}w_0 > (a-\bar{c})w_\infty$. There exists an equilibrium in the assistance market where $Q = w_0 + w_\infty$ only if c_s is small enough, more precisely if $c_s \leq \frac{1}{2}e^2$. Assuming it is the case, such an equilibrium exists if and only if $\frac{w_\infty}{m}a(a-2\bar{c}) + 2w_0c_s \leq w_0(e^2 + \frac{m-1}{m}a(a-2\bar{c}))$, with uniqueness if and only if $\frac{w_\infty}{m}a(a-2\bar{c}) + 2w_0c_s < w_0e^2$. We obtain:

Proposition 8.5. *Assume $\bar{c}w_0 > (a-\bar{c})w_\infty$.To decrease the average electricity consumption, it is optimal to provide assistance to both type 0 and ∞ consumers. If m is large enough so that*

$$\frac{w_\infty}{m}a(a-2\bar{c}) + 2w_0c_s < w_0e^2, \tag{40}$$

the unique Cournot equilibrium on the assistance market will lead to the optimal decrease of average electricity consumption.

Notice that the condition (40) implies $c_s < \frac{1}{2}e^2$. And if there is no type-∞ consumer, or infinitely many assistance providers, (40) reduces to $c_s < \frac{1}{2}e^2$. The reason is that if there are many assistance providers, the price of assistance will be small enough to attract also type 0 consumers. And if $\frac{w_\infty}{w_0}$ is small, type ∞ consumers will play little role on the assistance market and providers will essentially target type 0 consumers.

8.3.2 The case $\mathbb{E}(NS_0) < 0$

Optimal sales of assistance When the electricity prices are very high, consumers of type 0 have a negative surplus after billing. Since $\mathbb{E}(NS_0) < 0$, if consumers with type $+\infty$ buy the assistance, then consumers with type 0 also do. Again we have three cases:

1) if $p_s \leq \mathbb{E}(NS_1)$, types 0 and $+\infty$ buy the assistance, so that the new distribution of types becomes $w(p_s) = (0,1,0)$. Case A will always apply with this new distribution (see Lemma 5.1), and the new expected total consumption of electricity will be

$$\bar{Q}_1 = \frac{n}{n+1}(a-\bar{c}).$$

2) If $\mathbb{E}(NS_1) - \mathbb{E}(NS_0) \geq p_s > \mathbb{E}(NS_1)$, only type 0 buys the assistance, so that $w(p_s) = (0, w_0 + w_1, w_\infty)$. Again, case A will always apply with this new distribution, and the new expected total consumption of electricity will be

$$\bar{Q}_2 = \frac{n}{n+1}(a - \bar{c})(w_0 + w_1).$$

3) If $p_s > \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$, nobody buys the assistance and the expected total consumption remains:

$$\bar{Q}_3 = \int_{c_L}^{a(1-\frac{w_0}{nw_1})} \frac{n}{n+1}(a(w_0 + w_1) - cw_1)dG(c) + \int_{a(1-\frac{w_0}{nw_1})}^{c_H} aw_0 dG(c).$$

It is clear that $\bar{Q}_2 \leq \bar{Q}_1$. To compare $\bar{Q}_2$ and $\bar{Q}_3$, one can write

$$\bar{Q}_2 = \int_{c_L}^{a(1-\frac{w_0}{nw_1})} \frac{n}{n+1}(a-c)(w_0+w_1)dG(c) + \int_{a(1-\frac{w_0}{nw_1})}^{c_H} \frac{n}{n+1}(a-c)(w_0+w_1)dG(c).$$

$(a-c)(w_0+w_1) \leq a(w_0+w_1) - cw_1$ holds for all c. Assume now that $c \geq a(\frac{nw_1 - w_0}{nw_1})$, then $nc(w_0 + w_1) \geq ncw_1 \geq a(nw_1 - w_0)$, so that $n(a-c)(w_0+w_1) \leq aw_0(n+1)$, and $\frac{n}{n+1}(a-c)(w_0 + w_1) \leq aw_0)$. We conclude that $\bar{Q}_2 \leq \bar{Q}_3$.

Lemma 8.6. *When $\mathbb{E}(NS_0) < 0$, the best outcome in terms of decrease of electricity consumption is $\bar{Q}_2$, that is when $\mathbb{E}(NS_1) - \mathbb{E}(NS_0) \geq p_s > \mathbb{E}(NS_1)$, so that only type 0 buys the assistance.*

The assistance market The assistance service is sold by a Cournot oligopoly of $m \geq 1$ identical assistance providers with positive unit cost c_s. The demand function is now:

$$Q_s(p_s) = \begin{cases} 1 & \text{if} \quad p_s = 0 \\ w_0 + w_\infty & \text{if} \quad 0 < p_s \leq \mathbb{E}(NS_1) \\ w_0 & \text{if} \quad \mathbb{E}(NS_1) < p_s \leq \mathbb{E}(NS_1) - \mathbb{E}(NS_0) \\ 0 & \text{if} \quad p_s > \mathbb{E}(NS_1) - \mathbb{E}(NS_0) \end{cases}$$

We now apply Proposition 8.1 in the case, where $Q_1 = w_0 + w_\infty$, $Q_2 = w_0$, $p_2 = \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$ and $p_1 = \mathbb{E}(NS_1)$.

Proposition 8.7. *There always exists a Cournot equilibrium, and at each equilibrium the total quantity of sales Q is 0, w_0 or $w_0 + w_\infty$.*

a) There exists a Cournot equilibrium with sales $w_0 + w_\infty$ if and only if:

$$c_s \leq \mathbb{E}(NS_1) \text{ and } -w_0\mathbb{E}(NS_0) \leq w_\infty \left(m\mathbb{E}(NS_1) - (m-1)\mathbb{E}(NS_0) - mc_s\right). \tag{41}$$

b) There exists a Cournot equilibrium with sales w_0 if and only if:

$$c_s \leq \mathbb{E}(NS_1) - \mathbb{E}(NS_0) \text{ and } -w_0\mathbb{E}(NS_0) \geq mw_\infty(\mathbb{E}(NS_1) - c_s). \tag{42}$$

c) There exists a Cournot equilibrium with no sales if and only if: $(c_s \geq \mathbb{E}(NS_1) - \mathbb{E}(NS_0))$ *or* $(w_0 = 0$ *and* $c_s \geq \mathbb{E}(NS_1))$*.*

It follows that:
- *if* $c_s > \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$*, there is a unique equilibrium with* $Q = 0$*.*
- *if* $c_s = \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$*, there are two equilibrium sales with* $Q = 0$ *or* $Q = w_0$*.*
- *if* $\mathbb{E}(NS_1) < c_s < \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$*, the equilibrium sales at each equilibrium are* $Q = w_0$*.*
- *if* $c_s \leq \mathbb{E}(NS_1)$ *and* $-w_0\mathbb{E}(NS_0) > w_\infty(m\mathbb{E}(NS_1) - (m-1)\mathbb{E}(NS_0) - mc_s)$*, the equilibrium sales at each equilibrium are* $Q = w_0$*.*
- *if* $c_s \leq \mathbb{E}(NS_1)$ *and* $-w_0\mathbb{E}(NS_0) < mw_\infty(\mathbb{E}(NS_1) - c_s)$*, the equilibrium sales at each equilibrium are* $Q = w_0 + w_\infty$*.*
- *if* $c_s \leq \mathbb{E}(NS_1)$ *and* $mw_\infty(\mathbb{E}(NS_1) - c_s) \leq -w_0\mathbb{E}(NS_0) \leq w_\infty(m\mathbb{E}(NS_1) - (m-1)\mathbb{E}(NS_0) - mc_s)$*, there are 2 equilibrium sales with* $Q = w_0 + w_\infty$ *or* $Q = w_0$*.*

Under the objective of decreasing the average consumption of electricity, we would like to have an assistance market with equilibrium sales $Q = w_0$. The unique equilibrium sales are $Q = w_0$ if and only if:

$$\mathbb{E}(NS_1) < c_s < \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$$

or $\left(c_s \leq \mathbb{E}(NS_1) \text{ and } -w_0\mathbb{E}(NS_0) > w_\infty(m\mathbb{E}(NS_1) - (m-1)\mathbb{E}(NS_0) - mc_s)\right)$.

Proposition 8.8.

Assume $\mathbb{E}(NS_1) < c_s < \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$*. Then the unique Cournot equilibrium on the assistance market will lead to the optimal decrease of average electricity consumption (for each value of* m*).*

Assume $c_s \leq \mathbb{E}(NS_1)$*. When* m *is large enough, at the unique Cournot equilibrium both types 0 and* ∞ *will buy the assistance, which is sub-optimal with respect to the objective of a decrease in electricity consumption.*

Assume $c_s \leq \mathbb{E}(NS_1)$*, the unique Cournot equilibrium induces the optimal decrease of average electricity consumption if and only if:*

$$-w_0\mathbb{E}(NS_0) > w_\infty(m\mathbb{E}(NS_1) - (m-1)E(NS_0) - mc_s).$$

In particular when $m = 1$*, the above inequality reduces to* $-w_0\mathbb{E}(NS_0) > w_\infty(\mathbb{E}(NS_1) - c_s)$*.*

The intuition is that since we want only type 0 to buy the assistance, it will help if there is a small number of firms m and/or a cost c_s not too small so that the price of assistance will be high.

Application: a monopoly on the electricity market Suppose there is Cournot competition between m identical firms on the market for assistance, followed by an independent monopoly on the electricity market ($n = 1$). Thanks to proposition 5.3, we have:

$$\mathbb{E}(NS_0) = \int_{c_L}^{a(1-\frac{w_0}{w_1})} -\frac{a}{2}\left(c + a\frac{w_0}{w_1}\right) dG(c) - \int_{a(1-\frac{w_0}{w_1})}^{c_H} \frac{a^2}{2} dG(c),$$

$$\mathbb{E}(NS_1) = \int_{c_L}^{a(1-\frac{w_0}{w_1})} \frac{1}{8}\left(a(1-\frac{w_0}{w_1}) - c\right)^2 dG(c).$$

Here $\mathbb{E}(NS_0) < 0$, and if consumers with type $+\infty$ buy the assistance, then consumers with type 0 also do.

Lemma 8.6 and Propositions 5.3 and 8.8 then perfectly apply.

Notice that if $c_H \leq a(1-\frac{w_0}{w_1})$, the formula simplify to $\mathbb{E}(NS_0) = -\frac{a}{2}(\bar{c}+a\frac{w_0}{w_1})$ and $\mathbb{E}(NS_1) = \frac{1}{8}\mathbb{E}(a(1-\frac{w_0}{w_1}) - c)^2$. Computations show that when $n = m = 1$ and costs are nil $c = c_s = 0$, the unique Cournot equilibrium induces the optimal decrease of average electricity consumption if and only if:

$$w_0 \geq w_1 \text{ or } (w_0 < w_1 \text{ and } 4w_0^2 w_1 > w_\infty (w_1 - w_0)^2). \tag{43}$$

8.3.3 Integrated monopoly

Consider now the case where the same single firm

• first proposes to sell an assistance to consumers, at price p_s with cost c_s. A subset of consumers will buy the assistance without anticipating that their decision can influence the energy prices.

• then, the monopoly observes the current cost of energy c and fixes a price p for energy. Each consumer will then choose his consumption according to his current perception of prices.

Since we want to contrast this market structure with the case of two successive independent monopolies, we assume that the expressions for $\mathbb{E}(NS_0)$ and $\mathbb{E}(NS_1)$ are the same as in section 8.3.2, in particular $\mathbb{E}(NS_0) < 0$. By section 8.3.2, to decrease the consumption of electricity it is optimal to sell the assistance to type 0 only.

Let us come back to the electricity market of section 5.2. The profit of the firm from energy sales is $\Pi_c = \frac{(a(w_0+w_1)-cw_1)^2}{4w_1}$ in case A, and $\Pi_c = aw_0(a-c)$ in case B. So the expected profit of the firm on the electricity market is:

$$\mathbb{E}(\Pi_c) = \int_{c_L}^{a(1-\frac{w_0}{nw_1})} \frac{(a(w_0+w_1) - cw_1)^2}{4w_1} dG(c) + \int_{a(1-\frac{w_0}{nw_1})}^{c_H} (a-c)aw_0 dG(c).$$

The integrated monopoly will maximize its global profit by solving $\mathrm{Max}_{p_s}\pi(p_s)$, where

$$\pi(p_s) = (p_s - c_s)D(p_s) + \int_{c=c_L}^{a(1-\frac{w_0(p_s)}{w_1(p_s)})} \frac{1}{4w_1(p_s)}\big(a(w_0(p_s)+w_1(p_s)) - cw_1(p_s)\big)^2 dG(c)$$
$$+ \int_{c=a(1-\frac{w_0(p_s)}{w_1(p_s)})}^{c_H} aw_0(p_s)(a-c)dG(c),$$

with $D(p_s)$ the proportion of consumers who buy the assistance at price p_s.

Like in section 8.3.2, we have 3 cases:

1) If $p_s \le \mathbb{E}(NS_1)$, types 0 and ∞ buy the assistance, so that $w_1(p_s) = 1$ and

$$\pi(p_s) = (p_s - c_s)(w_0 + w_\infty) + \frac{1}{4}\mathbb{E}((a-c)^2).$$

This expected profit is maximized for $p_s = \mathbb{E}(NS_1)$ with value

$$\pi_1 := (\mathbb{E}(NS_1) - c_s)(w_0 + w_\infty) + \frac{1}{4}\mathbb{E}((a-c)^2).$$

2) If $\mathbb{E}(NS_1) - \mathbb{E}(NS_0) \ge p_s > \mathbb{E}(NS_1)$, only type 0 buys the assistance. It results $w_1(p_s) = w_0 + w_1$ and

$$\pi(p_s) = (p_s - c_s)w_0 + \frac{(w_0 + w_1)}{4}\mathbb{E}((a-c)^2).$$

The expected profit is maximized for $p_s = \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$ with value

$$\pi_2 := (\mathbb{E}(NS_1) - \mathbb{E}(NS_0) - c_s)w_0 + \frac{(w_0 + w_1)}{4}\mathbb{E}((a-c)^2).$$

3) Finally, if $p_s > \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$, nobody buys the assistance service then the expected profit reduces to

$$\pi_3 := \int_{c=c_L}^{a(1-\frac{w_0}{w_1})} \frac{1}{4w_1}\big(a(w_0 + w_1) - cw_1\big)^2 dG(c) + \int_{c=a(1-\frac{w_0}{w_1})}^{c_H} aw_0(a-c)dG(c).$$

It remains to compare π_1, π_2 and π_3.

Proposition 8.9. *Integrated Monopoly: To decrease the consumption of electricity it is optimal to sell the assistance to type 0 only (case 2 above). This is profitable to the integrated monopoly if and only if* $\pi_2 \ge \max\{\pi_1, \pi_3\}$, *where* π_1, π_2 *and* π_3 *are defined above.*

Let us now compare the integrated monopoly with the case of two independent monopolies determined in subsection 8.3.2 for $m = n = 1$. On the one hand, the energy department of the integrated monopoly is in competition with its assistance department, which may decrease the sales of assistance. On the other hand, the integrated monopoly might be able to sell assistance at a loss to attract consumers and increase its profit on the electricity market. One can show the following.

Lemma 8.10. *If* $\pi_2 > \max\{\pi_1, \pi_3\}$*, then* $c_s < \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$ *and* $-w_0\mathbb{E}(NS_0) > w_\infty(\mathbb{E}(NS_1) - c_s)$*,*

The fact that $\pi_2 > \max\{\pi_1, \pi_3\}$ implies $c_s < \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$ shows that whenever the integrated monopoly induces the optimal sales, it does not sell at a loss. This can be explained as follows: selling assistance at a loss for the monopoly is interesting only to attract type ∞ consumers to the electricity market, but this is not socially desirable here since we want assistance to be provided to type 0-consumers only. Using propositions 8.7 and 8.8, the lemma shows that whenever the integrated monopoly supplies the optimal provision of assistance, then two disjoint monopolies ($n = m = 1$) can also achieve it. It is then more demanding to achieve the optimal sales with an integrated monopoly.

Proof of Lemma 8.10. Assume that $\pi_2 > \max\{\pi_1, \pi_3\}$.

The inequality $\pi_2 > \pi_1$ implies that $(\mathbb{E}(NS_1) - \mathbb{E}(NS_0) - c_s)w_0 > (\mathbb{E}(NS_1) - c_s)(w_0 + w_\infty)$ so $-w_0\mathbb{E}(NS_0) > w_\infty(\mathbb{E}(NS_1) - c_s)$.

It remains to show that $c_s < \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$. Assume by contradiction that $c_s \geq \mathbb{E}(NS_1) - \mathbb{E}(NS_0)$, then $\pi_2 \leq \frac{1}{4}(w_0 + w_1)\mathbb{E}(a-c)^2$. We now distinguish 2 cases.

1) Assume $w_0 \geq w_1$. then $\pi_3 = w_0 a(a - \bar{c}) \geq \frac{1}{4}(w_0 + w_1)a(a - \bar{c}) \geq \frac{1}{4}(w_0 + w_1)\mathbb{E}(a - c)^2 \geq f_2$. Contradiction.

2) Assume $w_0 \leq w_1$. Computations show that

$$\frac{1}{4w_1}(a(w_0 + w_1) - cw_1)^2 > \frac{w_0 + w_1}{4}(a - c)^2.$$

Moreover, if $c \geq a(1 - \frac{w_0}{w_1})$ then $a - c < 4a\frac{w_0}{w_0 + w_1}$ (using $w_0 < 3w_1$) and $aw_0(a-c) > \frac{w_0+w_1}{4}(a - c)^2$. We obtain that

$$\pi_2 \leq \frac{1}{4}(w_0+w_1)\mathbb{E}(a-c)^2 = \int_{c=c_L}^{a(1-\frac{w_0}{w_1})} \frac{w_0 + w_1}{4}(a-c)^2 dG(c) + \int_{c=a(1-\frac{w_0}{w_1})}^{c_H} \frac{w_0 + w_1}{4}(a-c)^2 dG(c)$$

$$< \int_{c=c_L}^{a(1-\frac{w_0}{w_1})} \frac{1}{4w_1}\big(a(w_0 + w_1) - cw_1\big)^2 dG(c) + \int_{c=a(1-\frac{w_0}{w_1})}^{c_H} aw_0(a - c)dG(c) = \pi_3.$$

Hence a contradiction, ending the proof of lemma 8.10.

Particular case: Assume $c = c_s = 0$ and $0 < w_0 < w_1$. With the notation $\theta := \frac{w_0}{w_1}$, we get $\mathbb{E}(NS_1) = \frac{1}{8}a^2(1 - \theta)^2, \mathbb{E}(NS_0) = -\frac{1}{2}a^2\theta$, $\pi_1 = \frac{1}{8}(1 - w_1)a^2(1 - \theta)^2 + \frac{1}{4}a^2$, $\pi_2 = w_1\theta(\frac{1}{8}a^2(1 - \theta)^2 + \frac{1}{2}a^2\theta) + \frac{1}{4}w_1(1 + \theta)a^2$, $\pi_3 = \frac{1}{4}w_1 a^2(1 + \theta)^2$. After some computation we obtain $\frac{\pi_2 - \pi_3}{\theta w_1} = \frac{1}{8}a^2(1 - \theta)(-1 - \theta) < 0$. In this case an integrated monopoly does not induce the optimal decrease of electricity consumption whereas two independent monopolies do it if condition (43) is met, which is very likely as w_∞ is small.

The conclusion is that, under our set of hypotheses, when the social objective is to decrease total consumption, a private vertically integrated monopoly should be unbundled.