The Life Cycle Origins of the Investment Puzzle

Martin Aragoneses *

September 15, 2024

Abstract

Weak US investment after the 1980s is puzzling because rising profitability and falling interest rates should have stimulated investment. I find the decline in the startup rate of new businesses is behind this missing investment puzzle. Confidential US Census micro data shows a striking divergence between micro and macro trends. Investment increased for the average firm despite a decline in aggregate investment, but changes in the firm age distribution masked this investment boom from aggregate data. Fewer startups being born aged firms and depressed aggregate investment because older firms, despite likely being more profitable, invest less intensely. In a calibrated firm dynamics model, firm aging due to falling startup rates explains 80% of the investment trend decline from 11.5% to 9% of GDP between 1980 and 2010. Given historical changes in startup rates, the life cycle model rationalizes the boom and bust in aggregate investment and its puzzling relation with profits and interest rates since the 1950s. Consistent with the model, cross-country data shows rising investment and falling profits amidst a resurgence in startup activity since 2010.

*Department of Finance, INSEAD. martin.aragoneses@insead.edu. I thank my advisors at Harvard Ludwig Straub, Xavier Gabaix, Gabriel Chodorow-Reich, Josh Lerner, Marc Melitz, and Emmanuel Farhi (whom this paper is dedicated to) for their guidance. I also thank Philippe Aghion, Robert Barro, Greg Mankiw, Adrien Bilal, Elisa Rubbo, Martin Eichenbaum, Christian Wolf, Benjamin Friedman, Ivan Werning, Bill Kerr, Josh Lerner, Joel Flynn, Juanma Castro-Vincenzi, Spencer Kwon, Pierfrancesco Mei, Jon Cohen, and participants at Harvard's macro lunch seminars for comments. Any views expressed are those of the author and not those of the U.S. Census Bureau. The Census Bureau has reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 2590. (Disclosures CBDRB-FY23- P2590-R10905 and CBDRB-FY24- P2590-R11393). All errors are my own.

1 Introduction

Following a boom in the aftermath of World War II, US aggregate investment has been weak since the 1980s. Given falling interest rates and corporate valuations reaching all-time highs, *why did the US not experience an investment boom since the 1980s?* In standard models, we would expect forces driving up returns to stimulate investment. Rather than booming, investment in physical capital–the main focus of this work, which includes computers or robots–has trended down by over 2.5% of GDP since the 1980s.[1]

The existing literature on the aggregate investment puzzle has debated which firm-level changes could have made firms invest less.[2] Some argue incentives to invest fell among firms following the rise in market power perhaps driven by a deterioration of anti-trust institutions. Others argue new technologies increased returns to scale of some firms at the expense of tangible capital and firms that cannot keep up with intangible-intensive superstars. While the decline in startup rates is viewed as one more sign of weak investment, my work shows falling entry is behind the missing investment boom. This work builds on the firm dynamics tradition of Hopenhayn (1992) to explore the life cycle origins of the aggregate investment puzzle placing the decline in startup activity at the center stage.

In this paper, I find the aging of firms that followed this decline in the startup rate of new businesses explains around 80% of the aggregate investment decline despite high corporate profitability since the 1980s. In the neoclassical life cycle model calibrated to firm-level data, the co-movement between aggregate investment, entry, and interest rates and their inverse relation with profitability is neither puzzling nor a sign of changing competition or technology.[3] These seemingly puzzling phenomena are a feature of perfectly competitive economies with neoclassical technology where the supply of startups and the firm age distribution change over time. My work builds on a rising literature, including Hopenhayn *et al.* (2022) and others, that argues modeling heterogeneous firm dynamics is important to understand the macro-economy.[4]

[1] Broader measures that include intangibles (e.g., intellectual property) have stagnated.

[2] Explanations include Gutiérrez and Philippon (2017), Gutiérrez *et al.* (2021); Gutiérrez and Philippon (2019), intangibles, Crouzet and Eberly (2018), falling rates Liu *et al.* (2022), and technology Autor *et al.* (2020, 2017), Aghion *et al.* (2019).

[3] Clementi and Palazzo (2016) and Clementi *et al.* (2014) have shown the firm life cycle is an important form of investment propagation and amplification in the short run.

[4] My work inherits from the firm demographic literature pioneered by Hopenhayn *et al.* (2022), Peters and Walsh (2022), Pugsley and Şahin (2019), Alon *et al.* (2018) and Karahan *et al.* (2019), who all speak about firm aging but do not feature explicit capital investment. Bilal *et al.* (2021) is an exception in a model with labor market rigidities and ideas getting harder to find. Gutiérrez and Philippon (2019) have shown skepticism about whether firm demographic explanations, although consistent with the decline in the mass of entrants and potentially the decline in dynamism, could account for the puzzling divergence of entry, investment, and profitability. My work shows incorporating firm the life cycle into a model with investment addresses this skepticism due to age composition effects.

I start by documenting empirically that firm aging has been a powerful force behind the aggregate investment decline using confidential data from the US Census. I combine micro data on the investment behavior of firms in the Census of Manufacturing universe and firm age from the recently-redesigned Longitudinal Business Database (LBD). I show the share of output firms re-invest into building physical capital (structures and equipment) systematically declines with age in the cross-section of firms, mimicking the negative investment-to-GDP trend.[5] The data also shows the median size-weighted firm has aged from around 20 to 30 years old since the late 1990s. A simple aggregation formula implies that the changing firm age composition depressed investment-to-GDP like in the national accounts.

Surprisingly, the data shows firm aging has undone an investment boom conditional on age in manufacturing, where a 10% investment increase for the average firm has been overpowered by a 14.5% decline in aggregate investment.[6] This evidence suggests changes in startup rates and the firm age distribution can make micro and macro trends diverge.[7]

I further show that the depressing effects of firm aging on investment may apply more broadly than for physical capital in manufacturing by measuring *intangible* capital investment intensity of firms across *all sectors* in the LBD. Consistent with the evidence of tangible investment, citation-weighted patents per employee also systematically declines with firm age while increasing conditional on age. This suggests firm aging may have been putting the breaks to the rise of intangibles, slowing their positive impact on aggregate investment.[8]

I build a neoclassical theory with heterogeneous firms to understand why the firm age distribution matters for aggregate investment dynamics. Firms enter the

---

This shows the investment literature could benefit from explicitly modeling the firm life cycle given recent advances in the literature such as Sterk *et al.* (2021); Jaimovich *et al.* (2023).

[5]This result is consistent with Ottonello and Winberry (2023) who shows younger public firms in Compustat invest more intensely and exhaust returns to capital as they grow with age.

[6]This result is consistent with why measures of startup *quality* in Guzman and Stern (2020) have increased while startup *quantity* have declined. This missing investment conditional on age may not be just a feature of manufacturing. Survival and investment are related in life cycle models, and Census data on all sectors shows younger firms survive at higher rates today than those in the past.

[7]This finding that investment and survival policies conditional on age seem to have shifted stands in sharp contrast to the findings in the firm life cycle literature, e.g. Hopenhayn *et al.* (2022), that argue nothing has changed at the micro level. I also find that structural transformation has been another force dampening the investment decline due to aging because the economy has reallocated away from sectors with low startup rates and old firms such as manufacturing.

[8]I focus on data on the "output" of investment expenditures in R&D such as patents granted and the citations of those patents. The patent intangibles data is available for all other sectors in the economy, including services (however, only until 2001 instead of 2017). While more work remains to be done here, there is suggestive evidence that the bulk of the increase in intangible investment intensity is due to changes conditional on age larger than those seen for tangible capital. Given the evidence in Aragoneses (2023b) which uses data on the "inputs" of intangible investment, my current best guess if that the rise in intangibles may be operating through new cohorts of young firms that are more likely to adopt new technologies as they appear, not through firm aging.

economy as startups and invest in capital experiencing productivity dynamics. The model combines elements of seminal models of firm life cycle dynamics–Hopenhayn (1992)–and of capital investment–Hayashi (1982)'s *Q*-theory. Endogenous changes in startup activity driven by population or technology need not matter for the macroeconomy: if firms do not change much over the life cycle, the firm age distribution becomes irrelevant for aggregate investment. However, in the empirically relevant case where firms start small and older firms grow slower as they approach their productivity frontier, firm aging induced by a startup deficit depresses aggregate investment. Lower investment demand depresses interest rates as long as the startup deficit is partly driven by technology.[9]

I calibrate the model to quantify how historical changes in the firm age distribution affected aggregate investment in the post-war era. To discipline the economic forces in the model I employ the same back-of-the envelope calculation formula I used to analyze the data. In the model, investment intensity decline with firm age as in US data since younger firms front-load investment in anticipation of future profits. Firms receive these back-loaded profits if they survive to be old as rewards to past investments. However, despite this forward looking firm-level investment, aggregate investment in the model becomes backward-looking as the firm age distribution is affected by past startup rates.

Strikingly, the post 1980 investment puzzle is not so puzzling through the lens of the life cycle model. Quantitatively, firm aging predicts a substantial part of the 2.5% decline in tangible investment to GDP in the data since 1980, with my preferred model predicting 80% (2%).[10] The investment effect of firm aging post-1980 is comparable to the magnitude of the Marshall Plan in 1948-1951. In the model, the steady decline in firm entry since the 1980s shifted the composition of firms towards those that invest less intensely despite being more profitable. Firm aging reallocates economic activity from young firms focused on re-investing their earnings to fuel future growth towards older firms profiting from past investments while having less room to grow. In the life cycle model, we should not expect high profitability on aggregate to stimulate investment, since profits are earned by different firms (the old) than the ones carrying out the investments (the young).

Going back in time, I find this neoclassical life cycle model calibrated to post-2000 firm age micro data effectively reproduces relevant aggregate dynamics observed since the 1950s, particularly the rise and fall of investment-to-GDP. Inferring shocks to the supply of business ideas that make the model reproduce the historical startup rate boom and bust the US experienced since WWII, the model generates the desired co-movement between entry, tangible investment and (inverse) profits in US aggregate data. Surprisingly, I find little evidence that the historical relation between these macroeconomic aggregates changed much around 1980. This continuity casts

[9]This driving force is known as "ideas getting harder to find" following Bloom *et al.* (2020).

[10]This comes from non-linear balanced growth path comparisons. A lower bound was 1% of GDP, which come from linearized transitional dynamics.

doubt on investment puzzle explanations that rely on post-1980 changes depressing incumbent firm investment *directly*.[11] While there is mounting evidence that changes in anti-trust institutions or production technologies took place, these likely affected investment *indirectly* through their effects on the changing firm age distribution and the startup stage.

I find the startup deficit has also been driving 70% of the 4.7% secular decline in interest rates since the 1986–when rates start to trend down in the data.[12] Model exercises also predicts rising interest rates between 1950 until 1980, a period in which firms in the US economy turned younger and investment boomed. Consequently, lower interest rates may not be expected to stimulate investment and startup activity, but may instead be a byproduct of a depression in both.[13] In fact, the best fit of the model is achieved when firms do not directly respond to interest rate changes at all as in Hopenhayn (1992).

The model also predicts that a brighter future may lie ahead. Recent US Census data and cross-country data from the OECD show there has been a resurgence in startup activity in the aftermath of the Great Recession[14]. This startup surge, which accelerated during and after the recent pandemic, may be generating an investment boom as firms rejuvenate through the lens of the model. Consistent with the model, investment has been rising and profits falling in the cross section of countries experiencing a startup surge. The model also highlights that this rise in startup activity–potentially due to recent technological advances such as remote work operating against falling population growth–may be one of the forces helping the economy grow out of the zero lower bound.

This paper yields a lesson for policy-makers interested in raising aggregate investment as urged by as Mario Draghi (2024): prioritize boosting startup activity

[11]Some examples include Gutiérrez *et al.* (2021) and Crouzet and Eberly (2018).

[12]The 70% number comes from varying household rates due to per capita growth without affecting firm discount rates; allowing for some sensitivity of firms to interest rates shrinks this number to 40%, while making firms fully sensitive to rates further lowers it to 20%. Gormsen and Huber (2023) argue falling interest rates have not affected firms much perhaps due to hurdle rates arising from behavioral frictions or competition. This resonates with Zwick and Mahon (2017), Koby and Wolf (2020), and Winberry (2021) who argue against high responsiveness of firms investment decisions to interest rate changes in neoclassical investment models like Khan and Thomas (2008).

[13]Auclert *et al.* (2021b) perform a similar analysis for savings and interest rates and household aging focusing on the household side. Relative to them, I focus on the firm side of the economy, and find taking the fall in firm entry as an input generates the puzzling relation between investment, interest rates, and profitability via changes in the firm age distribution. However, although short-run investment elasticities to interest rates exist, little is known about their long-run counterparts, with Gormsen and Huber (2023) arguing firms may not react much to interest rate changes.

[14]This is a fact recently documented by Haltiwanger and Decker (2023), a fact that could not be seen with older vintages of the Census data, which has been recently re-designed. Since I use their same most-up-to date data from the US Census, I am able to detect the effects of the recent rejuvenation of firms, which none of the studies in the literature tend to emphasize. This result is also consistent with the evidence in Guzman and Stern (2020) showing the surge in growth-oriented startups after the Great Recession, challenging the notion that business dynamism is still on decline.

to take advantage of the fact that young firms invest more intensely.

Outline. I first present a decomposition formula used to look at the data. I then gather the different terms of this formula in US Census micro data, in particular, how investment behavior changes by firm age and the shifting firm age distribution. Then, I document how this firm aging empirically predicts the aggregate investment decline. I then present a model where changes in the firm age distribution matter for aggregate investment and calibrates it to firm age micro data post 2000. The paper then induces the model to have startup rate dynamics like in post 1950s data, generating the seemingly puzzling dynamics of aggregate investment through the changing age distribution. Then I discuss implications for profitability and interest rates. Finally, the paper provides cross-country evidence of the investment channel highlighted in this paper and the resurgence in entry since the Great Recession.

2 Data

2.1 Investment dynamics and firm age: a formula

How can we exploit micro data on the changing firm age distribution to understand macro investment trends? In the micro data, we observe the number of firms N_a by age a, their average size y_a in terms of output, and how much they invest on average i_a. Given this data, the following equation–which will reappear in a proposition later in the paper–can be used to dis-aggregate the ratio of investment to GDP I_t/Y_t in macro data as a weighted average of investment-to-output ratios by firm age i_a/y_a in the micro data.

$$\overbrace{I_t/Y_t}^{\text{Investment/GDP}} = \sum_a \frac{\overbrace{N_{at}}^{\text{firms}}\overbrace{y_{at}}^{\text{size}}}{\sum_a N_{at}y_{at}} \overbrace{(i_{at}/y_{at})}^{\text{inv/output share}}, \quad a = \text{firm age} \tag{1}$$

Since firms grow with age, economic activity is concentrated on a smaller number of older firms, so equation (1) reflects that total GDP of firms of a given age ($Y_a = N_a y_a$) is a combination of how many of those firms there are and their size. Further, the macro-relevant investment share i_a/y_a in equation (1) is not a simple average investment ratio among firms of a given age, but one that weights by their size[15]. For aggregate dynamics, one wants to compare the production and investment behavior of relatively big, young firms against large, mature ones (e.g., 15 year-old firms like Uber or Moderna against 50 year-old firms like Starbucks or

[15]Using the same size-weighted logic of equation (1), we can go from firm-level to age level ratios

$$\underbrace{i_a/y_a}_{\text{macro ratio}} = \sum_f \underbrace{\left(y_{fa}/Y_a\right)}_{\text{firm } f \text{ share in } a} \cdot \underbrace{\left(i_{fa}/y_{fa}\right)}_{\text{firm-level inv/output}} \neq \underbrace{\mathbb{E}\left[i_{fa}/y_{fa}\right]}_{\text{average ratio}}$$

Microsoft) rather that the limited investment of old "mom-and-pop" shops that are just as small as they were when they were young.

2.2 Data sources to dis-aggregate investment/GDP

The main measures and sources for each term in equation (1) are described below.

Aggregate data. For the macro ratio (I/Y) on the left hand side, I gathered the share of aggregate investment in physical capital (gross private domestic non-residential equipment and structures fixed investment) relative to GDP. The data is from the national accounts (BEA/NIPA from September 2023) for the post-war era. From the BEA, I also gather other forms of corporate investments in intellectual property products (i.e. intangibles), the share of corporate profits after taxes, and the labor share of GDP for applications; I will discuss this later throughout the dissertation. I also gathered their counterparts for other countries from the OECD. I also collect historical real interest rate data from Rogoff *et al.* (2022).

Micro ratios. For the micro ratios (i_a/y_a) on the right hand side, I obtained how the share of physical (equipment and structures) investment expenditures in value added–since sales minus intermediate costs aggregates up to GDP–varies across firms of different ages. I obtained access to this for the universe of manufacturing producers in the U.S. Census. The main data used, known as the Census of Manufacturers (CM), contains 2.7 million firm-year observations of $338,000$ firms every five years between 1977 and 2017, and an additional 1.1 million subset of it for the period after 2002 to combat censoring–I highlight under each fact what sample is used. I have kept data treatment as minimalist as possible to let the data speak for itself. I did not want to drop any firms or trim measures in order to not introduce selection (i.e. I did not want to keep only the "good" looking younger firms). Only a negligible number of small plants in Census of Manufacturing were excluded from the analyses because they could not be matched to the LBD which contained the measure of firm age I use. I aggregated plants at the firm level. Instead of dropping either firms or plants I kept "quality" control variables for several empirical issues (imputation, right-censoring, zero-observations) known to exist in Census. Since Census contains the universe of manufacturers, I do not need to use any sampling weights. All age-level ratios are size-weighted as described in Footnote 15.

Firm age. I measure the 1978-2021 firm age (a) distributions (N_a) following the methods Census used for their public-use Business Dynamics Statistics (BDS) from September 2023.[16] Rather than tracking incorporations, Census imputes firms' age by linking administrative and tax records that are constantly curated to adjust for changes to firms' organizational and legal structure. Census tracks firms as a collection of units (establishments whose age is the years since they first report

[16]In this article I used the most up to date available measure of firm identifiers in the Longitudinal Business Database (LBD) and the latest release of the BDS disclosed in late September of 2023. Note that since 2020, Census greatly updated their measurement of firm linkages and age.

positive employment). It then uses a relatively conservative measure of firm age: when a "firm" first appears in government records, they are assigned an initial age by determining the age of the oldest unit that belongs to the firm at time of birth. Firm age then accumulates naturally. This is done because some firms have multiple units at time of birth, hence generating concerns that mergers and acquisitions may lead firms to appear as new entrants when they really are older firms. Thus, measuring age based the oldest of an organization's establishments when the firm first appears prevents any spurious age changes that would result if an organization's age was updated based on M&A activity.[17] A startup is a firm for which all units (if there are more than one) are new to the economy in that year.[18]

Intangible investment intensity. While the main objective of this paper is to understand the dynamics of physical capital investment intensity as a function of firm age, the approach developed here can also include "intangible" capital investment. For all sectors in the economy, I measure patenting activity using what is known as the "SSEL NBER patent match" and merge it to LBD which contains information on the number of employees of firms.[19] I measure intangible investment intensity relative to the number of employees in terms of the number of patents granted to a firm weighted by forward citations across all patents granted in a year. This citation-weighted measure captures the fact that not all patents are worth the same, weighting more heavily more scientifically meaningful patents. Even though we do not know how much firms are spending to get these patents, these measures provide a sense of how intensively firms of different ages are investing in building intangible capital through R &D.

Censoring. Because the data of incorporation is not available, firm age is "censored" for the oldest cohort of firms: Census cannot provide the age distribution at or prior to 1977. I deal with this issue in several standard ways: excluding censored age groups, excluding earlier periods, and controlling for censored firms. For public BDS data, I use the raw variable of age but *exclude* the very oldest cohort of censored firms in the economy in every year, and estimate relations for uncensored ages in later years of the sample where sufficiently many uncensored ages exist. For confidential Census data, I use the latest periods, and I have disclosed results where I estimate a separate mean for censored age groups and found controlling for censoring often increases the strength of the relations highlighted here[20].

Historical entry series. To build a historical startup rate series, I gather historical

[17]See https://www.kauffman.org/wp-content/uploads/2019/12/bds_handout_011209.pdf

[18]I acknowledge this is a crude measure of age, but my goal here was to make my estimates able to be combined with public data from the BDS so my results were portable to other studies. I am working on a companion study describing the firm age anatomy of the economy using different measures of firm age, Aragoneses (2023a).

[19]A major advantage of this data is that it covers not just manufacturing but also data on services and other sectors. However the data stops in 2001 and thus, it cannot be used to do the same split-sample longitudinal analyses as the main investment data.

[20]This has been done due to disclosure requirements, since dropping firms would require "cutting" the data. I have however checked results inside Census dropping the oldest firms.

Table 1: Summary statistics of main data sources

Data Source	Sector	Variable	77-97	02-17	Δ	% Δ
BEA (Macro)	All	Investment/GDP I/Y	10.6	8.8	-1.7	-17%
	Manu.		10.9	9.4	-1.5	-14%
Census (Micro)	All	Startup Rate N_0/N	11.6	8.8	-2.9	-25%
	Manu.		8.8	5.3	-3.5	-40%
		Investment/GDP I/Y	8.3	7.1	-1.2	$-$**14.5**%
		"Mean" $E(i/y)$	8.0	8.8	$+0.8$	$+$**10**%

Note. This table highlights that aggregated Manufacturing Census data exhibits a similar levels and declines in investment-to-GDP I/Y as the broader economy across all sectors. It also shows that the within-sector decline in entry N_0/N has been larger than the aggregate one. Despite the decline in aggregate investment in Census, at the micro level average investment-to-output $E(i/y)$ increased indicating a disconnect between micro and macro trends.. It shows public data from the BEA and data from the US Census Bureau's public BDS for all sectors and for manufacturing. In addition, it also shows data on restricted-access micro-data from the Census of Manufacturers with ≈ 2.7 million firm-year observations in the universe of US manufacturing firms. See Section 2.1 for further data details.

firm entry from the Survey of Current Businesses pre 1963 as digitized by Hopenhayn *et al.* (2022), and match it to the the post 1977 BDS (2023) entry rate using an imputation of firm entry rates for the years between 1963 and 1977 from Karahan *et al.* (2019) that imputes establishment entry rates using the County Business Patterns.[21] I also gathered a measure of the entry rate from the Global Business Monitor.

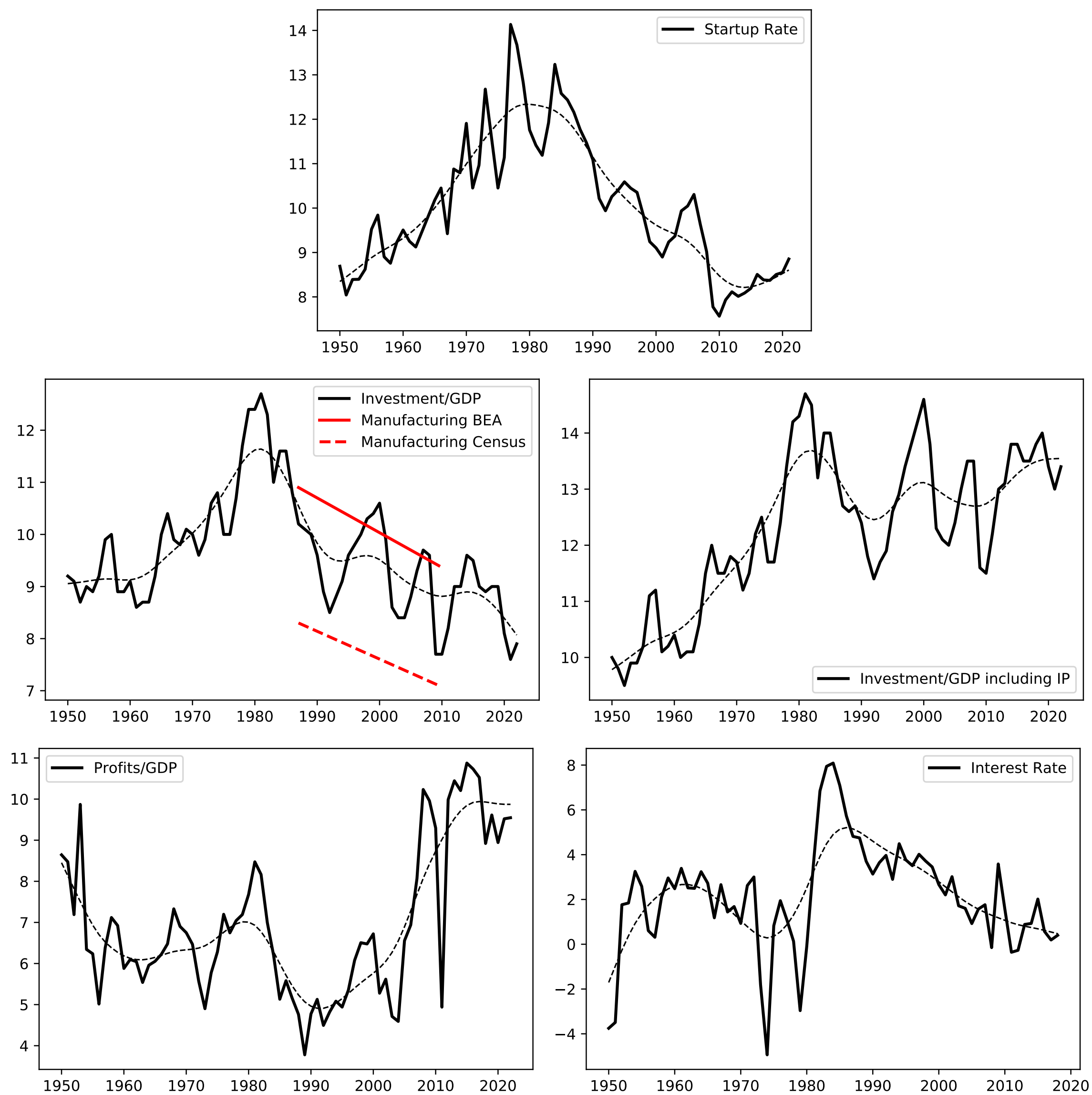
2.3 Puzzling macro trends

Figure 1 shows that aggregate physical capital investment-to-GDP I/Y move with firm startup rates: both increased after WWII and have declined after 1980.

Economy-wide v.s. Manufacturing investment trends. Table 1 shows manufacturing exhibits trends similar to the aggregate economy, with declining investment and firm entry rates since the 1980s; manufacturing and the aggregate exhibit similar level of tangible I/Y in the period of 1977-1997 and a similar magnitude of the decline since then (10.9% vs 10.6% and -17% and -14% respectively). Manufacturing

[21] For a companion article that will be available shortly (Aragoneses (2023a)) I also digitized records from Crum (1953) "The Age Structure of the Corporate System" on the 1945-1946 firm age distribution right after WWII from tax records by incorporation dates going back to 1860. Using this initial age distribution together with the historical path of the firm startup rate I have been able to impute the firm age distribution in 1977 using an iterative procedure. Although this distribution cannot be used to determine the age of particular firms in the 1977 spell of the micro data, it can be used for aggregation.

Figure 1: Trends in startup activity, investment, profits, and r^*



Note. Investment boomed after WWII accompanied by rising startup rates and interest rates until the 1980s and started under-performing afterwards. Manufacturing also experienced a decline in investment, as shown by the red line. The Census of Manufacturing exhibits a similar trend to the BEA data, although with different levels. This figure shows BEA GDP ratios of investment and GDP and corporate profits after tax. The restricted-access data from the Census of Manufacturers is from $\approx$ 2.7 million firm-year observations in the universe of US manufacturing firms. All trends include a HP filtered trend with smoothing parameter $\lambda = 100$. Data on US interest rates is from Rogoff *et al.* (2022). Data on startup rates come from Census BDS data on firm entry after 1978 and from the SCB prior to 1963, following Karahan *et al.* (2019) to interpolate years in between. See Section 2.1 for data details.

does however have a lower entry rate and a larger entry decline, likely due to the process of structural transformation, as the economy shifted towards services that had higher entry rates, mitigating the aggregate entry decline. However, as shown in the Data Appendix, despite having lower entry and higher initial size y_0, manufacturing firms do grow just as fast over the life cycle as the average firm across all sectors y_a/y_0, suggesting that firm dynamics patterns in manufacturing can be extrapolated to learn about patterns in the rest of the economy.

Aggregate (BEA) v.s. Aggregated (Census) manufacturing investment decline. The table also shows that although the measure of I/Y aggregated inside Census is lower than the one captured in BEA data (8.3% vs 10.9% in the pre-period), the time trends of both measures are identical: -14%. This suggests that although Census might systematically under-measure investment relative to BEA, this does not affect the time trends that are of interest for this study.

Figure 1 also shows broader measures of investment that include intellectual property (intangibles) increased before 1980, but have flat-lined afterwards. This break in the investment series is puzzling because real interest rates have been on decline since the 1980s, which should have stimulated investment on aggregate. Not only did total investment not rise, but its physical component fell strongly. The literature has proposed broadly three different explanations. First, a rise in intangibles made firms substitute away from physical capital Crouzet and Eberly (2018). Second, a fall in competition made firms need to invest less to remain market leaders Gutiérrez and Philippon (2017). Third, falling rates may benefit market leaders more Liu *et al.* (2022). All view the startup trend as an outcome: startups are being "discouraged" from entering due to incumbents.

The summary statistics Table 1 shows the striking disconnect: while *aggregate* physical I/Y fell by 14% along with the startup rate, the *average* investment share of firms in Census $\mathbb{E}\left(i_f/y_f\right)$ increased by 10%.[22] As will become clear throughout the rest of the section, while aggregate investment has been weak, in fact, firms at the micro level seem to be investing more in physical capital than they did before.

In what follows, the trends in the startup rate will be thought of as an "input" depressing aggregate investment by shifting economic activity towards older firms while the average firm remains young and dynamic.

2.4 Investment and growth by age and over time

How does investment behavior and firm growth vary with firm age and over time?

Fact 1.1: Older firms invest less intensely: investment/output falls with age

Table 2 shows in the universe of manufacturing firms after 2002 the relationship between firm age and the investment expenditure shares of value added is negative,

[22] This divergence between the average firm and the aggregate trend has also been highlighted by Kehrig and Vincent (2021) and Hubmer and Restrepo (2021) within the Census of Manufacturers for a different trend, the labor share.

Table 2: Older firms invest less intensely but investment intensity increased given age (for both tangible and intangible investment)

$\frac{\text{investment}}{\text{output}}$	(1)	(2)	(3)	(4)	(5)	(6)
	Base.	t FE	a FE	Sec FE	**Controls**	**Intangibles**
β_{age}	$-.09$	$-.09$		$-.08$	$-.10$	$-.12$
	(.01)	(.01)		(.01)	(.01)	(.03)
β_{time}	$+.05$		$+.05$	$+.02$	$+.05$	$+.06$
	(.01)		(.01)	(.01)	(.01)	(.03)

$$i_{fat}/y_{fat} = \alpha + \beta_{age}\, a + \beta_{time}\, t + \Theta' X + \varepsilon_{fat}$$

Note. This table hows that investment/output falls with age ($\beta_{age} < 0$) while it has increased over time ($\beta_{time} > 0$), motivating why aging could be behind the aggregate trend in Table 1 and reflecting a disconnect between the declining macro investment and the increasing average investment in Table 1. (1) pools all data 1977-2017 data running a regression on age and year without controls, (2) uses year fixed effects, (3) age fixed effects, (4) adds fine sector fixed effects, and (5) adds a mean for firms in the 1976 cohort whose age is unknown, showing the age relationship strengthens when controlling for the censored cohort of oldest firms, (6) uses a measure of intangible investment intensity (patents/employee weighted by citations). A figure in the data appendix uses only 2002-2017 data to address censoring. The main data used is from the Census of Manufacturers, $\approx$ 2.7 million firm-year observations pooling 1977-2017. The last column's NBER patents data sample covers firms in all sectors of the economy (not just manufacturing) for which information on the SSEL patent match exists and was able to be matched to the Longitudinal Business Database LBD. While the economic censuses cover he 1977-2017 period in five year spells, the LBD-NBER patent match in the SSEL is annual but only covers the 1976-2001 spell. Variables: for tangible capital intensity I uses total investment expenditures in physical capital (structures, equipment) relative to value added (the micro counterparts of investment/GDP) by firm age (measured by definition in LBD used in BDS); for intangible capital intensity (last column) I use the number of patents granted relative to the number of employees. Weights: share of the firm in total value added of its age group per equation, patent citations to capture scientific value (last column). See Section 2.1 for data details.

mimicking the decline in aggregate investment-to-GDP. Investment expenditures tends to grow with firm age at a slower rate than output grows with age, and as a result.[23] The table shows the share of output that is re-invested back into the business robustly declines with age, and adding controls for data quality issues (such as right-censoring) only strengthens this relation. Aging a decade decreases investment/output by 1%. [24] The last column of Table 2 also shows that intangible investment falls with firm age, measured using the number of new patents granted per employee weighted by citations to capture scientific importance. The sign and magnitude of the decline roughly mimics what one sees for tangible capital. Importantly, this data covers all sectors in the economy (not just manufacturing).[25]

Fact 1.2 Investment intensity increased conditional on age over time

Table 2 shows that in fact firms today seem to be investing more than firms did in the past once we condition on age, 0.5% more every decade. The Empirical Appendix suggests this might be driven by new cohorts of younger firms. This fact is unlikely to be driven by either censoring or manufacturing specificities. In fact, the last column shows the investment boom conditional on age is also present for intangible investment measures based on patenting LBD firms across all sectors. The next section also will demonstrate that survival rates (a form of investment) conditional on age have increased across all sectors in non-censored younger cohorts.

Fact 1.3: Firm size increases with age in a stationary way

Figure 3 shows that while the size of a business increases with age economy-wide, this size-age relationship has remained surprisingly stationary over time. A table in the Appendix also shows this holds regardless of whether employment or value added is used as a measure of size[26]. This stationarity is striking because there has been an aggregate slowdown since the 1980s (which would make firms grow faster) but also because investment conditional on age increased (which would make firms grow slower).[27] I will exploit the strength of the output by age relation and its apparent stationarity hereafter.

2.5 Micro facts on the changing firm age distribution

Since firms survive longer and fewer started, the US firm population has aged.

[23]This is consistent with independent work by Ottonello and Winberry (2023) using Compustat.

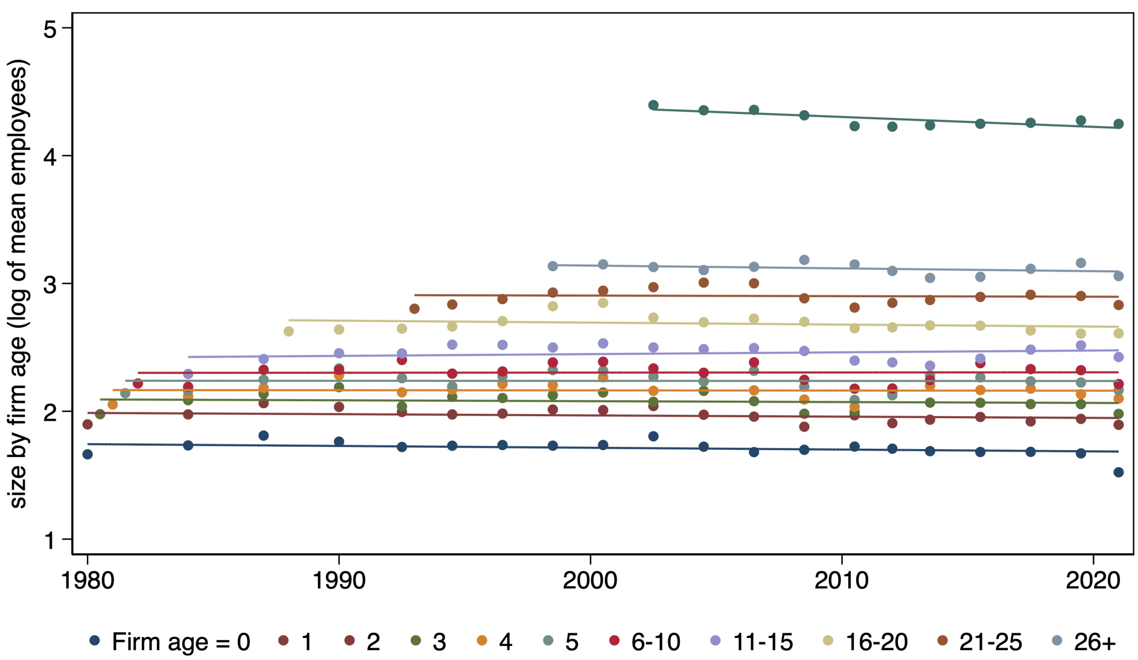
[24]See empirical appendix for further evidence that censoring is not an issue.

[25]I focus on tangible investment rather total investment because the measure of intangible investment has many caveats: it ends in 2001 and measures only the output of investment (patents granted) rather than inputs (dollar-valued expenditures), that is, is available only for innovating firms, and thus it misses the extensive margin of investment (the choice of whether to patent or not) as well as the intensity at which firms spend in patenting.

[26]The Table also shows this property holds for value added, which scales faster than employment

[27]Prior studies using old vintages of US Census data found the size-age relation seemed to be flattening

Figure 2: Size increases with age in a stationary way



Note. This figure shows size by age has been stationary so firms grow with age but not faster or slower than in the past. The figure uses Census BDS data on employment by age across all sectors, but the Data Appendix shows this same pattern holds in the Manufacturing Census 1977-2017 for output as well as employment. See Section 2.1 for data details.

Fact 1.4: Younger firms today survive at higher rates than in the past Aging can take place if fewer firms are being born or existing firms survive longer. Declining firm entry was one of the trends in Figure 1. Figure 17 in the Appendix shows that while in every year older firms tend to exit less, younger firms today are more likely to survive than those in the past, a fact that has recently emerged in BDS after the LBD methodology was re-designed to better measure firms identities. Nearly 30% of newborn startups did not survive a year, while close to 20% exit in the 2010s. This echoes the result in Table 2: firms are investing more intensely in capital but also in survival.

Fact 1.5: US firms have aged significantly Figure 3 shows the US age distribution has shifted towards old firms: the median firm age rose from 6 to 9 since the 1980s.[28] Firm aging has generated an even stronger shift in workers towards older firms (given that size increases with age in a stable way). Following the red line in Figure 3, we see that the median worker nowadays is employed at a 30 rather than 20 year old firm as they were in the late 1990s. The Appendix demonstrates that aggregates underestimate within-sector aging because the economy is moving away from sectors with less entry (and older firms) such as manufacturing.[29] The empirical predictive analyses that follow are thus performed within sectors.

2.6 Firm aging implications: a back-of-the envelope calculation

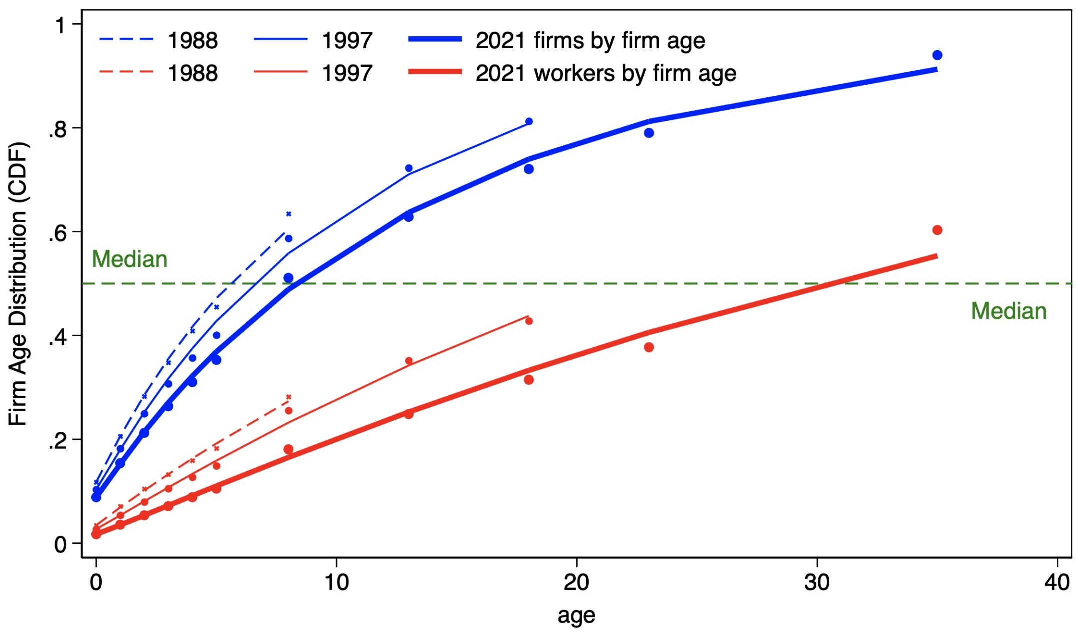
The shift in the age distribution of US firms documented above predicts the trend in aggregate investment to GDP. I do so using a back-of-the-envelope calculation that shifts only the composition of firms by age holding constant how micro investment behavior and size varies by age. This age-based de-composition isolates how the three terms in equation (1) change over time.

$$\Delta\frac{I}{Y} = \underbrace{\sum_a \left(\Delta\frac{N_a}{N}\right)\frac{y_a}{y}\frac{i_a}{y_a}}_{\text{firm age composition}} + \underbrace{\sum_a \frac{N_a}{N}\left(\Delta\frac{y_a}{y}\right)\frac{i_a}{y_a}}_{\text{size by age}\approx 0} + \underbrace{\sum_a \frac{N_a}{N}\frac{y_a}{y}\left(\Delta\frac{i_a}{y_a}\right)}_{\text{investment intensity by age}} \tag{2}$$

[28]When studying life cycle dynamics, most data sources share a similar problem: age since founding date is not directly observed. Age must be estimated within the available years for which longitudinal linkages are available. In US Census, we do not know the true age of businesses founded in 1976 or earlier. Thus, the maximum age observed in 1987 will be 10. To get around this problem, I follow Axtell (forthcoming) in approximating the business age distribution via a Weibull function (thinner tailed than exponential) separately for each year, sector, and type of business (plants and firms). My reliable estimates start at 1992 because this is when the coefficients from a Weibull fit $F_{st}(a) = 1 - e^{-(a/p_{st})^{q_{st}}}$.

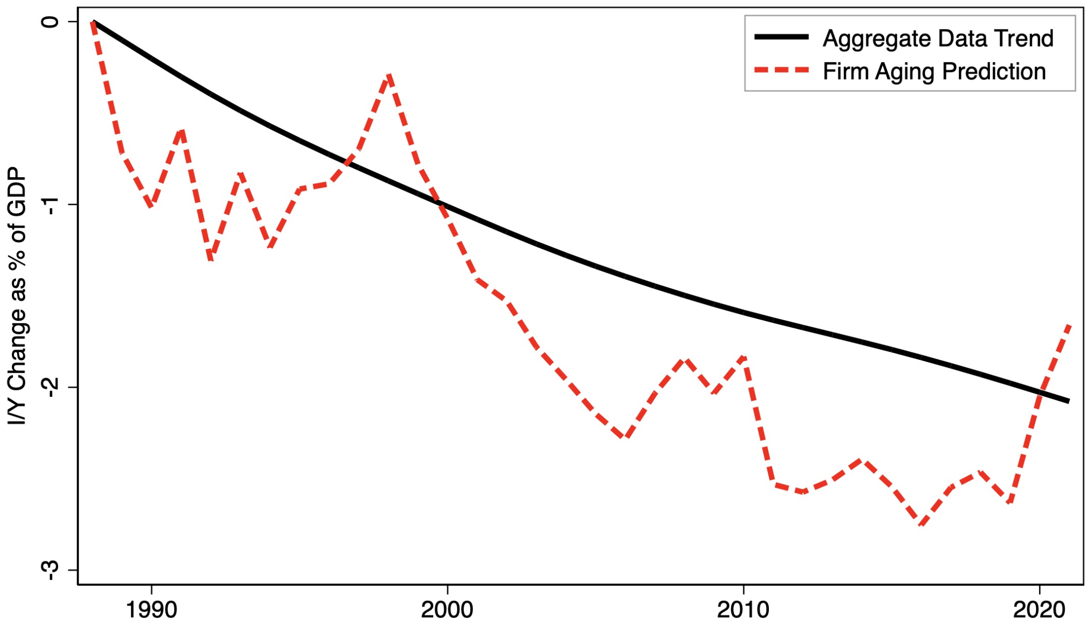
[29]In the appendix I show how within each year-sector, I estimate the share of firms using uncensored age data, and fill in the missing empirical distribution with the estimated one. I estimate micro ratios and the distribution with data truncated at the maximum age observable (i.e. 40 in 2018) and extrapolate the terms for the formula up to age 200 since that is the age of some of today's largest firms. See the Empirical Appendix for details on the implementation. In addition, I also find plants tend to be younger on average but exhibit similar aging patterns to firms.

Figure 3: US firms have aged significantly



Note. This figure shows firms have aged significantly in the US since the late 1980s. Following the red line, we see that the median worker nowadays is employed at a 30 rather than 20 year old firm as they were in the late 1990s. Figure shows a Weibull cumulative distribution function $F(a) = 1 - \exp\left(-\left(a/p\right)^q\right)$ overlaid on data from BDS (09/2023) with coefficients estimated separately for each year. I mitigate the effect of censoring by estimating this age distribution with only the non-censored age groups (dropping the firms in the first year of BDS), and do not use the pre 1988 years where little age variation is available. The figure uses Census BDS data on employment by age across all sectors, but the Data Appendix shows this same pattern holds in the Manufacturing Census 1977-2017 for output as well as employment. See Section 2.1 for data details. See Section 2.1 for data details.

Figure 4: Changing firm age composition predicts the aggregate investment decline



$$\underbrace{\Delta_{t_0}^{T} I_t/Y_t}_{\text{aggregate trend}} \approx \underbrace{\sum_a \left(\Delta_{t_0}^{T} \frac{N_{at}\bar{y}_a}{\sum_a N_{at}\bar{y}_a}\right)\left(\bar{i}_a/\bar{y}_a\right)}_{\text{firm aging prediction}}$$

Note. This figure shows the aging of US firms depressed aggregate investment, while a recent rejuvenation may have put a stop to this downward pressure. The "aggregate data trend" is from the BEA's investment-to-GDP ratio in Figure 1 while "firm aging prediction" dashed red line in Figure 6 plots the "firm age composition" term in the back-of-the-envelope equation. Firm aging shifts the age distribution of firms N_{at}–estimated from Census BDS data on each year like in Figure 3–towards firms that have lower investment-to-output ratios (i_a/y_a), measured in restricted-access micro-data from the Census of Manufacturers as shown in Fact 1. Importantly, the aging prediction reached its lowest point in the mid-2010s but has since then has experienced a rebound due to a resurgence in entry. Throughout I hold the size-by-age y_a and investment-by-age i_a ratios are held constant, where y_a is estimated in the BDS in terms of labor but adjusted to value added using estimates of the value added to labor relation from the Census of Manufacturing. In my implementation I isolate *within-sector aging* effects by controlling for structural transformation in the economy $\Delta I_t/Y_t \propto \sum_s Y_{s0}/Y_0 \cdot \Delta I_{st}/Y_{st}$ holding constant the 1980s sectorial composition.

Fact 1.6: Changing firm age composition predicts aggregate investment decline The "firm aging prediction" dashed red line in Figure 4 plots the "firm age composition" term in the back-of-the-envelope equation (2). It suggests firm aging (shown in Fact 3) has been a powerful force depressing aggregate investment-to-GDP in the US since the 1980s. The prediction is remarkably close to the actual trend in black, and in fact, predicts that if aging was the only force, there would have been an even larger decline. Firm aging drove down investment by 2.5% of GDP between the late 1980s and the 2010s (125% of the actual 2% decline).

Why firm aging depressed investment after the 1980s. Intuitively, aging shifts economic activity towards firms that have lower investment-to-output ratios (i_a/y_a), as shown in Fact 1 for the Manufacturing Census. The prediction weights the change in the *age distribution of firms* (N_a/N) in Fact 5 by the stable size-by-age relation (y_a/y) shown in Fact 3 to arrive to the change in the *firm age distribution of GDP* ($\Delta N_{at}y_a / \sum_a N_{at}y_a$). Given the stable size-by-age relation, the middle term has likely been negligible ($\Delta_t y_a/y \approx 0$), implying an increase in investment intensity given age likely dampened the aggregate decline. This is consistent with the evidence in Fact 2 that investment increased condition on age.

The recent rejuvenation of US firms. Importantly, the aging prediction reached its lowest point in the mid-2010s but has since then has experienced a significant rebound. This is due to the resurgence of firm entry after the Great Recession that accelerated during the COVID-19 pandemic.

Implementation details. I extrapolate my estimates inside Census for manufacturing i_a/y_a profile (Fact 1) in Compustat across all sectors (see Appendix) a similar negative profile exists across all sectors[30]. In the empirical appendix, I show further evidence that manufacturing is not very different from the rest of the economy, including that size-by-age in manufacturing mimics the average firm in all sectors, justifying why we can use the extrapolation from manufacturing for the decomposition. Manufacturing indeed is still one of the largest sectors although it has been on decline due to structural transformation shifting the economy towards services: in 1947 manufacturing was 1/3 of the non-finance economy while today is only around 1/6.[31] Because the level of i/y might be different in the manufacturing Census than in the rest of the economy, I use the schedule in manufacturing relative to a startup (i_a/i_0) / (y_a/y_0) to multiply the changing weight $N_{at}y_a / \sum_a N_{at}y_a$ where the estimate used controls for censoring ages. I also isolate *within-sector aging* effects by controlling for structural transformation in the economy. I do so applying this formula for $\Delta I_{st}/Y_{st}$ at the *sector* level and aggregating up $\Delta I_t/Y_t \propto \sum_s Y_{s0}/Y_0 \cdot \Delta I_{st}/Y_{st}$, holding sectorial composition of output Y_{s0}/Y_0 unchanged since the 1980*s* to prevent the reallocation from older to younger sectors (manufacturing to services)

[30] A more serious cross-sector investigation of differences in investment behavior over the life cycle is left for future work.

[31] It is common to remove Finance, Insurance, and Real Estate (FIRE), see Ottonello and Winberry (2020), from analyses because firms in these sectors tend to behave differently from the rest of the economy.

biasing down the aggregate dynamics. In the BDS I estimate N_a changes and the l_a employment by age profiles within each sector and period. I also use the adjustment between labor and value added inside Census to adjust the BDS schedule. I have left the details of the decomposition are in the Data Appendix.

Intangibles. Despite the different approaches to measure investment in physical capital (in manufacturing) and intangible capital (across all sectors), both types of data show that by the time firms reach 25 years old, they invest 60% as intensely as a young startup. Furthermore, we have seen conditional on age intangibles exhibit the same investment boom over time observed in physical capital. This adds to the robustness of the main channel highlighted in this study–that firm aging quantitatively depresses investment intensity at the aggregate. Further exploring these changes conditional on age and deepening our understanding for intangibles investment using broader measures is left for future work.[32]

This age decomposition, while illustrative, supports the assessment that changing age composition alone, while holding constant the micro relations estimated above, can help predict the aggregate investment-to-GDP decline observed after the 1980*s*.

3 Theory

Does entry and the firm age distribution matter for aggregate investment dynamics? I build a neoclassical theory that combines elements of models of firm life cycle dynamics–Hopenhayn (1992)–and of capital investment-Hayashi (1982)'s Q-theory. The model is calibrated to the age heterogeneity in the back-of-the-envelope formula to assess the aggregate implications of changes in startup activity since 1950.

3.1 Environment

Time is discrete and infinite, with no aggregate risk. The close economy is populated by a household and entrepreneurs that can launch startups to become firms.

Household. Each of a growing mass $\bar{L}_t = \prod_{k=0}^{t} g_{Lk}$ of symmetric individuals supplies inelastically one unit of labor. They do not own any capital, own all firms in the economy, including startups, and receive the flow profits of firms (net of all

[32]As an alternative measure, I also collected information on spending into advertising, communications, and non-production worker salaries as a proxy for organizational capital. Some of this spending of non-production workers could be used in R&D activities to produce patents. This "input" rather than "output" approach to the measurement of intangible investment shows clearly an *increase in intangible investment conditional on age that is especially concentrated on young firms*. This could imply that while for tangible investment the aging force has been quantitatively the dominant depressing force while for intangible investment the negative impact of aging has been offset by an increase conditional on age that may not be visible using existing measures of patenting firms. While these aging effects seem to also potentially be present in intangible measures of investment based on advertising/patenting, it seems like direct changes conditional on age may have been more powerful for intangibles.

investment costs) as dividends π_t. They have access to bonds b_t (in zero net supply) paying interest rate r_t. The household maximizes lifetime utility over consumption c_t of all its members given a sequence of per capita budget constraints:

$$
\begin{aligned}
V_0(b_0) &= \max_{\{c_t, b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \bar{L}_t \log(c_t) && (3) \\
c_t + g_{Lt+1} b_{t+1} &= (1+r_t) b_t + w_t + \pi_t && (4)
\end{aligned}
$$

Firms. Output, Y_t, the numeraire, is produced by a stock of firms of mass N_t. Firms produce using labor l_t, which they hire in a competitive market, their own tangible capital stock k_t, and productivity z_t, which can be thought of as their intangible capital stock. Firms are heterogeneous in z_t and k_t. The production technology features a growing common level of productivity $\bar{Z}_t = \prod_{k=0}^{t} g_{Zk}$, and decreasing returns to scale θ, so that $\alpha\theta$ is the capital share. Decreasing returns allows unequal firms to coexist in equilibrium rather than all the output being produced by the most productive firm in the economy.

$$
y_t = \bar{Z} z_t \left(k_t^{\alpha} l_t^{1-\alpha}\right)^{\theta}, \quad \alpha, \theta < 1 \tag{5}
$$

Firms will enter the economy drawing a relatively small productivity z_e, accumulating productivity and capital as they age. Their productivity evolves as a Markov process with persistence ρ accumulating log-normal shocks ε with dispersion σ if they survive. Their tangible capital stock depreciates at rate δ_k and is accumulated explicitly via costly investment i_t–our key object of interest. Convex adjustment costs $\phi(i,k)$ slow down this capital accumulation process. This captures the fact that it may take firms some time to build up to their desired capital level that matches their level of productivity.

$$
\begin{aligned}
\log z_{t+1} &= \rho \log z_t + \sigma \varepsilon_{t+1} \quad , \qquad \varepsilon_{t+1} \sim N(0,1), \quad z_0 = z_e \quad \text{given} && (6) \\
k_{t+1} &= (1-\delta_k) k_t + i_t \quad , \qquad \phi(i,k) = \kappa^i (i/k)^{\gamma} k && (7)
\end{aligned}
$$

After production takes place and workers are paid, firms continuously decide whether to invest in continuing in operation or shutting down. I capture this exit-survival choice by the discrete choice $p_{ft} \in \{0,1\}$ of paying an additional fixed cost of production in goods κ_f. If the firm shuts down, their intangible capital is lost forever because it cannot be sold in the market; in contrast, they can sell their capital stock net of depreciation and costs required to uninstall their capital stock $\lambda = 1 - \kappa^i \left(-(1-\delta_k)\right)^{\gamma-1}$. Their dynamic optimization problem given (5)-(7) is:

$$
\begin{aligned}
v_t(z,k) &= \max_{p_{f,t} \in \{0,1\}, i, k', y, l} \left(1 - p_{ft}\bar{p}_f\right)(1-\delta_k)\lambda k \quad (\leftarrow \text{shut-down choice value}) \\
&+ p_{ft}\bar{p}_f \left\{ y - w_t l - k' - \phi(i/k) - \kappa^f + \frac{1}{1+r_{t+1}^f} \mathbb{E}_t\left[v_{t+1}(z', k')\right] \right\} \qquad (8)
\end{aligned}
$$

Note that current z and k are the only state variables because firms start with a different initial $z_e < 1$ described below, but converge to a common long run mean $\bar{z} = 1$ over the life cycle.[33]

Startups. I allow both for free entry as well as entrant selection and investment. A non-negative mass of entrepreneurs $M_t \geq 0$ decides to enter the economy in every period. Each entrepreneur M_t pays a cost of entry κ_e, which can be thought of as the cost of "exploring" the market. Once they pay the cost, they draw an "idea" of their future intangible productivity z_e from a Pareto distribution $F_e(z_e) = 1 - (z_e/\underline{z}_e)^{-\xi}$ such that $z_e \geq \underline{z}_e$. This endogenous mass of entrepreneurs choose to enter as long as the net expected value of doing so is non-negative:

$$-\kappa^e + \int v_e(z_e) F_e(dz_e) \geq 0 \tag{9}$$

Given their initial idea z_e, each potential entrepreneur in mass M_t decides whether to start the business or not and if they do so, they invest in an initial level of capital k'_e with which to start the firm. Similarly to incumbents' survival, their entry choice $p_{st} \in \{0, 1\}$ is subject to an overhead cost κ^s so their optimization program is:

$$v_e(z_e) = \max_{k'_e, p_{st} \in \{0,1\}} p_{st} \left\{ -\kappa^s - k'_e + \frac{1}{1 + r^f_{t+1}} \mathbb{E}_{z'|z_e} \left[v_{t+1}(z', k'_e) \right] \right\} \tag{10}$$

Entrants are entrepreneurs with a signal above an endogenous threshold $z_e > z^*_e$ will choose to launch their startup.

Crucially, M_t does not directly affect firms idiosyncratic problem: entry affects the economy only through the distribution of firms. The level of M_t will scale up the economy. A faster growing M_t will result in relatively younger firms.

Discount rates. The firm dynamics literature often works with two extremes regarding firms' discount rates: either r^f_t is fixed at an exogenous rate r^* as in Hopenhayn *et al.* (2022)or sets it equal to the household rate r_t as in the neoclassical model in Khan and Thomas (2008). I allow for an intermediate case where $r^f_t = (1 - m) r^* + m r_t$. The constant $m \in [0, 1]$ allows firms' investment behavior to depend on household interest rates but in a less sensitive fashion than in the neoclassical model as argued by Gormsen and Huber (2023), Gabaix (2020), Winberry (2021) and Koby and Wolf (2020). $m < 1$ allows the model to be relatively insensitive to interest rates without having to calibrate investment adjustment costs to be so high that firms do not grow enough over the life cycle.

Law of Motion of the Distribution μ_t. There is an endogenous measure of firms $\mu_t(s)$ where $s = (z, k)$ is a short hand for the state vector and t is a short hand for

[33] I have also develop an extension where firms draw a long-run mean $\bar{z}$ where $\log z_{t+1} = (1 - \rho) \log \bar{z} + \rho \log z_t + \sigma \varepsilon_{t+1}$ which required an additional state variable so $(\bar{z}, z, k)$. Because the presence of k due to adjustment costs already breaks the perfect correlation between size and productivity, I leave this extension for the Appendix.

all aggregates: $\{w_h, r_h, M_h, \bar{L}_h, \bar{Z}_h\}_{h\geq t}$. There is an underlying age distribution:

$$\mu_t(s) := \sum_{a=0}^{\infty} \mu_{at}(s) \tag{11}$$

Let $ds = (dz, dk)$. The distribution evolves according to the endogenous entry and exit of firms as well as their capital investment decisions captured by ϖ_k and exogenous productivity dynamics captured by ω_z:

$$\begin{aligned} \mu_{a+1t+1}\left(s'\right) &= \bar{p}_f \int p_{ft}(s)\, \varpi_k\left(k' = k'_t(s)\right) \omega_z\left(z'|z\right) \mu_{a,t}(ds) \quad \text{if} \quad a > 1 & (12) \\ &= M_{t+1} \int p_{st}(z_e)\, \varpi_k\left(k' = k'_{et}(z_e)\right) \omega_z\left(z'|z_e\right) F_e(dz_e) \quad \text{if} \quad a = 0 & (13) \end{aligned}$$

Consequently, the mass of firms evolves as:

$$N_{t+1} = \int \mu_{t+1}\left(ds'\right) = \bar{p}_f \int p_{ft}(s)\, \mu_t(ds) + M_{t+1} \int p_{st}(s)\, F_e(ds) \tag{14}$$

I define the general equilibrium of this economy and its balanced growth path.

3.2 Equilibrium

I define a recursive competitive equilibrium given paths of exogenous shocks $\{\bar{L}_t, \bar{Z}_t\}$ where all agents optimize given sequences of prices $\{w_t, r_t\}$ that adjust to clear markets:

Household. Given prices w_t, r_t and profits π_t, the household chooses individual consumption sequences $\{c_t\}$ to maximize lifetime utility (3) given their constraints which reflect that bonds are in zero net supply so $b_t = 0\ \forall t$.

$$(1 + r_{t+1})\,\beta = c_{t+1}/c_t, \quad c_t = w_t + \pi_t \tag{15}$$

Firms. Given prices w_t, r_t, firm value and policy functions satisfy (8). Firms choose output and labor statically, and given output and profits, firms make their dynamic investment decisions $p_f(s)$, $k'(s)$. Because all costs are paid in output, firms optimally choose to have a constant labor share of output $\bar{\theta} = \theta(1 - \alpha)$.

$$w_t l_t(s) / y_t(s) = \bar{\theta}, \quad y_t(s) = \left(\bar{\theta}/w_t\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}} \left(\bar{Z}_t z k^{\alpha\theta}\right)^{\frac{1}{1-\bar{\theta}}} \tag{16}$$

Startups. Given prices w_t, r_t and the value of a firm $v_t(s)$ implied by (8), $M_t > 0$ entrepreneurs enter until free entry is satisfied. Once each draws z_e, each entrepreneur makes their investment decisions: $p_s(z_e), k'_e(z_e)$ to satisfy (10). Free entry requires:

$$\kappa^e = \int v_e(z_e)\, F_e(dz_e) \tag{17}$$

Distribution. $\{\mu_t, M_t\}$ satisfy the law of motion of the distribution (11)-(13).

Market clearing. Given optimal choices, w_t, r_t, and M_t adjust to clear *labor markets*

$$\bar{L}_t = \int l_t(s)\,\mu_t(ds) = \bar{\theta} Y_t / w_t \tag{18}$$

as well as to clear *goods markets* where consumption is what is left of total production after the costs of investment and operations are paid.

$$Y_t = C_t + I_t + X_t \tag{19}$$

These costs include costs of entry, operations, and investment paid in goods (which could be thought of as a "broader" notion of investment that is not emphasized here)

$$I_t = M_t \int k_e(z)\, p_s(z)\, F_e(dz) + \int k'(s)\, p_f(s)\, \mu_t(ds) - \int (1-\delta_k)\, k p_f(s)\, \mu_t(ds) \tag{20}$$

$$X_t = M_t \left(\kappa_e + \kappa_s \int p_s(s)\, F_e(ds) \right) + \int \left[\kappa_f + \phi(s) \right] p_f(s)\, \mu_t(ds) \tag{21}$$

Aggregate output scales with the stock of firms and their average productivity

$$Y_t = \left[\int \left(z k^{\alpha\theta} \right)^{\frac{1}{1-\bar{\theta}}} \bar{\mu}_t(ds) \right]^{1-\bar{\theta}} \bar{Z}_t \bar{L}_t^{\bar{\theta}} N_t^{1-\bar{\theta}}$$

where $\int \bar{\mu}_t(ds) = 1$ defines a density. Thus the model jointly endogenizes Total Factor Productivity (TFP) as well as I_t/Y_t as a function of entry and the age distribution of firms. The model also highlights why profitability and investment are inversely related: when firms spend less in investment, they retain more earnings since the dividends distributed to individuals $\pi_t = \Pi_t/\bar{L}_t$ depend on the profit share:

$$\frac{\Pi_t}{Y_t} = 1 - \bar{\theta} - \frac{I_t}{Y_t} - \frac{X_t}{Y_t} \tag{22}$$

We are now ready to define a balanced growth path.

(Balanced Growth Path) Along a BGP where population and technology grow at constant rates $\bar{L}_t = g_L^t$ and $\bar{Z}_t = g_Z^t$, the mass of entrants M_t grows with GDP as a function of population growth and income per capita growth driven by technology

$$g_M = g_Y = g_c g_L, \quad g_c = g_Z^{1/\bar{\theta}} \tag{23}$$

so that $\bar{M}_t = M_t / \bar{Z}_t^{1/\bar{\theta}} \bar{L}_t = \bar{M}$ and other de-trended aggregates are stationary along the BGP. Startup activity only affects interest rates in the long run if it is not only driven by household demographics, but also due to technological change g_Z.

$$(1 + r(g_c))\,\beta = g_c \tag{24}$$

Proof. For goods markets to clear, total consumption $C_t = \bar{L}_t c_t$ needs to be growing at the same rate as output Y_t so per capita consumption grows $g_c = g_Y / g_L$. If costs of entry κ_e are constant, then M_t also needs to be growing with Y_t, $g_M = g_Y$. This means that consumption per capita, wages, and profits per capita need to be growing at $g_c = g_w = g_\pi$, which determines the equilibrium interest rate $(1 + r(g_c))\beta = g_c$. For labor markets to clear, wages need to be growing with technology:

$$w_t = \bar{\theta} Y_t / \bar{L}_t = g_w^t w \quad \text{where} \quad g_w = g_Y / g_L = g_Z^{\frac{1}{\bar{\theta}}}$$

Due to this general equilibrium offsetting, output by age is stationary over time (Fact 3) but grows with age as z, k accumulate.

$$y_t(s|w) = \mathbb{Y}_t(w_t)\left(zk^{\alpha\theta}\right)^{\frac{1}{1-\bar{\theta}}}, \quad \mathbb{Y}_t(w) = \left(\bar{\theta}^{\bar{\theta}} \bar{Z}_t / w_t^{\bar{\theta}}\right)^{\frac{1}{1-\bar{\theta}}} = \left(\bar{\theta}^{\bar{\theta}} \bar{Z} / w^{\bar{\theta}}\right)^{\frac{1}{1-\bar{\theta}}} = \mathbb{Y}(w)$$

Given r and w, because neither M_t nor $\bar{L}_t$ directly affect either the firm or startup problem (there are no direct competition or congestion effects), these problems become stationary as a function of aggregates $\mathbb{Y}(w)$ and $r^f(r)$.

$$\begin{aligned} v(s|w,r) &= \max\left(1 - p_f(s)\right)(1-\delta_k)\lambda k \\ &+ p_f(s) \left\{(1-\bar{\theta})\,\mathbb{Y}(w)\left(zk^{\alpha\theta}\right)^{\frac{1}{1-\bar{\theta}}} - k' - \phi - \kappa_f + \frac{\mathbb{E}v(s')}{1+r^f(r)}\right\} \quad (25) \\ v_e(z_e|w,r) &= \max p(z_e)\left\{-\kappa_s - k_e' + \mathbb{E}v(s') \,/\, \left(1 + r^f(r)\right)\right\} \quad (26) \end{aligned}$$

$r(g_c)$ is determined by households while w is determined by the free entry condition:

$$\mathbb{E}_{z_e} v_e(z_e | w, r(g_c)) = \kappa^e$$

Let the stationary distribution be $\bar{\mu}$ such that $\mu_t = N_t \bar{\mu}$ where N_t is growing with Y_t for goods markets to clear. Then, labor market clearing determines the number of firms the economy can sustain:[34]

$$N(w) = (w/\bar{\theta})^{\frac{1}{1-\bar{\theta}}}\, \bar{L} / (\bar{Z})^{\frac{1}{1-\bar{\theta}}} \,/ \int \left(zk^{\alpha\theta}\right)^{\frac{1}{1-\bar{\theta}}} \bar{\mu}(ds)$$

Therefore, given that w is determined by free entry independently of N, N and thus, M adjusts to clear the labor market. Given that M_t, μ_t and all aggregates are growing with technology and labor supply, I de-trend $\bar{M}_t = M_t / \bar{Z}_t^{1/\bar{\theta}} \bar{L}_t$ and

[34]Note that because N_t will be growing with Y_t, we have that:

$$1 = \int l_t(s)\, \mu_t(ds) \,/ \bar{L}_t = (N_t / w_t \bar{L}_t)\, \mathbb{Y}(w)\, \bar{\theta} \int \left(zk^{\alpha\theta}\right)^{\frac{1}{1-\bar{\theta}}} \bar{\mu}(ds)$$

$\bar{\mu}_t = \mu_t / \bar{Z}_t^{1/\bar{\theta}} \bar{L}_t$. Along the BGP with $g_M = g_Z^{1/\bar{\theta}} g_L$, $\bar{M}$ and $\bar{\mu}$ are stationary, and the normalized distribution solves:

$$\bar{\mu}\left(s'\right) \;=\; \frac{1}{g_M} \int p_f\left(s\right) \varpi_k\left(k'|s\right) \omega_z\left(z'|z\right) \bar{\mu}\left(ds'\right) + \bar{M} \int p_s\left(z_e\right) \varpi_k\left(k'|z_e\right) \omega_z\left(z'|z_e\right) F_e\left(dz_e\right)$$

I now calibrate the BGP of the model to firm age micro data.

3.3 Calibration

Figure 5 shows the model is able to match the key relations in the data quite closely. I parameterize the model in a BGP that the economy is assumed to have reached right before 2020. I discipline the parameters of the model given g_M by forcing the model to reproduce the three central relations in the data used for the age decomposition from equation (1) using a Simulated Method of Moments (SMM) algorithm that minimizes the distance between moments in the model and in the data[35]. The model is complex and non-linear so no one-to-one mapping between moments and parameters exists. However, several moments are more useful in identifying certain parameters.

Age distribution of firms N_a. Figure 5 shows the model reproduces well the entry rate around 8%, median firm age around 7, and the fact that the vast majority of firms (around 80%) are under 25 in the data. Intuitively, the age distribution of firms is most influenced by g_M directly affecting entry and the operating cost and by exogenous p_f that drive exit, with the former mattering more for young firms and the latter more for old firms.

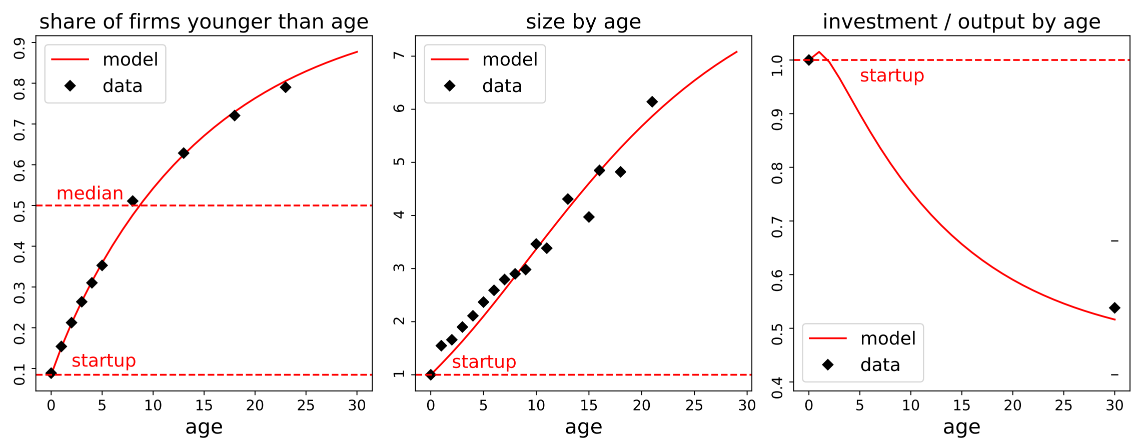
Size by age y_a/y_0. Figure 5 also shows the model matches well how size grows with age. The model reproduces that firms grow by a factor of 6 by the time they reach 25. These growth dynamics of firms relative to entry are influenced by the productivity process. The key driver of these patterns are that entrants are born relatively unproductive and small given z_0 and given $\rho \in (0,1)$ and $\kappa_x \neq 0$ their productivity process takes time to converge to their steady state.

Investment/output by age i_a/y_a. Figure 5 shows the model matches the overall negative relation between investment intensity and age, where a 30 year old firm is expected to invest roughly half out of output relative to the youngest firms in the economy. Due to positive selection from exit, initially investment intensity increases with age, but eventually diminishing returns and convergence make investment intensity decline with age. The key driver of these patterns is that firms begin with

[35]Formally, I search iteratively for a parameter vector Θ–listed in Table X–which jointly minimizes the distance between empirical moments and their theoretical counterpart which change with Θ and depend on the endogenous variables $\mathbf{X}$ that must satisfy the equilibrium conditions of the model given by $H\left(\cdot\right)$.

$$\Theta^* = \text{argmin}_\Theta \left\| m\left(\mathbf{X}|\Theta\right) - \hat{m}^{\text{data}} \right\| \quad \text{s.t.} \quad H_\Theta\left(\mathbf{X}\right) = 0$$

Figure 5: Model calibrated targeting firm life cycle dynamics in the micro data



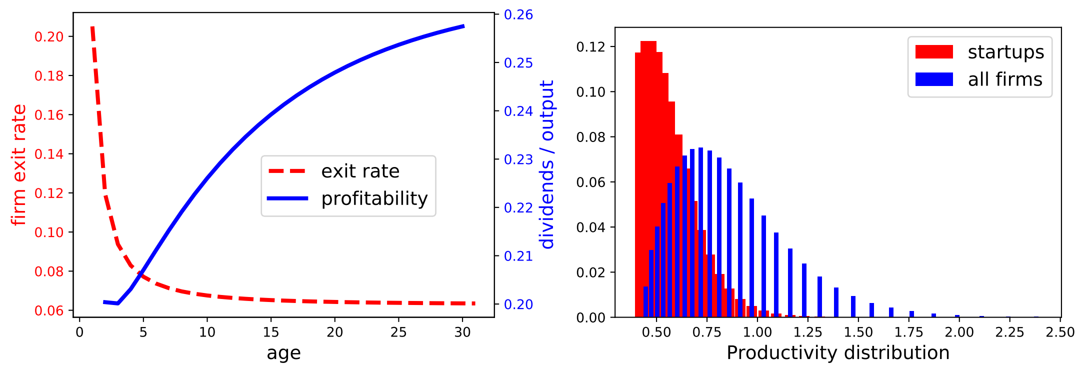
Note. This figure shows the model intentionally matches the three terms of the back-of-the-envelope decomposition formula used to retrace I/Y in the empirical section. These three objects are the main targets for the baseline calibration under the parameters in Table 3 along a 2020 BGP. The figure depicts the model's counterparts of the data's share of firms N_a economy-wide from the BDS, output by age y_a/y_0 from the BDS adjusted to value added using an adjustment factor from the Census of Manufacturing, and investment-to-output by age i_a/y_a relative to startups, extrapolating from the manufacturing data. For instance, a 30 year old firm invests roughly half of what the youngest firms invest relative to their size.

Table 3: Model Calibration

Θ	Parameter	Value	$m(\Theta)$	Targeted moment
ρ	AR(1) persistence	0.935	y_a/y_0	size by age
σ	AR(1) dispersion	0.29	N_a/N	age distribution of firms
δ_k	depreciation rate	0.096	i_a/y_a	investment/output by age
δ_f	exogenous exit rate	0.02	$1-S$	aggregate exit rate
α	capital share	0.168	I/Y	aggregate investment share
θ	returns to scale	0.72	WL/Y	aggregate labor share
κ_f	operating cost	0.013	$1-S_a$	exit rate by age
κ_x	investment cost	0.307	i_0/y_0	entrant inv. share
κ_e	entry "idea" cost	0.034	$\mathbb{E}v_e$	entrant value
κ_s	entry startup cost	0.013	y_0	entrant mean size
g_M	startup growth	4.8%	N_0/N	entry rate
δ_r	interest rate spread	0.07	r_0	0% real rate in 2010
ξ	entrant's pareto tail	20.5	$1-S_0$	entrant exit rate
g_L	population growth	1%		Externally calibrated
γ	investment cost exp	2		Externally calibrated
β	discount rate	0.99		Externally calibrated

Note. I used a Simulated Method of Moments (SMM) algorithm to jointly calibrate model parameters. See Figure 5 for more details.

Figure 6: Older firms are more profitable and exit less as they are better selected



Note. This figure shows older firms exit less and are more profitable as they are better selected relative to the youngest firms. This figure plots firm policies and the marginal production stationary distribution $\bar{\mu}(z) := \int_k \bar{\mu}(z,k)\,dk$ under the baseline BGP calibration in Table 3. Startups are firms that decide to enter into production, while all firms include firms across different ages.

little capital and and low productivity, but accumulate both capital and productivity as they age experiencing idiosyncratic shocks.

In addition to the three empirical schedules, I also target key moments of the life cycle of firms and aggregates.[36] Table 2 lists the parameters that emerge from the calibration and the moments used.

3.4 Why do Young Firms Invest More Intensely?

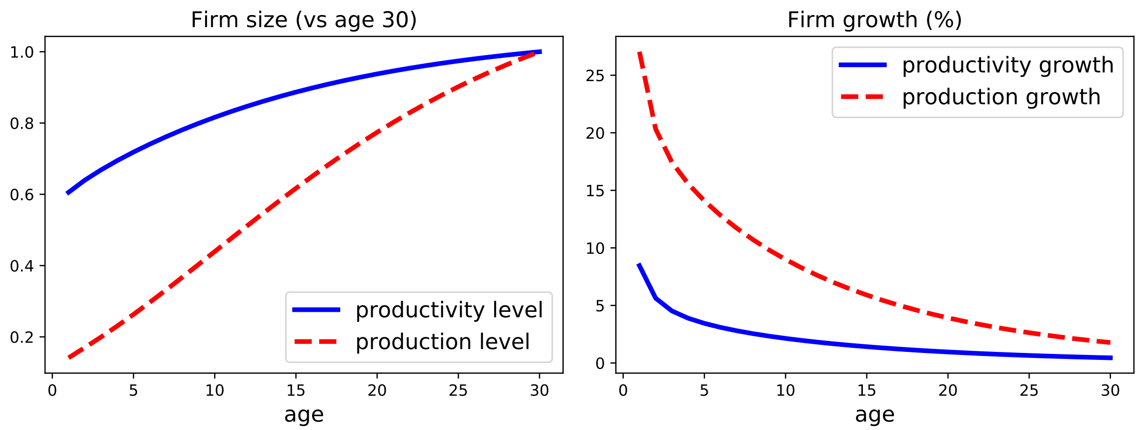
Figure 6 shows older firms are more profitable than younger firms. Since younger firms also exit more, as in Fact 4, a firm surviving to old age must have accumulated good productivity shocks. Given the larger size, productivity, and profitability of older firms, why do they invest less intensely in the model calibrated to US data?

Several economic forces lead firms to front-load investment when young and postpone profits until they are old.

(1) Productivity growth slows down with age. Figure 7 illustrates firms start unproductive and small but grow and slow down as they converge to their steady state. When firms are young, their z is low, but since firms anticipate high growth potential, they invest strongly as their expected returns are high. Older firms are larger but their z is closer to their frontier and thus they decelerate investment since

[36] I have also calibrated the spread to match a micro-elasticity of investment to match the interest rate to be $\mathbb{E}\partial_r i = 6\%$ from quasi-experimental evidence provided by Koby and Wolf (2020) (5%) and Winberry (2021) (7%), to make sure investment demand is downward sloping instead of flat.

Figure 7: Firms have higher returns on investment and grow faster when young



Note. This figure plots average productivity z level and growth as well as production $y(z,k)$ level and growth under the stationary age distribution $\bar{\mu}_a$ from the baseline BGP calibration in Table 3.

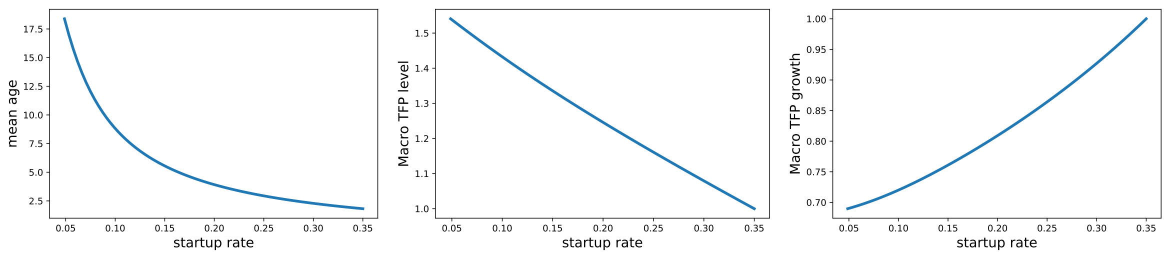
they do not expect to grow as dramatically in the future. Absent adjustment costs and endogenous exit, output would be driven only by productivity $y^*(z)$, and this would be the only force at play.

(2) Capital takes time to build. Figure 7 illustrates that firms start smaller than the optimal size their productivity would imply because capital takes time to build. Adjustment costs raise the incentives of relatively productive young firms to accelerate their growth via explicit investment and "catch-up" to their desired productivity-implied size. Older firms have reached higher valuation, z, and have had the time for their k to catch up to their productivity, but their growth is slowing and returns of capital have been exhausted.

(3) Age limits precautionary motives. Younger firms face a imminent threat of a shock forcing them to exit. This increases precautionary incentives for relatively high productivity young firms to speed up their growth via investment to escape exit. Older firms do not face this high exit threat any longer, generating lower incentives to invest.

Firm aging reallocates economic activity from young "growth" firms focused on re-investing their earnings into the businesses to fuel future growth to older "value" firms that are more established and have less room to grow and focus on reaping the benefits from their past investments and ease off their investment expenditures.

Figure 8: Lower startup rates ages firms, depressing TFP growth despite concentration



Note. This figure is purely illustrative. It shuts down adjustment costs $\phi = 0$ and plots mean age $\sum_a \left(p_f/g_M\right)^a a$ and the level and growth rate of macro TFP $\sum_a \left(p_f/g_M\right)^a z_a^k$ defined in the Appendix by varying g_M and thus the startup rate $f\left(g_M\right) = \bar{M}/\bar{N} = g_M - p_f$

3.5 A Tractable Special Case

I simplify the model to show aggregates depend on the firm age distribution through the endogenous dynamics of aggregate TFP. In the cost-less limit $\kappa_f, \kappa_i, \kappa_e, \kappa_s \to 0$ where exit can only happen exogenously and investment is flexible, the age distribution becomes exponentially decaying

$$N_t = p_f N_{t-1} + M_t = \sum_{a=0}^{\infty} N_{ta} = \sum_{a=0}^{\infty} p_f^a M_{t-a} \tag{27}$$

M_t enter exogenously at each t, starting with $k_0 = 0$. Their investment demand reflects uncertainty about their future survival and productivity

$$r_t^f + \delta_k = p_f \left(1 - \bar{\theta}\right) \mathbb{E}_{z|z_-} \partial_k y_t \tag{28}$$

Like in Figure 7, firms grow with productivity as they age and slow down as they converge to a steady state. It turns out that in this simple model, aggregates can be expressed as averages of all dynamic heterogeneity across firms weighted by the age distribution[37]. Underlying this is the fact that firm-level objects–including the path of capital $k_{t+1}\left(z_t\right)$–are linear in terms related to "macro" (e.g., w_t) and "micro" sources of dynamics heterogeneity (i.e., z_t):

$$Y_t = \int y_t\left(s\right) \mu_t\left(ds\right) = \mathbb{Y}_t \sum_a p_f^a M_{t-a} z_{at}^y, \quad K_t = \int k \mu_t\left(ds\right) = \mathbb{K}_t \sum_a p_f^a M_{t-a} z_{at-1}^k \tag{29}$$

[37]This follows from policy functions' linearity in idiosyncratic states (e.g. Gorman (1959) and Werning (2015)

Macro indices reflect technology and general equilibrium variables common to all firms:

$$\mathbb{K}_t = \left(\bar{Z}_t \left(\bar{\theta}/w_t\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}} p_f \alpha\theta\kappa_z / \left(r_t^f + \delta_k\right)\right)^{\frac{1-\bar{\theta}}{1-\theta}}$$

$$\mathbb{Y}_t = \left[\bar{Z}_t \left(\bar{\theta}/w_t\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}}\right]^{\frac{1-\bar{\theta}}{1-\theta}} \left[p_f \alpha\theta\kappa_z / \left(r_t^f + \delta_k\right)\right]^{\frac{\alpha\theta}{1-\theta}}$$

What happens when startup activity slows down? In the BGP, the mass of entrants grows with population and technology $M_t = g_M^t \bar{M}$ where $g_M = g_Z^{1/\bar{\theta}} g_L$. Lower g_M depresses the startup rate $s\left(g_M\right)$, shifting the age distribution towards older firms:

$$\int \left(g_M\right) = \bar{M}/\bar{N} = g_M - p_f, \quad N_{at}/N_t = \left(p_f/g_M\right)^a \tag{30}$$

As firms in the economy age, Figure 8 shows aggregate TFP *growth* slows down while concentration, linked to the *level* of TFP, rises. As we will see below, this decline in TFP growth that takes place when firms age due to slower startup activity will drive down investment intensity in the economy.

4 The life cycle origins of the investment puzzle

4.1 How does aggregate investment change with startup activity?

This section shows a standard neoclassical model, when enriched to incorporate firm life cycle dynamics present in US micro data, can predict the seemingly puzzling investment, interest rate, and value dynamics observed in aggregate data. Through the lens of the model driven by startup supply growth shocks we see these relations are no longer puzzling: they are just a feature of any economy where changes in startup activity affect not only the scale of the economy, but also its age distribution of firms.

I present a general result on how startup activity affects aggregate investment, which I illustrate analytically within tractable model of section 3.5 before providing simulations with the full model calibrated in the previous section.

The main surprising message of this theoretical result will be that despite investment being a *forward-looking* object at the micro level, at the macro level it is also partially *backward-looking* object due to the age distribution of firms. Furthermore, the proposition highlights that the age distribution matters for investment to the extent that there are meaningful age differences across firms: if firms were heterogeneous throughout their lives but they did not have life cycle profiles, the channel highlighted here would disappear.

Consider first the full model with a growing supply of startup ideas $M_t = g_M^t \bar{M}$ (driven by either technology g_L or population g_Z), so the mass of firms N_t also grows at rate g_M. Then, the mass of firms of age a declines exponentially with

g_M (i.e. $\bar{M}_{t-a} = \bar{M}_t g_M^{-a}$) and the accumulation of exit/survival choices, which also depend on other firms policies. An important assumption in the model is that M_t does not directly affect any firms' policies, only indirectly thorough how the price vector $x = w, r$ is affected by the distribution of firms, $\mu_t = N_t \bar{\mu}$. Let $y_a(x) = \int y(s|x)\, \bar{\mu}_a(s|x)$ be the average output by age (weighted by the state space). Similarly, $i_a(x)$ is average investment by age, and $S_a(x)$ is the survival function (the probability that firms survive to a) implied recursively by their optimal exit choices given x.

Given this notation, the following result shows that one can write the aggregate investment to GDP ratio as a function of the growth rate in startup activity given underlying age profiles. (Aggregate Investment with a Firm Age Distribution Given Prices). *Given prices $x = w, r$ for different growth rates of the mass of startups $g_M = g_L g_Z^{1/\bar{\theta}}$, firm policy functions remain unchanged and we can express aggregate investment-to-GDP as a backward-looking function of g_M via the age distribution of firms and their life cycle profiles:*

$$\overbrace{\frac{I_t}{Y_t}}^{\text{Investment/GDP}} = \sum_a \frac{\overbrace{\left(\frac{\bar{p}_f}{g_{Mt}}\right)^a S_a(x)}^{\text{firms by age } N_a} \cdot \overbrace{y_a(x)}^{\text{size by age}}}{\sum_a \left(\frac{\bar{p}_f}{g_{Mt}}\right)^a S_a(x) \cdot y_a(x)} \cdot \overbrace{\left(\frac{i_a(x)}{y_a(x)}\right)}^{\text{inv/output by age}}$$

Given x, I_t/Y_t falls when g_M falls as long as $\int i(s|x)\, \bar{\mu}_a(ds|x) \,/\, \int y(s|x)\, \bar{\mu}_a(ds|x)$ investment-to-output by age is a g_M-invariant declining function of firm age. Falling g_M ages the distribution of firms like an increase in the probability of survival $\bar{p}_f$ that does not affect firm policies.

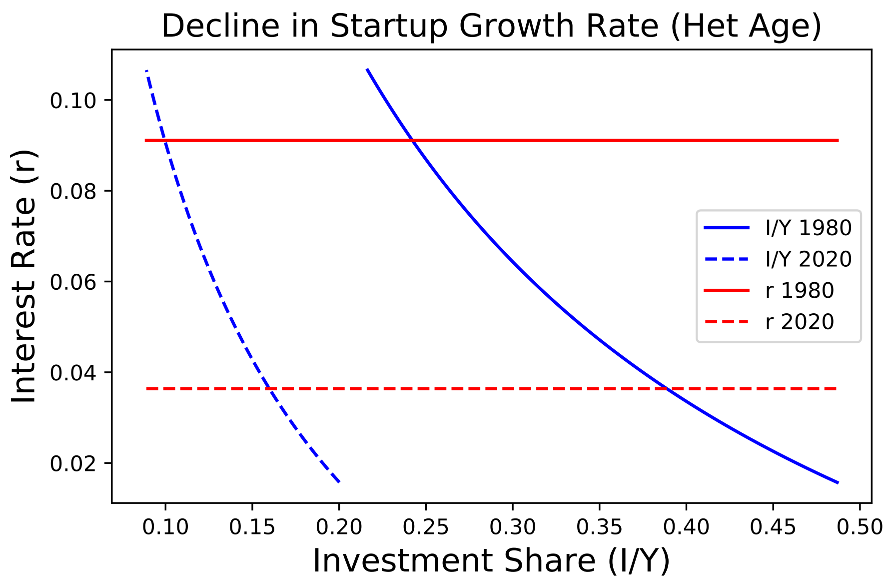
The proposition implies that if we calibrate a model to the same size-by-age y_a, how investment-to-output falls with age i_a/y_a, and the age distribution N_a at some point, then given prices, this model would behave similarly in respond to changes in startup rates driven by g_M. Underlying it is the strong assumption that changes in entry do not directly affect incumbent firms partial equilibrium problems, which would not hold in oligopolistic models where firms directly internalize competition. Given this assumption, entry changes affects the economy only through the age distribution of firms and through its impact on general equilibrium prices. In what follows, I hold these constant at first.

It is easier to see the implications of this result through the tractable model of section 3.5 where the model admits explicit aggregation despite its complex micro-structure.

Life cycle differences make the age distribution matter for I/Y:

$$\frac{I_t}{Y_t} = \left(p_f \frac{\sum_a \left(\frac{p_f}{g_{Mt}}\right)^a z_a^k}{\sum_a \left(\frac{p_f}{g_{Mt}}\right)^a z_a^{k-}} - 1 + \delta_k \right) \frac{\sum_a \left(\frac{p_f}{g_{Mt}}\right)^a z_a^{k-}}{\sum_a \left(\frac{p_f}{g_{Mt}}\right)^a z_a^{y}} \left(\frac{p_f \alpha \theta \kappa_z}{r_t - 1 + \delta_k} \right) \tag{31}$$

Figure 9: Firm aging depresses investment demand despite lower interest rates



Note. This figure is purely illustrative. It shuts down adjustment costs $\phi = 0$ and plots I/Y in (31) and the equilibrium interest rate (24) for two values of g_M corresponding to high and low startup rates: $\int (g_M)$

Falling startup activity need not matter for aggregates. Without age differences in productivity, even if firms differ in size, $z_a = z$, and if only population drives entry, $\bar{M}_t \propto \bar{L}_t$ so $g_c = g_M/g_L = 1$, interest rates and I/Y would have both remained fixed:

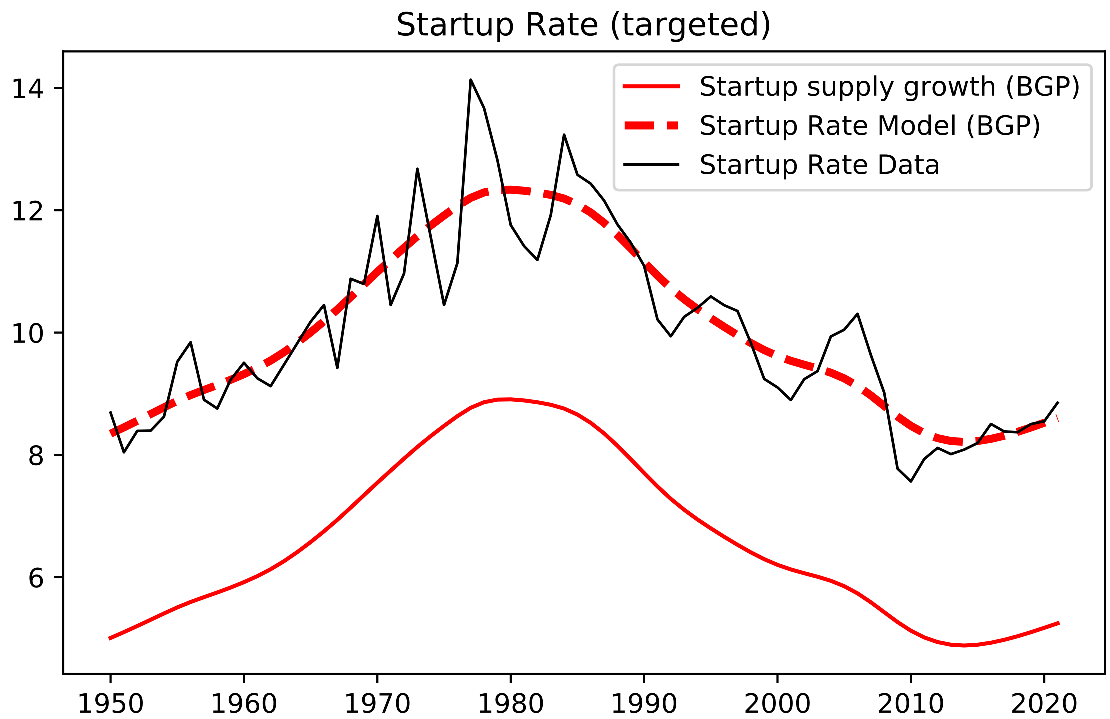
$$\frac{I}{Y} = \left(\delta_f + \delta_k\right) \frac{z^k}{z^y} \left(\frac{p_f \alpha \theta}{1/\beta + \delta_k}\right) \quad \text{if } z_a = z \; \forall a \quad \text{and} \quad g_M = g_L \tag{32}$$

Figure 9 illustrates the post 1980s slowdown in business idea growth linked to technology or population $g_M = (g_Z)^{\frac{1}{\theta}} g_L$. By 2020, the investment demand curve $\frac{I}{Y}(g_M|r)$ has fallen because lower entry, by aging firms, depresses TFP growth. Furthermore, interest rate $r(g)$ declined with entry as output per capita fell as long as $g_M \neq g_L$. Overall these combined effects of the startup rate decline can lead to a lower aggregate I/Y due to firm aging and a decline in interest rates as output per capita slows down.

4.2 Aggregate Investment given post-WWII Startup Rates

I use historical changes in the startup rate to show a neoclassical model with firm life cycle dynamics and a changing firm age distribution is enough to explain why

Figure 10: Startup rate targeted with the growth of business ideas $g_{M,t}$



Note. This figure the main counterfactual exercise performed with the model: I use the growth rate in startup activity ($g_M = g_Z^{1/\bar{\theta}} g_L$) to target the actual path of the startup rate since 1950, showing that the model perfectly hits this path by varying g_M only. The source of entry rate data is the BDS (September 2023) after 1978, historical records from SCB and CBP prior to 1978 as described in section 2.1.

the US had an investment boom after WWII and a decline after 1980.

Figure 10 depicts my first quantitative exercise in which I hold constant all parameters in the model except for one: the growth rate of startup supply flows regardless of whether it is driven by technology or labor supply (since $g_M = g_Z^{1/\bar{\theta}} g_L$). In the model, I infer the growth path required to match the data on the startup rate dating back to 1950, $\int_t (\hat{g}_{Mt}) \approx \int_t^{\text{data}}$. In historical records dating back to the 1960s, we see there was a rising influx of new startups after WWII—including Walmart (1950), Dunkin Donuts (1950), and Burger King (1953)—and this "boom" in the startup rate continued for three decades. Some of the largest companies still around today—including Starbucks (1971), Microsoft (1975), Apple (1976), and Costco (1983) —were founded at the peak of startup activity around 1980. The model economy generates a 40% decline in the startup rate since the 1980*s* as a result of a slightly larger decline in the growth rate in the number of new business ideas.[38]

Figure 11 depicts the aggregate investment-to-GDP dynamics generated by the inner equilibrium relationship in the model economy experiencing movements in the startup rate from Figure 10. This can be seen as an "out of sample" exercise, since the parameters of the model are all calibrated using data mostly coming from after 2000.

Figure 11 shows that in the model, consistent with aggregate data, the surge in startup activity after WWII translated into an aggregate investment boom as the average firm became younger. It also shows the post-1980 fall in investment-to-GDP that followed the steady decline in the birth rate of new companies, which shifted the composition of firms towards older firms that invest less intensely as in the model despite these being larger, better selected, and more profitable on average.

Finally, the model predicts that there could be an investment rebound following the Great Recession due to a recent resurgence of firm entry that accelerated during the COVID-19 pandemic. This matches the result found in the empirical decomposition.

The previous analyses showed what happened to the economy across balanced growth paths as we varied the rate of growth of startup flows independently across each point in time. This produced an effect size of the age composition effect of around 2.2% of GDP, potentially explaining over 80% of the 2.6% effect seen in the aggregate data trend in BEA.[39] The Appendix shows robustness to general equilibrium transitional dynamics under perfect foresight.

To summarize, I find that, depending on the method used, the aging effects are likely between 1.3% and 2.2% of GDP ($50\% - 85\%$) of the aggregate boom and bust in physical investment to GDP. The remaining ($15 - 50\%$) is likely explained by other forces discussed in the literature, such as weakening of anti-trust institutions or the shift in production functions towards intangibles, but quantifying this is left for future work.

The fact that the model shows no break in the series while perfect competition and neoclassical technology are held constant highlight no "direct" micro changes to the firms' problems are needed to generate these boom and bust aggregate investment dynamics. However, the remaining 50% to 15% of the effect might still be explained by some of the hypotheses in the literature.[40]

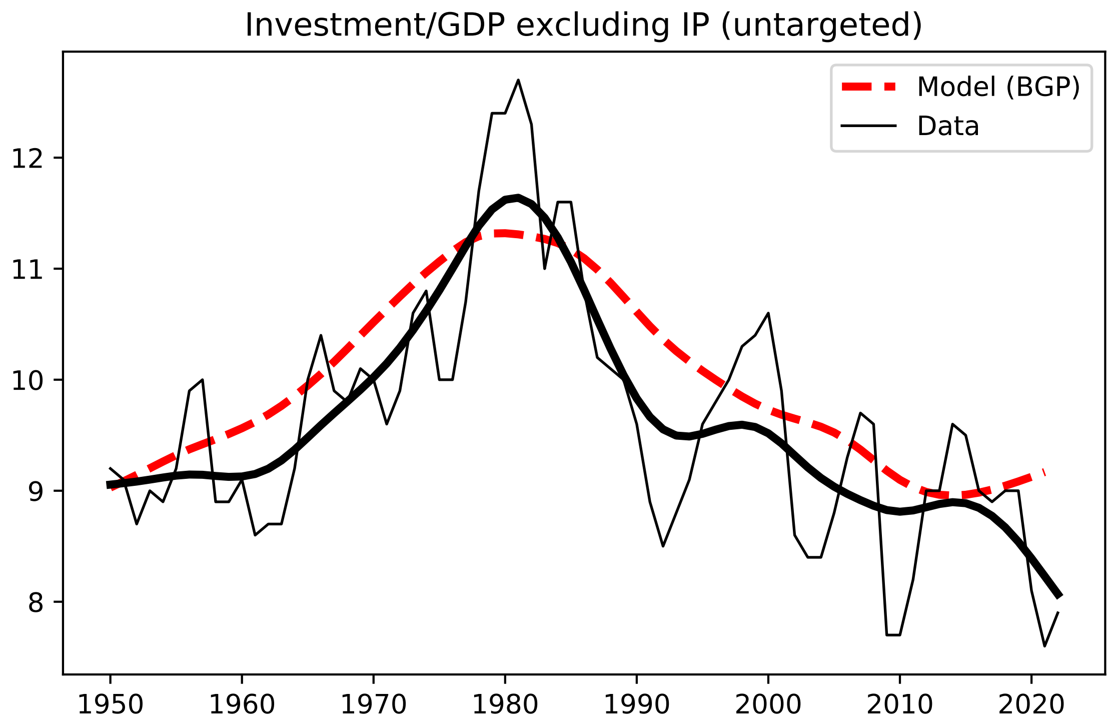
---

[38]As shown in equation (**??**) the full model, it is no longer the case that startup rates are purely driven by the exogenous supply of startups in the exogenous exit model, $\int (g_M) = g_M - \bar{p}_f$, because a slowdown in g_M lowers *aggregate* exit through indirect channels (i.e. through shifting the age distribution towards the old that exit less and by lowering $r(g)$ which increases survival).

[39]Note this is lower than the 125% found in the empirical section for two reasons: first, the entry series fed through the model does not adjust for structural transformation as it was done in the data; second, entry is the only driver of aging in the model, while in the data, increasing survival of younger firms further ages firms.

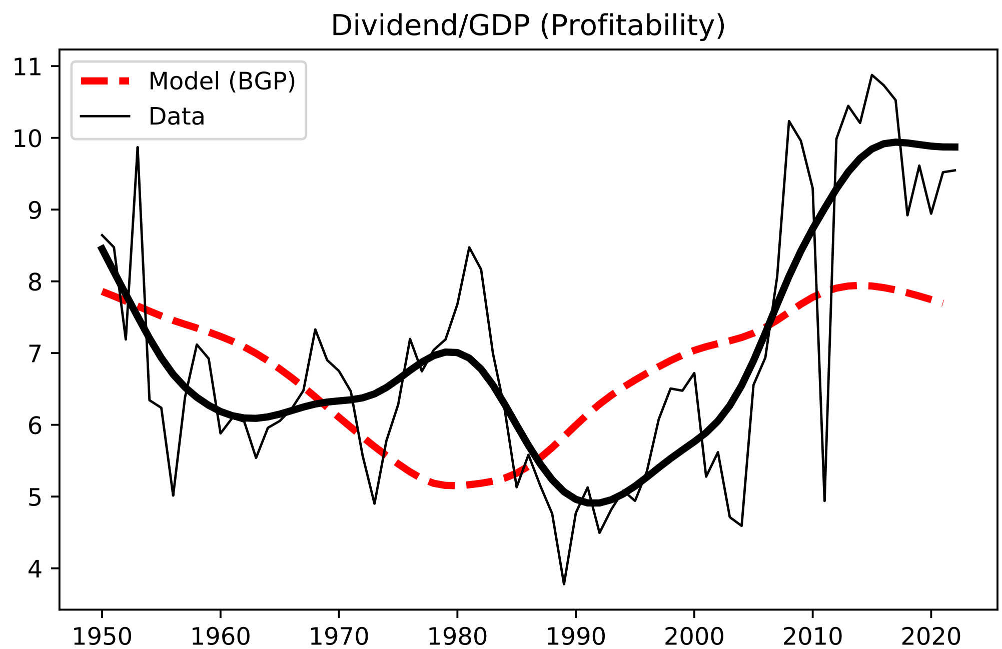
[40]For example, the deterioration in anti-trust institutions highlighted in Gutiérrez *et al.* (2021); Gutiérrez and Philippon (2019, 2017) or changes in production technologies towards intangible capital highlighted in Crouzet and Eberly (2018). In Appendix Figure 37 I show that aging of firms may

Figure 11: Investment to GDP driven by Startup Rates via Firm Age Distribution



Note. Illustrating the main result of this paper, this figure shows changes in firm entry matter for aggregate investment presenting a counterfactual exercise performed with the model across balanced growth paths. I calibrate startup ideas growth g_{Mt} to target the actual startup rate time series (Figure 10), and solve non-linearly for the resulting BGP. This figure plots the resulting (untargeted) path of I_t/Y_t implied by the model where the entry rate and the age distribution of firms change over time. Note here I do not let the interest rate adjust setting $g_M = g_L$, and the combination of free entry and market clearing hold the wage $w_t = w$ constant as described in Hopenhayn *et al.* (2022) and others in the firm life cycle literature. The source of entry rate data is the BDS (September 2023) after 1978, historical records from SCB and CBP prior to 1978 as described in section 2.1. Source: BDS (September 2023) after 1978, historical records from SCB and CBP prior to 1978. See section 2.1 for details.

Figure 12: Aggregate profitability: Dividends/GDP



Note. This figure qualitatively shows BGP changes in firm entry affect aggregate profitability: when entry falls firms get older, investment falls as in Figure 11, and profits rise. Corporate profits after taxes comes from the BEA as in Figure 1.

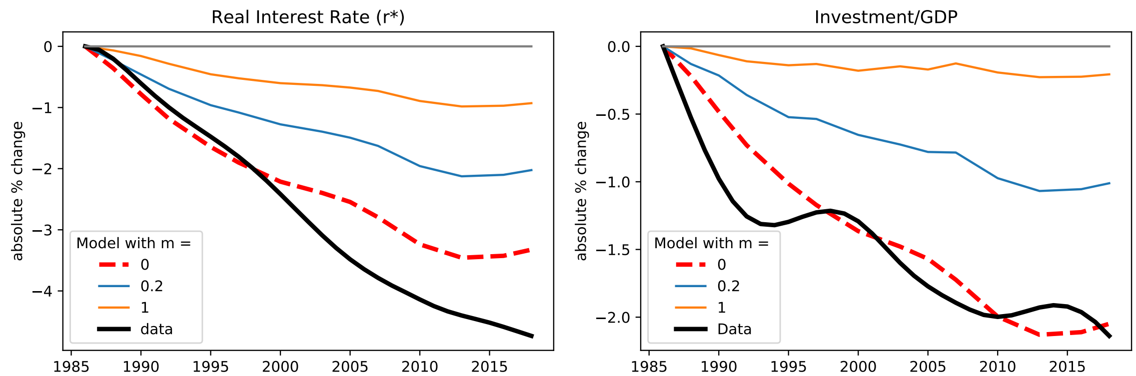
4.3 Low Investment Despite High Profitability and Low Rates

The literature has been puzzled on why investment has not increased given rising corporate profitability and lower interest rates: If firms have higher earnings why are they not investing more? And how do changes in startup activity and the swings in investment demand relate to the equilibrium interest rates given the supply and demand logic in Figure 9?

Life cycle origins of the profitability puzzle. The perfectly competitive neoclassical life cycle model provides a simple explanation: falling startup rates make the average firm older, reallocating economic activity towards firms that are more profitable due to selection (Figure 6) but have less growth potential so invest less intensely as in

have been again one of the forces keeping total investment stagnant despite the rise of intangibles. In work in preparation, Aragoneses (2023b), I use US Census data to find that the changing composition of startups (not aging) has been an important driver in the increase of intangible capital: relative to startups in the past, recent cohorts of firms are significantly more likely to invest in intangible capital. This suggests that any intangible-related technology that started to become available after 1980 may be affecting the economy through the startup stage. But this is left for future work.

Figure 13: Interest rate dynamics consistent with startup rate trend



Note. This figure shows the equilibrium response of (household) interest rates $r_t(g)$ and $I_t/Y_t(g)$ for different levels of sensitivity of firms' discount rates to the household interest rate parametrized by m in $r_t^f = (1-m)\bar{r} + mr_t$. Since the 1986, the model is able to generate 70% of the 4.7% absolute decline of interest rates in the data (note investment fell an extra 0.5% in the data relative to the model before 1986). The version of the model that is best able to match the data is one with $m = 0$, where firms discount rate is insensitive to interest rates as argued by Gormsen and Huber (2023). With partial sensitivity as in Gabaix (2020) with $m = 0.2$, the investment and rates falls to 50% and 40% relative to the data. In the extreme case where $m = 1$, as in Khan and Thomas (2008), small (1%) declines in interest rates are able to almost completely neutralize the negative effects on investment.

Fact 1. In the model, this generates a natural inverse relation between investment expenditures and the share of dividends in GDP:

$$\uparrow \frac{\Pi_t}{Y_t} = 1 - (1-\alpha)\theta - \downarrow \frac{I_t}{Y_t} - \frac{X_t}{Y_t}$$

Figure 12 shows the life cycle model can reproduce qualitatively the seemingly puzzling inverse relation between profitability and investment on aggregate. The source of the puzzle is that the higher returns are earned by different firms (the old) than the ones carrying out the investing (the young).

Life cycle origins of the interest rate puzzle. Figure 13 shows the equilibrium response of the interest rates facing households $r_t(g_t) = g_t/\beta - 1$ and the investment share of GDP $I_t/Y_t(g_t, r_t(g_t))$ among firms where g_t is calibrated to match the path of the firm startup rate. I plot responses across different levels of sensitivity of firms' discount rates to the household interest rate parametrized by m in $r_t^f = (1-m)\bar{r} + mr_t$. The version of the model that is best able to match the data is one with $m = 0$, where firms discount rate is insensitive to interest rates as argued by Gormsen and Huber (2023) so that $I_t/Y_t(g_t)$ only depends on g_t through the firm age distribution. Under this version, since 1986, the model is able to generate 70% of the 4.7% absolute decline of interest rates in the data and a decline in investment of 2% (note investment fell an extra 0.5% in the data relative to the model before 1986 which the model cannot generate). With partial sensitivity, for example due to inattention as in Gabaix (2020) with $m = 0.2$, the investment and rates falls to 50% and 40% relative to the data. In the extreme case where $m = 1$, as in Khan and Thomas (2008), small (1%) declines in interest rates are able to almost completely neutralize the negative effects on investment.

As an alternative exercise, I allow for the exogenous component of real rates $\bar{r}_t$ to adjust $r_t^f = mr_t\left(g_M/g_L\right) + (1-m)\,\bar{r}_t$ to adjust to match the *actual* path of *investment/GDP* in the data; $(1-m)\,\bar{r}_t$ can be thought of as a spread faced by firms.[41] Since $r_t\left(g_t\right) = g_t/\beta - 1$, this requires that population growth is not the only driver of the startup deficit. I infer the g_{Mt} and $\bar{r}_t$ paths that generates the *actual* path of *startup rates* in the model $\int\left(g_M, r\left(g\right)\right)$ as well as the actual path of investment-to-GDP. Figure 38 shows the implied path of household interest rate r_t as well as the one faced by firms r_t^f in the model forced to match jointly the startup and investment-to-GDP dynamics resembles the path of real interest rates in US data. Through the lens of the neoclassical model, the post 1980 decline in interest rates is not so puzzling given that the fall in the startup rate has been depressing investment demand and increasing supply of savings. I find the model requires movements in $\bar{r}_t$ that offset interest rate movements since the 1950s (shown in the Appendix). Despite r_t changing, firm discount rates r_t^f do not change much consistent with Gormsen and Huber (2023).

In conclusion, we should expect times when the average firm age is rising to exhibit higher dividends/GDP and low interest rates instead of being puzzled about why these two forces do not incentivize investment and startup activity. Lower interest rates may be a symptom that startup activity depressing investment and economic activity while high valuations may be a symptom that the older average firm is receiving rewards for *past* investments.

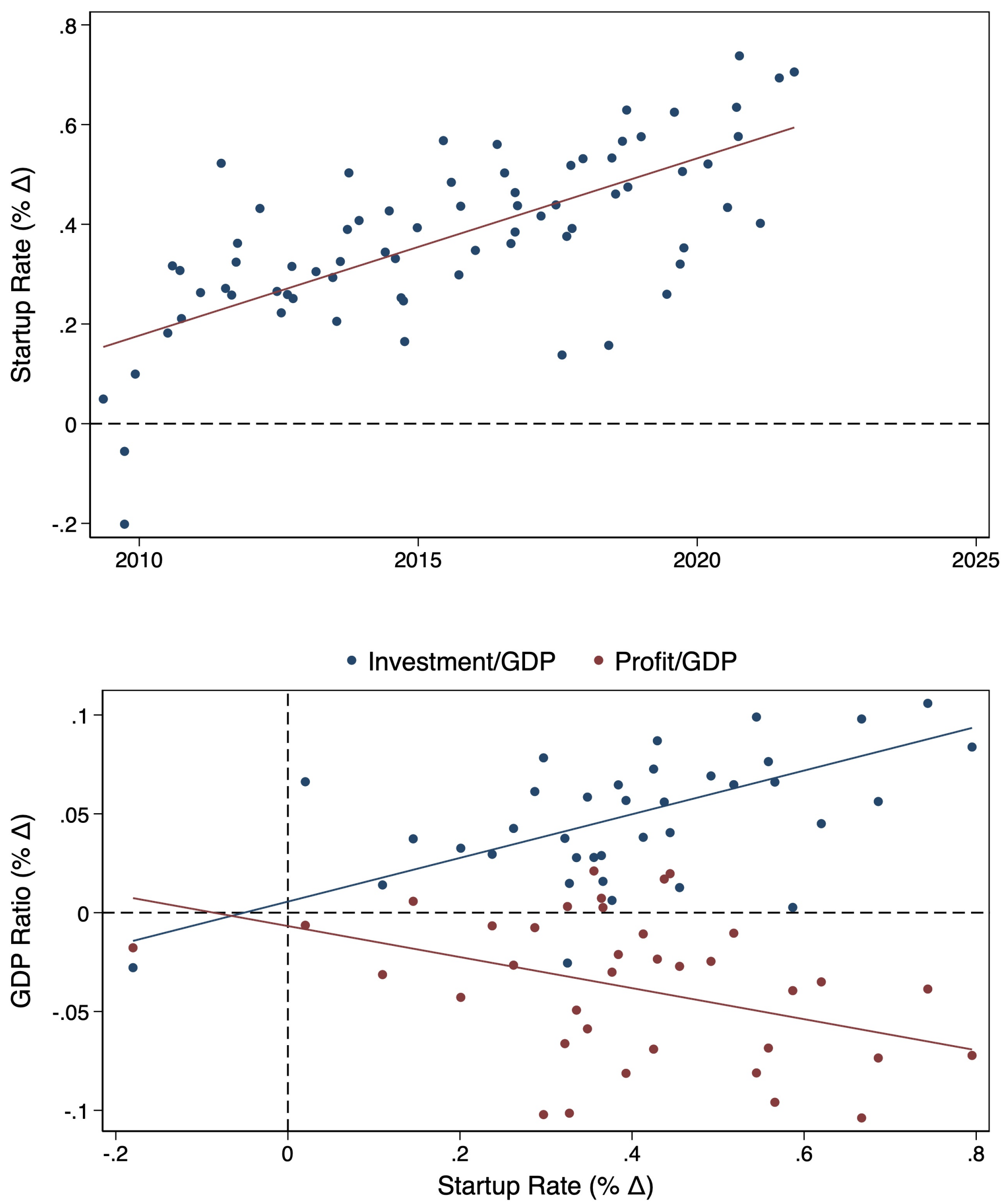
4.4 The Resurgence in Startup Activity across Countries

The firm life cycle channel discussed in this paper predicts that booming startup activity from 1950 to 1980 increased US aggregate investment and lowered profits as the average firm turned younger. I now show these relationships arise across countries.

Early signs of a startup resurgence across countries. Figure 14 shows that the recent resurgence in startup activity highlighted throughout this paper after the Great Recession, a finding discussed in Haltiwanger and Decker (2023) and Guzman

[41]This is inspired by Gormsen and Huber (2023) and the mounting evidence that the interest rate faced by households is not the same one faced by firms and that firms' investment are overly sensitive to interest rates in the neoclassical model.

Figure 14: Startup surge linked to higher investment and lower profits across countries



$$\Delta_{2010}^{T} \log \left[\frac{\text{Investment}_{ct}}{\text{GDP}_{ct}}, \frac{\text{Profit}_{ct}}{\text{GDP}_{ct}} \right] = \alpha_c + \beta_X \Delta_{2010}^{T} \log \left(\text{Startup Rate}_{ct} \right) + \varepsilon_{ct}$$

Note. OECD Data on gross physical capital formation and operating surplus relative to aggregate value added. Startup rate across countries comes from the Total Early-Stage Entrepreneurial Activity (TEA) indicator from the GEM. The combined dataset features 33 countries and has been weighted by GDP.

and Stern (2020), is a common pattern across countries. Figure 14 plots a positive trend in the index of Total early-stage Entrepreneurial Activity (TEA) Rate, defined as the percentage of 18-64 population who are either a nascent entrepreneur or owner-manager of a new business from the Global Entrepreneurship Monitor, which has continued to rise even through and after the COVID-19 pandemic.

Concurrently with this rising startup trend across countries, countries where startup activity increased also saw an increase in investment to GDP measured from gross capital formation data from the OECD and a decline in aggregate profitability measured as the share of operating surplus relative to aggregate value added in the OECD. Although this is not shown here, there has been early signs that interest rates have been on the rise bringing different economies out of the zero lower bound after the Great Recession. It is still early to tell, but my work suggests that a resurgence in rise in firm entry observed after the Great Recession, which especially accelerated during the COVID pandemic, could be one of the secular forces driving the economy away from the zero lower bound.

5 Conclusion

The post-1980 US investment slowdown *was* puzzling because interest rates fell and corporate profitability increased (which should have stimulated investment in standard models). I argue falling startup rates go a long way in explaining the investment puzzle. Lower firm entry (1) depressed aggregate investment while rising valuations by aging firms and (2) depressed interest rates by lowering output per capita growth. This mechanism arises in a life cycle model where older firms have been selected by market forces and are more profitable but younger firms invest more intensely consistent with restricted-access US Census micro data. In this framework, firms front-load investment in anticipation of back-loaded profits.

I study what happens when the model experiences shocks to the supply of business ideas that generate the actual boom and bust in the startup rate observed in the US since the 1950s. I find the model can rationalize the historical relation between tangible investment, interest rates, and profitability. It shows aggregate investment could have fallen after 1980 even without changes in anti-trust institutions or production technology that the literature argues *directly* depressed firm investment incentives.

Indeed, the data presented in this paper shows little signs of an investment decline at the micro level nor that firms grow any slower over the life cycle. In fact, younger firms today seem to invest more intensely and survive at higher rates than firms in the past. In manufacturing, where the most detailed investment data we have comes from, there has been a 10% investment *increase* for the average firm despite a 14.5% decline in aggregate investment.

This missing increase in investment is likely not just happening in manufacturing for physical capital. In all sectors, one measure correlated with investment, survival rates, has increased conditional on age: while 30% of newborn startups exited after a year in 1980s, only around 20% exit in the 2010s. I also present evidence across all sectors based on citation-weighted patents per employee. This suggests that just like for physical capital, intangible investment intensity, which also systematically decline with firm age, has also experienced an increase conditional on age. Thus, across different forms of investment, firm aging seems to have overpowered a micro investment boom, hiding it from the aggregate statistics and creating a puzzling disconnect between firms and the macroeconomy. This highlights that changes in the firm age distribution driven by shifts in the startup rate can make micro and macro trends diverge and go a long way in explaining the dynamics of aggregate investment since WWII.

The results of this paper do not imply institutional or technological changes did not take place, but rather that they could indirectly have depressed aggregate investment through the startup stage, making understanding the startup deficit all the more important. The literature has pointed to slowing population growth, business ideas getting harder to find, or rising entry costs as some of the drivers consistent with my model's startup supply shock, but more work here remains to be done.

There is reason for optimism about the years to come. Recent US Census data and data across OECD countries show the startup decline may have ended, with startup rates recovering since the Great Recession and even surging throughout the recent pandemic. Given the population growth continues to slow it is likely that technical change has accelerated in recent years (e.g., the discovery of remote-work technologies allowing startups to hire talent globally). I find signs that firm aging trends have started to reverse. Although it may take time to undo three decades of aging, the rejuvenation of firms is already delivering an increase in investment and a decline in profits across countries, consistent with the model presented in this paper.

This channel presents an opportunity for policymakers such as Mario Draghi (2024) interested in rising aggregate investment in the European Union. Rather than trying to incentivize established firms to invest more, policy could be better targeted towards boosting startup activity to take advantage of the fact that young firms invest more intensely in search for future profits. Exploring these normative implications for the cross section of countries is left for future work.

References

Aghion, P., Bergeaud, A., Boppart, T., Klenow, P. J. and Li, H. (2019). *A theory of falling growth and rising rents*. Tech. rep., National Bureau of Economic Research.

Alon, T., Berger, D., Dent, R. and Pugsley, B. (2018). Older and slower: The startup deficit's lasting effects on aggregate productivity growth. *Journal of Monetary Economics*, **93**, 68–85.

Aragoneses, M. (2023a). The firm age anatomy of the us economy in the post war era.

— (2023b). Us startup creation and immigration policy.

Auclert, A., Bardóczy, B., Rognlie, M. and Straub, L. (2021a). Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, **89** (5), 2375–2408.

—, Malmberg, H., Martenet, F. and Rognlie, M. (2021b). *Demographics, wealth, and global imbalances in the twenty-first century*. Tech. rep., National Bureau of Economic Research.

Autor, D., Dorn, D., Katz, L. F., Patterson, C. and Reenen, J. V. (2017). Concentrating on the fall of the labor share. *American Economic Review*, **107** (5), 180–185.

—, —, —, — and Van Reenen, J. (2020). The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics*, **135** (2), 645–709.

Bilal, A., Engbom, N., Mongey, S. and Violante, G. L. (2021). *Labor market dynamics when ideas are harder to find*. Tech. rep., National Bureau of Economic Research.

Bloom, N., Jones, C. I., Van Reenen, J. and Webb, M. (2020). Are ideas getting harder to find? *American Economic Review*, **110** (4), 1104–1144.

Clementi, G. L., Khan, A., Palazzo, B. and Thomas, J. K. (2014). Entry, exit and the shape of aggregate fluctuations in a general equilibrium model with capital heterogeneity. *Unpublished Working Paper*.

— and Palazzo, B. (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics*, **8** (3), 1–41.

Crouzet, N. and Eberly, J. (2018). Intangibles, investment, and efficiency. In *AEA Papers and Proceedings*, American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203, vol. 108, pp. 426–431.

Crum, W. L. (1953). The age structure of the corporate system.

Gabaix, X. (2020). A behavioral new keynesian model. *American Economic Review*, **110** (8), 2271–2327.

Gorman, W. M. (1959). Separable utility and aggregation. *Econometrica: Journal of the Econometric Society*, pp. 469–481.

Gormsen, N. J. and Huber, K. (2023). *Corporate discount rates*. Tech. rep., National Bureau of Economic Research.

Gutiérrez, G., Jones, C. and Philippon, T. (2021). Entry costs and aggregate dynamics. *Journal of Monetary Economics*, **124**, S77–S91.

— and Philippon, T. (2017). *Declining Competition and Investment in the US*. Tech. rep., National Bureau of Economic Research.

— and Philippon, T. (2019). *The failure of free entry*. Tech. rep., National Bureau of Economic Research.

Guzman, J. and Stern, S. (2020). The state of american entrepreneurship: New estimates of the quantity and quality of entrepreneurship for 32 us states, 1988–2014. *American Economic Journal: Economic Policy*, **12** (4), 212–243.

Haltiwanger, J. and Decker, R. A. (2023). Surging business formation in the pandemic: Causes and consequences?

Hayashi, F. (1982). Tobin's marginal q and average q: A neoclassical interpretation. *Econometrica: Journal of the Econometric Society*, pp. 213–224.

Hopenhayn, H., Neira, J. and Singhania, R. (2022). From population growth to firm demographics: Implications for concentration, entrepreneurship and the labor share. *Econometrica*, **90** (4), 1879–1914.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, pp. 1127–1150.

Hubmer, J. and Restrepo, P. (2021). *Not a typical firm: The joint dynamics of firms, labor shares, and capital–labor substitution*. Tech. rep., National Bureau of Economic Research.

Jaimovich, N., Terry, S. J. and Vincent, N. (2023). *The Empirical Distribution of Firm Dynamics and Its Macro Implications*. Tech. rep., National Bureau of Economic Research.

Karahan, F., Pugsley, B. and Şahin, A. (2019). *Demographic origins of the startup deficit*. Tech. rep., National Bureau of Economic Research.

Kehrig, M. and Vincent, N. (2021). The micro-level anatomy of the labor share decline. *The Quarterly Journal of Economics*, **136** (2), 1031–1087.

Khan, A. and Thomas, J. K. (2008). Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. *Econometrica*, **76** (2), 395–436.

Koby, Y. and Wolf, C. (2020). Aggregation in heterogeneous-firm models: Theory and measurement. *Manuscript, July*.

Liu, E., Mian, A. and Sufi, A. (2022). Low interest rates, market power, and productivity growth. *Econometrica*, **90** (1), 193–221.

Ottonello, P. and Winberry, T. (2020). Financial heterogeneity and the investment channel of monetary policy. *Econometrica*, **88** (6), 2473–2502.

— and — (2023). Investment, innovation, and financial frictions.

Peters, M. and Walsh, C. (2022). Population growth and firm-product dynamics. *NBER Working Paper*, **29424**.

Pugsley, B. W. and Şahin, A. (2019). Grown-up business cycles. *The Review of Financial Studies*, **32** (3), 1102–1147.

Rogoff, K. S., Rossi, B. and Schmelzing, P. (2022). *Long-run trends in long-maturity real rates 1311-2021*. Tech. rep., National Bureau of Economic Research.

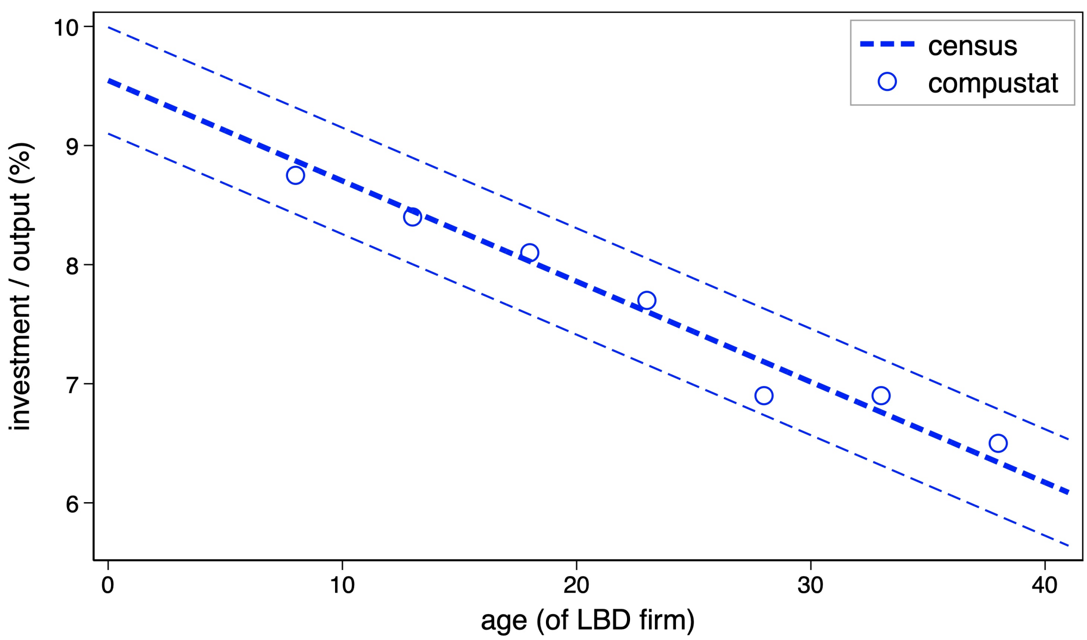
Sterk, V., Sedláček, P. and Pugsley, B. (2021). The nature of firm growth. *American Economic Review*, **111** (2), 547–579.

Werning, I. (2015). *Incomplete markets and aggregate demand*. Tech. rep., National Bureau of Economic Research.

Winberry, T. (2021). Lumpy investment, business cycles, and stimulus policy. *American Economic Review*, **111** (1), 364–396.

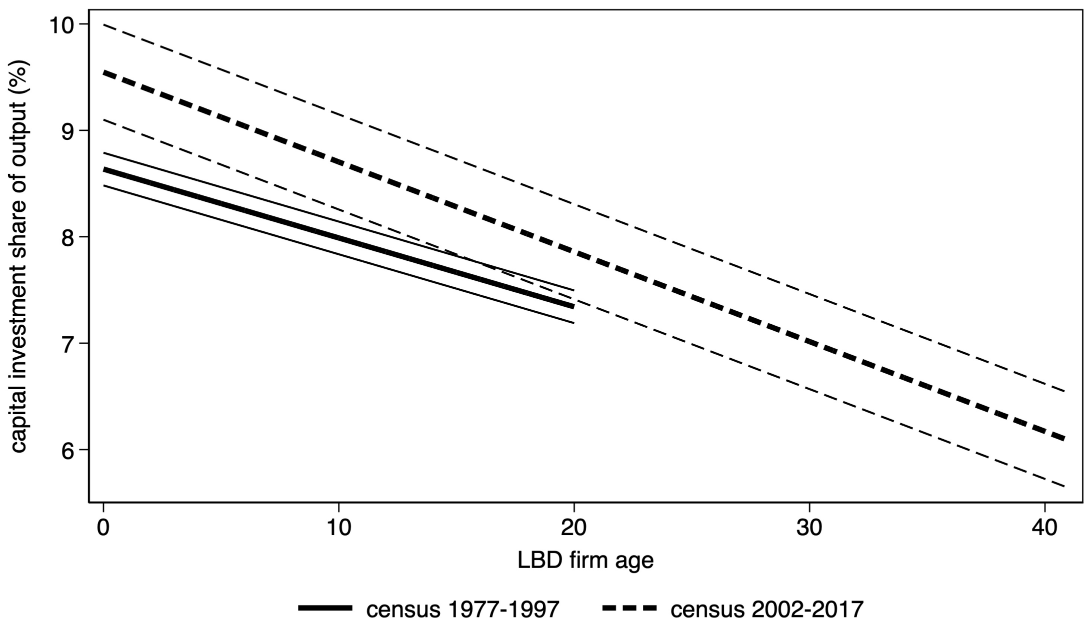
Zwick, E. and Mahon, J. (2017). Tax policy and heterogeneous investment behavior. *American Economic Review*, **107** (1), 217–248.

Figure 15: Capital investment intensity falls with firm age



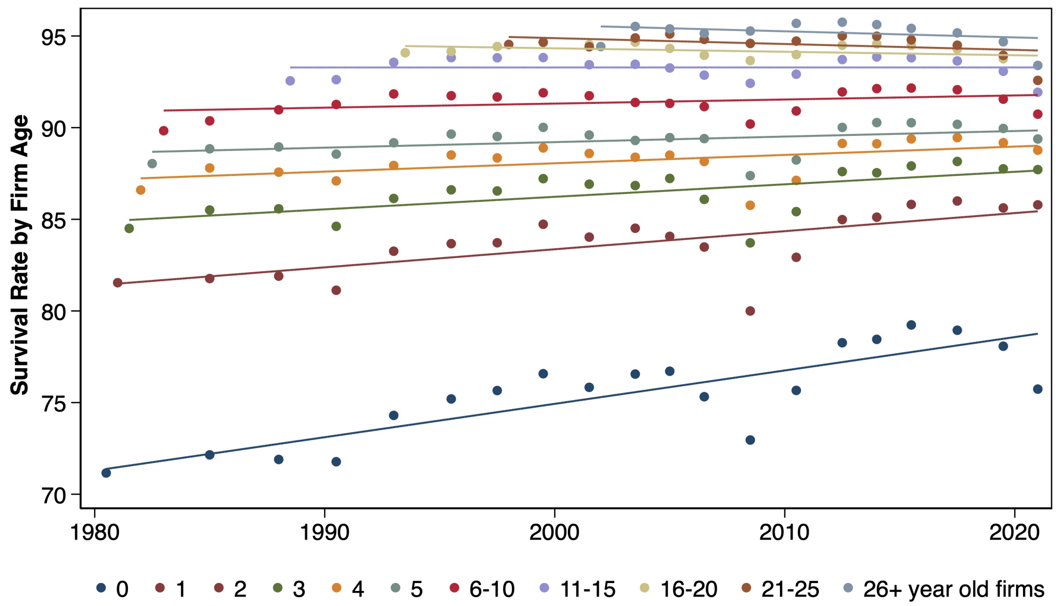
This figure uses only 2002-2017 data to address censoring; it shows investment/output falls with age. The data used is from the Census of Manufacturers, $\approx$ 2.7 million firm-year observations pooling 1977-2017.Variables: total investment expenditures in physical capital (structures, equipment) relative to value added (the micro counterparts of investment/GDP) by firm age (measured by definition in LBD used in BDS). Weights: share of the firm in total value added of its age group per equation (1). It also plots data from Compustat North America on CAPX/SALE for firms in all sectors to show this declining relation between investment intensity and firm age holds outside of manufacturing. See Section 2.1 for data details.

Figure 16: Capital investment intensity increased conditional on age



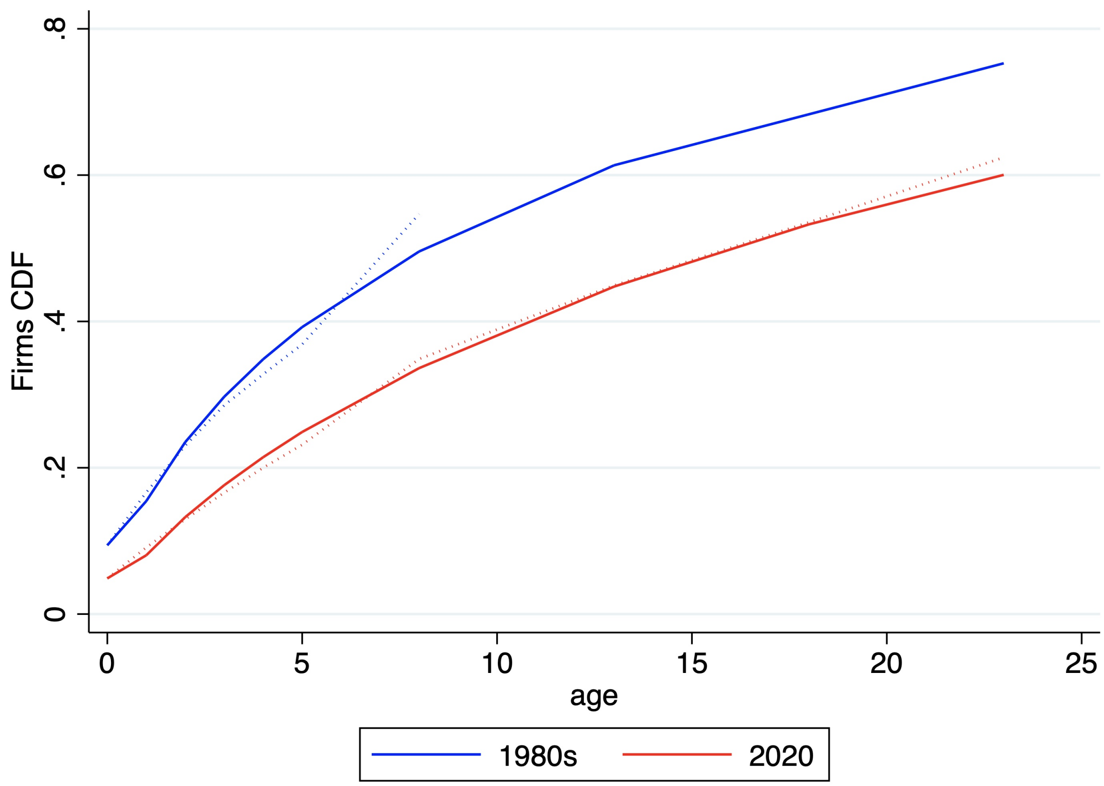
This figure splits the sample into pre-post 2000, showing that investment to output increased over time, while strongly declines with age. The data used is from the Census of Manufacturers, ≈ 2.7 million firm-year observations pooling 1977-2017. Variables: total investment expenditures in physical capital (structures, equipment) relative to value added (the micro counterparts of investment/GDP) by firm age (measured by definition in LBD used in BDS). Weights: share of the firm in total value added of its age group per equation (1). See Section 2.1 for data details.

Figure 17: Younger firms today survive at higher rates than in the past



Note. This figure shows survival rates (1 - exit rates) among younger firms have been increasing, exacerbating the aging of firms caused by declining entry. The figure uses Census BDS data on employment by age across all sectors, but the Data Appendix shows this same pattern holds in the Manufacturing Census 1977-2017 for output as well as employment. See Section 2.1 for data details.

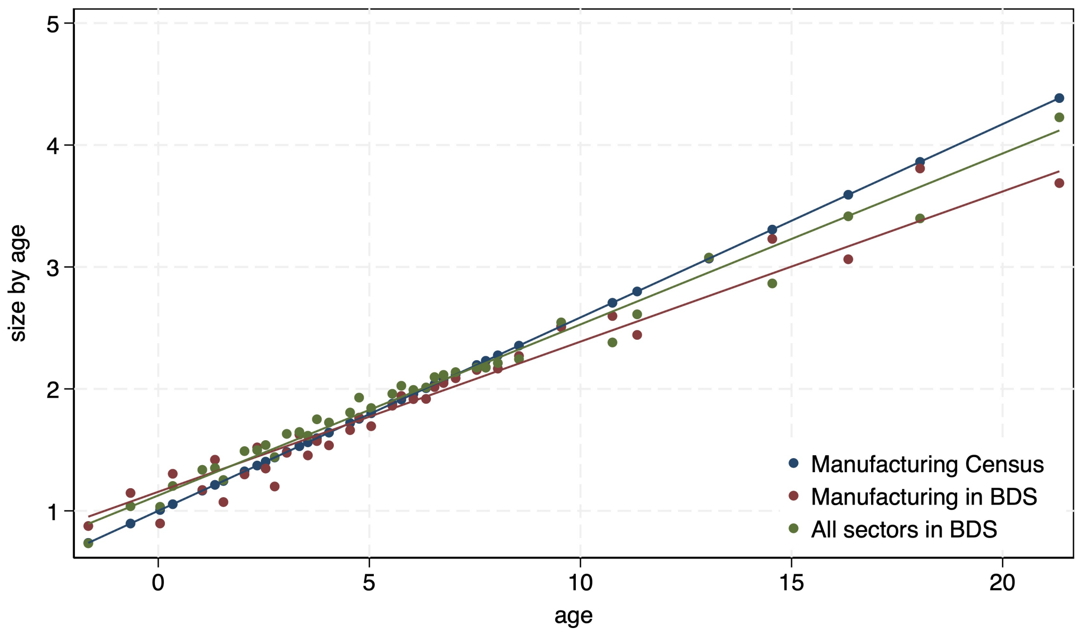
Figure 18: CDF changes from 1980 to 2020



Estimate the firm age distribution and firm aging $\Delta N_a/N$ as changes in p, q from the following equation that predicts CDFs

$$LHS = \log\left(-\log\left(1 - CDF_{st}\left(a\right)\right)\right) = -\log \hat{p}_{st} + \hat{q}_{st} \log\left(a\right)$$

Figure 19: Manufacturing is fairly similar to the average sector in terms of its scaling of employment by age.



In both manufacturing and economy-wide, firms grow by $4x$ their workforce by the time they pass 20 years old. Within each sector/year, calculate the slope of employment by age in BDS based on the regression where

$$\frac{l_{ast}}{l_{0st}} = \gamma_s + \gamma_t + \beta_{age}^{emp} a$$

Figure 20: The growth over the life cycle for manufacturing and the aggregate is similar.

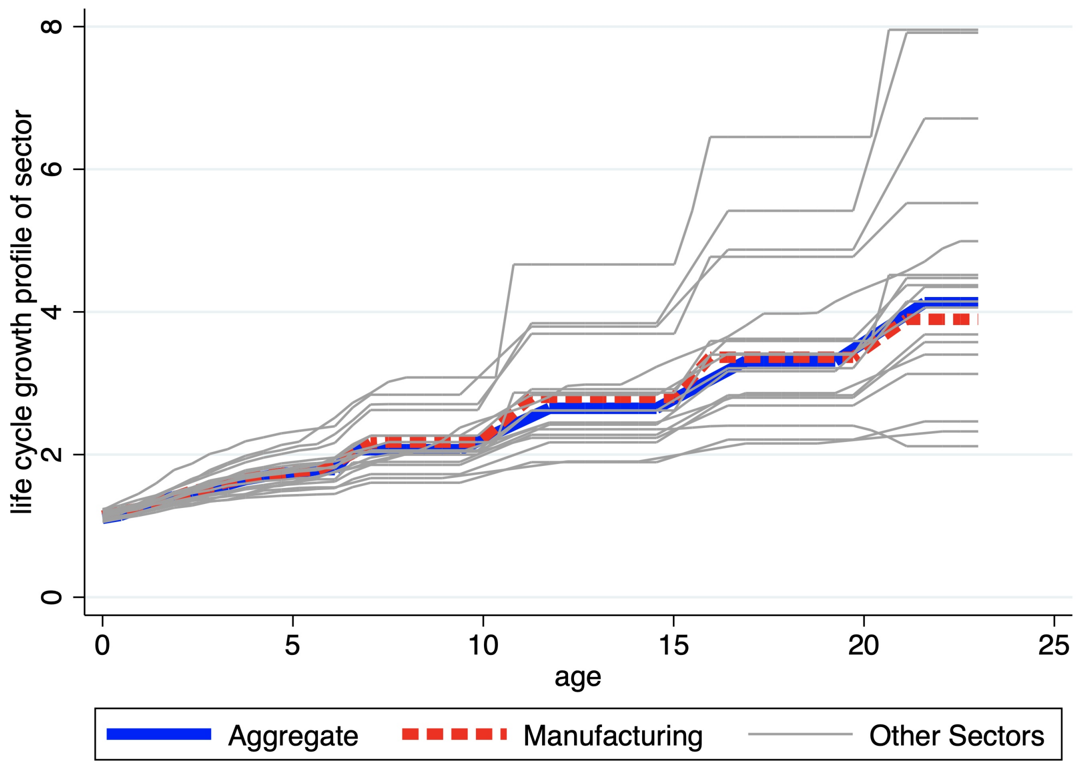
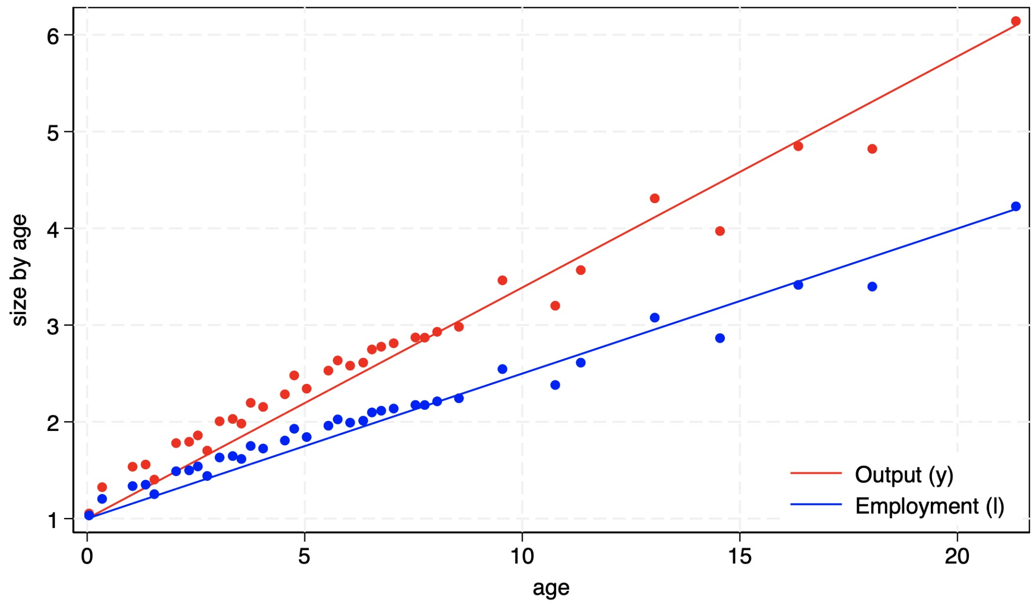


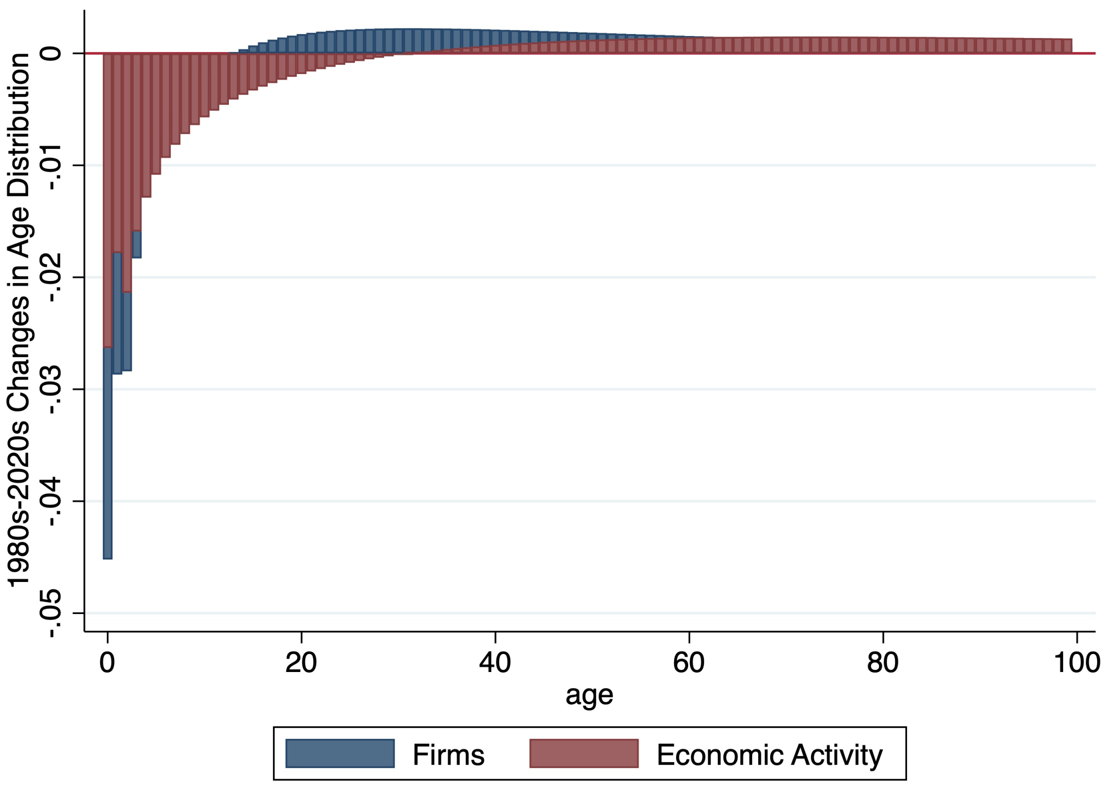
Figure 21: Converting employment to output (value added)



Note that for the formula, what matters is how output increases with age. In many models, employment and output move one for one, but in the data, output is more concentrated than employment. To adjust between the two, I estimated the slope of output by age for output and employment by age inside Census–without constant–so that one can adjust between the two using the following factor. Output by age is one of my main calibration targets.

$$\lambda_{y/l} = \frac{\beta_{age}^{out}}{\beta_{age}^{emp}} \approx 1.6 \implies \frac{y_a}{y_0} = 1 + \overbrace{\lambda_{y/l}\beta_{age}^{emp}}^{0.238} a$$

Figure 22: Estimating the firm age shift in economic activity



Estimate aging of economic activity $\Delta Y_a/Y$ requires re-weighting by the distribution according to the size by age coefficients to transform aging of firms into aging of activity. The intuition is what would happen if we held constant the output by age but shifted only the age composition of firms. To build changes in I/Y due to age composition effects holding i_0/y_0 and $\frac{i_a/y_a}{i_0/y_0}$ constant we need to estimate the following change

$$\Delta_{\text{aging}}^{t_0\to T} I/Y \quad = \quad \sum_a \left(\frac{(N_{aT}/N_{0T})\,\overline{y_a/y_0}}{\sum_a (N_{aT}/N_{0T})\overline{y_a/y_0}} - \frac{(N_{at_0}/N_{0t_0})\,\overline{y_a/y_0}}{\sum_a (N_{at_0}/N_{0t_0})\overline{y_a/y_0}} \right) \quad \frac{i_0}{y_0} \left(\frac{i_a/y_a}{i_0/y_0} \right)$$

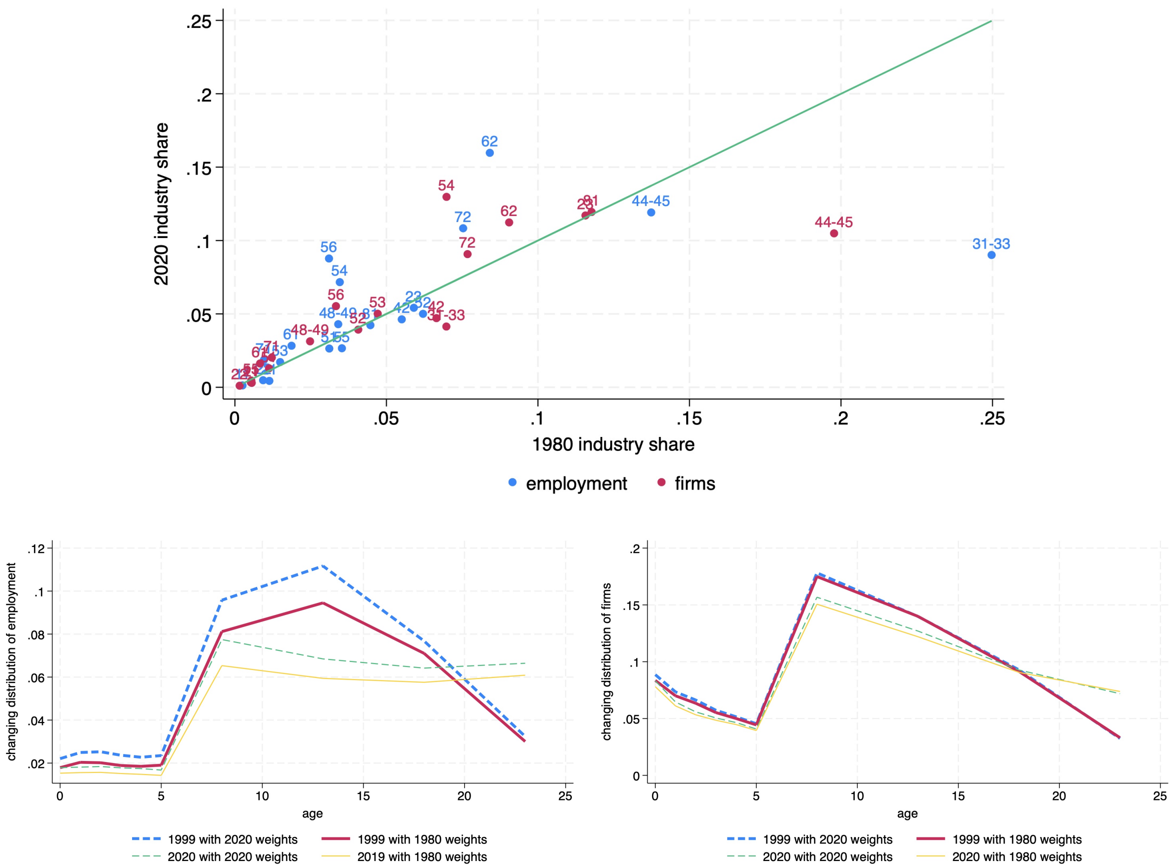
Table 4: 30 year old firms invest nearly half as intensely as startup firms on a given year

		output y = value added		output y = total sales	
		2002 − 2017	1977 − 2017	2002 − 2017	1977 − 2017
Investment to output of 30 year old firms (startup = 1)					
$\frac{i_{30t}/y_{30t}}{i_{0t}/y_{0t}}$	Upper CI	0.6737101	0.6739792	0.70106965	0.7014271
	Mean	**0.553**	**0.5737**	**0.5659**	**0.5896**
	Lower CI	0.4322899	0.4734208	0.43073035	0.4777729
Investment to output of 30 year old firms (startup = 1)					
$\frac{i_{at}/y_{at}}{i_{0t}/y_{0t}}$	Slope $\beta_0^{i/y}$	-0.0149	-0.01421	-0.01447	-0.01368
	SE $\sigma_0^{i/y}$	(0.002446)	(0.002032)	(0.002739)	(0.002266)

$$\frac{i_{at}}{y_{at}} = \left(1 + \beta_0^{i/y} a\right) \frac{i_{0t}}{y_{0t}}$$

Note. See Section 2.1 for data details. Full sample subject to censoring, I prefer the estimates from the post period only. In the pre-period, some young firms might appear unusually old because they are really old, minimizing the drop. Upper and lower bounds of the drop by a years old are constructed as $1 + (\beta \pm \gamma_c \cdot \sigma) \cdot a$ where γ_c is the critical value for a $90th$ confidence interval.

Figure 23: Within-sector variation.

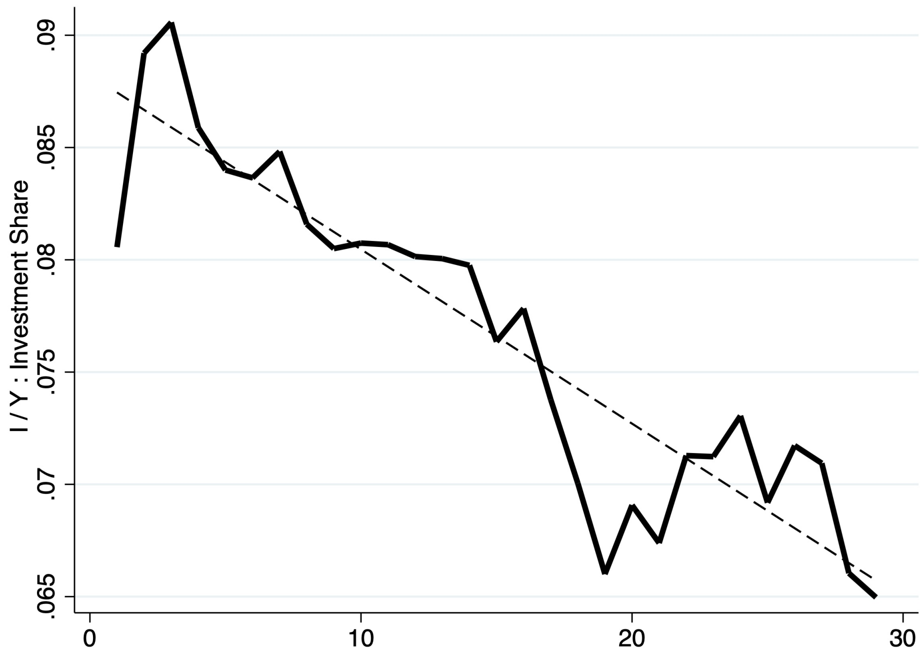


Further, in order to prevent structural changes to affect the age composition and focus within-sector aging, I hold the sectorial composition of output constant in 1980.

$$\Delta \frac{I_t}{Y_t} \approx \sum_s \frac{Y_{s0}}{Y_0} \sum_a \Delta_t \frac{y_{sa}\, N_{sa}}{\sum_\alpha\, y_{sa} N_{sa}} \frac{i_{sa}}{y_{sa}}$$

use these industry shares to compute the decomposition for years $1980 - 2021$, re-weighting sectors "as if" their composition had stayed constant in 1980. This prevents aggregate trends in aging from being biased by the fact that the economy has been shifting towards services which have younger firms on average. This weighting, thus, exploits the full extent of within-sector aging.

Figure 24: Investment falls with age (Compustat)



This figure uses data from Compustat North America (relative to sales).

Figure 25: Profits rise with age (Compustat)

This figure uses data from Compustat North America (relative to sales).

Figure 26: Size increases with age in a stationary way

size	$output_{at}$		$employees_{at}$	
$\frac{y_{at}}{y_{0t}}$	β_{age}	β_{time}	β_{age}	β_{time}
	$+.253$	$-.008$	$+.159$	$-.011$
	(.020)	(.016)	(.014)	(.016)

$$\frac{y_{at}}{y_{0t}} = \alpha + \beta_{age}\, a + \beta_{time}\, t + \Theta' X + \varepsilon_{fat}$$

Note. This table shows size by age has been stationary so firms grow with age but not faster or slower than in the past in the Manufacturing Census 1977-2017 for output as well as employment. See Section 2.1 for data details. In the table, I control for censoring by adding a mean for firms in the 1976 cohort whose age is unknown.

Figure 27: Firms (top) v.s. Establishments (bottom): Employees per business by age group with (left) and without (right) NAICS 2 industry fixed effects. Stationarity holds approximately even for establishments as long as we look within sectors.

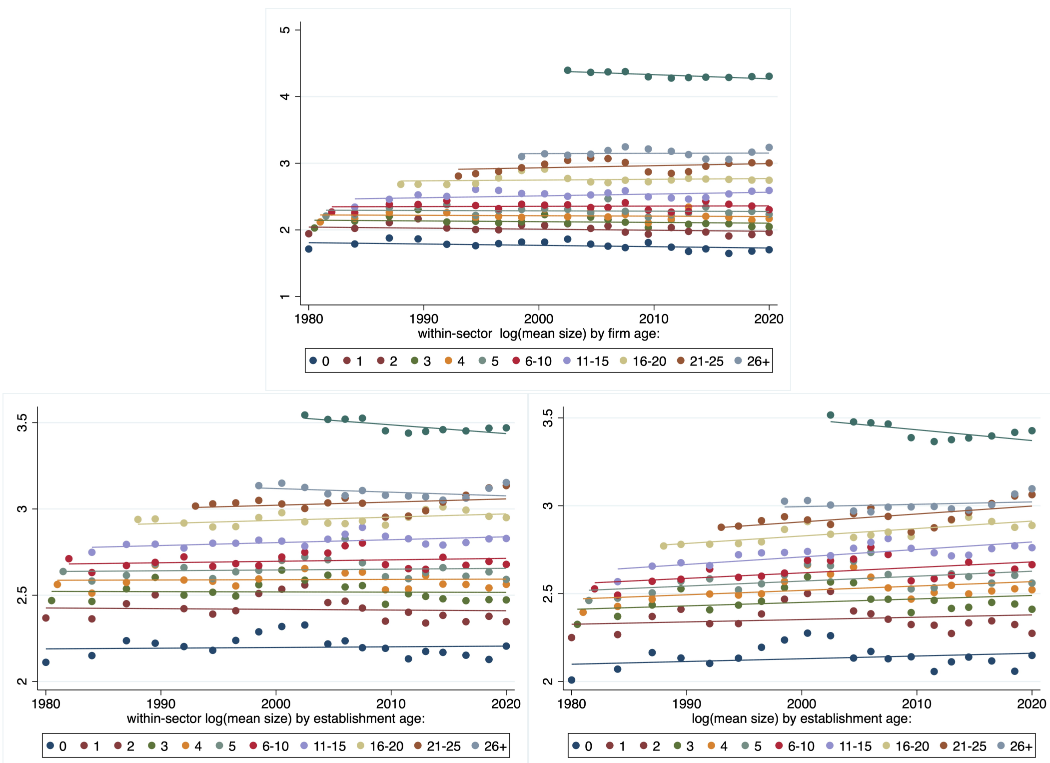
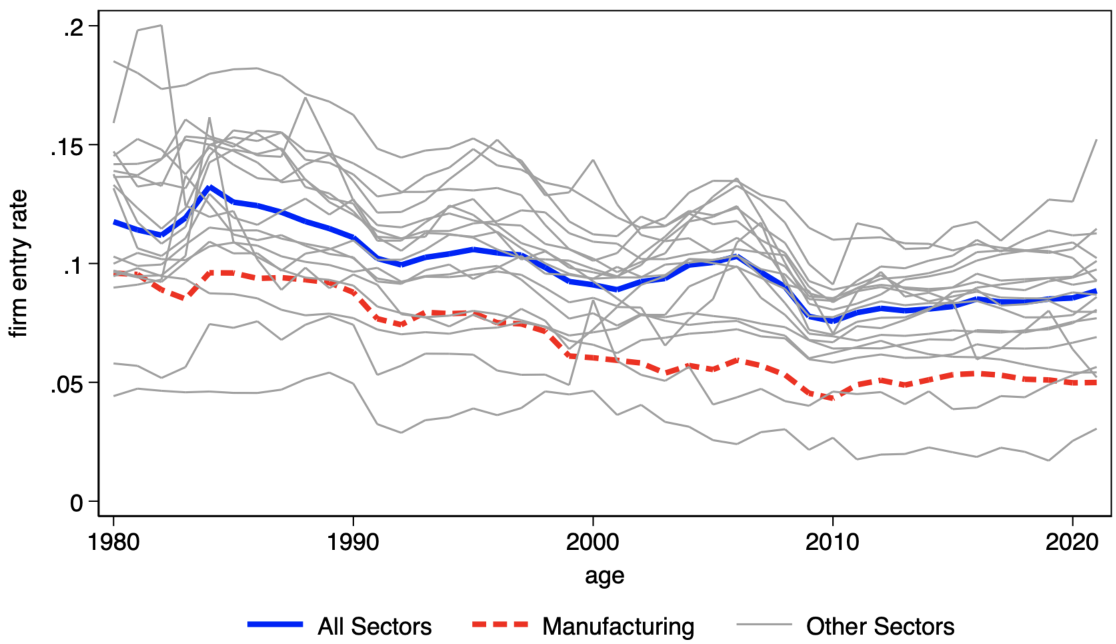


Figure 28: Entry rate has steadily fallen



Note. See Section 2.1 for data details.

Appendix to Chapter **??**

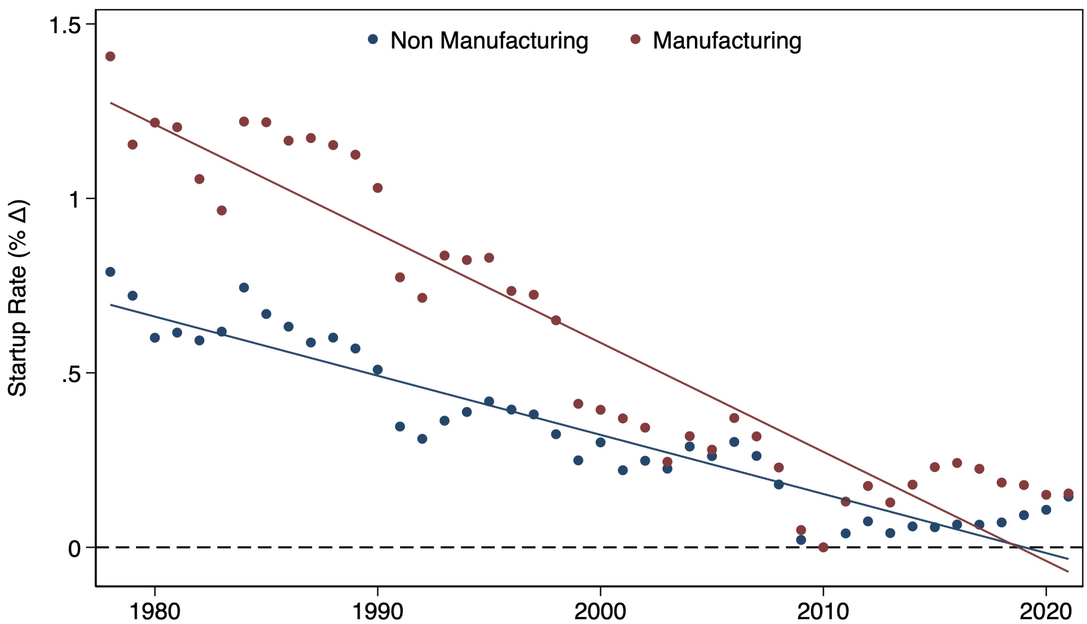
Measuring intangible investment intensity

While in the aggregate data from the BEA there are measures of investment that include or exclude intellectual property, in the US Census micro data are two ways to measure intangible investment intensity. Each way has it's own advantages and dis-advantages given the structure of the data which I discuss below.

Relative to tangible capital, measuring intangible capital is much more difficult to measure in administrative data. Thus, I collect multiple, all imperfect, proxies of intangible investment of firms.

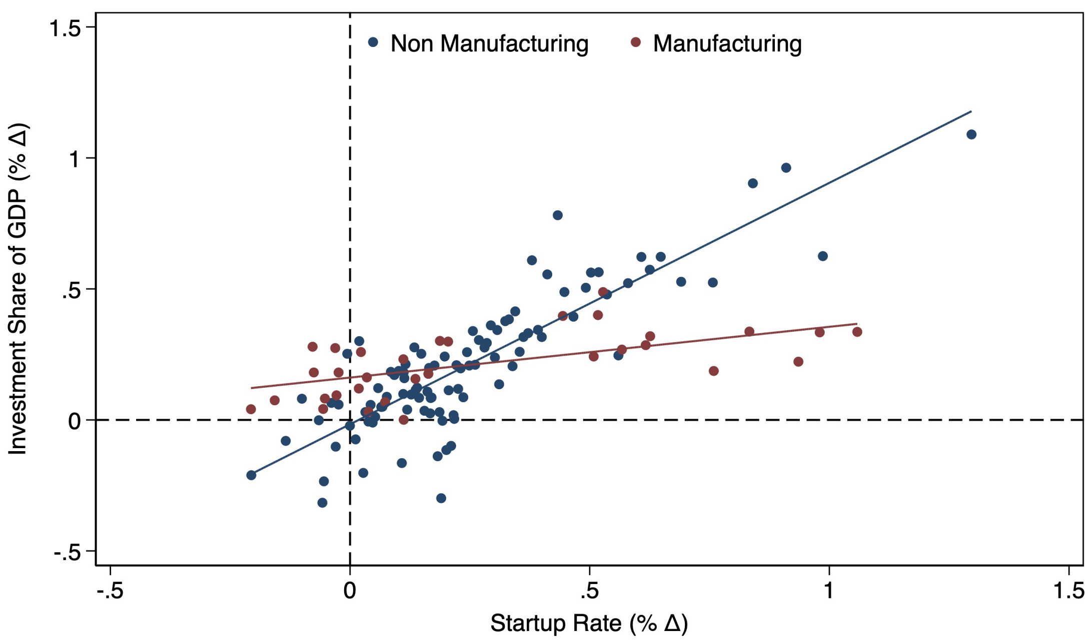
Intangible "input" measurement in manufacturing using advertising and non-production workers data. First, within the Census of Manufacturing (recall, available from 1977 to 2017), I am able to observe spending in communications and advertising as a share of output. Even though this is a small expense relative to physical capital investment, it is one of the commonly used proxies for intangible investment (e.g. marketing) firms use to build their brand. Further, also in manufacturing, I am able to measure the spending on non-production labor, which potentially includes managers and R & D workers that increase the firm's organizational capital and intellectual property. I measure "input" based intangible investment intensity like I

Figure 29: Entry fell more in manufacturing



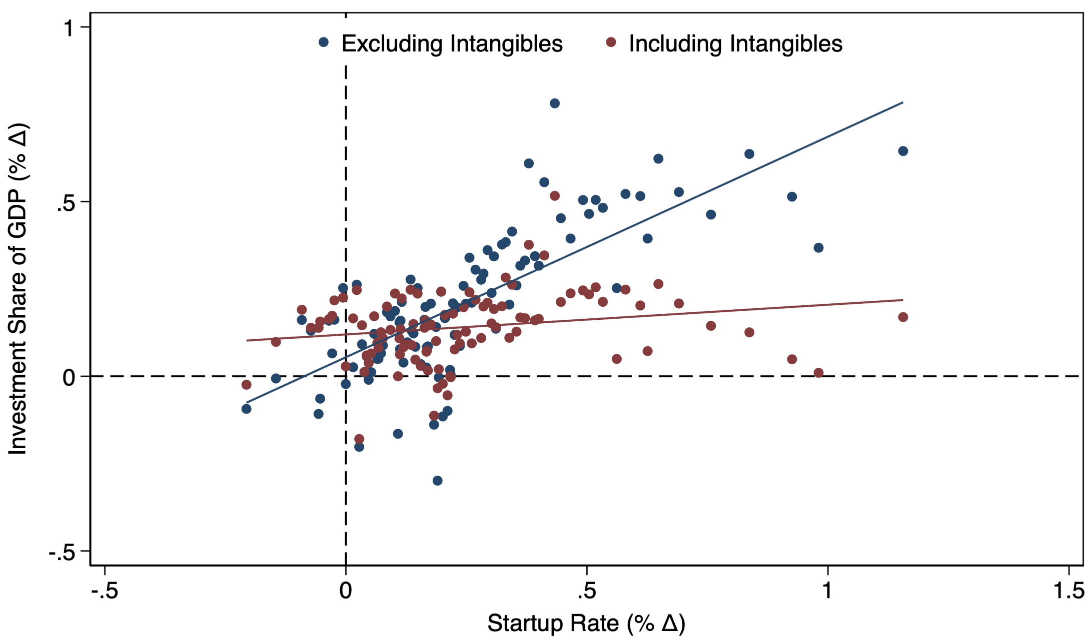
Note. Source: BDS.

Figure 30: Systematic relation between investment and entry across sectors



Note. Source: BDS and BEA data.

Figure 31: Systematic relation between investment and entry across sectors



Note. Source: BDS and BEA data.

Table 5: "Output" based intangible investment intensity fall with age but increase given age

Variable	age β	period β	constant
Citations/emp	-0.1194 (0.02677)	0.06012 (0.02696)	16.7 (5.362)
	-0.1201 (0.02687)		15.9 (5.95)
Patents/emp	-0.005022 (0.0003618)	0.0002715 (0.0003645)	0.5831 (0.07248)
	-0.004941 (0.0003618)		0.5228 (0.08011)
	N = 1078000		

$$\left[\frac{i_c^*}{l}, \frac{i_p^*}{l}\right]_{a,\tau} = \alpha + \beta_{\text{period}}\tau + \beta_{\text{age}}\, a + \varepsilon_{a,\tau}$$

Note. This table shows that patent investment intensity relative to the number of employees consistently declines with firm age and that increases conditional on age. The NBER patents data sample covers firms in all sectors of the economy for which information on the SSEL patent match exists and was able to be matched to the Longitudinal Business Database LBD. While the economic censuses cover he 1977-2017 period in five year spells, the LBD-NBER patent match in the SSEL is annual but only covers the 1976-2001 spell.

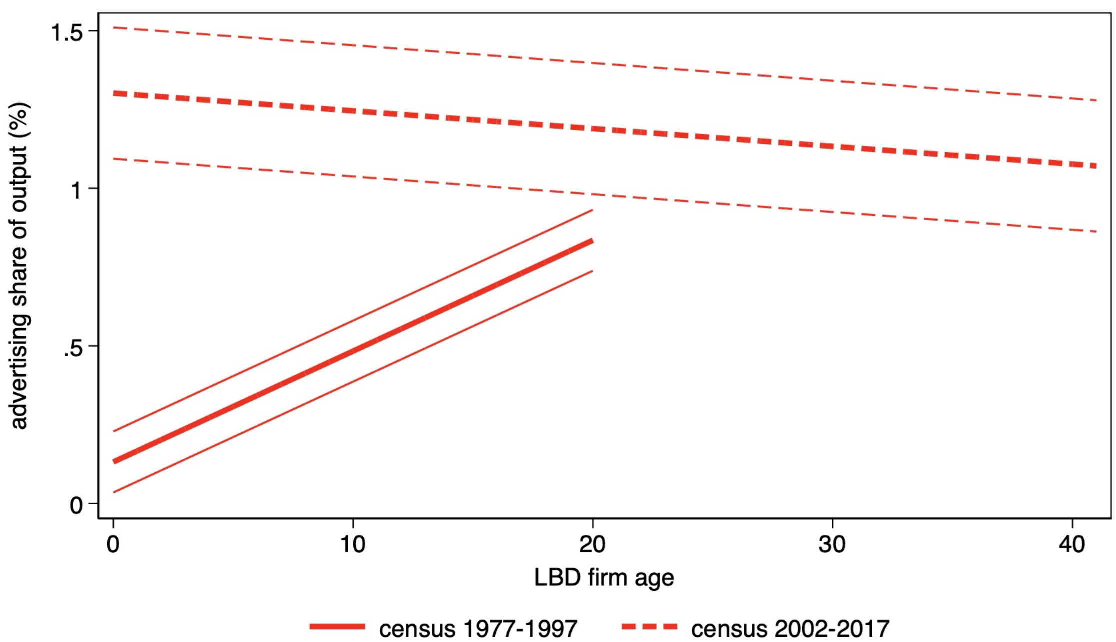
did for physical capital, as expenditure divided by output both in nominal terms.

Intangible "output" measurement in the entire economy using patent data. In addition, for all sectors in the economy, I measure patenting activity using what is known as the "SSEL NBER patent match" and merge it to LBD which contains information on the number of employees of firms. A major advantage of this data is that it covers not just manufacturing but also data on services and other sectors. However the data stops in 2001 and thus, it cannot be used to do the same split-sample longitudinal analyses as the main investment data. I measure intangible investment intensity relative to the number of employees i_a^*/l_a in terms of the number of patents granted to a firm as well as their total forward citations across all patents granted in a year. This citation-weighted measure captures the fact that not all patents are worth the same, and thus, weights more heavily more meaningful patents. Even though we do not know how much firms are spending to get these patents, these measures provide a sense of how intensively firms of different ages are investing in building intangible capital through R & D.

I try to measure these forms of intangible investment directly via "input" expenditures in advertising, non-production worker salaries and indirectly via "output" of expenditures in R & D such as patents granted and the citations of those patents.

Fact 1.7: Intangible investment/output also fall with firm age and rises over time

Figure 32: "Input" based intangible investment intensity shifts conditional on age



Note. This data comes from the Census of Manufacturers from 1977 - 2017. The measure on the left hand side is total advertising expenditures as a fraction of value added. It shows that there has been an increase in intangible investment intensity over time that has been disproportionately large for young firms.

Appendix to Chapter **??**

The most up to date version of this appendix can be found in the job market paper in my Harvard website: https://sites.harvard.edu/martin-aragoneses/job-market-paper/.

A Explicit aggregation model derivations

Consider the model in section 3.5. The goal of this section is to show that one can write out aggregate I/Y as a function of the age distribution of firms driven by *growth* rates of supply-side forces (technology and the supply of startups and workers).

Assumptions. Let us simplify the richer quantitative model described above with (1) exogenous exit only $p_{ft} = p_f$ and (2) no adjustment costs $\kappa_j \to 0$.

Firms. The stock of firms evolves as

$$N_{t+1} = p_f N_t + \bar{M}_t$$

Age. While their age distribution is

$$N_{ta} = p_f^a \bar{M}_{t-a}$$

Goods markets. Final output is used for consumption and investment

$$Y_t = C_t + I_t$$

Production. Firms produce with an exogenous $\bar{Z}_t$ and an exogenously driven idiosyncratic z_t. Let $\bar{\theta} = \theta(1-\alpha)$

$$y_t = (\bar{Z}_t z_t)^{1-\bar{\theta}} \left(l_t^{1-\alpha} k_t^{\alpha} \right)^{\theta}$$

Investment problem. Firms enter exogenously with $k_0 = 0$ investing thereafter $k' = (1-\delta_k) k_t + i_t$ unless they are forced to exit, in which case $k' = 0$. Their Bellman is

$$v_t(z,k) = (1-\delta_k) k + p_f \left\{ \max_{l_t, k'} y_t - w_t l_t - k' + \frac{1}{1 + r_{t+1}^f} \mathbb{E}_{z'|z} \left[v_{t+1}(z', k') \right] \right\}$$

Static choice. Optimality yields constant labor shares.

$$w_t = \bar{\theta} \frac{y_t}{l_t}, \quad y_t - w_t l_t = \left(1 - \bar{\theta}\right) y_t$$

Output can be written as a separable function between macro and micro terms

$$y_t = y(z,k) = \left(\frac{\bar{\theta}}{w_t} \right)^{\frac{\bar{\theta}}{1-\bar{\theta}}} \bar{Z}_t z k^{\frac{\alpha\theta}{1-\bar{\theta}}}$$

Dynamic choice. Firms choose investment based on first order/envelope conditions

$$1 + r_{t+1}^{f} = \mathbb{E}_{z'|z}\partial_{k'}v_{t+1}\left(z', k'\right), \quad \partial_k v_t\left(z, k\right) = \left(1 - \delta_k\right) + p_f\left(1 - \bar{\theta}\right)\partial_k y_t$$

Capital is driven by productivity $k_t\left(z_{t-1}\right)$–with $k_{t+1}\left(z_t\right)$ defined similarly

$$k_t = \left[\frac{p_f \alpha\theta}{r_t^f + \delta_k}\left(\frac{\bar{\theta}}{w_t}\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}}\bar{Z}_t\mathbb{E}_{z_t|z_{t-1}}z_t\right]^{\frac{1-\bar{\theta}}{1-\theta}}$$

Thus, we can further simplify output as

$$y_t = y\left(z_t, z_{t-1}\right) = \left[\bar{Z}_t\left(\frac{\bar{\theta}}{w_t}\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}}\right]^{\frac{1-\bar{\theta}}{1-\theta}}\left[\frac{p_f\alpha\theta}{r_t^f + \delta_k}\right]^{\frac{\alpha\theta}{1-\theta}} z_t\mathbb{E}_{z|z_{t-1}}z$$

AR(1) Process. Notice that if z_t follows an $AR(1)$ process $\kappa_z = e^{\frac{\sigma^2}{2}}$ and $\mathbb{E}_{z_t|z_{t-1}}z_t = \kappa_z z_{t-1}^{\rho}$

$$y_t\left(z_t, z_{t-1}\right) = \left[\bar{Z}_t\left(\frac{\bar{\theta}}{w_t}\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}}\right]^{\frac{1-\bar{\theta}}{1-\theta}}\left[\frac{p_f\alpha\theta\kappa_z}{r_t^f + \delta_k}\right]^{\frac{\alpha\theta}{1-\theta}} z_{t-1}^{\rho\frac{\alpha\theta}{1-\theta}} z_t$$

$$k_t\left(z_{t-1}\right) = \left[\frac{p_f\alpha\theta\kappa_z}{r_t^f + \delta_k}\left(\frac{\bar{\theta}}{w_t}\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}}\bar{Z}_t z_{t-1}^{\rho}\right]^{\frac{1-\bar{\theta}}{1-\theta}}$$

$$k_{t+1}\left(z_t\right) = \left[\frac{p_f\alpha\theta\kappa_z}{r_{t+1}^f + \delta_k}\left(\frac{\bar{\theta}}{w_{t+1}}\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}}\bar{Z}_{t+1} z_t^{\rho}\right]^{\frac{1-\bar{\theta}}{1-\theta}}$$

A.1 Aggregation

The key benefit of this analytical model is that the objects that are common "macro" (e.g. w_t) scale the idiosyncratic "micro" ones (e.g. z_t), allowing for linear aggregation.

GDP. Exploiting that $N_{at} = p_f^a \bar{M}_{t-a}$ is the age distribution

$$Y_t = \mathbb{Y}_t Z_t^y$$

Capital.

$$K_t = \int k\mu_t = \mathbb{K}_t Z_t^0$$

Investment. Notice that firms start with zero capital so $i_t^s = 0$. Further, because of exit, only p_f firms choose to leave capital for the future.

$$I_t = p_f \mathbb{K}_{t+1} Z_t^1 - (1 - \delta_k) \mathbb{K}_t Z_t^0$$

Micro indices. Where the "micro" indices vary by age but not with aggregates, and summarize all dynamic heterogeneity in the model: (1) capital *investment* at a reflects only the current state (2) capital *inherited* at a reflects prior state, (3) output today which reflects both today and yesterday's state.

$$z_{at}^y = \int z_t \left(z_{t-1}^\rho\right)^{\frac{\alpha\theta}{1-\bar{\theta}}} \bar{\mu}_{at}, \quad z_{at-1}^k = \int \left[z_{t-1}^\rho\right]^{\frac{1-\bar{\theta}}{1-\bar{\theta}}} \bar{\mu}_{at}, \quad z_{at}^k = \int \left[z_t^\rho\right]^{\frac{1-\bar{\theta}}{1-\bar{\theta}}} \bar{\mu}_{at}$$

$$Z_t^y = \sum_a p_f^a \bar{M}_{t-a} z_{at}^y, \quad Z_t^0 = \sum_a p_f^a \bar{M}_{t-a} z_{at-1}^k, \quad Z_t^1 = \sum_a p_f^a \bar{M}_{t-a} z_{at}^k$$

Macro indices. Common technology and general equilibrium variables

$$\mathbb{K}_t = \left(\bar{Z}_t \left(\frac{\bar{\theta}}{w_t}\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}} \frac{p_f \alpha\theta\kappa_z}{r_t^f + \delta_k}\right)^{\frac{1-\bar{\theta}}{1-\theta}}, \quad \mathbb{Y}_t = \left[\bar{Z}_t \left(\frac{\bar{\theta}}{w_t}\right)^{\frac{\bar{\theta}}{1-\bar{\theta}}}\right]^{\frac{1-\bar{\theta}}{1-\theta}} \left[\frac{p_f \alpha\theta\kappa_z}{r_t^f + \delta_k}\right]^{\frac{\alpha\theta}{1-\theta}}$$

Intuitively, the common capital to output ratio is determined by r:

$$\frac{\mathbb{K}_t}{\mathbb{Y}_t} = \frac{p_f \alpha\theta\kappa_z}{r_t^f + \delta_k}$$

A.2 Balanced Growth Path (BGP) in Partial Equilibrium

Consider a BGP where entry grows at a constant rate: $\bar{M}_t = g_M^t \bar{M}$. Then $\bar{M}_{t-a} = (g_M)^{t-a} \bar{M}$. Then, N_t also grows at a constant rate, it must grow with entry.

$$g_M = \frac{N_{t+1}}{N_t} = p_f + \frac{(g_M)^t \bar{M}}{N_t} = p_f + \frac{\bar{M}}{\bar{N}}$$

Entry rate. In this BGP entry is endogenously driven by growth in startup supply

$$\frac{\bar{M}}{\bar{N}}(g_M) = g_M - p_f$$

Output. In the BGP, output is homogenous of degree one with respect to $\bar{M}_t$ so it's level decreases when entry falls, however, because the age distribution shifts towards the old (which are larger) average output increases with aging (and there is concentration)

$$Y_t = \mathbb{Y}_t \bar{M}_t \sum_a \left(\frac{p_f}{g_M} \right)^a z^y_{at}$$

Output-per-firm. Notice along the BGP, we can write output and output per firm as

$$\frac{Y_t}{N_t} = \mathbb{Y}_t \left(g_M - p_f \right) \sum_a \left(\frac{p_f}{g_M} \right)^a z^y_{at}$$

Investment to GDP.

$$\frac{I_t}{Y_t} = \frac{I_t}{K_t} \frac{K_t}{Y_t} = \left(p_f \frac{\mathbb{K}_{t+1}}{\mathbb{K}_t} \frac{\sum_a \left(\frac{p_f}{g_M} \right)^a z^k_{at}}{\sum_a \left(\frac{p_f}{g_M} \right)^a z^k_{at-1}} - (1 - \delta_k) \right) \frac{\mathbb{K}_t}{\mathbb{Y}_t} \frac{\sum_a \left(\frac{p_f}{g_M} \right)^a z^k_{at-1}}{\sum_a \left(\frac{p_f}{g_M} \right)^a z^y_{at}}$$

In general, the aggregate can change due to technology or costs

$$\frac{\mathbb{K}_{t+1}}{\mathbb{K}_t} = \left(\frac{\bar{Z}_{t+1}}{\bar{Z}_t} \left(\frac{w_t}{w_{t+1}} \right)^{\frac{\bar{\theta}}{1-\bar{\theta}}} \frac{r^f_t + \delta_k}{r^f_{t+1} + \delta_k} \right)^{\frac{1-\bar{\theta}}{1-\theta}}$$

But in PE, only $\bar{Z}_t$ and $\bar{M}_t$ change, while GE costs w and r are constant so

$$\frac{\mathbb{K}_{t+1}}{\mathbb{K}_t} = \frac{\mathbb{Y}_{t+1}}{\mathbb{Y}_t} = (g_Z)^{\frac{1-\bar{\theta}}{1-\theta}}, \quad \frac{\mathbb{K}_t}{\mathbb{Y}_t} = \frac{p_f \alpha \theta \kappa_z}{r^f + \delta_k}$$

Growth. Because the micro-economy does not change, we have that the rate of growth of the economy depends on technology and entry supply only.

$$g_Y = \frac{Y_{t+1}}{Y_t} = \frac{\mathbb{Y}_{t+1}}{\mathbb{Y}_t} \frac{\bar{M}_{t+1}}{\bar{M}_t} = (g_Z)^{\frac{1-\bar{\theta}}{1-\theta}} g_M$$

In equilibrium given r and w, a decline in productivity growth depresses investment, while the aging from entry shifts the distribution towards older firms.

$$\frac{I_t}{Y_t} (g_M, g_Z | r, w) = \left(\frac{p_f \alpha \theta \kappa_z}{r + \delta_k} \right) \left(\delta_k + p_f (g_Z)^{\frac{1-\bar{\theta}}{1-\theta}} \frac{\sum_a \left(\frac{p_f}{g_M} \right)^a z^k_{at}}{\sum_a \left(\frac{p_f}{g_M} \right)^a z^k_{at-1}} - 1 \right) \frac{\sum_a \left(\frac{p_f}{g_M} \right)^a z^k_{at-1}}{\sum_a \left(\frac{p_f}{g_M} \right)^a z^y_{at}}$$

Stationary output-per-firm given age. For the result in the data that output per firm given age is constant, the following condition must hold:

$$y = \frac{Y_t}{N_t} = \mathbb{Y}_t \left(g_M - p_f \right) \sum_a \left(\frac{p_f}{g_M} \right)^a z^y_{at}$$

With constant w and r, this is constant along the BGP if $g_Z = 1$ and so output growth is driven by entry

$$\mathbb{Y}_t = \mathbf{Y}, \quad g_Y = g_M$$

Which if w and r are fixed can only happen if $g_Z = 1$. Thus, we recover:

$$\frac{I_t}{Y_t}\left(g_M|r,w\right) = \left(\frac{p_f \alpha\theta\kappa_z}{r+\delta_k}\right)\left(\delta_k + p_f \frac{\sum_a \left(\frac{p_f}{g_M}\right)^a z^k_{at}}{\sum_a \left(\frac{p_f}{g_M}\right)^a z^k_{at-1}} - 1\right)\frac{\sum_a \left(\frac{p_f}{g_M}\right)^a z^k_{at-1}}{\sum_a \left(\frac{p_f}{g_M}\right)^a z^y_{at}}$$

Further, the profit share will be inversely related to investment to GDP

$$\frac{\Pi_t}{Y_t} = 1 - \bar{\theta} - \frac{I_t}{Y_t}$$

No age heterogeneity. Imagine there is no age heterogeneity. Then z_{at} may capture size heterogeneity. Thus, changes in g_M leave I/Y unchanged.

$$\frac{I}{Y} = \left(\frac{p_f \alpha\theta}{r+\delta_k}\right)\left(\delta_k + p_f \frac{Z^1}{Z^0} - 1\right)\frac{Z^0}{Z_y}$$

A.3 BGP in General Equilibrium of a Small Open Economy

In a SOE, r is fixed, but labor market clearing requires the wage to adjust in order to meet demand.

$$w_t \bar{L}_t = \bar{\theta} Y_t$$

Thus, aggregate output solves the fixed point

$$Y_t = \mathbb{Y}_t\left(Y_t\right) \sum_a p_f^a \bar{M}_{t-a} z^y_{at}$$

Aggregate production function. $Y_t\left(\bar{Z}_t, \bar{L}_t, \bar{M}_t\right)$

$$Y_t = \left[\bar{Z}_t\right]^{\frac{1-\bar{\theta}}{1-\alpha\theta}} \left[\bar{L}_t\right]^{\frac{\bar{\theta}}{1-\alpha\theta}} \left[\sum_a p_f^a \bar{M}_{t-a} z^y_{at}\right]^{\frac{1-\theta}{1-\alpha\theta}} \left[\frac{p_f \alpha\theta\kappa_z}{r^f + \delta_k}\right]^{\frac{\alpha\theta}{1-\alpha\theta}}$$

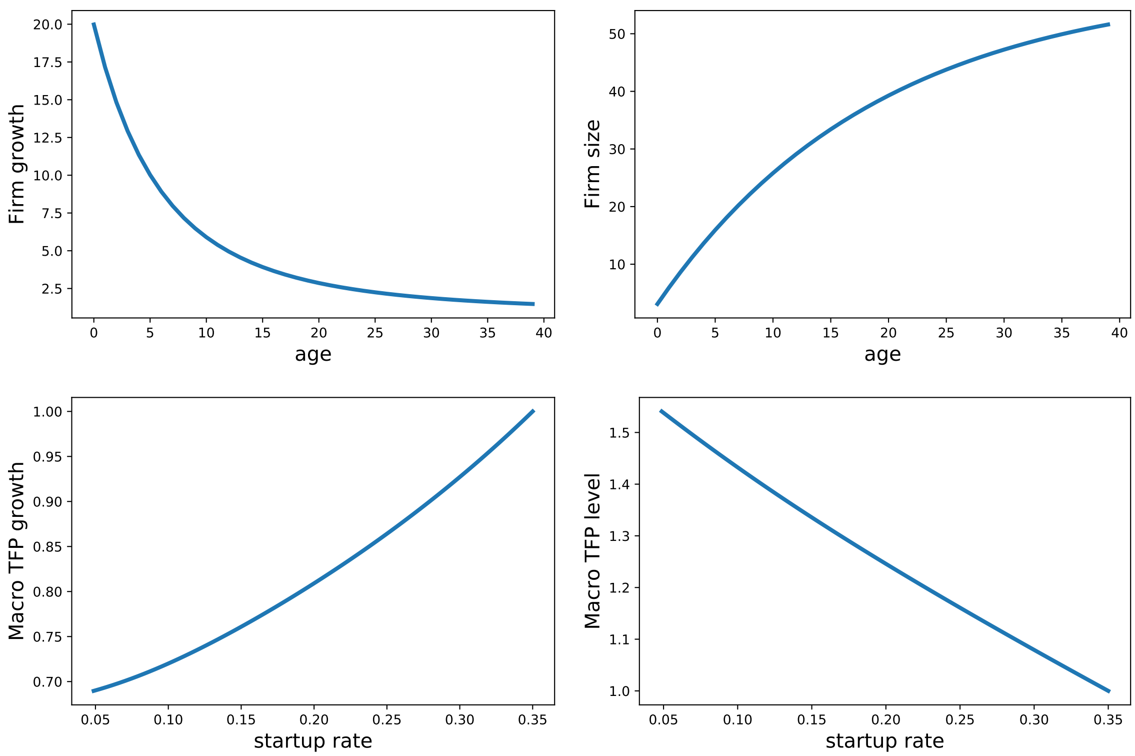
BGP output growth of the economy is driven by technology, labor, and startup supply.

$$g_Y = \left[g_M^{1-\theta} g_Z^{1-\bar{\theta}} g_L^{\bar{\theta}}\right]^{\frac{1}{1-\alpha\theta}}$$

Because r is constant, g_M increases competition and lowers the wage. Note this negative effect of w is offset by the positive mass effect. Since r is fixed

$$\frac{\mathbb{Y}_{t+1}}{\mathbb{Y}_t} = \frac{\mathbb{K}_{t+1}}{\mathbb{K}_t} = \left(g_Z\right)^{\frac{1-\bar{\theta}}{1-\alpha\theta}} \left[g_L\right]^{\frac{\bar{\theta}}{1-\alpha\theta}} \left[g_M\right]^{-\frac{\bar{\theta}}{1-\alpha\theta}}$$

Figure 33: At the **micro** level, older firms grow slower despite being larger. At **macro** level, a lower startup rate ages firms, increasing the TFP *level* but slowing TFP *growth*.



Size-by-age stationarity. Recall our condition that GDP per firm needs to remain constant so that firms do not shrink with less entry.

$$Y_t = \mathbb{Y}_t \bar{M}_t \sum_a \left(\frac{p_f}{g_M}\right)^a z^y_{at}, \quad \mathbb{Y}_t = \mathbf{Y}, \quad g_Y = g_M$$

Because r is fixed, then it must be that wages offset technology

$$\frac{\mathbb{Y}_{t+1}}{\mathbb{Y}_t} = \text{constant} \iff g_w = g_Z^{\frac{1-\bar{\theta}}{\bar{\theta}}} \implies g_Z^{\frac{1}{\bar{\theta}}-1} g_L = g_M = g_Y$$

Thus, under this scenario, I/Y does not shift with g_Z, but it will change with g_M due to aging.

$$\frac{I_t}{Y_t}(g_M|r_f) = \frac{I_t}{K_t}\frac{K_t}{Y_t} = \left(p_f \frac{\sum_a \left(\frac{p_f}{g_M}\right)^a z^k_{at}}{\sum_a \left(\frac{p_f}{g_M}\right)^a z^k_{at-1}} - (1-\delta_k) \right) \frac{p_f \alpha\theta\kappa_z}{r^f + \delta_k} \frac{\sum_a \left(\frac{p_f}{g_M}\right)^a z^k_{at-1}}{\sum_a \left(\frac{p_f}{g_M}\right)^a z^y_{at}}$$

Thus, in general equilibrium along the BGP, as long as the "right" micro stationarity condition holds, we recover the same shift-share without much changes within age.

A.4 Aggregation in Full General Equilibrium (r changes)

Note that as the economy slows down, r falls. Then

$$(1 + r_{t+1})\,\beta = \left(\frac{c_{t+1}}{c_t}\right)^{\varsigma} = \left(\frac{C_{t+1}}{C_t}\frac{\bar{L}_t}{\bar{L}_{t+1}}\right)^{\varsigma}$$

Because $Y_t = C_t + I_t$, C_t must grow with Y_t so along the BGP

$$(1 + r_{t+1})\,\beta = \left(\frac{c_{t+1}}{c_t}\right)^{\varsigma} = \left(\frac{Y_{t+1}}{Y_t}\frac{\bar{L}_t}{\bar{L}_{t+1}}\right)^{\varsigma} \implies (1+r)\,\beta = \left(\frac{g_Y}{g_L}\right)^{\varsigma}$$

so $r = \frac{1}{\beta}\left(\frac{g_Y}{g_L}\right)^{\varsigma} - 1$ becomes endogenous, and varies with growth in per capita consumption (if the economy moves towards a new GBP where g_y is lower, so will be r).

Wage growth.

$$w_t \bar{L}_t = \bar{\theta} Y_t \implies g_w = \frac{g_Y}{g_L}$$

Output growth. Along a BGP, r is fixed by parameters so it affects the *level* but not the growth of Y_t directly

$$Y_t = [\bar{Z}_t]^{\frac{1-\bar{\theta}}{1-\alpha\theta}} (\bar{L}_t)^{\frac{\bar{\theta}}{1-\alpha\theta}} \left[\frac{p_f \alpha\theta\kappa_z}{r^f_t + \delta_k}\right]^{\frac{\alpha\theta}{1-\alpha\theta}} \left[\sum_a p_f^a \bar{M}_{t-a} z^y_{at}\right]^{\frac{1-\theta}{1-\alpha\theta}}$$

$$g_Y = \left[g_M^{1-\theta} g_Z^{1-\bar{\theta}} g_L^{\bar{\theta}} \right]^{\frac{1}{1-\alpha\theta}}$$

Stationarity in size-by-age. This requires that output is homogeneous of degree one with $\bar{M}_t$. Since r is not changing:

$$\frac{\mathbb{K}_{t+1}}{\mathbb{K}_t} = \frac{\mathbb{Y}_{t+1}}{\mathbb{Y}_t} = (g_Z)^{\frac{1-\bar{\theta}}{1-\alpha\theta}} [g_L]^{\frac{\bar{\theta}}{1-\alpha\theta}} [g_M]^{-\frac{\bar{\theta}}{1-\alpha\theta}} = 1 \implies g_M = (g_Z)^{\frac{1-\bar{\theta}}{\bar{\theta}}} g_L$$

Technology-driven rates. Hence, $g_Y = g_M$. Thus, the real rate simply tracks technology

$$(1+r)\,\beta = \left(\frac{g_Y}{g_L}\right)^{\varsigma} = \left(\frac{g_M}{g_L}\right)^{\varsigma}$$

Investment to GDP. When the economy exhibits both a slowdown in the startup supply and slowdown in economy-wide TFP, but the TFP slowdown is offset by wages, this generates the result that (1) the aggregate investment share falls with aging (due to composition effects), (2) while partially offset by the interest rate decline due to the slowdown in TFP.

$$\frac{I_t}{Y_t} = \frac{I_t}{K_t}\frac{K_t}{Y_t} =\downarrow \left(p_f \frac{\sum_a \left(\frac{p_f}{g_M}\right)^a z^k_{at}}{\sum_a \left(\frac{p_f}{g_M}\right)^a z^k_{at-1}} - (1-\delta_k) \right) \frac{p_f \alpha\theta\kappa_z}{\downarrow \frac{1}{\beta}\left(\frac{g_M}{g_L}\right)^{\varsigma} - (1-\delta_k)} \frac{\sum_a \left(\frac{p_f}{g_M}\right)^a z^k_{at-1}}{\sum_a \left(\frac{p_f}{g_M}\right)^a z^y_{at}}$$

Conditional on g_M, we would expect the lower g_Z to boost investment shares as what happened in PE. However, this does not take place because the wages adjust. The only reason I_t/Y_t falls is because of aging. Thus, a slowdown in technology has been offset by a decline in wages. If technology had slowed down, we would have seen a decline in micro growth by age. However, the decline in growth has fed back into a decline in macro interest rates, stimulating investment. At the same time, we see a second effect that dampened the I_t/Y_t decline: the fall in r.

We can also re-write this as follows

$$\frac{I_t}{Y_t}(g) = \sum_a \frac{\left(\frac{p_f}{g_M}\right)^a z^y_{at}}{\sum_a \left(\frac{p_f}{g_M}\right)^a z^y_{at}} \frac{i_{at}}{y_{at}}(g)$$

$$\frac{i_{at}}{y_{at}} = \left[p_f \frac{z^k_{at}}{z^y_{at}} - (1-\delta_k)\frac{z^k_{at-1}}{z^y_{at}} \right] \frac{p_f \alpha\theta\kappa_z}{\frac{1}{\beta}\left(\frac{g_M}{g_L}\right)^{\varsigma\frac{1-\bar{\theta}}{\bar{\theta}}} - 1 + \delta_k}$$

where changes in Z and L feedback into changes in entry: $g_M = (g_Z)^{\frac{1-\bar{\theta}}{\bar{\theta}}} g_L$.

To summarize: a slowdown in g_M (associated with a slowdown in L, Z, both...) impacts I_t/Y_t in two ways:

Figure 34: The lower startup rate depresses r while aging firms. r decline from slower entry generates investment increase conditional on age.

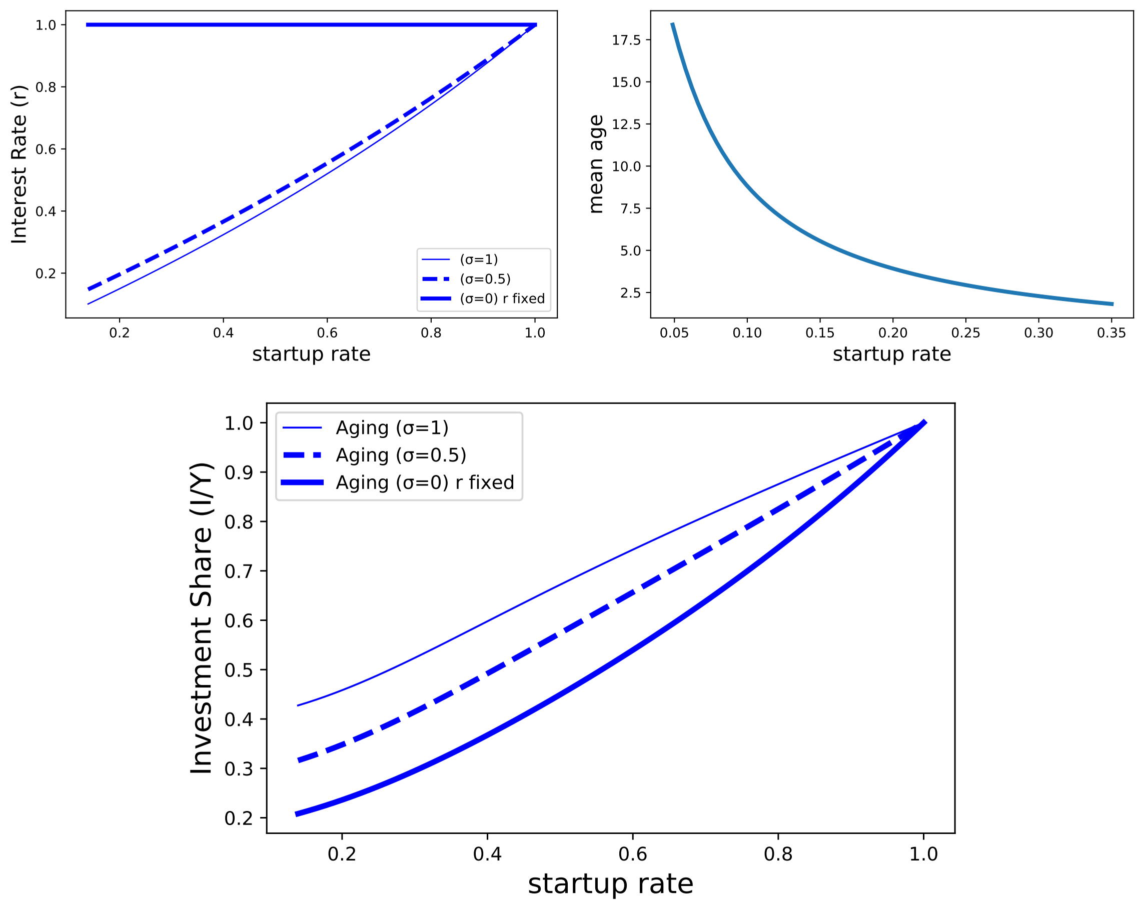
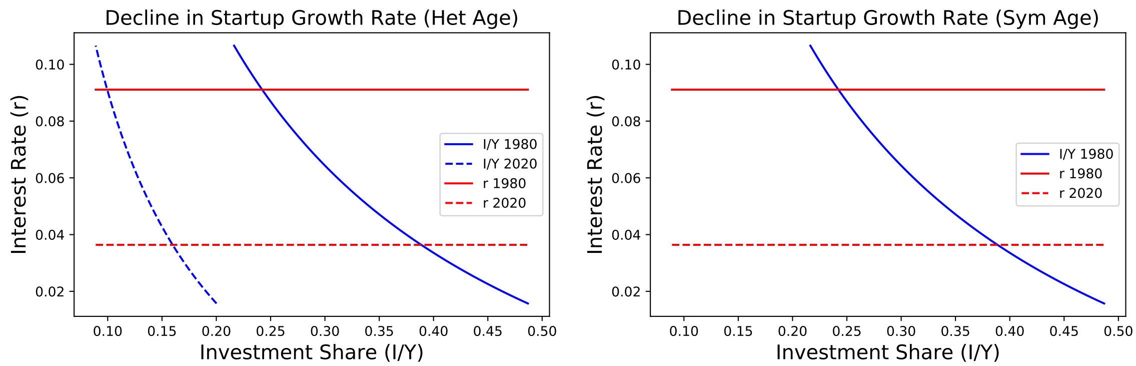


Figure 35: Firm aging depresses investment demand despite lower interest rates. Absent aging effects, the slower startup rate would have made investment increase.



(1) Aging depresses investment by reallocating GDP towards firms that invest less intensely.

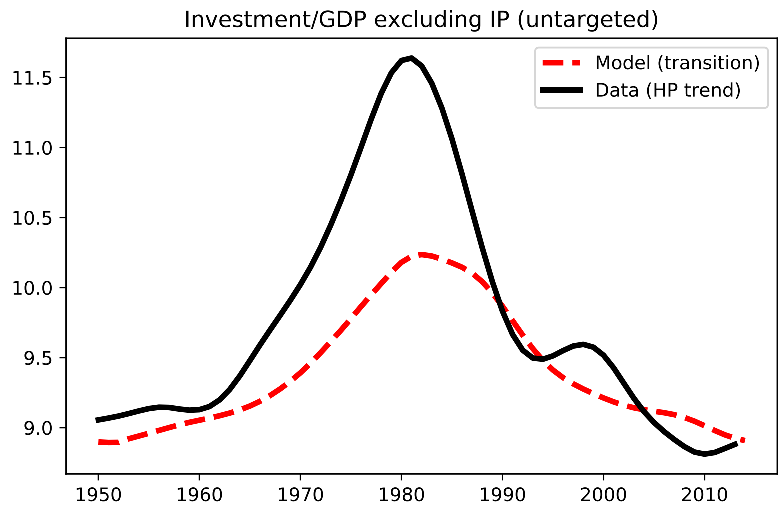
(2) Investment increases conditional on age (across all age groups) because r declines.

Crucial here is that aggregates are homogenous of degree one in $\bar{M}_t$ and relatedly that size-by-age is stationary.

A.5 Perfect foresight transition in general equilibrium.

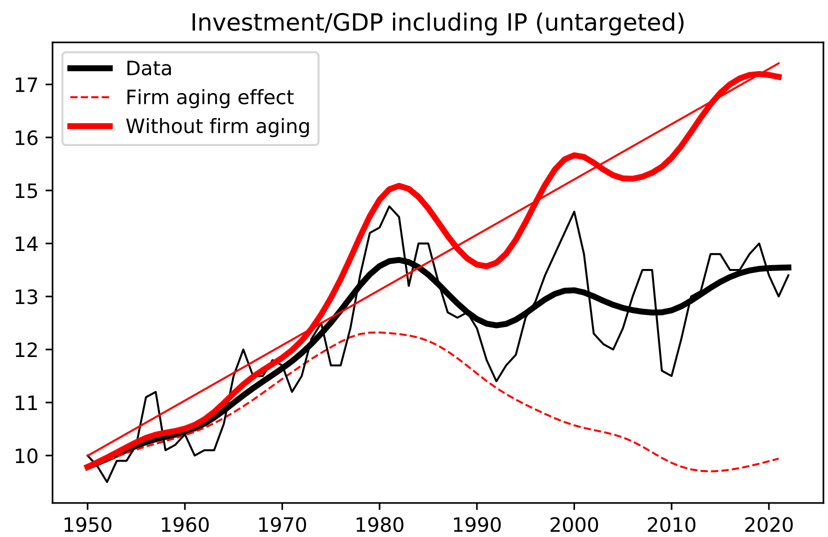
Transition. This is likely an upper bound on the effects of changes in startup activity in transition. Due to the backward-looking nature of the distribution, changes in startup activity will affect the age distribution less dramatically than what is implied by cross-BGP comparisons. Furthermore, because of the forward-looking nature of firms, firms expecting increases in general equilibrium prices will dampen their investment. These backward and forward looking smoothing forces are common across models with heterogeneous firms. Figure 36 presents the transitional dynamics in the model using the sequence-space methods in Auclert *et al.* (2021a). In this general equilibrium transition, the effect age composition effects are still present but are smaller, of around 1.3% of GDP, but still explain around 50% of the 2.6% actual trend. The startup rate path is presented in the Theory Appendix. Notice importantly that relative to the cross-BGP results, the aging effects are delayed since it takes time for the age distribution to shift.

Figure 36: Robustness to transition in general equilibrium



Note. This figure investment to GDP in the model experiencing a perfect foresight transition between 1950 and 2020 BGPs. I use sequence-space methods in Auclert *et al.* (2021a) to solve for g_{Mt} that matches the data's boom and bust of the startup rate, and solve for the linearized transition path of I_t/Y_t.

Figure 37: Investment to GDP driven by Startup Rates via Firm Age Distribution



Note. This figure qualitatively shows firm aging may have slowed down the effects of intangibles in the aggregate. Total investment to GDP including intellectual property comes the BEA measure used in Figure 1. The dashed line is the effects of firm aging reproducing the analyses in Figure 11 but recalibrating the model to match the total investment to GDP. The solid red line shows what the implied data trend would have been removing this aging effect.

A.6 Application: Why did the Rise of Intangibles not lead to an Investment Boom?

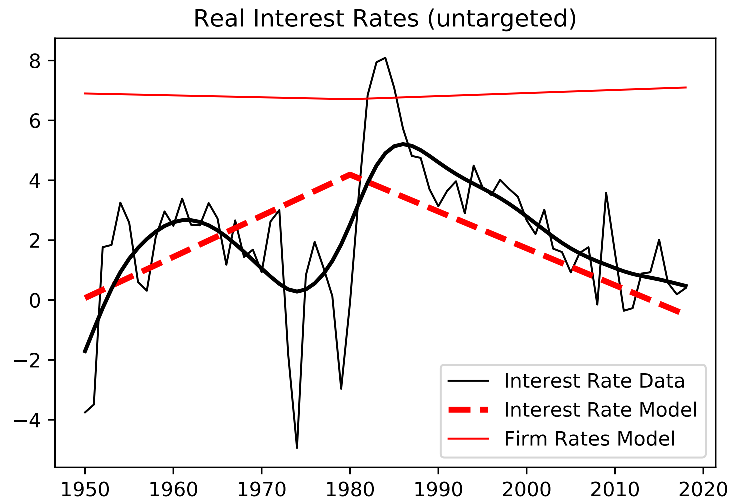
Thus far the model has been applied to only tangible capital. In this section, I broaden my view of capital and find that the firm aging force highlighted in this paper could have been slowing down the aggregate impact of intangibles.

I re-calibrate the model to match the total investment share instead of the tangible one assuming that the investment-to-output by age relation that holds for tangibles holds for total investment. I then perform the same exercise of matching the startup rate dynamics in the data and looking at the implied path of aggregate investment to GDP. Figure 37 shows that aging of firms may have been again one of the forces keeping total investment stagnant despite the rise of intangibles. In fact, the figure shows that if firm aging had not taken place, investment would have continued on its pre-1980 upward trajectory, increasing by 70% since 1950 instead of by just 30% as it did in aggregate data.

Consistent with this result, I show in the Appendix that while tangible investment to GDP across sectors is very correlated with the sector-level entry rate since the 1980, intangible-inclusive investment is less correlated with entry. This suggests that there has been an increase in intangible investment *given* age among US firms, which is the "wedge" highlighted by Crouzet and Eberly (2018). This would be consistent with my empirical finding that investment to output increased given age (Fact 1).

In a companion paper, Aragoneses (2023b), I investigate the life cycle dimension of the rise in intangibles using the same confidential data from the US Census used in this paper. I find that the changing composition of startups has been an important driver in the increase of intangible capital. Relative to startups in the past, recent cohorts of firms are significantly more likely to invest in intangible capital. This suggests that any intangible-related technology that started to become available after 1980 may be affecting the economy through the startup stage.

Figure 38: Interest rate dynamics consistent with startup and investment time series



Note. This qualitatively shows BGP changes in firm entry matter for interest rates: intuitively, since the BGP interest rate is tied to the growth in per capita income, $r_t(g_t) = g_t/\beta - 1$, whenever $g_M \neq g_L$ a boom and bust in entry growth will translate into a rise and fall in interest rates as we have seen in historical data, which comes from Rogoff *et al.* (2022).

A.7 Interest rate dynamics consistent with startup and investment time series