Game Theory (Théorie des jeux)

Course title - Intitulé du cours	Game Theory – Théorie des jeux
Level / Semester - Niveau /semestre	M1 / S1
School - Composante	Toulouse School of Economics – Ecole d'Economie de Toulouse
Teacher - Enseignant responsable	Bertrand GOBILLARD
Other teacher(s) - Autre(s) enseignant(s)	
Other teacher(s) - Autre(s) enseignant(s)	
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Other teacher(s) - Autre(s) enseignant(s)	
Lecture Hours - Volume Horaire CM	30
TA Hours - Volume horaire TD	12
TP Hours - Volume horaire TP	0
Course Language - Langue du cours	English (international track) French (standard track)
TA / TD Language - Langue des TD	Anglais

Teaching staff contacts - Coordonnées de l'équipe pédagogique :

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Teaching assistants: Xin Zhang, Dario Gori, Pedram Pooyafar, Jingyi Zhang, En Qi Teo, Wen-Hao Wang.

Course's Objectives - Objectifs du cours :

Objectives. Brodadly speaking the game theory class is an introductory course (at an intermediate level) to Economic theory (namely the modelisation and analysis of economic problems using rigorous theoretical reasoning) when the focus concerns situations of strategic interaction. Its prime objective is to familiarize students with basic, key concepts and techniques developped in modern microeconomic / equilibrium / game theory.

On a more concrete level its objective is to provide students with a solid skills base in game theory that, have become an important area in modern microeconomics, and develops a theoretical framework for investigating circumstances where interdependent decision-makers intereract strategically using mathematical models. To this end it builds on a precise description of foundations to the framework of game theory, enabling students: i) To acquire knowledge and expertise on why and how apply the approach; ii) To be more confortable with standard techniques used in economic theory to investigate a wide variety of situations involving strategic decision making.

At the end of the class it is expected students will be able to represent an economic situation as a game they can solve using appropriate solution concepts and method. In that respect, besides the ability to understand a problem and formalize and study it as a game, that of developing a clear and precise argument justifying a suggested solution is to the least as important as finding a right solution. Note, that it is even more prevalent here since the class won't focus on arduous technical (mathematical) aspects that quickly arise when conceiving and studying games, to restrict attention to thoroughly understanding core concepts and methods; with no compromise on accurate formalism, logic and rigour of argument, nonetheless. This also means, in should be added, it is central not to be passive in the learning process for this class: practice is key to get an in-depth understanding of fundameltal

notions and methods studied in the course of the semester, which includes (individual and collective) home study and class participation; shall you be a couch or field game theory player.

Course outline. Essential notions and tools of game theory are studied in turn (more or less extensively): from definitions of games to solution concepts; from static games under complete information to dynamic games under incomplete information.

The course is illustrated with examples (such as bargaining games, signalling games, coordination games, voting games, repeated games…) and economic applications (including simultaneous and sequential models of competition, tacit collusion, auctions, bank runs, investment races, and so on). Precisely, to develop the theory and help students be more familiarized with its core concepts, specific, first (very) basic, examples will be used. More advanced instances of games will then be studied, to further understanding. Unfortunately we won't have time, during the lectures, to develop in greater details economic aspects of general classes of problems such as those mentionned above (it is in fact common in class to introduce very stylised variants; for illustration purposes of the theory firstly). Nonetheless, to evidence how these tools can be used to approach standard economic issues, typical such examples will be included in problem sets. Lectures might nonetheless sometimes be the opportunity for brief discussions on specific applications and references to economics.

The theoretical material covered over the semester is outlined in a collection of slides, entitled *Main slides* or *Core material*, already at the disposal of the students on the moodle platform. These notes will be complemented with others that can be used during the lectures and for personal work; notes containing examples, illustrations, and suggestions on how to proceed to define and solve (examples of) games. A more detailed description of the structure of the course, and a tentative schedule of the lectures, is given afterwards; its main content is first summarized below.

O. Defining and representing games (Normal form game; Mixed extension of a normal form game; Extensive form game, Extensive form and normal form). 1 – 2 weeks

I. Static games of complete information (Games under normal form; The notion of strict dominance; The best response correspondence and Nash equilibria in pure strategies; The mixed extension of a normal form game; The best response correspondence and Nash equilibria in mixed strategies; Link between Nash equilibria and strict dominance). 2 – 3 weeks.

II. Dynamic games of complete information (Games under extensive form; Extensive form and normal form; Nash equilibria and backward induction; Subgame perfect Nash equilibria). 2 – 3 weeks

III. Repeated games (The framework; Finitely repeated games; Infinitely repeated games), 2 weeks

IV. Games with incomplete information . 1 – 2 weeks

Prerequisites - Prérequis :

There is no strong prerequisite for this class, apart from most basic mathematical tools (basic calculus and analysis; basic set theory language; expected utility; basic proof techniques), and a taste for rigorous reasoning. One there refers to the "pre-work material" that better defines prerequisites.

Practical information about the sessions - Modalités pratiques de gestion du cours :

(Practical organisation of the classes). Lectures: 30 hours; language: English (International track) and French (Standard track). Tutorials: 12 hours (7 "Standard sessions" and (possibly) one "Question session"); language: English. The organisation of the lectures is as follows. Three lectures are taught each week during the first 5 weeks (Monday afternoon, Wednesday afternoon and Friday afternoon). During the remaining 5 weeks there is one lecture taught on Friday afternoon.

(Pedagological organisation and usage of Teaching material). A mixed mode of teaching articulating "more standard" parts with "more interaction based" moments.

The primary idea is that students (pre-)work on the content material of a given lecture before going to class. In this way they will have a first familarity of new notations or abstract concepts of great help in learning. Moreover lessons can then be more dedicated to "key issues" and "concerns and questions" students may have on a specific topic. Following experience, and a first trial in recent past based on "flipped classroom" mode of teaching, the plan is to experience an alternative (partial-flipped-classes) protocole, where each lecture combines "more standard" parts one the one hand, at the same time building on "interaction based" study; relying on preparatory (individual or group) work by students on fundamentals of the framework of game theory and the definition of examples of games used in a given lecture. The idea is that the teacher won't "teach per se" the entire material, and will assume students have some familiarity with it. The objective is to include discussions, focus on specific aspects driven by student participation, and concerns, also considering interaction between students and active learning will be as important (for this course in particular new knowledge fully builds on past knowledge).

In more pratcial terms the lectures (and course material) will be (accordingly) organized as follows.

1. It will be announced in advance which part of the theoretical material will be taught during a lecture (a first schedule is provided below).
2. To prepare for the class students will have access to a set of (short) notes with guidelines, entitled *Preparatory material* (as for "Material for preparatory work"), that corresponds to problems and questions precisely defined that can be used to primarily investigate general notions and concepts outlined in the *Core material* (Main slides).
3. The study of these questions will precisely be pursued in class but when the focus will concern only key principles or issues (anticipated by "the teacher" but also those raised by students).

It is key that students work on the preparatory material before coming to class; why attend the lecture otherwise. To avoid wrong incentives however, students are not expected to do thorough analysis or work on all the (detailed) material provided to them in end. The preparatory work is designed so that students can experiment a first understanding of the main concepts and issues related to how to study specific situations. It is as well designed for students to have a sufficient (first) understanding of examples of games used during a lecture before coming to class; what will be most than helpful. Finally, it may also be instructive for students to go back to issues initially identified in the preparatory material after the dedicated class.

In any case, notes contained in the *Compementary material* will provide a comprehensive examination of the material covered in class; including the analysis of games introduced in *Preparatory material*, with suggestions on how to study them in addition to detailed answers and explanations.

Students will have access to a lot a material that can be used for training and understanding of the principles and tools of game theory studied in this class, including past mid-term and final exams, and a collection of problem sets consisting of a variety of exercises and problems; each with detailed solutions. These problem sets build the material for tutorials, but teaching assistants will only (are asked to) study in quite detail a few of those each session (there are too many of them). Students will however have access to detailed solutions to all of them (each being a potential problem for home study), and are free to ask questions about problems not solved in the classroom. Problem sets primarily contain "classic or typical games" for practice and preparation of the exams; although many examples of such "typical games (exams)" will also be included in the material used during the lectures. An additional objective of the problem sets is to include (at the end) some "economic applications" (written is less stylised game theory format); by reason that we won't have time to cover thoroughly

such applications in class, but it is as well important to point out how useful game theory can be to economists aiming at modelling, and studying, various economic questions.

Students will also have access to help desk sessions, with two formats provided.

(i) *One to one meetings* (or one to small groups meetings) with a teaching assistant. Each meeting has to be scheduled and lasts fifteen minutes, where students ask for discussion of a (possibly several) specific point related to what has been done in class (including tutorials). This is also the opportunity to approach difficulties related to a lack of knowledge in students' background required to study the course material.

(ii) *(ii) Group meetings*. A session of two or three hours, with two teaching asssitants, when a specific problem (that students can study in advance) is discussed and analysed; usually a problem from a past exam. There, TAs will bring suggestions on how to understand the problem and proceed, say how to approach and solve the questions asked; in addition to providing right answers (complete, detailed written solutions being also provided nonetheless). Primarily here the objective is to build on questions students will have, bust also on issues arising during the class from discussions between/with TAs.

(Organisation under Covid measures; just in case). As it shall be expected teaching will take place "in class." Its organisation however will adapt to possible sanitary restrictions if the evolution of the Covid pandemic were to reverse. If due do Covid measures it is not possible to organize the teaching as usual, we shall aim to proceed as follows. The prime idea would be to have lectures taught on site (if possible; in smaller groups if need be), with the possibility for students to follow the lectures "100% on line" and switch from "online" to "onsite" mode of teaching depending on their personnal condition or situation. The overall principle is that we will adapt, in the most appropriate manner given all possible kinds of constraints and possibilities.

Grading system - Modalités d'évaluation :

Midterm exam (20%) and final exam (80%).

Bibliography/references - Bibliographie/références :

There is no compulsory textbooks, but we would recommend:

- Robert Gibbons, "A primer in Game Theory", Wheatsheaf Books, 1992; this book can also be found under the title "Game Theory for Applied Economists", Princeton University Press.
- Steven Tadelis, "Game Theory: An Introduction", Princeton University Press.

More exhaustive and advanced material can be found in:

- Martin Osborne and Ariel Rubinstein, "A course in Game Theory", The MIT Press,
- Martin Osborne, "Introduction to Game Theory: International Edition", OUP Oxford,
- Drew Fudenberg and Jean Tirole, "Game Theory", The MIT Press.

Lectures notes (mostly provided in the format of slides) contain all the material necessary to study this course:

1. *Core material* (Main (blue) slides)
2. *Preparatory material* (PMa)
3. *Complementary (more detailed) material* (CMa)

which is complemented with *Problem sets* and *Past Final* and *Mid-term exams*; with solutions.

APPENDIX: MORE DETAILED PRESENTATION OF THE STRUCTURE OF THE COURSE

A/ DETAILED DESCRIPTION OF THE CONTENTS OF THE CLASS AND HOW IT IS COVERED IN THE MATERIAL MADE AVAILABLE TO STUDENTS

INTRODUCTION

- Preparatory material (I): Questions for an introduction
- Complementary material (I): Introductory notes

CHAPTER 0 / DEFINING AND REPRESENTING GAMES

Part 1: Games under normal form (or strategic form)

- Preparatory material (II) and Complementary material (II): Normal form games

Part 2: The mixed extension of a normal form game

- Preparatory material (III) and Complementary material (III): The mixed extension

Part 3: Games under extensive form

- Preparatory material (IV) and Complementary material (IV): The extensive form

Part 4: Extensive form and normal form

- Preparatory material (V) and Complementary material (V): From extensive form to normal form
- Preparatory material (VI) and Complementary material (VI): Playing at random in extensive form games

Part 5: Introducing uncertainty

- Preparatory material (VII) and Complementary material (VII): Uncertainty and nature player

CHAPTER 1 / STATIC GAMES UNDER COMPLETE INFORMATION (STUDYING NORMAL FORM GAMES)

Part 1: Important preliminaries

Part 2: Games under normal form (or strategic form)

Part 3: The notion of strict dominance

- Preparatory material (VIII) and Complementary material (VIII): (A first discussion) On Strictly Dominated and Best Response strategies
- Preparatory material (IX) and Complementary material (IX): Dominance in pure strategies

Part 4: The Best-Response correspondence and Nash equilibria (in pure strategies)

- Preparatory material (VIII) and Complementary material (VIII): (A first discussion) On Strictly Dominated and Best Response strategies
- Preparatory material (X) and Complementary material (X): Nash equilibria in pure strategies
    - *Take home exercise*

Part 5: Analysis of games in mixed strategies

- Preparatory material (XI) and Complementary material (XI): (On) Solutions under mixed strategies in normal form games
    - Preparatory material (XI Bis) and Complementary material (XI Bis): Typical exam problems on Mixed strategies / Simultaneous games; with solutions

CHAPTER 2 / DYNAMIC GAMES UNDER COMPLETE INFORMATION

Part 1: Representing dynamic games

Part 2: Nash equilibria, Backward Induction, Subgame-Perfection

- Preparatory material (XII) and Complementary material (XII): Solutions in dynamic games I/ Nash Equilibria

- Preparatory material (XIII) and Complementary material (XIII): Solutions in dynamic games II/ Backward induction and Subgame perfect Nash equilibrium in perfect information games
- Preparatory material (XIV) and Complementary material (XIV; Part I, On the meaning / understanding / usage of the concept using previous basic games; Part II, Two standard games / Finite horizon and pure strategies): Solutions in dynamic games III/ Subgame perfect Nash Equilibria

CHAPTER 3 / DYNAMIC GAMES UNDER COMPLETE INFORMATION

Part 1: The model

- Preparatory material (XV) and Complementary material (XV): On repeated games

Part 2: Finitely repeated games

- Preparatory material (XVI) and Complementary material (XVI): Solutions in repeated games I/ First equilibrium constructions and solution in finite horizon games

Part 3: Infinitely repeated games

- Preparatory material (XVII) and Complementary material (XVII): Solutions in repeated games II/ Infinite horizon games

CHAPTER 4 / GAMES WITH INCOMPLETE INFORMATION

- Preparatory material (XVIII) and Complementary material (XVII): On incomplete information games

B/ TENTATIVE SCHEDULE OF THE LECTURES

Week 1: Introduction and Practicalities; Normal form game and its mixed extension (Ch 0; PMa & CMa I-III); Extensive form game (PMa & CMa IV).

Week 2: Links between extensive and normal form; Mixed and behavioral strategies in dynamic games; Nature player (Ch 0; PMa & CMa V-VII); On Best Response (BR) and Strict Dominance (SD) (Ch 1; PMa & CMa VIII); Introduction to solutions concepts and Strict dominance in pure strategies (Ch 1; PMa & CMa IX).

Week 3: BR and Nash equilibria (NE) in pure strategies; (Ch 1; PMa & CMa X); Summary of fundamentals learned so far (based on past take home exam).

Week 4: BR and NE in mixed strategies (Ch 1; PMa & CMa XI); End of Chapter 1; NE in dynamic games (Ch 2; PMa & CMa XII).

Week 5: Backward Induction (BI) and Subgame perfect Nash Equilibria (SPNE) in perfect information games (Ch 2; PMa & CMa XIII); Preparation for mid-terms (Based on a past mid-term)

Week 6: Discussion of what has been done so far based on the mid-term exam; On solving for SPNE [Chapter 2; PMa & CMa XIV); SPNE (Ch 2; PMa & CMa XIV')

Week 7: SPNE, Continued (Ch 2; PMa & CMa XIV'); Introduction to repeated games and finite horizon games (Ch 3; PMa & CMa XV and XVI)

Week 8: Infinite horizon repeated games (Ch 2; PMa & CMa XVII)

Week 9: End of the analysis of dynamic and repeated games; Introduction to incomplete information games (Ch 4; PMa XVIII)

Week 10: Incomplete information games (Ch 4; CMa XVIII); Conclusion / Discussion