Vertical Forward Commitments

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PRELIMINARY: COMMENTS WELCOME

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Abstract

This paper presents model of wholesale and retail competition in "subscription" settings where the resolution of the retail market precedes that of the wholesale market. Retailers compete on prices while wholesale producers compete via Nash-Cournot quantities. For vertically integrated firms, retail market shares take on characteristics similar to forward commitments in the wholesale market. Retail commitments are therefore reduce both wholesale and retail prices wholesale market. The social surplus implications will depend upon the scope import/export opportunities on the wholesale market.

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1 Introduction

In Industrial Organization theory, it has long been recognized that credible commitment represents a key dimension in determining oligopoly outcomes in dynamic settings. Certainly establishing a level of production capacity represents at least a short-run commitment to limit output in industries where time-lags in adding capacity are long. As highlighted by the work of Allaz and Vila (1994), forward financial or contractual arrangements can provide a commitment to increase output, and therefore be pro-competitive in settings such as Cournot competition, where firms are producing strategic substitutes.

In restructured electricity markets, forward commitments have been demonstrated to be a key determinant of firm behavior in short-run markets.[1] In many of these cases, however, the forward commitments could be considered to be exogenously determined, an artifact of transition policies put in place at the time of deregulation. For example, in the United Kingdom and many markets of the eastern United States, the divestitures of power plants were coupled with fixed-price supply commitments from those plants. A much more complex question is the endogenous level of commitment one might expect in such markets after these transition arrangements have run their course.

This question is particularly relevant at the moment as the power industry is going through a period of consolidation, raising new concerns about the types of market structures and levels of concentration that can produce satisfactorily competitive markets. Since past analysis indicates that forward commitments are critical in determining the competitiveness of electricity markets, any analysis of the price impacts of mergers needs to consider this dimension as well.

To address the issue of forward commitments, one has to consider the dominant form of commitment that is emerging in the power sector, namely vertical integration between production and retailing. Despite the efforts of regulators in some markets, most notoriously California, to discourage integration between production and retail, consolidation in many markets has led to a structure where almost all major producers have a presence in retail,

[1] Green, 1999, Wolak 2000, Bushnell *et al.* 2008

and vice-versa. While this is a quite common organizational structure in energy markets, it is important to recognize the distinct nature of retail provision of a utility service such as electricity, natural gas, or telecommunications. Unlike gasoline or durable goods customers, retail customers in utility industries are essentially "locked in" for some period of time. Additionally, there are both technological and traditional restrictions on the frequency with which prices can change in these industries. These factors combine to create the conventional arrangement where retail customers "subscribe" to service at prices that are fixed for at least a month, and in many cases much longer.

The result is a vertical dynamic in which the timing of pricing is reversed from the conventional model where producers determine wholesale prices first, and retails mark-up those prices to end-use customers. Instead, in utility markets, retailers often commit to a price *before* the wholesale price is determined. For a vertically integrated firm in such a setting, taking on a retail customer therefore represents a forward commitment with analogous incentives to a forward contract. Several papers have demonstrated the important pro-competitive role that vertical positions (Mansur 2007, and Bushnell *et al.* 2008), or fixed-price forward contracts (Wolak 2000, Bushnell *et al.* 2008, and Gans and Wolak 2011) play in electricity markets.

However, many of these papers draw upon historic or confidential data sources to establish the vertical position of the firms in the analysis. Firms acquiring divested assets were mandated to sign such contracts as part of the restructuring transition. As these contracts expire, and retail choice becomes viable, researchers and regulators have much less external visibility into the forward and vertical positions of these firms. Without this information, it is very difficult to get an accurate picture of the competitive potential of a market. This dearth of information on retail market positions of suppliers makes it extremely difficult to assess the wholesale price effects of proposed merger between suppliers of wholesale electricity. As we demonstrate in this paper, ignoring the impact of forward market positions in competitive effects analysis can yield biased conclusions about the potential harm to competition from a merger.

2 Vertical Integration and Forward Commitments

In this paper, we develop a model that combines two strains of literature; the vertical integration literature and the forward commitments literature. The literature on vertical integration is extensive and we highlight here the papers most relevant to the characteristics of our setting. First, our setting involves a homogenous wholesale product that is available through an impartial and transparent wholesale market at a uniform price. While much of the literature on vertical restraints has utilized settings where bilateral contracts between vertically separated agents allow for price discrimination, several papers have examined this uniform price setting. Salinger (1988) models a vertical structure with M upstream and$N$ downstream firms, where both the upstream and the downstream compete in Cournot quantities. This is one of the first papers to demonstrate the trade-off between reducing double marginalization on the one hand and the incentive to raise rival's costs on the other. He finds that the impacts of vertical integration are mixed and the net impact depends upon the parameters of the model. Ordover, Salop, and Saloner (1990) study a setting where a single upstream firm is dealing with several downstream firms, who each compete in prices. They also identify a trade-off between the pro and anti-competitive effects of integration.

Both of these papers apply the standard assumption that the upstream prices are determined before the downstream, making the upstream firms "leaders" of the downstream firms. One paper that applies the opposite sequencing is Gans (2008), but he does not study an open, uniform price setting and instead utilizes a model of Nash bargaining. Salinger (1989) considers a uniform price setting with general conjectures about the impact of output at one stage on the strategies of firms in the other stage. McAffe and Hendriks (2009) apply a model similar in spirit to ours, with multiple firms at both the wholesale and retail levels. Firms compete with supply (and demand) functions reduced to a single parameter space by a functional form assumption on supply offers and market bids, but Cournot can be considered a subset of their structure. They do not assume that the upstream leads the downstream, but rather represent an equilibrium where in effect both markets are cleared simultaneously. Integrated firms in the MH model do internalize how their retail position modifies their "net" wholesale position, but do not anticipate what effect this commitment (e.g. reduction in net positions) can have on other upstream producers.

One important difference from other papers examining the vertical integration quesiton istherefore is the treatment of this commitment effect, which builds on a literature that studies the impacts of forward commitments (e.g. futures contracts) on oligopoly outcomes. Starting with Allaz and Vila (1988), a line of theoretical work has explored the extent to which the existence of forward markets can impact competition in oligopolistic markets. Much of this work has focused on the electricity industry, in part because it features three elements present in the Allaz and Vila model, oligopoly suppliers, homogenous commodity products, and robust forward markets (Powell, 1993; Newbery, 1998; Green, 1999). Importantly, the pro-competitive effects of forward commitment arise in a strategic-substitutes environment. At an extreme opposite of the AV model, Mahenc and Salanie (2004) find that when firms engage in differentiated Bertrand competition in the spot market, the ability to sign forward contracts can reduce competition. Ferreira (2003) examines a context in which there are infinite forward contracting rounds and demonstrates that a kind of folk-theorem result can arise, supporting a range of equilibria. Liski and Monterro (2006) demonstrate conditions in which repeated contracting can faciliate tacit collusion. Green and Le Coq (2006) argue that the risk of facilitating collusion is greatly reduced when the contracts are of longer term (i.e. cover many spot periods).

3 Two-Stage Model of Electricity Market Competition

In this section we develop a theoretical framework for evaluating the relative effects of vertical and horizontal mergers in our institutional setting. The general strategies and timing of the market is as follows. Firms $j \in 1..J$ first offer retail rates to customers and subscribe a quantity of customers q^r_{jt} for t time periods. The retail quantity can vary for every t, but the level is established in advance of $t = 1$. Once a retail commitment has been established in advance of $t = 1$, firms compete on the wholesale market to buy and sell power to each other, to an import/export market, and to unsubscribed customers. Importantly, retail commitments are considered sunk by the firms by the time the wholesale market outcomes are determined.

Given this sequence of market commitments, a subgame perfect equilibrium can be described by wholesale market equilibria that are nested within a first-stage retail market equilibrium. We begin by characterizing the second stage of this sequence, the wholesale market, following the structure of Allaz-Vila. We assume that firms, as producers of a homogenous wholesale product, engage in Cournot competition at the wholesale level. The Cournot best-response functions of each firm will be strongly influenced by the retail commitments those firms have made in the first stage.

Let q_{jt} be the wholesale electricity production, produced with cost $C_j(q_{jt})$, of firm j. Conditioned on a specific vector of retail commitment q^r_{jt}, the profits of firm j will be

$$\pi_{jt} = \left[p^r_j - p_t(q_{it}Q_{-j,t})\right] q^r_{jt} + p_t(q_{jt}, Q_{-j,t})q_{jt} - c_j(q_{jt}) \tag{1}$$

where for firm j, the term $Q_{-j,t}$ represents the aggregate output of all other firms. The first term of (3) represents the firm's revenue from its retail commitments. Note that the "costs" to retailers are assumed to be dominated by the wholesale cost (either real or opportunity) of power, which is captured in the second term.

The second and third terms in (3) form the profit from net sales of $q_{jt} - q^r_{it}$ in the short-term market. Note that the second term can be positive or negative depending on the extent of firm j's retail commitments (q^r_{jt}) relative to the amount of output (q_{jt}) it produces during hour t. The critical point here is retail revenues are *sunk* at time t, because the prices for these commitments have been determined before time t.

A firm that produces more output than its retail commitments is essentially "long" in wholesale energy and therefore has an incentive to raise wholesale market prices in order to increase the revenues it earns from wholesale market sales. Conversely, a firm that produces less output than its retail quantity can be thought of as short in energy. A supplier that is short maximizes the profits it earns from selling energy by using its ability to exercise unilateral market power to *lower* the short term wholesale electricity price.

In the following section, we discuss approaches for modeling the retail quantity, but we begin with a numerical solution to the second stage of the two-stage game: the wholesale market. More formally, we assume that at the wholesale level, firms engage in Cournot competition. Each strategic firm chooses output level to maximize profits, given the output choices of their competitors. This generates the following first-order condition for each strategic firm:

$$\frac{d\pi_{jt}(q_{jt}, Q_{-jt})}{dq_{jt}} = p^w_t(q_{jt} + Q_{-jt}) + \frac{dp_t}{dq_{jt}}(q_{jt} - q^r_{jt}) - C'_j(q_{jt}) \geq 0. \tag{2}$$

The market equilibrium is derived from the simultaneous solution of the first-order conditions of all firms. The impact of forward retail commitments on wholesale market behavior is intuitively represented in (2). The first two terms represent the wholesale market marginal revenue of firm j, which is increasing with q^r_{jt}. A larger retail commitment raises marginal revenue and therefore the output of firm j. This phenomenon is by now well established in power markets, where forward and retail commitments have been identified as key influences on firm behavior.[2]

Most of these papers utilize external data on retail or forward commitments. However, when q_{it} is observed, and external data on marginal costs are available, equation (2) can be used to estimate the retail commitments q^r_{it}. In a related paper, we do this with PJM market

[2]Bushnell, Mansur, and Saravia (2008), Green (1998), Wolak (2001).

data for 2010. However, in order to develop a prediction of how retail commitments might change in the event of a merger or other market shock, a model of the (first stage) retail market is required.

3.1 Retail Competition and Retail Commitments

Predictions of merger effects need to incorporate an assumption about the impact of the merger on retail commitments. One such assumption would be that the retail commitments we are estimating here are quite 'sticky' and may not be dramatically altered in the immediate months or years following a merger. In other words, retail market shares would remain unchanged. As a result, in order to calculate the price of effects of the merger, the merged firm would take on the aggregate retail commitments of its component entities.

However, it is natural in the long run to expect a merged entity to adjust its retail position, and for the other firms to respond to that adjustment. In order to assess the question of the possible long-term consequences of the merger, we therefore require a model that makes the retail price (or equivalently the retail commitment level) an endogenous choice variable for the firms.

We begin with a discrete choice model of retail electricity demand where each consumer choses a specific retail provider $i \in 1..I$. When consumer i drawn from a general population of consumers purchases from retailer j this yields utility of the form

$$u_{ij} = \alpha(\delta_j - p_j) + \epsilon_{ij}$$

Because of vertical integration many, but not necessarily all, of these firms are also represented in the $1..J$ wholesale firms. The "outside good" in this case therefore constitutes a purchase from retailers not integrated into the wholesale market. Given the standard assumptions about the distributions of ϵ_{ij} we can express retail demand as the market share, S_i of firm i, which is equivalent to

$$S_i = \frac{q^r_{it}}{\sum_{l \in I} q^r_{lt}} = \frac{e^{-\alpha\left(p^r_i + \delta_i\right)}}{e^{\alpha V_o} + \sum_{l \in I} e^{-\alpha\left(p^r_l + \delta_l\right)}}$$

where V_o represents the value of consuming the outside good. The retail position of a given retailer will then be its market share times the potential market size during a given time period, $q_{it}^r = S_i M_t$.

In order to predict wholesale market impacts, we combine this retail market model with a variant of the model of Allaz and Vila, who formulate a two-stage game of Cournot contracting. The sequence of firm decisions is as before. Firm's first make retail price commitments for a specific future time period, t, at some time prior to t. It is further assumed that each firm knows the retail price commitments of the other firms. When the spot market is held at time t, firms set production to maximize profits, subject to the advanced retail commitments they have already made. Each firm knows that a higher retail price (and therefore larger retail obligation) will commit it to produce more (perhaps beyond its unconstrained unilateral best-response) in the subsequent wholesale spot markets. However, firms may also internalize the fact that a credible commitment to produce more in the spot market will stimulate a response by other wholesale firms to produce *less*. Finally, higher retail prices will cause some customers to exit the market completely, thereby reducing the size of the wholesale market. In reaching an optimal forward commitment, therefore, firms balance these effects.

More formally, consider again the profit function of the combined markets

$$\pi_{it}(q_{it}, Q_{-it}) = p_{it}^r q_{it}^r + p_t^w(q_{it} + Q_{-jt})[q_{jt} - q_{jt}^r] - C(q_{jt}), \tag{3}$$

If we consider retail commitments to precede wholesale production decisions, then the retail commitment can be represented as the first stage the a two-stage game and the subsequent spot market production is determined through the retail commitments. Vertically integrated firms first choose a quantity of retail commitments, and then subsequently determine their wholesale production levels. In the second stage, retail commitments are sunk, and the first-order conditions described above capture the firm's wholesale market decisions. These second-stage production decisions are nested within the first-stage problem to derive an equilibrium level of retail commitments.

If we consider wholesale production $q_{it}(q_{it}^r, q_{-it}^r)$ to be a well-defined function of the commitments of all firms, we can express the first order condition for optimal retail commitments by differentiating 3 by p^r_{it}

$$\begin{aligned}\frac{d\pi_{it}}{dp^r_i} &= q^r_{it} + [p^r_i - p^w_t]\,\frac{\partial q^r_{it}}{\partial p^r_i} + [p^w_t - c'_i(q_{it})] * \left(\sum_j \frac{\partial q_{it}}{\partial q^r_{jt}}\frac{\partial q^r_{jt}}{\partial p^r_i} + \frac{\partial q_{it}}{\partial a_t}\frac{\partial a_t}{\partial p^r_i}\right) \\ &\quad + p^{w\prime}_t(Q) * [q_{it} - q^r_{it}] * \left(\sum_l \frac{\partial q^r_{lt}}{\partial p^r_i}\sum_j \frac{\partial q_{jt}}{\partial q^r_{lt}} - [1 - \sum_l \frac{\partial q_{lt}}{\partial a_t}]\frac{\partial a_t}{\partial p^r_i}\right) = 0.\end{aligned} \tag{4}$$

We have left the retail price in a general form above, but with the further restriction to our logit retail demand structure, we have $q^r_i = S_i M_t$, $\frac{\partial q^r_{it}}{\partial p^r_i} = -\alpha S_i(1 - S_i)M_t$, and $\frac{\partial q^r_{jt}}{\partial p^r_i} = \alpha S_i S_j M_t$. We use the notation Q_{ot} to represent the "size" of the wholesale market, before accounting for import or fringe wholesale supply. The size of the wholesale market is based upon the share of customers electing to not buy power from any retail provider (e.g. consumption of the outside retail good), $Q_{ot} = M_t * (1 - S_o)$. The cross-elasticity with regards to the outside good is $\frac{\partial Q_{0t}}{\partial p^r_i} = -\alpha S_0 S_i M_t$.

Equation 4 becomes

$$\begin{aligned}\frac{d\pi_{it}}{dp^r_i} &= q^r_{it} - [p^r_i - p^w_t]\,\alpha S_i(1 - S_i)M_t - [p^w_t - c'_i(q_{it})]\,\frac{\partial q_{it}}{\partial q^r_{it}}\alpha S_i(1 - S_i)M_t && (5) \\ &\quad + [p^w_t - c'_i(q_{it})]\sum_{j\neq i}\frac{\partial q_{it}}{\partial q^r_{jt}}\alpha S_i S_j M_t \\ &\quad - [p^w_t - c'_i(q_{it})]\,\frac{\partial q_{it}}{\partial Q_{0t}}\alpha S_o S_i M_t && (6) \\ &\quad - p^{w\prime}_t * [q_{it} - q^r_{it}]\alpha S_i(1 - S_i)M_t \sum_j \frac{\partial q_{jt}}{\partial q^r_{it}} \\ &\quad + p^{w\prime}_t * [q_{it} - q^r_{it}]\sum_{l\neq i}\alpha S_i S_l M_t \sum_j \frac{\partial q_{jt}}{\partial q^r_{lt}} \\ &\quad + p^{w\prime}_t * [q_{it} - q^r_{it}]\alpha S_o S_i M_t \left[1 - \sum_j \frac{\partial q_{jt}}{\partial Q_{ot}}\right] = 0.\end{aligned}$$

We now take a moment to discuss the elements of this combined two-stage first-order condition represented in equation (5). The first two elements are the direct impacts on retail profits, both from a higher price and through the impact of that price on retail quantity. The third term (on the first line) represents the wholesale marginal profit impact working indirectly through the change in spot market production by firm i in response to a change in retail position. The second line of (5) is the wholesale marginal profit impact working indirectly through the change in spot market production by firm i in response to changes in retail positions by other firms $j \neq i$. The third line reflects the change in $i's$ wholesale production in response to a change in the size of the wholesale market as retail customers move to the outside good. The last 3 lines capture the impact of a lower wholesale price. This price changes because of changes in production by both firm i and other firms $j \neq i$ in response to changes in the retail position of i (line 4) as well as changes in production by all firms (including i) in response to changes in the retail positions of all other firms $l \neq i$ (line 5). Last prices change in response to a shift in the size of the wholesale market as customers move to the outside good (line 6).

The conditions in (2) and (5) apply to general numbers of firms, costs, and demand structures. Using these two sets of conditions one can numerical calculate the two-stage equilibrium for a broad set of functional forms and parameter values describing the wholesale market. The following Lemmas describe several convenient properties of the Cournot game with forward commitments for demand structures of the form $D_t(p) = a_t - F(p)$, with inverse demand $P_t(Q) = G(a_t - Q_t)$ where $G = F^{-1}$ under the standard assumption that $G'' - C'' \leq 0$. We use the notation $\Phi_i = \sum_i \frac{\partial q_j}{\partial q_i^r}$ to describe the total change in market quantity resulting from a change in the retail position of firm i.

Lemma 1 *The change in the wholesale quantity of firm i with respect to a change in the size of the wholesale market,* $\frac{\partial q_i}{\partial Q_0} = \sum_j \frac{\partial q_j}{\partial q_i^r} = \sum_j \frac{\partial q_i}{\partial q_j^r} = \Phi_i$.

Lemma 2 Φ_i *is decreasing in the convexity of costs* C''.

Lemma 3 *For the case with constant marginal costs,* $\frac{\partial q_i}{\partial q_i^r} = n\Phi_i, \Phi_i = \frac{1}{(n+1)}$.

Using these results, we can greatly simply condition (5) under an assumption of n symmetric integrated firms. With symmetry we have that $\Phi_i = \Phi_j = \Phi \forall i, j$. By noting that

$q_{it}^r = S_i M_t$ and dividing (5) by $\alpha S_i M_t$, we have

$$\begin{aligned}\frac{d\pi_{it}}{dp_i^r} &= \frac{1}{\alpha} - [p_i^r - p_t^w](1 - S_i) - [p_t^w - C_i'(q_{it})]\left([(n+1)S - 1]\Phi - \frac{\partial q_i}{\partial q_j}\right) \\ &\quad + p_t^{w\prime} * [q_{it} - q_{it}^r] S_o [1 - (n+1)\Phi] = 0.\end{aligned} \tag{7}$$

The first two terms in (10) capture the standard retail marginal revenue terms. The third term captures the profit impacts of a net change in firm i's wholesale position. The last term captures the profit impacts of changes in the wholesale price, which arise indirectly from changes in the retail commitments as well as the size of the wholesale market. Note that when a firm is long at the wholesale level, $q_{it} > q_{it}^r$, the last term above will lower prices. This is because the firm internalizes the effect of its retail price on the size of the wholesale market, and therefore on the price in the wholesale market. If the firm is net short on the wholesale market, this term turns positive and the retailer will raise prices above the classic level where the sequential effect on the upstream market is ignored.

Finally, with constant marginal cost, from Lemma 3 we have $-[(n+1)S-1]\Phi - \frac{\partial q_i}{\partial q_j} = 1 - S$, rearranging in terms of retail price, the retail pricing equation becomes

$$p_i^r = \frac{1}{\alpha(1-S)} + C'.$$

For integrated firms that are net-long on the wholesale market, the last two terms in (10) provide offsetting incentives for retail prices. Through one channel, lower retail prices provides more wholesale market share. However, this also lowers wholesale prices, which lowers profits for the net-long firm. For general cost functions these two effects will counterbalance each other, but not exactly. In the special case with constant marginal costs and symmetric firms, these two offsetting effects from the two-stage game exactly cancel out. This means the retail pricing equation resembles that of an integrated retailer in a conventional vertical framework where wholesale market precede retail.

4 Linear Demand and Constant Marginal Cost

While numerical solution is required for more complex functional forms, we can derive several analytic results when restricting the model to linear demand and constant marginal cost. To develop the intuition behind our results we first demonstrate the effect of vertical integration in the basic model of Alaz and Vila (1988). Market demand is linear with slope of -1, so $Q(p_t) = a - p_t$. There are two symmetric firms, each with constant marginal production costs of c. Under these assumptions, they show that a two-stage model with perfect arbitrage between forward (e.g. in this case, retail) prices and spot prices produces the following equilibrium conditions in the spot (wholesale) market.

$$q_i^*(q_{\neq i}^r, q_i^r) = \frac{a - q_{\neq i}^r + 2q_i^r - c}{3}. \tag{8}$$

Given the closed form solution to $q_i^*(q_{\neq i}, q_i^r)$ it is straightforward to derive the sensitivity of the spot market equilibria, $\frac{\partial q_i}{\partial dq_j^r}, \frac{\partial q_i}{\partial a}$ by differentiating (8). This yields the vectors

$$\begin{pmatrix} \frac{\partial q_1}{\partial q_1^r} & \frac{\partial q_1}{\partial q_2^r} \\ \frac{\partial q_2}{\partial q_1^r} & \frac{\partial q_2}{\partial q_2^r} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix}; \begin{pmatrix} \frac{\partial q_1}{\partial a} \\ \frac{\partial q_2}{\partial a} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} \tag{9}$$

For reasons that will become clear later, we decompose the wholesale market intercept term $a = \hat{a} + Q_{exp}$, where $\hat{a} = \sum_i q_i^r$, the size of the "domestic" retail market and Q_{exp} represents the size of the export wholesal market (e.g. the portion of the wholesale market for which the local retail firms have no ability to access).

Incorporating the values from (9) into the first-stage first-order condition (10), and noting that $sum_j \frac{\partial q_i}{\partial q_j^r} = \Phi_i = \frac{1}{3}$ and $n = 2$ allows for

$$\begin{aligned} \frac{d\pi_{it}}{dp_i^r} &= \frac{1}{\alpha} - [p_i^r - p_t^w](1 - S_i) - [p_t^w - C_i'(q_{it})]\left([3S - 1]\frac{1}{3} - \frac{2}{3}\right)(1 - S_i) \\ &\quad + p_t^{w\prime} * [q_{it} - q_{it}^r]S_o\left[1 - (3)\frac{1}{3}\right] = 0. \end{aligned} \tag{10}$$

or

$$p_i^r = \frac{1}{\alpha(1 - S_i)} + c_i. \tag{11}$$

As we demonstrated earlier the integrated firm retail price equation (11) applies to any case with n firms and constant marginal cost. For an integrated retailers in our model, the feedback to the wholesale market affects retail profits in two ways. The first effect is the increase in wholesale production quantities when retail shares rise (the commitment effect). The second effect is the reduction in the the wholesale price when retail prices rise.

The first effect will provide retailers with a marginal incentive to lower prices, as it would increase retail (and therefore wholesale) market shares, while the second effect provides net-long firms with a marginal incentive to lower retail prices (and thereby raise wholesale prices). As we shall see, this contrasts with the incentives of an unintegrated retailer, for whom the first effect does not apply.

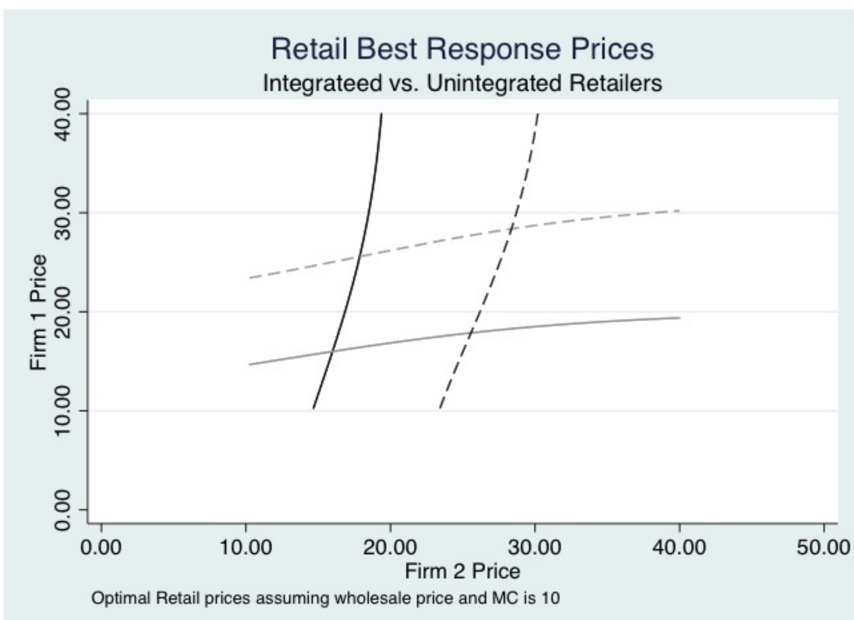

Figure 1: Retail best reply functions with and without integration

Figures 2 and 1 illustrate this dynamic for the specific case with $c = 0, \alpha = 0.1$, and $\delta = 2$.

Figure 1 illustrates the retail market equilibrium. The solid lines trace out the best response retail prices for the unintegrated firms, while the dashed lines do the same for 2 integrated firms. The added elements to the unintegrated retail firms price-response both shift out and increase the slope of these functions, producing a higher retail price equilibrium even if wholesale prices were assumed to be identical. However, wholesale prices also are impacted by the vertical integration, as illustrated in Figure 2. This Figure traces out the best-response production quantities for 2 Cournot firms with and without a vertical commitment for the case where there is no export

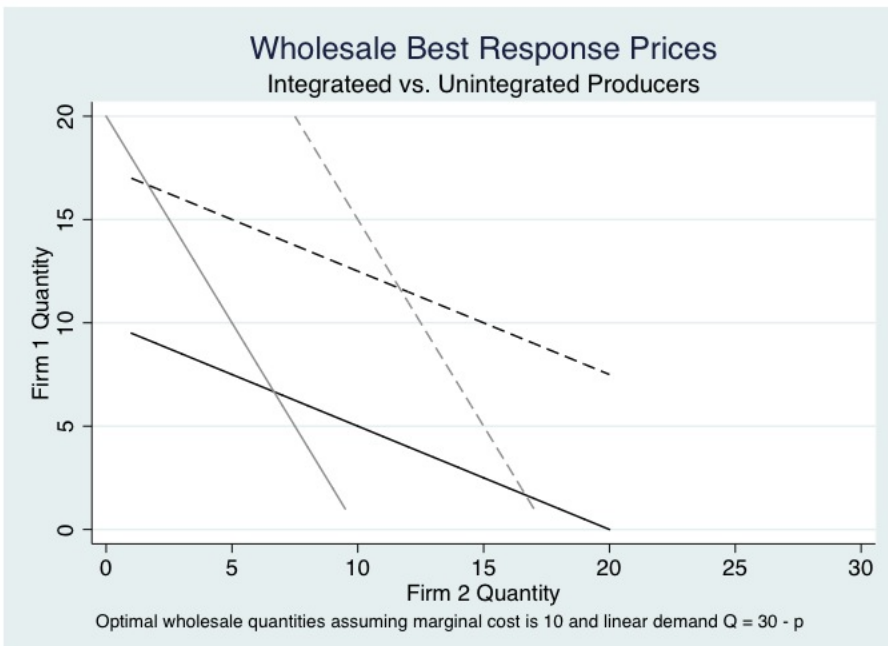

Figure 2: Wholesale response functions with and without integration

It will be convenient to express retail prices in terms of the market fundamentals, a and c. If the wholesale market has import, but not export possibilities, then the size of the upstream market (including imports) will equal the size of the downstream market, $a = \sum_i q_i^r = \sum_i S_i M$, which in a symmetric 2 firm case $a = 2S_i M$.

Lemma 4 *In the integrated case with no export market $a_t = (S_1 + S_2)M_t$, the following characterizes the wholesale market equilibrium.*

$$q_i^w = S_i M - \frac{c}{3};\; p^w = \frac{2}{3}c;$$

This result starkly illustrates the effect of integration. Prices at the wholesale level are driven below marginal cost as the integrated firms, who each are net short on the wholesale market in equilibrium, drive down prices at the expense of the importers. As demonstrated above, retail prices reflect a standard mark-up above marginal wholesale cost.

$$p_i^r = \frac{1}{\alpha(1 - S_i)} + c_i.$$

4.1 Impact of Vertical Integration

We can now assess the relative impact of vertical position on retail pricing. There are several cases to consider. First consider the case where both downstream and upstream move simultaneously. In a standard price-competition setting, where p^w represents marginal "wholesale" cost of the product, the FOC produce the following pricing relationship.

$$p_U^r = \frac{1}{\alpha(1 - S_U)} + p^w.$$

We instead focus on a second case of interest which features unintegrated retail firms who, following the market dynamics that are featured in this paper, commit to their retail prices before the wholesale market is resolved. The unintegrated retail firm will still internalize how its actions can influence wholesale prices by influencing the size of the wholesale market. We use the subscript U to denote the firm-level and market-level outcomes of the unintegrated case. Without any wholesale position by any retailers, $\frac{\partial q_l^w}{\partial q_j^r} = 0 \forall i, j$, and $q_{it} = 0$ for all i retailers, who would have no wholesale position. The first-order condition for retail pricing (5) becomes

$$\frac{d\pi_{it}}{dp_U^r} = \frac{1}{\alpha} - [p_U^r - p_U^w](1 - S_U) - p_U^{w\prime} q_U^r S_{oU}[1 - \sum_j \frac{\partial q_j^w}{\partial a}] = 0.$$

where $S_i = S_U \forall i$ when all retailers are identical. Under the assumptions of this example this further reduces to

$$p_U^r = \frac{1}{\alpha(1-S_U)} + p_U^w + \frac{n}{(n+1)}\frac{S_{oU}}{(1-S_U)}q_U^r = 0.$$

At the wholesale level, there is standard Cournot competition. Under the assumptions of this model the results are

$$q_w = \frac{a-c}{n+1} = \frac{nS_uM + Q_{exp} - c}{(n+1)};\ p_U^w = \frac{a+nc}{n+1} = \frac{Q_{exp} + nS_uM + nc}{n+1}.$$

Combining wholesale and retail level results we can establish the following

Proposition 1 *For the symmetric market with linear demand and n firms with constant marginal cost, if the unintegrated market results in positive production ($a > bc$), then both retail and wholesale prices will be lower with n vertically integrated firms than n vertically separated firms at each level.*

Proof. See Appendix.

This proposition extends the example of Lemma 4 to a slightly more general environment. The result is true regardless of the slope of fringe/import supply on the wholesale market, and for any size of the "local" retail market relative to the entire wholesale market. In cases where the local retail market determines a large portion of the overall wholesale market, as in the example of Lemma 4, the integrated firms reduce their wholesale production and drive prices below their marginal costs. The losses on the wholesale level are more than compensated by increased retail margins. In effect the local integrated firms exert monopsony power on the importing or fringe wholesale producers.

This result contrasts to Salinger's 1988 result where the price effect of integration is indeterminate. That previous result relied in part upon the fact that, once firms integrated they in effect exited the wholesale market or, equivalently, set their wholesale production exactly equal to their retail demand leaving themselves with a net zero position on the wholesale market. That is not the case here, as integrated firms are still anticipating a wholesale market reaction from the import/fringe producers. When the local retail market is large relative to the wholesale market, integrated firms become net short (wholesale less than retail), thereby producing the incentive to lower wholesale prices. When the local retail market is not the majority of the wholesale market, integrated firms will remain net long and sell to both their local retailers as well as into the export market.

Given the fact that integration lowers prices on both markets, the next natural question to ask is whether firms would find it profitable to pursue vertical integration. The following results speak to this question.

Lemma 5 *The wholesale production quantity of a single vertically integrated firm is greater than the optimum Stackleberg leader quantity when* $Q_{exp} - bC' < n(2n-1)S_{int}M$.

Proof: For Cournot problems of this functional form, the optimal Stackleberg leader quantity will be $\frac{a-bc}{n} = \frac{Q_{exp}+nS_UM-bc}{n}$. Vertical integration results in production of $\frac{a+nS_{int}M-bc}{(n+1)} = \frac{Q_{exp}+2nS_{int}M-bc}{(n+1)}$. This latter value will be larger when

$$[n^2S_{int} - n^2S_U - nS_U]M > n(n-1)S_{int}M > Q_{exp} - bC'$$

Proposition 2 *For the linear wholesale market with n symmetric wholesale firms and n symmetric retail firms, when one pair of retail and wholesale firms vertically integrates, the profit of the merging pair will increase.*

Proof: See Appendix.

As these results demonstrate, the integrated firm is able to take advantage of its strong commitment on the wholesale market to take on a form of Stackelberg leader role. Unlike the standard forward commitment model, however, the integrated firm may in some cases take on a larger forward commitment, and therefore produce more, than the optimal Stackelberg leader quantity. The net result is such that in some cases, when the export market is small relative to the domestic retail market, or there are a large number of firms, profits can be lower at the wholesale level for the integrated firm. In all cases this reduction in wholesale profit is more than compensated for, however, by the reduction in retail costs. If it were not, the retail level of the integrated firm would raise its prices until the forward commitment better aligned with the profits of the combined firm.

These results imply a plausible sequence of integration choices. The first firm to integrate is able to benefit - at least relative to other producers - from its leadership role on the wholesale market. Note that all the other wholesale producers are then strictly worse off. Wholesale prices *and* quantities decline for these firms. For these firms, the choice to integrate becomes more attractive, perhaps imperative, once other firms have taken the path to integration. Unlike the stand-alone wholesale producers, independent retailers may benefit from the reduction in wholesale costs brought about by the integration of a rival. The other group that is clearly harmed by the integration are the fringe sellers in the import/export market. Prices decline and the net production of the Cournot firms increases, meaning both lower prices and market share for the remaining producers.

4.2 Impact of Horizontal Aggregation

Given the apparently pro-competitive, or at least pro-consumer, elements of vertical integration in this setting, it is interesting to examine how the consolidation of vertically integrated firms compares to analogous concentrations of separated ones. In order to address this question we again turn to our assumption of linear demand and constant marginal cost. We examine a case where two of $n+1$ firms with identical marginal costs and retail quality wishes to merge. We then compare this case to one in which $n+1$ separated wholesale producers and $n+1$ retailers wish to merge and remain vertically separated.

First, it is interesting to note that in some cases, horizontal combination of vertically combined firms will *lower* prices. This is the situation where the retail market is a net importer from "outside" fringe firms. Concentration of "domestic" oligopoly firms allows for a more effective oligopsony strategy at the wholesale level.

Lemma 6 *When two of n integrated firms merge leaving $n-1$ identical integrated firms, wholesale prices will decline if $bC' > Q_{exp}$.*

Proof. Recall that the equilibrium wholesale and retail prices for the integrated case are $p^w_{int} = \frac{a - nS_{int}M + nbc}{n+1} = \frac{Q_{exp} + nbc}{n+1}$. Differentiating p^w_{int} with respect to n yields

$$\frac{\partial p^w_{int}}{\partial n} = \frac{bc - Q_{exp}}{(n+1)^2}$$

which is positive (meaning more firms raises prices) if $bc > Q_{exp}$. ■

In other words, high costs and high import elasticity favor prices below costs, where local firms use their concentration to oligopsonize imports. Larger export markets favor prices above costs, and firms utilize their market power to exert conventional oligopoly power on the export market. Note that concentration in the retail market will only raise prices in the integrated paradigm, where firms mark up marginal production costs. So the oligopsony effect at the wholesale level is not reflected in retail rates, but rather the profits of the integrated firms.

The more nuanced question is not whether concentration will lower prices absolutely but what the effect is *relative* to an analogous merger in the vertically separated paradigm.

Proposition 3 *For the constant marginal cost and linear demand case, a combination of mergers that reduce both the wholesale and retail markets from n to $n-1$ symmetric firms will increase prices more in the vertically separated environment than in the vertically integrated one.*

Proof. We use the terms Δ^w_p and Δ^r_p to denote the difference in prices $p_U - p_{int}$ between the two paradigms. From previous results we have that $\Delta^w_p = \frac{nS_{int}M}{n+1}$ and
$\Delta^r_p = \frac{1}{\alpha(1-S_U)} - \frac{1}{\alpha(1-S_{int})} + (p^w_U - c) + \frac{1}{b}[1 - \sum_J \frac{\partial q_j}{\partial a}]\frac{So_U}{(1-S_U)}q^r_{Ut} > (p^w_U - c) + \frac{1}{b}[\frac{1}{n+1}]\frac{So_U}{(1-S_U)}q^r_{Ut}$.

Since $\frac{\partial \Delta^r_p}{\partial n} = -\frac{1}{(n-1)^2}$ the wholesale price difference decreases with n. Since $\frac{\partial \Delta^w_p}{\partial n} = \frac{\partial p^w_U}{\partial n} - \frac{1}{(n-1)^2 * \frac{1}{b}\frac{So_U}{(1-S_U)}q^r_{Ut}}$, even if we assume that the market share did decrease with price, the difference in retail prices also decreases with n. Therefore if n decreases the gap between prices in the integrated and separated models gets larger at both the wholesale and retail levels. ■

One implication of Proposition 3 is that conventional merger analysis, which might consider the sector level impacts of consolidation without addressing the vertical interactions, would overstate the price-impacts of a merger. For example, a wholesale only model would overstate prices for n firms if vertical impacts are ignored. Proposition 3 demonstrates that upward bias in prices is even larger for $n-1$ firms. Thus the change in prices from n to $n-1$ must also be overstated.

4.3 Generalizations

The results in the previous sub-sections have been established for the case with symmetric firms, linear demand and constant marginal cost. We next consider the implications of relaxing one or more of these assumptions. First, we consider the assumption that firms at the wholesale level compete in Cournot quantities. The other oligopoly framework that has been frequently been applied to electricity markets is the supply-function equilibrium (SFE) first developed by Klemperer and Meyer (1989). We believe that the effects described here would manifest similarly under a wholesale SFE framework. As long as wholesale strategies are strategic substitutes, there would be an advantage to a credible commitment to produce more in the spot market. As demonstrated by Green (1998), this is true in the context of linear supply functions. The main issue in extending the Allaz-Vila framework to SFE has related to the conjectures firms apply to each other in the forward market. As Green demonstrated, if firms apply SFE conjectures to each-others forward commitments, the equilibrium can produce no commitments at all. In our environment the commitments are not necessarily sought after for their strategic impact on the spot market, but rather pursued because they are by themselves profitable. The wholesale effects are in some ways a side-effect retail rent seeking. Thus the complication that Green identified (source of forward commitment) is not relevant to our framework, where commitments stem from price-competition in retailing.

Convex Costs

From Lemma 2 we know that Φ is decreasing in C'' and is therefore maximized with constant marginal costs. Intuitively, as costs grow more convex, firms grow less responsive to both their own forward commitments and the forward commitments of other firms. Recall from equation (10) that a general expression for retail prices is

$$\begin{aligned} p_i^r \quad = \quad & \frac{1}{\alpha(1-S_i)} + p_t^w - [p_t^w - C_i'(q_{it})] \left([(n+1)S - 1]\Phi - \frac{\partial q_i}{\partial q_j} \right) \\ & + p_t^{w\prime} * [q_{it} - q_{it}^r] \frac{S_o}{(1-S_i)} [1 - (n+1)\Phi] = 0. \end{aligned} \tag{12}$$

One can identify conflicting effects, as a firms own costs grow highly convex, the effect of larger retail commitments on both its own and other firms output declines. However, the effect on the size of the market is unchanged. With wholesale quantities essentially fixed, shifting the wholesale demand curve inward will have a more dramatic impact on wholesale prices. As Φ grows smaller, the middle term in equation (12) grows smaller and the last term in (10) approaches $p_t^{w\prime} * [q_{it} - q_{it}^r] \frac{S_o}{(1-S_i)}$. The effect of this term on retail prices will depend upon the net position of integrated firms on the wholesale market $q_{it} - q_{it}^r$, which in a symmetric setting will in turn depend upon the size of the import/export market. If there is no export market, $q_{it} \leq q_{it}^r = 1/(nS_{int}M)$ and integrated firms will act to decrease wholesale prices by increasing retail rates, although by less than would independent retailers. With a sufficiently large export market, integrated will have with long positions that will bias retail prices downward, particularly relative to vertically separated firms.

Elasticity of Importing and Fringe Producers

One departure in our model from previous work is the presence of an "outside" wholesale market providing elasticity to the local wholesale market. In our electricity context, the outside wholesale market captures opportunities to buy or sell electricity from neighboring regions in which the local firms have no retail presence. In order to examine the implications of this feature of the model, we note the properties of (10) when $p^{w\prime}$ approaches zero. As shown above, with constant marginal costs of production at the wholesale level, $p^{w\prime}$ has no effect on retail prices. When $C'' > 0$, however, $p^{w\prime}$ has the effect on Φ and $\frac{\partial q_i}{\partial q_i^r}$ as increasingly convex costs. As $p^{w\prime}$ approaches zero, Φ_i and $\frac{\partial q_i}{\partial q_i^r}$ will also approach zero. Unlike the highly convex costs case, note that the last term in (12) will also approach zero. In this case integrated retailers will behave as if they have no impact on the wholesale market and will set retail prices accordingly, as if they were simply followers taking the wholesale price as constant.

5 Conclusions

In this paper we have developed a model intended to capture the dynamics of competition in utility or "subscription" settings where the downstream, or retail sector, precedes price formation in the upstream sector. Unlike other papers that have examined this problem we address a setting where transactions between wholesale and retail sectors are both transparent and non-excludable. As we demonstrate, when wholesale products are strategic substitutes, retail commitments allow firms to establish a leadership position. Since retail market share may be valuable in its own right, this commitment on the wholesale market may be greater or less than what would be considered optimal if only wholesale profits were considered.

For the setting with linear demand and constant marginal cost we have demonstrated several pro-consumer implications of vertical integration in this setting. Vertically separated firms will both lower retail prices and expand wholesale output when they vertically integrate. Because of the wholesale commitment created by integration, a firm that vertically integrates grabs market share from its unintegrated competitors, leading to increased profits for the integrated firm. Finally for a general number of symmetric firms, the "gap" between unintegrated prices and those that maintain under vertical integration grows larger as the number of firms decreases. This implies that a merger analysis that ignores these effects would overstate the price impact of a merger between two vertically integrated firms.

There are several obvious and important caveats to make about these results. First, we assume that both the costs and quality of wholesale and retail firms does not depend upon its integrated status. To the extent that vertical integration increased (or perhaps decreased) retail quality, these results may no longer hold. We have also assumed that retail integration is the only means through which wholesale firms can make forward commitments. Finally, while we have focused on the price-effects of vertical and horizontal mergers, the efficiency effects are more complicated. Vertically integration in many cases allows the integrated firms to exercise oligopsony power over competitive fringe or importing producers on the wholesale market. Thus, while "local" customers and producers benefit, the overall social surplus effect is ambiguous.

While analytical solutions to this modeling framework require strict assumptions about the functional forms of wholesale demand and production costs, we also present a more general formulation that can be solved numerically. In general, the more convex are costs, the less of a commitment is implied by integration into retailing. The strategic interactions are therefore minimized when costs are highly convex, while the efficiency benefits from reducing the oligopsony distortions of independent retailers grow to dominate. In either case, local consumers will benefit from integration, but the negative distortions on the wholesale market (such as oligopsony power over importers) is less severe with more convex costs. Future work will address questions about asymmetries of production cost and retail quality and examine which attributes of retail and wholesale firms most favor profitable vertical integration.

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Appendix

General Derivation of Best Responses to Forward Commitments

In order to solve the two-stage model, it is necessary to derive values for the changes in wholesale production in response to changes in retail positions, $\frac{\partial q_{jt}}{\partial q_{lt}^r}$. These can be obtained from the first-order conditions of the first-stage, which in terms of retail quantity can be written as

$$\frac{d\pi_{it}}{dq_{it}^r} = p_t^r + [p_t^w - c_i'(q_{it})]\frac{\partial q_{it}}{\partial q_{it}^r} + p_t^{w\prime}(Q_t) * [q_{it} - q_{it}^r]\sum_j \frac{\partial q_{jt}}{\partial q_{it}^r} = 0. \tag{13}$$

Allaz and Vila, working with a simple linear demand and constant marginal cost framework, are able to derive analytic solutions to the equivalents to $q_{it}(\overrightarrow{q^r})$, where we utilize vector notation to denote the full retail positions of all firms. Following Fowlie (2009), we note that we only require the derivatives of these functions, and can utilize the implicit function theorem to derive $\frac{\partial \overrightarrow{q_{it}}}{\partial \overrightarrow{q^r}} \quad s.t.\ \pi'(q(q^r)) = 0$.

Suppressing notation for time for the moment, note that the implicit function theorem tells us:

$$\frac{\partial q}{\partial q^r} = -\left(\frac{\partial \pi'}{\partial q}\right)^{-1} * \frac{\partial \pi'}{\partial q^r}.$$

Or in matrix form

$$\begin{pmatrix} \frac{\partial q_1}{\partial q_1^r} & \cdots & \frac{\partial q_1}{\partial q_J^r} \\ \vdots & \ddots & \vdots \\ \frac{\partial q_J}{\partial q_1^r} & \cdots & \frac{\partial q_J}{\partial q_J^r} \end{pmatrix} = -J(\nabla\pi)_q^{-1} \begin{pmatrix} \frac{\partial^2 \pi_1}{\partial q_1 \partial q_1^r} & \cdots & \frac{\partial^2 \pi_1}{\partial q_1 \partial q_J^r} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi_J}{\partial q_J \partial q_1^r} & \cdots & \frac{\partial^2 \pi_J}{\partial q_J \partial q_J^r} \end{pmatrix}$$

$$-J(\nabla\pi)_q^{-1} = A^{-1} = -\begin{pmatrix} 2p' - C_1''(q_{1t}) & \cdots & p' \\ \vdots & \ddots & \vdots \\ p' & \cdots & 2p' - C_J''(q_{Jt}) \end{pmatrix}^{-1}$$

and

$$\begin{pmatrix} \frac{\partial^2 \pi_1}{\partial q_1 \partial q_1^r} & \cdots & \frac{\partial^2 \pi_1}{\partial q_1 \partial q_J^r} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi_J}{\partial q_J \partial q_1^r} & \cdots & \frac{\partial^2 \pi_J}{\partial q_J \partial q_J^r} \end{pmatrix} = \begin{pmatrix} p' & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & p' \end{pmatrix}$$

For the symmetric case, note that the matrix A above can be decomposed as (A + B) where A is a $n \times n$ diagonal matrix with $p' - C''$ forming the diagonal elements and B is a

$n \times n$ symmetric matrix with p' in every element. The general formula for inverting matrices of this type produces the following structure.

$$\begin{pmatrix} \frac{\partial q_1}{\partial q_1^r} & \cdots & \frac{\partial q_1}{\partial q_J^r} \\ \vdots & \ddots & \vdots \\ \frac{\partial q_J}{\partial q_1^r} & \cdots & \frac{\partial q_J}{\partial q_J^r} \end{pmatrix} = \begin{pmatrix} \frac{n-\frac{C''}{p'}}{(n+1-\frac{C''}{p'})(p'-C'')} & \cdots & \frac{-1}{(n+1-\frac{C''}{p'})(p'-C'')} \\ \vdots & \ddots & \vdots \\ \frac{-1}{(n+1-\frac{C''}{p'})(p'-C'')} & \cdots & \frac{n-\frac{C''}{p'}}{(n+1-\frac{C''}{p'})(p'-C'')} \end{pmatrix}$$

Above we defined $\Phi_i = \sum_j \frac{\partial q_i}{\partial q_j^r}$ Given the general expression above we have

$$\Phi_i = \frac{1}{n+1-\frac{C''}{p'}}$$

For demand functions of the form $Q_t(p) = Q_{0t} - F(p)$ the reaction function with respect to the horizontal intercept of demand, Q_{0t} can be derived in a similar fashion. In this case, the implicit function theorem tells us:

$$\frac{\partial q}{\partial Q_{0t}} = -\left(\frac{\partial \pi'}{\partial q}\right)^{-1} * \frac{\partial \pi'}{\partial Q_{0t}}.$$

Note that with demand of the form $a_t - F_t(p)$ with inverse demand $p_t(Q) = F^{-1}(a_t - Q_t)$ we have $Q_{0t} = a_t$ and

$$\begin{pmatrix} \frac{\partial^2 \pi_1}{\partial q_1 \partial Q_{0t}} \\ \vdots \\ \frac{\partial^2 \pi_J}{\partial q_J \partial Q_{0t}} \end{pmatrix} = \begin{pmatrix} -p' \\ \vdots \\ -p' \end{pmatrix}$$

Implying

$$\frac{\partial \overrightarrow{q_t}}{\partial Q_{0t}} = A^{-1} \begin{pmatrix} -p' \\ \vdots \\ -p' \end{pmatrix}$$

Proof of Lemma 1. First note from the general derivation above that $\frac{\partial q^i}{\partial q_j^r} = -A_{ij}^{-1}B$, where B is a $n \times n$ diagonal matrix with p' as the diagonal elements, and $\frac{\partial q_i}{\partial Q_o} = -A_{ij}^{-1}\bar{b}$, where $\bar{b}$ is a $n \times 1$ vector with $-p'$ as every element of the vector. The fact that A^{-1} is a symmetric matrix yeilds $\sum_j \frac{\partial q_j}{\partial q_i^r} = \sum_j \frac{\partial q_i}{\partial q_j^r}$ and cross multiplication demonstrates that $\sum_j -A_{ij}^{-1}B = -A_{ij}^{-1}\bar{b}$.

Proof of Lemma 2. From Lemma 1 $\Phi_i = \sum_j \frac{\partial q_i}{\partial q_j^r} = \frac{\partial q_i}{\partial Q_0}$. From above $\frac{1}{n+1-\frac{C''}{p'}}$. Note that $\frac{\partial \Phi}{\partial C''} = p'/(n+1-\frac{C''}{p'})^2$, which is negative since $p' < 0$.

Proof of Lemma 3. The general best response function in terms of forward commitments for linear demand and affine marginal cost of the form $C(q) = c + c_2 q$ is derived in Bushnell (2008) and equals

$$q_i^*(q_i^r, q_j^r) = \frac{a - bc + \left((n + bc_2)q_i^r - \sum_{j \neq i} q_j^r\right)/(1 + bc_2)}{(n + 1 + bc_2)}. \tag{14}$$

With constant marginal cost, $c_2 = 0$. Differentiating (14) with respect to a and q_i^r, respectively produces

$$\frac{\partial q_i}{\partial q_i^r} = \frac{n}{n+1}, \frac{\partial q_i}{\partial q_j^r} = \frac{-1}{n+1}, \frac{\partial q_i}{\partial a} = \frac{1}{n+1}.$$

from this one can see that $\sum \frac{\partial q_i}{\partial q_j^r} = \frac{n-(n-1)}{n+1} = \frac{1}{n+1} = \Phi_i$, therefore $\frac{\partial q_i}{\partial q_i^r} = n\Phi_{i.}$

Proof of Proposition 1. With vertical integration wholesale market quantities and prices are

$$q_{int}^w = \frac{a + S_{int}M - bc}{(n+1)};\ p_{int}^w = \frac{a - nS_{int}M + nbc}{(n+1)b} = \frac{Q_{exp} + nbc}{(n+1)b};$$

at the retail level they are

$$p_{int}^r = \frac{1}{\alpha(1 - S_i)} + c_i \tag{15}$$

At wholesale level prices in the unintegrated case follow the solution to the standard linear Cournot Model, where the single-firm production quantity would be $q_U^w = \frac{a-c}{(n+1)}$.

$$p_U^w = \frac{a + nbc}{(n+1)b} = \frac{nS_U M + Q_{exp} + nbc}{(n+1)b}$$

where the last equivalence follows from $a = Q_{exp} + nS_U M$. Note that for any non-zero domestic retail market, $S_U M > 0$, wholesale prices will be higher than in the integrated case where $p_{int}^w = \frac{Q_{exp}+2bc}{3b}$.

Considering that for n symmetric retailers, $S_o = 1 - nS_U$, the retail level prices are

$$p_U^r = \frac{1}{\alpha(1 - S_U)} + p_U^w + \frac{1}{b}[1 - \sum_J \frac{\partial q_j}{\partial a}] \frac{So_U}{(1 - S_U)} q_{Ut}^r \tag{16}$$

It remains to show that $p_{int}^r < p_U^r$. By combining (16) and (15) this means

$$\tfrac{1}{\alpha(1-S_{int})} + c < \tfrac{1}{\alpha(1-S_U)} + p_U^w + \tfrac{1}{b}[1 - \textstyle\sum_J \tfrac{\partial q_j}{\partial a}] \tfrac{So_U}{(1-S_U)} q_{Ut}^r \tag{17}$$

For the linear demand and constant marginal cost case, $\Phi_i = \frac{\partial q_j}{\partial a} = \frac{1}{(n+1)}$, therefore the last term in (16) must be positive and prices would be higher than in a conventional Bertrand retail model where $p_U^w$was treated as exogenous. Further, in the unintegrated case wholesale price is higher than marginal cost, $p_U^w > c$.

Assume that despite this, retail prices were lower in the unintegrated case, $p_{int}^r \geq p_U^r$. Then $S_{int} < S_U$, and $\frac{1}{\alpha(1-S_U)} > \frac{1}{\alpha(1-S_{int})}$. However, for $p_{int}^r \geq p_U^r$ it must be the case that $\frac{1}{\alpha(1-S_{int})} > \frac{1}{\alpha(1-S_U)} + (p_U^w - c) + \frac{1}{b}[1 - \sum_J \frac{\partial q_j}{\partial a}]\frac{So_U}{(1-S_U)}q_{Ut}^r > \frac{1}{\alpha(1-S_U)}$.Where the latter inequality follows from the fact that $p_U^w > c$ and $[1 - \sum_J \frac{\partial q_j}{\partial a}] > 0$. Since $\frac{1}{\alpha(1-S_{int})} < \frac{1}{\alpha(1-S_U)}$ if $p_{int}^r \geq p_U^r$,this is a contradiction.

Proof of Proposition 2. First assume that the integrated firm makes no change to its retail strategy, but internalizes that retail position into its wholesale production decision. With one firm integrated, its wholesale production is $\frac{a+nq_i^r-bc}{(n+1)}$, which equals $\frac{Q_{exp}+2nS_UM-bc}{(n+1)}$, since $a = Q_{exp} + nS_UM$ if the retail quantity did not change. All the other firms produce $\frac{a-S_UM-bc}{(n+1)}$. The wholesale price is therefore

$$p_{int}^w = \frac{a}{b} - \frac{na + S_UM - nbc}{b(n+1)} = \frac{a - S_UM + nbc}{(n+1)b} = \frac{Qexp + (n-1)S_uM + nbc}{(n+1)b} +$$

Profit at the wholesale level for this firm would therefore be $(p_{int}^w - c) * q_{int}^w$ or

$$\left(\frac{a - S_UM}{(1+n)b} - \frac{bc}{(n+1)b}\right) * \left(\frac{a + nS_UM - bc)}{(n+1)}\right).$$

$$= \frac{(a^2 - 2abc + b^2c^2)}{(n+1)^2b} + \frac{(n-1)(a-bc) - nS_U^2M^2}{(n+1)^2b}.$$

As we shall see, the first term above is the same as the unintegrated wholesale profit. If retail prices were not changed, then retail revenues would be the same under both cases and retail costs (from wholesale purchases) would be $p_{int}^w * q^r = \left(\frac{a-S_UM+nbc}{(n+1)b}\right) S_UM$.

Profit in the unintegrated market is

$$(p_U^w - c)q_U^w = \frac{1}{b}\left[\frac{(a-bc)}{(n+1)}\right]^2 = \frac{1}{(n+1)^2b^2}\left(a^2 - 2abc + b^2c^2\right) = \frac{1}{9b^2}\left(n^2S_U^2M^2 - 2nS_UMbc + b^2c^2\right)$$

Retail revenues are by assumption the same in both cases and retail costs in the unintegrated case would be$\left(\frac{a+nbc}{(n+1)b}\right) S_UM$. Comparing profits for the two scenarios (retail prices unchanged, one firm integrated vs. no firm's integrated) we have

$$\pi_{int} - \pi_U = \frac{S_U^2M^2}{(n+1)b} + \left(\frac{(n-1)(a-bc) - nS_U^2M^2}{(n+1)^2b}\right) = \left(\frac{S_u^2M^2 + (n-1)(a-bc)}{(n+1)^2b}\right).$$

We have assumed that the wholesale market produces positive prices in the unintegrated case, which means that $a > bc$. The last term in (5) above must therefore be positive and the integrated firms profits increase from integration even if the integrated firm makes no change to its retail price. Of course the integrated firm can and would make an adjustment to its retail price, but only if such a change increases profits. Therefore the expression in (5) is a lower bound on the increase in profit of a firm from integrating and this lower bound is positive.