Green Capital Requirements *

Martin Oehmke[†] Marcus Opp[‡]

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Abstract

We study bank capital requirements as a tool to address financial risks and externalities caused by carbon emissions. Capital regulation can effectively address financial risks but doing so does not necessarily reduce emissions (e.g., higher capital requirements for carbon-intensive loans may crowd out clean lending). Relative to a planner who also has access to a carbon tax, reducing emissions via capital requirements alone may require sacrificing financial stability or may be altogether infeasible. However, if the government cannot commit to future environmental policies, capital requirements can make carbon taxes credible by ensuring banks have sufficient capital to absorb losses from stranded asset risk.

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[†]London School of Economics and CEPR, e-mail: m.oehmke@lse.ac.uk.

[‡]Stockholm School of Economics and CEPR, e-mail: marcus.opp@hhs.se.

Climate change is at the center of an active policy debate at central banks and financial regulators.[1] From the perspective of bank regulators, climate change is potentially relevant along two dimensions. First, as a consequence of climate change, the banking sector could be exposed to financial risks that are not captured by the current regulatory framework. Second, in the absence of a global carbon tax, some policymakers have argued that capital requirements could serve as a means to reduce carbon emissions and associated externalities.[2]

To investigate these issues, we embed climate-related risks into an otherwise standard model of bank capital regulation. Our positive results show that the effects of green tilts to capital regulation on credit allocation can be subtle. For example, higher capital requirements for dirty loans can reduce clean lending. Conversely, decreases in capital requirements for clean loans can increase dirty lending. These results obtain because changes in capital requirements affect credit allocation via the marginal loan, which can be clean or dirty.

From a normative perspective, our analysis shows that capital requirements can be an effective tool to address prudential risks arising from climate change. However, addressing climate-related financial risks via capital requirements is not equivalent to reducing emissions. For example, it can be optimal for a prudential regulator to increase capital requirements on loans affected by climate risk even if this crowds out clean lending. When carbon taxes are set optimally, a purely prudential mandate for the bank regulator is welfare-maximizing. In contrast, in the absence of optimal carbon taxes, a welfare-maximizing bank regulator may use capital requirements to target emissions in addition to prudential risks. However, our analysis shows that capital requirements alone are an imperfect tool to discourage the funding of carbon-intensive activities. When bank capital is ample, capital regulation is powerless to deter the funding of financially profitable dirty loans even if they generate negative social value. When bank capital is scarce, inducing banks not to fund dirty loans can require lowering capital requirements for clean loans below the prudentially optimal level, thereby sacrificing financial stability. In addition, even if capital regulation can successfully remove dirty loans from the banking system, high-emitting activities will likely attract funding elsewhere as long as they offer a positive return to investors.

Comparing the planner's solution with carbon taxes and capital requirements to a welfare-maximizing bank regulator who sets capital requirements in the absence of carbon taxes demonstrates that interventions that directly reduce the profitability of carbon-

[1] See, e.g., van Steenis (2019), ECB (2021), and Financial Stability Board (2022).

[2] See Dombrovskis (2017).

intensive investments (a carbon tax) are a more effective tool to reduce carbon emissions. However, capital requirements can play an indirect role in facilitating a reduction in carbon emissions: By ensuring sufficient loss-absorbing capital in the banking sector, they can help facilitate the introduction of carbon taxes or other measures, which governments may be reluctant to introduce as long as the resulting revaluation of bank assets ("stranded assets") could trigger a banking crisis.

We develop these insights in the context of a model in which banks extend loans to two types of borrowers, dirty (high carbon emissions) and clean (no emissions). Loans to both types of firms are risky and, when banks cannot repay deposits in full, the deposit insurance steps in. Because deposit insurance is not fairly priced, a deposit insurance subsidy arises, distorting banks' investment incentives. Capital requirements reduce the deposit insurance subsidy (a common feature in many models of bank capital regulation following Kareken and Wallace, 1978) but also reduce lending when bank equity is scarce.

We first provide a positive analysis of exogenous policy interventions, focusing on the two most commonly proposed tools, the *green supporting factor* (lower capital requirements for clean loans) and the *brown penalizing factor* (higher capital requirements for dirty loans). Both of these interventions decrease the relative profitability of making a dirty loan (similar to a substitution effect). However, they have opposite marginal effects on credit allocation. Whereas a brown penalizing factor crowds out the bank's marginal loan, a green supporting factor leads to crowding in at the margin (similar to an income effect). Raising capital requirements for dirty loans, therefore, crowds out clean lending if the marginal loan is clean. Conversely, lowering capital requirements for clean loans can crowd in dirty lending. While our baseline model with two types separates these two effects particularly cleanly, the economic insights carry over to more general settings: The net effect of green tilts to prudential capital requirements on bank funding decisions depends on the relative size of income and substitution effects.

Building on our positive analysis of exogenous policy changes, we then characterize how to optimally account for climate-related financial risks under a strictly prudential mandate. The prudential regulator's objective is to maximize the NPV generated by bank-funded firms net of deadweight costs arising from the deposit insurance put. Because the prudential regulator does not care about carbon emissions per se, emissions are reflected in capital requirements only insofar as they correlate with the NPV of the firm's investment and the associated deposit insurance put. Our analysis shows that capital requirements can effectively address climate-related financial risks. This reflects the broader insight that, conceptually, this is no different from managing "traditional" risks. In practice, the main difference is, therefore, likely one of measurement, given that historical data series contain limited information about the nature of climate-related financial risks.

We illustrate optimal prudential capital requirements in a transition risk scenario in which, due to changes in consumer preferences or environmental regulation, dirty firms become less profitable and riskier relative to a pre-climate risk calibration. The prudential regulator responds to these additional risks by increasing capital requirements for dirty loans, while the effect on capital requirement for clean loans is, in general, ambiguous. Notably, in the case when dirty firms are inframarginal and clean firms are marginal, it is optimal for the prudential regulator to increase capital requirements for dirty loans while keeping capital requirements for clean loans unaffected (since the trade-off for the marginal clean loan is unaffected). Building on our results on the effects of a brown penalizing factor, this implies that lending to marginal clean firms is reduced – under optimal prudential regulation. In this case, the prudential regulator does not act to reduce lending to dirty firms, but simply finds it optimal to require more capital for these loans in order to reduce their deposit insurance put.

We then turn our attention to welfare-optimal regulation that accounts for carbon externalities in addition to prudential considerations. These include direct externalities of carbon emissions on agents in the economy (including future generations) as well as financial risks that are generated by emissions of bank-funded firms but materialize outside of the regulator's perimeter, so that they are not captured by the regulator's prudential mandate (e.g., physical risks that mainly affect firms and banks in other parts of the world).

We first solve for the optimal allocation when a planner has access to both carbon taxes and capital requirements. This case demonstrates that once capital requirements are set optimally, a strict prudential mandate for bank regulators becomes optimal. In effect, carbon externalities and prudential concerns are dealt with separately. We then investigate welfare-maximizing capital requirements in a setting in which carbon taxes are absent (or subject to frictions). In the absence of carbon taxes, a welfare-maximizing regulator can implement the planner's allocation only in some cases. In particular, when the banking sector is relatively well-capitalized, it becomes necessary to lower capital requirements for clean loans below those set by the planner in order to prevent the funding of dirty loans. In fact, in some cases it becomes optimal for the welfare-maximizing regulator with one tool to "give up" on the goal of lowering carbon emissions and act as if its mandate was purely prudential. If (some) dirty firms have access to alternative sources of financing (e.g., via the bond market) the ability to reduce carbon externalities via capital requirements is constrained even further due to substitution to other funding markets. (In contrast, a prudential regulator would welcome substitution to the bond market because it removes risk from the banking sector.)

While capital requirements alone are therefore not an effective tool to address carbon externalities, they can play in indirect role. In particular, governments may be reluctant to introduce carbon taxes if the resulting revaluation of legacy assets leads to stranded asset risk that could trigger a banking crisis. Even worse, anticipating this, banks have no incentive to reduce their carbon exposure, leading to an inefficient regulatory standstill. If government inaction results from such a commitment problem, capital requirements can make carbon taxes credible by removing stranded asset risk from the banking sector. Therefore, even though our results do not support the use of capital requirements to replace carbon taxes or other forms of direct government intervention, they point to one specific channel in which they can facilitate government action by removing stranded asset risk.

Related literature. Our model builds on the large literature on prudential bank capital regulation.[3] This literature has focused on capital regulation in the presence of distortions introduced by deposit insurance, but has not considered how climate change affects capital requirements, which is the central focus of our paper. Introducing climate change leads two major departures from this literature. First, climate-related risks (see, e.g., Giglio, Kelly and Stroebel (2021)) become relevant for prudential bank capital regulation insofar as they affect financial risks in the banking sector. Second, climate change may lead to a change in the regulatory objective function to include carbon externalities, in addition to prudential risks in the banking sector. In this respect, our model is related to Thakor (2021), who develops a model of bank capital regulation in which the regulator's objective includes political considerations.

Our analysis of optimal capital regulation is complementary to Dávila and Walther (2022), who develop a general model of optimal second-best regulation, with an application to financial regulation in the presence of environmental externalities. Two recent papers have investigated positive effects of green capital requirements but do not consider optimal capital regulation: Dafermos and Nikolaidi (2021) study green differentiated capital requirements in a dynamic macrofinance model. Thomä and Gibhardt (2019) estimate the effect of green supporting and brown penalizing factors on required bank capital, assuming that the composition of bank balance sheets is unaffected by such a policy change.

[3] This literature includes, among others, Kareken and Wallace (1978), Rochet (1992), Repullo (2004), Pennacchi (2006), Repullo and Suarez (2004), Allen, Carletti and Marquez (2011), Admati, DeMarzo, Hellwig and Pfleiderer (2011), Martinez-Miera and Suarez (2012), Acharya, Mehran and Thakor (2016), Bahaj and Malherbe (2020, 2024), Malherbe (2020), Begenau (2020) and Harris, Opp and Opp (2025).

While the focus of our paper is on bank capital regulation, Papoutsi, Piazzesi and Schneider (2022) study the environmental impact of central bank asset purchases. Whereas bond purchases affect mainly firms that rely disproportionately on bond financing, bank capital regulation has the strongest effect on bank-dependent firms. Our result that capital requirements have limited ability to deter loans to dirty companies is reinforced if banks are worried that investing in (new) green loans will devalue dirty legacy assets, as pointed out by Degryse, Roukny and Tielens (2022). Jondeau, Mojon and Monnet (2021) propose a liquidity backstop to prevent runs on brown assets that may occur as part of the transition toward a greener economy.

1 Model

1.1 Model Setup

We consider a model with two dates ($t = 0, 1$), universal risk-neutrality, and no time discounting. The economy consists of three types of agents: a continuum of firms with investment opportunities, a continuum of competitive banks, and a regulator.

Firms. Each firm is of infinitesimal size and born as one of two observable types, $q \in \{C, D\}$, which we will refer to as *clean* and *dirty*.[4] We normalize the total mass of firms to one and denote the population fraction of type q as $\bar{\pi}_q$. For both types, production requires an investment of fixed scale I at $t = 0$. At date $t = 1$, random cash flows X_q and emissions ϕ_q are realized. Production by dirty firms causes higher carbon emissions, $\phi_D > \phi_C = 0$, where we normalize emissions by clean firms to zero. We also normalize the social cost of carbon to one, so that emission levels equal their social cost. We denote the mean cash flow of a firm of type q by $\overline{X}_q$. We assume that cash flows are perfectly correlated within each type but can have arbitrary correlation across types. Both firm types have profitable investment opportunities in the absence of carbon taxes, i.e.,

$$\text{NPV}_q := \overline{X}_q - I > 0 \qquad \forall q.$$

Firms have no internal funds, so they need to raise I units of outside financing to produce.

Banks. Firms can raise funds for production by obtaining a loan from a continuum of competitive and ex-ante identical banks (also of mass one). Each bank is endowed with inside equity $E \leq I$. Because there is a unit mass of banks, E also corresponds to the aggregate amount of equity in the banking sector. Upon raising D units of deposits from

[4] In Section 3, we discuss the implications of a large number of firm types and the possibility that firms can change their type and become cleaner at a cost, as in Oehmke and Opp (2025).

competitive depositors, a bank can finance an amount A of loans to firms, where

$$A = E + D \tag{1}$$

represents the bank's book value of assets (or loans).

Bank capital structure matters because the model features two deviations from the Modigliani-Miller benchmark. First, we assume that outside equity issuance is subject to frictions. For ease of exposition, we assume that the associated issuance cost is prohibitively high, so that bank equity is fixed at E.[5] Second, deposit insurance (or, equivalently, an implicit or explicit bailout guarantee for debtholders) results in an effective subsidy for deposit financing and rationalizes capital regulation (see below).[6] In our model, deposit insurance is not priced, so that total payouts to bank security holders are increasing in the deposit-to-asset ratio $\frac{D}{A}$. The results would be similar if deposit insurance were priced imperfectly, as in Chan, Greenbaum and Thakor (1992).[7]

Banks maximize the expected payoff to bank equityholders at date 1,

$$V = \max_{e,\mathbf{w}} E\left[1 + r_E\left(\mathbf{w}, e\right)\right], \tag{2}$$

where we define $e := \frac{E}{A}$ as the bank's (book) equity ratio and $r_E\left(\mathbf{w}, e\right)$ as the bank's expected return on equity (ROE), and where $\mathbf{w} = (w_C, w_D)$ denotes the portfolio weights of clean and dirty loans, respectively. Given that bank equity E is fixed, this objective function boils down to maximizing the bank's expected ROE. (Note that in our risk-neutral setting, any ROE exceeding 0 reflects a scarcity rent rather than a risk premium.)

Bank Regulator. The bank regulator sets capital requirements $\underline{e}_q$ as a function of the (observable) firm type q.[8] Given loan portfolio weights w_q, a bank then faces an equity ratio constraint

[5] Our results remain qualitatively unchanged if banks can issue additional outside equity at a positive but non-prohibitive marginal cost (see the discussion in Section 3). Moreover, even though banks could use their equity capital to pay dividends, as we will show below this is never optimal under optimal capital regulation.

[6] For ease of exposition, we simply assume the presence of deposit insurance or, equivalently, an implicit or explicit bailout guarantee. Deposit insurance arises naturally in banking models with fragility, following Diamond and Dybvig (1983). Dávila and Goldstein (2023) propose a model of optimal deposit insurance. Acharya and Yorulmazer (2007), Farhi and Tirole (2012), Bianchi (2016), Chari and Kehoe (2016), and Philippon and Wang (2022), among others, develop models of endogenous bailouts.

[7] One may wonder why we assume both a cost of outside equity and a (private) benefit of debt, given that either of these frictions would be sufficient to ensure that banks favor debt financing. The reason is that, in the absence of costly equity issuance, the regulator could simply eliminate bailout distortions by setting capital requirements to 100%.

[8] It is not crucial for our results that firm types are perfectly observable. The main results continue to hold if the regulator observes a noisy signal of firm type (see Section 3).

$$e \geq e_{\min}(\mathbf{w}) := \sum_q w_q \cdot \underline{e}_q. \tag{3}$$

Capital requirements have two main effects. First, by absorbing loan losses, higher capital requirements reduce transfers from the deposit insurance fund. We assume that such transfers are associated with a deadweight cost due to a positive shadow cost of public funds λ (see e.g., Farhi and Tirole (2021)). Second, higher capital requirements for a firm of type q affect banks' loan decisions and, therefore, the mass of funded firms, which we denote as $\pi_q \leq \bar{\pi}_q$.

1.2 Equilibrium with Exogenous Capital Requirements

As a preliminary step to our policy analysis in Section 2, we first characterize the equilibrium for exogenously given capital requirements. The analysis in this subsection draws on Harris et al. (2025), and we therefore present the results in a heuristic fashion. All proofs can be found in Appendix A.

We first characterize optimal decisions by individual banks and then characterize equilibrium lending by the banking sector as a whole.

Result 1 (Maximum Leverage and Specialization) *The regulatory equity ratio constraint binds, $e^* = e_{\min}(\mathbf{w}^*)$. Moreover, each individual bank finds it optimal to specialize in funding either exclusively clean or dirty firms.*

Result 1 states that individual banks maximize the amount of deposit funding and choose specialized portfolios. Maximum deposit funding is optimal because deposit insurance generates a subsidy for deposits. Specialization increases this subsidy by reducing diversification across loan types.[9] Because deposits are priced competitively, i.e., depositors require a net return of zero, the value of the deposit insurance put accrues to bank equityholders.

We now turn to the equilibrium lending decisions of the banking sector. It is useful to frame the banking sector equilibrium in terms of aggregate bank equity E, which is the scarce resource in the economy: When a firm of type q demands a loan of size I, this translates into demand for $I\underline{e}_q$ units of bank equity.

Given objective function (2), banks rank borrowers according to the maximum ROE associated with a loan. This maximum ROE is determined by the maximum interest

[9] While the prediction of fully specialized portfolios is somewhat extreme, it is analytically convenient because it allows us to derive closed-form solutions. Economically, all main insights carry through to the case in which banks' loan portfolios are not specialized.

rate a borrower would be willing to pay for the loan. As in standard consumer theory, the demand curve is then characterized by reservation prices, in this case in the form of the maximum ROE a borrower can offer to a bank.

Result 2 (Maximum ROE) *At the maximum interest rate that a borrower of type q is willing to pay, the bank equityholders' expected ROE is given by*

$$r_q^{\max}\left(\underline{e}_q\right) = \frac{NPV_q + PUT_q\left(\underline{e}_q\right)}{I\underline{e}_q}, \tag{4}$$

where $PUT_q\left(\underline{e}_q\right)$ *denotes the loan's contribution to the bank's deposit insurance put in an optimal loan portfolio,*

$$PUT_q\left(\underline{e}_q\right) = \mathbb{E}\left[\max\left\{I(1-\underline{e}_q) - X_q, 0\right\}\right]. \tag{5}$$

At the borrower's reservation interest rate, all expected surplus generated by the loan accrues to bank equityholders.[10] This surplus consists of the NPV of the firm's project and the value of the deposit insurance put associated with the loan under optimal (maximum) leverage and specialization, see Result 1.

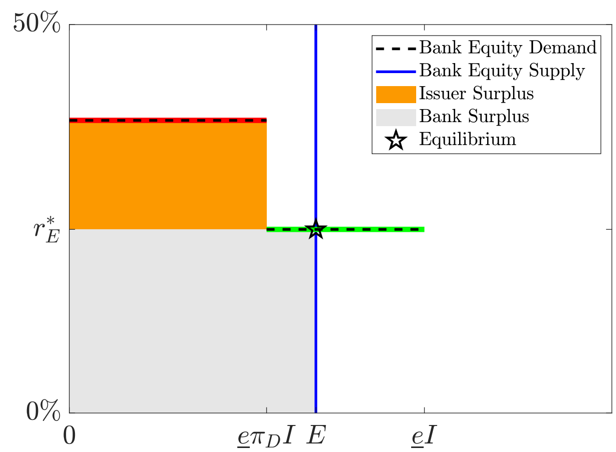

Figure 1. Banking Sector Equilibrium. This figure illustrates the banking sector equilibrium for an example economy where dirty firms can offer a higher maximum ROE to banks. Dirty firms are depicted in red, clean firms in green. The equilibrium ROE is denoted by r_E^*.

Because banks behave competitively in the lending market, they typically cannot extract all surplus from borrowers. Instead, the equilibrium return on bank equity r_E^* is

[10] If borrowers had access to non-bank financing, say via a competitive bond market, then this outside option would pin down the maximum interest rate the borrower is willing to pay for a bank loan (see the proof of Result 2 and Section 3 for a discussion). In our baseline model firms are bank-dependent for simplicity. Therefore, the outside option is not to invest at all and, therefore, equal to zero.

pinned down by the intersection of the aggregate demand for bank equity (from funded loans) and its (fixed) supply E.

The resulting equilibrium is illustrated in Figure 1 for an example specification in which dirty firms can offer a higher maximum ROE to banks. Since the borrower types feature distinct maximum ROE, the demand curve is a step function. In the illustrated equilibrium, dirty borrowers (red) are fully funded (they are inframarginal), whereas clean borrowers (green) are only partially funded (they are marginal). As can be seen, only the aggregate supply of bank equity matters of the equilibrium allocation of funds to firms, consistent with the baseline assumption of Philippon and Wang (2022). Since both types are funded in equilibrium, Result 1 implies that a subset of banks will specialize in funding all dirty firms and the remaining banks will finance exclusively clean firms. The loan rate for the marginal green borrowers is set such that all surplus accrues to banks (i.e., there is no consumer surplus for marginal loans). Inframarginal borrowers, on the other hand, obtain some consumer (or "issuer") surplus, which ensures that banks are indifferent between funding either type. More generally, we obtain

Result 3 (Banking Sector Equilibrium) *If $E < I \sum_q \bar{\pi}_q \cdot \underline{e}_q$, bank capital is scarce, so that $r_E^* > 0$. Marginal borrower types, satisfying $r_q^{\max} = r_E^*$, are partially funded. Borrowers with $r_q^{\max} > r_E^*$ are fully funded. If $E \geq I \sum_q \bar{\pi}_q \cdot \underline{e}_q$, all types are fully funded and bank capital is not scarce so that $r_E^* = 0$.*

Result 3 highlights the importance of the marginal borrower type, which pins down r_E^* and, therefore, the funding terms for all inframarginal types with $r_q^{\max} > r_E^*$. Which borrower type is marginal depends not only on exogenous firm or bank characteristics (such as the firm's NPV, and the capitalization of the banking sector) but also on the regulator's choice of capital requirements.

1.3 First-Best Benchmark

To clarify the distortions that arise in the decentralized banking economy, we briefly discuss the first-best allocation and how it could be implemented by a planner. We define welfare as total surplus, consisting of the total financial NPV of firm investments net of externalities and the deadweight cost of the deposit insurance put. We assume that this deadweight cost is linear in the size of the fiscal transfer to the banking sector, reflecting a constant marginal cost of public funds λ. Welfare can then be expressed as

$$\Omega = \sum_q \pi_q\left(\underline{\mathbf{e}}\right)\left[\mathrm{NPV}_q - \phi_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)\right].^{11} \tag{6}$$

The decentralized banking equilibrium features two distortions. First, the externality ϕ_q is unaccounted for in the bank's objective function. Second, the subsidy arising from the deposit insurance put $\mathrm{PUT}_q\left(\underline{e}_q\right)$ enters the banking sector's decision metric $r_q^{\max}\left(\underline{e}_q\right)$ with the opposite sign when compared to the planner's objective (6). The planner only wants those firm types to be funded, $\pi_q\left(\underline{\mathbf{e}}\right) > 0$, whose financial NPV exceeds the externality and the deadweight cost arising from the deposit insurance put.

We now highlight one particularly simple case in which the first-best allocation can be achieved with two simple tools, a carbon tax and a capital requirement $\underline{e}_q$. We suppose the carbon tax $\tau_q \geq 0$ is collected when cash flows are realized so that $\tau_q \leq X_q$ for every realization of X_q. We denote the expected carbon tax payment by $\bar{\tau}_q := \mathbb{E}\left(\tau_q\right)$. Because of the carbon tax, the after-tax financial NPV becomes $\mathrm{NPV}_{q,\tau_q} := \mathrm{NPV}_q - \bar{\tau}_q$. The after-tax deposit insurance put is $\mathrm{PUT}_{q,\tau_q}\left(\underline{e}_q\right) := \mathbb{E}\left[\max\left\{I(1-\underline{e}_q) - \left(X_q - \tau_q\right), 0\right\}\right]$, so that the banking sector's decision metric, the after-tax maximum ROE, becomes $r_{q,\tau_q}^{\max}\left(\underline{e}_q\right) = \frac{\mathrm{NPV}_{q,\tau_q} + \mathrm{PUT}_{q,\tau_q}\left(\underline{e}_q\right)}{I\underline{e}_q}$. (Going forward, we omit the subscript τ_q when carbon taxes are absent.)

Observation 1 *If the banking sector is sufficiently well capitalized, $E > \sum_q \bar{\pi}_q \mathbb{I}_{NPV_q > \phi_q} I$, the first-best allocation can be implemented by a combination of a carbon tax and a capital requirement of $\underline{e}_q = 100\%$.*

Given sufficient equity E, capital requirements and a carbon tax completely eliminate both distortions. Given a capital requirement of 100%, the risk-taking distortion due to the deposit insurance put vanishes given that $r_q^{\max}\left(1, \tau_q\right) = \frac{\mathrm{NPV}_q - \bar{\tau}_q}{I}$. Then, under the optimal carbon tax, banks find it optimal to only fund borrowers whose social NPV is positive, $\mathrm{NPV}_q - \phi_q > 0$.

There are two potential issues with this simple implementation of the first-best allocation. First, if banking sector equity is scarce it is impossible to ensure that all socially valuable firms are funded without tolerating a positive probability of bank failure. Second, environmental regulation can be inefficiently lax due to policy failure (see, e.g., Tirole (2012)). We now address these issues in our policy analysis.

[11] This specification does not account for consumer surplus. This would obtain, for example, if firms are able to extract all the surplus in the (unmodeled) product market. However, accounting for consumer surplus would not qualitatively change our main results.

2 Policy Analysis

Our policy analysis proceeds in three steps. In Section 2.1, we investigate the effects of exogenous changes to borrower-specific capital requirements on bank funding decisions. This analysis informs the debate regarding the effects of ad-hoc green tilts to capital requirements, as currently discussed in policy circles. Building on these insights, Section 2.2 then analyzes how a banking regulator would optimally set borrower-specific capital requirements under a prudential mandate (i.e., only considering financial stability) and how such a regulator adjusts regulation in response to climate-related financial risks. In Section 2.3, we then consider welfare-maximizing regulation, considering both carbon taxes and capital regulation. This section clarifies that a prudential mandate for a banking regulator is welfare optimal if environmental policy is not subject to frictions. It also investigates the role of capital regulation when carbon taxes are absent or subject to a commitment problem.

2.1 Green Tilts to Capital Requirements

Green tilts to capital requirements can take the form of a reduction in the capital requirement for clean loans (a *green supporting factor*) or an increase in capital requirement for dirty loans (a *brown penalizing factor*). Even though the focus of our paper is on clean and dirty borrowers, the conceptual insights from this analysis apply in any situation in which a regulator changes capital requirements (or risk weights) for a subset of firms.[12] To emphasize the broader applicability of our results, we first state a general proposition on how changes in capital requirements affect the banking sector equilibrium described in Section 1.2.

Proposition 1 (Tilts to Capital Requirements) *A sufficiently small increase in capital requirements for* ***any funded*** *borrower only reduces the funding of the* ***marginal*** *borrower type. If the increase (decrease) in capital requirements for inframarginal (marginal) borrowers firms exceeds a cut-off, the banking sector's ranking of borrower types reverses.*

Proposition 1 follows from the observation that a change in the capital requirement for one borrower type has two effects. First, it changes the affected borrower's maximum ROE, leading to an upward or downward shift in the respective segment of the demand curve. In particular, an increase in the capital requirement lowers the affected borrower's ROE via both the numerator and the denominator in Equation (4). This first effect

[12] Examples include the reduction in capital requirements for small and medium enterprises (the "SME supporting factor") introduced in 2014 and the infrastructure supporting factor (ISF) introduced in 2020.

induces a change in relative (reservation) prices, which can lead to a *substitution effect* from one borrower type to another. Second, a change in capital requirements changes the horizontal length of the relevant segment of the demand curve. If capital requirements increase, each loan to the affected borrower type requires more equity, so that the respective segment of the demand curve lengthens. This second effect is akin to an *income effect* that arises from the tightening of the banking sector's budget constraint.

Note that sufficiently small changes in capital requirements leave the ranking of borrowers unchanged. In this case, only the income effect is at play, leading to a crowding out of the marginal borrower (the first statement of Proposition 1). For sufficiently large changes in capital requirements, the ranking of borrower types can switch (the second statement of Proposition 1) due to the substitution effect.

We now apply Proposition 1 to illustrate the effects of a brown penalizing factor and green supporting factor, respectively.

2.1.1 Brown Penalizing Factor

Proposition 1 suggests that it is instructive to analyze the effects of a brown penalizing factor depending on which firm type is marginal. We first describe the case in which, prior to the intervention, clean firms are marginal (as was the case in Figure 1). Figure 2 illustrates the effect of a brown penalizing factor for this case. The left panel plots how the equilibrium changes in response to a small brown penalizing factor that leaves the ranking of borrowers unchanged. After the introduction of the BPF, funding the same number of dirty loans requires more bank equity, so that the dirty-loan segment of the demand curve lengthens (comparing the dotted and solid red lines). As a result, less equity is available to fund clean loans. The marginal clean loan is crowded out, as described in part 1 or Proposition 1. Conversely, if prior to the introduction of the brown penalizing factor the dirty firm is marginal (not pictured), then the brown penalizing factor reduces the funding of dirty loans.[13] Note that in both cases, the effects of a small brown penalizing factor are entirely driven by the income effect.

Corollary 1 (Brown Penalizing Factor) *If dirty firms are inframarginal, a marginal BPF reduces lending to clean firms and leaves lending to dirty firms unchanged. If dirty firms are marginal, a marginal BPF reduces lending to dirty firms and leaves lending to clean firms unchanged.*

The right panel of Figure 2 shows that, if the brown penalizing factor is sufficiently

[13] This case arises if either only the dirty type is funded (e.g., sufficiently low E for otherwise identical parameters as in Figure 2) or if the clean firm is ranked first and inframarginal, i.e., $r_C^{\max} > r_D^{\max}$.

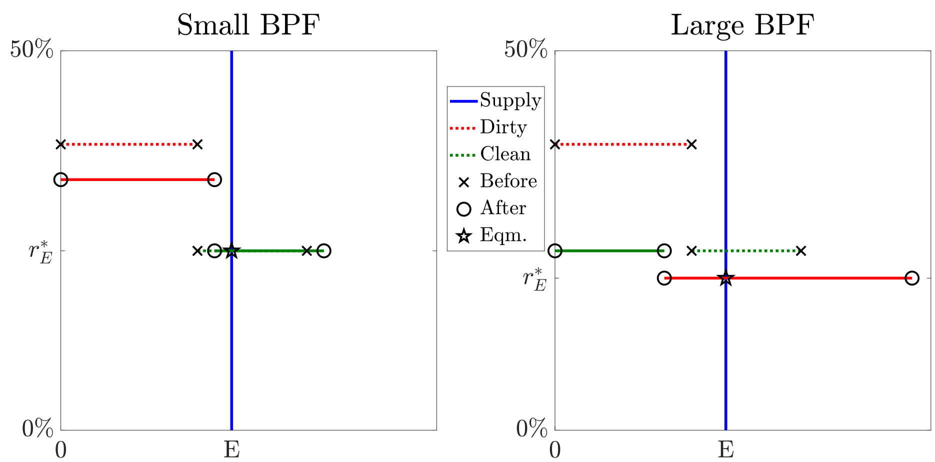

Figure 2. Brown penalizing factor (illustrated case: marginal firm is clean). The left panel illustrates the equilibrium impact of a small brown penalizing factor that leaves the relative ranking of firm types unchanged. The right panel illustrates the equilibrium impact of a large brown penalizing factor that reverses the relative ranking of firm types. Dotted lines and segment endpoints marked ∘ denote the benchmark equilibrium. Solid lines and segment endpoints marked × denote the equilibrium after the introduction of the brown penalizing factor.

large, the ranking of clean and dirty loans in terms of the borrower reservation price can be reversed (see Part 2 of Proposition 1). In this case, banks react by exhausting all clean lending opportunities before they start funding of dirty firms. Therefore, clean lending increases and dirty lending decreases. This result is driven both by the substitution effect (clean loans get funded first) and the income effect (the lengthening of the dirty-loan segment of the demand curve).

2.1.2 Green Supporting Factor

We now turn to the introduction of a green supporting factor. Mirroring the analysis of a brown penalizing factor, a small green supporting factor leaves the ranking of firms unaffected, so that the effect on the equilibrium allocation is driven entirely driven by the income effect. However, because the green supporting factor is a reduction in capital requirements, the income effect goes in the opposite direction, *crowding in* the marginal borrower. Hence, we obtain

Corollary 2 (Green Supporting Factor) *If clean firms are marginal, a marginal GSF increases lending to clean firms and leaves lending to dirty firms unchanged. If clean firms are inframarginal, a marginal GSF increases lending to dirty firms and leaves lending to clean firms unchanged.*

In sum, Corollaries 1 and 2 show that the brown penalizing and green supporting factors induce directionally equivalent substitution effects. In contrast, the resulting income effects go in different directions. Depending on which firm is marginal, these income effects can lead to counterintuitive effects. In particular, a brown penalizing factor crowds out clean lending when clean firms are marginal. Conversely, a green supporting factor crowds in dirty lending when dirty firms are marginal.

2.2 Prudential Capital Requirements

Up to now, our analysis has focused on two ad-hoc interventions, the brown penalizing and green supporting factors, starting from a benchmark equilibrium with exogenously given capital requirements. In this section, we analyze under which conditions these tools are employed as part of optimal capital regulation in response to emerging climate-related risks. In Section 2.2.1, we derive optimal capital requirements under a prudential mandate and characterize comparative statics with respect to changes in firm cash-flow distributions. In Section 2.2.2, we then apply these comparative statics to investigate the optimal prudential policy response to climate-related financial risks.

2.2.1 The Principles of Optimal Prudential Regulation

The prudential mandate trades off real activity, as measured by the financial value (or NPV) created by bank lending, against the deadweight costs generated by deposit insurance. The regulator's objective function is to maximize prudential surplus Ω_P given by

$$\Omega_P := \sum \pi_q\left(\underline{\mathbf{e}}\right)\left[\mathrm{NPV}_{q,\tau_q} - \lambda \cdot \mathrm{PUT}_{q,\tau_q}(\underline{e}_q)\right], \tag{7}$$

where the mass of funded firms $\pi_q\left(\underline{\mathbf{e}}\right)$ and the deposit insurance put $\mathrm{PUT}_{q,\tau}(\underline{e}_q)$ depend on the capital requirements for clean and dirty firms, $\underline{\mathbf{e}} = (\underline{e}_C, \underline{e}_D)$. As before, the deposit insurance put is associated with a linear deadweight cost. Even though prudential surplus Ω_P does not account for externalities (and, therefore, differs from the planner's objective Ω given in Equation (6)), we show in Corollary 3 that a strictly prudential objective leads to welfare-maximizing capital requirements under socially optimal carbon taxes.

To characterize optimal prudential capital requirements, it is instructive to rewrite the regulator's objective function as

$$\max_{\underline{\mathbf{e}}} \Omega_P = E \max_{\underline{\mathbf{e}}} \sum \kappa_q \mathrm{PPI}_{q,\tau_q}(\underline{e}_q), \tag{8}$$

where $\kappa_q := \frac{\pi_q(\underline{\mathbf{e}}) I \underline{e}_q}{E} \in [0,1]$ reflects the fraction of total equity that the banking sector allocates to funding type q. $\mathrm{PPI}_q(\underline{e}_q)$ denotes the *prudential profitability index.* In analogy to the banker's maximum ROE given in equation (4), the PPI reflects the surplus created per unit of bank equity from the prudential regulator's perspective,

$$\mathrm{PPI}_{q,\tau_q}(\underline{e}_q) = \frac{\mathrm{NPV}_{q,\tau_q} - \lambda \cdot \mathrm{PUT}_{q,\tau_q}(\underline{e}_q)}{I\underline{e}_q}. \tag{9}$$

Equation (9) reveals that carbon taxes feed into the PPI via its effect on firms' after-tax cash flows, which affect both the NPV and the deposit insurance put. Comparing equations (4) and (9), we see that there are two main differences between the regulator's PPI and the bankers' maximum ROE. First, the deposit insurance put enters with opposite sign, reflecting the wedge between the regulator's preferences and those of the banking sector. Second, whereas banks take ROEs as given, the regulator affects the PPI for each loan type via the chosen capital requirements.

We impose regularity conditions such that the capital requirement that maximizes the PPI for each type q, $\underline{e}_q^{\mathrm{PPI}}$, is interior and characterized by the first-order condition

$$\mathrm{PPI}_{q,\tau_q}(\underline{e}_q) = -\lambda \frac{\partial \mathrm{PUT}_{q,\tau_q}(\underline{e}_q)}{\partial \underline{e}_q} / I.\text{[14]} \tag{10}$$

The left-hand side of Equation (10) captures the marginal cost of increasing capital requirements. Fewer firms of a given type can be financed, resulting in a loss of prudential surplus PPI_q. The right-hand side captures the marginal benefit of higher capital requirements for type q, in the form of a lower deposit insurance put per unit of investment I (note that $\partial \mathrm{PUT}_{q,\tau_q(\underline{e}_q)}/\partial \underline{e}_q < 0$).

From the prudential regulator's perspective, a borrower with a higher PPI delivers more "bang for the buck" (prudential value per unit of equity capital) and is therefore preferred.

Definition 1 (The Prudential Regulator's Preferred Type) *The prudential regulator's preferred type is the one that achieves the highest possible PPI, i.e.,* $\max_q PPI_{q,\tau_q}(\underline{e}_q)$.

As shown in Proposition 2, the PPI plays an important role in characterizing optimal prudential capital regulation.

Proposition 2 (Principles of Optimal Prudential Regulation) *Optimal prudential regulation is characterized by the following four principles.*

[14] Lemma A.3 in the appendix shows that this regularity condition is satisfied for the log-normal distribution as long as λ is sufficiently high.

P1: Capital requirements are set sufficiently high so that banks allocate all equity towards funding real activity (rather than paying a dividend at date 0).

$$\sum_q \pi_q(\mathbf{e})\, \underline{e}_q I = E. \tag{11}$$

P2: For sufficiently low levels of bank equity, the regulator sets capital requirements such that banks lend exclusively to the regulator's preferred type $\max_q PPI_q(\underline{e}_q^{PPI})$.

P3: If firm type q is partially funded, its capital requirement maximizes PPI_q,

$$\hat{\underline{e}}_q = \underline{e}_q^{PPI}. \tag{12}$$

P4: If multiple firm types are funded, marginal deposit insurance puts are equalized across funded types,

$$\frac{\partial PUT_D}{\partial \underline{e}_D} = \frac{\partial PUT_C}{\partial \underline{e}_C}. \tag{13}$$

Principle P1 reflects that it is optimal to use all bank equity to generate prudential surplus. Principle P2 states that the first funded type is the prudential regulator's preferred type. Principle P3 states that the optimal capital requirement for the marginal type is set to maximize its PPI, $\hat{\underline{e}}_q = \underline{e}_q^{\text{PPI}}$, as in Equation (10). Finally, Principle P4 links capital requirements *across* funded types.

Principle P4 applies when both types are fully funded and when one type is partially funded (marginal). When both types are fully funded, marginal changes in either capital requirement do not affect lending decisions in the economy. In that case, capital requirements only serve to decrease the deadweight cost arising from the deposit insurance put, which is optimally done by equating marginal puts as indicated by Equation (13).

When one type is marginal, the regulator trades off financial stability against the value of additional lending at the margin. In this region, higher capital requirements for any funded type q crowd out lending to the marginal type q_m, with associated $\text{PPI}_{q_m,\tau_{q_m}}(\underline{e}_{q_m})$. Capital requirements for all funded types q then satisfy the optimality condition

$$-\lambda \frac{\partial \text{PUT}_{q,\tau_q}(\underline{e}_q)}{\partial \underline{e}_q} / I = \text{PPI}_{q_m,\tau_{q_m}}(\underline{e}_{q_m}). \tag{14}$$

Optimality condition (14) implies that $\underline{e}_{q_m}^* = \underline{e}_{q_m}^{PPI}$ and that marginal puts are equalized *across* funded types as indicated by Principle P4.

Based on these four core principles, Figure 3 plots optimal prudential capital re-

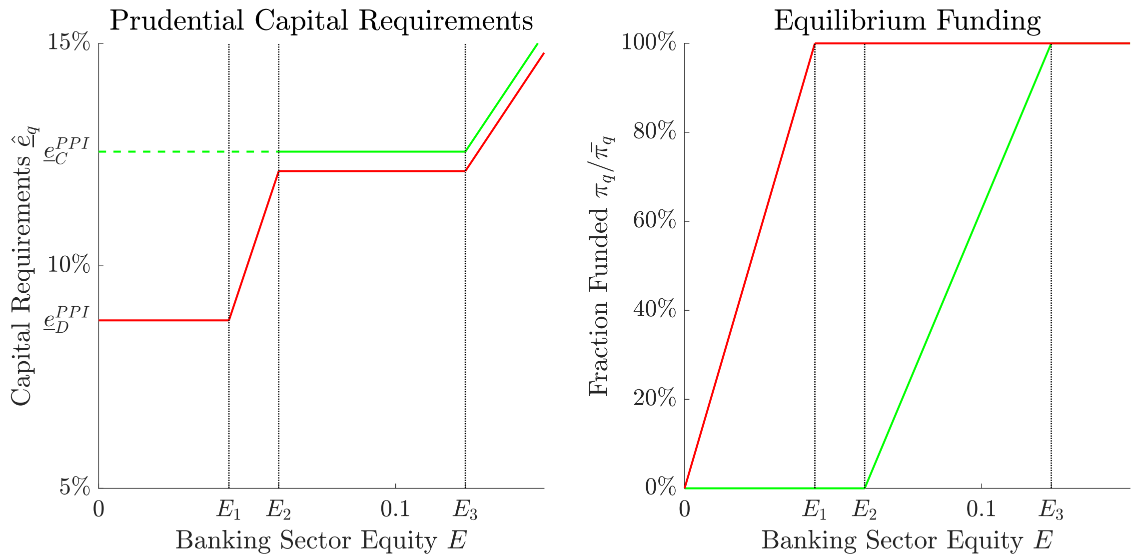

Figure 3. Optimal prudential capital regulation. This figure plots capital requirements (left panel) and equilibrium funding decisions (right panel) under optimal prudential bank capital regulation. Clean firms and their capital requirements are plotted in green, dirty in red. The dotted green line indicates that in this region only dirty loans are funded. This figure assumes the following baseline parameters: The mean log return on assets is $\mu_D = 3\%$ for dirty types and $\mu_C = 2.5\%$ for clean firms. The asset volatility for both types is $\sigma = 20\%$ as in Collin-Dufresne and Goldstein (2001). The population proportion of both types is $\bar{\pi}_C = \bar{\pi}_D = 50\%$. The required fixed scale investment for both types is normalized to $I = 1$, so that $E = 0.1$ implies that aggregate banking sector equity represents 10% of overall investment opportunities in the economy. The marginal cost of public funds is $\lambda = 1$. Carbon taxes for both types are set to zero. This parameterization yields capital requirements of $\underline{e}_D^{\text{PPI}} = 8.8\%$ and $\underline{e}_C^{\text{PPI}} = 12.6\%$.

quirements and the corresponding equilibrium funding decisions as a function of the capitalization of the banking sector E.[15] For illustrative purposes, the figure plots the case in which dirty firms are more profitable, $\overline{X}_D > \overline{X}_C$. In the opposite case, $\overline{X}_C > \overline{X}_D$, the figure would look identical with types reversed. For this graph and the remainder of this section we assume a log-normal cash-flow distribution with expected cash flow $\overline{X} = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$ and volatility parameter σ.

Figure 3 shows that optimal prudential capital requirements are weakly increasing in the capitalization of the banking sector E. This follows from the fact that the prudential value generated by bank lending declines as the most valuable types are funded first. In particular, for sufficiently scarce equity, $E < E_1 := \bar{\pi}_D \underline{e}_D^{\text{PPI}} I$, only the regulator's preferred type (in this illustration, the dirty type) is funded. The dirty type is marginal, so that the optimum prudential capital requirement is pinned down by Principle P3,

[15] Because we normalize the required fixed scale investment for both types to $I = 1$, E can be interpreted as banking sector equity relative to total investment opportunities in the economy. For example, $E = 0.1$ implies that aggregate banking sector equity represents 10% of overall investment opportunities in the economy.

$\hat{\underline{e}}_D = \underline{e}_D^{\text{PPI}}$. The mass of funded of dirty types, $\pi_D = E/\underline{e}_D^{\text{PPI}}$, is linearly increasing in banking sector equity until all dirty types are funded. Capital requirements for the unfunded clean type must be sufficiently high to deter lending to clean firms (e.g., by setting them to $\underline{e}_C^{\text{PPI}}$, as indicated by the green dashed line).

In the second region, $E \in (E_1, E_2)$, dirty firms are fully funded, $\pi_D = \bar{\pi}_D$. However, rather than inducing banks to fund clean firms, in this region it is optimal to use all bank equity to lower the deposit insurance put of dirty loans (i.e., $\hat{\underline{e}}_D = \frac{E}{\bar{\pi}_D I}$), e.g., Principle P1 applies. This is optimal since the marginal benefit of funding the next best investment opportunity, the clean type, is lower by a discrete amount.

Once the capitalization of the banking sector reaches $E = E_2$, the marginal reduction in the deadweight cost associated with the deposit insurance put is equal to the marginal value of funding a clean firm. Therefore, in the third region, $E \in (E_2, E_3)$, it becomes optimal to induce banks to fund some clean firms. Clean firms are now the marginal type, so that $\hat{\underline{e}}_C = \underline{e}_C^{PPI}$ by Principle P3. The capital requirement for the inframarginal (dirty) type is then determined by Principle P4, the equalization of marginal puts.

Finally, in the fourth region, $E > E_3$, both types are fully funded. In this region, any additional bank equity is used to reduce the deadweight costs arising from deposit insurance while maintaining the equalization of marginal puts (Principles P1 and P4).

We now investigate the comparative statics of optimal capital requirements with respect to firm profitability $\overline{X}$ and cash-flow volatility σ. (To obtain clean comparative statics with respect to σ in the lognormal specification, we adjust μ to keep $\overline{X}$ constant.)

Proposition 3 (Comparative Statics) *Assume that either the clean or the dirty type is partially funded.*

1. *A decrease in profitability $\overline{X}$ or an increase in riskiness σ of the **marginal** type leads to higher optimal prudential capital requirements for **all** funded types.*

2. *A decrease in profitability $\overline{X}$ or an increase in riskiness σ of the **inframarginal** type leads to higher optimal prudential capital requirements for the **inframarginal** type only. The optimal prudential capital requirement for the marginal type remains unchanged.*

Proposition 3 focuses on the most interesting cases in which one firm type is partially funded, implying that marginal changes to capital requirements affect bank lending decisions.[16] This corresponds to the first and the third regions illustrated in Figure 3. Part 1 of Proposition 3 reflects that, if the marginal bank-funded type becomes riskier or less

[16] For completeness, note that in the second and fourth region, $E \in (E_1, E_2)$ and $E > E_3$, neither type

profitable, the marginal benefit of bank lending (viewed from the prudential regulator's perspective) is reduced. Since optimal capital requirements are determined by a trade-off between enabling prudentially valuable lending and the social cost of levered bank financing (see the above discussion of Principle P4), a lower PPI of the marginal loan makes it to optimal to increase buffers across the entire banking sector by raising capital requirements for all types.

In contrast, part 2 of Proposition 3 states that, when an inframarginal type becomes riskier or less profitable, only the capital requirement for that type is affected. This is the case because the optimality condition that determines the capital requirement for the marginal type (10) is unaffected by changes to the cash-flow distribution of the inframarginal type. Note that these results readily extend to settings with more than two types. In particular, changes in marginal investment opportunities for banks feed back into optimal capital requirements for all funded types.

2.2.2 Climate-Related Financial Risks and Prudential Regulation

We now apply Propositions 2 and 3 to investigate how a prudential regulator optimally accounts for climate-related financial risks when setting capital requirements.

According to survey evidence by Stroebel and Wurgler (2021), the top five climate-related financial risks are regulatory risks (e.g., carbon taxes or other environmental regulation), stakeholder risks (e.g., changes in consumer or employee preferences), physical risks (e.g., floods and droughts), technological risks (e.g., technological obsolescence), and legal risks (e.g., legal exposures related to emissions or pollution).

In general, climate-related financial risks could affect the cash-flow distributions of both firm types. In our log-normal specification, climate risks can affect firms via changes in expected profitability $\overline{X}_q$, shocks to cash-flow volatility σ_q, or a combination of the two. Depending on the specific changes in cash-flow distributions, Propositions 2 and 3 characterize how prudential capital requirements should be adjusted in order to incorporate the effects of climate-related risks.[17]

To convey the economics of our model in the most transparent fashion, it is instruc-

is marginal. Here, capital requirements are determined by Principle P1 and P4: All equity capital is allocated to fund loans, and marginal deposit insurance puts are equalized if both types are fully funded. When both types are funded, a decrease in profitability or increase in riskiness of one type increases that type's optimal capital requirement and lowers that of the other type.

[17] While Propositions 2 and 3 are fairly general, one restriction to note is that they treat climate-induced changes in firm cash flows as being exogenous from the bank regulator's perspective. Given that climate change is driven by global emissions determined by many different factors, assuming that the bank regulator treats climate-related risk as exogenous seems like the most relevant case. For example, physical risks due to global warming are likely (approximately) independent of EU bank capital regulation. In Section 3, we discuss how our model can be extended to endogenous risks.

tive to zoom in on a subset of climate risks that exclusively affect one type. In fact, regulatory risks, stakeholder risks, and legal risks are all examples of transition risks that predominantly affect dirty types.[18] In Proposition 4, we characterize how optimal prudential capital requirements respond to risks that reduce the expected cash flows $\overline{X}_D$ and/or increase the cash-flow volatility σ_D of dirty types. (With appropriate relabeling, the proposition also covers the case in which clean firms become more profitable or less risky).

Proposition 4 (Incorporating Transition Risk) *A marginal increase in the cash-flow volatility of dirty firms σ_D or reduction in their expected cash flow $\overline{X}_D$*

1. *increases the optimal capital requirement for loans to dirty firms $\underline{\hat{e}}_D$;*
2. *has an ambiguous spillover effect on capital requirements for loans to clean firms $\underline{\hat{e}}_C$.*
   - *(a) If clean firms are marginal, their capital requirements are unaffected.*
   - *(b) If clean firms are inframarginal, their capital requirements increase.*
   - *(c) If both types are fully funded, capital requirements for loans to clean firms $\underline{\hat{e}}_C$ decrease.*

Part 1 of Proposition 4 states that the prudential regulator optimally responds to transition risks that affect dirty firms by raising capital requirements for loans dirty firms, corresponding to a brown penalizing factor. Intuitively, higher cash-flow volatility increases the put value associated with dirty loans and, hence, makes loans to dirty firms less attractive to the prudential regulator. For reductions in $\overline{X}_D$, the effect on the deposit-insurance put is reinforced by a reduction in NPV.

Part 2 of Proposition 4 investigates the spillover effects of transition risks that affect dirty firms on capital requirements for clean firms. When clean firms are marginal, their capital requirements are set to maximize their PPI, $\underline{\hat{e}}_C = \underline{e}_C^{PPI}$. Because the clean firms' PPI is unaffected by transition risk that only affects dirty firms, optimal prudential capital requirements for clean firms remain unchanged. If clean firms are inframarginal and dirty firms are marginal, transition risks that affect dirty firms decrease the prudential surplus generated by the marginal (dirty) loan. This reduction in the value of the marginal lending opportunity makes it optimal to increase capital requirements also for clean loans

[18] Regulatory transition risks are considered the top climate risk over the next five years (Stroebel and Wurgler, 2021). Given the average maturity of bank loans, this corresponds to the horizon most relevant for bank capital regulation.

in order to reduce the associated deposit insurance put. In addition to a brown penalizing factor, in this case it becomes optimal for the prudential regulator to also increase capital requirements for clean loans. Finally, if both firms are fully funded, the equalization of marginal puts (Principle P4) implies that capital requirements for clean firms decrease while capital requirements for dirty firms increase. In addition to a brown penalizing factor, in this case it becomes optimal for the prudential regulator to implement a green supporting factor.

Figure 4. Effects of transition risks on optimal prudential capital regulation. The figure plots the effect of transition risks on optimal prudential capital requirements (left panel) and equilibrium funding decisions (right panel). Clean firms and their capital requirements are plotted in green, dirty in red. In this illustration, transition risk takes the form of a percentage point reduction in the expected profitability of dirty firms from their initial log return (absent transition risk) of $\mu = 3\%$. Aggregate banking sector equity is set to $E = 0.1$, which corresponds to 10% of overall investment opportunities in the economy given an investment cost of $I = 1$. The remaining parameter values are as in Figure 3.

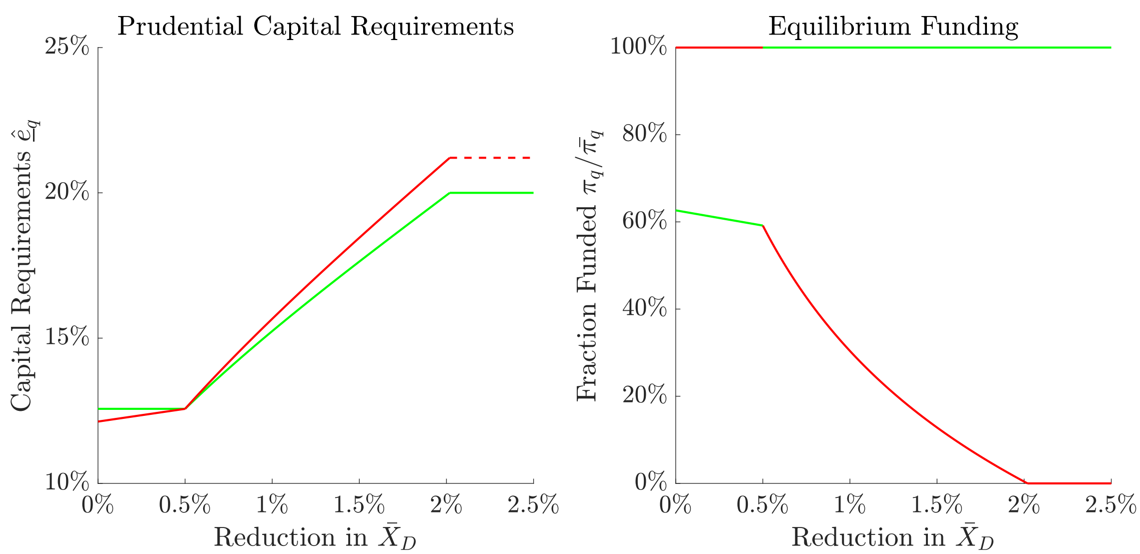

Figure 4 illustrates the effect of transition risk on optimal prudential capital requirements. Specifically, the figure plots the effects of a reduction in the profitability of dirty firms $\overline{X}_D$ on optimal capital requirements (left panel) and equilibrium funding decisions (right panel). The parameters are as in Figure 3 except that we fix the value of aggregate bank equity to $E = 0.1$. This implies that, absent transition risks, dirty firms are fully funded and, therefore, inframarginal. Clean firms are partially funded and, therefore, marginal (see Figure 3 at $E = 0.1$).

Figure 4 shows that as long as the reduction in $\overline{X}_D$ is sufficiently small (less than 0.5%), dirty firms remain inframarginal. In this region, the optimal policy response to higher transition risk is to increase capital requirements for dirty firms (i.e., a brown penalizing factor) while optimal capital requirements for clean firms remain unchanged. Thus, our ad-hoc policy analysis of a brown penalizing factor in Section 2.1 is relevant even under optimal policy. In particular, we see that a prudential regulator may choose to increase capital requirements for dirty firms even though clean firms are crowded out at the margin, as illustrated in the right panel of Figure 4.

Once the reduction in $\overline{X}_D$ exceeds the initial productivity advantage of dirty firms, the prudential regulator's preferred type switches. Beyond this point, clean firms are the prudential regulator's preferred type and, therefore, fully funded and inframarginal. Dirty firms are partially funded and marginal. In this region, a further reduction in the profitability of dirty firms leads to an increase in capital requirements for both dirty and clean firms (left panel). Therefore, in addition to a brown penalizing factor, the optimal policy features a "green penalizing factor" which arises because the marginal lending opportunity (a loan to a dirty firm) becomes less attractive. Because dirty firms are marginal, the increase in capital requirements for both clean and dirty firms is associated with crowding out of funding to dirty firms (right panel). At some point, dirty firms are no longer funded under optimal prudential capital requirements, i.e., a reduction of $\overline{X}_D$ by more than 2%.[19]

In summary, the prudential regulator's optimal response to transition risks that affect only dirty firms unambiguously features a brown penalizing factor, but the policy implications for clean firms are more subtle. If clean firms are marginal, their capital requirements are unaffected so that the brown penalizing factor causes crowding out of lending to clean firms (as highlighted in our adhoc analysis in Section 2.1). If clean firms are inframarginal, the brown penalizing factor is accompanied by a "green penalizing factor" to account for the deterioration of the marginal lending opportunity. Finally, when both firm types are fully funded, the prudential regulator uses a brown penalizing and a green supporting factor to ensure the equalization of marginal deposit-insurance puts.

2.3 Welfare-Maximizing Regulation

In the previous section, we characterized optimal capital regulation and the optimal policy reaction to climate risks under a classical prudential mandate. We now compare capital regulation under a prudential mandate with welfare-maximizing regulation. In

[19] While at that point dirty firms are still profitable, their prudential value is insufficient and the prudential regulator prefers to set capital requirements for clean firms so high that no equity is left for funding dirty firms.

addition to the trade-off between real activity and financial stability, welfare-maximizing regulation accounts for the social costs of emissions ϕ.

In Section 2.3.1, we characterize how a planner would optimally use two tools, capital regulation and carbon taxes, to maximize welfare. In practice, regulation is often conducted by multiple regulators with more narrowly defined objectives (e.g., environmental regulation and financial regulation). Based on the planner's solution, we characterize conditions under which separate regulators (i.e., an environmental regulator and a banking regulator) with distinct mandates can achieve welfare-maximizing regulation. We show that a prudential mandate for the banking regulator maximizes social welfare if (and only if) carbon taxes are set at the optimal level.

In Section 2.3.2, we then consider frictions to environmental regulation that lead to suboptimally low carbon taxes. We first consider environmental policy failures that are exogenous to banking regulation (e.g., caused by lobbying of firms). Should a banking regulator who can set capital requirements adapt its mandate? Here, our analysis points out significant shortcomings of capital regulation to address environmental externalities. We then address environmental policy failure that is endogenous to bank capital regulation. Specifically, endogenous policy failures can occur if the environmental regulator (e.g., the government) is subject to a commitment problem. In this case, capital requirements can solve the government's commitment problem, making stricter environmental regulation credible.

2.3.1 Benchmark: Optimal Carbon Taxes and Capital Requirements

As a benchmark, we first consider the optimal policy of a planner who sets both capital requirements and carbon taxes (indicated by two asterisks).

Proposition 5 (Optimal Joint Regulation) *Maximum welfare with two tools* $\left(\underline{e}_q^{**}, \tau_q^{**}\right)$ *can be achieved as follows:*

1. *For projects with negative social value,* $NPV_q < \phi_q$*, the planner imposes capital requirements of 100% and a sufficiently high carbon tax (e.g.,* $\tau_q^{**} = X_q$*).*

2. *For socially valuable projects,* $NPV_q > \phi_q$*, the planner imposes a carbon tax* τ_q^{**} *(collected whenever the firm is profitable) such that the expected tax payment matches the social cost* $\overline{\tau}_q^{**} = \phi_q$*. Given after-tax cash flows* $X_q - \tau_q^{**}$*, optimal capital requirements,* $\underline{e}_q^{**}$*, are set according to Proposition 2.*

For projects with negative social value, the planner's optimal policy ensures that banks do not find it profitable to fund these firms. Intuitively, the "harshest" use of the two tools, capital requirements of 100% and $\tau_q^{**} = X_q$, achieves this objective.

For socially valuable projects, $\text{NPV}_q > \phi_q$, it is possible to set the carbon tax in such a way that it reflects the externalities, $\overline{\tau}_q^{**} = \phi_q$, and is only collected in states in which the firm is profitable, thereby ensuring that the carbon tax is not ultimately borne by the tax payer.[20] As a result, the after-tax financial NPV of the project is $\text{NPV}_q - \phi_q$. Socially optimal capital requirements then correspond to those set under the prudential mandate using tax-adjusted cash flows.

An immediate corollary of Proposition 5 is that the optimal joint policy can be implemented by two separate regulators with distinct mandates. Specifically, once the appropriate carbon tax τ_q^{**} has been set, it is welfare maximizing for the banking regulator to adopt a prudential mandate.

Corollary 3 (Endogenous Prudential Mandate) *Under the carbon tax scheme τ_q^{**}, as characterized in Proposition 5, a banking regulator with a prudential mandate sets welfare-maximizing capital requirements*

$$\underline{e}_q^{**} = \hat{\underline{e}}_q\left(\tau_q^{**}\right).$$

Corollary 3 implies that the presence of externalities alone does not justify a departure from a purely prudential mandate for banking regulators. Considering a broader mandate for the banking regulator requires that environmental regulation is subject to frictions leading to suboptimal carbon taxes.

2.3.2 Frictions to Environmental Regulation

We now consider settings in which environmental regulation is subject to frictions. We first consider the case in which carbon taxes absent for exogenous reasons.[21] We then analyze endogenous policy failures that can arise when the environmental regulator cannot commit to future carbon taxes.

Exogenously lax environmental regulation. Tirole (2012) discusses various reasons for policy failures in the context of environmental regulation. For ease of exposition, we consider an extreme case and assume for the remainder of this section that carbon taxes are absent altogether. Can a banking regulator who can set capital requirements

[20] It is always feasible to set the state-contingent carbon tax that way since the projects have positive social value (even after accounting for externalities). See details in Proof of Proposition 5.

[21] The results are qualitatively unaffected if carbon taxes are positive but too low.

but not carbon taxes make up for this policy failure? To answer this question, it is most instructive to consider case 1 of Proposition 5, i.e., the case in which externalities caused by dirty firms are so large that they generate negative social value,

Assumption 1 $\phi_D > NPV_D > 0.$

In the benchmark case with carbon taxes described in Proposition 5, Assumption 1 implies an unconstrained optimal policy with a prohibitive carbon tax and capital requirements of 100% for dirty firms, $\underline{e}_D^{**} = 100\%$, so that projects by dirty firms are not funded. The carbon tax for clean firms is optimally zero (since $\phi_C = 0$), and optimal capital requirements for clean loans depend on the capitalization of the banking sector: $\underline{e}_C^{**} = \underline{e}_C^{PPI}$ as long as clean firms are partially funded ($E \leq \underline{e}_C^{PPI} \bar{\pi}_C I$) and $\underline{e}_C^{**} = \min\left\{\frac{E}{\bar{\pi}_C I}, 100\%\right\}$ when clean firms are fully funded, as illustrated by the dashed line in Figure 5.

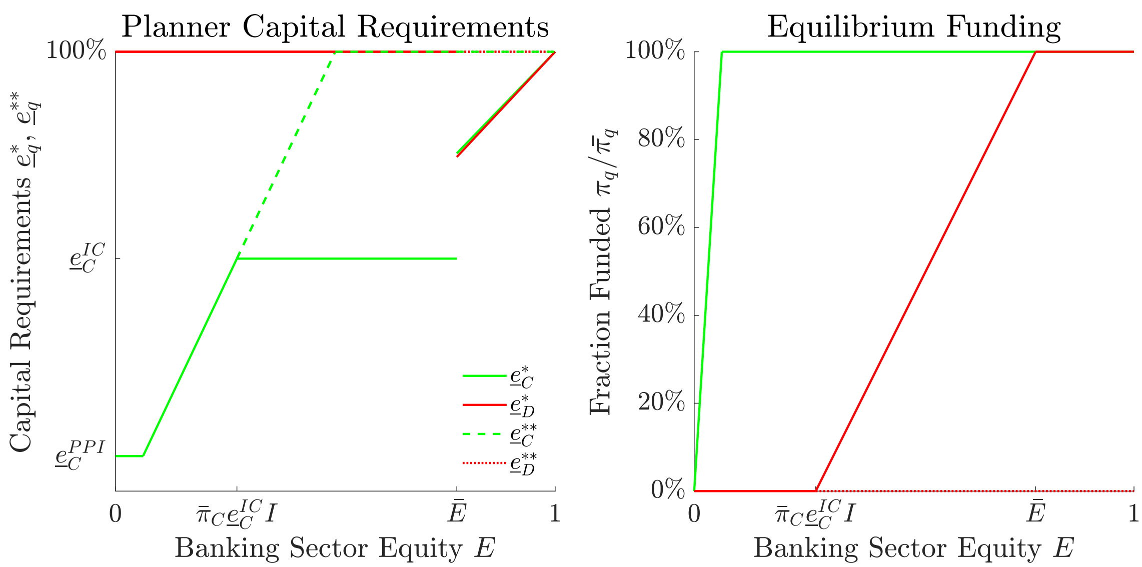

Figure 5. Planner capital requirements (One vs two tools). This figure plots capital requirements (left panel) and equilibrium funding decisions (right panel) under the planner's objective. The solid lines refer to the case where the planner can only avail herself to capital regulation, $\underline{e}_q^*$, the dashed lines refer to the case with two tools (capital requirements and carbon taxes) with associated capital requirements $\underline{e}_q^{**}$. Clean firms are plotted in green, dirty in red. This figure assumes the same parameters as in Figure 3, except that the mean log return on assets for dirty types is now $\mu_D = 6\%$ and their externality is $\phi_D = 0.1$ so that $\text{NPV}_D - \phi_D < 0$.

When carbon taxes are not available as a policy tool, the key change is that dirty loans remain financially profitable for banks even at maximum capital requirements of 100%, because

$$r_D^{\max}(1) = \frac{\text{NPV}_D}{I} > 0. \tag{15}$$

Since banks' ranking of borrowers is driven by profit maximization, the planner's choice of capital requirements is now constrained by banks' privately optimal lending decisions. To understand the constraints resulting from the inability to set carbon taxes, we introduce the concept of ranking alignment between banks and the welfare-maximizing regulator:

Definition 2 (Ranking Alignment) *With only one tool, there is ranking alignment between banks and the welfare-maximizing regulator if banks prefer to invest in clean firms at the optimal capital requirements $\underline{e}_q^{**}$ (see Proposition 5) even in the absence of carbon taxes. Ranking alignment is satisfied if and only if*

$$r_C^{\max}\left(\underline{e}_C^{**}\right) \geq r_D^{\max}(1). \tag{16}$$

Intuitively, whether there is ranking alignment depends on the relative profitability of clean and dirty firms and the capital requirement for the clean firm (recall that $r_C^{\max}$ is decreasing in capital requirements for clean firms). Because $\underline{e}_C^{**}$ is increasing in the capitalization of the banking sector (see Figure 5), ranking alignment also depends on E.

Lemma 1 (Determinants of Ranking Alignment) *Ranking alignment is impossible if clean firms are sufficiently unprofitable, $r_C^{\max}\left(\underline{e}_C^{PPI}\right) < r_D^{\max}(1)$. Ranking alignment always occurs if clean firms are more profitable than dirty firms, $r_C^{\max}(1) \geq r_D^{\max}(1)$. In the intermediate region, $r_C^{\max}(1) < r_D^{\max}(1) < r_C^{\max}\left(\underline{e}_C^{PPI}\right)$, there exists a cutoff for banking sector equity E below which there is ranking alignment.*

Based on the concept of ranking alignment, the following proposition compares capital requirements set by a welfare-optimizing capital regulator in the absence of a carbon tax, $\underline{e}_q^{*}$, with those set by a planner who sets both capital requirements and a carbon tax, $\underline{e}_q^{**}$.

Proposition 6 (The Limits of Green Capital Requirements) *Under ranking alignment, optimal capital requirements set by the welfare-maximizing regulator coincide with those set by the planner in Proposition 5. Without ranking alignment, the regulator sets lower capital requirements than the planner, $\underline{e}_q^{*} \leq \underline{e}_q^{**}$. Regardless of ranking alignment, dirty loans receive funding when banking sector equity E is sufficiently high.*

We illustrate the intuition behind Proposition 6 by focusing on the most interesting case, in which ranking alignment is satisfied for low values of banking sector equity E and violated once E reaches a threshold. This case is illustrated in Figure 5.[22]

[22] The economic insights and intuition gained from the other possible cases is similar. We discuss the remaining cases in the proof of Proposition 6.

For low levels of bank equity E, ranking alignment is satisfied and, therefore, the regulator can set the unconstrained optimal capital requirement described in Proposition 5 while also ensuring that clean loans are funded first, $\underline{e}_q^* = \underline{e}_q^{**}$. Specifically, the regulator sets the capital requirement for loans to dirty firms to 100%. For loans to clean firms, the optimal capital requirement is a function of equity. For low levels of aggregate bank equity ($E \leq \bar{\pi}_C \underline{e}_C^{PPI} I$), only clean firms are funded at the capital requirement that maximizes their PPI, $\underline{e}_C^{PPI}$. Once clean firms are fully funded ($E > \bar{\pi}_C \underline{e}_C^{PPI} I$), the regulator raises the capital requirements for clean loans to lower the deposit-insurance put for clean loans, thereby also making sure that no bank equity is left over to fund dirty firms.

However, if the regulator raised capital requirements for clean loans beyond $\underline{e}_C^{IC}$, where $\underline{e}_C^{IC}$ solves

$$r_C^{\max}\left(\underline{e}_C^{IC}\right) = r_D^{\max}\left(1\right), \tag{17}$$

ranking alignment would break down, and banks would prefer to fund dirty firms before funding clean firms. In order to prevent this ranking switch, the regulator initially responds by capping the capital requirement for clean loans at $\underline{e}_C^{IC}$. The regulator now effectively subsidizes clean loans by lowering their capital requirements below the level that would be set by a planner with access to a carbon tax, thereby sacrificing the prudential part of its mandate. Capping the capital requirement at $\underline{e}_C^{IC}$ also implies that, if $E > \bar{\pi}_C \underline{e}_C^{IC} I$, banks have equity left over to fund dirty firms, as illustrated in the right panel of Figure 5. Additional equity now translates one-for-one into additional funding of dirty firms, which the regulator cannot prevent if only capital requirements are available as a policy tool. Assumption 1 implies that, in this region, increases in bank equity reduce welfare, as illustrated in Figure 6.

Finally, when equity in the banking sector grows sufficiently large, $E > \bar{E}$, the prudential sacrifice required to ensure that clean loans are funded first is too costly, given that a large fraction of dirty firms are funded anyway. At that point, the regulator optimally gives up on steering bank lending towards green firms, leading to a discontinuous increase in capital requirements for clean firms. To avoid that clean lending is cut, capital requirements for inframarginal dirty firms are then optimally lowered to a level below 100%. Since both types are fully funded (see the right panel of Figure 5) the optimal choice of $\underline{e}_C^*$ and $\underline{e}_D^*$ is determined by the equalization of marginal puts (i.e., Principle 4 of Proposition 2). Bank equity is now solely used to phase out the deposit insurance put without affecting lending, so that welfare is strictly increasing in equity.[23] Interestingly, despite having a mandate that accounts for climate externalities, in this region the

[23] The positive slope is not visible in Figure 6 due to the scale of the y-axis.

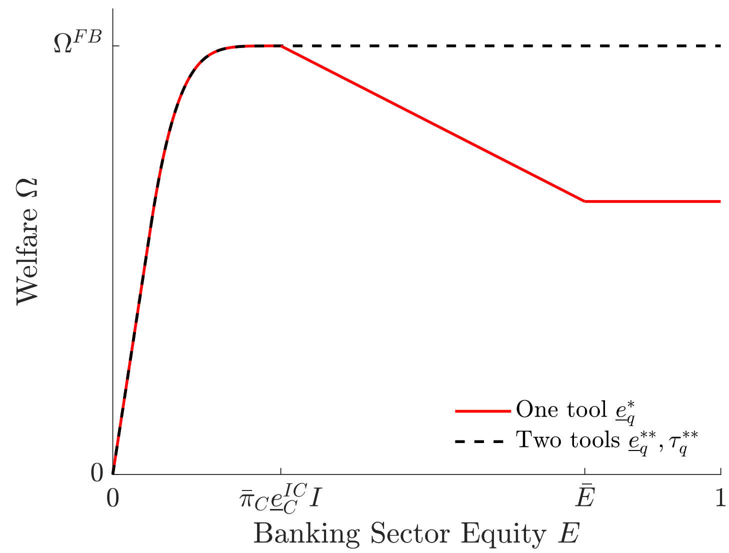

Figure 6. Welfare (One vs two tools). This figure plots welfare with one tool vs two tools as a function of banking sector equity (using the same parameters as in Figure 5). The solid red line refers to the case where the planner can only avail herself to capital regulation, $\underline{e}_q^*$, the black dashed line refers to the case with two tools (capital requirements and carbon taxes).

regulator optimally chooses to focus exclusively on the prudential part of its mandate.

In summary, our results demonstrate that capital requirements are not a suitable substitute for insufficient environmental policy. The limits of green capital requirements are evident when considering the relation between banking sector equity and total welfare. Conventional wisdom suggests that, all else equal, more equity in the banking sector increases welfare. Indeed, this is true when regulation consists of both capital requirements and a carbon tax. In this case, socially harmful projects are not funded, and additional bank equity increases credit provision and strengthens financial stability, as illustrated by the dashed black line in Figure 6. In line with Observation 1, first-best welfare is achieved for sufficiently high E. However, in the presence of externalities and weak environmental regulation, increased equity can reduce welfare by facilitating the financing of socially harmful projects, as illustrated by the red solid line in the region between $\bar{\pi}_C \underline{e}_C^{IC} I$ and $\bar{E}$). If we interpret bank equity as a proxy for firm funding constraints, this insight has broader implications: easing firms' access to capital—whether by allowing banks to raise more equity or by enabling firms to secure funding from non-bank sources—is not inherently beneficial for welfare when environmental regulation is suboptimal.

Endogenous Environmental Policy without Commitment In the previous subsection, we simply assumed that carbon taxes (or other environmental regulation) are absent for exogenous reasons. We now illustrate how this situation could arise endogenously due to a government commitment problem.

To capture the commitment problem, we consider a dynamic version of our baseline model with two rounds of bank financing and production, $t \in \{1, 2\}$. There are two players, the government and the bank regulator, both of which are long-lived. The government determines environmental policy (in the form of carbon taxes) to maximize welfare

$$\sum_{q,t} \delta^{t-1} \pi_{q,t} \left[\text{NPV}_q - \phi_q - \lambda \cdot \text{PUT}_q(\tau_{q,t}, \underline{e}_{q,t}) \right], \tag{18}$$

where δ denotes the government's discount factor. Because the government maximizes total welfare, it internalizes output, externalities, and financial stability. The bank regulator maximizes a *prudential* mandate which does not include carbon externalities,

$$\sum_{q,t} \delta^{t-1} \pi_{q,t} \left[\text{NPV}_q - \lambda \cdot \text{PUT}_q(\tau_{q,t}, \underline{e}_{q,t}) \right]. \tag{19}$$

There are two types of short-lived firms, clean and dirty. We maintain Assumption 1, so that dirty firms produce negative social value. Firms borrow from short-lived banks that start each period with aggregate equity capital E.[24] For ease of exposition, we also assume that E is sufficiently large so that both types are fully funded under optimal prudential capital requirements and no carbon taxes, i.e., $E > E_3$ (see Figure 3).

The timing within each period is as follows. First, the bank regulator sets capital requirements. Then banks make lending decisions given capital requirements. After lending decisions have been made, the government sets carbon taxes. Carbon taxes are downward rigid, i.e.,

$$\tau_{q,t} = \tau_{q,t-1} + \Delta\tau_{q,t}, \tag{20}$$

with $\Delta\tau_{q,t} \geq 0$ and an initial policy $\tau_{q,0} = 0$.[25] Finally, firm cash flows are realized. This sequence of moves ensures that some bank assets are determined before the government sets carbon taxes, capturing stranded asset risk.

We consider pure-strategy sub-game perfect equilibria. Solving by backward induction, we first note

Lemma 2 (No Carbon Taxes in Period 2) *The government does not raise carbon*

[24] The assumption that the banking sector starts each period with equity capital E is a simplifying assumption that rules out dynamic capital accumulation or losses in the banking sector, which are not the focus of this section.

[25] Without the assumption of downward-rigid carbon taxes, the government would find it optimal to set carbon taxes to zero in period 2, because at this point all financing has taken place. Downward rigidity rules out this artificial feature of our two-period model. A more literal interpretation of downward rigidity is that abolishing carbon taxes takes time due to institutional rigidity.

taxes in period $t = 2$, $\Delta\tau_{q,2} = 0$.

Intuitively, by the time the government moves in period 2, capital requirements $\underline{e}_{q,2}$ and funding decisions $\pi_{q,2}$ have been determined. Raising the carbon tax in period 2, therefore, increases the deposit insurance put without having any beneficial effects on funding decisions or emissions. Accordingly, if the government raises carbon taxes for dirty firms, it will do so in period 1.

In period 1, the government faces a trade-off if the banking sector is exposed to stranded asset risk from dirty legacy assets.

Lemma 3 (Stranded Assets Risk) *If there are no dirty legacy assets, $\pi_{D,1} = 0$, the government always raises the carbon tax in period 1. The optimal carbon tax $\tau^*_{D,1}$ ensures that dirty firms are not funded in period 2. If there are dirty legacy assets, $\pi_{D,1} > 0$, the government raises the carbon tax in period 1 if and only if the discount factor δ exceeds a strictly positive threshold $\delta^*(\underline{e}_{D,1}, \pi_{D,1})$.*

The key implication of Lemma 3 is that, if the banking sector is exposed to dirty legacy assets, the government raises carbon taxes only if the benefit of doing so exceeds the cost. The cost of raising carbon taxes stems from the resulting increase in the deposit insurance put for existing loans to dirty firms,

$$\lambda\pi_{D,1}\left[\mathrm{PUT}_D(\tau^*_{D,1}, \underline{e}_{D,1}) - \mathrm{PUT}_D(0, \underline{e}_{D,1})\right]. \tag{21}$$

This cost is strictly positive for any $\underline{e}_{D,1} < 1$. The period 2 gain from raising carbon taxes (which needs to be discounted by δ) is given by

$$\bar{\pi}_D\left[\phi_D + \lambda \cdot \mathrm{PUT}_D(0, \hat{\underline{e}}_D(0)) - \mathrm{NPV}_D\right] + \lambda\bar{\pi}_C\left[\mathrm{PUT}_C(0, \hat{\underline{e}}_C^{\text{No Tax}}) - \mathrm{PUT}_C(0, \hat{\underline{e}}_C^{\text{Tax}})\right]. \tag{22}$$

The first term in Equation (22) captures the avoided social cost of funding dirty firms in period 2. The second term reflects that, after introducing an appropriate carbon tax on dirty firms, clean firms can be fully funded at a higher capital requirement, because bank equity is now solely used to fund clean firms. This implies that $\mathrm{PUT}_C(0, \hat{\underline{e}}_C^{\text{No Tax}}) > \mathrm{PUT}_C(0, \hat{\underline{e}}_C^{\text{Tax}})$. The ratio of costs and benefits then determines the threshold value for the discount factor $\delta^*(\underline{e}_{D,1}, \pi_{D,1})$ in Lemma 3.

Stranded asset risk generates an endogenous commitment problem for the government. If there are no dirty legacy assets, $\pi_{D,1} = 0$, the government always follows through with a carbon tax that prevents the funding dirty firms in the next period. In contrast, if there is stranded asset risk, $\pi_{D,1} > 0$, then raising carbon taxes imposes a short-term cost (financial stability risks due to dirty legacy assets) and a long run benefit (reducing funding of dirty firms in the future). The discount factor δ then determines how the government weighs these costs and benefits. For example, a government that discounts the future more heavily is more likely not to introduce the carbon tax.

The following lemma shows that the government's commitment problem can result in multiple equilibria.

Lemma 4 (Bank Assets: Multiple Equilibria) *Suppose that $\delta < \delta^*(\underline{e}_{D,1}, \bar{\pi}_D)$. There are two equilibria. Either no bank invests in dirty firms, $\pi_{D,1} = 0$, and the government sets a positive carbon tax $\tau^*_{D,1} > 0$. Otherwise, dirty firms are fully funded $\pi_{D,1} = \bar{\pi}_D$, and the government sets no carbon tax $\tau_{D,1} = 0$.*

The multiplicity of equilibria arises because funding dirty firms in period 1 generates endogenous stranded asset risk, which makes it ex-post optimal for the government not to impose carbon taxes if $\delta < \delta^*(\underline{e}_{D,1}, \bar{\pi}_D)$. Conceptually, this result is similar in spirit to Acharya and Yorulmazer (2007) and Farhi and Tirole (2012), who highlight a related commitment problem with respect to collective bailouts. Biais and Landier (2022) analyze a government commitment problem with respect to emission caps in the presence of investment spillovers in green technology.

In the context of capital regulation, the key policy insight is that the no-carbon tax equilibrium exists only if the initial capital requirement for dirty firms $\underline{e}_{D,1}$ is sufficiently low.

Proposition 7 (Capital Requirements and Credible Carbon Taxes)

1. *The bank regulator can eliminate the no-carbon tax equilibrium by setting $\underline{e}_{D,1} = 1$.*

2. *A bank regulator with a strictly prudential mandate does not eliminate the no-carbon tax equilibrium. A bank regulator with a prudential mandate that is conditional on transition eliminates the no-carbon tax equilibrium.*

Proposition 7 demonstrates that, even though capital requirements are not an effective substitute for carbon taxes when it comes to directly reducing emissions (recall Proposition 6), they can be effective as a facilitator of stricter environmental policy when the government is concerned about stranded asset risk and subject to a commitment problem. In particular, if the bank capital regulator sets $\underline{e}_{D,1} = 1$, stranded asset risk from legacy assets disappears, and the government always finds it optimal to introduce the carbon tax. Sufficiently high capital requirements make carbon taxes credible.

However, for the bank regulator to eliminate the no-carbon tax equilibrium, the regulatory mandate needs to be slightly broader than a strict prudential mandate. A bank regulator with a strict prudential mandate prefers the equilibrium in which dirty firms are fully funded because, absent carbon taxes, these firms generate positive prudential value. As a first-mover, the bank regulator understands that this prudential value will not be reduced by future carbon taxes if $\pi_{D,1} = \bar{\pi}_D$, because in this case the government will not find it in its interest to introduce carbon taxes. In contrast to Corollary 3, a purely prudential mandate for the bank regulator is now no longer welfare-optimal because the government cannot commit to optimal carbon taxes.

Eliminating the no-carbon tax equilibrium via capital regulation requires a slightly broader mandate for the bank regulator. In particular, the regulatory mandate needs to be such that the regulator induces the transition by setting capital requirements that incorporate endogenous, forward-looking transition risks. Conditional on transition occurring, the bank regulator can then follow a regular prudential mandate. The regulator's mandate is, therefore, *prudential conditional on transition*.

3 Discussion and Extensions

In this section, we discuss how the model can be extended to capture a number of empirically relevant features not present in our baseline model.

Generalizing the Model: Many Firm Types and Costly Equity. For ease of exposition, our baseline model considers only two firm types and abstracts from the possibility that banks can raise additional equity. To highlight the robustness of our results, we now incorporate both of these features into our model.

First, we allow banks to raise equity ΔE in addition to the initial equity endowment E. The cost of issuing additional equity is given by $c(\Delta E)$, where the marginal cost of raising equity $c'(\Delta E)$ is strictly increasing and convex. The resulting inverse supply function for equity is plotted in blue in Figure 7.

Second, we extend the model to allow for heterogeneous productivity within both dirty and clean firms. This results in a model with many types, consisting of clean and dirty firms with many different productivity levels. For the illustration in Figure 7, we assume that the mean log return for both clean and dirty firms is drawn from a normal distribution with the same mean. In contrast to the illustration in the baseline model, there is therefore no difference in the average profitability of clean and dirty firms. Here, we assume instead that dirty firms exhibit significantly higher dispersion in productivity

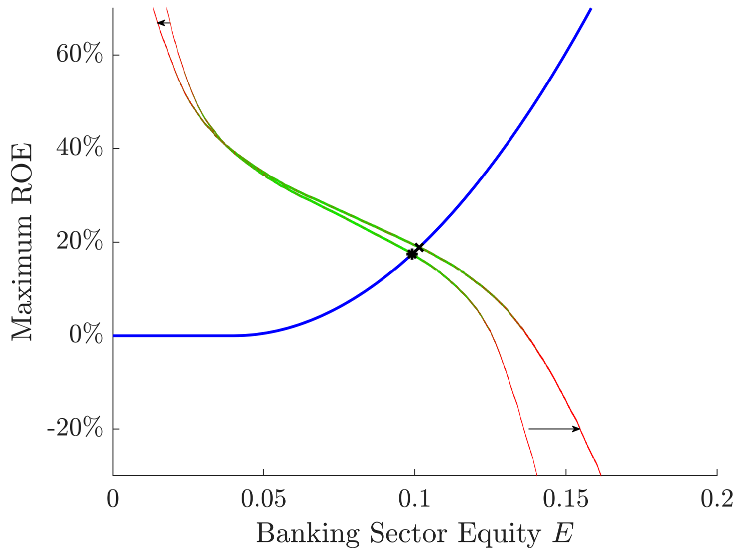

Figure 7. Effect of Brown Penalizing Factor with multiple types and equity raising. This figure illustrates an extension of our baseline model with heterogeneity within types (10,000 clean firms and 10,000 dirty firms). The mean log returns of clean and dirty firms, μ_q, are drawn from a normal distribution with equal mean but significantly higher dispersion for dirty firms, $\mu_C \sim N(0, 0.02)$ and $\mu_D \sim N(0, 0.1)$. We assume equal capital requirements of 15% for clean and dirty firms prior to the introduction of the BPF. The BPF raises the capital requirement for dirty loans to 20%. As a result of the BPF, the demand curve rotates as indicated by the arrows in the figure. The initial equity endowment is given by $E = 0.04.$, and the marginal cost of raising additional equity is $c'(\Delta E) = 50\Delta E^2$ for $\Delta E \geq 0$.

than clean firms.

As in the baseline model, the banking sector ranks firm types q according to their reservation price, the maximum ROE $r_q^{\max}$. Unlike our earlier graphs with two types, the resulting demand curve for equity is now approximately continuous. Because dirty firms exhibit higher dispersion in productivity, they are disproportionately represented at both the upper and lower end of the demand curve.[26]

We now consider the effect of a brown penalizing factor on equilibrium lending. The resulting equilibrium ROE, which determines which firms are funded by banks, is indicated by the "star" (pre BPF) and "x" (post BPF). The key difference to our baseline analysis is that any BPF now has both an income and a substitution effect. On the one hand, highly productive dirty firms remain inframarginal even after a significant BPF. The resulting increase in required equity to fund these productive dirty firms generates a negative income effect. This crowds out firms near the funding cutoff, which, in this

[26] Note that firms that receive sufficiently negative productivity shocks cannot offer positive ROEs to banks. These firms are never financed, regardless of the amount of equity in the banking system.

specification, are predominantly clean. On the other hand, dirty firms with intermediate productivity become less attractive to banks and fall below the cutoff funding r_E^*. This substitution effect frees up equity to fund additional clean firms.

The shift in the aggregate demand curve also influences banks' incentives to raise equity. In the illustration in Figure 7, the BPF raises the equilibrium ROE so that banks respond by issuing more equity. This attenuates the crowding-out effect at the margin.[27] In the illustration in Figure 7, the net effect of all these forces leads to crowding out of both clean and dirty firms, despite the mitigating effect of additional equity raising. Roughly one-third (410 out of 1253) of the firms that are crowded out are clean, even though capital requirements are only increased for dirty firms.

The main conclusion remains similar to that of the baseline model: Changes to capital requirements for inframarginal borrower types lead to crowding out of the marginal borrower type. The identity of the marginal borrower type depends on both the distribution of firm types and the characteristics of the banking sector (e.g., capital scarcity and frictions to raising additional equity). For example, even in the illustration in Figure 7, a significantly steeper (or shallower) equity supply curve would predominantly lead to crowding of dirty firms. This sensitivity to underlying parameters illustrates that the relevant marginal borrower types likely vary with business cycles and across countries with different financial development.

Non-bank financing. In our model, all firms are bank-dependent. If instead firms had access to competitive public markets (or another alternative source of financing), the formal analysis would be similar, except that this outside financing option would reduce the borrowers' reservation interest rate. This results in a lower maximum ROE of $r_q^{\max}\left(\underline{e}_q\right) = \frac{\mathrm{PUT}_q\left(\underline{e}_q\right)}{I\underline{e}_q}$ (for details, see the proof of Result 2). Intuitively, the only comparative advantage for banks now stems from government subsidies as reflected in the deposit insurance put.

The assumption of bank dependence gives capital requirements the best shot at addressing externalities: As long as capital requirements can ensure that banks do not fund dirty firms, emissions can be prevented. If (some) dirty firms have access to alternative sources of financing, a welfare maximizing regulator (equipped with only one tool) is further constrained by substitution to other funding markets. Whether substitution to non-bank financing is a concern for the regulator depends on the regulatory mandate:

[27] Note that depending on the shape of the equity supply function and the resulting intersection with the demand curve, it is possible that a BPF increases or decreases the equilibrium ROE. In the latter case, the reduction in equity would amplify (rather than mitigate) crowding out.

Observation 2 *The prudential regulator welcomes substitution because it removes risk from the banking sector. A regulator who cares about externalities can never reduce carbon emissions when dirty firms substitute bank loans with other funding sources.*

Endogenous Climate Risks. Our baseline model treats climate risks as exogenous from the perspective of the bank regulator. This assumption captures that the emissions caused by firms funded by even a large domestic banking sector are only a small fraction of total global emissions. However, some regulators may nevertheless view climate risks as endogenous. In this case, they internalize that carbon emissions caused by bank-funded firms feed back into the cash-flow distributions of bank-funded firms and, therefore, financial risks in the banking sector. For example, physical risks (floods and droughts) caused by the emissions of bank-funded firms can impose negative production externalities in the form of lower cash flows and higher volatility for other (clean and dirty) firms. Most of our analysis above carries over to a setting with endogenous climate risks. However, some additional considerations arise. For example, because competitive banks take carbon externalities as given, they do not take into account their own contribution to endogenous climate risks. In contrast, even a strictly prudential regulator would now counteract the externality on firm cash-flow distributions by increasing capital requirements for dirty firms.

Imperfect observability of firm types. For expositional clarity, we assumed that the bank regulator can perfectly observe both the riskiness and emissions of a firm. If the regulator only observed a noisy signal of firm quality, the main results would be qualitatively similar. There are, however, additional potential unintended consequences when types are only imperfectly observable. For example, if clean firms consist of both risky and safe clean firms, a uniform green supporting factor for all clean firms would disproportionately benefit risky clean firms, which would benefit from a larger increase in the value of the deposit insurance put. Accordingly, in this case a green supporting factor could incentivize banks to engage in "green risk-taking."

Firms' choice of production technology. For ease of exposition, we assumed that firm types are exogenously given, which should be interpreted as firms operating either in a clean or a dirty sector. In this baseline model, green tilts to capital regulation affect emissions via the banking sector's allocation of funding across sectors. If, in addition, firms within a given sector had access to a costly pollution-reducing technology, as in Oehmke and Opp (2025), they may have an incentive to invest in these technologies if capital requirements reward such investments. The incentives to become clean would depend on how much doing so increases in the maximum ROE firms can offer to banks.

4 Conclusion

How should climate change and associated climate risks be reflected in bank capital regulation? This paper has developed a flexible model of capital requirements to investigate both positive and normative aspects of this question.

Our positive results highlight that increases in capital requirements for dirty loans can reduce clean lending. Conversely, decreases in capital requirements for clean loans can crowd in dirty lending. This result obtains because changes in capital requirements affect credit allocation only via the marginal loan. Our model characterizes the conditions under which the marginal loan is clean or dirty.

From a normative perspective, our analysis shows that capital requirements can be an effective tool to deal with prudential risks arising from climate change. However, addressing climate-related financial risks via capital requirements is not equivalent to reducing emissions. For example, it can be optimal for a prudential regulator to increase capital requirements on loans affected by climate-related financial risk even if this crowds out clean lending. The insight that capital requirements can effectively deal with climate-related financial risks reflects that, conceptually, doing so is no different from managing "traditional" risks. However, in contrast to traditional risks, financial risks caused by climate change pose novel measurement challenges because historical data series contain limited information about these risks.

When carbon taxes are set optimally, a strictly prudential mandate for the bank regulator is welfare-maximizing. In contrast, in the absence of optimal carbon taxes, a welfare-maximizing bank regulator may use capital requirements to target emissions in addition to prudential risks. However, our analysis shows that capital requirements alone are a second-best tool to discourage the funding of carbon-intensive activities. When bank capital is ample, capital regulation is powerless to deter the funding of financially profitable dirty loans even if they generate negative social value. When bank capital is scarce, inducing banks not to fund dirty loans can require lowering capital requirements for clean loans below the prudentially optimal level, thereby sacrificing financial stability. In addition, even if capital regulation can successfully remove dirty loans from the banking system, high-emitting activities will likely attract funding elsewhere as long as they offer a positive return to investors.

Comparing the planner's solution with carbon taxes and capital requirements to a welfare-maximizing bank regulator who sets capital requirements in the absence of carbon taxes clearly demonstrates that interventions that directly reduce the profitability of carbon-intensive investments (a carbon tax) are a more effective tool to reduce carbon emissions. In this context, capital requirements can play an indirect role: By ensuring sufficient loss-absorbing capital in the banking sector, they can help facilitate optimal carbon taxes or stricter environmental regulation, which governments may otherwise be reluctant to introduce because the resulting revaluation of bank assets and the associated stranded asset risk could trigger a banking crisis.

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A Proofs

Proof of Result 1: Let $y_q \geq 0$ denote the interest rate that a borrower of type q promises to pay on the loan of size I. (This promised yield will be endogenous in equilibrium, see Results 2 and 3). Then, if a bank lends only to borrowers of type q (i.e., $w_q = 1$) and chooses a feasible equity ratio $e \geq \underline{e}_q$, its expected return on equity can be written as:

$$r_E = \frac{\mathbb{E}\left[\max\left\{\min\left\{I\left(1+y_q\right), X_q\right\} - \left(1-e\right)I, 0\right\}\right] - eI}{eI} \tag{A.1}$$

$$= \frac{\mathbb{E}\left[\max\left\{\min\left\{Iy_q, X_q - I\right\}, -eI\right\}\right]}{eI}. \tag{A.2}$$

Equation (A.1) reflects that the bank receives (from each borrower of type q) the minimum of the promised loan repayment, $I\left(1+y_q\right)$, and borrower's cash flows, X_q. Given an equity ratio of e, the amount of debt financing (per borrower) is $\left(1-e\right)I$. Since depositors require zero interest on their deposits (due to bailouts/deposit insurance), the bank needs to repay depositors a total of $\left(1-e\right)I$. Since bank shareholders are protected by limited liability, their gross-payoff is bounded below by zero. The numerator, therefore, reflects the expected payoff for bank shareholders net of their co-investment eI. Dividing by the co-investment yields the bank's expected return on equity. We can now decompose the numerator to write (A.2) as

$$r_E = \frac{\mathbb{E}\left[\min\left\{Iy_q, X_q - I\right\}\right] + \mathbb{E}\left[\max\left\{-eI - \min\left\{Iy_q, X_q - I\right\}, 0\right\}\right]}{eI}, \tag{A.3}$$

which follows from $\max\left\{a, b\right\} = a + \max\left\{b - a, 0\right\}$, setting $a = \min\left\{Iy_q, X_q - I\right\}$ and $b = -eI$. Here, $\mathbb{E}\left[\max\left\{-eI - \min\left\{Iy_q, X_q - I\right\}, 0\right\}\right]$ can be interpreted put value arising from a loan to a firm of type q. This put value can be further simplified, since $\max\left\{-eI - \min\left\{Iy_q, X_q - I\right\}, 0\right\} = 0$ whenever the borrower can repay the promised loan repayment, i.e., $X_q > I\left(1+y_q\right)$. We thus obtain

$$r_E = \frac{\mathbb{E}\left[\min\left\{Iy_q, X_q - I\right\}\right] + \mathbb{E}\left[\max\left\{I\left(1-e\right) - X_q, 0\right\}\right]}{eI}. \tag{A.4}$$

Equation (A.4) shows that the bank's ROE is strictly decreasing in e, so that the bank optimally chooses the minimum equity co-financing $e = \underline{e}_q$. This proves the statement about maximum leverage.

We now turn to the specialization result. Mixing two borrower types is strictly dominated because diversification lowers the bank's put value. This reflects the standard result that the option on a portfolio has a lower value than the corresponding portfolio of options. ■

Proof of Result 2: Let $y_q^{\max}$ denote the maximum interest rate that a borrower is willing to pay. The maximum ROE from lending to a borrower of type q is achieved by lending with maximum leverage, $e = \underline{e}_q$, at rate $y_q^{\max}$. Equation (A.4) then becomes

$$r_q^{\max}\left(\underline{e}_q\right) = \frac{\mathbb{E}\left[\min\left\{Iy_q^{\max}, X_q - I\right\}\right] + \mathrm{PUT}_q\left(\underline{e}_q\right)}{\underline{e}_q I}, \tag{A.5}$$

where $\mathrm{PUT}_q\left(\underline{e}_q\right) := \mathbb{E}\left[\max\left\{I(1-\underline{e}_q) - X_q, 0\right\}\right]$. Equation (A.5) covers both the case in which the firm type is bank bank-dependent (as in our baseline model) and the case in which the firm has access to an outside option (as in Section 3).

Case 1: If the firm is bank-dependent (and, thus, lacks an outside financing option) it is willing to pledge the entire NPV to the bank. (For unbounded cash flow distributions, such as the log-normal distribution, this corresponds to $y_q^{\max} = \infty$.) In this case, $\mathbb{E}\left[\min\left\{Iy_q^{\max}, X_q - I\right\}\right] = \mathbb{E}\left[X_q - I\right] = \mathrm{NPV}_q$. Then (A.5) simplifies to (4).

Case 2: If the firm has access to a competitive outside option, the reservation interest rate $y_q^{\max}$ equals the interest rate on the outside option. The value of $y_q^{\max}$ must be such that a competitive outside investor just breaks even on the investment,

$$\mathbb{E}\left[\min\left\{I\left(1 + y_q^{\max}\right), X_q\right\}\right] = I, \tag{A.6}$$

which implies that $\mathbb{E}\left[\min\left\{Iy_q^{\max}, X_q - I\right\}\right] = 0$. Therefore, the maximum ROE for bank equityholders (A.5) becomes:

$$r_q^{\max}\left(\underline{e}_q\right) = \frac{\mathrm{PUT}_q\left(\underline{e}_q\right)}{\underline{e}_q I}. \tag{A.7}$$

This expression reflects that the only comparative advantage of banks relative to competitive outside investors results from access to deposit insurance (or bailout guarantee). ■

Proof of Result 3: Given that equity is the (potentially) scarce resource, the banking sector prioritizes borrowers according to the maximum expected ROE (which act akin to reservation prices). We need to distinguish two cases.

Scarce equity: If not all firms can be financed with the available banking sector equity E, $E < I\sum_q \bar{\pi}_q \cdot \underline{e}_q$, then the marginal borrower type q_M pays the maximum interest rate $y_{q_M}^{\max}$ on her loan. A fraction of marginal firms with $r_{q_M}^{\max}\left(\underline{e}_{q_M}\right) = r_E^*$ is rationed. Even though banks are competitive, banks earn a scarcity rent of $r_E^* = r_{q_M}^{\max}\left(\underline{e}_{q_M}\right) > 0$. All borrower types with $r_q^{\max}\left(\underline{e}_q\right) > r_E^*$ are inframarginal and are fully financed. The interest rate on their loan $y_q < y_q^{\max}$ is set below their reservation interest rate, which ensures that banks also earn a ROE of r_E^* on loans to inframarginal borrower types (who, thus, obtain some borrower surplus from their projects).

Non-scarce equity: If $E \geq I\sum_q \bar{\pi}_q \cdot \underline{e}_q$, banks finance all firms since all firms possess, by assumption, access to a positive NPV project and, can, hence offer a positive ROE, $r_{q^*}^{\max}\left(\underline{e}_{q^*}\right) > 0$. Since banks are competitive and equity is not scarce, loan interest rates are set such that banks earn a ROE of $r_E^* = 0$ on all loans. All surplus (including the put value) is passed on to borrowers. ■

Proof of Proposition 1: We consider the typical case in which the maximum ROE of each type is distinct, i.e., not the knife-edge case where they are exactly equal. Since the maximum ROE, see (4), is continuous in capital requirements, sufficiently small changes in capital requirements will keep the ranking of borrowers by banks based on the maximum ROE intact. Since a small increase of capital requirements of any funded borrower type tightens the banking sector's equity budget constraint,

$$E = I\left(\sum_{q:r_q^{\max}>r_E^*} \pi_q \cdot \underline{e}_q + \pi_{q_M} \cdot \underline{e}_{q_M}\right),$$

fewer firms of the marginal type can be financed so that π_{q_M} must decrease.

Once capital requirements of inframarginal types increase sufficiently or capital requirements of marginal types decrease sufficiently, the banking sector's ranking based on $r_q^{\max}$ will change, which explains the second statement. ■

Lemma A.1 *Suppose a borrower's cash flow distribution is log-normal with mean cash flow* $\overline{X}_q = \exp\left(\mu_q + \frac{\sigma_q^2}{2}\right)$ *and return volatility* σ_q*. Then, if this borrower is funded by a bank in an optimal portfolio (see Result 1), the value of the deposit insurance put is given by:*

$$PUT_q\left(\underline{e}_q\right) = N\left(-d_2\right)\left(1-\underline{e}_q\right)I - N\left(-d_1\right)\overline{X}_q, \tag{A.8}$$

$$d_1 = \frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma_q} + \frac{\sigma_q}{2}, \tag{A.9}$$

$$d_2 = \frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma_q} - \frac{\sigma_q}{2}, \tag{A.10}$$

where N *denotes the standard normal cumulative distribution function.*

Proof of Lemma A.1: The textbook Black-Scholes-Merton formula, see e.g., Hull (2003), implies that the value of a put option on an asset with price S and volatility σ, given a strike price K, option maturity T, and risk-free rate r, is given by

$$P = e^{-rT}KN\left(-d_2\right) - SN\left(-d_1\right), \tag{A.11}$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_2 + \sigma\sqrt{T}. \tag{A.12}$$

Risk-neutrality and zero discounting imply that in our setting $S = \overline{X}_q$. The strike price of the put option generated by deposit insurance is $K = I\left(1-\underline{e}_q\right)$. Using $T = 1$, then yields Equations (A.8), (A.9), and (A.10). ■

Lemma A.2 *The following comparative statics of the put value apply:*

$$\frac{\partial PUT_q}{\partial \sigma} > 0, \tag{A.13}$$
$$\frac{\partial PUT_q}{\partial \overline{X}_q} = -N(-d_1) < 0, \tag{A.14}$$
$$\frac{\partial PUT_q}{\partial \bar{e}_q} = -I \cdot (1 - N(d_2)) < 0, \tag{A.15}$$
$$\frac{\partial^2 PUT_q}{\partial \bar{e}_q^2} = IN'(d_2)\frac{1}{\sigma}\frac{1}{1-\underline{e}_q} > 0, \tag{A.16}$$
$$\frac{\partial^2 PUT_q}{\partial \bar{e}_q \partial \overline{X}_q} = IN'(d_2)\frac{1}{\sigma}\frac{1}{\overline{X}_q} > 0, \tag{A.17}$$
$$\frac{\partial^2 PUT_q}{\partial \bar{e}_q \partial \sigma} = -IN'(d_2)\left(\frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma^2} + \frac{1}{2}\right) < 0. \tag{A.18}$$

Proof: The first three results are standard (see, e.g., Hull (2003)). To show the remaining results, it is useful to write

$$\frac{\partial PUT_q}{\partial \bar{e}_q} = -I + IN(d_2). \tag{A.19}$$

Since $d_2 = \frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma} - \frac{\sigma}{2}$, see (A.10), we obtain

$$\frac{\partial d_2}{\partial \bar{e}_q} = \frac{1}{\sigma}\frac{1}{1-\underline{e}_q} > 0, \tag{A.20}$$
$$\frac{\partial d_2}{\partial \overline{X}_q} = \frac{1}{\sigma}\frac{1}{\overline{X}_q} > 0, \tag{A.21}$$
$$\frac{\partial d_2}{\partial \sigma} = -\left(\frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma^2} + \frac{1}{2}\right) < 0. \tag{A.22}$$

Using $\frac{\partial^2 PUT_q}{\partial \bar{e}_q^2} = IN'(d_2)\frac{\partial d_2}{\partial \bar{e}_q}$ and (A.20), we obtain (A.16) and, analogously, (A.17) and (A.18). Note that (A.22) is unambiguously negative because both projects are, by assumption, positive NPV from a financial perspective, i.e., $\overline{X}_q > I > I\left(1-\underline{e}_q\right)$, and $\ln(x) > 0$ for any $x > 1$. ■

Lemma A.3 *If $\lambda > \frac{NPV_q}{PUT_q(0)}$, the maximizer of $PPI_q(\underline{e}_q) = \frac{NPV_q - \lambda \cdot PUT_q(\underline{e}_q)}{I\underline{e}_q}$ is finite and uniquely determined by the first-order condition*

$$IPPI_q(\underline{e}_q) = -\lambda\frac{\partial PUT_q(\underline{e}_q)}{\partial \underline{e}_q}. \tag{A.23}$$

Proof of Lemma A.3: The first-order condition $\frac{\partial PPI_q(\underline{e}_q)}{\partial \underline{e}_q} = 0$ implies

$$\frac{I\underline{e}_q\left(-\lambda \cdot \frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}\right) - \left(\text{NPV}_q - \lambda \cdot \text{PUT}_q(\underline{e}_q)\right) I}{I^2 \underline{e}_q^2} = 0. \tag{A.24}$$

Rearranging yields (A.23). To prove uniqueness, it is useful to rewrite (A.24) as

$$G\left(\underline{e}_q\right) = \text{NPV}_q, \tag{A.25}$$

where the function

$$G\left(\underline{e}_q\right) := \lambda\left[\text{PUT}_q(\underline{e}_q) - \underline{e}_q \frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}\right] \tag{A.26}$$

is defined on the domain $[0,1]$. It is now easy to verify that the function G takes on its maximum value at 0 with $G(0) = \lambda \text{PUT}_q(0) > 0$ and the minimum value at 1 with $G(1) = 0$. Moreover, G is differentiable and strictly decreasing with slope

$$G'\left(\underline{e}_q\right) = \lambda\left[\frac{\partial PUT}{\partial \underline{e}} - \left(\frac{\partial PUT}{\partial \underline{e}} + \underline{e}\frac{\partial^2 PUT}{\partial \underline{e}^2}\right)\right] = -\lambda \underline{e}\frac{\partial^2 PUT}{\partial \underline{e}^2} < 0, \tag{A.27}$$

where the last inequality uses $\frac{\partial^2 PUT}{\partial \underline{e}^2} > 0$, see (A.16). Since G is strictly decreasing and $G(1) = 0 < \text{NPV}_q$, (A.25) has a solution if and only if $G(0) >\text{NPV}_q$, which is equivalent to $\lambda > \frac{\text{NPV}_q}{PUT_q(0)}$. By continuity of G, the solution for $\underline{e}_q$ is unique. ■

Proof of Proposition 2: We prove each claim separately.

P1 We first prove that, under optimal prudential regulation, it is without loss of generality to restrict dividends to zero.

First, suppose that, at the optimal prudential capital requirements $\hat{\underline{\mathbf{e}}}$, banks earn a scarcity rent (i.e., $r_E^* > 0$). In this case, banks strictly prefer not to pay out dividends, since they can earn an excess return.

Second, consider the case in which bank equity is not scarce, so that all types are funded, $\pi_q(\hat{\underline{\mathbf{e}}}) = \bar{\pi}_q$, and banks do not earn a scarcity rent (i.e., $r_E^* = 0$). In this case the regulator's payoff is given by:

$$\sum \bar{\pi}_q \left[\text{NPV}_q - \lambda \cdot \text{PUT}_q(\hat{\underline{e}}_q)\right]. \tag{A.28}$$

Now suppose (by contradiction) that under optimal prudential regulation not all equity is used, $E - \sum \bar{\pi}_q \hat{\underline{e}}_q I > 0$, so that the banking sector finds it optimal to pay out the excess equity as dividends (as to ensure maximum leverage, see Result 1). Then the regulator could increase capital requirements for both types to $\tilde{\underline{\mathbf{e}}} > \hat{\underline{\mathbf{e}}}$ (where the inequality is strict for at least one type) until all equity is exhausted (i.e., $E = \sum \bar{\pi}_q \tilde{\underline{e}}_q I$). By construction, this would leave firm funding unaffected and strictly reduce the value of the deposit insurance put, thereby increasing the regulator's payoff. Hence, $\hat{\underline{\mathbf{e}}}$ could not have been optimal.

We now turn to the proofs of Principles 2 to 4. It is useful to phrase the regulator's problem in terms of the $\mathrm{PPI}_q(\underline{e}_q) = \frac{\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)}{I \underline{e}_q}$ and to denote the fraction of equity allocated to type q by κ_q. (For ease of exposition, we omit carbon tax τ_q subscripts in this proof.)

Problem 1 *The prudential regulator solves:*

$$\max_{\underline{\mathbf{e}}} \Pi_P = E \max_{\underline{\mathbf{e}}} \sum \kappa_q \left(\underline{\mathbf{e}}\right) PPI_q(\underline{e}_q), \tag{A.29}$$

s.t. to a short-selling constraint (i.e., the equity allocated to each type is non-negative),

$$\kappa_q \left(\underline{\mathbf{e}}\right) \geq 0, \tag{A.30}$$

the constraint that the mass of funded firms cannot exceed the supply of each type $\bar{\pi}_q$,

$$\kappa_q \left(\underline{\mathbf{e}}\right) E \leq \bar{\pi}_q \underline{e}_q I, \tag{A.31}$$

and the incentive constraint governing the banking sector's privately optimal allocation of equity,

$$\kappa_q \left(\underline{\mathbf{e}}\right) = \frac{\min\left\{ \max\left\{ E - \sum_{\breve{q}: r_{\breve{q}}^{\max} > r_q^{\max}} \bar{\pi}_{\breve{q}} \underline{e}_{\breve{q}} I, 0 \right\}, \bar{\pi}_q \underline{e}_q I \right\}}{E}. \tag{IC}$$

(IC) fully determines the funding decisions of the banking sector based on the ranking implied by $r_q^{\max}$.[28] For any given type q, the equity left after funding all types with higher ROE is given by $\max\left\{ E - \sum_{\breve{q}: r_{\breve{q}}^{\max} > r_q^{\max}} \bar{\pi}_{\breve{q}} \underline{e}_{\breve{q}} I, 0 \right\}$. The actual amount allocated to a given type is then the minimum of the residual equity for this type, $\max\left\{ E - \sum_{\breve{q}: r_{\breve{q}}^{\max} > r_q^{\max}} \bar{\pi}_{\breve{q}} \underline{e}_{\breve{q}} I, 0 \right\}$, and the amount of equity needed to fund all firms of type q, $\bar{\pi}_q \underline{e}_q I$. As is now clear, banks' optimal decisions according to (IC) automatically ensure that the constraints (A.30) and (A.31) are satisfied. However, it is still useful to add these constraints to prove Principles P2 to P4.

P2 Consider the regulator's relaxed problem, in which (A.31) and (IC) are ignored. This relaxed problem provides an upper bound to the regulator's payoff. In this relaxed problem, the regulator simply maximizes the convex combination of prudential profitability indices, $\sum \kappa_q \left(\underline{\mathbf{e}}\right) \mathrm{PPI}_q(\underline{e}_q)$. The optimal choice is given by allocating all equity to the regulator's preferred type (see Definition 1), which offers the

[28] Our assumptions ensure that $r_q^{\max} > 0$ for all types q. If this were not the case, we would obtain $\kappa_q \left(\underline{\mathbf{e}}\right) = 0$ for all types with $r_q^{\max} \leq 0$.

maximum PPI, $\max_q \text{PPI}_q(\underline{e}_q^{\text{PPI}})$, yielding a total payoff of $E \max_q \text{PPI}_q(\underline{e}_q^{\text{PPI}})$. We denote the regulator's preferred types as $\hat{q}$. We now prove that the regulator can achieve this upper bound payoff in the full problem (i.e., after including constraints (IC) and (A.31)) if and only if the equity needed to fund all firms of the preferred type, $\bar{\pi}_{\hat{q}}\underline{e}_{\hat{q}}^{\text{PPI}}I$, is greater than the supply of bank equity, i.e., $\bar{\pi}_{\hat{q}}\underline{e}_{\hat{q}}^{\text{PPI}}I > E$. To see this, set $\underline{e}_{\hat{q}} = \underline{e}_{\hat{q}}^{\text{PPI}}$ and $\underline{e}_q = 1$ for all other types $q \neq \hat{q}$. Given these capital requirements, banks rank type $\hat{q}$ highest (i.e., $r_{\hat{q}}^{\max}\left(\underline{e}_{\hat{q}}^{\text{PPI}}\right) > r_q^{\max}(1)$) so that (IC) implies that banks optimally allocate all equity to firm type $\hat{q}$, $\kappa_{\hat{q}} = \frac{\min\left\{E, \bar{\pi}_{\hat{q}}\underline{e}_{\hat{q}}^{\text{PPI}}I\right\}}{E} = 1$. To see why banks rank type $\hat{q}$ highest, note that

$$\begin{aligned}
r_{\hat{q}}^{\max}\left(\underline{e}_{\hat{q}}^{\text{PPI}}\right) &= \frac{\text{NPV}_{\hat{q}} + \text{PUT}_{\hat{q}}\left(\underline{e}_{\hat{q}}\right)}{I\underline{e}_{\hat{q}}} \\
&> \text{PPI}_{\hat{q}}(\underline{e}_{\hat{q}}) = \frac{\text{NPV}_{\hat{q}} - \lambda \text{PUT}_{\hat{q}}\left(\underline{e}_{\hat{q}}\right)}{I\underline{e}_{\hat{q}}} \\
&> \max_{\underline{e}_q} \text{PPI}_q(\underline{e}_q) \\
&\geq \text{PPI}_q(1) = \frac{\text{NPV}_q}{I} = r_q^{\max}(1),
\end{aligned}$$

where line 2 follows from the fact that the put is positive, and line 3 follows from the fact that $\hat{q}$ (rather than q) maximizes the PPI. Line 4 follows because the maximized value of the PPI must exceed $\text{PPI}_q(1) = \frac{\text{NPV}_q}{I}$, which is also the maximum ROE for type q if $\underline{e}_q = 1$. As a result, $r_{\hat{q}}^{\max}\left(\underline{e}_{\hat{q}}^{\text{PPI}}\right) > r_q^{\max}(1)$ and (A.31) is slack.

P3 Suppose that type q_M is marginal, i.e., $0 < \kappa_{q_M}(\underline{\mathbf{e}}) < \frac{\bar{\pi}_{q_M}\underline{e}_{q_M}I}{E}$. Then (A.29) and (IC) imply that the regulator's payoff is given by

$$\sum_{q: r_q^{\max} > r_{q_M}^{\max}} \bar{\pi}_q \left[\text{NPV}_q - \lambda \cdot \text{PUT}_q(\underline{e}_q)\right] + \left(E - \sum_{q: r_q^{\max} > r_{q_M}^{\max}} \bar{\pi}_q \underline{e}_q I\right) \text{PPI}_{q_M}(\underline{e}_{q_M}). \tag{A.32}$$

It is now easy to see that optimality of $\underline{e}_{q_M}$ requires that $\hat{\underline{e}}_{q_M} = \arg\max_{\underline{e}_{q_M}} \text{PPI}_{q_M}\left(\underline{e}_{q_M}\right) = \underline{e}_q^{\text{PPI}}$, since all other terms are independent of $\underline{e}_{q_M}$.

P4 We have to consider two cases. First, consider the case, in which all profitable types are financed. Then, the regulator's objective is

$$\sum_{q: r_q^{\max} > 0} \bar{\pi}_q \left[\text{NPV}_q - \lambda \cdot \text{PUT}_q(\underline{e}_q)\right], \tag{A.33}$$

s.t. to the (binding) equity capacity constraint (by Principle P1)

$$E - \sum \bar{\pi}_q \underline{e}_q I = 0. \tag{A.34}$$

(A.33) is a concave objective subject to a linear constraint (A.34). Denoting the associated Lagrange multiplier by η, we obtain the necessary and sufficient optimality condition

$$-\lambda\bar{\pi}_q \frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q} = \eta\bar{\pi}_q I, \tag{A.35}$$

which means that the marginal put value for all types is a constant,

$$-\frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q} = \frac{\eta I}{\lambda}, \tag{A.36}$$

implying (13).
Next suppose that not all types are fully financed, i.e., there is a marginal firm type $0 < \kappa_{q_M}(\underline{\mathbf{e}}) < \frac{\bar{\pi}_{q_M}\underline{e}_{q_M} I}{E}$. Then for all inframarginal types q, the first-order condition of (A.32) implies:

$$I\text{PPI}_{q_M}(\underline{e}_{q_M}) = -\lambda \frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}. \tag{A.37}$$

Since the marginal type's capital requirement maximizes its PPI, the associated first-order condition implies that

$$I\text{PPI}_{q_M}(\underline{e}_{q_M}) = -\lambda \frac{\partial \text{PUT}_{q_M}(\underline{e}_{q_M})}{\partial \underline{e}_{q_M}}. \tag{A.38}$$

Taken together, the two first-order conditions (A.37) and (A.38) imply that the marginal puts are equalized

$$\frac{\partial \text{PUT}_{q_M}(\underline{e}_{q_M})}{\partial \underline{e}_{q_M}} = \frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}. \tag{A.39}$$

■

Proof of Proposition 3: The proof will show the comparative statics separately for all regions 1-4, see Figure 3. Note, to prove the statements in Proposition 3, only regions 1 and 3 are relevant. We will prove the results for regions 2 and 4 for completeness.

1. Suppose firm type q is marginal in region 1. Using the results for the log-normal distribution with two parameters $\left(\overline{X}_q, \sigma_q\right)$, see Lemma A.3, the following first-order condition applies for the optimal capital requirement

$$\overline{X}_q - I - G\left(\underline{e}_q\right) = 0, \tag{A.40}$$

where

$$G\left(\underline{e}_q\right) := \lambda\left[\text{PUT}_q(\underline{e}_q) - \underline{e}_q \frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}\right]. \tag{A.41}$$

Since $G'\left(\underline{e}_q\right) < 0$ (see Proof of Lemma A.3), we obtain that $\left|G'\left(\underline{e}_q\right)\right| = -G'\left(\underline{e}_q\right)$. The comparative statics of the marginal type now follow from applying the implicit function theorem to (A.40),

$$\frac{\partial \underline{e}_q}{\partial \overline{X}_q} = -\frac{1 - \frac{\partial G(\underline{e}_q)}{\partial \overline{X}_q}}{\left|G'\left(\underline{e}_q\right)\right|} < 0, \tag{A.42}$$

$$\frac{\partial \underline{e}_q}{\partial \sigma_q} = \frac{\frac{\partial G(\underline{e}_q)}{\partial \sigma_q}}{\left|G'\left(\underline{e}_q\right)\right|} > 0, \tag{A.43}$$

where

$$\frac{\partial G\left(\underline{e}_q\right)}{\partial \overline{X}_q} = \lambda \left[\frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \overline{X}_q} - \underline{e}_q \frac{\partial^2 \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q \partial \overline{X}_q}\right] < 0, \tag{A.44}$$

$$\frac{\partial G\left(\underline{e}_q\right)}{\partial \sigma_q} = \lambda \left[\frac{\partial PUT_q}{\partial \sigma} - \underline{e}_q \frac{\partial^2 \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q \partial \sigma_q}\right] > 0. \tag{A.45}$$

The respective signs follow directly from Lemma A.2.

2. In region 2, where one type is fully financed and the other type is not financed (see Figure 3), the capital requirement of the funded type is just a function of the equity supply,

$$\underline{e}_q = \frac{E}{\bar{\pi}_q I}, \tag{A.46}$$

which is independent of $\overline{X}_q$ and σ_q for either type.

3. In region 3, the conditions for the marginal type are identical to case 1. Its capital requirement is only a function of its own cash-flow distribution by principle P3 (see Proposition 2).
For the inframarginal type, optimality condition (A.37) applies

$$- I\mathrm{PPI}_{q_M}(\underline{e}_{q_M}) = \lambda \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}. \tag{A.47}$$

We first consider the effect of an increase in the attractiveness of the marginal borrower, i.e., an increase of the $\mathrm{PPI}_{q_M}(\underline{e}_{q_M})$ of the marginal type, on the inframarginal type. An increase in the PPI of the marginal type occurs either because $\overline{X}_{q_M}$ increases or because the volatility of the marginal type shrinks. The implicit function theorem together with convexity $\frac{\partial^2 PUT_q}{\partial \bar{e}_q^2} > 0$ then imply that capital requirements of the inframarginal type are optimally lowered if the PPI of the marginal type is marginally higher, i.e., $\frac{\partial \underline{e}_q}{\partial \overline{X}_{q_M}} < 0$ and $\frac{\partial \underline{e}_q}{\partial \sigma_{q_M}} > 0$. This proves that capital requirements of both marginal and inframarginal types move in the same direction if the cash flow distribution of the marginal type is changed.
We next consider the comparative statics of optimum capital requirements of the

inframarginal type with respect to its own distribution parameters, i.e.,

$$\frac{\partial \underline{e}_q}{\partial \overline{X}_q} = -\frac{\frac{\partial^2 \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q \partial \overline{X}_q}}{\frac{\partial^2 \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q^2}} < 0, \tag{A.48}$$

$$\frac{\partial \underline{e}_q}{\partial \sigma_q} = -\frac{\frac{\partial^2 \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q \partial \sigma_q}}{\frac{\partial^2 \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q^2}} > 0, \tag{A.49}$$

which follows from the fact that $\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial X_q} > 0$, $\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2} > 0$ and $\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial \sigma_q} < 0$ (by Lemma A.2).

4. If both types are fully financed (see region 4 in Figure 3), Principle P4 implies that marginal puts are equalized,

$$\frac{\partial PUT_q}{\partial \underline{e}_q} - \frac{\partial PUT_{\tilde{q}}}{\partial \underline{e}_{\tilde{q}}} = 0. \tag{A.50}$$

For changes in the own distribution parameters, we obtain again conditions (A.48) and (A.49).
The comparative statics regarding changes in the cash flow distribution of the other type $\tilde{q}$ satisfy:

$$\frac{\partial \underline{e}_q}{\partial \overline{X}_{\tilde{q}}} = \frac{\frac{\partial^2 PUT_{\tilde{q}}}{\partial \underline{e}_{\tilde{q}} \partial X_{\tilde{q}}}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} > 0, \tag{A.51}$$

$$\frac{\partial \underline{e}_q}{\partial \sigma_{\tilde{q}}} = \frac{\frac{\partial^2 PUT_{\tilde{q}}}{\partial \underline{e}_{\tilde{q}} \partial \sigma_{\tilde{q}}}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} < 0. \tag{A.52}$$

Taking all these cases together, we obtain the claims in Proposition 3. ■

Proof of Proposition 4: The Proof follows directly from the proof of the general Proposition 3. ■

Proof of Proposition 5: Relative to the problem of the prudential regulator, see Problem 1, the welfare maximizing regulator with two tools solves a problem that is unchanged except for the objective function, which can be written as

$$\Omega = E \max_{\underline{\mathbf{e}}, \tau} \sum \kappa_q (\underline{\mathbf{e}}, \tau) \, \text{SPI}_q(\underline{e}_q), \tag{A.53}$$

where the "Social Profitability Index" (SPI) accounts for externalities ϕ_q on top the NPV and the social cost of bailouts, i.e.,

$$\mathrm{SPI}_q(\underline{e}_q) = \frac{\mathrm{NPV}_q - \phi_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)}{I\underline{e}_q}. \tag{A.54}$$

For firm types with $\mathrm{NPV}_q < \phi_q$, their SPI is negative, so it is socially optimal to prevent funding, i.e., $\kappa_q(\underline{\mathbf{e}}^{**}, \tau^{**}) = 0$. This can be achieved by a prohibitive carbon tax and capital requirements of 100%, which ensures that the put value is zero. This proves the first statement.

To obtain the second statement regarding types with $\mathrm{NPV}_q - \phi_q > 0$, recall that the PPI with a state-contingent carbon tax is defined as:

$$\mathrm{PPI}_{q,\tau_q}(\underline{e}_q) = \frac{\mathrm{NPV}_{q,\tau_q} - \lambda \cdot \mathrm{PUT}_{q,\tau_q}(\underline{e}_q)}{I\underline{e}_q}.$$

By setting $\bar{\tau}_q^{**} = \phi_q$, we ensure that $\mathrm{NPV}_{q,\tau_q^{**}} = \mathrm{NPV}_q - \phi_q$. Second, we need to ensure that $\mathrm{PUT}_{q,\tau_q^{**}}(\underline{e}_q) = \mathrm{PUT}_q(\underline{e}_q)$, i.e., that the carbon tax does not raise the put. Since $\mathrm{NPV}_q = X_q - I > \phi_q$, it is possible to find to find a tax scheme τ_q^{**} that raises on average $\bar{\tau}_q^{**} = \phi_q$ and is only collected in states where the firm is profitable, i.e., when $X_q > I$. As a result, the SPI and PPI coincide under optimal carbon taxes τ_q^{**}

$$\mathrm{SPI}_q(\underline{e}_q) = \mathrm{PPI}_{q,\tau_q^{**}}(\underline{e}_q). \tag{A.55}$$

Put differently, the objectives of the welfare-maximizing regulator, see (A.53), and the prudential regulator under socially efficient carbon taxes τ_q^{**} are identical. Hence, the socially optimal capital requirement satisfies $\underline{e}_q^{**} = \hat{\underline{e}}_q(\tau_q^{**})$. ■

Proof of Lemma 1: Since socially optimal capital requirements for clean firms coincide with optimal prudential capital requirements (because the optimal carbon tax is zero for clean types), we know that socially optimal capital requirements for clean firms with two tools are a weakly increasing function of banking sector equity with range $\underline{e}_C^{**}(E) \in [\underline{e}_C^{PPI}, 1]$, see Figure 5. Moreover, the maximum ROE for any type q, see (4), is a decreasing function of its capital requirements. Hence, if the maximum ROE for clean firms at the lower bound $\underline{e}_C^{PPI}$ is smaller than the maximum ROE for dirty firms at capital requirements of 100%, $r_C^{\max}(\underline{e}_C^{PPI}) < r_D^{\max}(1)$, ranking alignment is impossible. Likewise, if the maximum ROE for clean firms at the upper bound of 100% is higher than the maximum ROE for dirty firms at capital requirements of 100%, $r_C^{\max}(1) > r_D^{\max}(1)$, ranking alignment is always satisfied. These observations also imply that if $r_C^{\max}(1) < r_D^{\max}(1) < r_C^{\max}(\underline{e}_C^{PPI})$, then there exists a cutoff level for E below which there is ranking alignment and above which there is not. ■

Proof of Proposition 6: The problem of the welfare-maximizing regulator with one tool is to choose capital requirements $\underline{\mathbf{e}}$ to maximize

$$\Omega = E \max_{\underline{\mathbf{e}}} \sum \kappa_q(\underline{\mathbf{e}}) \, \mathrm{SPI}_q(\underline{e}_q),$$

where $\text{SPI}_q(\underline{e}_q)$ is defined in (A.54) subject to all the constraints given in Problem 1. By the definition of ranking alignment, see Lemma 1, socially optimal capital requirements e_q^{**} can be set under ranking alignment and banks will still prefer investing in clean loans. It is, thus optimal to set $e_q^* = e_q^{**}$.

When there is no ranking alignment, i.e., banks would prefer to invest in firms with dirty projects if capital requirements were set at e_q^{**}, the planner has two options: either to (strictly) lower capital requirements for clean firms to have them rank first, $e_C^* < e_C^{**}$, or to accept that dirty firms are funded first. In the latter case, this implies that after funding dirty firms, there is less equity left for clean firms. If clean firms are still funded, this means that capital requirements for clean firms must be strictly lower compared to the case with two tools in which all equity is used for funding clean firms, i.e., $e_C^* < e_C^{**}$. Moreover, when clean firms are funded, the optimal capital requirement for dirty firms will be strictly lower than 100% as dirty firms are inframarginal, i.e., $e_D^* < e_D^{**}$.

Finally, regardless of ranking alignment, if banking sector equity is sufficiently high, banks will start funding dirty firms since these are profitable. In particular, if $E = I$, all firms in the economy will be funded. ■

Proof of Lemma 2: By the time the regulator moves in period 2, $\pi_{q,2}$ and $\underline{e}_{q,2}$ are given. Raising the carbon tax at date 2 increases the put without having effects on funding decisions or emissions. Hence, the environmental regulator with objective (18) will not raise carbon taxes. ■

Proof of Lemma 3: We consider the regulator's incentives to raise carbon taxes for dirty firms.[29] Given a mass of funded dirty firms of $\pi_{D,1} \leq \bar{\pi}_D$, the relevant part of the objective function (18) is given by

$$-\lambda \pi_{D,1} \text{PUT}_D(\Delta\tau_{D,1}, \underline{e}_{D,1}) + \delta \sum_q \pi_{q,2} \left[\text{NPV}_q - \phi_q - \lambda \cdot \text{PUT}_q(\Delta\tau_{q,1}, \underline{e}_{q,2}) \right], \quad \text{(A.56)}$$

which uses the fact that $\Delta\tau_{q,2} = 0$, see Lemma 2, so that $\tau_{q,2} = \Delta\tau_{q,1}$.

Raising carbon taxes for dirty firms at $t = 1$ will increase the put in the first period, but may have the benefit of preventing emissions by dirty firms in the next period. Hence, if the environmental regulator raises the tax, it is optimal to raise it in such a way that just prevents funding of dirty types at $t = 2$. Let's call that tax $\Delta\tau^*$. (One never wants to raise it further since it would just increase the put without an additional future benefit). Setting a lower (but positive tax) would still result in a current-period cost in terms of a higher put, but would not prevent funding of dirty types. Thus, the regulator either sets a tax of $\Delta\tau^*$ or 0. The (relevant part of the) payoffs under these two policies are

$$\Omega^{\text{NO Carbon tax}} = -\lambda \pi_{D,1} \text{PUT}_D(0, \underline{e}_{D,1}) + \delta \sum_q \bar{\pi}_q \left[\text{NPV}_q - \phi_q - \lambda \cdot \text{PUT}_q(0, \underline{e}_{q,2}) \right],$$
$$\Omega^{\text{Carbon tax}} = -\lambda \pi_{D,1} \text{PUT}_D(\Delta\tau^*, \underline{e}_{D,1}) + \delta \bar{\pi}_C \left[\text{NPV}_C - \phi_C - \lambda \cdot \text{PUT}_C(0, \underline{e}_{C,2}) \right],$$

where we used the assumption that given sufficient equity E in both periods, all profitable

[29] Since clean firms do not produce emissions, their carbon tax is trivially zero.

types will be fully funded by banks.

Hence, we obtain a simple cost-benefit trade-off of raising carbon taxes. The period-1 cost of raising carbon taxes is

$$\lambda\pi_{D,1}\left(\mathrm{PUT}_D(\Delta\tau^*, \underline{e}_{D,1}) - \mathrm{PUT}_D(0, \underline{e}_{D,1})\right), \tag{A.57}$$

which is strictly positive since the put is strictly higher under $\Delta\tau^*$ for any $\underline{e}_{D,1} < 1$.

The period-2 benefit (which needs to be discounted by δ) is

$$\bar{\pi}_D\left[\phi_D + \lambda\mathrm{PUT}_D(0, \hat{\underline{e}}_D(0)) - \mathrm{NPV}_D\right] + \lambda\bar{\pi}_C\left[\mathrm{PUT}_C(0, \hat{\underline{e}}_C^{\text{No Tax}}) - \mathrm{PUT}_C(0, \hat{\underline{e}}_C^{\text{Tax}})\right] \tag{A.58}$$

where the first part captures the avoided social cost of funding the dirty type in period 2 (and the avoided put value reflects optimal prudential capital requirements given zero carbon taxes). The second term reflects the fact that clean firms can now be fully funded at higher capital requirements (since bank equity is solely used to fund clean types), i.e., $\mathrm{PUT}_C(0, \hat{\underline{e}}_C^{\text{No Tax}}) > \mathrm{PUT}_C(0, \hat{\underline{e}}_C^{\text{Tax}})$. The ratio of costs to benefits determines the threshold value $\delta^*\left(\underline{e}_{D,1}, \pi_{D,1}\right)$ for the discount factor.

The case of $\pi_{D,1} = 0$ is immediate since there is no current period cost and a strictly positive discounted benefit for any $\delta > 0$. ■

Proof of Lemma 4: Suppose that $\delta < \delta^*\left(\underline{e}_{D,1}, \bar{\pi}_D\right)$. If sufficiently many banks provide loans to dirty firms, i.e., $\pi_{D,1}$ is above a threshold, the environmental policy maker will indeed not impose carbon taxes as the cost in (A.57) exceeds the benefit in (A.58). In this case, dirty firms are profitable and will, hence, be fully funded, i.e., $\pi_{D,1} = \bar{\pi}_D$. If $\pi_{D,1}$ is below a threshold, the environmental policy maker will impose carbon taxes, so it's not profitable for banks to provide loans to dirty firms. Thus, it is also an equilibrium that no bank provides loans to dirty firms, i.e., $\pi_{D,1} = 0$.

Finally if $\delta \geq \delta^*\left(\underline{e}_{D,1}, \bar{\pi}_D\right)$, then the policy maker would impose carbon taxes even if dirty firms were fully funded. Since this would render providing loans to dirty firms unprofitable, we obtain $\pi_{D,1} = 0$. ■

Proof of Proposition 7: The first part of the proposition follows from the observation that the discount factor threshold is strictly decreasing in $\underline{e}_{D,1}$ with $\delta^*\left(1, \pi_{D,1}\right) = 0$. The result then follows from Lemma 4.

The second part of the proposition follows from the objective function of the prudential regulator in (19). If carbon taxes are not imposed, then prudential surplus is higher when dirty firms are funded. ■