ELECTRICITY RESOURCE ADEQUACY: RELIABILITY, SCARCITY, MARKETS, AND OPERATING RESERVE DEMAND CURVES

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Ultimate success is an open question for the international experiment in using electricity markets for public purposes.

From Down Under

"Plans for desperately needed new power generation are up in the air again. … [The New Zealand Labour Government's move] has the potential to up-end the electricity industry and turn back the clock to central planning. Electricity transmission is already centrally planned by state-owned Transpower.

"'The question is, should you centrally plan the alternatives,' Mr Hemmingway [Electricity Commission chairman] says. 'Do you give companies a leg-up in the form of a subsidy to undertake the alternatives? And, how would a package of centrally implemented alternatives distort the market?

"'How far we go down this slippery slope back toward central planning is a central question here. It's the key to our deliberations. We are aware of the slippery slope danger but we are also aware that if there are alternatives out there that are less expensive than the transmission line we ought not let them go to waste.'" (The Press, Christchurch, New Zealand, April 30, 2005.)

The New Zealand Energy Minister intervened with the "independent" Electricity Commission regulator, seeking more central direction of investment.

Roy Hemingway left his job as he entered it, fired with enthusiasm. (November 30, 2006.)

The Federal Energy Regulatory Commission (FERC) reform proposals for Order 888 arise from frustration with electricity restructuring efforts and providing open access to transmission needed to support competitive markets.

At its core, the debate identifies persistent disagreement about what open access means, and what models are available to achieve the purported benefits.

> "Now, the goal of the NOI in this proceeding is very clear. It is spelled out in the title: Preventing Undue Discrimination and Preference in Transmission Service. *We are not talking about market design. We are not talking about restructuring. We are talking about preventing undue discrimination and preference.*"
>
> (Statement of Joseph Kelliher, Chairman, Federal Energy Regulatory Commission, Regarding Notice of Inquiry on Preventing Undue Discrimination and Preference in Transmission Service, Docket No. RM05-25-000, September 16, 2005, emphasis added)

> "The first time the Commission found Order No. 888 allowed undue discrimination and preference in transmission service occurred in 1999. The solution advanced by the Commission was restructuring: encouraging voluntary RTO formation, in Order No. 2000. … The second time the Commission found Order No. 888 allowed undue discrimination and preference took place in 2002. The solution advanced by the Commission at the time was also restructuring, this time mandating RTO participation and a standard market design. … The solution we advance today is not restructuring, but more effective regulation, reform of the open access rules themselves, for the first time in nearly a decade."
>
> (Statement of Joseph Kelliher, Chairman, Federal Energy Regulatory Commission, Regarding Open Access Transmission Tariff (OATT) Reform (RM05-25-00), May 16, 2006.)

In the case of wholesale electricity markets, the choice between a pure market and central administrative solutions is a false dichotomy.

- **With current technology and system configuration, central administrative interventions are necessary.**
  - Limited control and metering.
  - Balancing, Dispatch and Security constraints.
- **Not all administrative interventions are equivalent.**
  - **Vicious circles**.

    Zonal pricing ► constrained-on and -off payments ► misplaced investment ► integrated resource procurements ► the ISO as the "utilities' utility."
  - **Virtuous circles**.

    Nodal pricing ► bid-based, security-constrained economic dispatch ► financial transmission rights.

Incomplete scarcity pricing leaves "missing money." This creates poor operational and investment incentives. There is a need for administrative intervention.

- **Installed Capacity**. Assuming it is impossible to provide adequate scarcity pricing, interventions focus on installed capacity requirements.
  - Emphasis on physical capacity and planning targets. This seems natural and innocuous, but the physical perspective leads to a host of market design problems.
  - Requirement for longer-term regulatory commitments and decisions. Substantial payments must come through the regulatory decision, investment requires the commitment.
  - Assumes there is some method for defining and ensuring transmission deliverability. If we knew how to do this, everything would be easier. But the electricity network makes this difficult.
  - Experience reveals unintended consequences and renews interest in better scarcity pricing.

- **Scarcity Pricing**. Suspending disbelief, consider better scarcity pricing.
  - An "energy only" market without an installed capacity requirement, but with alternative regulatory requirements.
  - Or "belts and suspenders" with better scarcity pricing that supports an installed capacity system.

The usual discussion of reliability planning standards refers to the loss of load probability (LOLP) and the ubiquitous 1 day in 10 years standard.

> "Loss of Load Expectation (LOLE) — LOLE is the expected number of days per year for which available generating capacity is insufficient to serve the daily peak demand (load). The LOLE is usually measured in days/year or hours/year. The convention is that when given in days/year, it represents a comparison between daily peak values and available generation. When given in hours/year, it represents a comparison of hourly load to available generation. LOLE is sometimes referred to as loss of load probability (LOLP), where LOLP is the proportion (probability) of days per year, hours per year, or events per season that available generating capacity/energy is insufficient to serve the daily peak or hourly demand. This analysis is generally performed for several years into the future and the typical standard metric is the loss of load probability of one day in ten years or 0.1 day/year."

(North American Electric Reliability Council, "Resource and Transmission Adequacy Recommendations," Prepared by the Resource and Transmission Adequacy Task Force of the NERC Planning Committee NERC Board of Trustees, June 15, 2004, p. 11.)

Ideally we would have consistent application where:

$$LOLE = LOLP * PERIOD = \frac{1\ day}{10\ yrs} 10\ yrs = 0.1 \tfrac{day}{year} * 10 years = 1\ day = 24 hours = 2.4 hrs / yr.$$

This is not the same as "events." With a modeled event of 2.4 hrs, *0.1 day/year* implies *2.4 hrs/decade*.

Despite the common reference to the 1 in 10 standard, there is not much standardization of reliability planning standards. This may not be much of a problem, but the same terms mean different things in different places.

> "Because utilities have historically planned generation reliability such that the expected number of days in a year with inadequate generation to meet load is well under one day, LOLP is typically expressed as 1-day-in-X-years; for example 1-day-in-10-years or 1-day-in-20-years. Note that "1-day-in-10-years" in this case does not mean that there is an expectation of 24 hours of outages in ten years. Rather, the metric indicates that there is a 1 in 10 chance that during the year there will be an outage during one of the 365 days."
>
> (Energy and Environmental Economics, Inc., BC Hydro, "Electric Reliability Primer," September 23, 2004, p. 7)

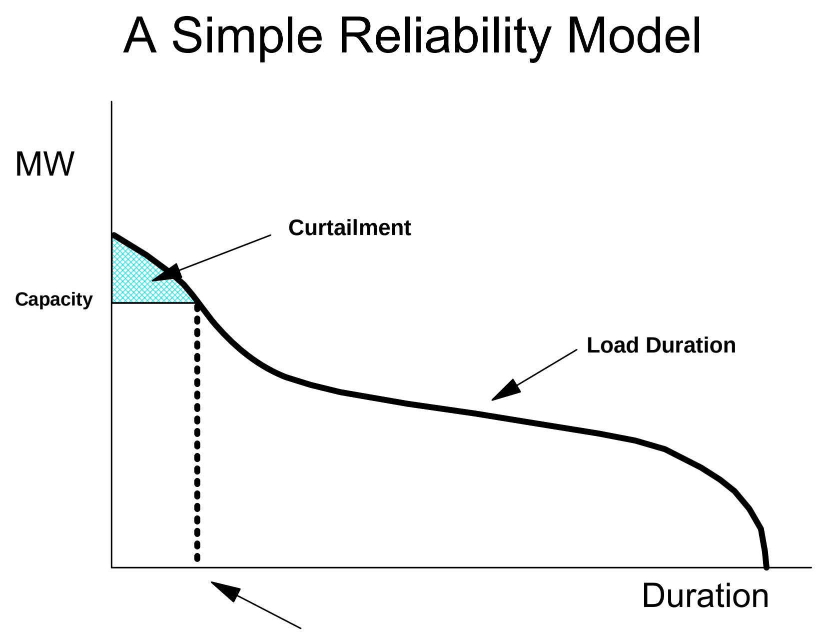
Other criteria include Expected Unserved Energy (EUE) and Value of Service (VOS).

Modeling for planning standards includes a range of approaches.

- Deterministic
- Probabilistic
    - Independent
    - Sequential

The many assumptions produce different reserve margin requirements, but the differences in definitions are small compared to the gap between the formulation of reliability standards and market design.

There is a simple stylized connection between reliability standards and resource economics. Defining expected load shedding duration, choosing installed capacity, or estimating value of lost load address different facets of the same problem.

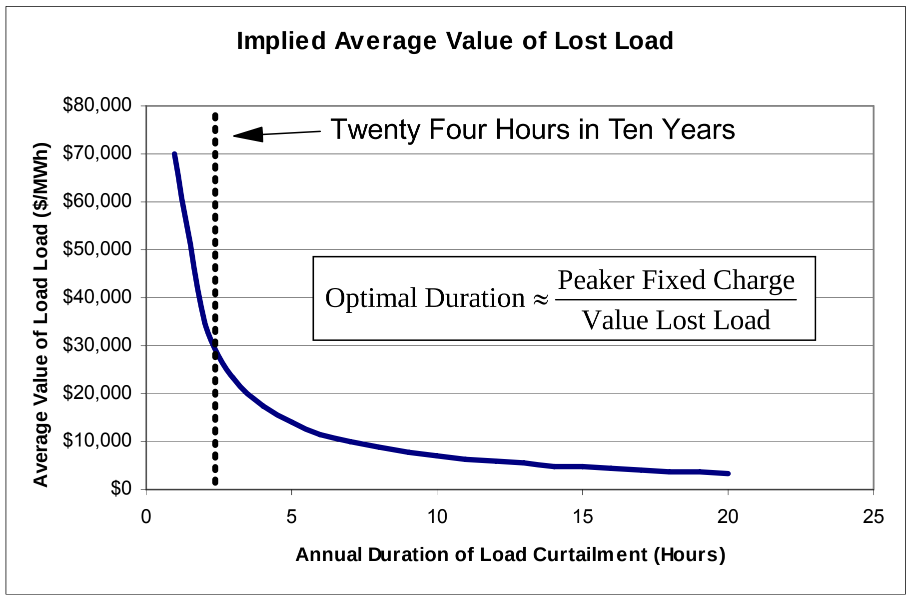


$$\text{Optimal Duration} \approx \frac{\text{Peaker Fixed Charge}}{\text{Value Lost Load}}$$

(Steven Stoft, Power System Economics, IEE Press, Wiley Interscience, 2002, p. 138)

The simple connection between reliability planning standards and resource economics illustrates a major disconnect between market pricing and the implied value of lost load.

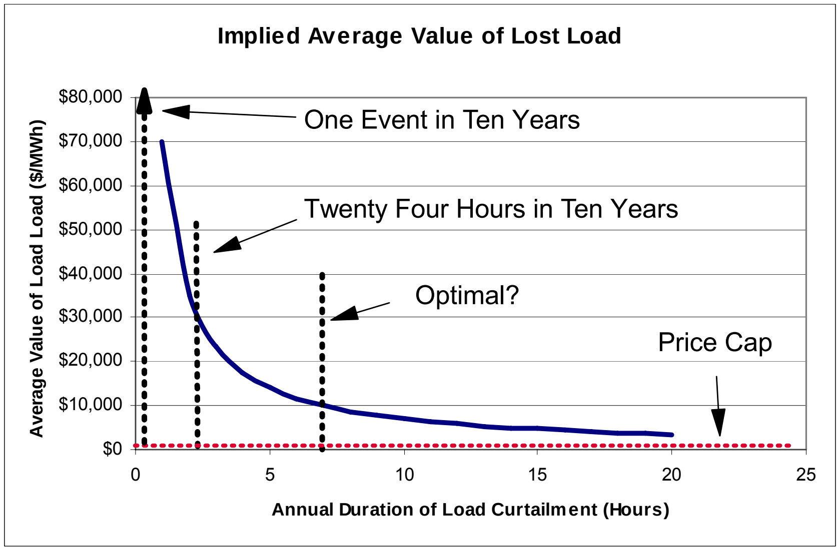
Reliability Planning Standard and Value of Lost Load



Peaker fixed charge at $65,000/MW-yr.

There is a large disconnect between long-term planning standards and market design. The installed capacity market analyses illustrate the gap between prices and implied values. The larger disconnect is between the operating reserve market design and the implied reliability standard.

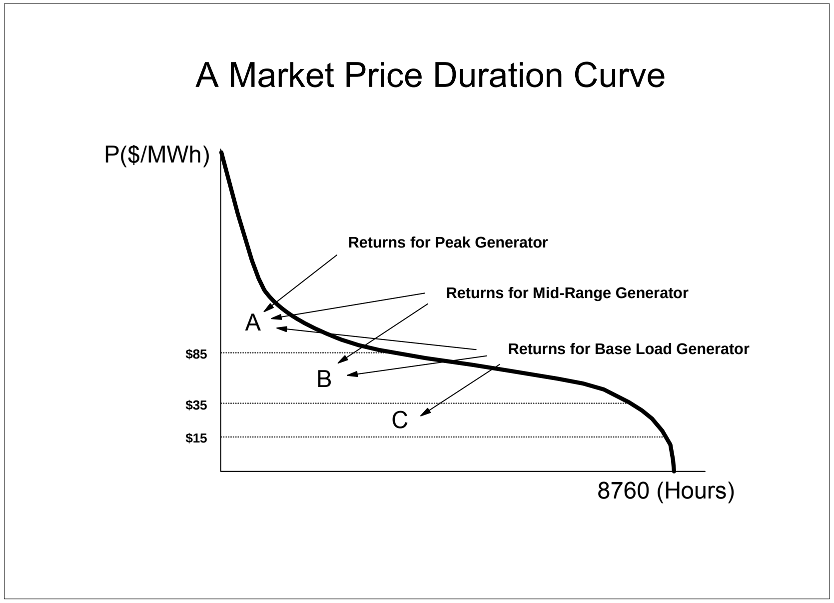
Reliability Standard and Market Disconnect

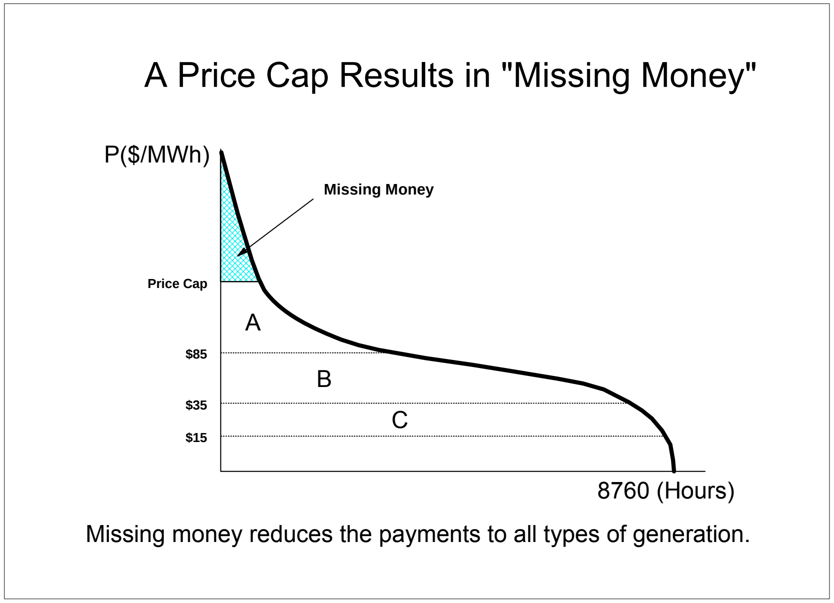


Peaker fixed charge at $65,000/MW-yr.

Implied prices differ by orders of magnitude. $\left(\text{Price Cap} \approx \$10^3;\ \text{VOLL} \approx \$10^4;\ \text{Reliability Standard} \approx \$10^5\right)$

A variety of market rules for spot markets interact to create *de jure* or *de facto* price caps. The resulting "missing money" reduces payments to all types of generation. The reduced payments affect operating and investment incentives for demand, generation and transmission.





If market prices do not provide adequate incentives for generation investment, the result is a market failure. The market design defect creates the pressure for regulators to intervene to mandate generation investment.

The obvious solution was to create a regulatory requirement that load serving entities purchase sufficient installed generation capacity to meet the projected load plus an adequate reserve margin.

- **Installed Capacity (ICAP) requirements through short-tem auctions or deficiency charges. A regulatory requirement to obtain "capacity" for peak load plus a reserve margin.**
  - PJM daily requirement.
  - NYISO monthly requirement.
  - ISONE monthly requirement.
- **The apparently obvious solution has not worked. ICAP is seen as a failed model. But it won't go away. Reforms of these reforms followed with further interventions.**
  - Locational variant (LICAP) in NYSIO with local installed reserve demand curve.
  - Peaking Unit Safe Harbor (PUSH) model for controlled exercise of market power in ISONE.
  - Reliability Must Run (RMR) and Out of Market (OOM) purchases, everywhere.

FERC recognizes the growing pressure for RMR contracts and similar interventions as part of the problem, not the solution.

The latest reforms of resource adequacy reforms move substantially in the direction of greater prescription and mandates from the central planners.

- **ISONE LICAP Proposal (August 31, 2004). FCM Settlement Proposal (March 6, 2006).**
  - LICAP: Locational Demand Curve. FCM: Fixed demand with pricing restrictions.
  - Zonal Transfer Limits.
  - LICAP: Month-Ahead Requirements. FCM: Three-year-Ahead Requirements.
  - Rules for Demand, Generation and Transmission Tradeoffs.

- **PJM Reliability Pricing Model (RPM) Proposal (August 31, 2005).**
  - Locational Variable Resource Requirement (VRR).
  - Zonal Transfer Limits.
  - Four-year-Ahead Requirements.
  - Rules for Demand, Generation and Transmission Tradeoffs.

Both proposals face substantial opposition over jurisdictional, cost and complexity issues.

However, given the defects in the electricity market designs, the direction established in these proposals is natural and inevitable. Given the assumptions, many of the elements of the proposals are logical and sophisticated. But the programs are unlikely to be enough to meet the objectives. And not all the pieces fit, or are even yet defined. More prescriptions will follow.

Given the expanding prescriptions of generation resources adequacy programs built on installed capacity requirements, there is a greater willingness to step back and look at the assumptions.

- **Focus on the market failure.**
    - Missing money arises from *de facto* price caps.
    - A market-based resource adequacy program would not slide down the slippery slope.

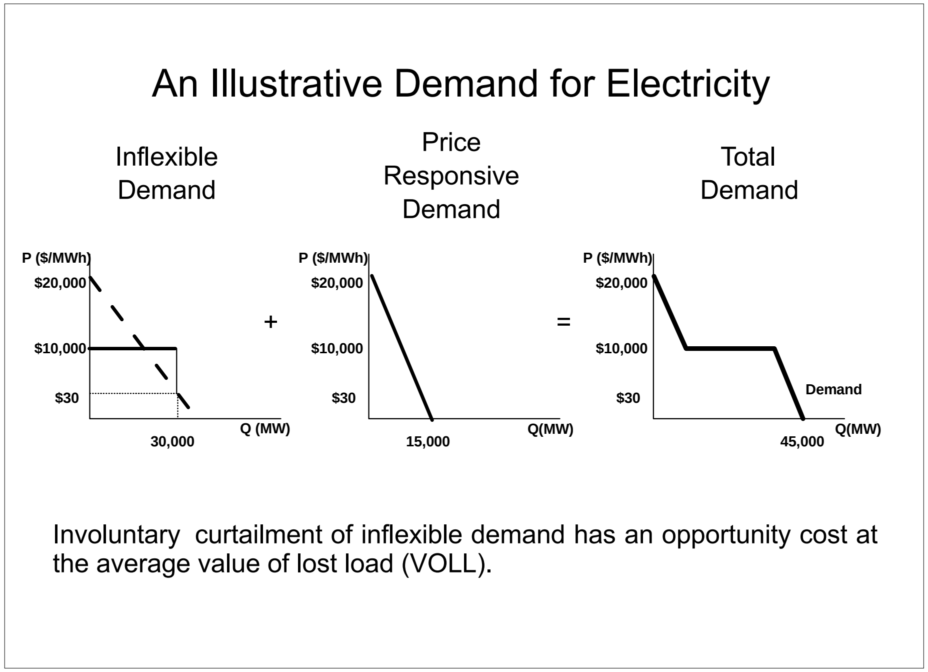
- **An energy only market alternative with no installed capacity mandate.**
    - Texas White Paper and PUC Staff proposal. (July 2005).
    - MISO White Paper and Staff proposal (August 2005).

- **An "energy only" market alternative with compatible interventions.**
    - Target operating conditions rather than planning standards.
    - Create a workable electricity spot market without the missing money.
    - Design other compatible interventions with hedging and market power mitigation to address the problems that motivated the *de facto* price caps.
    - Think "market based" rather than "command and control."

A workable "energy only" market would eliminate the "missing money" problem and provide an alternative to the growing prescriptions of installed capacity markets. The concept is not that there should be no market interventions. But the interventions should not overturn the market.

An "Energy Only" Market Outline

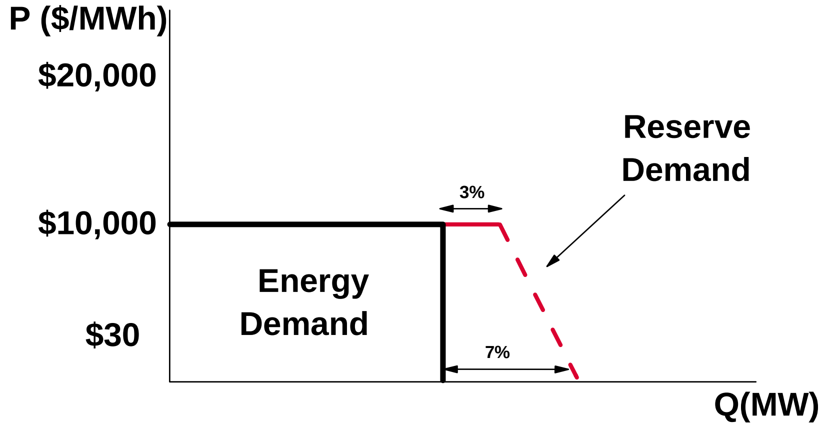
- **Implicit demand for inflexible load would define the opportunity costs as the average value of lost load (VOLL).**



… An "Energy Only" Market Outline

- **Operating reserve demand curve would reflect capacity scarcity.**

Illustrative Reserve Demand

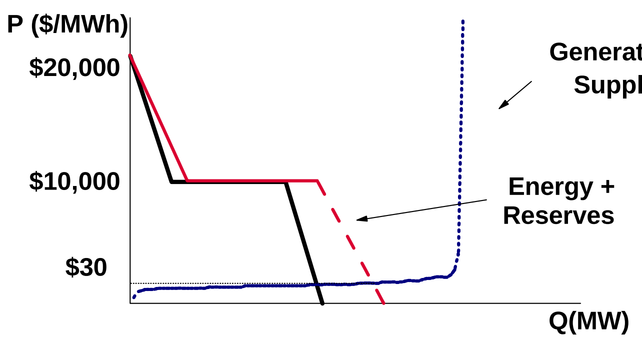


There is a minimum level of operating reserve (e.g., 3%) to protect against system-wide failure. Above the minimum reserve, reductions below a nominal reserve target (e.g., 7%) are price senstive.

… An "Energy Only" Market Outline

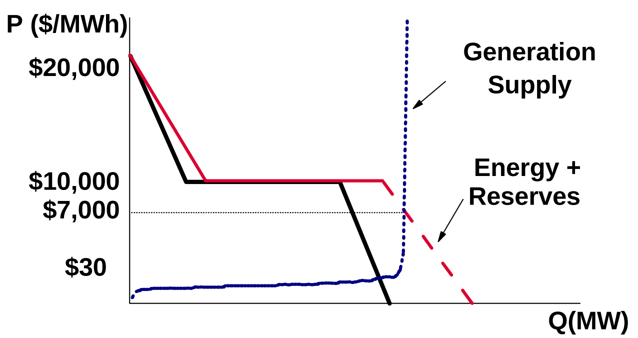
- **Market clearing eliminates the "missing money."**

Normal "Energy Only" Market Clearing



When demand is low and capacity available, reserves hit nominal targets at a low price.

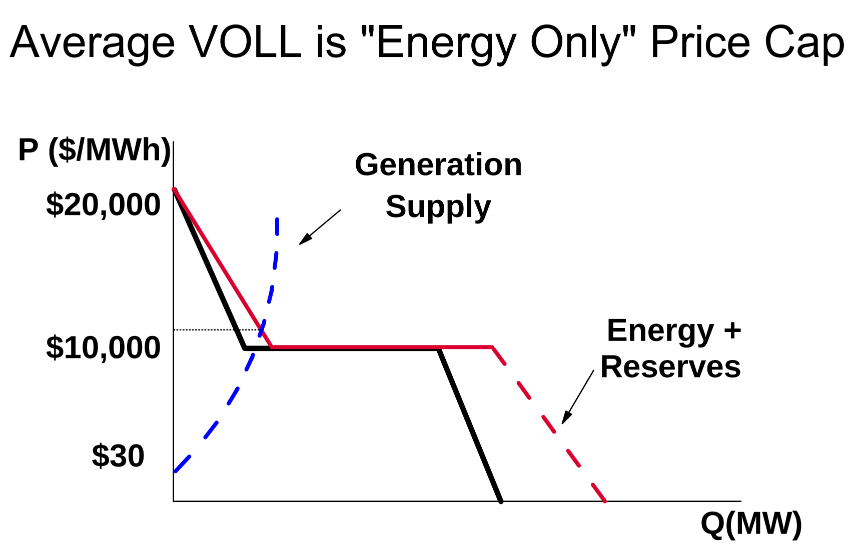
Scarcity "Energy Only" Market Clearing



When demand is high and reserve reductions apply, there is a high price.

… An "Energy Only" Market Outline

- **The average VOLL becomes the *de facto* price cap. But it is not a price cap that limits bids or leads to out of market purchases.**



Prices do not rise above the average VOLL unless ***all*** inflexible demand is subject to involuntary curtailment.

Locational fixed operating reserve minimums are already familiar practice. The detailed operating rules during reserve scarcity involve many steps. Improved scarcity pricing would accompany introduction of an operating reserve demand curve under dispatch based pricing. Consider a simplified setting.

- **Dispatched-Based Pricing.** Interpret the actual dispatch result as the solution of the reliable economic dispatch problem. Calculate consistent prices from the simplified model.
- **Single Period.** Unit commitment decisions made as though just before the start of the period. Uncertain outcomes determined after the commitment decision, with only redispatch or emergency actions such as curtailment over the short operating period (e.g. less than an hour).
- **Single Reserve Class.** Model operating reserves as committed and synchronized.
- **DC Network Approximation.** Focus on role of reserves but set context of simultaneous dispatch of energy and reserves. A network model for energy, but a zonal model for reserves.

The purpose here is to pursue a further development of the properties of a market model that expands locational reserve requirements to include operating reserve demand curve(s).

The NYISO market design includes locational operating reserve demand curves. The ISONE market design plan calls for locational operating reserve requirements with violation penalties that operate like a demand curve.[1]

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1 Independent Market Advisor, to the New York ISO, “2004 State of the Market Report New York ISO,” NYISO, July 2005, p. 59. ISO New England, “2006 Wholesale Markets Plan,” September 2005, pp. 16-17.

An alternative approach is to consider the expected unserved energy (*EUE*) and the Value of Lost Load (*VOLL*).

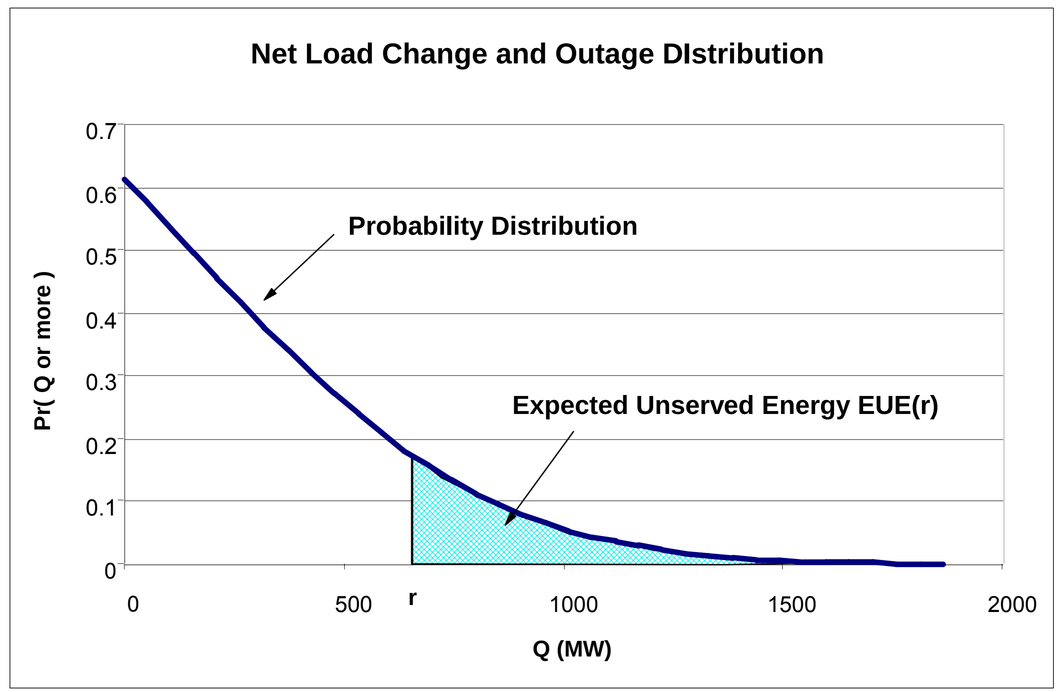
Suppose the *VOLL* per MWh is v. Then we can obtain the *EUE* and its total value (*VEUE*) as:

$$EUE(r) = \int_r^\infty \overline{F}_{LOL}(x)\,dx.$$

$$VEUE(r) = v\int_r^\infty \overline{F}_{LOL}(x)\,dx.$$

There is a chance that no outage occurs and that net load is less than expected, or $\overline{F}_{LOL}(0) < 1.$

The real changes may not be continuous, but it is common to apply continuous approximations.



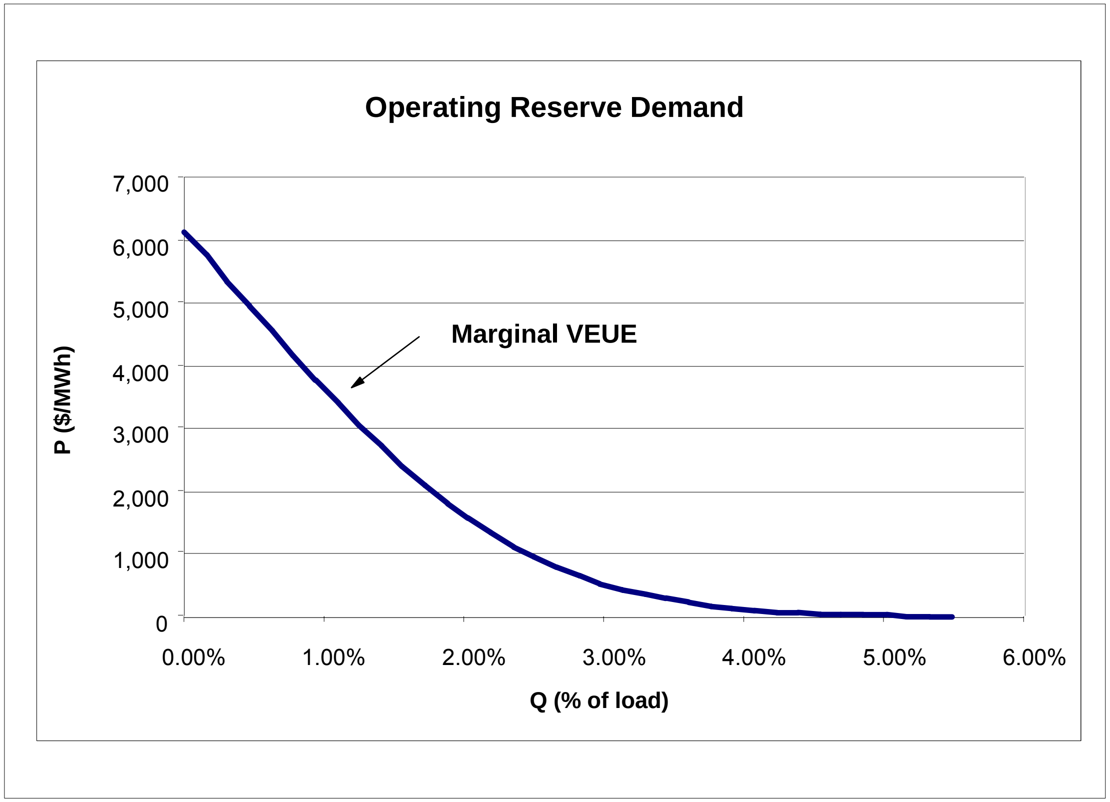
The probabilistic demand for operating reserves reflects the cost and probability of lost load.

$$Operating\ Reserve\ Demand\ Price(r) = P_{OR}(r) = v\bar{\Phi}\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right).$$

Example Assumptions

Expected Load (MW)	34000
Std Dev %	1.50%
Expected Outage %	0.45%
Std Dev %	0.45%
Expected Total (MW)	153
Std Dev (MW)	532.46
VOLL ($/MWh)	10000

Under the simplifying assumptions, if the dispersion of the LOLP distribution is proportional to the expected load, the operating reserve demand is proportional to the expected load. Total value is of same magnitude as the cost of meeting load.



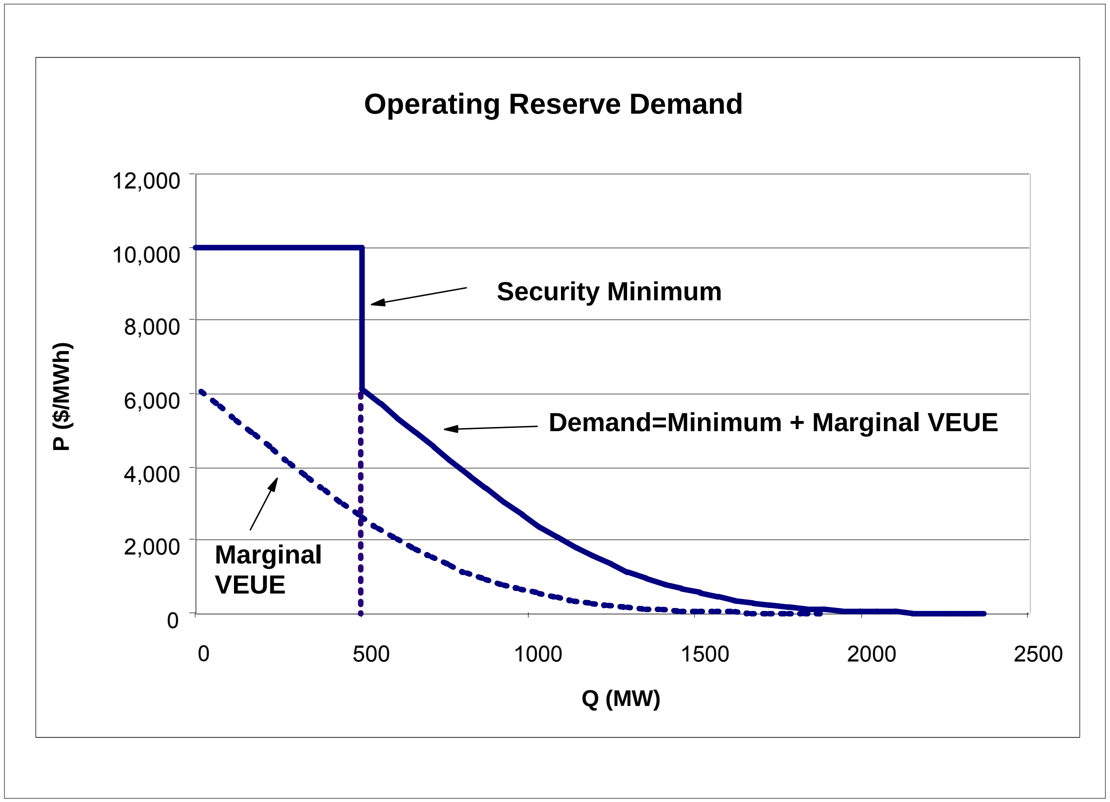
The deterministic approach to security constrained economic dispatch includes lower bounds on the required reserve to ensure that for a set of monitored contingencies (e.g., an n-1 standard) there is sufficient operating reserve to maintain the system for an emergency period.

Suppose that the maximum generation outage contingency quantity is $r_{Min}\left(d^0, g^0, u\right)$. Then we would have the constraint:

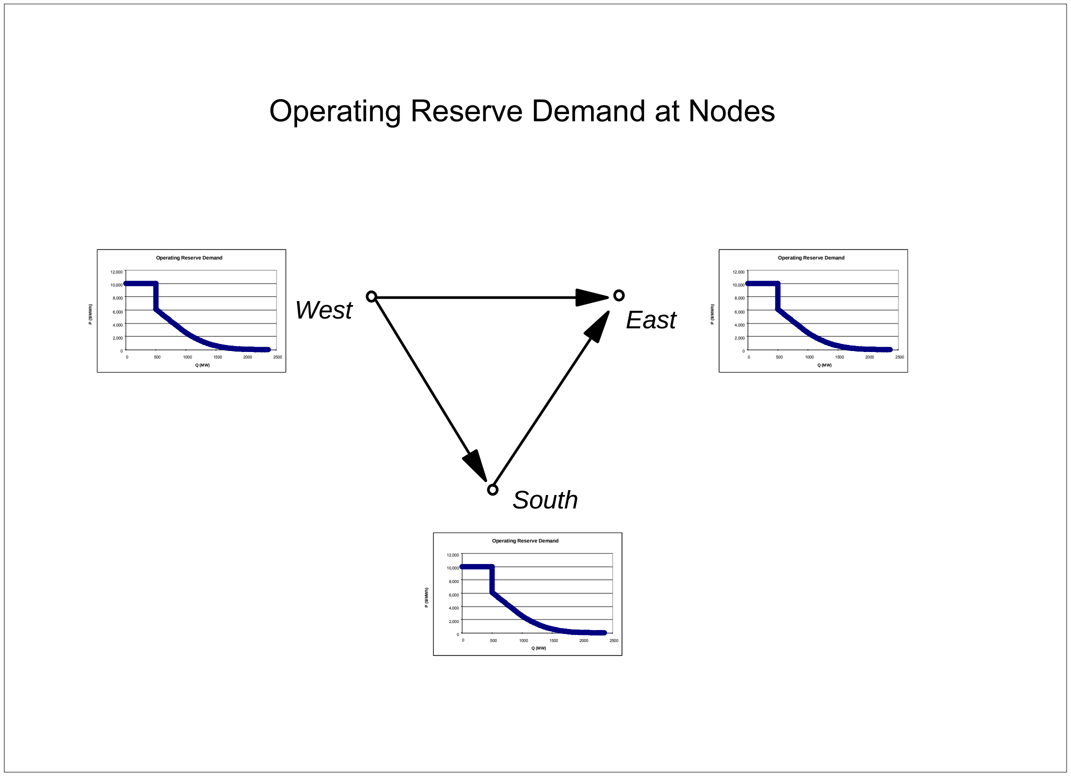
$$r \geq r_{Min}\left(d^0, g^0, u\right).$$

In effect, the contingency constraint provides a vertical demand curve that adds horizontally to the probabilistic operating reserve demand curve.

If the security minimum will always be maintained over the monitored period, the VEUE price at r=0 applies. If the outage shocks allow excursions below the security minimum during the period, the VEUE starts at the security minimum.

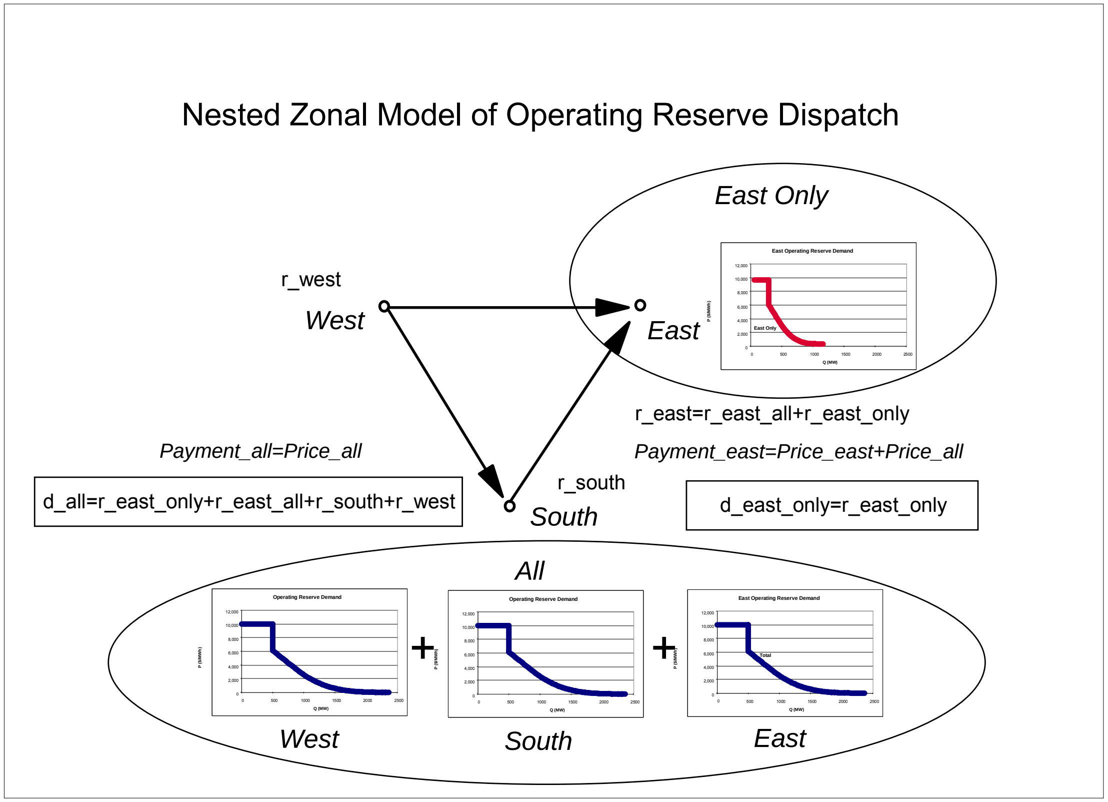


Suppose that the *LOLP* distribution at each node could be calculated.[2] This would give rise to an operating reserve demand curve at each node.



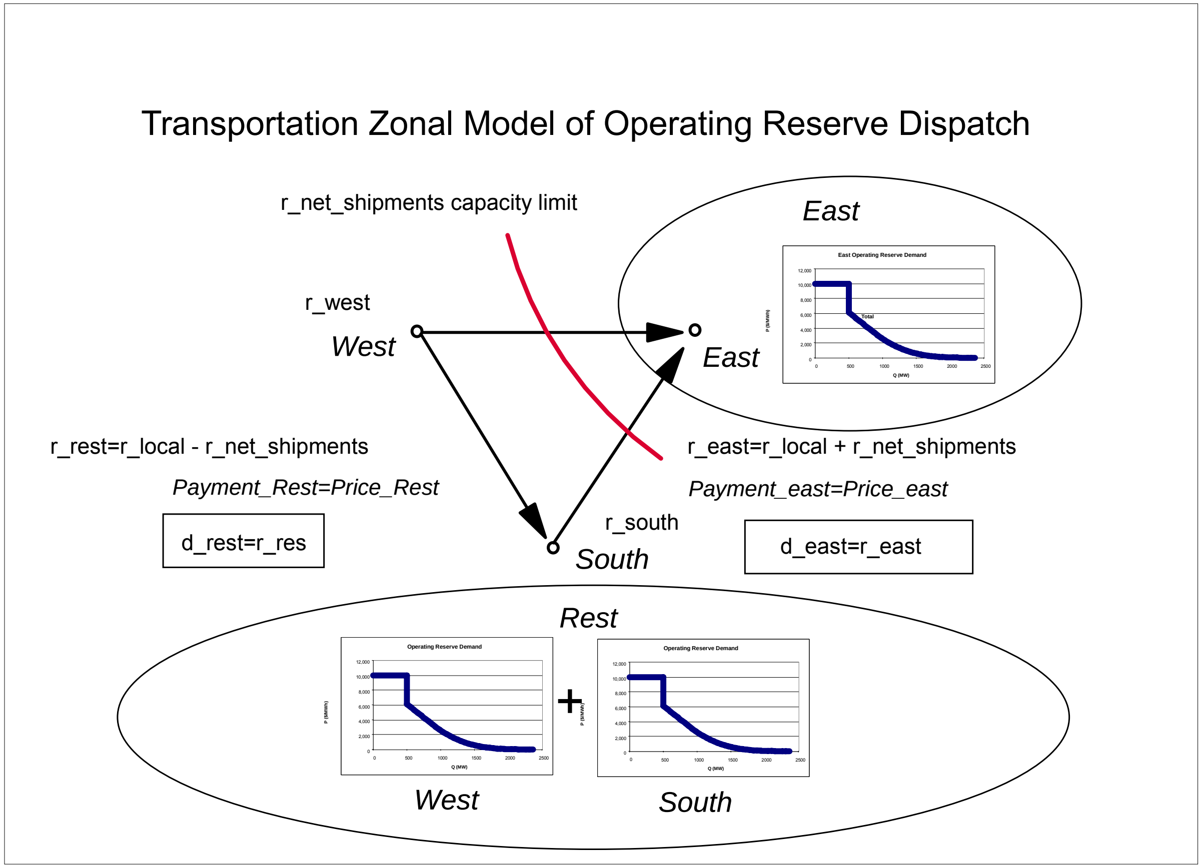
2 Eugene G. Preston, W. Mack Grady, Martin L. Baughman, "A New Planning Model for Assessing the Effects of Transmission Capacity Constraints on the Reliability of Generation Supply for Large Nonequivalenced Electric Networks," IEEE Transactions on Power Systems, Vol. 12, No. 3, August 1997, pp. 1367-1373. J. Choi, R. Billinton, and M. Futuhi-Firuzabed, "Development of a Nodal Effective Load Model Considering Transmission System Element Unavailabilities," IEE Proceedings - Generation, Transmission and Distribution, Vol. 152, No. 1, January 2005, pp. 79-89.

The next piece is a model of simultaneous dispatch of operating reserves and energy. One approach for the operating reserve piece is a nested zonal model (e.g., NYISO reserve pricing).

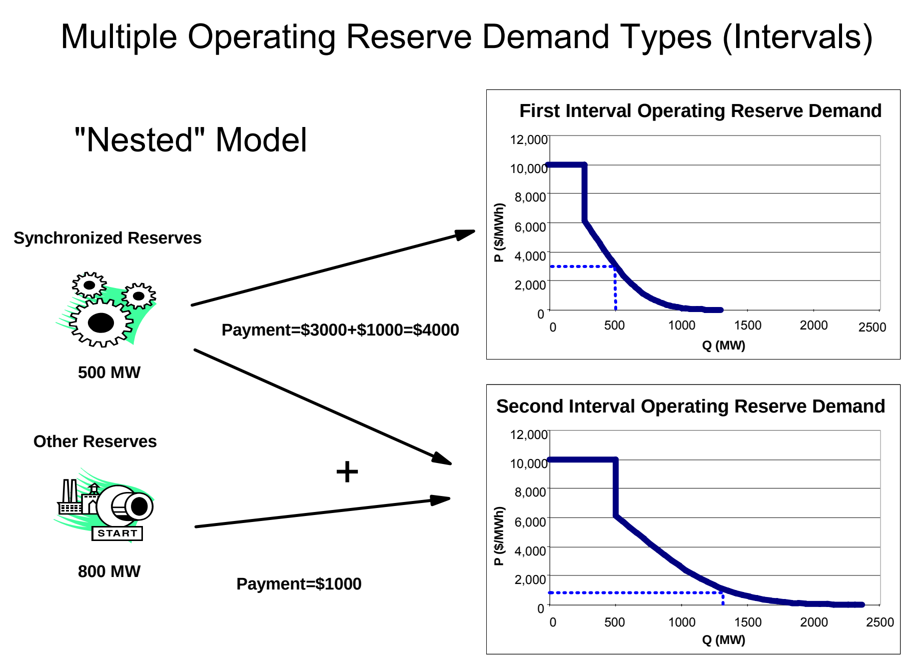


The result is that the input operating reserve price functions are additive premiums that give rise to an implicit operating reserve demand curves with higher prices.

An alternative approach would be to overlay a transportation model with interface transfer limits on operating reserve "shipments." The resulting prices are on the demand curves, but the model requires estimation of the (dynamic) transfer capacities. This is similar to the PJM installed capacity deliverability model, but specified an hour ahead rather than years ahead.



Multiple types of operating reserves exist according to response time. A nested model divides the period into consecutive intervals. Reserve schedules set before the period. Uncertainty revealed after the start of the period. Faster responding reserves modeled as available for subsequent intervals. The operating reserve demand curves apply to intervals and the payments to generators include the sum of the prices for the available intervals.



Nested model with two intervals, decisions made before uncertainty revealed.

Compared to a perfect model, there are many simplifying assumptions needed to specify and operating reserve demand curve. Compared to what is done in current market designs, using the operating reserve demand framework for consistent dispatch-based pricing should be an improvement. The sketch of the operating reserve demand curve(s) in a network could be extended.

- **Empirical Estimation.** Use existing LOLP models or LOLP extensions with networks to estimate approximate LOLP distributions at nodes.
- **Multiple Periods.** Incorporate multiple periods of commitment and response time. Handled through the usual supply limits on ramping.
- **Operating Rules.** Incorporate up and down ramp rates, deratings, emergency procedures, etc.
- **Pricing incidence.** Charging participants for impact on operating reserve costs, with any balance included in uplift.
- **Minimum Uplift Pricing.** Dispatch-based pricing that resolves inconsistencies by minimizing the total value of the price discrepancies.
- **…**

Appendix

Dispatch and Operating Reserve

Begin with an expected value formulation of economic dispatch that might appeal in principle. Given benefit (*B*) and cost (*C*) functions, demand (*d*), generation (*g*), plant capacity (*Cap*), reserves (*r*), commitment decisions (*u*), transmission constraints (*H*), and state probabilities (*p*):

$$\underset{y^i,d^i,g^i,r,u\in\{0,1\}}{Max}\; p_0\left(B^0\left(d^0\right)-C^0\left(g^0,r,u\right)\right)+\sum_{i=1}^{N}p_i\left(B^i\left(d^i,d^0\right)-C^i\left(g^i,g^0,r,u\right)\right)$$

$$s.t.$$

$$y^i=d^i-g^i,\quad i=0,1,2,\cdots,N,$$

$$\iota^t y^i=0,\quad i=0,1,2,\cdots,N,$$

$$H^i y^i\le b^i,\quad i=0,1,2,\cdots,N,$$

$$g^0+r\le u\bullet Cap^0,$$

$$g^i\le g^0+r,\quad i=1,2,\cdots,N,$$

$$g^i\le u\bullet Cap^i,\quad i=0,1,2,\cdots,N.$$

Suppose there are *K* possible contingencies. The interesting cases have $K\gg 10^3$. The number of possible system states is $N=2^K$, or more than the stars in the Milky Way. Some approximation will be in order.[3]

[3] Shams N. Siddiqi and Martin L. Baughman, "Reliability Differentiated Pricing of Spinning Reserve," IEEE Transactions on Power Systems, Vol. 10, No. 3, August 1995, pp.1211-1218. José M. Arroyo and Francisco D. Galiana, "Energy and Reserve Pricing in Security and Network-Constrained Electricity Markets," IEEE Transactions On Power Systems, Vol. 20, No. 2, May 2005, pp. 634-643. François Bouffard, Francisco D. Galiana, and Antonio J. Conejo, "Market-Clearing With Stochastic Security—Part I: Formulation," IEEE Transactions On Power Systems, Vol. 20, No. 4, November 2005, pp. 1818-1826; "Part II: Case Studies," pp. 1827-1835.

Introduce random changes in load ε^i and possible lost load l^i in at least some conditions.

$$\underset{y^i,d^i,g^i,l^i,r,u\in\{0,1\}}{Max}\; p_0\left(B^0\left(d^0\right)-C^0\left(g^0,r,u\right)\right)+\sum_{i=1}^{N} p_i\left(B^i\left(d^o+\varepsilon^i-l^i,d^0\right)-C^i\left(g^i,g^0,r,u\right)\right)$$

$$\begin{aligned}
&s.t.\\
&\quad y^0=d^0-g^0,\\
&\quad y^i=d^0+\varepsilon^i-g^i-l^i,\quad i=1,2,\cdots,N,\\
&\quad \iota^t y^i=0,\quad i=0,1,2,\cdots,N,\\
&\quad H^i y^i\le b^i,\quad i=0,1,2,\cdots,N,\\
&\quad g^0+r\le u\bullet Cap^0,\\
&\quad g^i\le g^0+r,\quad i=1,2,\cdots,N,\\
&\quad g^i\le u\bullet Cap^i,\quad i=0,1,2,\cdots,N.
\end{aligned}$$

Simplify the benefit and cost functions:

$$B^i\left(d^o+\varepsilon^i-l^i,d^0\right)\approx B^0\left(d^0\right)+k_d^i-v^t l^i \qquad , \qquad C^i\left(g^i,g^0,r,u\right)\approx C^0\left(g^0,r,u\right)+k_g^i\,.$$

This produces an approximate objective function:

$$p_0\left(B^0\left(d^0\right)-C^0\left(g^0,r,u\right)\right)+\sum_{i=1}^{N}p_i\left(B^i\left(d^o-l^i,d^0\right)-C^i\left(g^i,g^0,r,u\right)\right)=B^0\left(d^0\right)-C^0\left(g^0,r,u\right)+\sum_{i=1}^{N}p_i\left(k_d^i-k_g^i\right)-v^t\sum_{i=1}^{N}p_i l^i\,.$$

The revised formulation highlights the pre-contingency objective function and the role of the value of the expected undeserved energy.

$$
\begin{aligned}
&\underset{y^i,d^i,g^i,l^i,r,u\in\{0,1\}}{Max} B^0\left(d^0\right)-C^0\left(g^0,r,u\right)-v^t\sum_{i=1}^{N} p_i l^i \\
&s.t. \\
&\quad y^0 = d^0 - g^0, \\
&\quad y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1,2,\cdots,N, \\
&\quad \iota^t y^i = 0, \quad i = 0,1,2,\cdots,N, \\
&\quad H^i y^i \le b^i, \quad i = 0,1,2,\cdots,N, \\
&\quad g^0 + r \le u \bullet Cap^0, \\
&\quad g^i \le g^0 + r, \quad i = 1,2,\cdots,N, \\
&\quad g^i \le u \bullet Cap^i, \quad i = 0,1,2,\cdots,N.
\end{aligned}
$$

There are still too many system states.

Define the optimal value of expected unserved energy (VEUE) as the result of all the possible optimal post-contingency responses given the pre-contingency commitment and scheduling decisions.

$$VEUE\left(d^0, g^0, r, u\right) = \underset{y^i, d^i, g^i, l^i, r}{Min} \; v^t \sum_{i=1}^{N} p_i l^i$$

$$\begin{aligned}
&s.t. \\
&\quad y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N, \\
&\quad \iota^t y^i = 0, \quad i = 1, 2, \cdots, N, \\
&\quad H^i y^i \le b^i, \quad i = 1, 2, \cdots, N, \\
&\quad g^0 + r \le u \bullet Cap^0, \\
&\quad g^i \le g^0 + r, \quad i = 1, 2, \cdots, N, \\
&\quad g^i \le u \bullet Cap^i, \quad i = 1, 2, \cdots, N.
\end{aligned}$$

This second stage problem subsumes all the redispatch and curtailment decisions over the operating period after the commitment and scheduling decisions.

The expected value formulation reduces to a much more manageable scale with the introduction of the implicit VEUE function.

$$\underset{y^0, d^0, g^0, r, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - VEUE\left(d^0, g^0, r, u\right)$$

$$s.t.$$

$$y^0 = d^0 - g^0,$$

$$H^0 y^0 \leq b^0,$$

$$g^0 + r \leq u \bullet Cap^0,$$

$$\iota^t y^0 = 0,$$

$$g^0 \leq u \bullet Cap^0.$$

The optimal value of expected unserved energy defines the demand for operating reserves. This formulation of the problem follows the outline of existing operating models except for the exclusion of contingency constraints.

Ignore the network features for the first illustration. Assume all the load and generations is at a single location. Unserved energy demand is a random variable with a distribution for the probability that load exceeds available capacity.

$$Unserved\ Energy = Max\left(0, Load - Available\ Capacity\right)$$

Hence

$$\begin{aligned}
&Unserved\ Energy = Max\Big(0, E\left(Load\right) + \Delta\ Load - \big(Committed\ Capacity - \Delta Capacity\big)\Big) \\
&= Max\Big(0, \Delta\ Load + Outage + \big(E\left(Load\right) - Committed\ Capacity\big)\Big) \\
&= Max\big(0, \Delta\ Load + Outage - Operating\ Reserve\big).
\end{aligned}$$

This produces the familiar loss of load probability (*LOLP*) calculation, for which there is a long history of analysis and many techniques. With operating reserves (*r*),

$$LOLP = \Pr\left(\Delta\ Load + Outage \geq r\right) = \overline{F}_{LOL}\left(r\right).$$

A common characterization of a reliability constraint is that there is a limit on the *LOLP.* This imposes a constraint on the required reserves (*r*).

$$\overline{F}_{LOL}\left(r\right) \leq LOLP_{Max}.$$

This constraint formulation implies an infinite cost for unserved energy above the constraint limit, and zero value for unserved energy that results within the constraint.

An alternative approach is to consider the expected unserved energy (*EUE*) and the Value of Lost Load (*VOLL*).

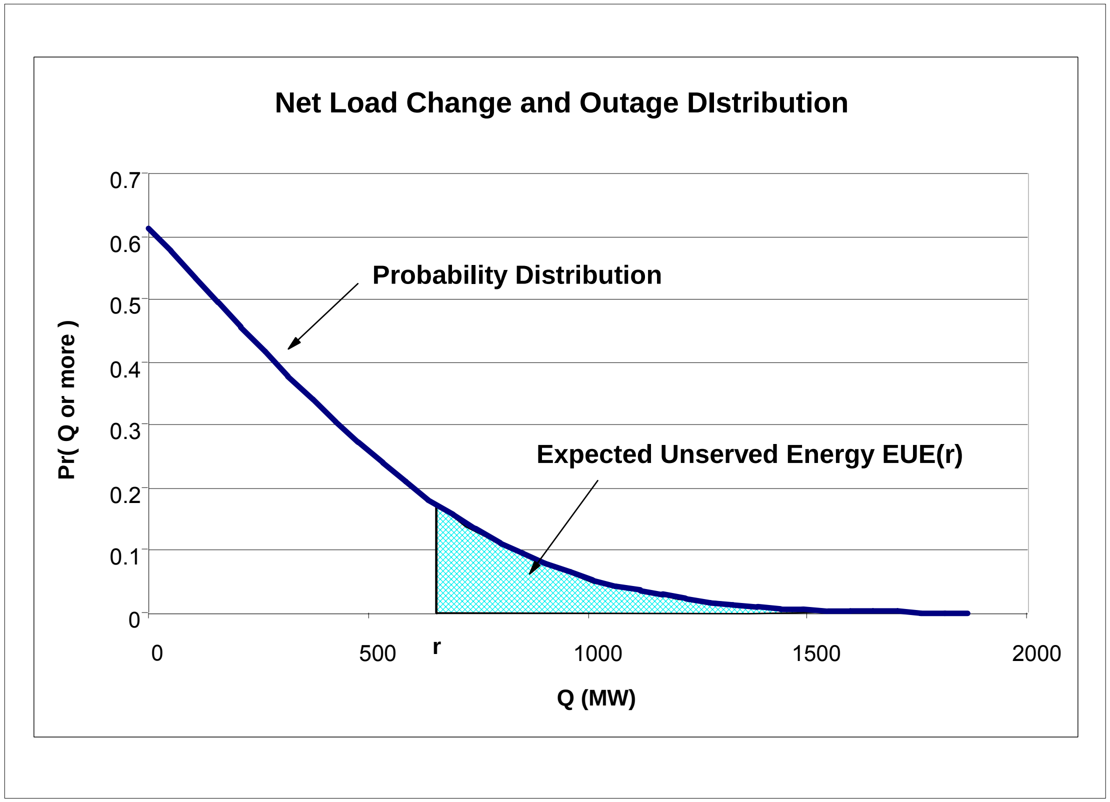
Suppose the *VOLL* per MWh is v. Then we can obtain the *EUE* and its total value (*VEUE*) as:

$$EUE(r) = \int_r^\infty \overline{F}_{LOL}(x)\,dx.$$

$$VEUE(r) = v\int_r^\infty \overline{F}_{LOL}(x)\,dx.$$

There is a chance that no outage occurs and that net load is less than expected, or $\overline{F}_{LOL}(0) < 1.$

The real changes may not be continuous, but it is common to apply continuous approximations.



The distribution of load and facility outages compared to operating reserves determines the LOLP.

A reasonable approximation is that the change in load is normally distributed: $\Delta\, Load \sim N\left(0,\sigma_L^2\right)$.

The outage distribution is more complicated and depends on many factors, including the unit commitment. Suppose that $o_j = 0,1$ is a random variable where $o_j = 1$ represents a unit outage. The probability of an outage in the monitored period, given that plant was available and committed at the start of the period ($u_j = 1$) is ω_j, typically a small value on the order of less than 10^{-2}:

$$Outage = \sum_j u_j Cap_j o_j,$$

$$\Pr\left(o_j = 1 \middle| u_j = 1\right) = \omega_j.$$

A common approximation of $\Pr\left(Outage\right)$ is a mixture of distributions with a positive probability of no outage and a conditional distribution of outages that follows an exponential distribution.[4]

$$\Pr\left(Outage = 0\right) = p_0, \Pr\left(Outage > x\right) = \left(1 - p_0\right)e^{-\lambda x}.$$

The combined distribution for change in load and outages can be complicated.[5] In application, this distribution might be estimated numerically, possibly from Monte Carlo simulations.

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4 Debabrata Chattopadhyay and Ross Baldick, “Unit Commitment with Probabilistic Reserve,” IEEE, Power Engineering Society Winter Meeting, Vol. 1, pp. 280-285.

5 Guy C. Davies, Jr., and Michael H. Kuttner, “The Lagged Normal Family Of Probability Density Functions Applied To Indicator-Dilution Curves,” Biometrics, Vol. 32, No. 3, September 1976, pp. 669-75.

For sake of the present illustration, make a simplifying assumption that the outage distribution is approximated by a normal distribution.

$$Outage \sim N\left(\mu_O, \sigma_O^2\right).$$

Then with operating reserves *r*, the distribution of the lost load is

$$LOLP = \Pr\left(\Delta\ Load + Outage \geq r\right) = \bar{F}_{LOL}\left(r\right)$$
$$= \bar{\Phi}\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right) = 1 - \Phi\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right).$$

Here $\Phi\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right)$ is the cumulative normal distribution with mean and variance $\mu_O, \sigma_O^2 + \sigma_L^2$.

$$EUE\left(r\right) = \int_r^\infty \bar{\Phi}\left(x \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right) dx.$$

$$VEUE\left(r\right) = v\int_r^\infty \bar{\Phi}\left(x \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right) dx.$$

This gives the implied reserve inverse demand curve as

$$Operating\ Reserve\ Demand\ Price\left(r\right) = P_{OR}\left(r\right) = v\bar{\Phi}\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right).$$

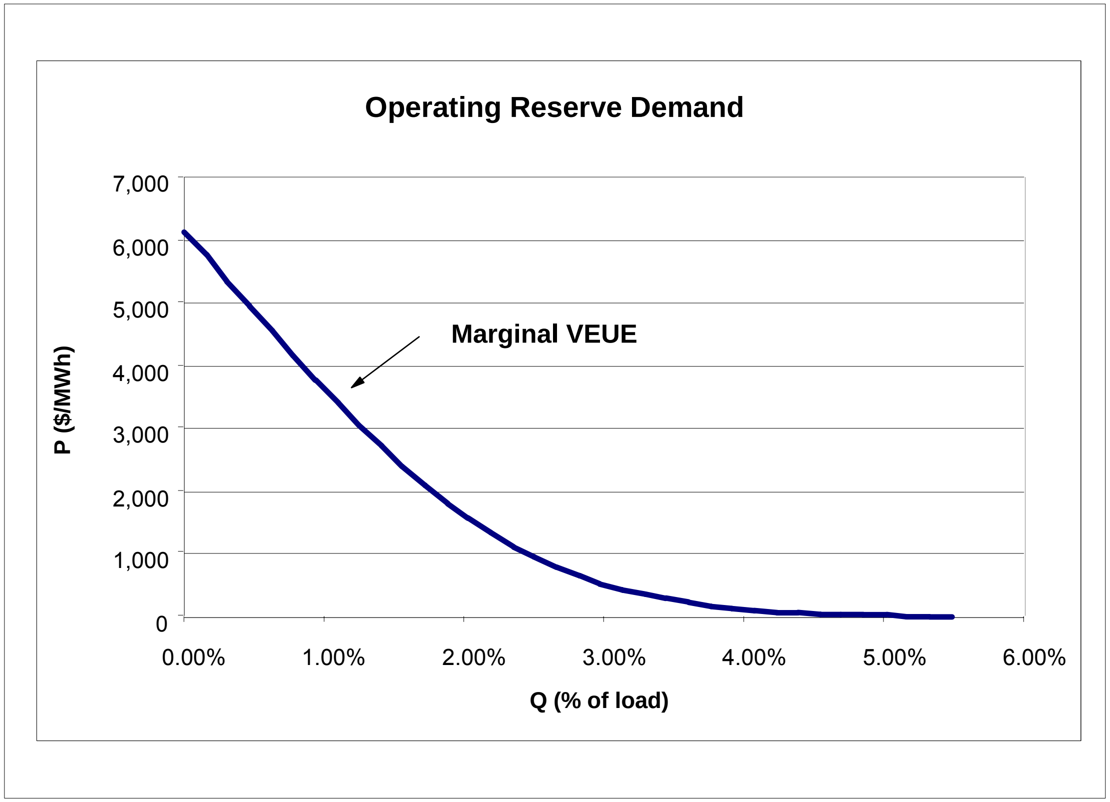
The probabilistic demand for operating reserves reflects the cost and probability of lost load.

$$Operating\ Reserve\ Demand\ Price(r) = P_{OR}(r) = v\bar{\Phi}\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right).$$

Example Assumptions

Expected Load (MW)	34000
Std Dev %	1.50%
Expected Outage %	0.45%
Std Dev %	0.45%
Expected Total (MW)	153
Std Dev (MW)	532.46
VOLL ($/MWh)	10000

Under the simplifying assumptions, if the dispersion of the LOLP distribution is proportional to the expected load, the operating reserve demand is proportional to the expected load. Total value is of same magnitude as the cost of meeting load.



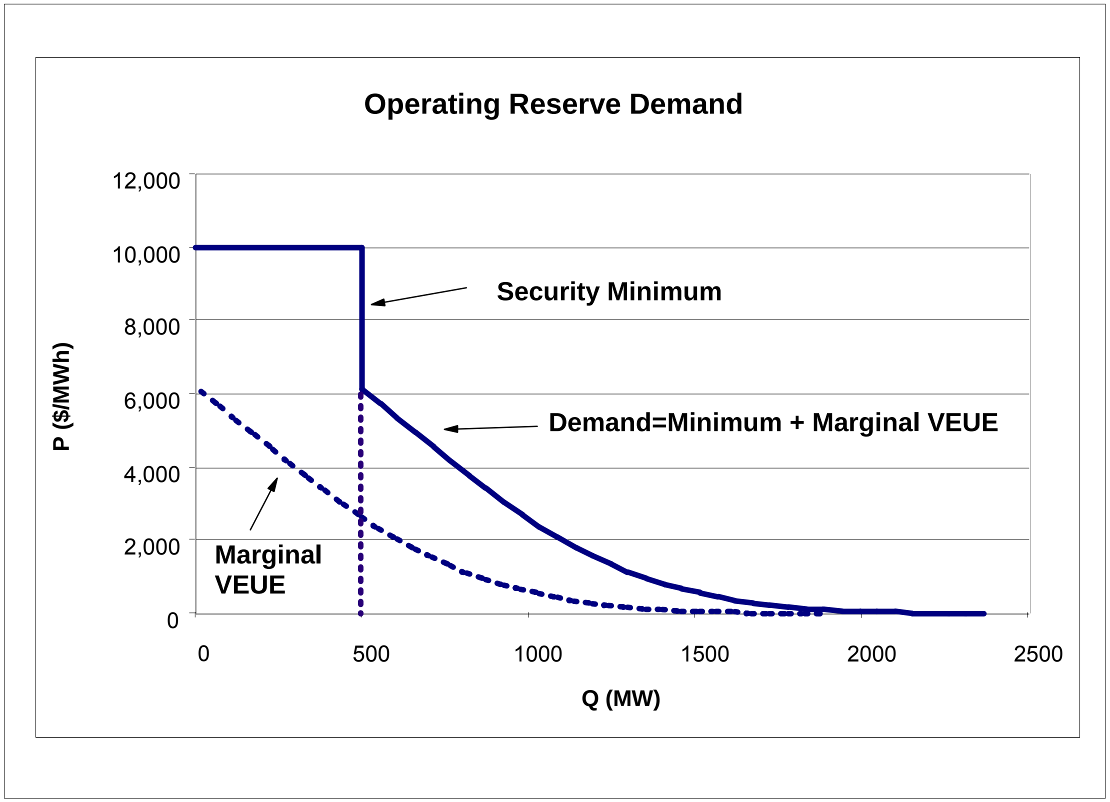
The deterministic approach to security constrained economic dispatch includes lower bounds on the required reserve to ensure that for a set of monitored contingencies (e.g., an n-1 standard) there is sufficient operating reserve to maintain the system for an emergency period.

Suppose that the maximum generation outage contingency quantity is $r_{Min}\left(d^0, g^0, u\right)$. Then we would have the constraint:

$$r \geq r_{Min}\left(d^0, g^0, u\right).$$

In effect, the contingency constraint provides a vertical demand curve that adds horizontally to the probabilistic operating reserve demand curve.

If the security minimum will always be maintained over the monitored period, the VEUE price at r=0 applies. If the outage shocks allow excursions below the security minimum during the period, the VEUE starts at the security minimum.



In a network, security constrained economic dispatch includes a set of monitored transmission contingencies, K_M, with the transmission constraints on the pre-contingency flow determined by conditions that arise in the contingency.

$$H^i y^0 \le \tilde{b}^i, \quad i = 1, 2, \cdots, K_M.$$

The security constrained economic dispatch problem becomes:

$$\begin{aligned}
&\underset{y^0, d^0, g^0, r, u \in (0,1)}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - VEUE\left(d^0, g^0, r, u\right) \\
&s.t. \\
&\quad y^0 = d^0 - g^0, \\
&\quad H^0 y^0 \le b^0, \\
&\quad H^i y^0 \le \tilde{b}^i, \quad i = 1, 2, \cdots, K_M, \\
&\quad g^0 + r \le u \bullet Cap^0, \\
&\quad r \ge r_{Min}\left(d^0, g^0, u\right) \\
&\quad \iota^t y^0 = 0, \\
&\quad g^0 \le u \bullet Cap^0.
\end{aligned}$$

If we could convert each node to look like the single location examined above, the approximation of *VEUE*, would repeat the operating reserve demand curve at each node.

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