European Sovereign Debt Risk Management: the Role of a European Debt Agency*

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Abstract

This paper proposes the establishment of a European Debt Agency (EDA) as a tool for the efficient management of Eurozone public debt , to address two primary risks: roll-over and sustainability risk. The proposed EDA would price its loans using a transparent formula that would anchor the price to fundamental economic factors. This approach would encourage fiscal discipline among Member States and avoid inefficient costs resulting from market price deviations from fundamentals, without resorting to debt mutualization. In addition, the paper suggests that adopting flexible fiscal rules alongside the EDA could result in a smoother path towards debt stabilization, by mitigating the macroeconomic effects of excessive fluctuations in risk premia. The simulations indicate that this combination could offer a comprehensive strategy for managing sovereign debt in the Eurozone that would promote fiscal responsibility and stability.

JEL codes: H12, H63, H81.
Keywords: European Debt Agency, European Safe Assets, Debt Management.

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1 Introduction

The current macroeconomic conditions in Europe, characterized by high government debts, low growth, uncertain inflation, and interest rates, make effective and efficient risk management for government debt more important than ever before.

This paper analyses the potential of a European Debt Agency (EDA) as an efficient debt management institution for the Euro area to reduce and manage the two main sources of risk for sovereign debt: roll-over risk and sustainability risk.

Sovereign debt roll-over risk refers to the risk that a government may not be able to refinance its existing debt obligations as they come due or roll them over into new debt, causing a default. This risk materializes very rapidly with a collapse in the price of sovereign bonds. Roll-over risk can serve as a tool to deter excessive debt when bond prices are firmly linked to underlying fundamentals. However, its efficiency as a discipline device is hampered when bond prices diverge from these fundamentals. Roll-over risk could be fully neutralized if the central bank were allowed to buy back government debt at maturity. This is not a feasible option for the ECB.

Sustainability risk refers to the risk that the government debt to GDP ratio gets on an explosive path. This risk materializes when the primary surplus to GDP ratio is permanently lower than the debt-stabilizing primary surplus. The debt stabilizing primary surplus is calculated by multiplying two factors: the debt-to-GDP ratio, and the difference between the average cost of financing the government's debt and the rate of GDP growth. Sustainability risk builds more slowly than roll-over risk as it takes time for fluctuations in bond prices to be reflected in the average cost of financing the debt.

Both risks are very relevant in the current European macroeconomic scenarios. The pandemic, along with the suspension of the Stability and Growth Pact (SGP) (Council of the European Union 2020), has led to record-high deficits and public debts in Europe. As a result, reducing the debt stock and bringing deficits back to acceptable levels have become critical objectives for European policymakers. The challenge of sustainability is further heightened by the potential increase in spending related to the ongoing geopolitical crisis, such as investments in energy to decrease reliance on Russian gas, to strengthen the European grid and promote the transition to renewable energy, and for common defense. The task is particularly arduous, since the growth of European economies is threatened by the inflation, caused by the increase in energy costs, by the rearrangement of value chains as well as by supply bottlenecks and by the size of euro area government debts. Persistence in the mismatch between observed inflation and the inflation target increases the risk of de-anchoring inflation expectations.[1] These dynamics, combined with expansive fiscal and monetary policies, generate the risk of driving the eurozone into a stagflationary debt crisis in the medium term.[2] When public debts are perceived to be riskier than in "normal times", the emergence of the risk of multiple equilibria with a collapse in bond prices leading to a sudden spike in roll-over risk may require a faster adjustment path, as already happened during the European sovereign debt crisis. In this context, a severe misalignment between the credit risk of MSs and the yields paid on their respective sovereign debts was observed. The challenge lies in identifying the optimal policy mix, which can implement a deleveraging process without jeopardizing the growth path of European economies that began in 2021. On the one hand, attempting to reduce high public debt through a long series of primary surpluses could be self-defeating when GDP growth rates (g) exceed the average cost of financing the debt (r). On the other hand, the difference between r and g may become non-negative, making fiscal policy incapable of implementing "tearless" deleveraging, especially in contexts characterized by high inflation and high debt.[3]

There is an ongoing broad debate on the changes to be implemented to the Stability and Growth Pact (SGP) in order to prevent these phenomena. Recently, the growth estimates have been revised downward (Lagarde (2022)). The fragility of European growth, combined with the containment of inflation, does not allow for sudden fiscal consolidation. Such high debts can only increase the risk of downgrading of the sovereigns of some eurozone countries, a prospect that will be further reinforced as soon as the ECB returns to applying the capital key rule. The debt normalization path could exacerbate the problems related to the scarcity of safe assets in the European financial system and to the doom loop.[4] In order to manage such a delicate situation, several proposals have been put forward to introduce schemes of collaboration and coordination between Member States and European institutions and a debate has emerged on the reform of the current framework for fiscal rules. This paper describes the effect of the joint implementation of growth-friendly fiscal rules for debt sustainability and the establishment of EDA as an efficient debt management institution. In particular, we consider the fiscal framework proposed by Giavazzi et al. (2021) in which the 60 percent debt reference value becomes a long-term objective, but a medium-term target is introduced driving the expenditure rule with different speeds of adjustment for different

[1] Bernanke et al. (2007); Corsello et al. (2021); Lane (2022), Lagarde (2022); Woodford and Walsh (2005)

[2] See Beckmann et al. (2022); Cochrane (2022) Cottarelli (2021); ECB (2022); Roubini (2021)

[3] See Amato and Saraceno (2022); ERSB (2021); Blanchard and Brancaccio (2019); Eichengreen and Panizza (2016); De Grauwe and Ji (2013); Lian et al. (2020)

[4] Alogoskoufis and Langfield (2019); Golec and Perotti (2017)

types of debt.

The Agency operates under the given fiscal framework and it raises liquid funds from the market by issuing bonds with finite maturity and continuously rolling them over to pay principal and capitalized interests. It offers credit to MSs through perpetual loans, which are priced based on their creditworthiness. As the cash flows from these loans are higher than the interest payments on EDA bonds, the Agency accumulates reserves. EDA bonds are traded, while perpetual loans are not, and they are priced using a transparent algorithm that considers them as risky perpetual loans. EDA amortizes its loans by recording a liability on its balance sheet under the Expected Loss Provision. The Expected Loss Provision is higher for countries with lower creditworthiness. The Agency's long-run equilibrium is maintained by matching its assets and liabilities. The Agency has seed capital, and because loans to MSs are priced differently based on their creditworthiness, there is no mutualization of debt. However, the pricing of loans generates a pooling effect that feeds the Solvency Capital of EDA on top of the Endowment. The size of this Solvency Capital justifies the low-risk status of EDA. The presence of EDA loans reduce roll-over risk and help MSs hedge their financing from market sentiment vagaries.

The paper is structured as follows: in Section 2, we place our contribution in the literature. Sections 3 and 4 provide an explanation of the working of EDA in detail, and in Section 5, we illustrate how the presence of EDA loans reduces roll-over by simulating the fluctuations of EDA loan prices over the period 2001-2021. In Section 6, we evaluate debt sustainability risk by simulating the implementation of fiscal rules with a forward horizon of twenty years from 2022 onwards, in two scenarios. The first scenario presupposes the implementation of the rules in the absence of EDA, the second considers presence of a "small" EDA, taking over the "slow" portion of MSs' debt.

2 Literature review

We place our paper in the literature by organizing available contributions around three main issues.

First, during the early stages of eurozone operation, there were several instances of excessive volatility of sovereign debt spreads. These episodes were characterized by prices diverging from their underlying fundamentals and sudden bursts of roll-over risk.

Second, the creation of a European safe asset is considered crucial for the smooth daily operations of financial market participants and for solving the "doom loop" problem that currently hampers debt sustainability for European banks. Various proposals have been made for creating such a safe asset, the most recent ones being associated with the establishment of a European Debt Agency.

Third, there is currently a debate surrounding the fiscal rule framework, which aims to reduce the risk of debt sustainability while avoiding an excess of restrictive fiscal policies. Several proposals for reforming this framework have been put forward.

2.1 Credit risk and Government Bond pricing in the Euro Area

The official press release of 21 July 2022 (ECB (2022a)) announcing the establishment of the Transmission Protection Mechanism (TPI) enunciates the principle of an ECB market intervention conditional on the macroeconomic compliance of the Member States to the existing rules and the presence of fluctuations in yields not justified by fundamentals. The TPI is explicitly intended to counter the formation of bad equilibria characterized by misalignments between expectations and MSs' fundamentals. While government bond pricing in line with fundamentals promotes market discipline, misalignments impact negatively on the debt dynamics, increase roll-over risk, and make debt stabilizing fiscal policy more costly. The empirical literature on the misalignment between government bond prices and fundamentals in the euro area is abundant.[5]

At the time of the establishment of the eurozone, economists and policymakers of the European institutions believed that the process of standardisation of issuance techniques and regulations, the elimination of exchange rate risk and the harmonisation of tax regimes would trigger a process of convergence and of greater integration in public debt markets. Indeed, in the pre-Great Financial Crisis (GFC) period, spreads between sovereign debt yields showed considerable convergence to low levels. The size of spreads, while still minor, could be ascribed to three main components: credit risk,[6] international risk factors[7] and liquidity risk.[8]

In the wake of the GFC and the European sovereign debt crisis, several empirical works focused on how the convergence process of government bond yields had broken down and how spreads displayed the emergence of idiosyncratic risk components.[9] In

[5] Afonso et al. (2014); Cantore et al. (2019); Corsetti et al. (2014); Favero and Missale (2012); Kim et al. (2015); Lane (2012); Lorenzoni and Werning (2019)

[6] Bernoth and Wolff (2008); Faini (2006);Von Hagen et al. (2011); Afonso and Strauch (2007) and Manganelli and Wolswijk (2009)

[7] Codogno et al. (2003), Geyer et al. (2004); Sgherri and Zoli (2009); Favero et al. (2010)

[8] Schuknecht et al. (2004); ECB (2003); Gomez-Puig (2006); Beber et al. (2009); Manganelli and Wolswijk (2009)

[9] Favero and Missale (2012); Afonso et al. (2014); Kim et al. (2015)

particular, evidence emerged that spreads dynamics were more driven by variables which more representative of economic sentiment than of fiscal fundamentals.[10]

The literature emphasised how this risk triggered self-fulfilling speculative attacks on the debts of those countries perceived to be riskier, as well as a capital migration to countries perceived to be safer (flight to quality). This phenomenon illustrates how changes in market sentiment are able to shift the sovereign debt market from a good to a bad equilibrium very quickly.[11]. Evidence of a sizeable redenomination risk, i.e. the possibility that some Member State would abandon the euro and return to a local currency, capable of causing contagion effects also emerged.[12] Analysis of the effects on sovereign debt yields of the ECB's conventional and unconventional monetary policies (QE) following the "whatever it takes" speech (Draghi (2012)) confirm the evidence for the presence of redenomination risk and point also to a persistent mismatch between yields and fiscal fundamentals[13].

A recent study Ceci and Pericoli (2022) proposes an empirically successful econometric model for government spreads of eurozone by estimating a a multi-country model in which the spreads of the government bond yields of Italy, France and Spain with respect to the German bond yield are regressed on a set of fundamental macroeconomic variables and a set of variables approximating the risk attitude of investors, for the period January 2007 – June 2022. The model provides an estimate of the fair value of the Italian ten-year sovereign spread, defined as a value consistent with the country's macroeconomic fundamentals. Since the second half of 2010, the fair value has often been lower than the observed spread, with significant differences in periods of market tension and political uncertainty, such as during the sovereign debt crisis in 2011-2012, political uncertainty in mid-2018, and the onset of the pandemic in March 2020.

In summary, there is significant evidence to suggest that sovereign debt yields in the eurozone have displayed inefficient and excessive volatility, consistent with levels of roll-over risk far higher that those justified by fundamentals. This evidence indicates that financing Member States of the European Union through the establishment of a European Debt Agency, with perpetual loans that are not traded but priced transparently to reflect fundamentals, can help to contain roll-over risk. This approach would insulate bond prices from fluctuations driven by market sentiment and help stabilization of financial markets.

[10] Georgoutsos and Migiakis (2013); Paniagua et al. (2017)

[11] Arghyrou and Tsoukalas (2011); De Grauwe and Ji (2013); Corsetti and Dedola (2016) and Lane (2012)

[12] De Santis (2019), Kremens (2018)

[13] Afonso and Jalles (2019); Favero (2013)

2.2 A European Safe Asset through a European Debt Agency

Establishing a European Debt Agency could contribute to increasing the low volume of euro-denominated safe asset, that currently does not exceed 25 per cent of the total volume of sovereign debt.

Insert Table1

A safe asset is a financial security embodying a payment promise with zero credit risk. Its high demand depends mainly on its use as high-quality collateral by financial operators on a daily basis in order to manage their liquidity needs. The need for safe assets can be driven by the need to comply with national and international regulations, as well as for portfolio building by risk-averse investors. Investment funds also use safe assets to price risky assets as well as a store of value. Moreover, safe assets are widely used by central banks in the implementation of their monetary policy.[14] In crisis time the financial system faces a shortage of safe assets and traders have to "accommodate" themselves with quasi-safe assets[15]. In these periods "flight to quality" causes a shift of portfolios from peripheral to core sovereign securities in the eurozone and the yield spreads widens not only because the price of riskier euro area government bonds decreases but also because the price of safer bonds rises. Moreover, in absence of a European safe asset, banks and insurance companies have been over-exposed to domestic government bonds and the value of their balance sheet has been considerably correlated with the value of government bonds. In this scenario a government debt crises induces a contraction of the supply of bank loans that increases the probability of a recession and of a downward spiral labelled as doom loop (Alogoskoufis and Langfield (2019)). The risk of a down loop might in turn induce governments to introduce bail-out possibilities that convey risk from the banking system to government bonds, despite in place regulatory provisions on banking resolution (e.g. the Bank recovery and Resolution Directive).[16]

Two crucial issues are relevant to design a common European safe asset, with or without establishing a Debt Agency.

The first one is the avoidance of debt mutualization, i.e. the pooling of national government debt within the euro area, which allows for joint liability among member countries, expressed by Article 125 TFEU, according to which "a Member State shall

[14] Bank of France (2021); Caballero et al. (2017); Golec and Perotti (2017); Greenwood and Vissing-Jorgensen (2018); Jank et al. (2022)

[15] Gorton and Ordonez (2014); Barro et al. (2022); Krishnamurthy and Vissing-Jorgensen (2012)

[16] Bolton and Jeanne (2011); Gennaioli et al. (2014); Gerlach et al. (2010); Dieckmann and Plank (2012)

not be liable for or assume the commitments of central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of another Member State". Debt mutualization helps reduce the borrowing costs for countries with higher debt levels, as the risk is shared across the entire euro area, but it also makes irresponsible behaviour profitable, as national government would not anymore subject to market discipline. Mutualization is currently considered as politically acceptable for a limited ('una tantum') number of common issues for special purposes.

The second issue is the avoidance of the "juniority effect", which occurs whenever debt is tranched in "senior" and "junior". The junior part is exposed to risk of mispricing and speculative attacks and the safety of the senior tranche could be jeopardized in the event of a systemic crisis.[17]

Before the pandemic crisis a number of proposals were made to introduce a safe asset without the explicit proposal for a European Debt Agency.

The first proposal is the issuance of Eurobonds using the European budget.[18] This type of asset is definitely a safe asset, as evidenced by the debt issuance to finance the Next Generation EU. However, it clearly involves government debt mutualization.

The second proposal [19], is the issuance by financial intermediaries of bonds covered by sovereign debt portfolios in two separate tranches: a senior one ("European senior bonds") and a junior one ("European junior bonds"). The payment of the coupons on the debt issued by the intermediaries are matched with the instalments paid on the sovereign debt portfolio. This proposal avoids mutualization but it does not avoid the "juniority effect".

The third proposed solution is the tranching of national debts[20]. In this case, a European agency would issue European bonds (E-bonds) in separate tranches and match them with senior loans granted to member states. No mutualization would occur and the senior tranches would constitute a European safe asset; however, the "juniority effect" is still a problem.

Dosi et al. (2021) propose a structural reform of the European Stability Mechanism (ESM). According to the authors "the ESM should abandon the current loan-based approach in favour of an insurance-based structure in which the Stability Mechanism becomes the guarantor of the public debts and the countries which get a direct and

[17] De Grauwe and Ji (2018)) and Gabor and Vestergaard (2018))
[18] Ubide et al. (2015); Zettelmeyer (2017)
[19] Brunnermeier et al. (2011); Brunnermeier et al. (2017); ESRB-HLTF (2018)
[20] Monti (2010); Juncker and Tremonti (2010)

immediate benefit from its guarantee pay an annual premium calculated at market prices". The insurance scheme would provide a European safe asset, however the insurance premium paid by MSs is calculated at market prices, hence its estimate could be affected by market mis-pricing.

Explicit proposals for a Debt Agency emerged in the wake of the pandemic.[21]

Giavazzi et al. (2021) propose that the Debt Agency gradually acquires over a period of five year a portion of the debt of each Member State. Total acquisitions are equal to the amount of "Covid Debt", measured as the increase in the debt-to-GDP ratio experienced by each country in 2020 and in 2021. In the years after the fifth the Agency would keep the ratio of debt issued on behalf of each country to GDP, i.e. $\frac{B_{i,t}^{EDA}}{Y_{i,t}}$, constant. The acquisition would be financed by the issuance of EDA debt, paying an interest r_t. This liability would be matched only by reserves. According to this proposal, reserves are cumulated with an annual outflow equal to the interest payment by the Agency, $r_t * B_t^{EDA}$ and an annual inflow made by the sum MS's contributions to the Agency set to $(r_t - g_{i,t}) * B_{i,t}^{EDA}$, with the proviso that in the case of $r_t < g_{i,t}$, the contribution is set to zero. Therefore, reserve dynamics for the Agency is counter-cyclical: reserves are accumulated in recessions and decumulated in expansions. However, MS's payments to the Agency are procyclical, (MS's contributions are higher in "bad times"): for any two countries with equal debt and opposite state of the cycle, the contribution of the country in recession will be higher than that of the country in expansion. Moreover, as recessions occur more rarely and last shorter than expansions, reserves are likely to deplete over time. Also, some form of debt mutualization is present, and it has been argued (Micossi (2022)) that this proposal is not compliant with Article 125 of TFEU.

Micossi (2021) and Avgouleas and Micossi (2021) propose to unburden the ECB balance sheet from the amount of sovereign debt purchased under unconventional monetary policy programmes with simultaneous purchase of the latter by the ESM, financed through the issuance of safe ESM bonds. The implementation of this proposal would free the ECB from fiscal dominance without mutualization, and would increase the volume of safe assets for the European and international financial system. However, this framework is a one-of measure with little potential for interaction with the dynamics of debt stabilizing rules over a long-horizon.

[21] Giavazzi et al. (2021) and Micossi (2021)

2.3 Fiscal rules: the current framework and proposals for reform

The European fiscal framework was built with the aim of settling together two different objectives. On the one hand, the goal has been to promote debt sustainability, while preserving independency of common monetary policy from fiscal policy; on the other hand, Member States should have been allowed enough flexibility as to be able to set up countercyclical fiscal measures. Both principles are sought to be incorporated into the EU fiscal framework in order to avoid the adverse effects connected with a sovereign debt crisis and equip MSs with an adequate instrument for macro-stabilization.

The basic architecture of the rules has been outlined in Article 126 TFEU and Protocol 12, annexed to the Maastricht Treaty, and it is articulated around two reference values: 3 percent for the deficit and 60 percent for the debt-to-GDP ratio. With the changes occurred over time in the economic environment, the simple layout of the Stability and Growth Pact (SGP) turned out to be unsatisfactory. As a consequence, periodically, there have been few attempts to change its structure[22]. More and more layers of complexity have been added in the hope of changing rules that constrained fiscal policy during downturns, while wrongly allowing for pro-cyclical fiscal expansions. All these additions tried to deal with the weaknesses of the SGP, introducing cyclical-adjusted variables and making the enforcement of fiscal limits less arbitrary. At the same time, the basic layout, i.e. the uniform debt target and the reference value for the allowed deficit, was not challenged by these successive modifications. So, despite the numerous subsequent amendments and improvements to the EU fiscal rules, their underlying logic and juridical structures have been retained.

However, the debate about rethinking the fiscal framework is now focusing on criticizing the level of common debt and deficit reference values. A strong majority of economists agrees that debt sustainability is not a predetermined concept that can be summarized through the adherence to fixed values. On the contrary, it should be assessed case by case, taking into consideration the country's fiscal track record, its ability to generate primary surplus, the current country-specific interest rates and its long-term growth. Furthermore, the effort of trying to reconcile time-invariant values with more state-contingent procedures is doomed to failure because of the overall intricacy. The implementation of such flexible measures is impaired by the estimation of badly-measured indicators and incorrect forecasts, as happens for example when

[22]Reforms occurred in 2005 with the first amendment; in 2011 through a collection of new laws, known as the "Six Pack"; in 2013 with the so-called "Fiscal Compact"; in 2014 with the review of the "Six Pack" and "Two Pack" rules; in 2015 with the issuance of guidance by the Commission

estimating structural budget balances. This can ultimately lead to misleading policy guidance.[23]

While it is widely shared that a deep reform of the EU fiscal rules is needed, what is controversial is the way in which this change should be implemented. Among all the proposed reforms, the one made by Blanchard et al. (2021) stands out for its radical approach. The authors, indeed, put forward the proposal of abandoning fiscal rules in favor of fiscal standards, i.e. qualitative prescriptions able to properly seize relevant contingencies. Within this framework, imposed limits would be state-contingent and country-specific and based on general objectives coupled with stochastic debt sustainability analysis, aimed at assessing how risky is debt sustainability for MSs. The task of assessing whether or not countries' fiscal adjustment plans are adequate should be assigned to independent national fiscal institutions (IFIs) and/or to the European Commission. In addition, as far as the enforcement is concerned, it could be conducted by the Council of the European Union or, alternatively, by an independent body, such as the European Court of Justice (ECJ) or a specialized EU-level court. In particular, the authors argue for this second option.

Although the revision of the EU fiscal framework proposed by Blanchard et al. (2021) is extremely attractive and well-designed, the problem lies in the implementation of the project within the current EU law. As a matter of fact, employing fiscal standards would require to amend Protocol 12, in order to remove the 60 percent and 3 percent debt and deficit benchmarks outlined there. Moreover, even the designed procedures of enforcement raise some issues. The enforcement via the Council of the EU could be seen as a variant of the current procedure, so it would not require legislative changes at European level. However, it may require changes in national constitutions as far as it is needed to give power in the national budget approval process to either a national or an EU entity. The adjudication by a new European designated body, instead, would for sure require a deep change of EU primary legislation, namely a Treaty change. These are quite ambitious challenges to overcome, since implementing some of these modifications would involve the unanimous consent of Member States.

Preserving these benchmarks, even if only partially, while trying to make the rules less convoluted and less procyclical is a more realistic approach. For this reason, most of the reform proposals[24] moved in this direction. Martin et al. (2021), for instance, proposed a less radical approach with respect to fiscal standards: the idea is to maintain the reference values, while giving up the country- and time-invariant aspect. Indeed,

[23] Claeys et al. (2016); Fatás (2019)

[24] Beetsma et al. (2018), Bénassy-Quéré et al. (2018), Claeys et al. (2016), Darvas et al. (2018), Thygesen et al. (2019) and more recently Giavazzi et al. (2021), Haroutunian et al. (2022)

each MS should have its own specific medium-term target and it would function as an indicator for the expenditure rule. Even though it avoids radical changes of the Treaties, it would still require the establishment of domestic IFI and, as a consequence, the critical amendment of national legislation.

In April 2023, after months of deliberations, the European Commission has finally officially disclosed its proposal for renovating the EU fiscal rules. Besides the numerous shortcomings, it is worth underlining two aspects that are on the right track. The first improvement is represented by the idea of treating debt sustainability as a whole, abandoning the year-after-year rule of the 3 percent deficit. Secondly, the proposed plan envisages the evaluation of the pre-existing debts of each country, allowing for a case by case analysis.

The interaction between fiscal rules to ensure debt sustainability and a European Debt Agency to ensure efficient debt management without mutualization could play a key role in giving flexibility to fiscal policy while preserving debt sustainability. Giavazzi et al. (2021) take a step in this direction pairing their proposal for a European Debt Agency with a new fiscal framework. Their plan maintains the 60 percent debt reference value as a long-term objective, but it introduces a medium-term target driving the expenditure rule and different speeds of adjustment for different type of debts: slow speed of adjustment and fast speed of adjustment. Slow speed debt is the results of the deficits accumulated in response to crises and to finance spending for the future. Crisis over the sample 2001-2021 are identified as years in which the escape clause is active (the Covid period and recessions in 2008-2009 and 2011-13). As part of the golden rule scheme, any spending with positive impact on medium-term growth and benefiting future generations is also included in the slow-speed debt. The fast-speed part is the residual stock of debt.

Our proposal for an EDA is part of this new debate. In the following sections we will recall its modus operandi and its ability to reduce roll-over and sustainability risks, to then simulate a management of the new rules reinforced by the operation of a EDA, tasked with managing only part of the Member States' debt.

3 The European Debt Agency

Our operational model for the EDA exhibits the following key characteristics:[25]

[25] We leverage on the framework elaborated in Amato et al. (2021), but we introduce important refinements, mostly concerning the dynamics of loan prices and that of the quantities of Reserves. For general considerations on the institutional role of the EDA, and its relationship with the debate on European rules, the reader can refer to Amato et al. (2022).

i) The Agency collects liquid funds on the market by issuing bonds with finite maturity and by continuously rolling them over to pay principal and capitalized interests.

ii) When EDA begins its operations it starts buying MSs bonds. The Agency provides credit to MSs to finance the repayment of their maturing bonds (principal plus interests) as well as their primary budget deficit. This credit facility takes the form of perpetual loans, entailing for the Agency a commitment to renew the loans perpetually ("perpetuity option clause") unless a MS partially refunds them through primary budget surpluses.

iii) The perpetual loans are priced using a risk-adjusted unit cost differentiated according to the MS's creditworthiness. The perpetuity is computed following a perpetual-amortization scheme. The EDA amortizes its loans by recording a liability on its balance sheet corresponding to the expected credit loss that has been priced in the perpetuity.

iv) The deferred perpetuities charged to MSs are collected annually by the EDA and accumulated under an "accrued interest reserve" item, intended to cover the Agency's future liabilities (EDA bond principal, bond accrued interests and expected losses). The reserve takes the form of a "Central Bank interest-bearing liability"; the interest rates used in revaluing the reserves are in line with the capitalized interests payable on the EDA bonds. Also, the agency is initially endowed with a seed capital injection.

v) The dynamics of assets and liabilities pin down solvency capital. This capital could be measured in terms of the number of forbearance years of a "stressed" annuity payment that it allows to each MS. The annuity payment is stressed in the sense that it is computed for credit grade "next to default".

The introduction of perpetuities with EDA, allows to move from a jump-to-default logic to a forbearance logic. Whilst default logic implies that a significant part of the exposure is non-performing, forbearance only suspends the payment of the interest due (i.e. of the perpetuity instalments) during a congruous but limited restructuring period, without implying any form of default. In the context of a perpetuity scheme, a finite payment suspension cannot undermine the soundness of the scheme itself. Solvency capital is then naturally measured in terms of the number of forbearance years of a "next to default" annuity payment for each MS. MSs cannot issue perpetuities directly, as these do not easily complement the liability structure prevailing in the market (according to a logic of liquidity preference), which implies a portfolio offering of assets with finite duration. This is why the EDA intermediation is needed, in order to decouple perpetuity and the issuance portfolio by offering bonds with finite duration and leveraging mechanisms to roll over issues while minimising repricing risks thanks to a high credit rating. EDA creditworthiness leverages on three key elements: 1) portfolio diversification, 2) solvency capital and 3) repricing of the perpetuity to address interest rate risk (change in the prevailing level of interest rates for relevant durations relative to the issuance portfolio). The scheme of the EDA balance sheet is summarized in Table 2.

Insert Table2

Bonds issued by EDA are traded on the markets and the availability of solvency capital and reserves gives them the status of European safe asset. Instead, there is no market for the perpetual loans, perpetuities are priced by EDA with a risk risk-adjusted interest rate made up of two basic components: the average cost of servicing the EDA issued bonds and an add-on cost reflecting the riskiness of each MS in line with its specific creditworthiness, i.e. proportional to its degree of compliance to the agreed EU rules. The cost of perpetuities for each MS is a function of the market cost of the EDA's issuing portfolio, plus a differential cost reflecting the MS's specific creditworthiness. This allows the EDA to avoid any component of mutuality in prices.

4 Pricing of the irredeemable mortgage scheme by EDA

To price the irredeemable mortgage scheme EDA computes the present value of an infinite stream of payments using the yield r_t^B as a discount rate, reflecting its annual cost of debt. Future payments are not deterministic but they occur only if MSs are not in "default". The probability with which a given MS enters the state of default in each future period is computed by i) assigning each MS to a specific *credit risk class j* based on its creditworthiness, from the safest (conventionally labelled AAA) to the default class (labelled D) ii) assuming that a country defaults only when it reaches state D, and modelling the transition from one state to the other via a transition matrix that depends on the state of the economic cycle iii) taking into account that, as the business cycle is stationary the predicted point-in-time transition matrix at each period in the future converges rapidly to a constant through-the-cycle transition matrix. Given the discount factor and the credit risk migration model the present value of a unitary perpetual annuity for a country i initially in credit risk class j can be then computed as $\tilde{a}_{ij,t}$ and the interest on the perpetual annuity is then set to $\frac{1}{\tilde{a}_{ij,t}}$ (see appendix A1).

Given the unitary perpetual annuity value, the annual instalment cost for each country, labelled as "idiomatic cost", is computed in order to preserve the intertemporal financial equilibrium of the EDA. To this end each country should pay an annual instalment, $c_{ij,t}$ that ensures the match between the present value of the perpetuity's payment and the difference between the value of bonds issued by EDA to finance the country, $B_{i,t}^{EDA}$, and reserves accumulated by the country with EDA, $R_{i,t}^{EDA}$:

$$c_{ij,t} = \frac{B_{i,t}^{EDA} - R_{i,t}^{EDA}}{\tilde{a}_{ij,t}}. \tag{1}$$

It is immediate to verify that the above formula guarantees that, for each country, the present value of total assets with EDA $\tilde{a}_{j,t}c_{ij,t}$ is equal to the total of current net liabilities with EDA $(B_{i,t}^{EDA} - R_{i,t}^{EDA})$.

"Idiomatic cost"has several important features.

1. Each Member State pays for the risk inherent to the specific credit risk class j to which it is assigned, without involving any form of solidarity or mutuality among Member States of different credit risk classes. Thanks to the irredeemable nature of the loan granted by the Debt Agency, the instalment corresponds to the risk-adjusted interest that a Member State of credit risk class j has to pay annually to finance its debt based on its creditworthiness.

2. The annual instalment cost is repriced in each period so that EDA's assets are shielded from interest rate risk and upgrades and downgrades in the merit credit of Member States are timely fully priced.

3. Each Member State debt is priced independently. Pricing the debt of each country independently generates a total payment to EDA higher than the case in which the Debt Agency prices at time t its loans portfolio using an average annuity cost computed as the weighted average of the annuities of the credit risk classes, with weigths determined by the relative loan exposure for each class. Average pricing assures in expectation the agency intertemporal equilibrium exploiting a "pooling effect" that it is not present under idiomatic pricing. Therefore, idiomatic fundamental pricing scheme generates a total payment that is structurally higher than the one implied by average pricing. Under idiomatic pricing EDA will accumulate reserves that can be precisely attributed to each country. The sum of reserves and loans will exceed the value of bonds and will form the expected losses component of the balance sheet. Under the pricing scheme adopted, all countries are expected to default on a given debt proportion at a (differently distant) finite time in the

future, however, EDA has always positive equity. In fact, at the time in which a country is expected to default, reserves will match bonds issued and expected losses will match outstanding loans. Note that the accumulation of reserves and expected losses will be related to the credit risk class to which countries are assigned. The worse the credit risk class, the faster the accumulation of reserves and expected losses.

4. By leveraging on the potentially irredeemable nature of sovereign debts, we have intended to price the overall cost by means of an amortizing scheme according to which every single Member State pays for an infinite period of time only a risk-adjusted interest, regularly re-priced. Reserves accumulated by EDA under the idiomatic pricing scheme will contribute (together with an initial endowment) to form its required risk capital. In the context of a perpetual long scheme where fiscal rules prevent the risk of exploding debts and deficits, capital can be used by MS to suspend the payments of the perpetual loans for a number of "forbearance" years in which reserves accumulated with EDA can be used to pay loans and, if necessary, to finance temporary primary deficits. It is therefore natural to evaluate risk capital in terms of forbearance years allowed by the reserves available with EDA for each MS.

5 EDA and Roll-over Risk

Fig.(1) illustrates the historical trends of the yields to maturity of 10-year bonds issued by Member States.

Insert Figure 1

Fig.(1) clearly shows that the initial convergence process following the inception of the euro in 2001, had been substituted by a process of divergence beginning after the US subprime lending crisis, reaching its peak during the sovereign debt crisis of 2011-2012. In fact, a "divergent symmetry" ("symmetrical divergence") emerges between countries with high credit rating (primarily Germany) and countries with a tight budget constraint (especially Italy and Greece). This pattern becomes inefficient if the resulting cost of debt service for MSs were different than the cost of debt service consistent with their fundamental risk. The effects of this inefficiency are worsened when member states banking system bond holdings are affected by home bias: in this case a "doom loop" emerges when falling government bond prices causes a reduction in bank loans that in turn increases roll-over risk via its recessionary impact.

What would have happened if the cost of servicing the debt of these Member States had been calculated on the basis of the idiomatic pricing scheme for EDA loans? Given a discount factor represented by the 10-year fixed interest rate swap in the euro area, the historical rating grades assigned by Credit Rating Agencies to MS's over the period 2001-2022, and the estimated point-in time- and through-the-cycle transition matrices of our credit risk migration model, we have simulated the idiomatic costs of loans with EDA for each Member States. Figure 2 shows the simulated series of idiomatic costs for Italy, Germany and a hypothetical country with the credit grade "next to default" together with the observed yields of 10-year Government Bonds for Germany and Italy.

Insert Figure 2

These costs are "risk sensitive", but the idiomatic pricing of risk is very different from the pricing observed in 10-year bond yields for Germany and Italy during the simulation sample. Importantly, idiomatic costs do not manifest "diverging symmetries" in favour or against a particular Member State, since they are calculated on the assumption that a "systemic risk factor" operates at the level of the entire eurozone. Note also that there are several episodes in which the observed 10-year bond yield for Italy is much higher than the idiomatic cost for a country with a credit grade "next to default" despite the fact the rating grades assigned to Italy never went any close to it. Although 10-year yields and idiomatic costs are not directly comparable because of the different duration of the underlying investment, their different fluctuations would eventually be reflected in different average costs of government debt servicing. The evidence suggests that the cost of debt service for MSs has been inefficient, i.e. different from the one consistent with their fundamentals.

In a recent paper Ceci and Pericoli (2022) provide an estimate of the fair value of the Italian ten-year sovereign spread, defined as the value consistent with the country's macroeconomic fundamentals. They estimate first a panel model with fixed effect for Italy, Spain and France over the period 2007-2022.

$$s_{i,t} = \alpha + \delta_i + \beta_0 Z_{i,t} + \beta_1 X_{i,t} + +\beta_2 F_t + \epsilon_{i,t}. \tag{2}$$

where $s_{i,t}$ is the end-of-period spread observed for country i at time t measured as the difference between the 10-year government bond yield of country i and the corresponding 10-year German bond yield; δ_i represents country fixed effects. $Z_{i,t}$ includes country specific macroeconomic variables; namely, the debt-to-GDP ratio, the expected average inflation over the next five years surveyed by Consensus Forecast; the average real GDP growth rate expected over the next twelve months surveyed by Consensus Forecast, the average real GDP growth expected over the next five years surveyed by Consensus

Forecast, and the unemployment rate. $X_{i,t}$ and F_t represent variables related to the risk attitude of investors which are, respectively, country specific (a market-based indicator of redenomination risk, the so-called ISDA basis) and common to all countries (the search volume of "euro break-up" or similar words using the Google search engine). After successful estimation of the model, they determine the fair value of the spread as

$$s_{i,t} = \hat{\alpha} + \hat{\delta}_i + \hat{\beta}_0 Z_{i,t}. \tag{3}$$

Figure 3 reports the fair value of the Italian ten year bond obtained using the fair value of the spread derived by Ceci and Pericoli (2022) (with bounds reflecting the uncertainty of the estimated parameters) and the simulated idiomatic cost derived by our methodology (with bounds determined by the idiomatic cost attributed to the rating grades of the next riskier and safer groups). The approaches are very different, in that in the case of the idiomatic cost risk adjusted returns are computed for the perpetuity, while in the case of the fair value only macroeconomic fundamentals are used to determined the fair value of spread without including any risk pricing uncorrelated with $Z_{i,t}$. However, there is a common clear indication of several episodes of inefficient pricing of 10 year Italian government bonds over the sample considered.

Insert Figure 3

6 EDA, Flexible Fiscal Rules and debt sustainability

To illustrate the potential of the role of EDA for efficient debt management we simulate over an horizon of twenty years two different scenarios in which fiscal sustainability is achieved through the implementation of flexible fiscal rules. In particular, the adoption of flexible fiscal rules without EDA, and the adoption of flexible fiscal rules with an EDA. The first scenario is a benchmark scenario in which EDA is not established. In the second scenario, EDA gradually acquires a portion of MS's debt equal to the amount of the slow-adjusting debt subject to the golden rule. In this scenario, the "fast" portion of government debt would be financed by sovereign bonds and the "slow" portion of debt would instead be financed by loans with EDA.

6.1 Flexible Fiscal Rules

We build on Giavazzi et al. (2021) who consider a golden rule paired with the separation of the debt into two components: slow speed of adjustment and fast speed of adjustment.

The proposed rule has a medium-term target for the debt-to-GDP ratio to be achieved by imposing a ceiling on the net primary spending. The medium-term is defined as 10 years and the medium-term target proposed is given by the current debt plus a correction that depends on two components: 20 percent of the gap between slow debt and its final target and 50 percent of the gap between fast debt and its final target. The medium-term target debt is then set as follows:

$$\hat{b}_{i,t} = b_{i,t} - 10 * \beta * \left(b_{i,F,t} - b^*_{i,F,t}\right) - 10 * \gamma * \left(b_{i,S,t} - b^*_{i,S,t}\right)$$

where $\beta = 0.05$ and $\gamma = 0.02$. $b_{i,t}$,$b_{i,S,t}$, and $b_{i,F,t}$ are respectively is the ratio to GDP of total debt, slow speed of adjustment debt, and fast speed of adjustment debt for country i at time t.

Net spending is then set by the fiscal reaction function that stabilizes the debt-to-GDP ratio towards the long-term targets. The reaction function is then specified as follows:

$$(g_{i,t} - t_{i,t}) = -\frac{r_{i,t} - \Delta y_{i,t}}{1 + \Delta y_{i,t}} b_{i,t-1} - \frac{1}{10} * \left(b_{i,t-1} - \hat{b}_{i,t-1}\right) \tag{4}$$

where $r_{i,t}$ is the cost of financing and it is equal to ratio of interest payment in year t to the stock of debt in year t-1, $\frac{IP_{i,t}}{B_{i,t-1}}$, $(g_{i,t} - t_{i,t})$ is the ratio of the primary deficit to GDP, $\Delta y_{i,t}$ is the rate of growth of nominal output, $Y_{i,t}$, for the Member State i at time t. Equation (4) can be equivalently written as:

$$(g_{i,t} - t_{i,t}) = -\frac{r_{i,t} - \Delta y_{i,t}}{1 + \Delta y_{i,t}} b_{i,t-1} - \beta \left(b_{i,F,t-1} - b^*_{i,F,t-1}\right) - \gamma \left(b_{i,S,t-1} - b^*_{i,S,t-1}\right) \tag{5}$$

The fiscal reaction function has two components: the first one is the primary surplus that would stabilize the debt to GDP ratio at the current level, while the second one implements the correction and ensures that the medium term target and the final target are reached within the chosen horizons.

Overall, this proposal maintains the long-run target for debt-to-GDP ratio at 60%, while operating on different speeds of adjustment for the debt. A golden rule is then implemented, with the precise role of avoiding premature consolidation coming out of recessions and stimulating “good” spending.

6.2 Debt Sustainability and EDA

The potential role of EDA in the implementation of flexible fiscal rules is assessed by simulating over the period 2023-2040 two scenarios for debt stabilization: a benchmark without EDA and an alternate scenario in which EDA is introduced[26]. Fiscal rules are first simulated without EDA to assess the pattern of debt stabilization and of primary surpluses necessary to achieve it. Then an alternative scenario is built in which EDA acquires progressively the slow debt and it takes 5 years to complete this operation. In our simulations EDA begins operating in 2022, by issuing bonds to make loans to MS's to acquire progressively their entire current and past slow debt. An initial capital, equal to a share of the ESM capital determined by the ratio of the total "slow" debt to total debt of MS's when EDA becomes operational is conferred to EDA via the ESM and it can be attributed within EDA to member countries according to the ESM weights (https://www.esm.europa.eu/esm-governance).

From the inception of EDA the fiscal rule is modified to take in account that the government debt is made of a mixture of Bonds and Loans with EDA and loans with EDA are treated as slow-debt.

After its establishment EDA continues to absorb pre-EDA slow debt until all past and new slow debt is financed through EDA loans. From 2023 onwards MSs start to pay deferred instalments to EDA. The EDA bonds increase because of interest rate payments and because of new lending. As payments on the perpetual loans flow in the Agency accumulates reserves and it also records expected loss provisions ($el_{i,t}^{EDA}$), computed as the difference between the sum of loan-debt and reserves and the bond-debt issued by the EDA. The initial endowment to GDP ratio ($en_{i,t}^{EDA}$) is kept constant over time. After the transition is completed there are two types of debt: government bonds, that finance exclusively fast debt, and EDA loans, that finance exclusively slow-debt.

In the baseline scenario debt and deficit dynamics are simulated by specifying first the stochastic processes for the euro area swap rate 1-year and 10-year swap rates, which are the drivers of the German ten-year yields. Different MS's ten-year yields spreads are driven by fiscal fundamentals and volatility around them. Fluctuations in yields at different maturities impact on the dynamics of the average cost of servicing the debt. GDP growth in all countries depends on the common monetary policy on the primary deficits and the country risk premia as captured by the spread of 10 year yields on government bonds on the 10 year German Bund.

In the alternative scenario the pricing equation for EDA loans is introduced, and the model is modified in that, whilst keeping all estimated parameters constant, the volatility of the country risk premium declines as the presence of EDA mitigates the deviations of bond prices from fundamentals. The simulation model contains six stochastic equations for GDP growth in the MS's countries (Germany, Italy, France, Spain, Portugal and Greece), six stochastic equations for the 10-year yields on government bonds, six

[26]See Appendix A2 for detailed description of the models we have used for simulation

stochastic equations for the average cost of financing the debt, two stochastic equations for the 1-year and the 10-year swap rates, one equation for the rate on EDA loans, six fiscal reaction functions determining the primary surplus for MS countries and six dynamic equations determining government debt to GDP dynamics for MS countries. Stochastic simulation is implemented by bootstrapping the rows of the matrix containing the estimated residuals for all the stochastic equations.[27] The results of our simulations are illustrated in Figures 4-5.

Insert Figures 4-5

Stabilization is achieved by the adopted fiscal rules both in the baseline and the alternative scenario. However, stabilisation costs are very similar in the two scenarios for low-debt countries. Still, they are much smaller when EDA is present for high-debt countries as the primary surpluses needed to stabilize the debt-to-GDP ratio are smaller and less volatile. Stochastic simulations also show that the upper bounds of the 95 per cent confidence intervals for primary surpluses implied by the fiscal rules in the worst-case scenario are much smaller when the debt agency is operational. This evidence depends on the importance of EDA loans to reduce the level and the volatility of the cost of financing the debt and witnesses the importance of EDA in reducing the risk associated with fiscal rules for debt stabilization. Importantly, Pareto efficiency is achieved within EDA in the sense that no MSs are worse off and some are better off.

7 Conclusions

The current economic situation in Europe has made it so that countries could be at high risk of two types of government debt risks: roll-over risk and sustainability risk. To help manage these risks, we suggest the establishment of an EDA (European Debt Management Agency). Our proposal addresses roll-over risk by pricing EDA loans with a transparent formula that anchors price to fundamentals. This discipline mechanism, avoids the inefficient costs generated by deviation of market prices from fundamentals while giving Member States incentives for fiscal virtues as it does not imply debt mutualization. Out-of-sample simulation analysis show that adopting flexible fiscal rules in the presence of EDA allows a smoother path towards debt stabilization by reducing the macroeconomic consequences of excessive fluctuations in risk premia. Overall our results show that EDA is efficient in two different ways. With EDA loans roll-over risk is addressed by issuing loans whose pricing is always efficient in the sense that it never

[27]See the appendix for a description of specification, estimation and simulation of the model.

deviates from fundamentals. Sustainability risk is addressed by the joint implementation of growth-friendly fiscal rules for debt sustainability and the establishment of EDA as an efficient debt management institution. The important concept of efficiency in this context is Pareto efficiency. Our out-of-sample simulations demonstrate that introducing EDA brings benefits to certain Member States without imposing any significant costs on the others.

8 Tables and Figures

rating 31/12/2020	Outstanding debt	Proportion
AAA	2,776,336	24.49%
AA	3,649,460	32.20%
A	1,699,107	14.99%
BBB	2,868,706	25.31%
BB	341,023	3.01%
total	11,334,631	100.00%

Table 1: Eurozone Public Debts, source ECB 2021

Assets	Liabilities
Perpetual Loans to Member States (L^{EDA})	EDA Bonds (B^{EDA})
Reserves (R^{EDA})	Expected Loss Provisions (EL^{EDA})
	Solvency Capital (SC^{EDA})

Table 2: EDA - stylized balance sheet

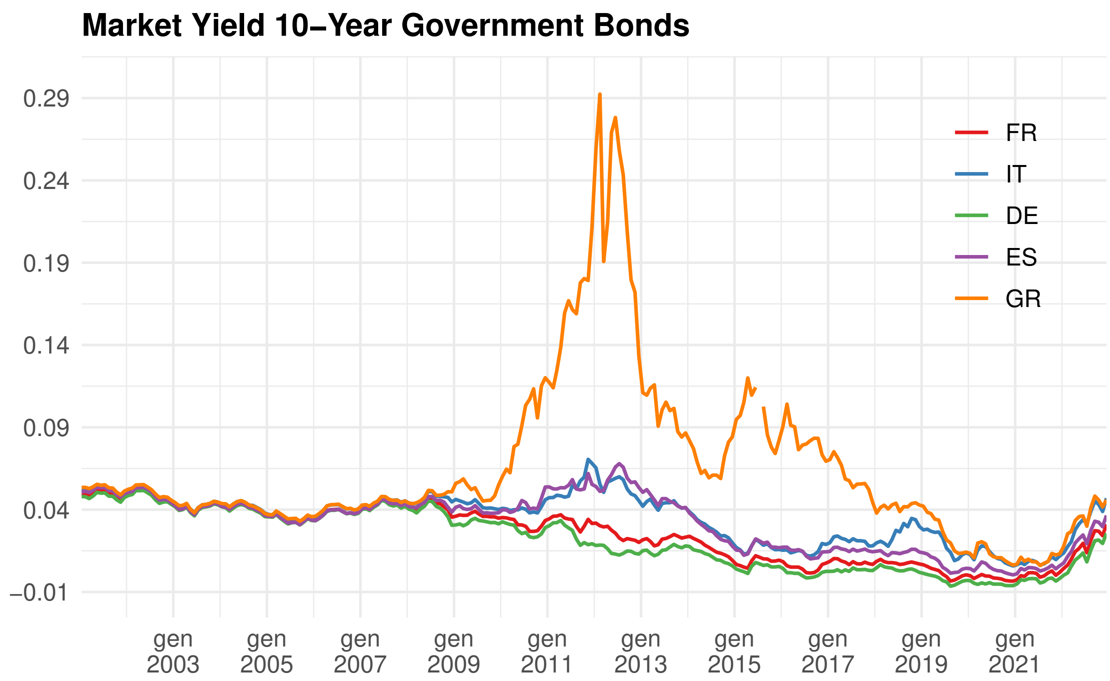

Figure 1: Yields on 10-Year Government Bonds

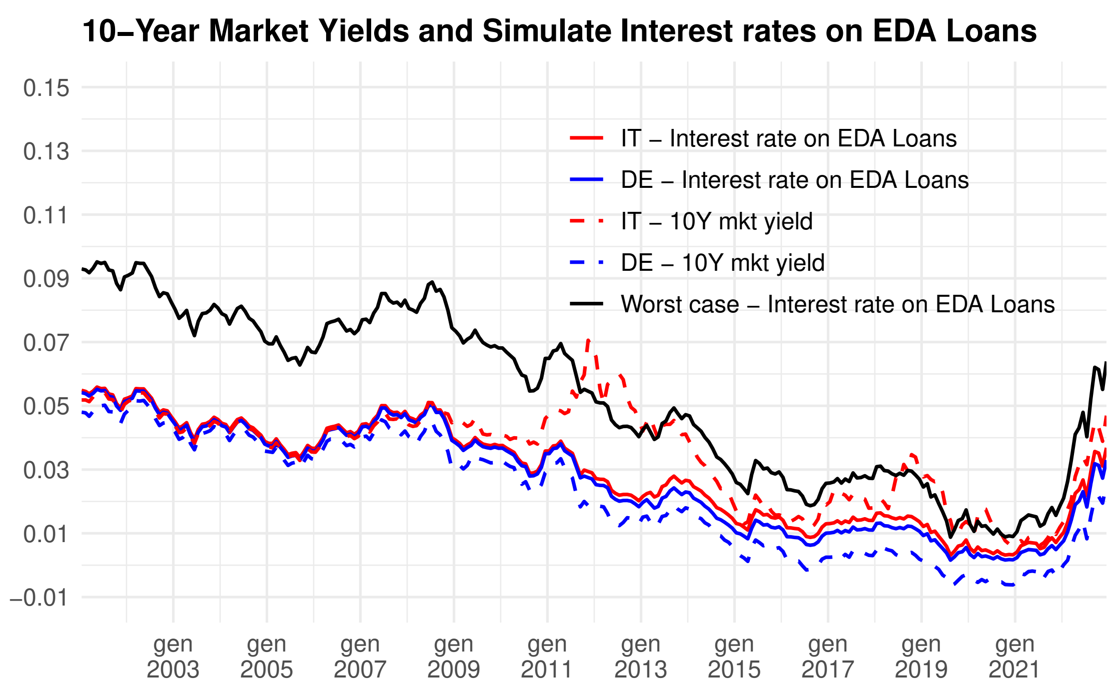

Figure 2: Yields on 10-year government bonds and simulated interest rate on perpetual EDA loans

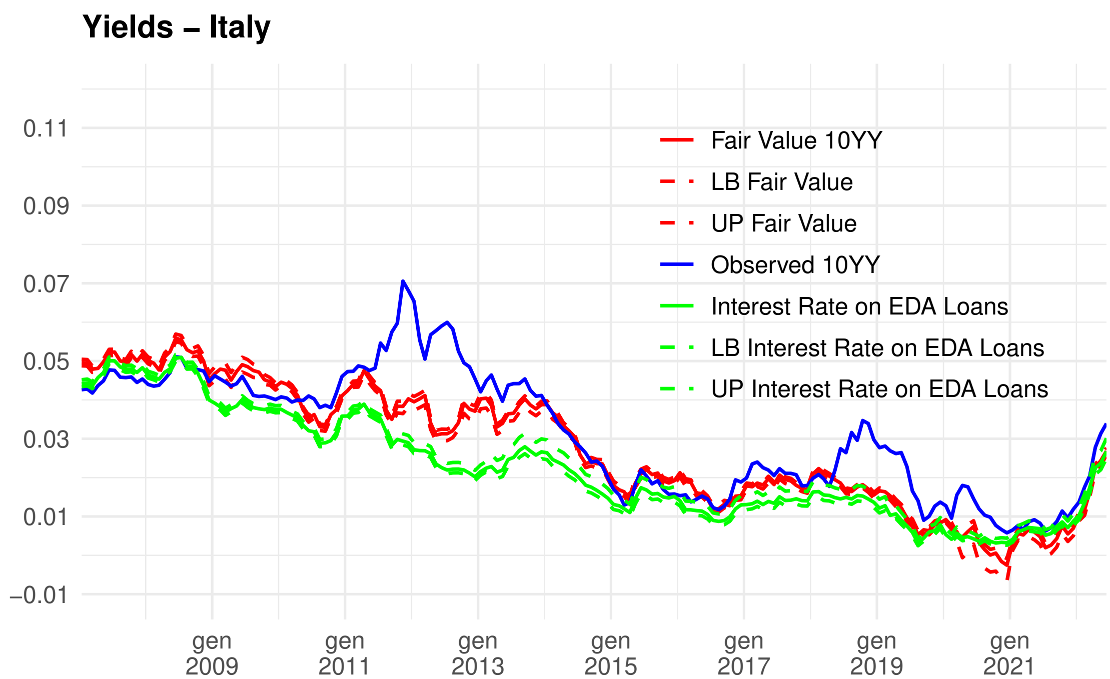

Figure 3: Italy: Observed 10 Year Yield, fair yield and interest on perpetual EDA loans

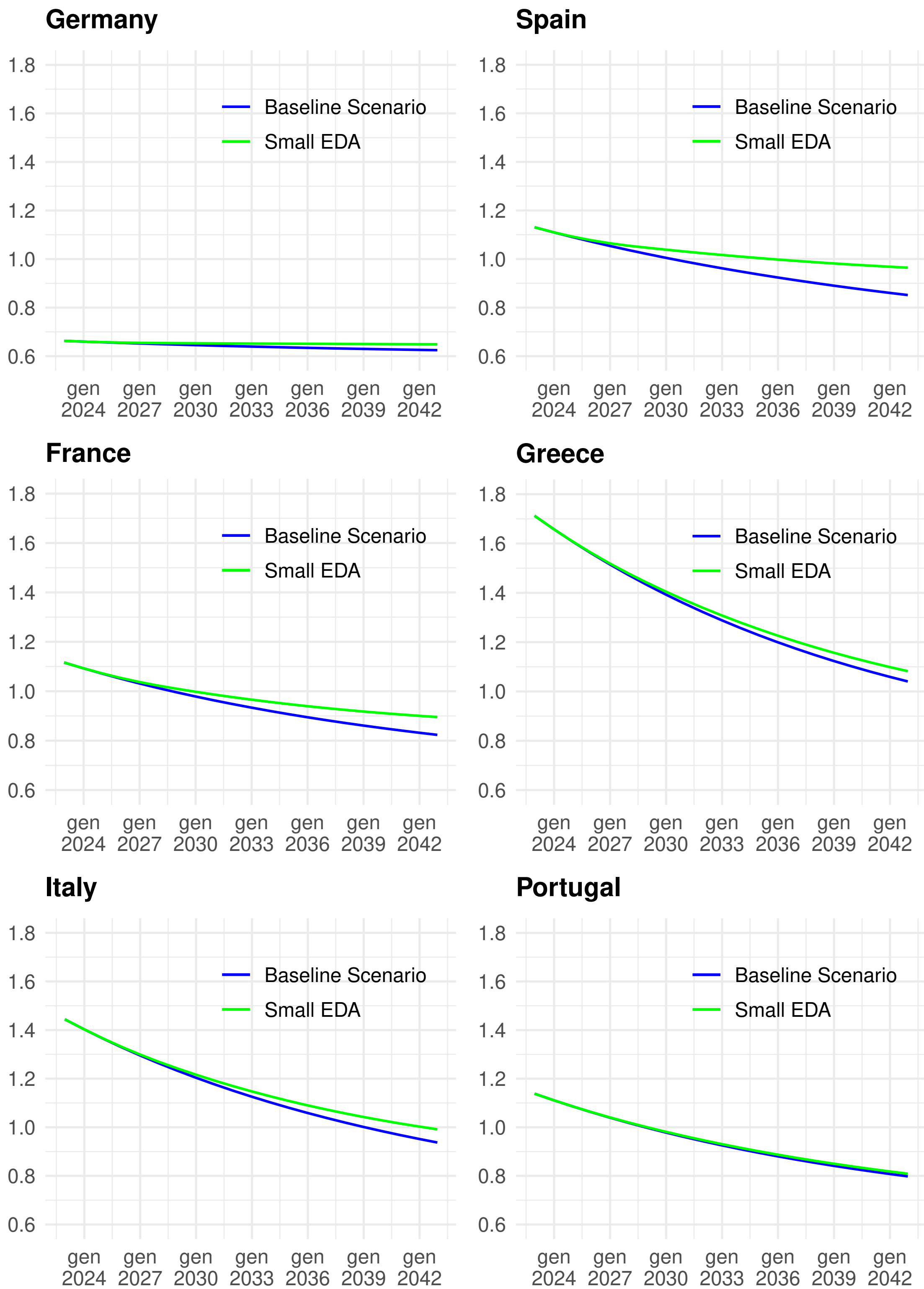

Figure 4: Simulated Total Debt to GDP ratio

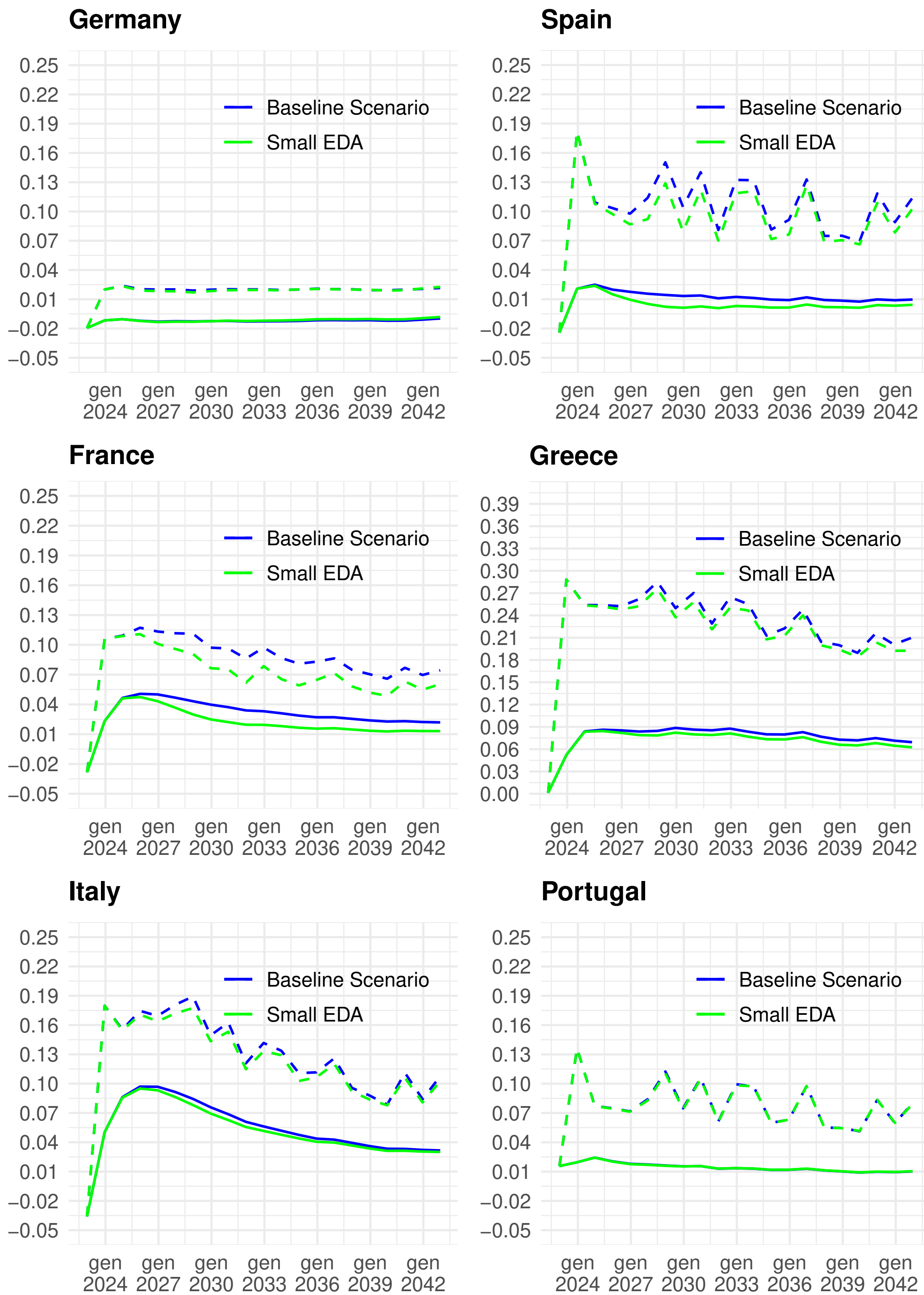

Figure 5: Simulated Stabilizing Primary Surplus. Solid lines denote the mean simulated values and dotted lines denote the upper bound of the 95 per cent confidence intervals.

References

Afonso, A., Arghyrou, M. G., and Kontonikas, A. (2014). "Pricing sovereign bond risk in the European Monetary Union area: an empirical investigation." *International Journal of Finance & Economics*, *19*(1), 49–56.

Afonso, A., and Jalles, J. T. (2019). "Quantitative easing and sovereign yield spreads: Euro-area time-varying evidence." *Journal of International Financial Markets, Institutions and Money*, *58*, 208–224.

Afonso, A., and Strauch, R. (2007). "Fiscal policy events and interest rate swap spreads: Evidence from the EU." *Journal of International Financial Markets, Institutions and Money*, *17*(3), 261–276.

Alogoskoufis, S., and Langfield, S. (2019). "Regulating the doom loop." *Available at SSRN 3453158*.

Amato, M., Belloni, E., Falbo, P., and Gobbi, L. (2021). "Europe, public debts, and safe assets: the scope for a european debt agency." *Economia Politica*, *38*, 823–861.

Amato, M., Belloni, E., Favero, C. A., and Gobbi, L. (2022). "Creating a Safe Asset without Debt Mutualization: the opportunity of a European Debt Agency." *CEPR Discussion Paper No. DP17217*.

Amato, M., and Saraceno, F. (2022). "Squaring the circle: How to guarantee fiscal space and debt sustainability with a european debt agency." *BAFFI CAREFIN Centre Research Paper*, *2022-172*.

Arghyrou, M. G., and Tsoukalas, J. D. (2011). "The greek debt crisis: Likely causes, mechanics and outcomes." *The World Economy*, *34*(2), 173–191.

Avgouleas, E., and Micossi, S. (2021). "On selling sovereigns held by the ECB to the ESM: institutional and economic policy implications." *CEPS Policy Insights PI2021-04/MARCH*.

Bank of France (2021). Le Bulletin de la Banque de France 234/6.

Barro, R. J., Fernández-Villaverde, J., Levintal, O., and Mollerus, A. (2022). "Safe assets." *The Economic Journal*, *132*(646), 2075–2100.

Beber, A., Brandt, M. W., and Kavajecz, K. A. (2009). "Flight-to-quality or flight-to-liquidity? Evidence from the euro-area bond market." *The Review of Financial Studies*, *22*(3), 925–957.

Beckmann, J., Klaus-Jürgen, G., Jannsen, N., Sonnenberg, N., and Stolzenburg, U. (2022). "Inflation expectations: quo vadis?" *EPRS - Monetary Dialogue Papers, February 2022*.

Beetsma, R., Thygesen, N., Cugnasca, A., Orseau, E., Eliofotou, P., and Santacroce, S. (2018). "Reforming the EU fiscal framework: A proposal by the european fiscal board." *VoxEU.org*.

Bénassy-Quéré, A., Brunnermeier, M., Enderlein, H., Farhi, E., Fuest, C., Gourinchas, P.-O., Martin, P., Pisani-Ferry, J., Rey, H., Schnabel, I., et al. (2018). "Reconciling risk sharing with market discipline: A constructive approach to euro area reform." *Policy Insight No. 91, Centre for Economic Policy Research*.

Bernanke, B. S., et al. (2007). "Inflation expectations and inflation forecasting." In *Speech at the Monetary Economics Workshop of the National Bureau of Economic Research Summer Institute, Cambridge, Massachusetts*, vol. 10.

Bernoth, K., and Wolff, G. B. (2008). "Fool the markets? Creative accounting, fiscal transparency and sovereign risk premia." *Scottish Journal of Political Economy*, *55*(4), 465–487.

Blanchard, O., and Brancaccio, E. (2019). "Crisis and revolution in economic theory and policy: A debate." *Review of Political Economy*, *31*(2), 271–287.

Blanchard, O., Leandro, A., and Zettelmeyer, J. (2021). "Redesigning eu fiscal rules: From rules to standards." *Economic Policy*, *36*(106), 195–236.

Bolton, P., and Jeanne, O. (2011). "Sovereign default risk and bank fragility in financially integrated economies." *IMF Economic Review*, *59*(2), 162–194.

Brunnermeier, M., Garicano, L., Lane, P. R., Pagano, M., Reis, R., Santos, T., Thesmar, D., Van Nieuwerburgh, S., and Vayanos, D. (2011). "European safe bonds (ESBies)." *Euro-nomics.com*.

Brunnermeier, M. K., Langfield, S., Pagano, M., Reis, R., Van Nieuwerburgh, S., and Vayanos, D. (2017). "ESBies: Safety in the tranches." *Economic Policy*, *32*(90), 175–219.

Caballero, R. J., Farhi, E., and Gourinchas, P.-O. (2017). "The safe assets shortage conundrum." *Journal of economic perspectives*, *31*(3), 29–46.

Cantore, C., Levine, P., Melina, G., and Pearlman, J. (2019). "Optimal fiscal and monetary policy, debt crisis, and management." *Macroeconomic Dynamics*, *23*(3), 1166–1204.

Ceci, D., and Pericoli, M. (2022). "Sovereign spreads and economic fundamentals: an econometric analysis." Tech. rep., Bank of Italy,Occasional Paper 713.

Claeys, G., Darvas, Z. M., and Leandro, A. (2016). "A proposal to revive the European Fiscal Framework." Tech. rep., Bruegel Policy Contribution.

Cochrane, J. (2022). "The fiscal theory of the price level." In *The Fiscal Theory of the Price Level*, Princeton University Press.

Codogno, L., Favero, C., and Missale, A. (2003). "Yield spreads on EMU government bonds." *Economic Policy*, *18*(37), 503–532.

Corsello, F., Neri, S., and Tagliabracci, A. (2021). "Anchored or de-anchored? that is the question." *European Journal of Political Economy*, *69*.

Corsetti, G., and Dedola, L. (2016). "The mystery of the printing press: Monetary policy and self-fulfilling debt crises." *Journal of the European Economic Association*, *14*(6), 1329–1371.

Corsetti, G., Kuester, K., Meier, A., and Müller, G. J. (2014). "Sovereign risk and belief-driven fluctuations in the euro area." *Journal of Monetary Economics*, *61*, 53–73.

Cottarelli, C. (2021). "Euro area fiscal policies and capacity in post-pandemic times." *IN-DEPTH ANALYSIS Requested by the ECON committee*.

Darvas, Z., Martin, P., and Ragot, X. (2018). "European fiscal rules require a major overhaul." *Notes du conseil danalyse economique*, *47*(2), 1–12.

De Grauwe, P., and Ji, Y. (2013). "Self-fulfilling crises in the eurozone: An empirical test." *Journal of International Money and finance*, *34*, 15–36.

De Grauwe, P., and Ji, Y. (2018). "How safe is a safe asset?" *CEPS Policy Insight*, *2018-08*.

De Santis, R. A. (2019). "Redenomination risk." *Journal of Money, Credit and Banking*, *51*(8), 2173–2206.

Dieckmann, S., and Plank, T. (2012). "Default risk of advanced economies: An empirical analysis of credit default swaps during the financial crisis." *Review of Finance*, *16*(4), 903–934.

Dosi, G., Minenna, M., Roventini, A., and Violi, R. (2021). "Making the eurozone work: a risk-sharing reform of the european stability mechanism." *Annals of Operations Research*, *299*, 617–657.

Draghi, M. (2012). Speech by Mario Draghi, President of the European Central Bank at the Global Investment Conference in London, 26 July 2012, Available at https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html.

ECB (2003). European Central Bank-Monthly Bulletin-December.

ECB (2022). Economic Bulletin Issue.

ECB (2022a). The Transmission Protection Instrument, Press release.

Eichengreen, B., and Panizza, U. (2016). "A surplus of ambition: can europe rely on large primary surpluses to solve its debt problem?" *Economic Policy*, *31*(85), 5–49.

ERSB (2021). Lower for longer. Macroprudential policy issues arising from the low interest rate environment.

ESRB-HLTF (2018). "Sovereign bond-backed securities: A feasibility study (January)." *Frankfurt: European Systemic Risk Board*.

Faini, R. (2006). "Fiscal policy and interest rates in europe." *Economic policy*, *21*(47), 444–489.

Fatás, A. (2019). "Fiscal policy, potential output, and the shifting goalposts." *IMF Economic Review*, *67*(3), 684–702.

Favero, C., and Missale, A. (2012). "Sovereign spreads in the eurozone: which prospects for a Eurobond?" *Economic Policy*, *27*(70), 231–273.

Favero, C., Pagano, M., and Von Thadden, E.-L. (2010). "How does liquidity affect government bond yields?" *Journal of financial and quantitative analysis*, *45*(1), 107–134.

Favero, C. A. (2013). "Modelling and forecasting government bond spreads in the euro area: a gvar model." *Journal of Econometrics*, *177*(2), 343–356.

Gabor, D., and Vestergaard, J. (2018). "Chasing unicorns: The european single safe asset project." *Competition & Change*, *22*(2), 139–164.

Gennaioli, N., Martin, A., and Rossi, S. (2014). "Sovereign default, domestic banks, and financial institutions." *The Journal of Finance*, *69*(2), 819–866.

Georgoutsos, D. A., and Migiakis, P. M. (2013). "Heterogeneity of the determinants of euro-area sovereign bond spreads; what does it tell us about financial stability?" *Journal of Banking & Finance*, *37*(11), 4650–4664.

Gerlach, S., Schulz, A., and Wolff, G. B. (2010). "Banking and sovereign risk in the euro area." *Available at SSRN 2785373*.

Geyer, A., Kossmeier, S., and Pichler, S. (2004). "Measuring systematic risk in emu government yield spreads." *Review of Finance*, *8*(2), 171–197.

Giavazzi, F., Guerrieri, V., Lorenzoni, G., and Weymuller, C.-H. (2021). "Revising the European fiscal framework." *VOX, CEPR Policy Portal*.

Golec, P., and Perotti, E. (2017). "Safe assets: a review." *ECB working paper No. 2035*.

Gomez-Puig, M. (2006). "Size matters for liquidity: Evidence from EMU sovereign yield spreads." *Economics Letters*, *90*(2), 156–162.

Gorton, G., and Ordonez, G. (2014). "Collateral crises." *American Economic Review*, *104*(2), 343–378.

Greenwood, R. M., and Vissing-Jorgensen, A. (2018). "The impact of pensions and insurance on global yield curves." *Harvard Business School Finance Working Paper*, *18-109*, 19–59.

Haroutunian, S., Hauptmeier, S., Leiner-Killinger, N., and Mergenthaler, P. (2022). "A central bank view of reforming Europe's fiscal framework." *VoxEU.org*.

Jank, S., Moench, E., and Schneider, M. (2022). "Safe asset shortage and collateral reuse." *SAFE Working Paper*.

Juncker, J.-C., and Tremonti, G. (2010). "E-bonds would end the crisis." *Financial Times*.

Kim, S.-J., Salem, L., and Wu, E. (2015). "The role of macroeconomic news in sovereign cds markets: Domestic and spillover news effects from the US, the Eurozone and China." *Journal of Financial Stability*, *18*, 208–224.

Kremens, L. (2018). "Currency redenomination risk." *Available at SSRN 3132064*.

Krishnamurthy, A., and Vissing-Jorgensen, A. (2012). "The aggregate demand for treasury debt." *Journal of Political Economy*, *120*(2), 233–267.

Lagarde, C. (2022). Speech by Christine Lagarde, President of ECB, at "The ECB and Its Watchers XXII" conference, Available at https://www.ecb.europa.eu/press/key/date/2022/html/ecb.sp220317 9d2f052c92.en.html.

Lane, P. R. (2012). "The european sovereign debt crisis." *Journal of economic perspectives*, *26*(3), 49–68.

Lane, P. R. (2022). Interview with Philip R. Lane, Member of the Executive Board of the ECB, conducted by Robert Shortt, Available at https://www.ecb.europa.eu/press/inter/date/2022/html/ecb.in220107 054220fc2c.en.html.

Lian, W., Presbitero, A., and Wiriadinata, U. (2020). "Public debt and r-g at risk." *IMF Working Paper*.

Lorenzoni, G., and Werning, I. (2019). "Slow moving debt crises." *American Economic Review*, *109*(9), 3229–3263.

Manganelli, S., and Wolswijk, G. (2009). "What drives spreads in the euro area government bond market?" *Economic Policy*, *24*(58), 191–240.

Martin, P., Pisany-Ferry, J., and Ragot, X. (2021). "A new template for the European fiscal framework." *VoxEU.org*.

Micossi, S. (2021). "On the Selling of Sovereigns held by the ESCB to the ESM: A revised proposal." *Available at SSRN 3964574*.

Micossi, S. (2022). "Managing sovereign debts held by the escb: operational and legal constraints." *CEPS Policy Insights*, *2022-02*.

Monti, M. (2010). "A new strategy for the single market." *Report to the President of the European Commission José Manuel Barroso, May*, *10*.

Paniagua, J., Sapena, J., and Tamarit, C. (2017). "Sovereign debt spreads in EMU: The time-varying role of fundamentals and market distrust." *Journal of Financial Stability*, *33*, 187–206.

Roubini, N. (2021). "The looming stagflationary debt crisis." *Project Syndicate*.

Schuknecht, L., von Hagen, J., Bernoth, K., et al. (2004). "Sovereign risk premia in the european government bond market." Tech. rep., ECB Working Paper No. 369.

Sgherri, M. S., and Zoli, M. E. (2009). *Euro area sovereign risk during the crisis*. International Monetary Fund.

Thygesen, N., Beetsma, R., Bordignon, M., Duchêne, S., Szczurek, M., et al. (2019). "Assessment of EU fiscal rules with a focus on the six and two-pack legislation." *European Fiscal Board Report*.

Ubide, A., et al. (2015). "Stability bonds for the euro area." *PIIE Policy Brief*, *15*, 18.

Von Hagen, J., Schuknecht, L., and Wolswijk, G. (2011). "Government bond risk premiums in the EU revisited: The impact of the financial crisis." *European Journal of Political Economy*, *27*(1), 36–43.

Woodford, M., and Walsh, C. E. (2005). "Interest and prices: Foundations of a theory of monetary policy." *Macroeconomic Dynamics*, *9*(3), 462–468.

Zettelmeyer, J. (2017). "A new fiscal governance for the Eurozone." *Quo Vadis? Identity, policy and the future of the European Union*, 127.

Appendix

A.1. Credit Risk Migration Model

To measure the credit-standing migration risk to which each credit risk class is exposed, we propose a methodology with which to calculate perpetual annuities based on a theoretical *through-the-cycle transition matrix* to show the feasibility of the Debt Agency framework proposed. Given a *point-in-time transition matrix* TM_t at time t, its generic element a_{ji} represents the annual probability that an obligor of the credit risk class j in year t will pass to a credit risk class i in the following year. The matrix has dimension $n \times n$ and the elements of row j, $a_{j1}, ..., a_{jn}$ must sum to unity, since every obligor with rating j will certainly be assigned to some credit risk class $z \in \{1, ..., j, ...n\}$ from year t to $t+1$; including the case of being reassigned to the same class j. As a convention, the rows and the columns of TM_t are ordered according to safety class, from the safest (conventionally labeled AAA) to the default (label D: default state). Following the standard diagonalization method for square matrices, we assume that the TM_t matrix can be decomposed in a Q matrix and a L_t diagonal matrix so that:

$$TM_t = QL_tQ^{-1} \tag{6}$$

The L_t matrix depends on t and shows correlations with the business cycle in its elements $\lambda_j(t)$. In particular we assume that these values depend on the European output gap according to the following generalized logistic function:

$$\begin{aligned} TM_t &= Q\Lambda_t Q^{-1} \\ \lambda_1(t) &= 1 \\ \lambda_{j,t} &= \frac{1}{1+\theta_{j1} exp(\theta_2(y_t - y_t^*))} \quad for \quad j > 1 \end{aligned}$$

As the output gap is a stationary process with zero mean, we have that $E(\lambda_{j,t}) = \lambda_j$, with λ_j being the element j of the eigenvalues diagonal matrix Λ in the decomposition:

$$\begin{aligned} TTC &= Q\Lambda Q^{-1} \\ \lambda_1 &= 1 \\ \lambda_j &= \frac{1}{1+\theta_{j1}} \quad for \quad j > 1 \end{aligned}$$

The *through-the-cycle transition matrix* TTC was estimated averaging publicly available data[28] of rating grades assigned to sovereign debts by Credit Rating Agencies in the period 1993-2015. This period has been chosen to include aspects of major institutional changes (e.g. events such as the introduction of the euro or the Eurozone sovereign debt crisis). Our estimated TTC matrix is reported in Table 3 Given the

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.9599	0.0401	0	0	0	0	0	0
AA	0.0179	0.9107	0.0643	0.0071	0	0	0	0
A	0	0.0281	0.8989	0.0730	0	0	0	0
BBB	0	0	0.0528	0.8746	0.0561	0.0132	0.0033	0
BB	0	0	0	0.0490	0.8529	0.0784	0.0131	0.0065
B	0	0	0	0	0.0706	0.8853	0.0294	0.0147
CCC	0	0	0	0	0	0.3846	0.4231	0.1923
D	0	0	0	0	0	0	0	1

Table 3: Estimated TTC transition matrix

estimate of TM_t, TTC can be computed as $E(TM_t)$.

Since the decomposition is unique unless linear transformations, then Q represents the eigenvectors matrix of the above linear functional and we have $E(\Lambda_t) = \Lambda$.

Proposition 1 *Given the filtered probability space $(\mathbf{\Omega}, \mathbf{\Sigma}, \mathcal{F}_t, \mathbf{P})$, the matrix TTC, interpreted as a* ***through-the-cycle matrix***[29]***,*** *is the expectation of the stochastic process $\{TM_t\}_{t\geq 0}$ adapted to the natural filtration $\mathcal{F}_t$ generated by y.*

Proof. Take the expectation of TM_t and substitute $E(l_j)$ with λ_j:

$$\begin{aligned} TTC &= E(TM_t) \\ &= E(QL_tQ^{-1}) \\ &= QE(L_t)Q^{-1} \\ &= Q\Lambda Q^{-1} \end{aligned}$$

■

[28]Standard & Poor's Sovereigns Ratings have been downloaded from Bloomberg using a query with parameters:
- RTG_SP_LT_LC_ISSUER_CREDIT
- RATING_AS_OF_DATE_OVERRIDE
- Sovereign Issuer Ticker.

[29]The matrix TTC can be estimated by averaging all the TM_t element-wise. Being TM_t right stochastic matrices, i.e. real square matrix with each row summing to 1, it is straightforward to show that TTC is still a right stochastic matrix and it models how each class of credit moves on average (i.e. in the absence of any specific market cycle) to the other credit classes. As a consequence, its decomposition has eigenvalues ≤ 1 with $\max(\lambda_j) = 1$.

On the basis of these results θ_{j1} are obtained from λ_j and θ_2 can be estimated (with a resulting value of -18.2) via a restricted non linear system of the equations linking $\lambda_{j,t}$ and $(y_t - y_t^*)$. We use the OECD Leading Indicator for the Euro Area (available at monthly frequency)as the proxy for the output gap. Its annual persistence is estimated at 0.54. We then have

$$\begin{aligned} E_t TTC_{t+1} &= Q E_t \Lambda_{t+1} Q^{-1} \\ E_t TTC_{t+2} &= Q E_t \Lambda_{t+2} Q^{-1} \\ E_t TTC_{t+i} &\approx Q \Lambda Q^{-1} \quad for \quad i > 2 \end{aligned}$$

The value over time of the eigenvalues in Λ_t is reported in Figure 6.

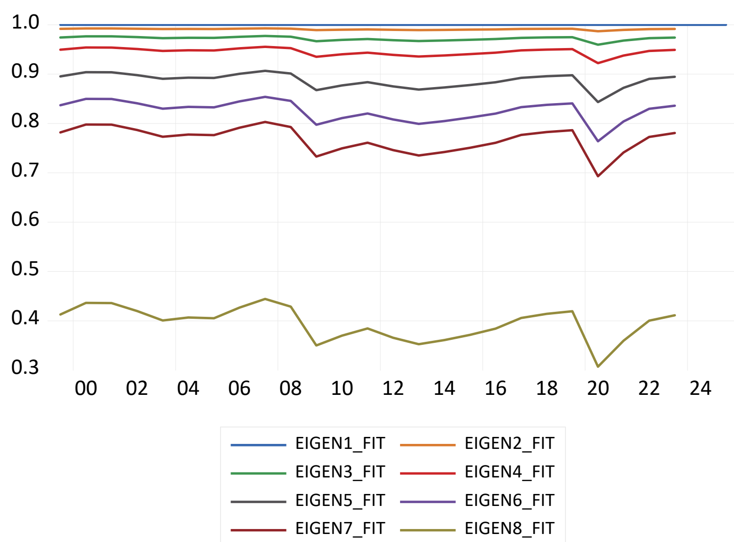

Figure 6: Time varying eigenvalues in the Λ_t matrix

Following this model, the **expected cumulative default probability** in the interval $[t, t + \tau]$ is the linear operator given by:

$$\begin{aligned} \mathbf{cdp}(t, t+1) &= Q E_t \Lambda_{t+1} Q^{-1} \mathbf{v} \\ \mathbf{cdp}(t, t+2) &= Q \Lambda_{t+2} E_t \Lambda_{t+1} Q^{-1} \mathbf{v} \\ \mathbf{cdp}(t, t+\tau) &= Q \Lambda^{\tau-2} E_t \Lambda_{t+2} E_t \Lambda_{t+1} Q^{-1} \mathbf{v} \quad for \quad \tau > 2 \end{aligned} \tag{7}$$

where $\mathbf{cdp}(t)$ is an n-components stochastic process, the j-th element of which, $cdp_{\mathbf{j}}(t)$, represents the cumulative default probability that an obligor of rating grade class $j = 1, ..., n$ will have defaulted by time t, with $\mathbf{cdp}(0) = 0$ and $\mathbf{v}$ a null vector apart its last element equal to 1.

Proposition 2 *The process* $\mathbf{cdp}(t)$ *can be seen as a stochastic vector depending from the process* $(y_t - y_t^*)$. *Since the matrix* L_t *depends deterministically from* y, *this guarantees that* $\mathbf{cdp}(t)$ *is measurable given the filtration* $\mathcal{F}_t$ *generated by* $(y_t - y_t^*)$.[30]

A.2. Perpetual Annuities and Fundamental Pricing

Given the process $\mathbf{cdp}(t)$ in equations (7), the survival probability in the interval $\tau \in [t, t+\tau]$ of an obligor not in default is:

$$\mathbf{sp}(t, t+\tau) = E\left[\mathbf{1} - \mathbf{cdp}(t+\tau)\right] \quad for \quad \tau > 1 \tag{8}$$

where $\mathbf{1}$ is the unit vector.

The expected present value of a vector of unitary annuity maturing at time $t+\tau$ can be written as:

$$\mathbf{a}(t, t+\tau) = \sum_{j=1,\dots\tau} \frac{1}{(1+r_t^B)^j} \mathbf{sp}(t, t+j) \tag{9}$$

where r_t^B represents a common appropriate financial discount rate[31]. Note that the components of vector $\mathbf{a}(0,t)$ are ordered decreasingly, with the highest rating grades corresponding to higher annuity values since the present value of a unitary annuity is proportional to the survival probability of the corresponding credit risk class and a null value for the vector's last component. Using the expression of $\mathbf{sp}(t)$ in eq. (8), we can rewrite:

$$\begin{aligned} \mathbf{a}(t, t+\tau) &= \frac{\mathbf{1} - QE_t\Lambda_{t+1}Q^{-1}\mathbf{v}}{(1+r_t^B)} + \frac{\mathbf{1} - QE_t\Lambda_{t+1}Q^{-1}\mathbf{v}}{(1+r_t^B)^2} \\ &+ \sum_{j=3,\dots\tau} \frac{1}{(1+r_t^B)^j} \left(\mathbf{1} - Q\Lambda^{j-2}E_t\Lambda_{t+2}E_t\Lambda_{t+1}Q^{-1}\mathbf{v}\right) \end{aligned} \tag{10}$$

(11)

By letting $\alpha = \frac{1}{(1+r_t^B)}$ and $\beta_j = \frac{\lambda_j}{(1+r)}$, the expected present value of a vector of unitary annuity maturing at time t can now be written as:

[30] For a demonstration of this proposition, see the previous version of this work, available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3579496. For any further information on the analytics, please write to massimo.amato@unibocconi.it

[31] For simplicity's sake, it has been assumed that the purely financial rate does not exhibit a term structure. This hypothesis represents a mere simplification for calculation purposes which can easily be removed.

$$\mathbf{a}(t,t+\tau) = \alpha\frac{1-\alpha^t}{1-\alpha}\mathbf{1} - QB_{t+1}Q^{-1}\mathbf{v} - QB_{t+1}B_{t+2}Q^{-1}\mathbf{v} - Q\left(B_{t+1}B_{t+2}B(I+B+...B^{\tau-3})\right)Q^{-1}\mathbf{v} \quad (12)$$

where B is a diagonal matrix with generic element β_j. Since the terms in Λ are $\lambda_j \leq 1$, it follows $\alpha, \beta_j \in (0,1)$. By taking the limit for $t \to \infty$ we obtain the following perpetual annuity formula:

$$\mathbf{a}(t) = \lim_{\tau\to\infty}\mathbf{a}(t,t+\tau) = \frac{\alpha}{1-\alpha}\mathbf{1} - Q\left(B_{t+1} + B_{t+1}B_{t+2} + B_{t+1}B_{t+2}B'\right)Q^{-1}\mathbf{v}$$
$$B' = B(I-B)^{-1} \quad (13)$$

The vector $\mathbf{a}(t)$ in the eq. (13) represents expected present values at $t=0$ of **an irredeemable mortgage annuity** paid by each obligor according to its rating grade.

In order to consider the possibility to recover part of the credit if an obligor defaults, we should adjust the value of $\mathbf{a}(t)$ accordingly. To this end, eq. (13) should be modified to take this effect into account. Introducing the loss-given-default (LGD), $(1-rr)$, and letting rr be the recovery rate[32], the vector of **expected values of the recovery rate by credit risk class** for a perpetual annuity,$\mathbf{r} = \lim_{\tau\to\infty}\mathbf{r}(t,t+\tau)$, can be written as:

$$\begin{aligned}
\mathbf{r} &= rr\sum_{j=1}^{\tau}\frac{1}{(1+r_t^B)^j}\left(E_t\mathbf{cdp}(t+j) - E_t\mathbf{cdp}(t+j-1)\right) \quad (14)\\
&= \frac{rr}{(1+r_t^B)}Q\left(\Lambda_t\Lambda_{t+1}(I-\Lambda_{t+1}^{-1})\right)Q^{-1}\mathbf{v}\\
&+ \frac{rr}{(1+r_t^B)^2}Q\left(\Lambda_t\Lambda_{t+1}\Lambda_{t+2}(I-\Lambda_{t+2}^{-1})\right)Q^{-1}\mathbf{v}\\
&+ \frac{rr}{(1+r_t^B)^3}Q\left(\Lambda_t\Lambda_{t+1}\Lambda_{t+2}(I-\Lambda^{-1})B'\right)Q^{-1}\mathbf{v}\\
B' &= \lim_{\tau\to\infty}\sum_{j=1}^{\tau}\frac{1}{(1+r_t^B)^j}\Lambda^j = B(I-B)^{-1}
\end{aligned}$$

Following a unitary-payment perpetual amortizing scheme and allowing for partial recovery of funds in case of default, the present value of an **expected positive exposure** $\tilde{a}_j$ must always satisfy the equivalence $\tilde{a}_j(1-r_j) = a_j$, where $r_j < 1$ is j-th element

[32]The LGD parameter should be identified for each Member State in order to take its specific risk into account. Since our ultimate purpose is to provide an exemplification of a possible DA architecture based on an irredeemable cost configuration, in our calculations we assume a uniform LGD value for all MSs.

of the vector **r**. Bearing this in mind, the final expectation of a **unitary perpetual annuity value** at time $t = 0$ calculated for each obligor according to its rating grade j is then:

$$\tilde{a}_j = \frac{a_j}{1 - r_j} \tag{15}$$

The vector $\tilde{\mathbf{a}}(0)$, whose elements are the values $\tilde{a}_j$, can be interpreted as a set of perpetual annuities based on **fundamental risk metrics** inherent to obligors labelled with specific credit risk class. In our numerical exercise, we set $rr = 0.3$ in the baseline simulation.

A.2. The Simulation Model

Our simulation model is constructed in three steps: debt reclassification, baseline stabilization scenario without EDA, alternative simulation scenario with EDA.

A.2.1 Debt Reclassification

The first step of our simulation model is a within sample (2000-2021) simulation that allows to reclassify public debt into slow-adjusting debt, at the beginning of our out-of-sample simulation and fast-adjusting debt and Covid-debt was obtained by simulating the dynamics of fiscal variables of all MS's from 2001 to 2021.[33]

$$\begin{aligned}
B_{i,t} &= B_{i,t-1} + r_{i,t}B_{i,t-1} + (G_{i,t} - T_{i,t}) + SFA_{i,t} \qquad (1)\\
B_{i,COV,t} &= B_{i,COV,t-1} + I_{COV,i,t} * I_{+,i,t}\left(r_{i,t}B_{i,t-1} + (G_{i,t} - T_{i,t}) + SFA_{i,t}\right)\\
B_{i,REC,t} &= B_{i,REC,t-1} + I_{REC,i,t} * I_{+,i,t}\left(r_{i,t}B_{i,t-1} + (G_{i,t} - T_{i,t}) + SFA_{i,t}\right)\\
B_{i,S,t} &= B_{i,COV,t} + B_{i,REC,t}\\
B_{i,F,t} &= B_{i,t} - B_{i,S,t}\\
d_{i,F,t} &= \frac{B_{i,F,t}}{Y_{i,t}}, \qquad d_{i,S,t} = \frac{B_{i,S,t}}{Y_{i,t}}\\
d_{i,COV,t} &= \frac{B_{i,COV,t}}{Y_{i,t}}, \qquad d_{i,t} = \frac{B_{i,t}}{Y_{i,t}}
\end{aligned}$$

where $r_{i,t}$ is the cost of financing and it is equal to $\frac{IP_{i,t}}{B_{i,t-1}}$, $IP_{i,t}$ represents the interest payment, $G_{i,t} - T_{i,t}$ is the primary deficit, $SFA_{i,t}$ are the stock-flow adjustments, $\Delta y_{i,t}$ is the nominal output rate of growth for the Member State i at time t. Using the OECD data equation (16) allows to exactly reconstruct the government debt dynamics of each member state. As the Covid debt is identified by cumulating the positive primary deficits occurred in COVID years, $I_{+,i,t}$ is an indicator taking a value of 1 in presence of positive deficit and zero otherwise, $I_{COV,i,t}$ is an indicator taking a value of 1 during the years of the Covid pandemic and zero otherwise. Similarly, as the slow-debt is identified by cumulating the primary deficits occurred in crisis, i.e. when the escape clause was active, $I_{rec,i,t}$ is an indicator function for recession periods taking a value of 1 in 2008 and 2012, 0.5 in 2009 and 2011 (recession lasted half a year), 0.25 in 2013 (recession lasted one quarter) and zero otherwise.

A.2.2 The Baseline Scenario In the baseline scenario debt and deficit dynamics are simulated from 2022 onwards by specifying first the stochastic processes for the euro area swap rate which is the driver of the ten-year yields and of the average cost of debt for the different MS's, together with the GDP growth rate of the different MS's. The model is then closed by adding the deterministic reaction functions for the primary deficits and the stock-flow relationship determining the debt dynamics. Total debt is

[33] At the beginning of the sample set $B_{i,REC,t} = 0$, $B_{i,S,t} = 0$

reclassified in fast and slow adjusting.

$$
\begin{aligned}
\Delta y_{i,t} &= a_{0,i} + a_1 r_{t-1}^{1,swap} + a_{2,i}(r_{i,t-1}^{10} - r_{DE,t-1}^{10}) + a_{3,i}\left(t_{i,t-1} - g_{i,t-1}\right) + u_{1,i,t} \\
\Delta \bar{y}_t &= \sum_{j=1}^{n} w_{i,t-1}\Delta y_{i,t}, \quad w_{i,t} = \frac{y_{i,t}}{\sum_{i=1}^{n} y_{i,t}} \\
r_t^{1,swap} &= b_0 + b_1 \Delta \bar{y}_t + b_2 r_{t-1}^{1,swap} + u_{2,i,t} \\
r_t^{10,swap} &= c_0 + c_1 r_{t-1}^{10,swap} + c_2 r_t^{1,swap} + u_{3,t} \\
r_{i,t} &= d_{1,i} r_{i,t-1} + d_{2,i} r_{i,t}^{10} + d_{3,i} r_t^{1,swap} + u_{4,i,t} \\
r_{DE,t}^{10} &= h_{0,i} + h_1 r_{DE,t-1}^{10} + h_2 r_t^{1,swap} + u_{6,i,t} \\
r_{i,t}^{10} &= r_{DE,t}^{10} + k_{0,i} + k_1 b_{i,t-1} + \sigma_{i,t} u_{5,i,t} \\
\sigma_{i,t} &= \varphi b_{i,t-1} \\
(g_{i,t} - t_{i,t}) &= -\frac{r_{i,t} - \Delta y_{i,t}}{1 + \Delta y_{i,t}} b_{i,t-1} - \beta\left(b_{i,F,t-1} - b_{i,F,t-1}^{*}\right) - \gamma\left(b_{i,S,t-1} - b_{i,S,t-1}^{*}\right) \\
b_{i,t} &= b_{i,t-1} + \frac{r_{i,t} - \Delta y_{i,t}}{1 + \Delta y_{i,t}} b_{i,t-1} + (g_{i,t} - t_{i,t}) \\
b_{i,S,t} &= \frac{1 + r_{i,t}}{1 + \Delta y_{i,t}} * b_{i,S,t-1} + \phi_{i,t} * (g_{i,t} - t_{i,t}) \\
b_{i,F,t} &= b_{i,t} - b_{i,S,t} \\
b_{i,F,t}^{*} &= 0.6 * \frac{b_{i,F,t}}{b_{i,F,t} + b_{i,S,t}}, \qquad b_{i,S,t}^{*} = 0.6 * \frac{b_{i,S,t}}{b_{i,F,t} + b_{i,S,t}}
\end{aligned}
$$

where $\phi_{i,t}$ is the share of total primary deficit that contributes to slow debt for country i in period t, and $u_{5,i,t} \sim N(0,1)$. In this model the volatility of the country i risk premium changes as a consequence of the changes in debt, accordingly to the parameter φ (see below).

The parameters and the residuals of the stochastic equations are estimated over the sample 2001-2021. System estimation by SURE is implemented and panel restrictions on the coefficients are imposed whenever they are not rejected. The parameter φ is estimated using panel methods. Specifically, we constructed a measure of yearly spread volatility by computing the standard deviation from monthly data. Then, the following model has been estimated

$$\sigma_{i,t} = \varphi b_{i,t-1} + \epsilon_{i,t}^{\sigma} \tag{16}$$

Figure 7 shows the estimated relationship, while the estimation results are reported in Tables 4-9. Stochastic simulation is implemented by bootstrapping the rows of the matrix containing the estimated residuals for all the stochastic equations.

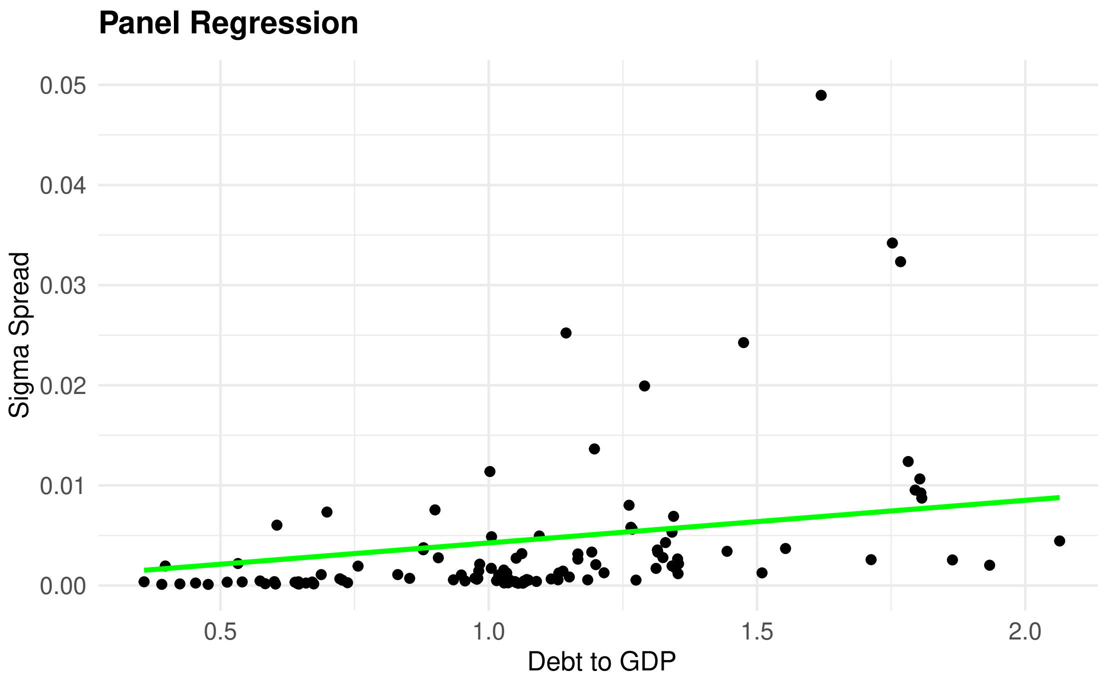

Figure 7: Spread volatility and Debt to GDP

Table 4: Estimation Results - OLS

	Dependent variable:	
	$r_t^{1,swap}$	$r_t^{10,swap}$
	(1)	(2)
$r_{t-1}^{1,swap}$	0.771^{***}	
	(0.118)	
$\Delta\bar{y}_t$	0.212^{**}	
	(0.065)	
$r_{t-1}^{10,swap}$		0.433^{***}
		(0.079)
$r_t^{1,swap}$		0.592^{***}
		(0.084)
Constant	-0.002	0.005^{**}
	(0.003)	(0.002)
Sample: 2000-2021		
R^2	0.747	0.935
Adjusted R^2	0.719	0.928
Residual Std. Error (df = 18)	0.009	0.004
Note	*p<0.1; **p<0.05; ***p<0.01	

$\Delta\bar{y}_t$ is the weighted average rate of GDP growth in the six countries with weights defined as the ratio of country i GDP to the aggregate GDP.

Table 5: Estimation Results, SURE

	Dependent variable: $\Delta y_{i,t}$					
	DE	ES	FR	GR	IT	PT
	(1)	(2)	(3)	(4)	(5)	(6)
$r^{1,swap}_{t-1}$	− 0.146	− 0.146	− 0.146	− 0.146	− 0.146	− 0.146
	(0.273)	(0.273)	(0.273)	(0.273)	(0.273)	(0.273)
$(r^{10}_{ES,t-1} - r^{10}_{DE,t-1})$		-2.033^{***}				
		(0.435)				
$(r^{10}_{FR,t-1} - r^{10}_{DE,t-1})$			-3.177^{*}			
			(1.235)			
$(r^{10}_{GR,t-1} - r^{10}_{DE,t-1})$				-0.785^{***}		
				(0.160)		
$(r^{10}_{IT,t-1} - r^{10}_{DE,t-1})$					-0.984^{***}	
					(0.261)	
$(r^{10}_{PT,t-1} - r^{10}_{DE,t-1})$						-0.717^{***}
						(0.167)
$(t_{DE,t-1} - g_{DE,t-1})$	-0.405^{**}					
	(0.089)					
$(t_{ES,t-1} - g_{ES,t-1})$						
$(t_{FR,t-1} - g_{FR,t-1})$			-0.405^{**}			
			(0.089)			
$(t_{GR,t-1} - g_{GR,t-1})$						
$(t_{IT,t-1} - g_{IT,t-1})$					-0.405^{**}	
					(0.089)	
$(t_{PT,t-1} - g_{PT,t-1})$						
Constant	0.033^{***}	0.056^{***}	0.032^{***}	0.052^{***}	0.038^{***}	0.044^{***}
	(0.006)	(0.011)	(0.008)	(0.014)	(0.008)	(0.010)
Sample: 2000-2021						
Note:						*p<0.1; **p<0.05; ***p<0.01

Table 6: Estimation Results, SURE

	Dependent variable:
	$r^{10}_{DE,t}$
$r^{10}_{DE,t-1}$	0.476***
	(0.084)
$r^{1,swap}_{t}$	0.556***
	(0.093)
Constant	0.002
	(0.002)
Sample: 2000-2021	
R^2	0.931
Adjusted R^2	0.923
Residual Std. Error	0.005 (df = 18)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 7: Estimation Results, SURE

	Dependent variable: $(r^{10}_{i,t} - r^{10}_{DE,t})$				
	ES	FR	GR	IT	PT
	(1)	(2)	(3)	(4)	(5)
$b_{ES,t-1}$	0.01*				
	(0.004)				
$b_{FR,t-1}$		0.01*			
		(0.004)			
$b_{GR,t-1}$			0.01*		
			(0.004)		
$b_{IT,t-1}$				0.01*	
				(0.004)	
$b_{PT,t-1}$					0.01*
					(0.004)
Constant	0.004	− 0.005	0.028 *	0.002	0.010
	(0.004)	(0.003)	(0.012)	(0.005)	(0.007)
Sample: 2000-2021					
Note:			*p<0.1; **p<0.05; ***p<0.01		

Table 8: Estimation Results, SURE

	Dependent variable: $r_{i,t}$					
	DE	ES	FR	GR	IT	PT
	(1)	(2)	(3)	(4)	(5)	(6)
$r_{DE,t-1}$	0.88***					
	(0.03)					
$r_{ES,t-1}$		0.82***				
		(0.03)				
$r_{FR,t-1}$			0.81***			
			(0.05)			
$r_{GR,t-1}$				0.87***		
				(0.06)		
$r_{IT,t-1}$					0.82***	
					(0.04)	
$r_{PT,t-1}$						0.85***
						(0.02)
$r_t^{1,swap}$	0.09	0.04	0.07	0.15	0.06	0.05
	(0.04)	(0.04)	(0.07)	(0.07)	(0.04)	(0.03)
$r_{DE,t}^{10}$	0.02					
	(0.06)					
$r_{ES,t}^{10}$		0.14**				
		(0.04)				
$r_{FR,t}^{10}$			0.13			
			(0.09)			
$r_{GR,t}^{10}$				−0.01		
				(0.02)		
$r_{IT,t}^{10}$					0.13**	
					(0.04)	
$r_{PT,t}^{10}$						0.07***
						(0.02)
Sample: 2000-2021						

Note: *p<0.1; **p<0.05; ***p<0.01

Table 9: Panel Estimation Results

	Dependent variable:
	$\sigma_{i,t}$
$b_{i,t-1}$	0.004***
	(0.000)
Sample: 2000-2021	
R^2	0.333
Adjusted R^2	0.327
Residual Std. Error	0.007 (df = 109)
Note:	*p<0.1; **p<0.05; ***p<0.01

A.2.3 The Alternative Scenario

In this scenario EDA acquires progressively the slow debt and it takes 5 years to complete this operation. The simulation of a scenario with EDA requires adding to the stochastic processes for the euro area swap rate, for the average costs of debt and for the GDP growth rates the processes for the interest rates on EDA bonds, r_t^{EDA} and on EDA loans, $\frac{1}{\tilde{a}_{ij,t}}$. r_t^{EDA} is simulated as the sum of the nominal swap rate and a risk-premium that depends on the Solvency Capital of the Agency SC_t^{EDA}. [34] The interest rate on EDA loans which is computed period by period given r_t^{EDA} and the empirical credit-risk migration model. The following model is then considered for the stochastic variables to be simulated by keeping the parameters and the residuals at

[34] In all simulations EDA Solvency Capital is sufficiently high to set r_t^{EDA} equal to the euro area swap rate.

their estimated values in the baseline scenario:

$$
\begin{aligned}
\Delta y_{i,t} &= a_{0,i} + a_1 r_{t-1}^{1,swap} + a_{2,i}(r_{i,t-1}^{10} - r_{DE,t-1}^{10}) + a_{3,i}\left(t_{i,t-1} - g_{i,t-1}\right) + u_{1,i,t} \\
\Delta \bar{y}_{i,t} &= \sum_{j=1}^{n} w_{i,t-1}\Delta y_{i,t}, \quad r_{i,t}^{EDA} w_{i,t} = \frac{y_{i,t}}{\sum_{i=1}^{n} y_{i,t}} \\
r_t^{1,swap} &= b_0 + b_1 \Delta \bar{y}_{i,t} + b_2 r_{t-1}^{1,swap} + u_{2,i,t} \\
r_t^{10,swap} &= c_0 + c_1 r_{t-1}^{10,swap} + c_2 r_t^{1,swap} + u_{3,t} \\
r_{i,t}^{EDA} &= f(TTC_i, r_t^{10,swap}) \\
r_{i,t} &= d_{1,i} r_{i,t-1} + d_{2,i} r_{i,t}^{10} + d_{3,i} r_t^{1,swap} + u_{4,i,t} \\
r_{DE,t}^{10} &= h_{0,i} + h_1 r_{DE,t-1}^{10} + h_2 r_{t-1}^{1,swap} + u_{6,i,t} \\
r_{i,t}^{10} &= r_{DE,t}^{10} + k_{0,i} + k_1 b_{i,t-1} + \sigma_{i,t} u_{5,i,t} \\
\sigma_{i,t} &= \varphi b_{i,t-1}
\end{aligned}
$$

The path of MS's fiscal variables and EDA's balance sheet are then simulated considering an initial transition period of five-years in which EDA progressively acquires the slow debt. In our simulations EDA begins operating at the end 2022, by issuing bonds to make a loan to member countries to cover a fifth of the existing slow debt, the total of interest payment on slow debt and the share $\phi_{i,t}$ of the primary deficit that qualifies for the golden rule. As payments of interest on the loans will begin in the following period the expected loss provision is zero and the reserves are equal to the initial endowment. We entertain the possibility that the initial capital conferred to EDA is the ESM capital, attributed within EDA to member countries according to the ESM weights (https://www.esm.europa.eu/esm-governance). From the inception of EDA the fiscal rule is modified to take in account that the government debt is made of a mixture of Bonds and Loans with EDA and loans with EDA are treated as slow-debt.

$$
\begin{aligned}
p_{i,t}^{EDA} &= 0 \\
nl_{i,t}^{EDA} &= \frac{1}{5} * \frac{1}{1+\Delta y_{i,t}} * b_{i,S,t-1} + \frac{r_{i,t}}{1+\Delta y_{i,t}} b_{i,S,t-1} + \phi_{i,t}\left(g_{i,t}-t_{i,t}\right) \\
l_{i,t}^{EDA} &= nl_{i,t}^{EDA} \\
b_{i,t}^{EDA} &= nl_{i,t}^{EDA} \\
el_{i,t}^{EDA} &= 0 \\
ry_{i,t}^{EDA} &= 0 \\
en_{i,t}^{EDA} &= \frac{C_0 w_i}{y_{i,t}} \\
SC_{i,t}^{EDA} &= l_{i,t}^{EDA} + ry_{i,t}^{EDA} + en_{i,t}^{EDA} - b_{i,t}^{EDA} - el_{i,t}^{EDA} \\
\left(g_{i,t}-t_{i,t}\right) &= -\frac{r_{i,t}-\Delta y_{i,t}}{1+\Delta y_{i,t}} b_{i,t-1} - \frac{1}{10} * \left(d_{i,t-1} - \hat{d}_{i,t-1}\right) \\
b_{i,t} &= b_{i,t-1} + \frac{r_{i,t}-\Delta y_{i,t}}{1+\Delta y_{i,t}} b_{i,t-1} + \left(1-\phi_{i,t}\right)\left(g_{i,t}-t_{i,t}\right) \\
b_{i,S,t} &= \frac{4}{5} * \frac{1}{1+\Delta y_{i,t}} * b_{i,S,t-1} \\
b_{i,F,t} &= b_{i,t} - b_{i,S,t} \\
d_{i,t} &= b_{i,t} + l_{i,t}^{EDA} \\
b_{i,F,t}^{*} &= 0.6 * \frac{b_{i,F,t}}{d_{i,t}}, \qquad b_{i,S,t}^{*} = 0.6 * \frac{b_{i,S,t}}{d_{i,t}}, \qquad l_{i,t}^{EDA,*} = 0.6 * \frac{l_{i,t}^{EDA}}{d_{i,t}} \\
\hat{d}_{i,t} &= d_{i,t} - 10 * \beta\left(b_{i,F,t} - b_{i,F,t}^{*}\right) - 10 * \gamma\left(b_{i,S,t} - b_{i,S,t}^{*}\right) - 10 * \gamma\left(l_{i,t}^{EDA} - l_{i,t}^{EDA,*}\right)
\end{aligned}
$$

where all variables are expressed as a ratio to country i GDP in period t. $p_{i,t}^{EDA}$ are payments made by country i to EDA for their loans with EDA, $nl_{i,t}^{EDA}$ is the new lending made by EDA to country i to EDA, $l_{i,t}^{EDA}$ are loans to country i from EDA, $b_{i,t}^{EDA}$ are bonds issued by EDA attributable to country i, $el_{i,t}^{EDA}$ is the expected loss provision attributable to country i within EDA, $ry_{i,t}^{EDA}$ are reserves within EDA attributable to country i, $SC_{i,t}^{EDA}$ is the Solvency Capital attributable to country i within EDA. After its establishment EDA continues to absorb pre-EDA slow debt until all past and new slow debt is financed through EDA loans. In the transition period ($2022 < t < 2027$), MSs start to pay deferred instalments to EDA. The EDA bonds increase because of interest rate payments and because of new lending. The Agency accumulates reserves[35] and also expected loss provisions ($el_{i,t}^{EDA}$), which are computed as the difference between

[35]Reserves are remunerated at a rate equal to r_t^{ECB}, that in this case, for conservative reasons, is set to be equal to the growth rate of each country $\Delta y_{i,t}$

the sum of loan-debt and reserves and the bond-debt issued by the EDA. The initial endowment to GDP ratio ($en_{i,t}^{EDA}$) is kept constant. Therefore we have:

$$
\begin{aligned}
m &= 4 \\
& for \quad (t \quad in \quad t_0+1 : t_{0+4})\{ \\
p_{i,t}^{EDA} &= \frac{b_{i,t-1}^{EDA} - ry_{i,t-1}^{EDA}}{\tilde{a}_{ij,t}(1+\Delta y_{i,t})} \\
nl_{i,t}^{EDA} &= \frac{1}{m} * \frac{1}{1+\Delta y_{i,t}} * b_{i,S,t-1} + \frac{r_{i,t}}{1+\Delta y_{i,t}} b_{i,S,t-1} + \phi_{i,t}\left(g_{i,t} - t_{i,t}\right) + p_{i,t}^{EDA} \\
l_{i,t}^{EDA} &= \frac{1}{1+\Delta y_{i,t}} l_{i,t-1}^{EDA} + nl_{i,t}^{EDA} \\
b_{i,t}^{EDA} &= \frac{1+r_t^{EDA}}{1+\Delta y_{i,t}} b_{i,t-1}^{EDA} + nl_{i,t}^{EDA} \\
ry_{i,t}^{EDA} &= \frac{1+r_t^{ECB}}{1+\Delta y_{i,t}} ry_{i,t-1}^{EDA} + p_{i,t}^{EDA} \\
el_{i,t}^{EDA} &= l_{i,t}^{EDA} + ry_{i,t}^{EDA} - b_{i,t}^{EDA} \\
en_{i,t}^{EDA} &= en_{i,t-1}^{EDA} \\
SC_{i,t}^{EDA} &= l_{i,t}^{EDA} + ry_{i,t}^{EDA} + en_{i,t}^{EDA} - b_{i,t}^{EDA} - el_{i,t}^{EDA} \\
(g_{i,t} - t_{i,t}) &= -p_{i,t}^{EDA} - \frac{r_{i,t} - \Delta y_{i,t}}{1+\Delta y_{i,t}} b_{i,t-1} + \frac{\Delta y_{i,t}}{1+\Delta y_{i,t}} l_{i,t-1}^{EDA} - \frac{1}{10} * \left(d_{i,t-1} - \hat{d}_{i,t-1}\right) \\
b_{i,t} &= b_{i,t-1} + \frac{r_{i,t} - \Delta y_{i,t}}{1+\Delta y_{i,t}} b_{i,t-1} + \left(1 - \phi_{i,t}\right)\left(g_{i,t} - t_{i,t}\right) - \frac{\frac{1}{m} + r_{i,t}}{1+\Delta y_{i,t}} * b_{i,S,t-1} \\
b_{i,S,t} &= \frac{m-1}{m} * \frac{1}{1+\Delta y_{i,t}} * b_{i,S,t-1} \\
b_{i,F,t} &= b_{i,t} - b_{i,S,t} \\
d_{i,t} &= b_{i,t} + l_{i,t}^{EDA} \\
b_{i,F,t}^{*} &= 0.6 * \frac{b_{i,F,t}}{d_{i,t}}, \qquad b_{i,S,t}^{*} = 0.6 * \frac{b_{i,S,t}}{d_{i,t}}, \qquad l_{i,t}^{EDA,*} = 0.6 * \frac{l_{i,t}^{EDA}}{d_{i,t}} \\
\hat{d}_{i,t} &= d_{i,t} - 10 * \beta\left(b_{i,F,t} - b_{i,F,t}^{*}\right) - 10 * \gamma\left(b_{i,S,t} - b_{i,S,t}^{*}\right) - 10 * \gamma\left(l_{i,t}^{EDA} - l_{i,t}^{EDA,*}\right) \\
m &= \quad m-1 \quad \}
\end{aligned}
$$

After the transition is completed there are only the types of debt: government bonds, that finance exclusively fast debt, and EDA loans, that finance exclusively slow-debt.

From 2028 onward we have:

$$
\begin{aligned}
p_{i,t}^{EDA} &= \frac{b_{i,t-1}^{EDA} - ry_{i,t-1}^{EDA}}{\tilde{a}_{ij,t}(1+\Delta y_{i,t})} \\
nl_{i,t}^{EDA} &= \phi_{i,t}\left(g_{i,t} - t_{i,t}\right) + p_{i,t}^{EDA} \\
l_{i,t}^{EDA} &= \frac{1}{1+\Delta y_{i,t}} l_{i,t-1}^{EDA} + nl_{i,t}^{EDA} \\
b_{i,t}^{EDA} &= \frac{1+r_t^{EDA}}{1+\Delta y_{i,t}} b_{i,t-1}^{EDA} + nl_{i,t}^{EDA} \\
ry_{i,t}^{EDA} &= \frac{1+r_t^{ECB}}{1+\Delta y_{i,t}} ry_{i,t-1}^{EDA} + p_{i,t}^{EDA} \\
el_{i,t}^{EDA} &= l_{i,t}^{EDA} + ry_{i,t}^{EDA} - b_{i,t}^{EDA} \\
en_{i,t}^{EDA} &= en_{i,t-1}^{EDA} \\
SC_{i,t}^{EDA} &= l_{i,t}^{EDA} + ry_{i,t}^{EDA} + en_{i,t}^{EDA} - b_{i,t}^{EDA} - el_{i,t}^{EDA} \\
(g_{i,t} - t_{i,t}) &= -p_{i,t}^{EDA} - \frac{r_{i,t} - \Delta y_{i,t}}{1+\Delta y_{i,t}} b_{i,t-1} + \frac{\Delta y_{i,t}}{1+\Delta y_{i,t}} l_{i,t-1}^{EDA} - \frac{1}{10} * \left(d_{i,t-1} - \hat{d}_{i,t-1}\right) \\
b_{i,t} &= b_{i,t-1} + \frac{r_{i,t} - \Delta y_{i,t}}{1+\Delta y_{i,t}} b_{i,t-1} + \left(1 - \phi_{i,t}\right)\left(g_{i,t} - t_{i,t}\right) \\
d_{i,t} &= b_{i,t} + l_{i,t}^{EDA} \\
b_{i,t}^{*} &= 0.6 * \frac{b_{i,t}}{d_{i,t}}, \qquad l_{i,t}^{EDA,*} = 0.6 * \frac{l_{i,t}^{EDA}}{d_{i,t}} \\
\hat{d}_{i,t} &= d_{i,t} - 10 * \beta\left(b_{i,t} - b_{i,t}^{*}\right) - 10 * \gamma\left(l_{i,t}^{EDA} - l_{i,t}^{EDA,*}\right)
\end{aligned}
$$