# Inflation Persistence, Noisy Information, and the Phillips Curve

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A vast literature has documented that US inflation persistence has fallen in recent decades, but this finding is difficult to explain in monetary models. Using survey data on inflation expectations, I document a positive co-movement between ex-ante average forecast errors and forecast revisions (suggesting forecast sluggishness) from 1968 to 1984, but no co-movement afterward. I extend the New Keynesian setting to include noisy and dispersed information about the aggregate state and show that inflation is more persistent in periods of greater forecast sluggishness. My results suggest that changes in firm forecasting behavior explain around 90% of the fall in inflation persistence since the mid-1980s. I also find that the changes in the dynamics of the Phillips curve (PC) can be explained by the change in information frictions. After controlling for changes in information frictions, I estimate only a modest decline in the slope. I find that a more significant factor in the dynamics of the PC is the shift towards greater forward-lookingness and less backward-lookingness. Finally, I find evidence of forecast underrevision in the post-COVID period, which explains the increase in the persistence of current inflation.

**Keywords:** Inflation persistence, Phillips curve, noisy information.
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# 1. Introduction

Since long, expectations have played a central role in macroeconomics. However, most of work considers a limited theory of expectation formation, in which agents are perfectly and homogeneously aware of the state of nature and others' actions. In this paper, I consider a theory of expectation formation that incorporates significant heterogeneity and sluggishness in agents' forecasts, thus relaxing the standard *full information rational expectations* (FIRE) benchmark.[1] I include such expectation formation features into an otherwise standard New Keynesian (NK) model by introducing noisy and dispersed information, rationally processed separately by each agent, and match the information-specific parameters to the observed sluggishness in forecasts. I use this framework to interpret two empirical challenges in the literature: the fall in inflation persistence and the change in the dynamics of the Phillips curve (PC).

As for the first empirical challenge, evidence suggests that the dynamic properties of US inflation have not been constant over time. In particular, inflation in the post-war period exhibits a high degree of persistence up until the mid-1980s, falling significantly since then. This fall in inflation persistence is not easily understood through the lens of monetary models, which has resulted in the "inflation persistence puzzle" (Fuhrer 2010).[2] This break coincides with a change in the US Federal Reserve's communication policy, which became more transparent and informative after the mid-1980s. Using survey data on US firms' forecasts, I document a significant sluggishness in responses to new information until the mid-1980s, but no evidence of sluggishness afterward.[3] The theoretical framework I build is consistent with this evidence. I argue that the change in the Fed communication improves firms' information and I use the model to show that the reduced stickiness in firms' inflation forecasts explains the fall in inflation persistence.

The second empirical challenge documents that the dynamics of the PC have changed in recent decades. The literature has mainly focused on the output gap coefficient, arguing for a flatter curve in recent decades (Rubbo 2019; del Negro et al. 2020; Ascari and Fosso 2021; Hazell et al. 2022). This finding indirectly implies that central bank (CB) actions, understood as nominal interest rate changes, are less effective in affecting inflation. I

[1] I define sluggishness as the stickiness of current expectations on past expectations. In the data, I measure sluggishness as a positive co-movement between ex-ante average forecast errors and forecast revisions.

[2] Persistence determines both the memory of any past shock on today's outcome and the unconditional volatility of an autoregressive dynamic process. See Fuhrer (2010) for a handbook literature review.

[3] Coibion and Gorodnichenko (2015a) find evidence for an *increase* in the level of information frictions since the 1980s, and explain such an increase from a rational inattention perspective. In section 2.2, using their data, I provide evidence of the *decrease* in information frictions *related to inflation*. I argue that this particularity about inflation expectations comes from changes in the Fed's communication policy.

estimate only a modest decline in the slope of the PC since the mid-1980s, once I control for the decrease in information frictions. Instead, I argue from the perspective of the model that the change in the dynamics of the PC can be explained by a lack of backward-lookingness and an increase in forward-lookingness after the mid-1980s.

In terms of the details of the model, I explain the fall in inflation persistence through a decrease in the degree of information frictions that firms face on CB actions. Since the late 1960s, there has been a gradual improvement in the US Federal Reserve's public disclosure and transparency, sending clearer signals of their actions and future intentions to the market.[4] In this framework, inflation is more persistent in periods of greater forecast sluggishness: noisy information generates an underreaction to new information because individuals shrink their forecasts toward prior beliefs when the signals they observe are noisy. This endogenous anchoring in forecasts causes firms to set prices to their existing prior, thus slowing the speed of price changes. Using micro-data on inflation expectations from the Survey of Professional Forecasters (SPF) and the Livingston Survey on Firms, I document that firms' forecasts used to react sluggishly before the mid-1980s. However, there is no evidence of sluggishness in recent decades. My results suggest that agents became more informed about inflation after the change in the Federal Reserve disclosure policy. Because inflation depends on the expectations of future inflation, the change in expectation formation feeds into inflation dynamics, which endogenously reduces inflation persistence. Quantitatively, I find that this change in firms' forecasting behavior explains around 90% of the fall in inflation persistence since the mid-1980s. Formally, I extend the textbook NK framework in Galí (2015); Woodford (2003b) to noisy information following Angeletos and Huo (2021), itself a simplified version of Lucas (1972); Woodford (2003a); Nimark (2008); Lorenzoni (2009) in which agents only observe signals of exogenous variables. I assume that firms do not have complete and perfect information about aggregate economic conditions.

[4]Before 1967 the *Federal Open Market Committee* (FOMC), the US Fed decision unit, only announced policy decisions once a year in its Annual Report. In 1967, the FOMC decided to release the directive in the Policy Report (PR), 90 days after the decision. In 1976, the PR was enlarged and its delay was reduced to 45 days. Between 1976 and 1993 the information contained in the PR increased, without any further changes in the announcement delay. In 1977, the *Federal Reserve Reform Act* officially entitled the Fed with 3 objectives: maximum employment, stable prices, and moderate long-term interest rates. In 1979, the first macroeconomic forecasts on real GNP and GNP inflation from FOMC members were made available. The "tilt" (the likelihood regarding possible future action) was introduced in the PR in 1983. Between 1985 and 1991, the Fed introduced the "ranking of policy factors", which after each meeting ranked aggregate macro variables in importance, signaling priorities about possible future adjustments. The minutes, a revised transcript of the discussions during the meeting, started being released together with the PR in 1993, 45 days after the meeting. In 1994 the FOMC introduced the immediate release of the PR after a meeting if there had been a change, coupled with an immediate release of the "tilt" (likelihood regarding possible future action) since 1999. Since January 2000 there has been an immediate announcement and press conference after each meeting, regardless of the decision. See Lindsey (2003) for a comprehensive historical review. I provide a more detailed historical analysis of the Fed's gradual increase in transparency in Online Appendix OA.3.

Firms can observe their granular conditions – the output they produce given their price,– but they do not have perfect information about aggregate variables like inflation, output, or interest rates. In place, they observe a noisy signal that provides information on the state of the economy, in this case, the monetary policy shock. With this piece of information, firms form expectations on inflation, aggregate output, and interest rates. This setting leads to a dynamic beauty contest in which firms need to form beliefs on what other firms believe about the economy.

This paper contributes to the literature that aims to explain the fall in inflation persistence observed in the US since the 1980s (Fuhrer and Moore 1995; Benati and Surico 2008; Cogley and Sbordone 2008; Fuhrer 2010; Cogley et al. 2010; Goldstein and Gorodnichenko 2019). The literature has proposed theoretical explanations from different angles, with the shared commonality that the fall in the persistence of inflation can be attributed to a change in the conduct of monetary policy and the time-varying nature of exogenous shocks hitting the economy. Motivated by the documented increase in the aggressiveness of the monetary authority toward excess inflation (Clarida et al. 2000; Lubik and Schorfheide 2004), Cogley et al. (2010) estimate an NK model, enlarged with trend inflation, using two different samples, before 1979 and after 1982. They find that the Taylor rule coefficient for inflation increased in the second subsample, cost-push shocks became less persistent, and the disturbances hitting the economy became less volatile. Davig and Doh (2014) employ a regime-switching approach with time-varying parameters, finding that the monetary policy was more aggressive before 1970 and after the Volcker disinflation, consistent with the lower inflation persistence observed in those periods. Their analysis shows that a 40% decline in the persistence of inflation can be attributed both to the reduction in the weight of exogenous shocks with high persistence, which experienced a relative fall in volatility compared to other disturbances, and the increase in the Taylor rule coefficient. Bianchi and Ilut (2017) incorporate fiscal policy, allowing for time-varying coefficients both in the monetary and fiscal rules and on the volatility of exogenous shocks. Their analysis suggests that persistent inflation can arise from fiscal imbalances when monetary policy accommodates fiscal policy. In contrast, I only consider a single parametric change – an increase in the signal-to-noise ratio – to isolate the transmission mechanism. However, the increase in the signal-to-noise ratio that I document could be endogenized in my framework through the documented increase in the Taylor rule coefficient for inflation, which dampens the general equilibrium dimension, reduces strategic complementarities across agents, increases the weight that agents optimally assign to news, and boosts the signal-to-noise ratio. Closer to the present paper, Erceg and Levin (2003) explain the high persistence of inflation during the Volcker disinflation using a noisy information framework, where agents have incomplete information on the CB's inflation target. In contrast, I obtain a closed-form solution for inflation dynamics, allowing for a trace of the key drivers of the fall in persistence after the Volcker disinflation. Moreover, I extend this analysis to the structural inflation dynamics, the noisy-hybrid PC, and find evidence for a fall in intrinsic persistence, but no evidence for a fall in the persistence of structural disturbances. Finally, another strand of the literature explores the relation between long-term inflation expectations and the persistence of inflation, arguing that inflation has become less persistent because long-term expectations are less sensitive to current developments (Gáti 2023; Carvalho et al. 2023). Such finding can be rationalized in the current paper as an increase in agents' information on the CB's inflation target.

I also contribute to the literature studying the time-varying properties of the PC. The literature has mainly focused on the output gap coefficient, arguing for a flatter curve in recent decades, implying a fall in the sensitivity of inflation and the real side of the economy, including changes in the policy rate or CB actions ("inflation disconnect" puzzle, see e.g., del Negro et al. 2020; Ascari and Fosso 2021). Instead of focusing solely on the slope, I show that under noisy information the PC is enlarged with a backward-looking term on lagged inflation and myopia towards expected future inflation, both of them dependent on the degree of information frictions. First, I show that the fall in inflation persistence can be explained by a fall in *intrinsic* persistence and myopia. Second, I show that under a *general* information structure, the PC is modified such that current inflation is related to current and future output through two different channels: the output gap coefficient *and* firms' expectation formation process. I estimate only a modest decline in the slope once I control for a decline in information frictions, using SPF forecasts (Coibion et al. 2018; Crump et al. 2019; Hazell et al. 2022).

***Roadmap.*** The paper proceeds as follows. Section 2 documents the two empirical challenges and the decrease in forecast sluggishness and information frictions in recent decades. In Section 3, I describe the theoretical framework, and I derive the main results in section 4. Section 5 concludes the paper.

## 2. Empirical Challenges and Information Frictions

In this section, I discuss the two empirical challenges and the change in inflation forecast underrevision. First, I provide empirical evidence on the fall in the persistence of inflation in recent decades. Second, I show that the change in persistence coincides with a decline in forecast underreaction in recent decades. Third, I argue that the documented changes

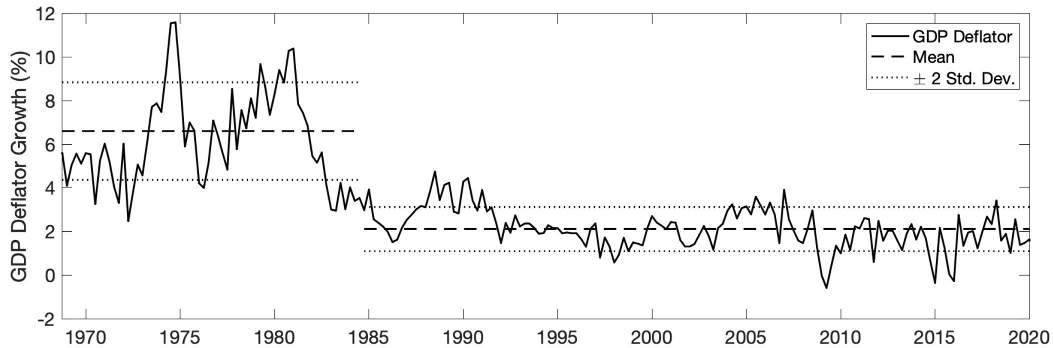



FIGURE 1. Time series of inflation, with subsample (pre- and post-1985) mean and standard deviation. *Source: U.S. Bureau of Economic Analysis, Gross Domestic Product: Implicit Price Deflator, https://fred.stlouisfed.org/series/GDPDEF*.

in the persistence of inflation and forecast underreaction can explain the changes in the dynamics of the PC over time.

### 2.1. The First Puzzle: Inflation Persistence

A vast literature has documented that US inflation persistence has fallen in recent decades. Fuhrer and Moore (1995); Cogley and Sbordone (2008); Fuhrer (2010); Cogley et al. (2010); Goldstein and Gorodnichenko (2019) find evidence of a structural break in the first-order autocorrelation of inflation in the 1980-1985 window, with persistence falling from around 0.75-0.8 to 0.5. In this section I revisit this empirical challenge and document a fall in inflation persistence since the mid-1980s.[5]

The inflation time series is reported in Figure 1. I follow Fuhrer (2010) and divide the sample into two sub-periods, pre- and post-1985:Q1 until 2020:Q2. I report the mean and 2 standard deviation bands by each subperiod. Inflation started its upward trend in the 1960s, continuing in the next decade with two peaks in the mid-1970s and the early 1980s. Then, inflation started its downward trend lasting until the early 1990s, and roughly remained at 2% afterward. The average level of inflation has fallen from 6% to 2%, and inflation has become less volatile.[6] In the monetary literature, inflation is generally assumed to follow an independent autoregressive stochastic process. In such a case, the

[5] I define the inflation rate at time t, $\pi_t$, as the (annualized) log growth in the index, $400 \times (\log P_t - \log P_{t-1})$, where $P_t$ is the quarterly GDP deflator at time $t$. Inflation data is available at a quarterly frequency since 1947:Q1. However, I will stick to the 1968:Q4-2020:Q2 sample since I seek to link the results presented in this section to surveys on expectations, which are available since 1968:Q4.

[6] I omit the fall in the average level and volatility of inflation from the analysis since both can be easily explained in a trend-inflation NK setup through a decrease in the inflation target of the CB, and an increase in the aggressiveness of the monetary authority towards the inflation gap, for which Clarida et al. (2000) provide empirical evidence.

| | Full Sample | 1968:Q4–1984:Q4 | 1985:Q1–2020:Q2 |
|---|---|---|---|
| Mean | 3.373 | 6.151 | 2.111 |
| Volatility | 2.399 | 2.218 | 1.017 |
| First-Order Autocorrelation | 0.880 | 0.770 | 0.511 |

TABLE 1. Summary statistics over time.

stationary mean depends both on the intercept, $\alpha_\pi$ in equation (1), and the lagged inflation coefficient, $\rho_\pi$. Stationary volatility depends both on the volatility of the innovation and lagged inflation coefficients. Table 1 reports summary statistics on the mean, volatility, and first-order autocorrelation by each subsample. In the following, I seek to investigate if these differences across subsamples are statistically significant.

Assume that inflation follows a simple AR(1) process with a drift. Once more, I follow Fuhrer (2010) and assume that the break date is 1985:Q1.[7] I test for a (potential) structural break on the intercept and persistence coefficients by estimating equation (1). Formally, I consider the regression

$$\pi_t = \alpha_\pi + \rho_\pi \pi_{t-1} + \alpha_{\pi*} \times \mathbb{1}_{\{t \geq t^*\}} + \rho_{\pi*} \times \mathbb{1}_{\{t \geq t^*\}} \pi_{t-1} + e_t \tag{1}$$

where $\mathbb{1}_{\{t \geq t^*\}}$ is an indicator variable equal to one if the period is within the post-1985 era and zero otherwise, and $e_t$ is the error term. I report my findings in table 2, panel A. First, I find that both the intercept and the persistence are significant when I consider the full sample with no structural break (column 1). Second, in a subsample analysis, I provide evidence of the fall in inflation persistence from around 0.78 to 0.51 (see columns 2 and 3). Third, I find evidence of a structural break in persistence, falling from 0.80 in the pre-1985 period to 0.5 afterward (columns 4 and 5). I do not find any evidence of a structural break in the intercept. I repeat the structural break analysis in columns 6 and 7, but instead consider 1991:Q1 as the break date. Results do not change.

***Robustness.*** In Online Appendix OA.1.1 I explore alternative analyses, obtaining similar results. I consider (*i*) two alternative measures of inflation, price inflation (CPI) and producer inflation (PCE), (*ii*) rolling-sample and time-varying estimates, and (*iii*) a subsample unit root analysis.

[7] I additionally test for the null of no structural break in inflation dynamics around 1985:Q1 (Bai and Perron 1998, 2003). I reject the null of no break ($p$-value = 0.000). An agnostic date test suggests that the break occurred in 1991:Q1. I report the results for the 1991:Q1 structural break in table 2, panel A columns 6 and 7.

Panel A. Estimates of regression (1).

| | (1) Full Sample | (2) 1968:Q4-1984:Q4 | (3) 1985:Q1-2020:Q2 | (4) Structural Break 1985:Q1 | | (5) Structural Break 1991:Q1 | |
|---|---|---|---|---|---|---|---|
| $\pi_{t-1}$ | 0.880*** | 0.779*** | 0.513*** | 0.816*** | 0.779*** | 0.805*** | 0.849*** |
| | (0.0466) | (0.0778) | (0.132) | (0.0481) | (0.0773) | (0.0493) | (0.0579) |
| $\pi_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$ | | | | -0.329*** | -0.266* | -0.384*** | -0.505*** |
| | | | | (0.0807) | (0.153) | (0.0843) | (0.167) |
| Constant | 0.400** | 1.353*** | 1.030*** | 1.097*** | 1.353*** | 1.075*** | 0.796*** |
| | (0.166) | (0.484) | (0.312) | (0.262) | (0.481) | (0.247) | (0.287) |
| Constant$\times \mathbb{1}_{\{t \geq t^*\}}$ | | | | | -0.323 | | 0.461 |
| | | | | | (0.574) | | (0.444) |
| Observations | 206 | 63 | 143 | 206 | 206 | 206 | 206 |

Panel B. Estimates of regression (3).

| | (1) Full Sample | (2) 1968:Q4-1984:Q4 | (3) 1985:Q1-2020:Q2 | (4) Structural Break 1985:Q1 | | (5) Structural Break 1980:Q3 | (6) Structural Break 1991:Q1 |
|---|---|---|---|---|---|---|---|
| Revision | 1.230*** | 1.414*** | 0.169 | 1.501*** | 1.414*** | 1.175*** | 1.286*** |
| | (0.250) | (0.283) | (0.193) | (0.317) | (0.281) | (0.358) | (0.264) |
| Revision $\times \mathbb{1}_{\{t \geq t^*\}}$ | | | | -1.111*** | -1.245*** | -0.854** | -0.850** |
| | | | | (0.379) | (0.341) | (0.392) | (0.377) |
| Constant | -0.0875 | 0.271 | -0.317*** | -0.135* | 0.271 | 0.700*** | 0.0886 |
| | (0.0696) | (0.185) | (0.0478) | (0.0690) | (0.184) | (0.231) | (0.140) |
| Constant $\times \mathbb{1}_{\{t \geq t^*\}}$ | | | | | -0.587*** | -1.084*** | -0.360** |
| | | | | | (0.190) | (0.237) | (0.149) |
| Observations | 197 | 58 | 139 | 197 | 197 | 197 | 197 |

HAC robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

TABLE 2. Estimates of regressions (1) and (3).

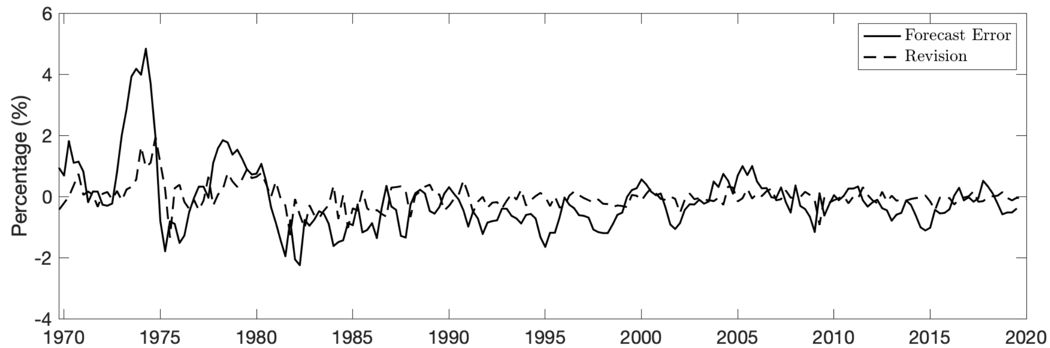



FIGURE 2. Time series of ex-ante average forecast errors and forecast revisions. *Sources: (1) First-Release Values, GDP Deflator, https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/p, and (2) Survey of Professional Forecasters, Median Forecast, GDP Deflator, https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/median-forecasts.*

### 2.2. Evidence on Information Frictions

As discussed in the introduction, the actions of the Fed have become more transparent over time. The delay between the Fed's action and the announcement to the public has been shortened from around a year to a few minutes, and the amount of information contained in the Policy Report and other documents released to the public has increased substantially. In this section, I document a contemporaneous change in beliefs around the same period on which inflation persistence is reported to break. Using survey data on US firms' forecasts, I document a significant sluggishness in responses to new information until the mid-1980s, measured by the positive co-movement of ex-ante forecast errors and forecast revisions, but no evidence of sluggishness afterward. Using expectations data from the Survey of Professional Forecasters (SPF), I study whether there is a significant change in different measures of information frictions around 1985:Q1.[8] The problem that the econometrician faces when trying to quantify or estimate the degree of information frictions is that she does not know what each agent, or the average agent, has observed at any given point in time. The literature has approached this regression design problem by measuring the change in actions after an inflow of information. Consider, for example, the average (cross-sectional mean) forecast of annual inflation at time $t$, $\overline{\mathbb{F}}_t\pi_{t+3,t}$, where $\pi_{t+3,t}$ is the GDP deflator growth between periods $t+3$ and $t-1$. One can think of this object as the

[8]The American Statistical Association and the National Bureau of Economic Research started the survey in 1968:Q4, which has been conducted by the Federal Reserve Bank of Philadelphia since 1990:Q1. Every three months, professional forecasters are surveyed on their forecasts of economic variables like output, inflation, or interest rates. These forecasters work at Wall Street financial firms, commercial banks, consulting firms, university research centers, and other private sector companies.

action that the average forecaster makes at time $t$. Let us now consider the average forecast of 4-quarters-ahead inflation at time $t-1$, $\overline{\mathbb{F}}_{t-1}\pi_{t+3,t}$. The difference between these two objects, the average forecast revision$_t \equiv \overline{\mathbb{F}}_t\pi_{t+3,t} - \overline{\mathbb{F}}_{t-1}\pi_{t+3,t}$, provides us with information about the average agent action *after* an inflow of information between periods $t$ and $t-1$. Coibion and Gorodnichenko (2012, 2015a) document a positive co-movement between ex-ante average forecast errors, denoted by forecast error$_t \equiv \pi_{t+3,t} - \overline{\mathbb{F}}_t\pi_{t+3,t}$, and average forecast revisions.[9] I plot the raw series in Figure 2. Formally, their regression design is

$$\text{forecast error}_t = \alpha_{\text{rev}} + \beta_{\text{rev}}\,\text{revision}_t + u_t, \tag{2}$$

where a positive co-movement ($\widehat{\beta}_{\text{rev}} > 0$) suggests that positive revisions predict positive forecast errors.[10] That is, after a positive revision of annual inflation forecasts, agents consistently under-predict inflation. Although I only focus on firms in the main text, this form of forecast stickiness or sluggishness is consistent across different agent types (see Coibion and Gorodnichenko (2012, 2015a) for evidence on consumers, firms, central bankers, etc.)

A positive $\beta_{\text{rev}}$ coefficient suggests that positive revisions predict positive (and larger) forecast errors, and thus, that agents underrevise their forecasts. The results, reported in the first column in table 2, panel B, reject the FIRE assumption: the measure of information frictions, $\beta_{\text{rev}}$, is significantly different from zero. A 1 p.p. revision predicts a 1.23 p.p. forecast error. The average forecast is thus smaller than the realized outcome, which suggests that the forecast revision was too small, or that forecasts react sluggishly. Following the previous analyses on inflation persistence, I assume that the break date is 1985:Q1.[11] Columns 2 and 3 report the subsample analysis, and provide preliminary evidence on a fall in the underrevision behavior since the mid-1980s.[12] Following a similar structural break analysis as in Section 2.1, I study if there is a change in expectation formation around the same break date. Formally, I test for a structural break in belief formation around 1985:Q1

[9] I use the first-release value of annual inflation, since forecasters do not have access to future revisions of the data when they provide their forecast.

[10] Under the FIRE assumption, $\beta_{\text{rev}}$ should be zero. Since an agent's information set is identical to each other agent's, the average expectation operator in (2) could be interpreted as a representative agent forecast, and one would be effectively regressing the forecast error of the representative agent on its forecast revision. Under rational expectations (RE), the forecast revision should not consistently predict the forecast error. Otherwise, the agent would incorporate this information in his information set. Therefore, a positive estimate of $\beta_{\text{rev}}$ in the above regression suggests that the FIRE assumption is violated.

[11] I test for the null of no structural break in underrevision dynamics around 1985:Q1. I reject the null of no break ($p$-value = 0.01). If I am instead agnostic about the break date(s), the test suggests that the break occurred in 1980:Q1.

[12] This result is consistent with the subsample analysis in Angeletos et al. (2020), table VI.

by estimating the following structural-break version of (2),

$$\text{(3)} \quad \text{forecast error}_t = \alpha_{\text{rev}} + \beta_{\text{rev}}\text{revision}_t + \alpha_{\text{rev}*} \times \mathbb{1}_{\{t \geq t^*\}} + \beta_{\text{rev}*} \times \mathbb{1}_{\{t \geq t^*\}}\text{revision}_t + u_t$$

A significant estimate of $\beta_{\text{rev}*}$ suggests a break in the information frictions. The results in the fourth and fifth columns in Table 2, panel B suggest that there is a structural break around 1985:Q1. The estimate $\widehat{\beta}_{\text{rev}*} < 0$ suggests that firms' forecasts underreact *less* since 1985. (In fact, I do not find any evidence of forecast stickiness.) I repeat the structural break analysis in columns 6 and 7, but considering either 1980:Q3 (the most probable date according to the unknown structural break test) or 1991:Q1 as the break dates, respectively. Results do not change.

In the lens of a noisy and dispersed information framework, this implies that agents became *more* more informed about inflation, with individual forecasts relying less on priors and more on news.[13] These structural break findings are consistent with alternative measures of information frictions, as discussed in Online Appendix OA.1.2.[14]

***Robustness.*** In Online Appendix OA.1.2 I explore alternative analyses, obtaining similar results. I consider (*i*) rolling-window and time-varying estimates of the underrevision coefficient, (*ii*) the unbalancedness in the number of respondents of the SPF, (*iii*) the response of forecast errors to monetary policy shocks, and show that they react before 1984 but not afterwards, (*iv*) the cross-sectional standard deviation of forecasts as disagreement, and show that it does not react to monetary policy shocks (inconsistent with alternative theories of belief formation), and (*v*) I repeat the analysis using the Livingston Survey on Firms, finding similar results.

***Comparison to Coibion and Gorodnichenko (2015a).*** Using the same data source, Coibion and Gorodnichenko (2015a) estimate equation (2) each quarter separately for *all* variables in the dataset, and then compute nonparametrically a local average of the estimated underrevision coefficients to report the low frequency variation in the degree of information rigidities.[15] They find that information frictions have increased sin the 1980s. They link this

[13]In Online Appendix OA.1.2, I find that ex-ante inflation forecast errors react to monetary shocks before 1985 and not after, confirming the presence of information frictions before the mid-1980s and not afterward.

[14]I conduct robustness checks studying the impulse response of ex-ante inflation forecast errors to ex-ante monetary policy shocks, the cross-sectional volatility of inflation forecasts over time, or using alternative datasets like the Livingston Survey.

[15]See section III.A (pp. 2672-74). The list of variables includes forecasts of real GDP growth, housing starts, industrial production growth, GDP deflator inflation, CPI inflation, real consumption growth, real federal government consumption and gross investment, real state and local government consumption and gross investment, real nonresidential fixed investment, real residential fixed investment unemployment rate,

finding to the onset of the *Great Moderation* (McConnell and Perez-Quiros 2000), arguing that the relative decrease in the volatility of macroeconomic variables, with respect to the increase (or moderate decrease) in microeconomic variables (Davis et al. 2006; Comin and Mulani 2006; Comin and Philippon 2005; Davis and Kahn 2008) can explain the increase in information frictions since the 1980s from a rational inattention perspective (Maćkowiak and Wiederholt 2012). I estimate equation (3) using their sample period and including forecasts of all variables included in the SPF. I report the results in table 3, panel A. I do not find evidence of information frictions before the mid-1980s, and an increase in information frictions thereafter. The findings in this paper, instead, relate *only* to inflation. Using their data on the expectations on the real GDP deflator, I find similar estimates to the ones in table 2, panel B. I report them in table 3, panel B.

These findings suggest that while information frictions (rationally decided or not) *increased* for most variables, they *declined* for inflation. A particularity of inflation is that CB actions have become more salient, either through the shortened lag and the information conveyed to the public (mid-1980s) or an explicitly set inflation target (1990s). In the theoretical framework, I show that the increase of the precision of CB actions explains the fall (*i*) in information frictions and (*ii*) in the persistence of inflation.

### 2.3. The Second Puzzle: The Phillips Curve

Unemployment in the US has fluctuated between historically large and low levels since 1985. During the Great Recession, unemployment increased to a level comparable to that of the Volcker disinflation. Shortly after that, unemployment decreased to unprecedented low levels. Throughout this period, inflation seemed to be unaffected and disconnected from the changes in the real side of the economy, with no disinflation during the Great Recession and no large inflation up to the COVID episode (Hall 2011; Ball and Mazumder 2011; Coibion and Gorodnichenko 2015b; del Negro et al. 2012; Lindé and Trabandt 2019). This contrasts with the Volcker disinflation experience, which caused a large increase in unemployment and gave rise to the concept of the "sacrifice ratio".[16]

Taking a model-oriented view, this second empirical challenge implies that the PC has flattened in recent decades, implying that inflation is no longer affected by other real variables (del Negro et al. 2020; Ascari and Fosso 2021; Atkeson and Ohanian 2001; Stock

three-month Treasury bill rate, and the quarterly average level of Moody's Aaa corporate bond yield.They find similar results if they estimate rolling window regressions for each variable and then average across these estimates.

[16]The sacrifice ratio measures the change in output per each 1 p.p. change in inflation.

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| | CG (2015) Sample | 1968:Q4-1984:Q4 | 1985:Q1-2014:Q3 | Structural Break | |
| Panel A. Sample includes forecasts of all variables in the dataset. | | | | | |
| Forecast revision | 0.0484 | -0.895 | 0.907** | -0.896 | -0.895 |
| | (0.445) | (0.623) | (0.449) | (0.620) | (0.623) |
| Revision × $\mathbb{1}_{\{t \geq t^*\}}$ | | | | 1.807** | 1.803** |
| | | | | (0.763) | (0.768) |
| Constant | -0.333** | -0.187 | -0.321** | -0.293** | -0.187 |
| | (0.134) | (0.342) | (0.130) | (0.125) | (0.341) |
| Constant × $\mathbb{1}_{\{t \geq t^*\}}$ | | | | | -0.134 |
| | | | | | (0.365) |
| Observations | 1887 | 399 | 1488 | 1887 | 1887 |
| Panel B. Sample includes forecasts of inflation. | | | | | |
| Revision | 1.193*** | 1.344*** | 0.224 | 1.447*** | 1.344*** |
| | (0.289) | (0.343) | (0.200) | (0.364) | (0.341) |
| Revision × $\mathbb{1}_{\{t \geq t^*\}}$ | | | | -1.023** | -1.120*** |
| | | | | (0.409) | (0.395) |
| Constant | 0.00200 | 0.257 | -0.223*** | -0.0552 | 0.257 |
| | (0.0835) | (0.196) | (0.0578) | (0.0803) | (0.195) |
| Constant × $\mathbb{1}_{\{t \geq t^*\}}$ | | | | | -0.479** |
| | | | | | (0.203) |
| Observations | 173 | 58 | 115 | 173 | 173 |

HAC robust standard errors in parentheses.* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE 3. Estimates of forecast underreaction, Coibion and Gorodnichenko (2015a) dataset.

and Watson 2020). The most well-known (structural) inflation equation is the NKPC,

$$\pi_t = \kappa \widetilde{y}_t + \beta \mathbb{E}_t \pi_{t+1} \tag{4}$$

which relates current inflation, $\pi_t$, to the current output gap, $\widetilde{y}_t$, and expected future inflation, $\mathbb{E}_t \pi_{t+1}$. Notice that, in this framework, inflation is *only* related to output *through* the PC slope, $\kappa$. The most prominent explanation for the lack of dependence of inflation on output is the fall in the output gap coefficient. The literature has extensively focused on this coefficient, arguing that this relation has flattened and that inflation is less dependent on any other (real) variable. The available empirical evidence is mixed, with the most recent evidence arguing for a modest decline over time (McLeay and Tenreyro 2020; Hazell et al. 2022).

In section 4.2, I argue that an extension to the benchmark model, in which the assumption of complete and full information (FI) is relaxed, enlarges the PC (4) with a lagged inflation term and myopia towards expected inflation. From the perspective of the model, the change in the dynamics of the PC can be explained by a lack of backward-lookingness and an increase in forward-lookingness after the mid-1980s, which is supported by the data. Once these additional terms have been controlled for, and I estimate a PC closer to the hybrid version implied by price-indexation settings, I find only modest evidence for a change in the slope of the PC.

### 2.4. Alternative Explanations within FIRE

In the Supplementary Material, section SM.1., I revisit different theories that produce a structural relation between inflation and other forces in the economy, and I show that they cannot explain the large fall in persistence. In the benchmark NK model, inflation inherits the properties of the exogenous driving forces. Hence, to explain the fall in inflation persistence documented in the data, a fall in the persistence of these exogenous shocks is required. I find that the persistence of exogenous monetary policy, total factor productivity, and other shocks have been remarkably stable in the post-war period. Additionally, a fall in the volatility of cost-push shocks with respect to the volatility of the other two shocks can explain a fall in the first-order autocorrelation from 0.8 to 0.745, insufficient to explain the documented fall.

I then consider changes in the reaction function of the monetary authority. The previous literature has considered the possibility of the Fed conducting a passive monetary policy before 1985, which in the lens of the theory would lead to a multiplicity of equilibria. Clarida et al. (2000) document that the inflation coefficient in the Taylor rule was well below one, not satisfying the Taylor principle, and Lubik and Schorfheide (2004) estimate an NK model under determinacy and indeterminacy and argue that monetary policy after 1982 is consistent with determinacy, whereas the pre-Volcker policy is not. I find that inflation dynamics are more persistent in the indeterminacy region, with an autocorrelation of 0.643, falling to 0.5 in the determinacy region after the mid-1980s. Another explanation put forward by McLeay and Tenreyro (2020) is that a monetary authority conducting optimal monetary policy under discretion could explain the disconnect without resorting to $\kappa$. I show that an increase in the inflation coefficient in the Taylor rule can be micro-founded through a change in the monetary stance in which the CB follows a Taylor rule in the pre-1985 period, while it follows optimal monetary policy under discretion in the post-1985 period. Persistence increases from 0.799 to 0.800. If we instead consider a change towards optimal commitment, the implied coefficient for inflation in the Taylor rule is required to increase from 1 to 4.5, which is inconsistent with empirical evidence.

Acknowledging the fact that purely forward-looking models cannot generate intrinsic persistence, I extend the benchmark and explore backward-looking frameworks. I first consider price indexation. A restricted firm resets its price (partially) indexed to past inflation, which generates anchoring in aggregate inflation dynamics. A fall in the degree of indexation could explain the fall in inflation persistence. However, the parameterization of such a parameter is not a clear one. Price indexation implies that every price is changed every period, and therefore one could not identify the Calvo-restricted firms in the data and estimate indexation. The parameter is usually estimated using aggregate data and trying to match the anchoring of the inflation dynamics, and its estimate will therefore depend on the additional model equations. Christiano et al. (2005) assume full indexation. Smets and Wouters (2007) estimate a value of 0.21 trying to match aggregate anchoring in inflation dynamics. One cannot credibly claim that indexation is the cause of the fall in inflation persistence since it needs to be identified from the macro aggregate data. The last extension is to include trend inflation, for which the literature has documented a fall from 4% in the 1947-1985 period to 2% afterward (see e.g., Ascari and Sbordone 2014; Stock and Watson 2007). Augmenting the model with trend inflation creates intrinsic persistence in the inflation dynamics through relative price dispersion, which is a backward-looking variable that has no first-order effects in the benchmark NK model. I find that the fall in trend inflation and the increase in the Taylor rule coefficients produce a small decrease in intrinsic persistence, from 0.91 to 0.84.

## 3. Noisy Information

In this section, I consider a theory of expectation formation that incorporates significant heterogeneity and sluggishness in agents' forecasts, thus relaxing the standard FIRE benchmark. I include such expectation formation features into an otherwise standard New Keynesian model by introducing noisy and dispersed information, rationally processed separately by each agent, and match the information-specific parameters to the observed sluggishness in forecasts.[17] I argue that the change in the Fed communication improved firms' information, and I use the model to show that the reduced underreaction in firms'

[17] I abstain from RE deviations. Bordalo et al. (2020) and Broer and Kohlhas (2019) find evidence of a violation of the RE assumption by regressing (2) at the individual level, finding evidence of agent over-confidence when forecasting inflation. I do not assume a departure from RE because over-confidence would have no effect on aggregate dynamics (see Angeletos et al. 2020) and would therefore not affect the main results.

inflation forecasts will translate into reduced persistence in inflation.[18] I show that in this framework, inflation is more persistent in periods of greater forecast sluggishness. Noisy information generates an underreaction to new information because individuals shrink their forecasts toward prior beliefs when the signals they observe are noisy. This endogenous anchoring in forecasts causes firms to set prices to their existing prior, thus slowing the speed of price changes. Because inflation depends on the expectations of future inflation, the change in expectation formation feeds into inflation dynamics, which endogenously reduces inflation persistence. I find that this change in firm forecasting behavior explains around 90% of the fall in inflation persistence since the mid-1980s.

### 3.1. The Noisy Information New Keynesian Model

In order to relate the previous empirical findings on inflation persistence to information frictions, I build a noisy information New Keynesian model based on the island setting by Angeletos and Huo (2021), whereas I assume that firms cannot observe past prices.[19] Firms observe the economic conditions on their island, but they do not have FI about the economic conditions in the archipielago. In particular, firms can observe their own granular conditions, such as their production given their price, but they do not have perfect information about aggregate macro variables like inflation, output, or interest rates. They observe a noisy signal that provides information on the state of the economy, in this case, the monetary policy shock. With this piece of information, firms form expectations on inflation, aggregate output, and interest rates.[20] For simplicity, I assume that households and the monetary authority have access to FI.[21]

Apart from this information friction, which I describe formally below, firms are subject to the standard Calvo-lottery price friction, which allows us to write the price-setting

[18] A concern on the gradual information disclosure argument is that market participants could observe the changes in interest rates and monetary aggregates induced by the action and could thus infer the action, in the spirit of the Grossman and Stiglitz (1980) paradox. To alleviate this concern, I measure information frictions using data from professional forecasters. The underlying assumption here is that professional forecasters are among the most informed agents in the economy since their job is to make predictions for private companies. Obtaining evidence on significant information frictions from these agents would therefore invalidate the previous criticism.

[19] As a result, firms' price-setting optimality condition can be written as a beauty contest in which firms forecast prices as opposed to inflation. Angeletos and Huo (2021) use the simplifying assumption that firms observe past prices but do not incorporate them to their information set, arguing that inflation contains little statistical information about real variables, while Vives and Yang (2016) motivate this through bounded rationality and inattention. Huo and Pedroni (2021) allow for endogenous information, but such a choice eliminates the benefit of closed-form dynamics, and the concept of persistence becomes less clear.

[20] The derivation of the model is relegated to Online Appendix OA.4.

[21] I relax the FIRE assumption on households in Online Appendix OA.2. The framework is consistent with a constant level of information frictions regarding expectations on output, and can explain a fall in the persistence of inflation from 0.808 to 0.709.

problem as a forward-looking one, and compete in a monopolistic economy. There is a continuum of firms indexed by $j \in \mathcal{I}_f = [0,1]$, each being a monopolist producing a differentiated intermediate-good variety with constant elasticity of substitution $\epsilon$, producing output $Y_{jt}$ and setting price $P_{jt}$. Technology is represented by the production function $Y_{jt} = N_{jt}^{1-\alpha}$, where $1-\alpha$ is the labor share.

***Aggregate Price Level Dynamics.*** In every period, each firm can reset its price with probability $(1-\theta)$, independent of the time of the last price change. That is, only a measure $(1-\theta)$ of firms can reset their prices in a given period, and the average duration of a price is given by $1/(1-\theta)$. Let $p_t = \log P_t$ denote the (log) aggregate price level and $p_t^* = \log P_t^*$ the (log) aggregate price set by firms that can act. Such an environment implies that aggregate price level dynamics are given (in log-linear terms) by

$$p_t = (1-\theta)\, p_t^* + \theta\, p_{t-1}, \qquad p_t^* = \int_{\mathcal{I}_f} p_{jt}^*\, dj \tag{5}$$

That is, the (log) aggregate price level at time $t$ is a weighted average of the average price set by resetters and the average price set by non-resetters, $p_{t-1}$.

***Optimal Price Setting.*** A firm re-optimizing in period $t$ will choose the price $P_{jt}^*$ that maximizes the current market value of the profits generated while the price remains effective. Formally, $P_{jt}^* = \arg\max_{P_{jt}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt} \left\{ \Lambda_{t,t+k} \frac{P_{jt} Y_{j,t+k} - W_{t+k} N_{j,t+k}}{P_{t+k}} \right\}$, where $\Lambda_{t,t+k} \equiv \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma}$ is the stochastic discount factor, and $\mathbb{E}_{jt}(\cdot)$ denotes firm $j$'s expectation conditional on *its* information set at time $t$, and subject to the sequence of demand schedules, $Y_{j,t+k} = \left( \frac{P_{jt}}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$, and their production technology. I assume that prices are set before wages. Log-linearizing the resulting first-order condition around the zero inflation steady-state, I obtain the familiar price-setting rule $p_{jt}^* = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_{jt} \left( p_{t+k} + \Theta \widehat{\text{mc}}_{t+k} \right)$, where $\widehat{\text{mc}}_t$ is the deviation of real marginal costs from steady-state, and $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$. The only difference between the price-setting rule arising in this framework and the one in the benchmark comes from the expectation operator. In the benchmark case, information sets are homogeneous and all firms allowed to act set the same price. Instead, in this framework, each firm will set a different price based on its information set.

***Aggregate Phillips curve.*** Market clearing in the goods and labor market implies that $c_t = y_t = (1-\alpha) n_t$. Using the equilibrium aggregate labor supply condition, one can write marginal costs in terms of output, $\text{mc}_t = w_t - p_t = \left( \sigma + \frac{\varphi+\alpha}{1-\alpha} \right) y_t$, where $\sigma$ is the elasticity of intertemporal substitution and $\varphi$ is the inverse Frisch elasticity. Rewriting output in terms of its gap with respect to the flexible-prices equilibrium, $p^*_{jt} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_{jt}\left[p_{t+k} + \Theta\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\widetilde{y}_{t+k}\right]$, which one can rewrite recursively as

$$p^*_{jt} = (1-\beta\theta)\mathbb{E}_{jt}\,p_t + \frac{\kappa\theta}{1-\theta}\mathbb{E}_{jt}\widetilde{y}_t + \beta\theta\mathbb{E}_{jt}\,p^*_{j,t+1} \tag{6}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. Condition (6) is intuitive: when a firm $j$ sets its price, it considers how competitive will its price be compared to the average price in the economy (playing a game of strategic complementarities with other firms), which will be the aggregate demand in the economy, and the future conditions since its price will be effective for an unknown number of periods. To derive the aggregate PC, one can aggregate condition (6) across firms. The aggregate PC can then be written as

$$\pi_t = \kappa\theta\sum_{k=0}^{\infty}(\beta\theta)^k\overline{\mathbb{E}}^f_t\,\widetilde{y}_{t+k} + (1-\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\overline{\mathbb{E}}^f_t\,\pi_{t+k} + (1-\theta)\left(\overline{\mathbb{E}}^f_t\,p_{t-1} - p_{t-1}\right) \tag{7}$$

where $\pi_t = p_t - p_{t-1}$ is the inflation rate and $\overline{\mathbb{E}}^f_t(\cdot) = \int_{\mathcal{J}_f}\mathbb{E}_{jt}(\cdot)\,dj$ is the average firm expectation operator. Compared to the standard framework, there is an additional term on the right-hand side, the result of firms not perfectly observing the previous price index. At this point, it is important to stress that to derive the aggregate PC (7) I have not yet specified an information structure. Therefore, it should be interpreted as a *general* aggregate PC.[22]

***Demand side.*** The demand side behaves as in the standard framework. Output gap dynamics are described by the standard Dynamic IS (DIS) curve (8), where the current output gap depends negatively on the expected real interest rate and positively on future aggregate demand; and nominal interest rates are set by the CB following a Taylor rule (9), in which the CB reacts to excessive inflation and output by reducing the nominal interest rates, and releases a monetary policy shock (10) that has an AR(1) structure:

$$\widetilde{y}_t = -\frac{1}{\sigma}\left(i_t - \mathbb{E}_t\pi_{t+1}\right) + \mathbb{E}_t\widetilde{y}_{t+1} \tag{8}$$

$$i_t = \phi_\pi\pi_t + \phi_y\widetilde{y}_t + \nu_t \tag{9}$$

$$\nu_t = \rho\nu_{t-1} + \sigma_\varepsilon\varepsilon^\nu_t, \qquad \varepsilon^\nu_t \sim \mathcal{N}(0,1) \tag{10}$$

[22]In the benchmark model, agents perfectly observe inflation and output, and face a symmetric Nash equilibrium game, and thus every firm acts as a representative agent firm. In such a case, the aggregate PC (7) can be simplified to the standard one, (4).

The monetary policy shock $v_t$ will be a key object in this economy. It is the only aggregate state variable, and I will assume that firms will have imperfect information on the CB's action $v_t$, consistent with the evidence on the transparency policy change by the Fed.[23]

***Information Structure.*** To generate heterogeneous beliefs and sticky forecasts, I assume that the information is incomplete and dispersed. Each firm $j$ observes a noisy signal $x_{jt}$ that contains information on the monetary shock $v_t$, and takes the standard functional form of "outcome plus noise". Formally, signal $x_{jt}$ is described as

$$x_{jt} = v_t + \sigma_u u_{jt}, \quad \text{with } u_{jt} \sim \mathcal{N}(0, 1) \tag{11}$$

where signals are agent-specific. This implies that each agent's information set is different, and therefore generates heterogeneous information sets across the population of firms.

An equilibrium must satisfy the individual-level optimal pricing policy functions (6), the aggregate DIS curve (8), the Taylor rule (9), and rational expectation formation should be consistent with the exogenous monetary shock process (10) and the signal process (11).

***Solution Algorithm.*** Here I outline the solution algorithm, and the interested reader is referred to the Proof of Proposition 1 in Appendix A. I first guess that the dynamics of the output gap are endogenous to the aggregate price index and the monetary shock: $\tilde{y}_t = a_y p_{t-1} + b_y p_{t-2} + c_y v_t$ for some unknown coefficients $(a_y, b_y, c_y)$. This allows me to write the individual price-setting condition (6) as a beauty contest in which each firm's decision will depend on its expectation of the fundamental and others' actions. I compute the expectations. For example, using the Kalman filter, one can write the expectation process as

$$\mathbb{E}_{jt}\boldsymbol{Z}_t = \boldsymbol{\Lambda}\mathbb{E}_{j,t-1}\boldsymbol{Z}_{t-1} + \boldsymbol{K}x_{jt} = (\boldsymbol{I} - \boldsymbol{\Lambda}L)^{-1}\boldsymbol{K}x_{jt} = \widetilde{\boldsymbol{\Lambda}}(L)x_{jt}, \qquad \boldsymbol{Z}_t = \begin{bmatrix} v_t & p_t & \tilde{y}_t \end{bmatrix}^\top \tag{12}$$

where I have made use of the lag operator $L$, and $\widetilde{\boldsymbol{\Lambda}}(z) = (\boldsymbol{I} - \boldsymbol{\Lambda}z)^{-1}\boldsymbol{K}$ is a polynomial matrix that depends on the guessed dynamics and the information noise $\sigma_u$.[24] I then insert these objects into firm $j$'s price policy function (6), and obtain aggregate price level dynamics. Finally, I verify the initial guess by introducing the implied price level dynamics into the

[23] Instead, the shock $v_t$ could be interpreted as an inflation target shock, such that $i_t = \phi_\pi(\pi_t - \bar{\pi}_t) + \phi_y \tilde{y}_t$ with $v_t = -\phi_\pi \bar{\pi}_t$, where $\bar{\pi}_t$ is an inflation target. Such an interpretation of the results presented in this paper would be consistent with the findings by Benati and Surico (2008), who find that countries with CBs that follow an inflation-targeting policy experience lower inflation persistence.

[24] In the case of the Kalman filter, I also need to guess the dynamics of the price level. To derive the results I use the Wiener-Hopf filter in Appendix A.

DIS curve (8). Notice that extending the benchmark framework to noisy and dispersed information generates anchoring in expectations, which now follow the autoregressive process (12). This additional anchoring will result in inflation being more intrinsically persistent in the noisy information framework, compared to the benchmark setting. The following proposition outlines inflation dynamics.

PROPOSITION 1. *Under noisy information, the price level dynamics are given by*

$$p_t = (\vartheta_1 + \vartheta_2)\, p_{t-1} - \vartheta_1\vartheta_2\, p_{t-2} - \psi_\pi \chi_\pi(\vartheta_1, \vartheta_2)\nu_t \tag{13}$$

*where* $\vartheta_1$ *and* $\vartheta_2$ *are the reciprocal of the two outside roots of the quartic polynomial*

$$\begin{aligned}\mathcal{P}(z) = -(\beta\theta - z)(1-\theta z)(z-\rho)\,(1-\rho z) - \tau z \Bigg[ &(\beta\theta - z)(1-\theta z) + z(1-\theta)(1-\beta\theta) \\ &+ z^2\kappa\theta \frac{\vartheta_1[\sigma(1-\vartheta_2) + \phi_y](\vartheta_1 + \vartheta_2 - 1 - \phi_\pi) + (1-\vartheta_2)(\phi_\pi - \vartheta_2)(\sigma + \phi_y)}{[\sigma(1-\vartheta_1) + \phi_y][\sigma(1-\vartheta_2) + \phi_y]} \\ &+ z^3\kappa\theta \frac{\vartheta_1\vartheta_2[\sigma(1-\vartheta_1)(1-\vartheta_2) - (\vartheta_1 + \vartheta_2 - 1 - \phi_\pi)\phi_y]}{[\sigma(1-\vartheta_1) + \phi_y][\sigma(1-\vartheta_2) + \phi_y]} \Bigg]\end{aligned}$$

*with*

$$\psi_\pi = \frac{\kappa}{(1-\rho\beta)[\sigma(1-\rho) + \phi_y] + \kappa(\phi_\pi - \rho)} \tag{14}$$

*and* $\chi_\pi$ *is a scalar endogenous to information frictions defined in the appendix, with* $\tau = \sigma_\varepsilon^2/\sigma_u^2$.

PROOF. See Appendix A. □

First differencing the price level dynamics (13), one can obtain the implied inflation dynamics as

$$\pi_t = (\vartheta_1 + \vartheta_2)\pi_{t-1} - \vartheta_1\vartheta_2\pi_{t-2} - \psi_\pi\chi_\pi(\vartheta_1, \vartheta_2)\Delta\nu_t \tag{15}$$

In the noisy information framework, inflation is intrinsically persistent and its persistence is governed by the new information-related parameters $\vartheta_1$ and $\vartheta_2$, as opposed to the benchmark framework in which it is only extrinsically persistent. The intuition for this result is simple: inflation is partially determined by expectations (see equation 7 under noisy information, or 4 under complete information). Under noisy information, expectations are anchored and follow an autoregressive process (see expression 12), which creates the

| Parameter | Description | Value | Source/Target |
|---|---|---|---|
| $\sigma$ | IES | 1 | Galí (2015) |
| $\beta$ | Discount factor | 0.99 | Galí (2015) |
| $\varphi$ | Inverse Frisch elasticity | 5 | Galí (2015) |
| $1-\alpha$ | Labor share | 0.75 | Galí (2015) |
| $\epsilon$ | CES between varieties | 9 | Galí (2015) |
| $\theta$ | Calvo lottery | 0.89 | Hazell et al. (2022) |
| $\rho$ | Monetary shock persistence | 0.5 | Galí (2015) |
| $\phi_\pi$ | Inflation coefficient Taylor rule | 1.5 | Galí (2015) |
| $\phi_y$ | Output gap coefficient Taylor rule | 0.125 | Galí (2015) |
| $\sigma_\varepsilon$ | Volatility monetary shock | 1 | Galí (2015) |

TABLE 4. Model parameters.

additional source of anchoring in inflation dynamics, measured by the information-related parameters $\vartheta_1$ and $\vartheta_2$.

### 3.2. Calibrating Information Frictions

In the theoretical framework, I rationalize the average forecast underreaction through expectation anchoring to priors. In this section, I calibrate the information friction parameter $\sigma_u$ to match the observed sluggishness in forecasts across time, given the rest of the parameters. I report the parameter values in Table 4. For the quantitative analysis, I use a standard parameterization in the literature, with the only exception of $\theta = 0.89$, which is calibrated to match a PC slope $\kappa = 0.03$ (mean value in the literature, reported in Hazell et al. 2022). Finally, I calibrate $\tau = 0.0715$ in the pre-1985 sample to match the empirical evidence on $\beta_{\text{rev}}$ in Table 2, panel B.

As argued before, the signal noise became more precise in the dispersed-information model lens. In the next proposition, I relate the previous empirical findings on expectations to model-implied inflation persistence.

PROPOSITION 2. *The theoretical counterpart of the coefficient $\beta_{rev}$ in (2) is given by*

$$\beta_{rev} = \frac{\lambda^3\rho(1-\vartheta_1\lambda)(1-\vartheta_2\lambda)}{(1-\lambda^4)(\rho-\lambda)}\left\{\lambda\frac{\prod_{j=1}^4(\lambda-\xi_j)}{\prod_{k=1}^2(\lambda-\vartheta_k)} - \frac{1-\lambda^2}{\vartheta_1-\vartheta_2}\sum_{k=1}^{2}\frac{\vartheta_k\prod_{j=1}^4(\vartheta_k-\xi_j)}{(1-\lambda\vartheta_k)(\lambda-\vartheta_k)}\right\} \tag{16}$$

*where $\lambda$ is the inside root of the quadratic polynomial $\mathcal{Q}(z) = (1-\rho z)(z-\rho)+\tau z$, and $(\xi_1, \xi_2, \xi_3, \xi_4)$ are the reciprocals of the roots of the quartic polynomial $\mathcal{Q}_2(z) = \phi_0 + \phi_1 z + \phi_2 z^2 + \phi_3 z^3 + \phi_4 z^4$,*

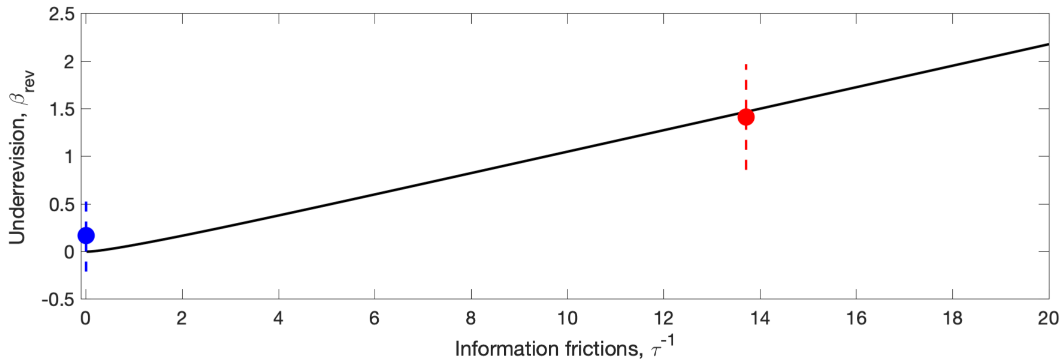



FIGURE 3. Coefficient $\beta_{rev}$ and information frictions $\tau^{-1}$. In red, is the estimated underrevision coefficient (with 95% confidence interval, dashed line) before 1985. In blue, is the estimated underrevision coefficient (with 95% confidence interval, dashed line) after 1985.

*where* $\phi_0 = -\psi_\pi \chi_\pi$, $\phi_1 = \left(\frac{1}{\lambda} - \frac{1}{\rho}\right)\phi_0$, $\phi_2 = \frac{(\rho-\lambda)\phi_0}{\lambda^2\rho}$, $\phi_3 = \frac{(\rho-\lambda)\phi_0[\lambda^3-\vartheta_1-\vartheta_2+\lambda\vartheta_1\vartheta_2]}{\lambda^2\rho(1-\lambda\vartheta_1)(1-\lambda\vartheta_2)}$, *and* $\phi_4 = \frac{-\lambda^3+\lambda^4\vartheta_2+\lambda^4\vartheta_1-\vartheta_1\vartheta_2[\lambda-(1-\lambda^4)\rho]}{\lambda^2\rho(1-\lambda\vartheta_1)(1-\lambda\vartheta_2)}$.

PROOF. See Appendix A. □

The empirical results reported in section 2.2 support a fall in information frictions in recent decades. Proposition 2 maps the theoretical degree of information frictions, $\sigma_u$, with the Coibion and Gorodnichenko (2015a) estimate. It introduces the model-implied $\beta_{rev}$ coefficient, which depends on the monetary policy shock persistence $\rho$ and on the information-related parameters $\vartheta_1$, $\vartheta_2$ and $\lambda$, where $\lambda$, in turn, depends on the persistence parameter and the signal-to-noise ratio. In the noisy information framework, $\beta_{rev}$ is strictly positive and increases with the degree of information frictions. I show this graphically in Figure 3. In the model lens, this underrevision is the consequence of individual anchoring to priors and generates forecast underreaction at the aggregate level.

## 4. Results

### 4.1. Inflation Persistence

In the noisy information framework, inflation persistence is governed by $\vartheta_1$ and $\vartheta_2$. Formally, one can write the inflation first-order autocorrelation as $\rho_1 = \frac{(1+\rho)(\vartheta_1+\vartheta_2)+(1-\rho)(\vartheta_1\vartheta_2-1)}{1+\rho\vartheta_1\vartheta_2}$, which is increasing in both $\vartheta_1$ and $\vartheta_2$. Since the ultimate goal is to understand the break in inflation persistence documented in Section 2.1, the following proposition exposes the determinants of $\vartheta_1$ and $\vartheta_2$, and provides analytical comparative statics.

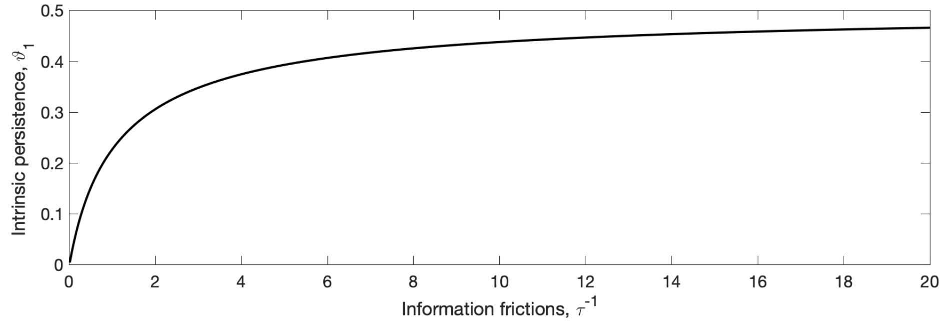
PROPOSITION 3. *The persistence parameters satisfy* (*i*) $\vartheta_1 \in (0, \rho)$, (*ii*) $\vartheta_1$ *is increasing in* $\sigma_u$, (*iii*) $\vartheta_2 \in (\theta, 1)$, *and* (*iv*) $\vartheta_2$ *is decreasing in* $\sigma_u$.

PROOF. See Appendix A. □

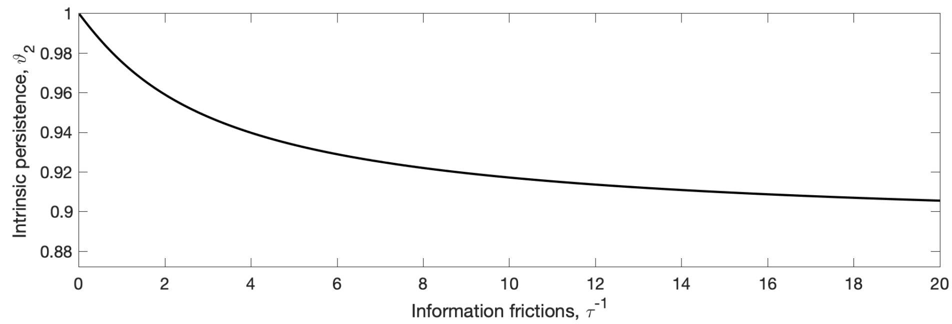
Inflation persistence and information frictions are related through $\vartheta_1$ and $\vartheta_2$. The above proposition is key to understanding the time-varying properties of inflation persistence. First, part (*i*) establishes that $\vartheta_1$ is bounded by 0 and $\rho$. Part (*ii*) states that $\vartheta_1$ is increasing in the degree of information frictions, formalized via the noise of the signal innovation $\sigma_u$. A decrease in information frictions reduces inflation first-order autocorrelation through a de-anchoring of individual inflation expectations, which would in turn de-anchor inflation dynamics. Figure 4A plots the level of intrinsic persistence $\vartheta_1$ for different degrees of information frictions, measured by $\tau^{-1}$. Part (*iii*) establishes that $\vartheta_2$ is bounded by $\theta$ and 1. Part (*iv*) states that $\vartheta_2$ is decreasing in the degree of information frictions. A decrease in information frictions increases inflation first-order autocorrelation through anchoring of individual inflation expectations, which would in turn anchor inflation dynamics. Figure 4B plots the level of intrinsic persistence $\vartheta_2$ for different degrees of information frictions. In the limit of no information frictions $\sigma_u \to 0$, $\vartheta_1 \to 0$ and $\vartheta_2 \to 1$.

Information frictions do, therefore, have opposing effects on persistence. On the one hand, information frictions lead to an additional persistence through an increase in $\vartheta_1$, the standard mechanism in Angeletos and Huo (2021). On the other hand, there is an additional component $\vartheta_2$ that is decreasing in information frictions. This element arises from the fact that I am solving the model in prices, instead of inflation as in Angeletos and Huo (2021) or as in the benchmark setting in Galí (2015) in which prices follow a unit root. Looking at the price level dynamics (5), when firm $j$ forecasts the aggregate price level $p_t$, she needs to forecast the average action by other firms $p_t^*$, but also backcast the aggregate price level in the past $p_{t-1}$ (see equation 5). Information frictions relax the forward-lookingness of the model equations, as formalized by Gabaix (2020); Angeletos and Huo (2021), resulting in price level dynamics no longer following a unit root. In the frictionless limit, prices follow a unit root, formalized by $\vartheta_2 \to 1$. However, as shown in Figure 4C, the net result of an increase in information frictions is an increase in the first-order autocorrelation. These key results, coupled with the next result introduced in Proposition 2, explain the overall fall in inflation persistence.

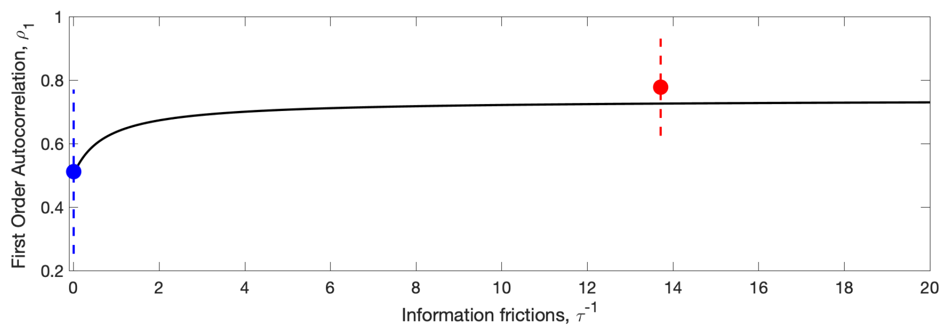
The key finding is that $\beta_{\text{rev}}$ and $\rho_1$, the theoretical counterparts of Coibion and Gorodnichenko (2015a) underrevision estimate and inflation persistence, are closely related as I show in Figure 4D, and the fall in the first-order autocorrelation can be explained by a fall in information frictions. Propositions 1-2 establish a direct relationship between the



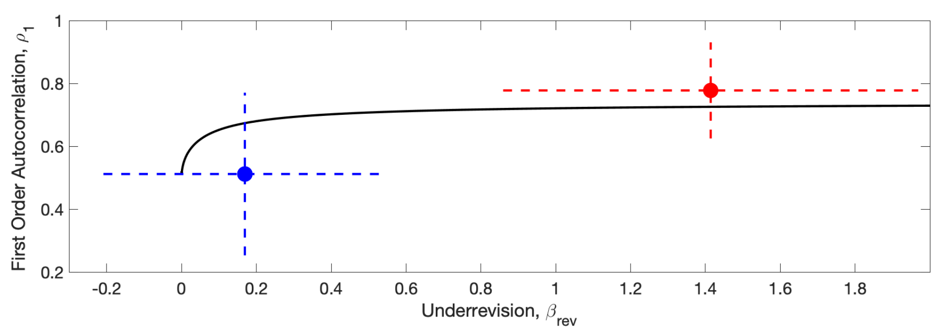

A. Intrinsic persistence $\vartheta_1$ and information frictions $\tau^{-1}$




B. Intrinsic persistence $\vartheta_2$ and information frictions $\tau^{-1}$




C. First-Order Autocorrelation $\rho_1$ and information frictions $\tau^{-1}$




D. First-Order Autocorrelation $\rho_1$ and information frictions $\beta_{rev}$

FIGURE 4. Comparative statics. In red, estimated first-order autocorrelation and underrevision coefficients (with 95% confidence interval, dashed line) before 1985. In blue, estimated first-order autocorrelation and underrevision coefficients (with 95% confidence interval, dashed line) after 1985.

first-order autocorrelation of inflation $\rho_1$ and $\beta_{\text{rev}}$, our empirical measure of information frictions. Figure 4D shows graphically the monotonically increasing relation between inflation persistence and $\beta_{\text{rev}}$. In the initial pre-1985 period, with $\beta_{\text{rev}} = 1.501$ from table 2, panel B, the model-implied inflation first-order autocorrelation is $\rho_1 = 0.728$. In the post-1985 period, with no information frictions, the first-order autocorrelation falls to $\rho_1 = \rho = 0.5$, which is the persistence of the monetary policy shock in the benchmark framework (see Galí 2015). Comparing our model results to the empirical analysis in Tables 1 and 2 (panel A), I find that the noisy information framework produces persistence dynamics that lie within the 95% confidence interval, and can explain around 90% of the fall in the point estimate.

***Role of Calvo Friction.*** In this framework, the first-order autocorrelation of inflation depends on the degree of information frictions, summarized by the two roots $\vartheta_1$ and $\vartheta_2$. A key parameter affecting the transmission of information frictions to the economy is the Calvo inaction probability $\theta$ since it regulates the degree of strategic complementarities on firms' actions. To see this, insert the aggregate price level dynamics (5) into firm *j*'s best response (6), $p^*_{jt} = (1-\beta\theta)(1-\theta)\sum_{k=0}^{\infty}\theta^k \mathbb{E}_{jt}\, p^*_{t-k} + \frac{\kappa\theta}{1-\theta}\mathbb{E}_{jt}\tilde{y}_t + \beta\theta\mathbb{E}_{jt}\, p^*_{j,t+1}$. An increase in the Calvo inaction probability has opposing effects on the degree of strategic complementarities within firms. On the one hand, an increase in $\theta$ reduces the impact of expected past aggregate actions through a smaller coefficient $(1-\beta\theta)(1-\theta)$. On the other hand, it increases the memory of past expectations on today's actions, $\sum_{k=0}^{\infty}\theta^k\mathbb{E}_{jt}\, p^*_{t-k}$. It seems natural, however, that this second effect dominates when I look at inflation persistence. Formally, I showed in proposition 3 that $\vartheta_2$ is bounded from below by $\theta$ and that it is decreasing in the degree of information frictions $\sigma_u$. Therefore, a high $\theta$ limits the sensitivity of the root $\vartheta_2$ to changes in $\sigma_u$ and helps in generating the increase in the first-order autocorrelation after an increase in $\sigma_u$, given that $\rho_1$ is increasing in $\vartheta_2$.

The calibration of the Calvo pricing friction implies a mean price duration of 7.8 quarters. This estimate is in the upper range in the micro literature, although aligned with the macro literature. Bils and Klenow (2004); Klenow and Kryvtsov (2008); Nakamura and Steinsson (2008); Goldberg and Hellerstein (2009) find a median price duration of 4.5-11 months in US micro data. Galí (2015) sets $\theta = 0.75$ to match an implied duration of 1 year. Christiano et al. (2011) set $\theta = 0.85$. Auclert et al. (2020); Afsar et al. (2021) estimate $\theta$ between 0.88 and 0.93 from macro data, implying a price duration of 12-14 quarters. In Figure 5, I plot the implied first-order autocorrelation for different values of the Calvo price friction in the range of the literature. Depending on this parameter, the noisy information framework explains between 40% and 100% of the fall in the point estimate in the

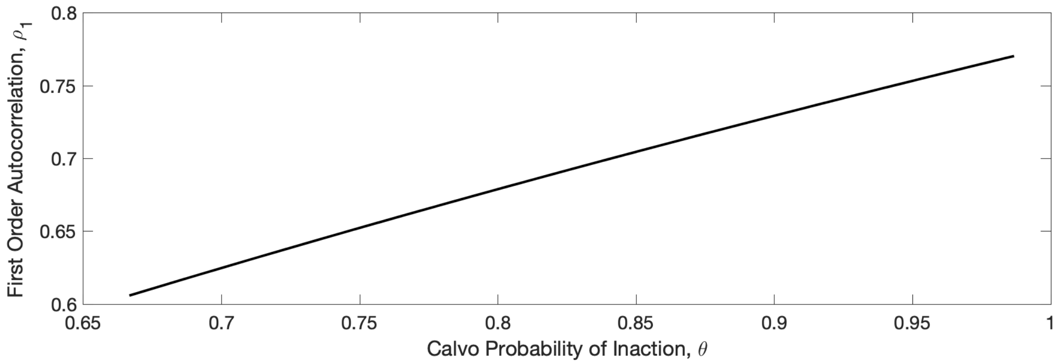



FIGURE 5. First-order autocorrelation $\rho_1$ and price friction $\theta$.

first-order autocorrelation.

### 4.2. The Phillips Curve

In this section, I argue that after controlling for changes in information frictions, the decline in the slope of the PC appears to be relatively modest. I find evidence that a more significant factor in the dynamics of the PC is the shift towards greater forward-lookingness and less backward-lookingness. I conduct two main exercises. First, in a more theoretical exercise, I use the noisy information framework to rewrite the inflation dynamics as an *as if* FIRE setting with wedges (Angeletos and Huo 2021). According to my theory, the PC (4) needs to be extended with a lagged inflation term and myopia towards expected inflation in the pre-1985 sample period.[25] Once these additional terms are controlled for, and I estimate a PC close to the hybrid version implied by price-indexation settings, I do not find any evidence of a change in $\kappa$, but rather a decrease in backward-lookingness. Second, by relaxing the FIRE assumption but without any belief structure restriction, the PC is instead given by (7). Instead of replacing expectations of future inflation by its realization, as the literature generally does when estimating condition (4), I use the survey forecasts to estimate (7), and I only find evidence of a modest change in the slope.

***The Wedge Phillips Curve.*** Next, I argue that once I consider a micro-founded PC that takes into account noisy information, I do not find any evidence of a change in the slope of the PC. Furthermore, I show that the key drivers behind the change in the dynamics of the PC are the fall in its backward-lookingness and the increase in its forward-lookingness after the mid-1980s

[25]The derivation of the PC relies on the FIRE assumption (and, implicitly, on the Law of Iterated Expectations at the aggregate level), for which I find a strong rejection in the data.

Let us first recall inflation dynamics in the standard model. In the benchmark model, the PC is given by (4), the DIS curve is given by (8), the Taylor rule is given by (9) and the monetary policy shock process is given by (10). Inserting the Taylor rule (9) into the DIS curve (8), one can write the model as a system of three first-order stochastic difference equations with reduced-form dynamics $\boldsymbol{x}_t = \delta \mathbb{E}_t \boldsymbol{x}_{t+1} + \varphi v_t$, where $\boldsymbol{x}_t = [\tilde{y}_t \quad \pi_t \quad p_t]^\top$ is a $3 \times 1$ vector containing output, inflation, and prices, $\delta = \begin{bmatrix} \frac{\sigma}{\sigma+\phi_y+\kappa\phi_\pi} & \frac{1-\beta\phi_\pi}{\sigma+\phi_y+\kappa\phi_\pi} & 0 \\ \frac{\sigma\kappa}{\sigma+\phi_y+\kappa\phi_\pi} & \frac{\kappa+\beta(\sigma+\phi_y)}{\sigma+\phi_y+\kappa\phi_\pi} & 0 \\ 0 & -1 & 1 \end{bmatrix}$, and $\varphi = \frac{1}{\sigma+\phi_y+\kappa\phi_\pi}\begin{bmatrix}-1 & -\kappa & 0\end{bmatrix}^\top$. Angeletos and Huo (2021) show that, using the noisy information dynamics (A.13) and (15), one can reverse engineer an *as if* system dynamics that mimics the dynamics of the noisy information model albeit maintaining the FIRE assumption, such that the following ad-hoc system of equations $\boldsymbol{x}_t = \omega_b \boldsymbol{x}_{t-1} + \delta\omega_f \mathbb{E}_t \boldsymbol{x}_{t+1} + \varphi v_t$ satisfies the model dynamics for some pair of $3 \times 3$ matrices $(\omega_b, \omega_f)$. The next proposition states that under a certain pair $(\omega_b, \omega_f)$, the ad-hoc economy produces the same dynamics of the noisy information framework.

PROPOSITION 4. *The ad-hoc hybrid dynamics $\boldsymbol{x}_t = \omega_b \boldsymbol{x}_{t-1} + \delta\omega_f \mathbb{E}_t \boldsymbol{x}_{t+1} + \varphi v_t$ produces identical dynamics to the noisy information model if $(\omega_b, \omega_f)$ satisfy*

$$
\begin{aligned}
B - \varphi &= \delta\omega_f(AB + \rho B) \\
\omega_b &= (I_3 - \delta\omega_f A)A
\end{aligned}
\tag{17}
$$

*where*

$$
A = \begin{bmatrix} 0 & -b_y & a_y + b_y \\ 0 & \vartheta_1\vartheta_2 & -(1-\vartheta_1)(1-\vartheta_2) \\ 0 & \vartheta_1\vartheta_2 & \vartheta_1 + \vartheta_2 - \vartheta_1\vartheta_2 \end{bmatrix}, \quad B = \begin{bmatrix} -\psi_y\chi_y(\vartheta_1,\vartheta_2) \\ -\psi_\pi\chi_\pi(\vartheta_1,\vartheta_2) \\ -\psi_\pi\chi_\pi(\vartheta_1,\vartheta_2) \end{bmatrix}
$$

$$
a_y = \frac{\vartheta_1[\sigma(1-\vartheta_2)+\phi_y](\vartheta_1+\vartheta_2-1-\phi_\pi) + (1-\vartheta_2)(\phi_\pi-\vartheta_2)(\sigma+\phi_y)}{[\sigma(1-\vartheta_1)+\phi_y][\sigma(1-\vartheta_2)+\phi_y]}
$$

$$
b_y = \frac{\vartheta_1\vartheta_2[\sigma(1-\vartheta_1)(1-\vartheta_2) - (\vartheta_1+\vartheta_2-1-\phi_\pi)\phi_y]}{[\sigma(1-\vartheta_1)+\phi_y][\sigma(1-\vartheta_2)+\phi_y]}
$$

*with $\psi_y$ and $\chi_y$ defined in Appendix A. In particular, the "as if" FIRE PC dynamics are described by*

$$
\pi_t = \omega_\pi \pi_{t-1} + \omega_p p_{t-1} + \kappa\tilde{y}_t + \delta_y \mathbb{E}_t \tilde{y}_{t+1} + \delta_\pi \beta \mathbb{E}_t \pi_{t+1} + \delta_p \mathbb{E}_t p_{t+1}
\tag{18}
$$

*where* $(\omega_\pi, \omega_p, \delta_y, \delta_\pi, \delta_p)$ *depend on the* $(\boldsymbol{\omega}_b, \boldsymbol{\omega}_f)$ *pair, and expectation operators satisfy the FIRE assumption.*

PROOF. See Appendix A. □

The FIRE wedge PC (18), together with a wedge IS curve derived in Appendix A, produces identical dynamics to the noisy information setup derived in section 3. Notice that, to derive similar dynamics in a FIRE setup, the PC needs to be extended with intrinsic persistence and myopia. Since all new terms depend on the degree of information frictions $\sigma_u$, the model predicts that changes in beliefs will affect the dynamics of the PC. In particular, the model predicts that as information frictions vanish, i.e., in the benchmark model with no information frictions, I have $\omega_{b,11} = \omega_{b,12} = \omega_{b,21} = \omega_{b,22} = \omega_{f,12} = \omega_{f,21} = 0$ and $\omega_{f,11} = \omega_{f,22} = 1$. As a result, $\omega_\pi = \omega_p = \delta_y = \delta_p = 0$, $\delta_\pi = 1$ and the PC is reduced to the *purely* forward-looking curve (4).

I now test this theoretical prediction in the data by estimating the wedge PC (18), allowing for a structural break in all coefficients after 1985. I proxy the output gap term using the CBO Output Gap, replace expectations of future variables with realized future variables and estimate the equation using the generalized method of moments (GMM). In the estimation exercise, I focus particularly on the theoretical prediction of a lack of backward-lookingness post-1985, instead of the slope analysis.[26]

In table 5 column 1, I report the estimated coefficients for the full sample exercise. I find that only inflation-related coefficients are significant, suggesting support for backward-lookingness and significant myopia (coefficient well below the discount factor $\beta = 0.99$). I report the structural break results in columns 2 and 3. In column 2 I only allow for a structural break on the contemporaneous output gap coefficient. I find no evidence of a structural break in the slope (i.e., no evidence of flattening in the PC). In column 3 I explore if there has been any other structural break in the dynamics of the PC. Consistent with my previous findings on belief formation, I find a structural break in lagged and forward inflation: in recent decades the PC has become *more* forward-looking and *less* backward-looking. This last result aligns well with the documented fall in the persistence of inflation and information frictions, and with the mechanism dynamics proposed by the noisy information framework, suggesting that the fall in the first-order autocorrelation of inflation can be explained by a lack of intrinsic persistence after the mid-1980s.

Notice that condition (17) does not uniquely determine the set of weights $\boldsymbol{\omega}_f$ that is consistent with the noisy information dynamics. Different weights in $\boldsymbol{\omega}_f$ are consistent

[26]As stressed in Hazell et al. (2022), aggregate macroeconomic data does not have enough power to detect the slope of the PC.

| | (1) Wedge PC | (2) Break Output | (3) Break All | (4) Hybrid PC | (5) Break Hybrid | (6) Unemployment | (7) Gap |
|---|---|---|---|---|---|---|---|
| $\pi_{t-1}$ | 0.447*** | 0.496*** | 0.738*** | 0.453*** | 0.720*** | 0.665*** | 0.643*** |
| | (0.0978) | (0.110) | (0.165) | (0.0891) | (0.131) | (0.120) | (0.120) |
| $\pi_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$ | | | -0.671** | | -0.597** | -0.573** | -0.555** |
| | | | (0.267) | | (0.232) | (0.260) | (0.254) |
| $p_{t-1}$ | 0.00354 | -0.0164 | 0.0363 | | | | |
| | (0.0162) | (0.0243) | (0.202) | | | | |
| $p_{t-1} \times \mathbb{1}_{\{t \geq t^*\}}$ | | | -0.105 | | | | |
| | | | (0.256) | | | | |
| $\widetilde{y}_t$ | 0.0633 | 0.134 | 0.378 | -0.000813 | 0.0566 | -0.0352 | -0.0303 |
| | (0.112) | (0.123) | (0.240) | (0.0166) | (0.0488) | (0.0374) | (0.0710) |
| $\widetilde{y}_t \times \mathbb{1}_{\{t \geq t^*\}}$ | | -0.0726 | -0.127 | | -0.0143 | -0.00470 | -0.0169 |
| | | (0.0632) | (0.291) | | (0.0781) | (0.0548) | (0.0988) |
| $\widetilde{y}_{t+1}$ | -0.0695 | -0.104 | -0.343 | | | | |
| | (0.126) | (0.125) | (0.212) | | | | |
| $\widetilde{y}_{t+1} \times \mathbb{1}_{\{t \geq t^*\}}$ | | | 0.0963 | | | | |
| | | | (0.328) | | | | |
| $\pi_{t+1}$ | 0.540*** | 0.509*** | 0.226 | 0.539*** | 0.273** | 0.354*** | 0.344*** |
| | (0.102) | (0.108) | (0.215) | (0.0885) | (0.129) | (0.115) | (0.121) |
| $\pi_{t+1} \times \mathbb{1}_{\{t \geq t^*\}}$ | | | 0.866*** | | 0.643*** | 0.687** | 0.603** |
| | | | (0.335) | | (0.244) | (0.322) | (0.269) |
| Observations | 202 | 202 | 202 | 202 | 202 | 202 | 202 |

HAC (1 lag) robust standard errors in parentheses. Instrument set: four lags of the effective federal funds rate, CBO Output gap, GDP Deflator growth rate, Commodity Inflation, M2 growth rate, spread between long- and short-run interest rate and labor share. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE 5. Estimates of regression (18).

with noisy information dynamics, although the dynamics are unique.[27] I explore which set of wedges $(\boldsymbol{\omega}_b, \boldsymbol{\omega}_f)$ is consistent with the documented dynamics and with the findings in table 5. Since I do not find any evidence of the relevance of the lagged price level and the forward output gap (column 1), I choose wedges such that they produce the well-known hybrid PC. The following corollary provides us with the hybrid wedge PC.

COROLLARY 1. *The hybrid PC* $\pi_t = \omega_\pi \pi_{t-1} + \kappa \widetilde{y}_t + \delta_\pi \beta \mathbb{E}_t \pi_{t+1} + \chi v_t$ *produces identical dynamics to the "as if" FIRE PC* (18), *where* $(\omega_\pi, \delta_\pi, \chi)$ *depend on the* $(\boldsymbol{\omega}_b, \boldsymbol{\omega}_f)$ *pair. As information frictions vanish,* $\omega_\pi = \chi = 0$ *and* $\delta_\pi = 1$.

PROOF. See Appendix A. □

As before, the noisy information model suggests that intrinsic persistence and myopia in the hybrid PC should vanish in the post-1985 sample. Estimating the micro-founded hybrid PC, reported in table 5 (columns 4 and 5), I fail to reject the null that, since the structural break in 1985:Q1, (*i*) anchoring has gone to zero and (*ii*) myopia has disappeared. I repeat the analysis by replacing the CBO output gap with the unemployment rate or the CBO unemployment gap, and I find similar results (see columns 6 and 7). The true "elephant in the room" is the shift towards greater forward-lookingness, rather than a (potential) modest decline in the slope of the PC.

***Controlling for Imperfect Expectations.*** To obtain the results on inflation persistence, I have assumed a particular belief structure, RE but noisy and dispersed information. In this section, I take an agnostic stance on expectation formation. I start the analysis from the aggregate PC (7), derived under no assumptions on beliefs. In this case, inflation is related to current *and future* output through two different channels: the slope of the PC, $\kappa$, and firms' expectation formation process, $\overline{\mathbb{E}}_t^f(\cdot)$. To test for a potential structural break in the slope *controlling for* non-standard expectations, I regress the *general* PC (7), truncated at $k = 4$, for which I do not assume a particular information structure, using real GDP and GDP Deflator growth forecast data from the SPF. I set $\beta$ and $\theta$ to their values in table 4, and regress

$$\pi_t = \alpha_1 + \alpha_2 \widetilde{y}_t^e + \alpha_3 \pi_t^e + \eta_t \tag{19}$$

where $\alpha_2 = \kappa$, $\alpha_3 = 1 - \theta$, $\widetilde{y}_t^e = \theta \sum_{k=0}^{4} (\beta\theta)^k \overline{\mathbb{E}}_t^f \widetilde{y}_{t+k}$ and $\pi_t^e = \sum_{k=0}^{4} (\beta\theta)^k \overline{\mathbb{E}}_t^f \pi_{t+k}$ denote the truncated sums of the expected output gap and inflation, respectively, and $\eta_t = (1 - \theta)\left(\overline{\mathbb{E}}_t^f p_{t-1} - p_{t-1}\right) +$ truncation error is the error term. I use standard GMM methods by

[27] Intuitively, agents' actions can be anchored/myopic concerning aggregate output or inflation.

| | Unemployment | | Real GDP Growth | |
|---|---|---|---|---|
| | Full Sample | Structural Break | Full Sample | Structural Break |
| $\widetilde{y}_t^e$ | -0.00519*** | -0.0231*** | -0.0128 | 0.0245 |
| | (0.00171) | (0.00679) | (0.0133) | (0.0224) |
| $\widetilde{y}_t^e \times \mathbb{1}_{\{t \geq t^*\}}$ | | 0.0133*** | | -0.0403** |
| | | (0.00493) | | (0.0201) |
| $\pi_t^e$ | 0.282*** | 0.342*** | 0.258*** | 0.251*** |
| | (0.0109) | (0.0261) | (0.00999) | (0.0108) |
| Observations | 199 | 199 | 199 | 199 |

HAC (1 lag) robust standard errors in parentheses. Instrument set: four lags of forecasts of annual real GDP growth and annual GDP Deflator growth. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE 6. Estimates of regression (19).

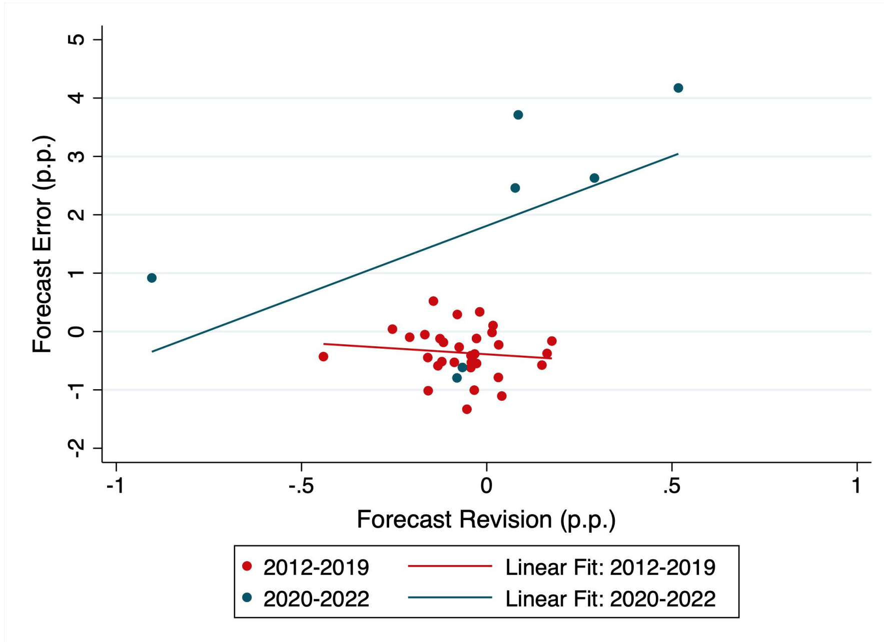
instrumenting for expectations with 4-quarter lagged annual inflation and output gap expectations. The results are reported in table 6. In column 1, I report the full sample estimation using unemployment expectations as a proxy for the output gap. I find that $\kappa$ is small and similar to the value found by Hazell et al. (2022). In column 2, I regress its (output) structural break version. I find evidence for a moderate fall in the slope of the PC. Columns 3 to 4 report the results of the same analysis, using real GDP growth expectations as a proxy for the output gap. I find similar results.[28]

***Summary.*** I find that once I control for imperfect expectations and a potential change in their dynamics, I only estimate a modest decline in the slope of the PC since the mid-1980s. First, I showed that the noisy information model can explain the change in the dynamics of the PC as a reshuffle between backward-lookingness and forward-lookingness via changes in belief formation. Second, I documented empirically that controlling for non-standard expectations, proxied by the forecasts submitted by professional forecasters, I find evidence for a fall in the slope of the PC from 0.023 to 0.010.

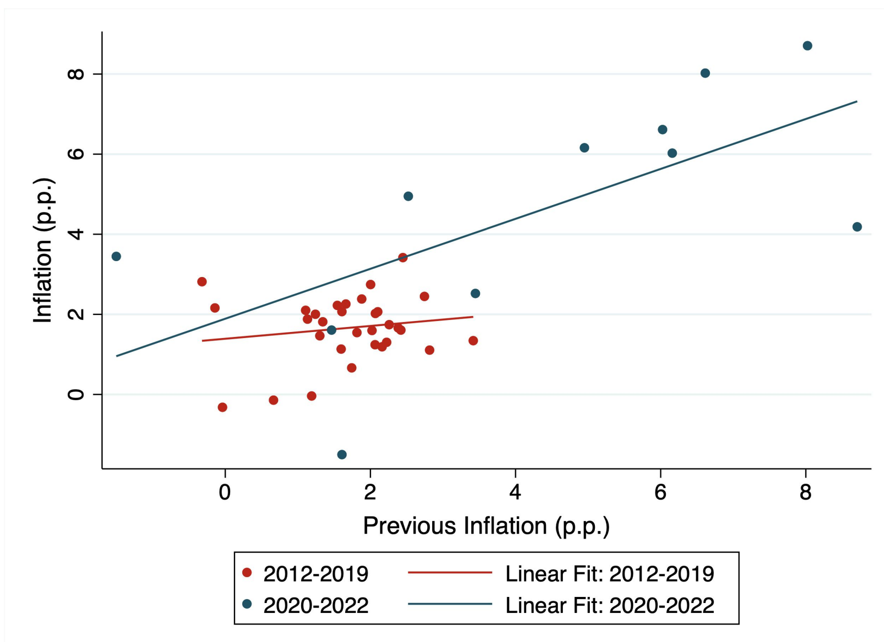
## 5. Conclusion

In this paper, I explain the fall in inflation persistence since the mid-1980s through changes in beliefs. State-of-the-art monetary models face significant challenges in explaining this fall in inflation persistence. I show that, by relaxing the FIRE assumption in the

[28] I repeat the analysis using the Livingston Survey in Online Appendix OA.1 and find similar results.




A. Scatter plot of ex-ante average forecast error (vertical axis) and average forecast revisions (horizontal axis), computed using the SPF and vintage GDP Deflator data. Red dots correspond to 2012-2019 observations, and blue dots correspond to observations after 2019.




B. Scatter plot contemporaneous inflation (vertical axis) and one-quarter lagged inflation (horizontal axis). Red dots correspond to 2012-2019 observations, and blue dots correspond to observations after 2019.

FIGURE 6. Scatter plots of forecast underrevision and inflation's first-order autocorrelation.

benchmark NK framework, the model can generate the documented fall in persistence. Using micro-data on inflation expectations from the Survey of Professional Forecasters (SPF), I argue that agents became more informed about inflation after the change in the Federal Reserve disclosure policy, which endogenously lowers the intrinsic persistence in inflation dynamics.

I revisit theories that produce a structural relation between inflation and other economic forces. I show that a variety of NK models cannot explain the fall in inflation persistence. Since the benchmark model is purely forward-looking, inflation exhibits no intrinsic persistence, and its dynamic properties are now inherited from monetary policy shocks. However, I document that the persistence of monetary policy shocks has not changed over time. Acknowledging that purely forward-looking models cannot generate anchoring or intrinsic persistence, I extend the benchmark model to incorporate a backward-looking dimension. I show that the change in the monetary stance now affects inflation's intrinsic persistence. The effect is small, however. Then, I show that the noisy and dispersed information extension is consistent with the micro-data evidence on belief formation, and generates anchoring or intrinsic inflation persistence. Using SPF data, I document that a structural break in expectation formation, resulting in agents being more informed about inflation, is contemporaneous with the fall in inflation persistence. The model can therefore explain the fall in inflation persistence in a micro-consistent manner.

I discuss the consequences of noisy and dispersed information on the dynamics of the PC and the lack of flattening. In the noisy information model, the PC is enlarged with anchoring and myopia. Consistent with the theory, I find that both anchoring and myopia vanish after the reduction of information frictions in the mid-1980s. Finally, taking an agnostic stance on expectations, I show that there is evidence of only a modest decline in the slope of the PC, once I control for imperfect expectations.

***Will the 2020-2022 inflation be persistent?*** In this paper, I have only considered data up until 2020:Q2. The evidence provided points towards a lessening in the underrevision behavior of agents and a fall in inflation persistence since the mid-1980s. Taking these results together would make the reader conclude that current inflation will only be temporary (or, at least, less persistent than before the mid-1980s). However, having a look at the 2020:Q2-2022:Q2 data, one could argue that the underrevision behavior (see figure 6A) and inflation persistence (see figure 6B) are striking back.[29] Although admittedly speculative, these

[29] I include the 2012-2019 period to show that the increase in information frictions and persistence is not driven by forward guidance, a major policy change that could, in principle, have dampened agents' understanding of monetary policy due to its novelty.

findings suggest that CBs should focus on their communication in the coming quarters if they want to reduce the current inflation persistence.[30] This theory is imperfect, however, since it abstains from cost-push shocks and the bottlenecks arising from the input-output network of the economy. This suggests avenues for follow-up research, in which belief formation frictions interact with the input-output structure of the economy.

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[30] Alternatively, one could interpret this finding from a rational inattention perspective: given lower inflation volatility in the recent period, agents optimally chose to pay less attention to inflation, giving rise to information frictions.

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# Appendix A. Proofs of Propositions

***Proof of Proposition 1.*** Under noisy information on the firm side, the individual price policy functions are given by (6). Let us guess that the equilibrium output gap dynamics will take the form of

$$\widetilde{y}_t = a_y p_{t-1} + b_y p_{t-2} + c_y v_t \tag{A.1}$$

Making use of the guess I can rewrite the price-setting condition as

$$p^*_{it} = \frac{\kappa\theta c_y}{1-\theta}\mathbb{E}_{it}v_t + \frac{\kappa\theta b_y}{1-\theta}\mathbb{E}_{it}p_{t-2} + \frac{\kappa\theta a_y}{1-\theta}\mathbb{E}_{it}p_{t-1} + (1-\beta\theta)\mathbb{E}_{it}p_t + \beta\theta\mathbb{E}_{it}p^*_{i,t+1} \tag{A.2}$$

I now turn to solve the expectation terms in (A.2). I can write the fundamental representation of the signal process as a system containing (10) and (11), which admits the following state-space representation

$$\begin{aligned} \boldsymbol{Z}_t &= \boldsymbol{F}\boldsymbol{Z}_{t-1} + \boldsymbol{\Phi}\boldsymbol{s}_{it} \\ x_{it} &= \boldsymbol{H}\boldsymbol{Z}_t + \boldsymbol{\Psi}\boldsymbol{s}_{it} \end{aligned} \tag{A.3}$$

with $\boldsymbol{F} = \rho$, $\boldsymbol{\Phi} = \begin{bmatrix}\sigma_\varepsilon & 0\end{bmatrix}$, $\boldsymbol{Z}_t = v_t$, $\boldsymbol{s}_{it} = \begin{bmatrix}\varepsilon^v_t \\ u_{it}\end{bmatrix}$, $\boldsymbol{H} = 1$, and $\boldsymbol{\Psi} = \begin{bmatrix}0 & \sigma_u\end{bmatrix}$. It is convenient to rewrite the uncertainty parameters in terms of precision: define $\tau_\varepsilon \equiv \frac{1}{\sigma^2_\varepsilon}$ and $\tau_u \equiv \frac{1}{\sigma^2_u}$. The signal system can be written as

$$x_{it} = \frac{\sigma_\varepsilon}{1-\rho L}\varepsilon^v_t + \sigma_u u_{it} = \begin{bmatrix}\frac{\tau_\varepsilon^{-\frac{1}{2}}}{1-\rho L} & \tau_u^{-\frac{1}{2}}\end{bmatrix}\begin{bmatrix}\varepsilon^v_t \\ u_{it}\end{bmatrix} = \boldsymbol{M}(L)\boldsymbol{s}_{it}, \qquad \boldsymbol{s}_{it} \sim \mathcal{N}(0, I) \tag{A.4}$$

The Wold theorem states that there exists another representation of the signal process (A.4), $x_{it}t = \boldsymbol{B}(L)\boldsymbol{w}_{it}$, such that $\boldsymbol{B}(z)$ is invertible and $\boldsymbol{w}_{it} \sim (0, \boldsymbol{V})$ is white noise. Hence, I can write the following equivalence

$$x_{it} = \boldsymbol{M}(L)\boldsymbol{s}_{it} = \boldsymbol{B}(L)\boldsymbol{w}_{it} \tag{A.5}$$

In the Wold representation of $x_{it}$, observing $\{x_{it}\}$ is equivalent to observing $\{\boldsymbol{w}_{it}\}$, and $\{x_i^t\}$ and $\{\boldsymbol{w}_i^t\}$ contain the same information. Furthermore, note that the Wold representation has the property that, using the equivalence (A.5), both processes share the autocovariance generating function, $\rho_{xx}(z) = \boldsymbol{M}(z)\boldsymbol{M}^\top(z^{-1}) = \boldsymbol{B}(z)\boldsymbol{V}\boldsymbol{B}^\top(z^{-1})$.

Given the state-space representation of the signal process (A.3), optimal expectations of the exogenous fundamental take the form of a Kalman filter, $\mathbb{E}_{it}v_t = \lambda\mathbb{E}_{it-1}v_{t-1} + \boldsymbol{K}x_{it}$, where $\lambda = (I - \boldsymbol{KH})\boldsymbol{F}$, and $\boldsymbol{K}$ is given by

$$\boldsymbol{K} = \boldsymbol{PH}^\top\boldsymbol{V}^{-1} \tag{A.6}$$

$$\boldsymbol{P} = \boldsymbol{F}[\boldsymbol{P} - \boldsymbol{PH}^\top\boldsymbol{V}^{-1}\boldsymbol{HP}]\boldsymbol{F} + \boldsymbol{\Phi}\boldsymbol{\Phi}^\top \tag{A.7}$$

I still need to find the unknowns $\boldsymbol{B}(z)$ and $\boldsymbol{V}$. Propositions 13.1-13.4 in Hamilton (1994) provide us with these objects. Unknowns $\boldsymbol{B}(z)$ and $\boldsymbol{V}$ satisfy $\boldsymbol{B}(z) = I + \boldsymbol{H}(I - \boldsymbol{F}z)^{-1}\boldsymbol{FK}$ and $\boldsymbol{V} = \boldsymbol{HPH}^\top + \boldsymbol{\Psi}\boldsymbol{\Psi}^\top$. I can write (A.7) as

$$\boldsymbol{P}^2 + \boldsymbol{P}[(1-\rho^2)\sigma_u^2 - \sigma_\varepsilon^2] - \sigma_\varepsilon^2\sigma_u^2 = 0 \tag{A.8}$$

from which I can infer that $\boldsymbol{P}$ is a scalar. Denote $k = \boldsymbol{P}^{-1}$ and rewrite (A.8) as $k = \frac{\tau_\varepsilon}{2}\left\{1 - \rho^2 - \frac{\tau_u}{\tau_\varepsilon} \pm \sqrt{\left[\frac{\tau_u}{\tau_\varepsilon} - (1-\rho^2)\right]^2 + 4\frac{\tau_u}{\tau_\varepsilon}}\right\}$. I also need to find $\boldsymbol{K}$. Now that I have found $\boldsymbol{P}$ in terms of model primitives, I can obtain $\boldsymbol{K}$ using condition (A.6), $\boldsymbol{K} = \frac{1}{1+k\sigma_u^2}$. I can finally write $\lambda$ as

$$\lambda = \frac{k\sigma_u^2\rho}{1 + k\sigma_u^2} = \frac{1}{2}\left[\frac{1}{\rho} + \rho + \frac{\tau}{\rho} \pm \sqrt{\left(\frac{1}{\rho} + \rho + \frac{\tau}{\rho}\right)^2 - 4}\right] \tag{A.9}$$

One can show that one of the roots $\lambda_{1,2}$ lies inside the unit circle and the other lies outside as long as $\rho \in (0,1)$, which guarantees that the Kalman expectation process is stationary and unique. I set $\lambda$ to the root that lies inside the unit circle (the one with the '–' sign). Notice that I can also write $\boldsymbol{V}$ in terms of $\lambda$, $\boldsymbol{V} = k^{-1} + \sigma_u^2 = \frac{\rho}{\lambda\tau_u}$, where I have used the identity $k = \frac{\lambda\tau_u}{(\rho-\lambda)}$. Finally, I can obtain $\boldsymbol{B}(z) = 1 + \frac{\rho z}{(1-\rho z)(1+k\sigma_u^2)} = \frac{1-\lambda z}{1-\rho z}$, and therefore one can verify that $\boldsymbol{B}(z)\boldsymbol{V}\boldsymbol{B}^\top(z^{-1}) = \boldsymbol{M}(z)\boldsymbol{M}^\top(z^{-1})$.

Let us now move to the forecast of *endogenous* variables. Consider a variable $f_t =$

$A(L)\boldsymbol{s}_{it}$. Applying the Wiener-Hopf prediction filter, I can obtain the forecast as $\mathbb{E}_{it} f_t = \left[A(L)\boldsymbol{M}^\top(L^{-1})\boldsymbol{B}(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}x_{it}$, where $[\cdot]_+$ denotes the annihilator operator.[31]

Recall from condition (A.2) that I am interested in obtaining $\mathbb{E}_{it}v_t$, $\mathbb{E}_{it}\,p_{t-2}$, $\mathbb{E}_{it}\,p_{t-1}$, $\mathbb{E}_{it}\,p_t$ and $\mathbb{E}_{it}\,p^*_{i,t+1}$. I need to find the $A(z)$ polynomial for each of the forecasted variables. I start from the exogenous fundamental, $v_t$, to verify that the Kalman and Wiener-Hopf filters result in the same forecast. I can write the fundamental as $v_t = \begin{bmatrix} \frac{\tau_\varepsilon^{-\frac{1}{2}}}{1-\rho L} & 0 \end{bmatrix} \boldsymbol{s}_{it} = A_v(L)\boldsymbol{s}_{it}$. Let me now move to the endogenous variables. In this case, I need to guess (and verify) that each agent $i$'s policy function takes the form $p^*_{it} = h(L)x_{it}$.[32] The aggregate price level, given by (5), can then be expressed as $p_t = (1-\theta)\int h(L)x_{it}\,di + \theta\,p_{t-1} = (1-\theta)h(L)\frac{\tau_\varepsilon^{-\frac{1}{2}}}{(1-\rho L)(1-\theta L)}\varepsilon^v_t$. Using the guesses, I have $p_{t-k} = \begin{bmatrix}(1-\theta)\tau_\varepsilon^{-\frac{1}{2}}\frac{h(L)L^k}{(1-\rho L)(1-\theta L)} & 0\end{bmatrix}\boldsymbol{s}_{it} = A_{pk}(L)\boldsymbol{s}_{it}$ and $p_{i,t+1} = \frac{h(L)}{L}\boldsymbol{M}(L)\boldsymbol{s}_{it} = \begin{bmatrix}\tau_\varepsilon^{-\frac{1}{2}}\frac{h(L)}{L(1-\rho L)} & \tau_u^{-\frac{1}{2}}\frac{h(L)}{L}\end{bmatrix}\boldsymbol{s}_{it} = A_i(L)\boldsymbol{s}_{it}$. I am now armed with the necessary objects to obtain the three different forecasts,

$$
\begin{aligned}
\mathbb{E}_{it}v_t &= \left[A_v(L)\boldsymbol{M}^\top(L^{-1})\boldsymbol{B}(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}x_{it} = \left[\frac{L}{\tau_\varepsilon(1-\rho L)(L-\lambda)}\right]_+ \frac{\lambda\tau_u}{\rho}\frac{1-\rho L}{1-\lambda L}x_{it} \\
&= \left[\frac{\phi_v(L)}{L-\lambda}\right]_+ \frac{\lambda\tau}{\rho}\frac{1-\rho L}{1-\lambda L}x_{it} = \frac{\phi_v(L)-\phi_v(\lambda)}{L-\lambda}\frac{\lambda\tau}{\rho}\frac{1-\rho L}{1-\lambda L}x_{it}, \qquad \phi_v(z) = \frac{z}{1-\rho z} \\
&= \frac{\lambda\tau}{\rho(1-\rho\lambda)}\frac{1}{1-\lambda L}x_{it} = \left(1-\frac{\lambda}{\rho}\right)\frac{1}{1-\lambda L}x_{it} = G_1(L)x_{it} \qquad \text{(A.10)}
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}_{it}\,p_{t-k} &= \left[A_{pk}(L)\boldsymbol{M}^\top(L^{-1})\boldsymbol{B}(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}x_{it} = \left[\frac{h(L)L^{k+1}}{(1-\rho L)(L-\lambda)(1-\theta L)}\right]_+ \frac{(1-\theta)\lambda\tau}{\rho}\frac{1-\rho L}{1-\lambda L}x_{it} \\
&= \left[\frac{\phi_\pi(L)}{L-\lambda}\right]_+ \frac{(1-\theta)\lambda\tau}{\rho}\frac{1-\rho L}{1-\lambda L}x_{it} = \frac{\phi_\pi(L)-\phi_\pi(\lambda)}{L-\lambda}\frac{(1-\theta)\lambda\tau}{\rho}\frac{1-\rho L}{1-\lambda L}x_{it}, \quad \phi_\pi(z) = \frac{h(z)z}{(1-\rho z)(1-\theta z)} \\
&= (1-\theta)\frac{\lambda\tau}{\rho}\left[\frac{h(L)L^{k+1}}{1-\theta L} - h(\lambda)\lambda^{k+1}\frac{1-\rho L}{(1-\rho\lambda)(1-\theta\lambda)}\right]\frac{1}{(1-\lambda L)(L-\lambda)}x_{it} \\
&= (1-\theta)\left(1-\frac{\lambda}{\rho}\right)\left[\frac{h(L)L^{k+1}(1-\rho\lambda)}{1-\theta L} - \frac{h(\lambda)\lambda^{k+1}(1-\rho L)}{1-\theta\lambda}\right]\frac{1}{(1-\lambda L)(L-\lambda)}x_{it} = G_2(L)x_{it} \qquad \text{(A.11)}
\end{aligned}
$$

$$
\mathbb{E}_{it}\,p_{i,t+1} = \left[A_i(L)\boldsymbol{M}^\top(L^{-1})\boldsymbol{B}(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}x_{it} = \left[\frac{h(L)}{\tau_\varepsilon(1-\rho L)(L-\lambda)} + \frac{h(L)(L-\rho)}{\tau_u L(L-\lambda)}\right]_+ \frac{\lambda\tau_u}{\rho}\frac{1-\rho L}{1-\lambda L}x_{it}
$$

[31] See Online Appendix OA.5 for more details on the Wiener-Hopf prediction filter and the annihilator operator.

[32] In this framework agents only observe signals, and the policy function can only depend on current and past signals.

$$
\begin{aligned}
&= \left\{ \left[ \frac{h(L)}{\tau_\varepsilon (1-\rho L)(L-\lambda)} \right]_+ + \left[ \frac{h(L)(L-\rho)}{\tau_u L (L-\lambda)} \right]_+ \right\} \frac{\lambda \tau_u}{\rho} \frac{1-\rho L}{1-\lambda L} x_{it} \\
&= \left\{ \left[ \frac{\phi_{i,1}(L)}{L-\lambda} \right]_+ + \left[ \frac{\phi_{i,2}(L)}{L(L-\lambda)} \right]_+ \right\} \frac{\lambda \tau_u}{\rho} \frac{1-\rho L}{1-\lambda L} x_{it}, \quad \phi_{i,1}(z) = \frac{h(z)}{\tau_\varepsilon (1-\rho z)}, \quad \phi_{i,2}(z) = \frac{h(z)(z-\rho)}{\tau_u} \\
&= \left\{ \frac{\phi_{i,1}(L) - \phi_{i,1}(\lambda)}{L-\lambda} + \frac{\phi_{i,2}(L) - \phi_{i,2}(\lambda)}{\lambda (L-\lambda)} - \frac{\phi_{i,2}(L) - \phi_{i,2}(0)}{\lambda L} \right\} \frac{\lambda \tau_u}{\rho} \frac{1-\rho L}{1-\lambda L} x_{it} \\
&= \frac{\lambda}{\rho} \left\{ \frac{h(L)}{L-\lambda} \left[ \frac{\tau_u}{\tau_\varepsilon (1-\rho L)} + \frac{L-\rho}{L} \right] - \frac{h(\lambda)}{L-\lambda} \left[ \frac{\tau_u}{\tau_\varepsilon (1-\rho\lambda)} + \frac{\lambda-\rho}{\lambda} \right] - \frac{\rho h(0)}{\lambda L} \right\} \frac{1-\rho L}{1-\lambda L} x_{it}
\end{aligned}
$$

(A.12)

$$
= \left\{ \frac{h(L)}{L-\lambda} \left[ \left(1 - \frac{\lambda}{\rho}\right) \frac{1-\rho\lambda}{1-\rho L} + \frac{\lambda (L-\rho)}{\rho L} \right] - \frac{h(0)}{L} \right\} \frac{1-\rho L}{1-\lambda L} x_{it} = G_3(L) x_{it}
$$

Recall the best response for a firm $i$, condition (A.2). To be consistent with firm optimization, the policy function $h(z)$ must satisfy (A.2) at all times and signals. Plugging the obtained expressions and rearranging by $h(z)$, I can write $\widetilde{C}(z) h(z) x_{it} = d[z; h(\lambda), h(0)] x_{it}$, where

$$
\begin{aligned}
\widetilde{C}(z) &= (z-\beta\theta)(1-\theta z)(z-\lambda)(1-\lambda z) - z^2 \kappa\theta \left( \frac{(1-\theta)(1-\beta\theta)}{\kappa\theta} + z a_y + z^2 b_y \right) \left(1 - \frac{\lambda}{\rho}\right)(1-\rho\lambda) \\
&= \lambda \left\{ (\beta\theta - z)(1-\theta z)(z-\rho) \left( z - \frac{1}{\rho} \right) - \frac{\tau}{\rho} z \left[ (\beta\theta - z)(1-\theta z) + \kappa\theta \left( \frac{(1-\theta)(1-\beta\theta)}{\kappa\theta} + z a_y + z^2 b_y \right) z \right] \right\} \\
&= \lambda C(z)
\end{aligned}
$$

and

$$
\begin{aligned}
d[z; h(\lambda), h(0)] = \frac{\kappa\theta c_y}{1-\theta} \left(1 - \frac{\lambda}{\rho}\right) z(z-\lambda)(1-\theta z) - h(0)\beta\theta(1-\rho z)(z-\lambda)(1-\theta z) \\
- h(\lambda) \frac{\lambda}{1-\theta\lambda} \left(1 - \frac{\lambda}{\rho}\right) \kappa\theta \left( \frac{(1-\theta)(1-\beta\theta)}{\kappa\theta} + \lambda a_y + \lambda^2 b_y \right) z(1-\rho z)(1-\theta z)
\end{aligned}
$$

Notice that I can write polynomial $\widetilde{C}(z)$ in terms of its roots as $\widetilde{C}(z) = \theta\lambda \left(1 - \frac{\tau\kappa b_y}{\rho}\right)(z - \zeta_1)(z-\zeta_2)(z-\vartheta_1^{-1})(z-\vartheta_2^{-1})$ where $\zeta_1, \zeta_2$ are the inside roots of $C(z)$, and $\vartheta_1$ and $\vartheta_2$ are the reciprocals of the outside roots. To have a causal $h(z)$ polynomial, I need to eliminate the inside roots in its denominator, $\lambda C(z)$. I choose $h(0)$ and $h(\lambda)$ so that $d[\zeta_1; h(0), h(\lambda)] = 0$ and $d[\zeta_2; h(0), h(\lambda)] = 0$. As a result, I can write $d[z; h(0), h(\lambda)] = \frac{\kappa\theta\lambda\tau c_y}{(1-\theta)\rho(1-\rho\zeta_1)(1-\rho\zeta_2)}(z-\zeta_1)(z-\zeta_2)(1-\theta z)$, and hence the policy function is

$$
h(z) = \frac{\kappa c_y}{1-\theta} \frac{\tau\vartheta_1\vartheta_2}{(\rho - \tau\kappa b_y)(1-\rho\zeta_1)(1-\rho\zeta_2)} \frac{1-\theta z}{(1-\vartheta_1 z)(1-\vartheta_2 z)}
$$

Finally, the aggregate price level dynamics follow $p_t = (1-\theta)\frac{h(L)}{1-\theta L}\nu_t = \kappa c_y \frac{\tau\vartheta_1\vartheta_2}{(\rho-\tau\kappa b_y)(1-\rho\zeta_1)(1-\rho\zeta_2)}\frac{1}{(1-\vartheta_1 L)(1-\vartheta_2 L)}\nu_t$. I can therefore write inflation dynamics as $\pi_t = (1-L)\,p_t = (\vartheta_1+\vartheta_2)\pi_{t-1} - \vartheta_1\vartheta_2\pi_{t-2} + c_p\Delta\nu_t$ where $c_p = \kappa c_y \frac{\tau\vartheta_1\vartheta_2}{(\rho-\tau\kappa b_y)(1-\rho\zeta_1)(1-\rho\zeta_2)}$. Inserting inflation dynamics into the DIS equation (8) I can obtain output gap dynamics

$$\begin{aligned}\widetilde{y}_t &= \frac{1}{\sigma}(-\phi_\pi p_t + \phi_\pi p_{t-1} + \sigma\mathbb{E}_t\widetilde{y}_{t+1} + \mathbb{E}_t p_{t+1} - p_t - \nu_t)\\ &= \frac{(\sigma a_y + \vartheta - \phi_\pi)(1+\vartheta) + \phi_\pi + \sigma b_y - \vartheta}{\sigma} p_{t-1} - \frac{(\sigma a_y + \vartheta - \phi_\pi)\vartheta}{\sigma} p_{t-2}\\ &\quad - \frac{1-\rho(c_p - \sigma c_y) - (\sigma a_y + \vartheta - \phi_\pi)c_p}{\sigma}\nu_t \end{aligned} \tag{A.13}$$

To be consistent with our earlier guess (A.1), it must be that

$$\begin{aligned} a_y &= \frac{\vartheta_1[\sigma(1-\vartheta_2)+\phi_y](\vartheta_1+\vartheta_2-1-\phi_\pi) + (1-\vartheta_2)(\phi_\pi-\vartheta_2)(\sigma+\phi_y)}{[\sigma(1-\vartheta_1)+\phi_y][\sigma(1-\vartheta_2)+\phi_y]}\\ b_y &= \frac{\vartheta_1\vartheta_2[\sigma(1-\vartheta_1)(1-\vartheta_2) - (\vartheta_1+\vartheta_2-1-\phi_\pi)\phi_y]}{[\sigma(1-\vartheta_1)+\phi_y][\sigma(1-\vartheta_2)+\phi_y]}\end{aligned}$$

and two additional coefficients $(c_p, c_y)$ irrelevant for persistence. Finally, I can rewrite the $\widetilde{C}(z)$ polynomial as

$$\begin{aligned}\widetilde{C}(z) = \frac{\lambda}{\rho}\Bigg\{ &-(\beta\theta - z)(1-\theta z)(z-\rho)\,(1-\rho z) - \tau z\Bigg[(\beta\theta - z)(1-\theta z) + z(1-\theta)(1-\beta\theta)\\ &+ z^2\kappa\theta\frac{\vartheta_1[\sigma(1-\vartheta_2)+\phi_y](\vartheta_1+\vartheta_2-1-\phi_\pi) + (1-\vartheta_2)(\phi_\pi-\vartheta_2)(\sigma+\phi_y)}{[\sigma(1-\vartheta_1)+\phi_y][\sigma(1-\vartheta_2)+\phi_y]}\\ &+ z^3\kappa\theta\frac{\vartheta_1\vartheta_2[\sigma(1-\vartheta_1)(1-\vartheta_2) - (\vartheta_1+\vartheta_2-1-\phi_\pi)\phi_y]}{[\sigma(1-\vartheta_1)+\phi_y][\sigma(1-\vartheta_2)+\phi_y]}\Bigg]\Bigg\}\end{aligned}$$

□

***Proof of Proposition 2***. I am interested in obtaining $\beta_{rev} = \frac{\mathbb{C}(\text{forecast error}_t, \text{revision}_t)}{\mathbb{V}(\text{revision}_t)}$. Using the results from the proof of Proposition 1 I can write the forecast error as

$$\begin{aligned}\pi_{t+3,t} - \overline{\mathbb{E}}_t^f \pi_{t+3,t} &= p_{t+3} - p_{t-1} - \overline{\mathbb{E}}_t^f(p_{t+3} - p_{t-1}) = \frac{\phi_0 + \phi_1 L + \phi_2 L^2 + \phi_3 L^3 + \phi_4 L^4}{(1-\lambda L)(1-\vartheta_1 L)(1-\vartheta_2 L)}\varepsilon_{t+3}^\nu\\ &= \phi_0\frac{(1-\xi_1 L)(1-\xi_2 L)(1-\xi_3 L)(1-\xi_4 L)}{(1-\lambda L)(1-\vartheta_1 L)(1-\vartheta_2 L)}\varepsilon_{t+3}^\nu\\ &= \frac{\phi_0(\lambda-\xi_1)(\lambda-\xi_2)}{(\lambda-\vartheta_1)(\lambda-\vartheta_2)}\sum_{k=0}^{k}\lambda^k[\varepsilon_{t+3-k}^\nu - (\xi_3+\xi_4)\varepsilon_{t+2-k}^\nu + \xi_3\xi_4\varepsilon_{t+1-k}^\nu]\end{aligned}$$

$$
\begin{aligned}
&- \frac{\phi_0(\vartheta_1 - \xi_1)(\vartheta_1 - \xi_2)}{(\lambda - \vartheta_1)(\vartheta_1 - \vartheta_2)} \sum_{k=0}^{k} \vartheta_1^k [\varepsilon^\nu_{t+3-k} - (\xi_3 + \xi_4)\varepsilon^\nu_{t+2-k} + \xi_3\xi_4\varepsilon^\nu_{t+1-k}] \\
&+ \frac{\phi_0(\vartheta_2 - \xi_1)(\vartheta_2 - \xi_2)}{(\lambda - \vartheta_2)(\vartheta_1 - \vartheta_2)} \sum_{k=0}^{k} \vartheta_2^k [\varepsilon^\nu_{t+3-k} - (\xi_3 + \xi_4)\varepsilon^\nu_{t+2-k} + \xi_3\xi_4\varepsilon^\nu_{t+1-k}]
\end{aligned}
$$

where $\phi_0 = c_p$, $\phi_1 = \left(\frac{1}{\lambda} - \frac{1}{\rho}\right) c_p$, $\phi_2 = \frac{(\rho-\lambda)c_p}{\lambda^2\rho}$, $\phi_3 = \frac{(\rho-\lambda)c_p[\lambda^3-\vartheta_1-\vartheta_2+\lambda\vartheta_1\vartheta_2]}{\lambda^2\rho(1-\lambda\vartheta_1)(1-\lambda\vartheta_2)}$, $\phi_4 = \frac{-\lambda^3+\lambda^4\vartheta_2+\lambda^4\vartheta_1-\vartheta_1\vartheta_2[\lambda-(1-\lambda^4)\rho]}{\lambda^2\rho(1-\lambda\vartheta_1)(1-\lambda\vartheta_2)}$ and $(\xi_1, \xi_2, \xi_3, \xi_4)$ are the reciprocals of the roots of the polynomial $\phi_0 + \phi_1 z + \phi_2 z^2 + \phi_3 z^3 + \phi_4 z^4$. The average forecast revision is given by

$$
\begin{aligned}
\overline{\mathbb{E}}^f_t \pi_{t+3,t} - \overline{\mathbb{E}}^f_{t-1}\pi_{t+3,t} &= \overline{\mathbb{E}}^f_t (p_{t+3} - p_{t-1}) - \overline{\mathbb{E}}^f_{t-1}(p_{t+3} - p_{t-1}) = \frac{c_p(\rho - \lambda)(1 - \lambda^4)}{\rho\lambda^3(1 - \vartheta_1\lambda)(1 - \vartheta_2\lambda)(1 - \lambda L)}\varepsilon^\nu_t \\
&= \frac{c_p(\rho - \lambda)(1 - \lambda^4)}{\rho\lambda^3(1 - \vartheta_1\lambda)(1 - \vartheta_2\lambda)} \sum_{k=0}^{\infty} \lambda^k \varepsilon^\nu_{t-k}
\end{aligned}
$$

and I can finally write $\beta_{rev}$ as

$$
\begin{aligned}
\beta_{\text{rev}} = \frac{\mathbb{C}(\text{forecast error}_t, \text{revision}_t)}{\mathbb{V}(\text{revision}_t)} &= \frac{\lambda^3\rho(1 - \vartheta_1\lambda)(1 - \vartheta_2\lambda)}{(1 - \lambda^4)(\rho - \lambda)} \left\{ \frac{\lambda(\lambda - \xi_1)(\lambda - \xi_2)(\lambda - \xi_3)(\lambda - \xi_4)}{(\lambda - \vartheta_1)(\lambda - \vartheta_2)} \right. \\
&\left. - (1 - \lambda^2) \left[ \frac{\vartheta_1(\vartheta_1 - \xi_1)(\vartheta_1 - \xi_2)(\vartheta_1 - \xi_3)(\vartheta_1 - \xi_4)}{(1 - \lambda\vartheta_1)(\lambda - \vartheta_1)(\vartheta_1 - \vartheta_2)} + \frac{\vartheta_2(\vartheta_2 - \xi_1)(\vartheta_2 - \xi_2)(\vartheta_2 - \xi_3)(\vartheta_2 - \xi_4)}{(1 - \lambda\vartheta_2)(\lambda - \vartheta_2)(\vartheta_1 - \vartheta_2)} \right] \right\}
\end{aligned}
$$

□

***Proof of Proposition 3***. I start from the more general case studied in proposition **??**. Under no information frictions on the household side, I can write the characteristic polynomial of (**??**) as

$$
\begin{aligned}
C(z) = &\frac{(1 - \theta z)z}{\rho\sigma} \left\{ (z - 1)z^2(1 - \phi_\pi z)\beta\theta\kappa(\rho - \lambda)(1 - \rho\lambda) \right. \\
&\left. +\beta[\phi_y z - (1 - z)\sigma][\rho(1 - \theta z)(z - \beta\theta)(z - \lambda)(1 - \lambda z) - z^2(1 - \theta)(1 - \beta\theta)(\rho - \lambda)(1 - \rho\lambda) \right\} \\
= &\frac{(1 - \theta z)z\lambda}{\rho\sigma} \left\{ (z - 1)z^2(1 - \phi_\pi z)\beta\theta\kappa\tau \right. \\
&\left. +\beta[\phi_y z - (1 - z)\sigma][(1 - \theta z)(z - \beta\theta)[(z - \rho)(1 - \rho z) + \tau z] - z^2(1 - \theta)(1 - \beta\theta)\tau \right\}
\end{aligned}
$$

where $C(z)$ has seven roots. Using the following relations,

$$C(0) = 0$$

$$C(\lambda) = -\frac{\beta\lambda^3(1-\theta\lambda)(\rho-\lambda)(1-\rho\lambda)}{\rho\sigma}\left\{\theta\kappa(1-\lambda)(1-\lambda\phi_\pi) + (1-\beta\theta)(1-\theta)[\lambda\phi_y - (1-\lambda)\sigma]\right\} < 0$$
$$C(\rho) = \frac{\beta\theta(\rho-\lambda)(1-\rho)\rho(1-\rho\theta)(1-\rho\lambda)}{\sigma}\left\{\rho\kappa(\rho\phi_\pi - 1) + (\beta-\rho)[(1-\rho)\sigma - \rho\phi_y]\right\} > 0$$
$$C\left(\frac{\sigma}{\sigma+\phi_y}\right) = -\frac{\beta\theta\kappa(\rho-\lambda)(1-\rho\lambda)\sigma^2[(1-\theta)\sigma+\phi_y][\sigma(\phi_\pi-1)-\phi_y]\phi_y}{\rho(\sigma+\phi_y)^6} < 0$$
$$C(1) = \frac{\beta(1-\theta)^2(1-\beta\theta)\lambda(1-\rho)^2\phi_y}{\rho\sigma} > 0$$
$$\begin{aligned}C(\beta/\theta) = \frac{(1-\beta)\beta^2}{\theta^5\rho\sigma}&\{-\beta\theta\kappa(\beta-\theta)(\rho-\lambda)(1-\rho\lambda)(\beta\phi_\pi-\theta)\\ &+[(\beta-\theta)\sigma+\beta\phi_y][\rho(1-\beta)(1-\theta^2)(\theta-\beta\lambda)(\beta-\theta\lambda)-\beta\theta(1-\theta)(1-\beta\theta)(\rho-\lambda)(1-\rho\lambda)]\}\\ &< 0\end{aligned}$$
$$C(\rho^{-1}) = \frac{\beta\theta(\theta-\rho)(1-\rho)(\rho-\lambda)(1-\rho\lambda)}{\rho^7\sigma}\left\{(\phi_\pi-\rho)\kappa + (1-\rho\beta)[\phi_y + (1-\rho)\sigma]\right\} > 0,$$

I show that the seven roots are all real, four of them are between 0 and 1, and three of them are larger than 1. To show that $\vartheta_1$ is less than $\rho$, it is sufficient to show that $C(\rho^{-1}) > 0$. Since $C(\vartheta_1) = 0$, it has to be that $\vartheta_1^{-1}$ is larger than $\rho^{-1}$, or $\vartheta_1 < \rho$. To show that $\vartheta_2$ is more than $\theta$, it is sufficient to show that $C(\beta/\theta) > 0$. Since $C(\vartheta_2) = 0$, it has to be that $C(z)$ has a zero between $z = 1$ and $z = \theta^{-1}$. And thus, $\vartheta_2^{-1}$ is smaller than $\rho^{-1}$, or $\vartheta_2 > \theta$.

Taking the derivative of $C(z)$ with respect to $\tau$, and evaluating that derivative at $z = \vartheta_g^{-1}$ for $g \in \{1, 2\}$, $\frac{\partial C(\vartheta_g^{-1})}{\partial\tau} = (\theta - \vartheta_g)\frac{\beta\theta_g(1-\theta_g)\lambda\{\kappa(\phi_\pi-\vartheta_g)+(1-\beta\vartheta)[\phi_y+\sigma(1-\vartheta_g)]\}}{\vartheta_g^6\rho\sigma}$. I obtain $\frac{\partial C(\vartheta_1^{-1})}{\partial\tau} > 0$ and $\frac{\partial C(\vartheta_2^{-1})}{\partial\tau} < 0$. Combining this with the earlier observation that $\frac{\partial C(\vartheta_g^{-1})}{\partial z} < 0$ for $g \in \{1, 2\}$, and using the Implicit Function Theorem, I infer that $\vartheta_1$ ($\vartheta_2$) is increasing (decreasing) in $\sigma_u$.

□

***Proof of Proposition 4.*** In the benchmark NK model the PC is given by (4), the DIS curve is given by (8), the Taylor rule is given by (9) and the monetary policy shock process is given by (10). Inserting the Taylor rule (9) into the DIS curve (8), one can write the model as a system of two first-order stochastic difference equations, $\widetilde{A}\boldsymbol{x}_t = \widetilde{B}\mathbb{E}_t\boldsymbol{x}_{t+1} + \widetilde{C}v_t$, where $\boldsymbol{x}_t = [\widetilde{y}_t \quad \pi_t \quad p_t]^\intercal$ is a $3 \times 1$ vector containing output, inflation and prices, $\widetilde{A}$ is a $3 \times 3$ coefficient matrix, $\widetilde{B}$ is a $3 \times 3$ coefficient matrix and $\widetilde{C}$ is a $3 \times 1$ vector satisfying

$$\widetilde{A} = \begin{bmatrix}\sigma+\phi_y & \phi_\pi & 0\\ -\kappa & 1 & 0\\ 0 & 0 & 1\end{bmatrix}, \quad \widetilde{B} = \begin{bmatrix}\sigma & 1 & 0\\ 0 & \beta & 0\\ 0 & -1 & 1\end{bmatrix}, \quad \text{and} \quad \widetilde{C} = \begin{bmatrix}-1\\ 0\\ 0\end{bmatrix}$$

Premultiplying the system by $\widetilde{A}^{-1}$ I obtain $\boldsymbol{x}_t = \boldsymbol{\delta}\mathbb{E}_t\boldsymbol{x}_{t+1} + \boldsymbol{\varphi}v_t$, where $\boldsymbol{\delta} = \widetilde{A}^{-1}\widetilde{B}$ and $\boldsymbol{\varphi} = \widetilde{A}^{-1}\widetilde{C}$. In the dispersed information framework, structural-form dynamics are given by $A_s\boldsymbol{x}_t = B_s\boldsymbol{x}_{t-1} + C_s v_t$, where

$$A_s = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \quad B_s = \begin{bmatrix} 0 & -b_y & a_y + b_y \\ 0 & 0 & -1 \\ 0 & \vartheta_1\vartheta_2 & \vartheta_1 + \vartheta_2 - \vartheta_1\vartheta_2 \end{bmatrix}, \quad \text{and} \quad C_s = \begin{bmatrix} c_y \\ 0 \\ -\psi_\pi\chi_\pi \end{bmatrix}$$

with $(a_y, b_y, c_y)$ defined in the proof of Proposition 1. Premultiplying by $A_s^{-1}$ I obtain the reduced-form dynamics $\boldsymbol{x}_t = A\boldsymbol{x}_{t-1} + Bv_t$, where $A = A_s^{-1}B_s$ and $B = A_s^{-1}B_s$.

Using the Method for Undetermined Coefficients, the ad-hoc dynamics and the noisy information dynamics are observationally equivalent if

$$\begin{aligned} A\boldsymbol{x}_{t-1} + Bv_t &= \boldsymbol{\varphi}v_t + \boldsymbol{\delta\omega}_f\mathbb{E}_t\boldsymbol{x}_{t+1} + \boldsymbol{\omega}_b\boldsymbol{x}_{t-1} = \boldsymbol{\varphi}v_t + \boldsymbol{\delta\omega}_f\mathbb{E}_t(A\boldsymbol{x}_t + Bv_{t+1}) + \boldsymbol{\omega}_b\boldsymbol{x}_{t-1} \\ &= \boldsymbol{\varphi}v_t + \boldsymbol{\delta\omega}_f(A\boldsymbol{x}_t + B\mathbb{E}_t v_{t+1}) + \boldsymbol{\omega}_b\boldsymbol{x}_{t-1} = \boldsymbol{\varphi}v_t + \boldsymbol{\delta\omega}_f(A\boldsymbol{x}_t + B\rho v_t) + \boldsymbol{\omega}_b\boldsymbol{x}_{t-1} \\ &= \boldsymbol{\varphi}v_t + \boldsymbol{\delta\omega}_f[A(A\boldsymbol{x}_{t-1} + Bv_t) + B\rho v_t] + \boldsymbol{\omega}_b\boldsymbol{x}_{t-1} = \left[\boldsymbol{\delta\omega}_f AA + \boldsymbol{\omega}_b\right]\boldsymbol{x}_{t-1} + \left[\boldsymbol{\varphi} + \boldsymbol{\delta\omega}_f(A + \rho)B\right]v_t \end{aligned}$$

They are thus equivalent if $B - \boldsymbol{\varphi} = \boldsymbol{\delta\omega}_f(AB + \rho B)$ and $\boldsymbol{\omega}_b = (I_3 - \boldsymbol{\delta\omega}_f A)A$ for certain matrices $\boldsymbol{\omega}_b$ and $\boldsymbol{\omega}_f$, with elements $\omega_{b,ij}$ and $\omega_{f,ij}$ in their $ij$ position, respectively. The system of restrictions (17) implies that $\omega_{b,11} = \omega_{b,21} = \omega_{b,31} = 0$. I need to multiply the system by $\widetilde{A}$ to back out the structural dynamics. In particular, I can write inflation dynamics as $\pi_t = \omega_1\pi_{t-1} + \omega_2 p_{t-1} + \kappa\widetilde{y}_t + \omega_3\mathbb{E}_t\widetilde{y}_{t+1} + \omega_4\mathbb{E}_t\pi_{t+1} + \omega_5\mathbb{E}_t p_{t+1}$, where $\omega_1 = \omega_{b,22} - \kappa\omega_{b,12}$, $\omega_2 = \omega_{b,23} - \kappa\omega_{b,13}$, $\omega_3 = \beta\omega_{f,21}$, $\omega_4 = \beta\omega_{f,22}$ and $\omega_5 = \beta\omega_{f,23}$. □

***Proof of Corollary 1.*** Using the model dynamics (A.13)-(13), I can write

$$\begin{aligned} &\omega_2 p_{t-1} + \omega_3\mathbb{E}_t\widetilde{y}_{t+1} + \omega_5\mathbb{E}_t p_{t+1} = \omega_2 p_{t-1} + \omega_3\left[-b_y\pi_t + (a_y + b_y)p_t - \psi_y\chi_y\rho v_t\right] + \\ &\quad + \omega_5\left[\vartheta_1\vartheta_2\pi_t + (\vartheta_1 + \vartheta_2 - \vartheta_1\vartheta_2)p_t - \psi_\pi\chi_\pi\rho v_t\right] \\ &= \left\{\omega_5\vartheta_1\vartheta_2 - \omega_3 b_y + [\omega_3(a_y + b_y) + \omega_5(\vartheta_1 + \vartheta_2 - \vartheta_1\vartheta_2)]\right\}\vartheta_1\vartheta_2\pi_{t-1} + \\ &+ \left\{\omega_2 - (\omega_5\vartheta_1\vartheta_2 - \omega_3 b_y)(1 - \vartheta_1)(1 - \vartheta_2) + [\omega_3(a_y + b_y) + \omega_5(\vartheta_1 + \vartheta_2 - \vartheta_1\vartheta_2)](\vartheta_1 + \vartheta_2 - \vartheta_1\vartheta_2)\right\}p_{t-1} + \\ &+ \left\{-(\omega_5\vartheta_1\vartheta_2 - \omega_3 b_y)\psi_\pi\chi_\pi - [\omega_3(a_y + b_y) + \omega_5(\vartheta_1 + \vartheta_2 - \vartheta_1\vartheta_2)]\psi_\pi\chi_\pi - \rho(\omega_3\psi_y\chi_y + \omega_5\psi_\pi\chi_\pi)\right\}v_t \end{aligned}$$

and I can use the two degrees of freedom to set $\omega_3 a_y + \omega_5(\vartheta_1 + \vartheta_2) = 0$ and $\omega_5\vartheta_1\vartheta_2 - \omega_3 b_y = \omega_2$, with $\chi = -[\omega_3 a_y + \omega_5(\vartheta_1 + \vartheta_2 + \rho)]\psi_\pi\chi_\pi - \omega_3\rho\psi_y\chi_y = -\rho(\omega_5\psi_\pi\chi_\pi + \omega_3\psi_y\chi_y)$. □