Design of Contingent Capital
with Stock Price Trigger for Conversion

Suresh Sundaresan[*]   Zhenyu Wang[†]

April 30, 2010[‡]

Abstract

The proposal for banks to issue contingent capital (CC) that is forced to convert into common equity when stock price falls below a certain specified low threshold ("trigger") does *not* in general lead to unique equilibrium in equity and CC prices. Multiple or no equilibrium arise because both equity and CC are claims on the assets of the underlying bank that has issued these claims. For a security to be robust to price manipulation, it must have a unique equilibrium. For a unique equilibrium to exist, it is necessary that mandatory conversion must not result in any value transfers between equity and CC holders. The necessary condition for unique equilibrium is usually not satisfied by CC with fixed coupon rate. Contingent capital with floating coupon rate is shown to have a unique equilibrium if the coupon rate is set to be equal to the risk-free rate. This structure of CC anchors its value to par all the time before conversion, making it implementable in practice. Although a CC with unique equilibrium is robust to price manipulation, the no value transfer condition may preclude it from generating the desired incentives for bank managers or demand from investors.

JEL classification: G12, G23

1 Introduction

One of the lessons learned from the credit crisis of 2007-2009 is that the architecture governing the financial insolvency of banks and other financial institutions needs major overhaul. The bankruptcy of Lehman Brothers, and the financial distress experienced by Citigroup, Bank of America, and AIG has demonstrated the need to revisit the financial insolvency procedures that should govern banks and other financial institutions. In particular, the extensive amount of implicit guarantees, outright infusion of tax-payer money, and other direct and indirect benefits extended to large financial institutions has come in for much scrutiny, and new framework for capital market regulation has been proposed.[1]

An integral part of this debate is the design of the capital structure of banks and related financial institutions. There has been considerable interest in designing debt securities for banks that are forced into equity via mandatory conversion in periods of distress so that a) the banks are relieved of servicing their debt obligations in bad states of the world, when costly financial distress may result if creditors continue to demand their interest and principal payments, and b) the equity buffer could increase, further bolstering the bank's capital as a result of the forced conversion of junior debt[2].

Recently there have been a few issues of junior debt with such conversion provisions. Lloyds Bank recently issued the so called contingent convertible (CC, or "Coco bonds"). These bonds will convert into ordinary shares if the consolidated core tier one ratio of Lloyds falls below 5%. The bonds themselves are subordinated bonds, which prior to conversion count as the lower tier 2 capital, but count as core tier 1 in the context of the Financial Services Authority (FSA) stress tests. They will count as core tier 1 for all purposes upon conversion. Swiss regulators are encouraging Swiss banks to issue contingent capital[3]. In

---

[1]See, The Global Financial Crisis, A Plan for Regulatory Reform May 2009, by committee on capital markets regulation.

[2]See Squam Lake Working Group on Financial Regulation (2009).

[3]"New capital instruments on Swiss banks' radar,"By Patrick Jenkins in London and Haig Simonian, Financial Times, April 14, 2010.

Germany, preferred stocks have been issued with similar features.

A number of papers have argued for the issuance of reverse convertible notes (RCN) very much in the spirit of CC for over 10 years. Doherty and Harrington (1997) had suggested an RCN whereby the payments to junior debt can be made either in cash or equity at the *discretion* of the issuer at *pre-determined* stock prices. Flannery (2002) proposed an RCN structure in which the conversion is *mandatory* and occurs *at current market price* of equity. In more recent papers, several authors have argued for varying types of contingent capital debt[4].

Several potential problems have also been identified with such contingent debt issues by many papers. Acharya, Cooley, Richardson, and Walter (2009) have suggested that while contingent capital does restore some market discipline, it fails to fully address the fact that banks have deposits, secured debt (repos), non-contingent debt of other types, liabilities to derivatives transactions that carry either explicit or implicit guarantees. Hence both contingent capital and equity capital may have incentives to take excessive risks at the expense of guaranteed debt (taxpayer money)[5]. Hart and Zingalis (2010) have suggested that CC may introduce inefficiency as conversion eliminates default, which forces inefficient businesses to restructure and incompetent managers to be replaced. By eliminating the threat of potential defaults, such bonds, after their conversion, increase inefficiency in the banking sector. If banks have repo and derivatives positions as well, such bonds may not prevent defaults on systemic obligations and thus the risk of systemic crises[6]. Bond, Goldstein and Prescott (2009) have argued that if agents were to use market prices when taking corrective actions, prices will adjust to reflect such a use and may become potentially less revealing. Several alternative proposals have also been made with regard to bank capital. For example, Kashyap, Rajan and Stein (2008) have proposed that banks buy "systemic risk insurance" and secure

---

[4]See papers by Flannery (2009) and McDonald (2009)

[5]Acharya, Cooley, Richardson, and Walter, Real Time Solutions for Financial Reform, An NYU Stern Working Group on Financial Reform, December 2009.

[6]Hart and Zingalis (2010) propose that CDS spreads be used in trigger mechanisms.

the pay outs on insurance[7]. Admati and Pfleiderer (2010) have argued for increasing the liability of owners (equity holders) and suggest that such a structure will mitigate the conflicts of interests between equity and debt holders and may serve to reduce the need for bail outs.

Valuation of CC can be performed using the analytical approach developed in structural models of default pioneered by Merton (1974) and significantly extended by Black and Cox (1976) who value default-risky senior and subordinated debt securities. These models work with the (unobserved) asset value of the issuing firm as the state variable and derive *simultaneously the equity and debt values*. The paper by Black and Cox (1976) is particularly relevant as they explicit model safety covenant as a trigger for the bond holders to take over the firm. The contingent claims approach has been standard for pricing corporate debt and hybrid securities, as presented in detail by Garbade (2001).

Broadly, the valuation of contingent capital falls into the following categories: first, papers that specify triggers on the asset value to trigger conversion. Second, papers that directly specify triggers on stock prices to force conversion. Finally, there are papers that specify triggers based on risk-weighted capital, which combines assets with some accounting information. The first category of papers are useful in developing economic intuition about the valuation of CC, but are difficult to implement in practice. The second category of papers by specifying the stock price process exogenously avoid the problem of *simultaneously* determining stock and CC prices.

In the first category of papers, the structural framework has been used in two recent papers to analyze CC debt design by Albul, Jaffee and Tchistyi (2010) and Pennacchi (2010). These papers are insightful in helping to understand the pricing of convertible debt securities but do not directly address triggers that are directly based on equity prices. They place triggers on the underlying (unobserved) asset values. Albul, Jaffee and Tchistyi (2010) consider perpetual debt to derive closed-form solutions for optimal capital structure. Pennacchi

---

[7]In some catastrophe bond structures, the bond investor (insurance buyer) can put the bond at par if and when a specified catastrophic event occurs during the life of the bond. Banks can also buy such assets from insurance companies so that they get cash when they need it. In return, they must accept a lower coupon rate.

(2010) considers a jump-diffusion formulation and shows that managers may have incentives to shift risk, but this incentive is less with CC structure.

In the second category of papers, McDonald (2009) addresses the issue of pricing CC when there is a direct equity (and a stock index) trigger. McDonald (2009) directly specifies a well-defined process for the stock price, and does not relate the stock price *and* the value of CC to the underlying asset values. In reality, both the stock price and the CC price are determined simultaneously as both are claims to the assets of the underlying bank. In McDonald's formulation, which ignores default, the coupon of the CC is increased by the "conversion premium" so that the CC sells at par at issue.

The main thrust of our paper is the following: when triggers for mandatory conversion are placed directly on equity prices, there is a need to ensure that conversion does not transfer value between equity and CC holders. The economic intuition behind CC design problem is as follows. In the contingent capital (CC) proposed in the literature, junior debt converts to equity shares when the stock price reaches a certain threshold at low level. This sounds like a normal and innocuous feature. However, the unusual part of the CC design is that conversion into equity is *mandatory* as soon as stock price hits a trigger level from above. Since common stock is the residual claim of bank's value, it must be priced *together with* the CC. Keeping firm value fixed, a dollar more for the CC value must be associated with a dollar less for the equity value.[8] Therefore, a value transfer between equity and CC disturbs equilibrium by moving the stock price up or down, depending on the conversion ratio specified. The design of the conversion ratio must ensure that there is no such value transfer. The design proposals in the literature usually ensure that there is no value transfer at maturity, but do not ensure it *before* maturity.

If the value transfer never pushes stock price across trigger, there is no problem because given each asset value, investors always know whether there will be conversion or not. However, it is always possible that the value transfer pushes the stock price across trigger from

---

[8]We assume for exposition that the senior bank debt value remains unaffected.

above to below. In this case, there are two possible equilibria. In the first one, all investors believe conversion will not happen, leading stock price to stay above the trigger. In the second one, all investors believe conversion will happen, leading stock price to hit the trigger. Since two prices are possible whenever firm value drops to certain level, by combining these dual equilibria around trigger at different times in the future, numerous expected equity values are possible even well before conversion. These numerous values can form a range, and the whole range can be above the trigger. In this range, there is no obvious market force to push the stock price to a particular one. As we will show later, there are also economic conditions in which a CC with stock price trigger may not even have any equilibrium price. The theoretical implication of this is that arbitrage opportunities may exist no matter what price one assigns to the stock and CC.

The only way to prevent this is to ensure that there is no value transfer at conversion. This requires that at all possible conversion times, the value of converted shares must be exactly equal to the market value of the un-converted CC. This requirement implies that the conversion ratio usually cannot be chosen ex-ante once the trigger level has been chosen: this is due to the fact that the trigger level multiplied by the conversion ratio must equal the market value of the un-converted CC. However, there is one scenario when we can select the conversion ratio ex-ante: this corresponds to the design of CC such that the coupon payments are indexed in such a way that the CC always sells at par. In this case, we can set the conversion ratio as simply the par value divided by the trigger level of stock price at which mandatory conversion will occur. We explore this design possibility further in the paper.

The requirement of no value transfer at conversion also means that the terms of the CC cannot be used to set incentives to either control managers or market manipulation. Albul, Jaffee and Tchistyi (2010) suggest using the conversion ratios to control manipulation. Some practitioners argue that a "dilutive" conversion ratio, may incentivize the bank to lower risk ex-ante, and pre-emptively issue equity ex-post. Albul, Jaffee and Tchistyi (2010), Pennachi

(2010) and Flannery (2009) suggest that a particular choice of conversion ratio may have implications for either risk-shifting or for incentives to manipulate. Our paper shows that the design of CC does not allow for such a freedom to choose the conversion ratio. We show that such provision of incentives may not be feasible with mandatory conversion as it will transfer value at conversion, and hence will not admit a unique equilibrium.[9]

The road map for the paper is as follows. In section 2, we provide an illustration of the problems that arise when conversion ratios are specified ex-ante. The illustrations demonstrates that there may be multiple equilibria or no equilibrium depending on conversion ratio and coupon rate. In section 3, we derive the condition for an equilibrium to exist and be unique. In section 4, we focus on how to design the CC so that the structure is less prone to multiple equilibria. We present a floating coupon structure, whereby the coupon of the CC is indexed to the risk-free rate so that the market value of the floater is anchored to par. With this structure, CC with stock price trigger becomes operational. In this context, we discuss the market imperfections that are often cited as reasons for the potential failure of CC. Section 5 concludes.

2   Illustration of the Pricing Problem

Before discussing the pricing of contingent capital, we need to describe a bank which has a capital structure with CC. Consider a bank that has senior bond holders and common equity holders who have claims on an asset (or a business). The asset requires an investment of A_0 dollars today (time 0). The asset is typically risky; its value at time t is a random number A_t. At time 0, the bank has also issued a security called "contingent capital." The security is in the form of a debt (or preferred equity) with face value $\bar{C}$, which is junior to the bond but converts to common equity when certain pre-specified conditions are met.[10]

---

[9]The possibility of "multiple equilibria" has been identified by Diamond and Dybvig (1983) in the context of bank runs. Chari and Jagannathan (1988) find a unique equilibrium with bank runs, and stress the role of suspension of convertibility.

[10]To keep the analysis simple, in this section we assume that the contingent capital does not pay coupon or dividend. We make similar assumption to the bond. Also, we assume that the asset does not generate

The *contingent capital with stock price trigger* sets conversion condition on the bank's stock price of the bank. Suppose S_t is the stock price of this bank. At any time t, the bank converts the junior debt under the contingent capital to m_t shares of common equity as soon as the share price S_t falls to level K_t. The quantity m_t is referred to as *conversion ratio*, and K_t as *trigger price*. They are either constant or pre-specified functions of observable variables. A particular contract specifies the trigger price $K(\cdot)$ and conversion ratio $m(\cdot)$ as functions of market or accounting variables over time.

The following are two examples of contingent capital contracts. The *simple form* of contingent capital can have a constant trigger K and a constant conversion ratio m. The contingent capital contracts proposed in the literature typically have time-varying trigger and conversion ratio. The one suggested by Flannery (2002) specifies that $K_t = z \cdot \mathrm{RWA}_t/n$, where RWA_t is the risk-weighted asset reported most recently, z is a constant related to regulatory capital ratios, and n is the shares outstanding before conversion. The conversion ratio is $m_t = \bar{C}/K_t$. Since risk-weighted asset changes only at the end of each quarter, this contingent capital takes the simple form after its last change of risk-weighted asset, if it is not converted by then.

An important issue with contingent capital is whether it is cheaper and more effective in protecting the bank from bankruptcy than subordinated debt or equity. In fact, The preferred equity can be viewed as contingent capital with a special trigger: $K_t = 0$ for all $t \leq T$. The preferred equity under this contingent capital will never convert to common equity before the firm default on senior bond. It will never cause the bank defaults either. Common equity can also be viewed as contingent capital with special trigger—we set trigger to infinity $K_t = +\infty$ and conversion ratio to m, the junior debt always converts to m shares of common stock. Then, holding this contingent capital is equivalent to holding m shares of common equity.

Although methods for pricing of subordinated debt and equity is established, the pricing

cash flow. We will relax these assumptions in the next section.

of a contingent capital that has a stock price trigger as proposed by Flannery (2002) poses special challenges or problems. In this section, we illustrate this challenge with examples and trinomial trees in discrete time, leaving the formal analysis in dynamic continuous-time models to the next section.

To have a unique equilibrium price, the trigger and conversion ratio cannot be chosen arbitrarily. Let us obtain intuition from an extremely simple example. Suppose a firm has an asset that has certain value today. On the next day, the value of the asset is a random number. The bank has a senior bond; its par value is $80 and maturity date is tomorrow. The bank has a contingent capital, and its par value is $10. The trigger of the contingent capital is assumed to be $5. The bank has one share of equity. As of today, suppose the contingent capital has not been converted because the asset value has been so high that the per-share price of equity is above the trigger.

Given the trigger, a conversion that is too high can have multiple equilibrium prices. Suppose the conversion ratio is $m = 3$ shares. If the asset value turns out to be $100 on the next day, the stock price without conversion is

$$\frac{100 - 80 - 10}{1} = \$10 \,,$$

which is above the trigger. Therefore, if all investors believe that there will be no conversion, they will be willing to pay $10 per share. As a result, $10 is an equilibrium price for the stock. However, the stock price with conversion is

$$\frac{100 - 80}{1 + 3} = \$5 \,,$$

which hits the trigger. If all investors believe that conversion will happen they will only want to pay $5 per share. This then results in a stock price of $5 as another equilibrium price. Therefore, there are two possible equilibrium prices when the asset value turns out to be $100. This case happens because conversion at a high ratio transfers value from stock holders to contingent capital investors, pushing down the stock price to trigger.

Similarly, a conversion with a ratio that is too low will pull stock price above the trigger,

leaving no equilibrium price for the market to settle. In the above example, assume that the conversion ratio is $m = 1$. If the asset value turns out to be \$95 tomorrow, the stock price without conversion is

$$\frac{95 - 80 - 10}{1} = \$5 \,,$$

which hits the trigger. However, the stock price with conversion is

$$\frac{95 - 80}{1 + 1} = \$7.5 \,,$$

which is above the trigger. In this case, it is difficult for market to settle down to a price because conversion is expected if all investors believe there is no conversion and vice versa. This case happens because conversion at a low ratio transfers value from contingent capital investors to equity holders.

To avoid multiple/no equilibriums, we need to set the conversion ratio exactly right so that there is never value transfer between equity holders and contingent capital investors. In the above example, we need to set conversion ratio to $m = 2$, which equals

$$m = \frac{\text{Value of un-converted CC}}{\text{Trigger}} = \frac{10}{5} = 2 \,. \tag{1}$$

With this ratio, conversion will not move the stock price to cause ambiguity for conversion. For example, if the asset value is \$95, stock price is $(95 - 80 - 10)/1 = 5$ without conversion and $(95 - 80)/3 = 5$ with conversion. Then, contingent capital must convert because stock price hits the trigger of \$5 anyway. If the asset value is \$100, the stock price is $(100 - 80 - 10)/1 = 10$ without conversion and $(100 - 80)/3 = 6.66$ with conversion. Then, contingent capital should not convert because the stock price stays above trigger regardless of conversion or not.

The contingent capital proposed by Flannery (2002) in fact satisfies this condition at the maturity date. If the contingent capital does not convert at maturity, the value of contingent capital is simply its face value $\bar{C}$. Then, conversion ratio $m = \bar{C}/K$ is the ratio that does not transfer value on maturity date because if stock price hits the trigger, the contingent

10

capital investors receive $mS_T = mK$ dollars, which is the same $\bar{C}$ dollars they will receive if it is not converted.

The conversion ratio that guarantees no value transfer at maturity may still transfer value at some time before maturity, causing multiple equilibria. In the following, we set $m = 2$ in the above example and examine pricing of the securities in a one-step trinomial model presented in Figure 1. In the one-step trinomial model, we assume that the asset value on the next day can be $110, $95, or $70 tomorrow with probability 0.3, 0.4, and 0.3, respectively, as in panel (A). To keep calculations simple, we assume all investors are risk-neutral, and interest rate is zero. With these assumptions, the asset value today is $92. The values of the senior bond on the tree are displayed in panel (B). On the top two nodes at maturity, the bond is valued at par ($80) because the firm does not default. On the bottom node at maturity, the firm defaults, and the bond value is the asset value ($70). These possibilities of bond value at maturity imply that the expected value in the earlier node is $77.

The values of the contingent capital and per share equity in the simple trinomial model are also displayed Figure 1. In panel (C), it is assumed that the CC is not converted today. As we have discussed, when conversion ratio is $m = 2$, the CC does not convert when asset value is above $100 at maturity and converts when asset value is $95. When asset value is $70, the CC has zero value because the bank defaults. Therefore, the three possible payoffs of CC at maturity are $10, $10, and $0, corresponding to the cases of no conversion, conversion, and default, respectively. The corresponding stock prices at maturity are $20, $5, and $0. Notice that the stock price hits the trigger on the middle node, on which the contingent capital converts to two shares at $5 each. Given that the CC does not convert on the node before maturity, its value should be the expected payoff at maturity, which is $C = 7$, using the given probabilities (0.3, 0.4, and 0.3). Then, the stock price is $S = (92 - 77)/1 = 8$, which is also the expected value of the stock. Notice that the stock price is above the trigger. Therefore, not converting today is consistent with the trigger rule, and $C = 7$ and $S = 8$ is a equilibrium if all contingent capital investors and equity holders expect this outcome.

11

Panel (D) of Figure 1 shows that conversion today is another equilibrium. If the contingent capital converts on the node before maturity, the total number of shares becomes 3, and the stock price is $S = (92 - 77)/3 = 5$, which hits the trigger. This price is also the expected value of the stock of the stock. Since the contingent capital converts to 2 shares today, its value on this node is $C = 2 \times 5 = 10$ dollars. Therefore, conversion today is consistent with the trigger rule, and $C = 10$ and $S = 5$ is rational expectations equilibrium.

The two equilibria presented in Figure 1 leave the values of contingent capital and stock price undetermined on the node before maturity. Multiple equilibria occurred on a node before maturity because the conversion ratio, $m = 2$, is too high on this node and transfers value from equity holders to contingent capital investors. To prevent such value transfer, we need to set conversion ratio so that on this node the value of the converted shares at trigger price equals to the value of the un-converted contingent capital. As shown in Figure 1, the un-converted contingent capital is valued as $7 dollars on this node. Then, the conversion ratio should be set to $m = 7/K = 7/5 = 1.4$. However, to use this conversion ratio, we need to know the value of the un-converted contingent capital on this node. However, to know the value of un-converted CC at every time and in every estate is impractical, especially when CC is newly introduced to the market.

The example above was meant to illustrate the potential problems with the design of CC. To formally establish the outcomes of the example, we develop in the next section a continuous-time framework where we show the condition under which we can get a unique equilibrium.

3   Analysis in Continuous-Time Model

We develop the ideas in the context of a structural model of default, along the lines of Merton (1974) and Black and Cox (1976). Consistent with these models, we operate in a risk-neutral economy. We denote the assets of the bank by A_t, which generates cash flow at the rate of a_t at time t. One example is a bank whose asset follows a geometric Brownian motion

and has a constant a_t. Another example is a bank whose cash flow α_t follows a geometric Brownian motion, and A_t is the value of these future cash flows. If interest rate is constant, it is well known that A_t follows a geometric Brownian motion, and $a_t = \alpha_t/A_t$. These specific assumptions on the stochastic process of A_t and a_t are not necessary for the analysis here. Our analysis allows the bank asset value A_t to have time-varying drift μ_t and volatility σ_t. We assume

$$dA_t = \mu_t A_t dt + \sigma_t A_t dZ_t . \tag{2}$$

In risk-neutral probability, we should have $\mu_t = r_t - a_t$, where r_t is the instantaneous risk-less interest rate at time t and Z_t is a Wiener process.

We assume that the bank has issued a senior bond with a par value $\bar{B}$ and maturity T. The coupon rate of the senior bond is b_t, which can be constant or time-varying. This allows us to explore both fixed and floating rate bank debt. Let δ be the time when the senior bond defaults. There are several ways to specify default condition, which then determines the default time. The framework proposed in this section can accommodate various specification of defaults. The only thing we need here is that default event wipes out the value of equity. To be specific, let the default barrier at time t be $\bar{B}P(t,T)$. Then,

$$\delta = \inf\{t \geq 0 : A_t \leq \bar{B}P(t,T)\} . \tag{3}$$

Let $r(t,s)$ be the risk-less zero-coupon rate from time t to s, and

$$P(t,s) = e^{-r(t,s)(s-t)} = e^{-\int_t^s r(u,u)du} \tag{4}$$

be the risk-less discount factor. Introduce the indicator function:

$$1_{\text{expression}} = \begin{cases} 1 & \text{if expression is true} \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

The value of senior debt before default $(t < \delta)$ should be

$$B_t \quad = \quad E_t\left[\bar{B}P(t,T)1_{\delta>T} + A_\delta P(t,\delta)1_{\delta\leq T} + I_t^B\right] , \tag{6}$$

where $E_t[\cdot]$ denotes the expectation conditional on the information up to time t, and I_t^B is the discounted value of interest income:

$$I_t^B = \int_t^{\min\{\delta,T\}} b_t \bar{B} P(t,s) ds. \tag{7}$$

There is $n = 1$ share of equity and contingent capital in the capital structure of the bank. The par value of contingent capital is $\bar{C}$ and it pays coupon at a rate c_t until equity price hits a trigger level K_t, which is referred to as the stock price trigger. At the time t when the stock price hits the trigger for the first time, the contingent capital converts to m_t shares. Thus, after conversion, the outstanding shares of common equity is $1 + m_\tau$. Both K_t and m_t are given functions of observable variables at time t, and it.

To specify the terms in contingent capital, we need the existence of a unique stock price S_t at any time t for every realization of firm asset value A_t. Assuming that the unique stock price S_t exists, the first time stock price hits trigger is

$$\tau = \inf\{t \geq 0 : S_t = K_t\}. \tag{8}$$

Since equity value drops to zero when the senior debt defaults, we have $\tau < \delta$ as long as the trigger is positive, i.e., if $K_t > 0$ for all $t \geq 0$.

After paying the contractual coupon on senior bond and contingent capital, the cash flow generated from the assets of the bank will be paid to equity holders as dividends. Therefore, before conversion the total dividends paid to equity holders during a short period dt is $(a_t A_t - b_t \bar{B} - c_t \bar{C})dt$. After conversion and before default of senior bond, the total dividends paid to equity holders (including those new equity holders after conversion) during an infinitesimal period dt is $(a_t A_t - b_t \bar{B})dt$.

At any time t before contingent capital converts ($t < \tau$), the per-share value of common equity should be, in rational expectation,

$$\begin{aligned}
S_t &= E_t\left[(A_T - \bar{B} - \bar{C})P(t,T)1_{\tau>T} + I_t\right] \\
&\quad + E_t\left[\frac{1}{1+m_\tau}\left\{(A_T - \bar{B})P(t,T)1_{\delta>T} + J_\tau P(t,\tau)\right\}1_{\tau\leq T}\right],
\end{aligned} \tag{9}$$

where I_t is the present value of per-share dividend income before trigger point, and J_τ is the time-τ value of per-share dividend income after conversion:

$$I_t = \int_t^{\min\{\tau,T\}} (a_s A_s - b_s \bar{B} - c_s \bar{C}) P(t,s) ds \tag{10}$$

$$J_t = \int_t^{\min\{\delta,T\}} (a_s A_s - b_s \bar{B}) P(t,s) ds. \tag{11}$$

The value of un-converted contingent capital should be

$$\begin{aligned} C_t &= E_t \left[\bar{C} P(t,T) 1_{\tau>T} + H_t \right] \\ &+ E_t \left[\frac{m_\tau}{1+m_\tau} \left\{ (A_T - \bar{B}) P(t,T) 1_{\delta>T} + J_\tau P(t,\tau) \right\} 1_{\tau \le T} \right], \end{aligned} \tag{12}$$

where H_t is the present value of interest income before conversion:

$$H_t = \int_t^{\min\{\tau,T\}} c_s \bar{C} P(t,s) ds. \tag{13}$$

After contingent capital converts to m_τ shares and before senior bond's maturity or default $(\tau \le t < \min\{\delta,T\})$, the share value becomes

$$S_t = \frac{1}{1+m_\tau} E_t \left[(A_T - \bar{B}) P(t,T) \cdot 1_{\delta>T} + J_t \right]. \tag{14}$$

Since the value function B_t defined in equation (6) exists and is continuous in t and A_t, we focus on the value function of equity share S_t and the value of un-converted contingent capital C_t. Given a set of trigger K_t and conversion ratio m_t, a pair of value functions, (S_t, C_t), that satisfy equations (8), (9), and (12) is called a *dynamic rational expectations equilibrium* or, simply, an *equilibrium*. The equilibrium is *unique* if each of S_t and C_t has a unique value for every realization of A_t at any time t. In fact, such an equilibrium does not always exists for arbitrary specification of m_t. The next theorem presents the restriction on conversion ratio to ensure that an equilibrium exists.

Theorem 1 *For any given trigger K_t and conversion ratio m_t, a necessary condition for the existence of a unique equilibrium (S_t, C_t) is $m_t = C_t/K_t$.*

This necessary condition is also sufficient in the following sense:

Theorem 2 *For any given trigger K_t, there exists a unique equilibrium (S_t, C_t) that satisfies the conversion ratio $m_t = C_t/K_t$.*

These theorems say that a condition for a unique equilibrium is that there should be no transfer of value from CC holders to equity holders or vice versa. To see this, we can rewrite the condition as $m_t K_t = C_t$. At conversion time τ, we should have $S_\tau = K_\tau$. Then, the value of m_τ shares of stock at conversion is $m_\tau S_\tau$, which equals C_τ.

It is useful to provide some perspective on why the multiple equilibria do *not* arise with convertible bonds or options. With a convertible bond, the investor has the "option" to convert and get pre-specified number of shares of common stock. On each node, the investor can compute the value of the bond when it is not converted and compare it with the value of the bond when converted and select the maximum of the two values. Likewise, the holder of the option can also make the optimal decision on each node. These optimal decisions can be modeled by the "smooth pasting" or the "high contact" condition pioneered by Merton (1973) and further elucidated by Dixit and Pindyck (1994). In such models, the exercise boundary itself is endogenous and not mandated. With mandatory conversion, no agent is allowed to optimally act at the trigger. This absence of a "smooth pasting" condition then leads to the problems that we have articulated above. The smoothness breaks down if the mandatory conversion transfers value between equity and CC holders.[11] The state-contingent conversion ratio presented in the theorems prevents the value transfer, and in effect keeps the prices "smooth" at conversion[12]. However, mandatory conversions that may only occur at maturity do not pose any essential difficulty as the bond trades at par at maturity and hence there will be no value transfer with conversion. The Treasury capital assistance program has such features at maturity.[13]

---

[11]The effect of value transfer at mandatory conversion is similar to an exogenous value transfer caused by tax distortion. Albul at al (2010) have shown that differential tax treatment of CC's coupon interest and equity's dividend can cause multiple equilibria for mandatory convertible debt with trigger on asset value.

[12]In the valuation of barrier options, the exercise boundary is exogenous and their structure shares some of the features of CC. But the exercise of such options does not influence the underlying stock price itself, as there are no dilutionary effects to consider. These options are also in zero net supply.

[13]See Glasserman and Wang (2009) who value the capital assistance program.

In order to examine just how pervasive the problem of a lack of a unique equilibrium might be, we provide some numerical results. We first examine the effect of various conversion ratios on the severity of multiple equilibria: we compute the range of prices that can come about as a consequence of setting the conversion ratio arbitrarily. The case where the conversion ratio is "too low" leads to no equilibrium, and therefore cannot be demonstrated numerically. To obtain a range of multiple equilibria, we implement the continuous-time model using a large binomial tree. The upper bound of the equity price is obtained by choosing the equilibrium with higher stock price on every node that has multiple equilibria. Likewise, the lower bound of the equity price is obtained by choosing the equilibrium with lower stock price. The bounds of the CC value are constructed in a similar way. One intuitive interpretation of these bounds is the following. The equity upper bound reflects circumstances where equity holders somehow have the market power to dictate conversion to maximize their values. The presence of such bounds and the range associated with the bounds is an indication of potenital market manipulation possibilities with poorly designed bank contingent capital.

The range of prices generated by multiple equilibria seems substantial. As a numerical example, which is presented in Table 1, we assume that the bank's asset volatility is 6%, and its senior bond is valued at 90.43% of the current asset value. The par value of the contingent capital is 6% of the bank's current asset value, and the conversion trigger is 2% of the current asset value.[14] The maturities of the senior bond and contingent capital are both five years. The riskless interest rate is assumed to be 2% and constant for five years. With the above parameters, multiple equilibria produce equity values ranging from 3.83% to 4.45% of current asset value. They are associated with CC values ranging from 5.12% to 5.74% of current asset value. For a good approximation of continuous-time model, our trinomial tree contains 2,500 steps, roughly corresponds to two steps per trading day. Figure 2 shows that the range of multiple equilibrium prices converges to a fix width as the number of steps increases beyond 2,500.

---

[14]To keep the calculation simple, we set the coupon of the contingent capital to zero. If the coupon is nonzero, we may run into situations in which there are no equilibrium prices.

The range of multiple prices depends on the characteristics of the bank and the contracts of the contingent capital. In Figure 3, we let one parameter vary to see how the range of multiple prices change. Panels (A) and (B) show how the range is related to the bank's leverage with senior debt and its asset volatility. The range is wider for bank that has higher leverage with senior debt. The range is an increasing function of bank's asset volatility. Increasing senior debt or asset volatility raises the risk for the part of the firm shared by equity and CC holders. An increase in the risk enlarges the range of possible multiple equilibriums. Panels (C) and (D) of Figure 3 show how the range is related the amount of contingent capital and its conversion trigger. The range starts from zero and widens as the par value of contingent capital increases. Therefore, the more contingent capital a bank takes, the wider its range of equity prices is. The range is wider for low trigger than for high trigger. This is due to the fact that a low trigger is expected to take a longer time to reach and thus has more multiple equilibria to incorporate, pushing up the upper bound and pushing down lower bound.

These findings demonstrate that the bank-specific information such as leverage and asset volatility play an important role in the nature and the severity of multiple equilibria associated with the amount of CC and its trigger. A security with wide range of multiple equilibrium will not have a market force to make the price of the security converge to a particular point in the range. With the absence of such market force, the security is likely to be subject to price manipulation by large market players. We can think of a model in which there is a distribution of beliefs by a large number of investors. With this higher-order of probability distribution, market may be able to settle to a particular equilibrium. However, the assumption of a stable distribution of large number of investors is difficult to apply for the case of contingent capital bonds. Therefore, it is desirable to design a contingent capital that gives a unique equilibrium. In the next section, we suggest a design for CC that overcomes the problem of multiple equilibria or no equilibrium.

4 Design of Implementable Contingent Capital

The previous section has shown that in order to have a unique equilibrium, conversion ratio must satisfy at conversion times the following condition: $m_t = C_t/K_t$. This presents a major roadblock to the implementation of the CC design: the conversion ratio depends on the market value of the contingent capital. Since we cannot tell what the future market value is, the value C_t of the un-converted CC can be different from a pre-specified $m_t K_t$ at time before t. If in CC contract we set conversion ratio to be $m_t = C_t/K_t$, we need to know the value of the un-converted CC. With CC newly introduced into the market, observing the value of CC is unrealistic. Practically, the only observable "value" of CC is probably only the par value[15].

To use the par value for conversion ratio, we need to focus on a structure that makes the market value of the CC immune to changes in interest rates and default risk. For example, if the CC had no default risk, then by selecting the coupon rate at each instant to be the instantaneously risk-free rate we can assure that the CC will trade at par. See Cox, Ingersoll and Ross (1980) for a proof of this assertion[16]. In this case, CC will work well. This is because we can determine the conversion ratio ex-ante as follows:

$$m_t = C_t/K_t = \bar{C}/K_t. \tag{15}$$

Since $\bar{C}$ and K_t are known ahead of time, we can specify the conversion ratio ahead as well.

With default risk, however, no design of floating coupons will actually guarantee that the CC will sell at par. However, by choosing the coupon to reflect the market rates on short-term default-risky bank obligations it is possible to keep the price close to the par value. For example, if the coupon is tied to London Inter-bank Offered Rates (LIBOR) then

---

[15]It is well known that secondary markets for corporate and bank debt is highly illiquid and opaque. Therefore, investors often have only access to quotes and indications. Only recently TRACE has enforced post-trade transparency to a subset of corporate bond markets. Hence it is reasoble to design a security that does not presuppose the availability of an active secondary market.

[16]In the context of a CC, which is exposed to default risk, the appropriate indexed coupon may also require a compensation for the mandatory conversion in addition to the risk-free rate

the price of CC, which is a bank floater should remain close to par. By putting trigger K_t on equity, the impact of default risk before conversion on the pricing of CC reduces. Under the assumption that the bank declares default when equity value is driven to zero, CC holders are default-free prior to conversion, and are exposed to default only if they had already converted. Clearly, CC holders still bear large default risk ex ante because conversion time is uncertain and after conversion they hold equity, which bears default risk.

In the context of these issues, we show that the CC can be designed to be default-free during its life, even though the bank may have a positive probability of default on its debt claims subsequent to the expiry of CC. This idea is formalized in Theorem 3 below.

Theorem 3 *CC designed with a floating coupon rate $c_t = r_t$ and a conversion ratio $m_t = \bar{C}/K_t$, will always sell at par. This CC design leads to a unique equilibrium.*

This theorem has two parts: the first part generalizes the immunization results of Cox, Ingersoll and Ross (1980) to a setting where there is mandatory conversion, and a positive probability of default after the expiry date of CC. Since the coupons float with the risk-free rate, and the principal is guaranteed at conversion, the CC is fully immunized and therefore sells at par. The economic intuition behind the second part of the result is also intuitive. Since the CC sells at par, we can design the CC with an ex-ante conversion ratio that guarantees that upon conversion, the CC-holders will get par.[17] Theorem 3 has an important practical implication: with the design proposed in Theorem 3, the contingent capital certificates suggested by Flannery (2009) becomes operational and free of multiple equilibria. When mandatory conversion triggers are placed on stock prices, the structure of CC in Theorem 3 offers a practical way to design contingent capital for banks because the specification of conversion ratio in the contract does not rely on variables that is difficult to observe.

---

[17]In reality, bank-issued CC will be less liquid than risk-free assets such as Treasury securities, and hence the coupon will have to include a component for liquidity premium. In addition, if the underlying assets price process can have discontinuous jumps, then the result will not hold.

What are the benefits brought about by CC? In order to address this question within the context of our framework, we can examine the probability of default with and without the CC in the capital structure of the bank. As a benchmark, we can consider two cases: the first case would be the Black and Cox (1976) benchmark with the modification that the junior debt is designed as a floater. In the absence of conversion as proposed in our paper, junior debt is subject to default risk, and hence will carry a higher coupon rate in order to sell at par. This design ensures that, at the time the junior debt (or CC) is issued, the bank gets the same level of capital, equal to the par value of debt issued. Both the junior debt and the CC will provide tax-shield benefits, but the CC will provide less tax benefits as its coupon will be lower than an otherwise identical junior debt, which is subject to credit risk. The probability of default associated with the junior debt is higher than the probability of default associated with the CC. This follows from the fact that default will occur in Black and Cox (1976) setting whenever the asset value drops below the safety covenant level, which is dependent on the sum of the par values of both senior and junior debt. With CC, default event depends only on the par value of senior debt. This is one avenue from where the welfare improvement comes about. Since the probability of default is lower with CC, it stands to reason that the creditors to the bank will benefit at the expense of the equity holders. Since we do not have frictions such as taxes, or costs associated with financial distress, it is clear that any welfare improvement for one subset of claimants must represent a welfare loss to the remaining claimants.

A second benchmark to assess the value added by CC, which is perhaps more relevant for policy purposes is the alternative that the bank issues equity as opposed to CC. With equity issuance, the bank will not be able to get the tax-shield associated with CC. This can be the source of welfare improvement for the bank with CC. As noted by Pennachi (2010) the issuance of CC may also serve to reduce the risk-shifting incentives, which can be another important source of welfare improvement.

Can we claim that the introduction of CC serves to reduce the overall default risk of the

issuing bank? In our model, default risk of the bank is determined by the asset volatility and the choice of leverage. Once we admit frictions such as taxes and bankruptcy costs, it is clear that our model will enable us to compute the optimal leverage, consistent with CC in the capital structure with an equity price trigger. If the optimal (overall) capital structure with CC is less than the optimal capital structure without CC, then one source of default risk would have been lowered as the bank issues more equity in equilibrium. In addition, if the presence of CC leads to lower risk-shifting incentives, that can also reduce the overall default risk in the bank. Note that although the CC earns the risk-free rate until conversion, once it is converted it becomes equity and fully bears the default risk of the issuing bank. In this sense, CC serves to mitigate default risk. By accepting a residual claim in the bank's capital structure, the CC-holders help the bank in overcoming a potential crisis. Finally, as noted by Hart and Zingalis (2010), the CC (once converted) may also have the implication that the bank is freed of the discipline imposed creditors. Once CC is converted the bank may have to issue new CC to be able to provide that additional layer of protection. Whether the bank may be able to do that will depend on the asset value of the bank at the time CC gets converted. It is conceivable that the CC simply "back loads" the default risk of the bank, rather than reducing the overall default risk by appropriately constraining risk-taking propensities of the issuing bank. These issues are potentially interesting areas for further research. Since we have developed a framework for valuing CC with an equity trigger, these welfare questions can be examined in such a framework. Also, we have not considered in our framework, any aggregate liquidity event, which is an important motivator for designing contingent debt capital. This issue is best modeled in an equilibrium setting with many banks, and is beyond the scope of this paper.

5   Conclusion

We have not treated the effect of frictions in our analysis. Clearly, one of the arguments as to why debt issuance may add value to the bank is that the interest payments made on debt

are subject to tax relief. It stands to reason that CC should be treated as a debt security until its mandatory conversion in order for such issuance to generate additional value as well as to provide the necessary incentives to the issuing banks (See Duffie, 2009). A number of papers have also noted that the CC design might set in motion some perverse incentives. For example, Duffie (2009) notes that CC design based on equity design might set in motion a potential attack by short sellers to trigger conversion and benefit from the resulting dilution. Hillion and Vermaelen (2001) have argued such securities come under selling pressure from the holders of convertible. Rajan (2009) has suggested that the conversion should depend both on bank-specific information and market-wide systemic risk relevant information, a point explicitly considered by McDonald (2009) in his paper.

Consistent with many other observers (e.g., Acharya, Thakor and Mehran, 2010), we note that the mandatory conversion of junior debt should automatically result in suspension of dividends to all common stock holders. Holding other factors the same, this should serve to alleviate the selling pressure: any attempt to short the stock by the holders of CC will also result in losses in foregone future dividends on their long positions.

In addition, we can use a "look back" period as suggested in the literature as the basis for conversion as opposed to using a specific trigger price. Such mechanisms are already used in practice in the context of barrier options. In addition, it may be worthwhile to include a "optional sinking fund" feature in the design of CC so that there is a periodic and orderly retirement of principal (balloon) payment, which should further serve to mitigate the incentive for speculative short sale attack[18]. The sinking fund structure reduces the volume of conversion that must occur once the equity trigger is breached by the stock price leaving a speculative attack more vulnerable.

We argue that equity and CC are claims on the same assets, and their prices must be determined simultaneously. The conversion ratios must ensure that at the equity trigger there are no value transfers between equity and CC investors. Our paper provides the condition

---

[18]McDonald (2009) suggests a gradual and random retirement policy for CC in order to limit gains from manipulation.

that the conversion ratio must satisfy in order for a unique equilibrium to exist. Guided by this condition, we present a design that mitigates the problem of multiple equilibrium. Since this design does not transfer value between equity and CC holders, the security design suggested in the paper does not provide incentives for manipulating stock prices around conversion. We do not suggest that our design is "manipulation-proof", but rather that under the assumptions of the model and in well functioning markets, this design is less prone to manipulation. In contrast, a security that introduces multiple equilibria adds the problem of price manipulation to the originally well-functioning market.

Finally, we cast some doubts on designing the conversion ratio to mitigate risk-shifting or the propensity for manipulation or for coercive equity issues. Our paper suggests that the conversion ratio that admits a unique equilibrium produces no value transfer. Hence it is not possible to design "dilutive" ratios in order to promote coercive equity issuance as such a ratio will lead to a lack of a unique equilibrium.

A   Appendix: Proofs of the Theorems

Before proving the theorems, it is useful to make the following observation. If there were no contingent capital, at any time t before maturity ($t \leq T$) and default ($t \leq \delta$), the equity value would have been

$$U_t \;=\; E_t \left[(A_T - \bar{B}) P(t, T) \cdot 1_{\delta > T} + J_t \right] , \tag{16}$$

where J_t is defined in equation (11). From the literature, we know U_t is a continuous function of t and A_t, we can define another hitting time based on U_t and the given K_t and m_t:

$$v = \inf \left\{ t \geq 0 : \frac{1}{1 + m_t} U_t = K_t \right\} . \tag{17}$$

A.1 Proof of Theorem 1

To prove Theorem 1, we start with a pair (S_t, C_t) that satisfies equations (8), (9) and (12). Equations (9) and (12) imply

$$S_t = U_t - C_t. \tag{18}$$

For $\tau \leq T$, equation (9) implies that $S_\tau = U_\tau/(1 + m_\tau)$. Since $S_\tau \leq K_\tau$ by equation (8), we have $U_\tau/(1 + m_\tau) \leq K_\tau$, which implies

$$v \leq \tau \tag{19}$$

in view of equation (17). On the other hand, equation (17) implies $U_v/(1 + m_v) = K_v$ and thus converting is an equilibrium price at time v. If $S_v > K_v$, then not converting is also an equilibrium price at time v, contradicting to the assumption of unique equilibrium. Thus, uniqueness of equilibrium implies $S_v \leq K_v$. It follows that

$$\tau \leq v, \tag{20}$$

in view of equation (8). Combining equations (19) and (20), we have

$$\tau = v. \tag{21}$$

It then follows from equations (8), (17) and (18) that

$$\inf\{t \geq 0 : U_t \leq K_t + C_t\} = \inf\{t \geq 0 : U_t \leq (1 + m_t)K_t\}. \tag{22}$$

The above equation holds if and only if $K_t + C_t = (1 + m_t)K_t$, which implies $m_t = C_t/K_t$.

A.2 Proof of Theorem 2

To prove Theorem 2, we use the hitting time v defined in (17) as the conversion time, assuming any give conversion ratio m_t. With this conversion rule, a stock price and contingent

capital value before conversion $(t < \upsilon)$ are

$$
\begin{aligned}
S_t^* &= E_t\left[(A_T - \bar{B} - \bar{C})P(t,T)1_{\upsilon>T} + I_t^*\right] \\
&\quad + E_t\left[\frac{1}{1+m_\upsilon}\left\{(A_T - \bar{B})P(t,T)1_{\delta>T} + J_\upsilon^*P(t,\upsilon)\right\}1_{\upsilon\leq T}\right], \quad (23) \\
C_t^* &= E_t\left[\bar{C}P(t,T)1_{\upsilon>T} + H_t^*\right] \\
&\quad + E_t\left[\frac{m_\upsilon}{1+m_\upsilon}\left\{(A_T - \bar{B})P(t,T)1_{\delta>T} + J_\upsilon^*P(t,\upsilon)\right\}1_{\upsilon\leq T}\right], \quad (24)
\end{aligned}
$$

$$
H_t^* = \int_t^{\min\{\upsilon,T\}} c_s\bar{C}P(t,s)ds \quad (25)
$$

$$
I_t^* = \int_t^{\min\{\upsilon,T\}} (a_sA_s - b_s\bar{B} - c_s\bar{C})P(t,s)ds \quad (26)
$$

$$
J_\upsilon^* = \int_\upsilon^{\min\{\delta,T\}} (a_sA_s - b_s\bar{B})P(\upsilon,s)ds. \quad (27)
$$

The stock price after conversion $(\upsilon \leq t < \min\{\delta, T\})$ is

$$
S_t^* = \frac{1}{1+m_\upsilon}E_t\left[(A_T - \bar{B})P(t,T)1_{\delta>T} + J_t^*\right]. \quad (28)
$$

Equations (23), (24), and (28) imply

$$
S_t^* = U_t - C_t^*. \quad (29)
$$

Now, we use S_t^* to define another hitting time:

$$
\tau^* = \inf\{t \geq 0 : S_t^* \leq K_t\}. \quad (30)
$$

In view of equation (29), we have

$$
\tau^* = \inf\{t \geq 0 : U_t \leq K_t + C_t^*\}. \quad (31)
$$

If $m_t = C_t^*/K_t$, then

$$
\begin{aligned}
\tau^* &= \inf\{t \geq 0 : U_t \leq K_t + C_t^*\} \\
&= \inf\{t \geq 0 : U_t \leq (1+m_t)K_t\} = \upsilon. \quad (32)
\end{aligned}
$$

Therefore, (S_t^*, C_t^*) satisfies equations (8), (9) and (12) and thus is an equilibrium.

If (S_t, C_t) is another equilibrium with conversion ratio is $m_t = C_t/K_t$, following a similar reasoning in the derivation of $\tau^* = v$, we can show that conversion time $\tau = \inf\{t \geq 0 : S_t \leq K_t\}$ equals v, which gives $\tau = \tau^*$. Therefore, the values of stock and contingent capital as calculated in equations (9), (12), (23), and (24) imply $S_t = S_t^*$ and $C_t = C_t^*$. This proves the uniqueness of the equilibrium.

A.3 Proof of Theorem 3

In the floater, we set the interest rate of the contingent capital as the riskless rate. That is, $c_t = r_t$ for all t. In addition, we set conversion ratio as $m_t = C_t/K$. It follows from Theorem 2 that there is a unique equilibrium. We will show that in this unique equilibrium the value of the contingent capital at and before conversion equals the par value, i.e., $C_t = \bar{C}$ for $t < \tau$. Then, the conversion ratio $m_t = \bar{C}/K$ will be equivalent to $m_t = C_t/K$ at and before conversion.

At or before conversion $(t < \min\{\tau, T\})$, the value of contingent capital is

$$
\begin{aligned}
C_t &= E_t\left[\bar{C}P(t,T)1_{\tau>T} + H_t\right] \\
&\quad + E_t\left[\frac{m_\tau}{1+m_\tau}\left\{(A_T - \bar{B})P(t,T)1_{\delta>T} + J_\tau P(t,\tau)\right\}1_{\tau\leq T}\right],
\end{aligned} \tag{33}
$$

where J_τ is defined in equation (11) and H_t defined in (13) with $c_t = r_t$. The riskless floating rate for the coupon of CC implies

$$
H_t = \int_t^{\min\{\tau,T\}} r_s\bar{C}P(t,s)ds = \bar{C}\left[1 - P(t,T)1_{\tau>T} - P(t,\tau)1_{\tau\leq T}\right]. \tag{34}
$$

Substituting the above expression for H_t back into the valuation function of C_t and using the properties of iterated expectations and $P(t,T) = P(t,\tau)P(\tau,T)$, we obtain

$$
\begin{aligned}
C_t &= \bar{C} - E_t\left[\bar{C}P(t,\tau)1_{\tau\leq T}\right] \tag{35} \\
&\quad + E_t\left[\frac{m_\tau}{1+m_\tau}E_\tau\left[(A_T - \bar{B})P(\tau,T)1_{\delta>T} + J_\tau\right]P(t,\tau)1_{\tau\leq T}\right] \tag{36}
\end{aligned}
$$

Noticing

$$
\frac{1}{1+m_\tau}E_\tau\left[(A_T - \bar{B})P(\tau,T)1_{\delta>T} + J_\tau\right] = S_\tau = K, \tag{37}
$$

27

the value of contingent capital with interest floating rate r_t equals

$$C_t = \bar{C} - E_t \left[\bar{C} P(t, \tau) 1_{\tau \leq T} \right] + E_t \left[m_\tau K P(t, \tau) 1_{\tau \leq T} \right]. \tag{38}$$

Therefore, $m_t = \bar{C}/K$ for all t implies that

$$C_t = \bar{C} - E_t \left[\bar{C} P(t, \tau) 1_{\tau \leq T} \right] + E_t \left[\frac{\bar{C}}{K} K P(t, \tau) 1_{\tau \leq T} \right] = \bar{C}. \tag{39}$$

References

Admati Anat and Paul Pfleiderer, (2010), "Increased-Liability Equity: A Proposal to Improve Capital Regulation of Large Financial Institutions," Stanford University.

Acharya, Viral, Cooley, Richardson, and Walter, Real Time Solutions for Financial Reform, An NYU Stern Working Group on Financial Reform, December 2009

Acharya, Viral, Anjan Thakor and Hamid Mehran (2010), "Caught Between the Scylla and Charybdis? Regulating bank leverage when there is rent seeking and risk shifting," Working paper, New York University.

Albul, Boris, Jaffee Dwight M., and Tchistyi, Alexei, (2010) "Contingent Capital Debt and Capital Structure Decisions," Working paper, Hass School of Business, UC Berkeley.

Black, Fischer and John C. Cox, (1976), "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions", Source: The Journal of Finance, Vol. 31, No. 2, Papers and Proceedings of the Thirty-Fourth Annual Meeting of the American Finance Association Dallas, Texas December 28-30, 1975 (May, 1976), pp. 351-367.

Bond, Philip, Goldstein, Itay, and E.S.Prescott (2009), "Market-Based Corrective Actions," Review of Financial Studies, (2009).

Chari and Jagannathan, (1988) Banking panics, information and rational expectations equilibrium, J. Fin. 43 (1988) (3), pp. 749761.

Committee on Capital Markets Regulation,"The Global Financial Crisis: A Plan for Regulatory Reform," May 2009

Cox, John C., Jonathan E. Ingersoll, Jr., Stephen A. Ross (1980), " An Analysis of Variable Rate Loan Contracts", The Journal of Finance, Vol. 35, No. 2, Papers and Proceedings Thirty-Eighth Annual Meeting American Finance Association, Atlanta, Georgia, December 28-30, 1979 (May, 1980), pp. 389-403

Diamond and Dybvig, 1983 D. Diamond and P. Dybvig, Bank runs, deposit insurance and liquidity, J. Pol. Econ. 91 (1983) (3), pp. 401419.

Dixit, Avinash and Pindyck, Robert (1994), "Investment Under Uncertainty," Princeton University press.

Duffie, Darrell (2009), "Contractual Methods for Out-of-Court Restructuring of Systemically Important Financial Institutions", Graduate School of Business, Stanford University.

Flannery, Mark J. (2002), "No Pain, No Gain? Effecting Market Discipline via Reverse Convertible Debentures" University of Florida, working paper.

Flannery, Mark J. (2009), "Stabilizing Large Financial Institutions with Contingent Capital Certificates,"

Garbade, Kenneth D. (2001), *Pricing Corporate Securities as Contingent Claims*, The MIT Press, Cambridge, Massachusetts.

Glasserman, Paul and Wang, Zhenyu, (2009), "Valuing the Treasury's Capital Assistance Program," Federal Reserve Bank of New York Staff Reports, no. 413 (2009.

Hart, Oliver and Luigi Zingales, (2010), "How to make a bank raise equity," Financial Times, February 7 2010.

Hillion, Pierre and Theo Vermaelen (2001), "Death Spiral Convertibles," Working paper,

INSEAD, France.

Kashyap, Anil K., Raghuram Rajan, and Jeremy Stein (2008), "Rethinking Capital Regulation," Federal Reserve Bank of Kansas City Symposium, September 2008.

McDonald, Robert, L. (2009):"Risks Contingent Capital with a Dual Price Trigger", Working paper, Kellogg School of Management, Northwestern University.

Merton, Robert C. (1973), "Theory of Rational Option Pricing" Bell Journal of Economics, The RAND Corporation, vol. 4(1), pages 141-183, Spring.

Merton, Robert, (1974): "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates", The Journal of Finance, Vol. 29, No. 2, Papers and Proceedings of the Thirty-Second Annual Meeting of the American Finance Association, New York, New York, December 28-30, 1973 (May, 1974), pp. 449-470.

Pennacchi, George, (2010): "A Structural Model of Contingent Bank Capital," University of Illinois, College of Business.

Rajan, Raghuram (2009) "More capital will not stop the next crisis", Financial Times, October 1, 2009.

Squam Lake Working Group on Financial Regulation (2009), "An Expedited Resolution Mechanism for Distressed Financial Firms: Regulatory Hybrid Securities", April 2009, working paper, Council on Foreign Relations, Center for Geoeconomic Studies.
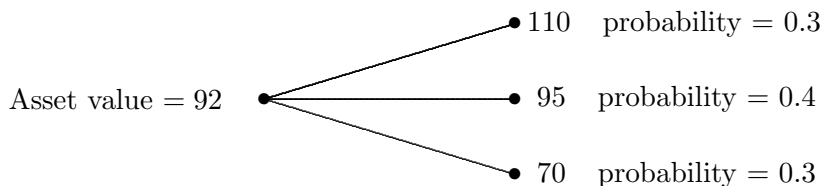
Table and Figures

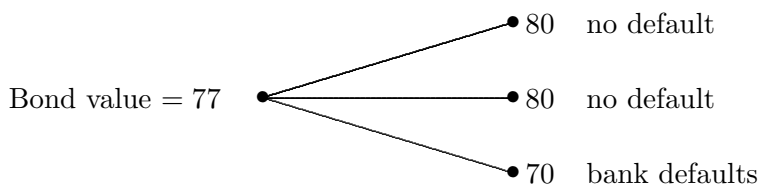Table 1: A Numerical Example for the Range of Multiple Equilibria

Name	Notation	Value
Parameters		
Asset value	A_0	100
Volatility	σ	6%
Senior bond value	B_0	90.43
Riskless rate	r	2%
Par value of CC	$\bar{C}$	6
Shares of common equity	n	1
equilibria		
Stock price	S_0	$[3.83\,,\,4.45]$
CC value	C_0	$[5.12\,,\,5.74]$

Figure 1: An One-Step Trinomial Tree Model

(A) *Bank asset value*

Asset value = 92

- 110   probability = 0.3
- 95   probability = 0.4
- 70   probability = 0.3

(B) *Senior bond value*

Bond value = 77

- 80   no default
- 80   no default
- 70   bank defaults

(C) *No conversion is an equilibrium*

CC value = 7

- 10   no conversion
- 10   convert to 2 shares
- 0   bank defaults

Stk price = 8

- 20 $= (110 - 80 - 10)/1$
- 5 $= (95 - 80)/3$
- 0   bank defaults
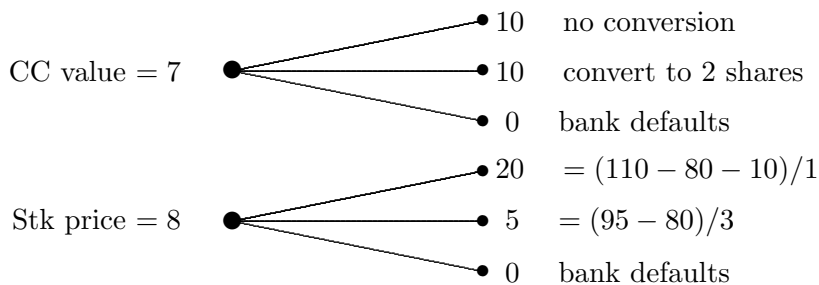
(D) *Conversion is another equilibrium*

CC value = 10

- 20 $= 2 \text{ shares} \times \$10$
- 10 $= 2 \text{ shares} \times \$5$
- 0 $= 2 \text{ shares} \times \$0$

Stk price = 5

- 10 $= (110 - 80)/3$
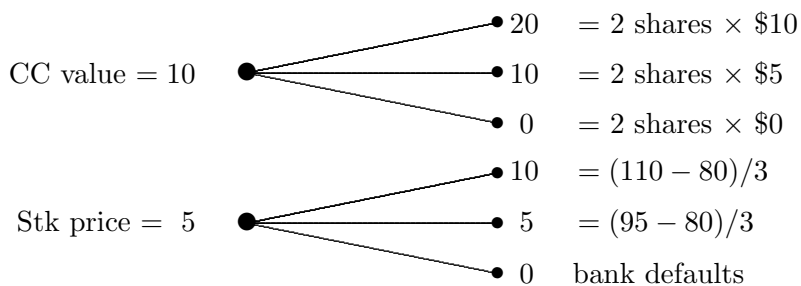- 5 $= (95 - 80)/3$
- 0   bank defaults

Figure 2: The range of multiple equity and CC prices converges to a fixed width as the number of steps in the binomial tree increases. The solid lines represent the upper and lower bounds of multiple equity prices and dot lines represent the bounds of CC values. The parameters used for the figure are the same as those in Table 1, except the number of steps. For the number of steps, its value in Table 1 is indicated by the vertical dash line.

Range of Multiple Equilibria
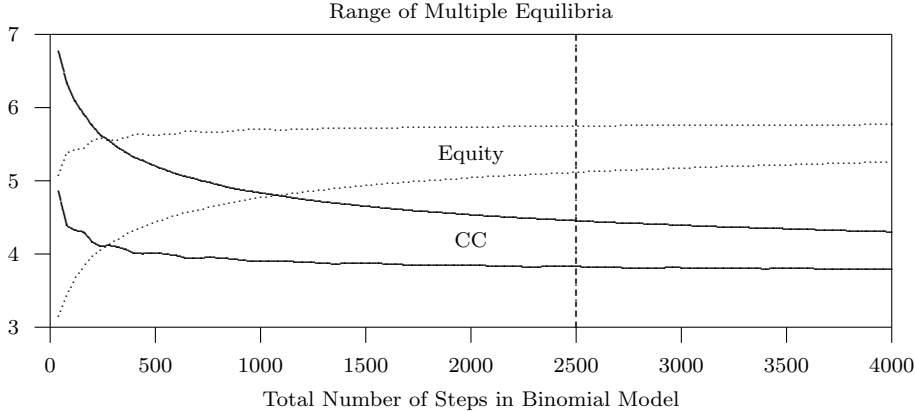
Equity

CC

Total Number of Steps in Binomial Model

Figure 3: The range of multiple equity and CC prices depend on bank characteristics and CC contracts. The solid lines represent the upper and lower bounds of multiple equity prices and dot lines represent the bounds of CC values. The parameters used for the figure are the same as those in Table 1, except the one that varies in a range indicated by the horizontal axis. For the varying parameter, its value in Table 1 is indicated by the vertical dash line.